Is space-time interaction real or apparent in seismic activity?

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Abstract: It is widely shared opinion that not only secondary (aftershocks) but also main earthquakes have the tendency to occur in space-time clusters. This assumption has affected the preferential choice of stochastic models in the studies on seismic hazard, like self-exciting (epidemic) models which imply the abrupt increase of the occurrence probability after a shock and the subsequent exponential decrease without the desirable increase before a forthcoming event. The importance of this assumption requires the application of statistical tools to evaluate objectively its coherence with the reality at different scale of magnitude-space-time. To this end we consider the earthquakes drawn from the historical Italian catalogue CPTI04 that geologists have associated with each of the eight tectonically homogeneous regions in which Italian territory is divided. Fixing different magnitude thresholds we perform statistical tests based on the space-time distance between pairs of earthquakes under the null hypothesis of uniform distribution in time and space and evaluate the significance of the possible clusters. Monte Carlo hypothesis testing is also used to obtain the null distribution and the simulated p-value.

Keywords: detection of space-time clusters, Knox test, K-nearest neighbour test, Mantel test

1 Introduction

Some occurrence patterns in the worldwise seismicity are ascribable to space-time clustering; the best-known is due to the aftershocks, smaller earthquakes that follow a previous large shock within a distance up to twice the rupture length from the mainshock and can continue over a period of weeks, months, or years. Some articles in the literature claim that also strong events occur in clusters (Kagan and Jackson (2000), Lombardi and Marzocchi (2007)); this feature, if validated, would have heavy consequences on the choice of models in hazard assessment. We think that it is necessary to pass from quantitative observations to inferential tests which assign the statistical significance to some assumptions. Three types of tests can be carried out (Rogerson and Yamada (2009)): general and focused tests and tests for the detection of clustering. General tests provide a global statistic that assesses the degree to which a pattern deviates from the null hypothesis of space-time randomness without giving informaton on the size and location of clusters, focused tests are used to know whether a cluster exists around prespecified foci, whereas in the third category many local tests are carried out simultaneously to uncover the location and size of any possible clusters by scan-type statistics. This article concerns the first step of a study on Italian seismicity in which we try to answer the question whether, for given magnitude thresholds, the global pattern of the past seismicity in tectonically homogeneous Italian regions is significantly clustered. In the future, where the answer is positive, we are going to establish, by scan-type statistics, whether the study region is homogeneous, and, where the answer is negative, to uncover isolated hot spots of increased activity and to look for geophysical explanations of this fact.

2 Space-time tests on tectonic regions in Italy

We consider three global tests: Knox, Mantel and Jacquez (or k NN) tests (Tango (2010)). Knox's statistic counts the number of observed pairs of n events close in both space and time:

$$T = \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} a_{ij}^S a_{ij}^T \tag{1}$$

where

$$a_{ij}^S = \begin{cases} 1, & i \neq j \text{ and } d_{ij}^S < \delta_1 \text{ (km)} \\ 0, & \text{otherwise} \end{cases} \qquad a_{ij}^T = \begin{cases} 1, & i \neq j \text{ and } d_{ij}^T < \delta_2 \text{ (years)} \\ 0, & \text{otherwise} \end{cases}$$

and δ_1 and δ_2 are unknown critical space and time limits to be prespecified. Under the null hypothesis H_0 - the temporal distances between pairs of events are independent of the spatial distances - it is proved that mean and variance of T are given by:

$$\begin{aligned} E(T) = & \ \frac{N_{1S}\ N_{1T}}{N} \\ Var(T) = & \ \frac{N_{1S}\ N_{1T}}{N} + \frac{4\ N_{2S}\ N_{2T}}{n(n-1)(n-2)} - \left(\frac{N_{1S}\ N_{1T}}{N}\right)^2 \\ & + \frac{\{N_{1S}(N_{1S}-1) - N_{2S}\} \times \{N_{1T}(N_{1T}-1) - N_{2T}\}}{n(n-1)(n-2)(n-3)} \end{aligned}$$

where $N = n(n-1)/2$, $N_{1S} = \frac{1}{2}\sum_{i=1}^{n} \sum_{j=1}^{n} a_{ij}^S$ and $N_{2S} = \frac{1}{2}\sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{k\neq j}^{n} a_{ij}^S\ a_{ik}^S$ (analogously we get N_{1T} and N_{2T} substituting a^S with a^T). Given values of δ_1, δ_2 and observed $T = t$, the null distribution of T and its p-value can be approximated by either one of the following:

• Poisson distribution when N_{1S} and N_{1T} are small compared with N (or $E(T)$ is roughly equal to $Var(T)$) with

$$\text{mid}-p-\text{value} = 1 \ - \ \sum_{k=0}^{t} \frac{E(T)^k}{k!}\ exp\{-E(T)\} \ + \ \frac{1}{2}\ \frac{E(T)^t}{t!}\ exp\{-E(T)\}$$

• Normal distribution with p-value given by: $1 - \Phi\left(\frac{t-E(T)}{\sqrt{var(T)}}\right)$

• Monte Carlo hypothesis testing: we simulate the null distribution of T calculating the same statistic for a large number N_{rep} of data sets obtained by permuting the times among the fixed spatial locations (or viceversa). In this way we get:

$$\text{Simulated } p\text{−value} = \frac{1 \ + \ \sum_{\nu=1}^{N_{rep}} I(T_\nu \geq T_{obs})}{N_{rep}+1} \quad . \tag{2}$$

