Scenario generation for long term fuel prices*

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Abstract

In this article we deal with the problem of scenario generation for fuel prices in the long term. The solution of many decision making problems in the energy sector such as the optimal mix of energy productions among different technologies, requires to model the dynamic of fuel prices and forecast their possible scenarios over time. We present two different approaches for scenario generation: a Vector autoregressive approach and a Monte carlo approach; The first one is based on the estimate of a Vector Auto Regressive model i.e. a set of simultaneous equations. The second one is based of the assumption that the returns dynamics follow a generalised weiner process. Using the two approaches we forecast prices' scenarios.

1 Introduction

In this paper we are interested in long term scenario generation of the fuel prices and EUA (CO2 allowances) price which affect the variable cost for electricity production in different technologies. In particular for production technologies like coal, nuclear,

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combined cycle gas turbine(CCGT) and biomass power plant the fuels that influence the variable costs are: gas, coal, nuclear and bio-oil; the EUA price influence the technologies that emits CO_2 that are coal and CCGT plants. We will concentrate our attention on this set of variables. In Section 2 we describe the data set, in Section 3 we analyze the statistical characteristic of fuel prices and returns using PCGive Software, in Section 4 we describe the econometric model VAR, in Section 5 we describe the scenario generation on VAR model's errors, in Section 6 we describe the Monte Carlo Simulation and in Section 7 we compare the results on the two different tecniques on scenario generation and present some possible further research.

2 Fuel prices description

In this section we describe the fuel prices, and EUA prices database; we used monthly data because we are interested in long period forecasting. We analyzed monthly database for

- gas prices from April 2003 to March 2011 expressed in €$/m^3$.
- coal prices from January 2004 to March 2011 expressed in €$/kg$.
- EUA prices from Dicember 2004 to May 2008 expressed in €$/t$.
- nuclear prices from April 2003 to May 2011 expressed in €$/kg$.
- biooil prices from May 2003 to October 2010 expressed in €$/kg$.

The source for the gas prices is a private contract of an italian GenCo, for the other fuel prices the source is the IEA World Energy Outlook 2010.

We have transformed the prices in €$/MWh$ dividing for the LHV (Lower Heating Value) of every fuel. This price is an indicator of sensitive of the technology to the fuel price variations: a high price means high sensitivity as for the gas plants, a low price means low sensitivity as for the nuclear plants. Notice that the total electricity cost for each technology depends on several parameters like the efficiency of the plant, the operating hours, the fixed costs, the investments costs, the CO_2 emission rate and the industrial life of the plant, so for various factors a technology with high variable costs might be more convenient respect to a technology with very low variable costs. The fuels and their characteristics are shown in Table 1. The behaviors of these prices against time are plotted in Figure 1.

The natural gas and the coal have a positive emission factor since they are fossil fuels; obviously the nuclear plants do not emit any CO_2 (but they produce nuclear waste which are very expensive and dangerous to treat); the plants that burn biooil have an emission rate equal to zero even if they emit CO_2 because the emission is considered a closed cycle: the CO_2 emitted in the atmosphere is the same that had been consumed by the plantations that produce the biomass. The cost of the nuclear fuel includes both the uranium cost and the processing costs.

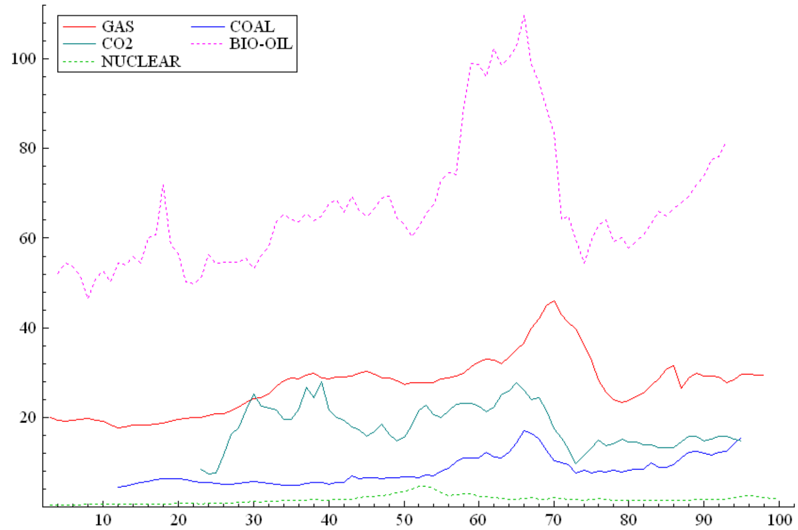

Figure 1: Behaviours of the four fuel prices and EUA prices considered from April 2003 to May 2011.

Fuel [*m.u.*]	Fuel cost year 0 [€/*m.u.*]	Lower Heating Value [MWh/*m.u.*]	Fuel cost year 0 [€/MWh]	CO_2 emission rate [t/GWh]
Gas $[Nm^3]$	0.29	9.58	30.27	200
Coal $[t]$	115	8141	14.13	338
Biooil $[Kg]$	0.