Mantel's test is a generalization of the Knox's test based on the same statistic (1) where reciprocal transformations of the distances are used to increase the influence of close distances and decrease that of the long distances, hence we have:

$$a_{ij}^S = \frac{1}{d_{ij}^S \ + \ c_1} \quad (a_{ii}^S = 0) \qquad\qquad a_{ij}^T = \frac{1}{d_{ij}^T \ + \ c_2} \quad (a_{ii}^T = 0)$$

with c_1 and c_2 unknown constants. To avoid the issues concerning the choice of the δ and c constants, Jacquez proposed a Knox-type test where the closeness is defined by the k nearest neighbours (k NN) such that:

$$a_{ij}^S = \begin{cases} 1, & \text{if event } j \text{ is a } k \text{ NN of event } i \ (\neq j) \text{ in space} \\ 0, & \text{otherwise} \end{cases}$$

Analogously we get a_{ij}^T. Monte Carlo hypothesis testing is required to obtain the null distribution of T and the simulated p-value (2) for both the Mantel's and the Jacquez's test.

3 Results

We have applied these tests to two data sets constituted by the 383 and 45 earthquakes of magnitude $M_w \geq 4.5$ and $M_w \geq 5.3$ respectively, occurred in the Central Northern Apennines West region characterized by normal faults. Figures 1 and 2 synthesize graphically some results of Knox's and Jacquez's tests showing the p-values obtained as the constants of the tests vary. We point out that space-time clustering of earthquakes of $M_w \geq 4.5$ is statistically significant for some values of δ_1, δ_2 and k, but it isn't when the threshold increases; consistent results are also obtained through the Mantel's test. This means that in the Italian tectonic context space-time clustering is not a property invariant to the magnitude threshold contrary to what is stated in the literature (Lombardi and Marzocchi (2007)). Hence this property must be verified through statistical tests so that the most appropriate stochastic model for hazard evaluation is proposed in each specific context.

References

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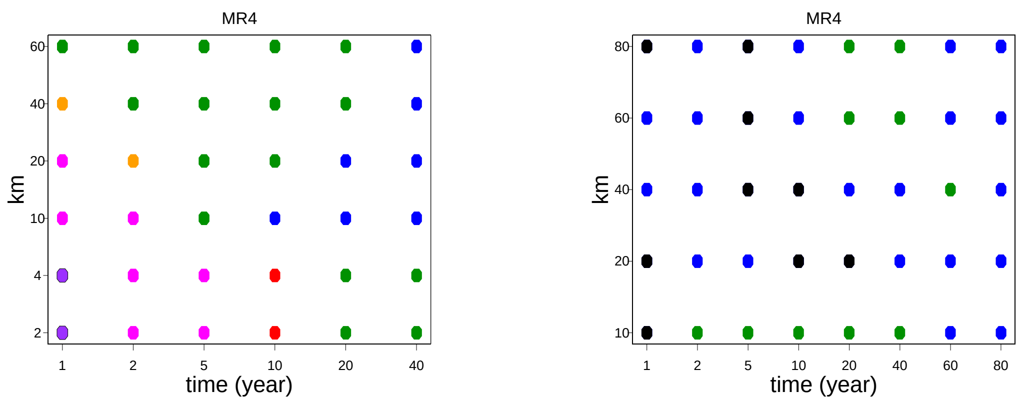

Figure 1: p-value of the Knox's test applied to earthquakes of $M_w \geq 4.5$ (left) and $M_w \geq 5.3$ (right) for different values of δ_2 (x-axis) and of δ_1 (y-axis): $p \leq 10^{-6}$ (violet), $10^{-6} < p \leq 0.01$ (magenta), $0.01 < p \leq 0.05$ (red), $0.05 < p \leq 0.10$ (orange), $0.10 < p \leq 0.50$ (green), $0.50 < p \leq 0.95$ (blue), $p > 0.95$ (black).

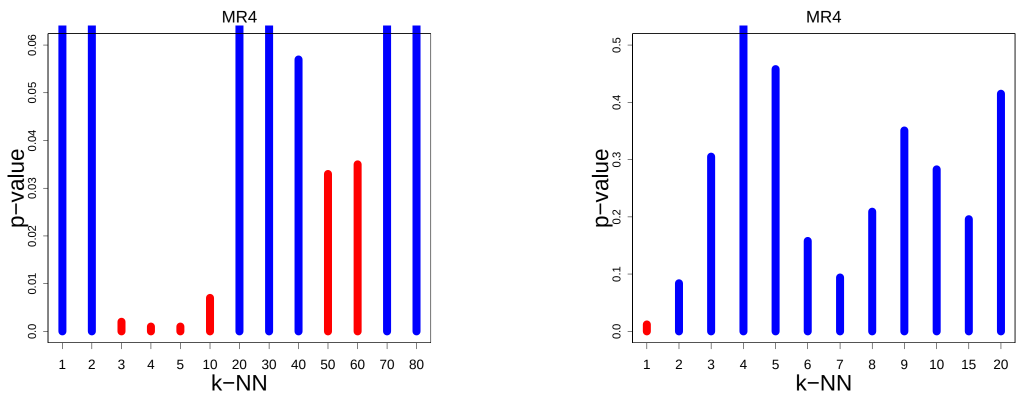

Figure 2: p-value of the Jacquez's test applied to earthquakes of $M_w \geq 4.5$ (left) and $M_w \geq 5.3$ (right): $p \leq 0.05$ (red), $p > 0.05$ (blue). Order k of the nearest neighbours on the x-axis.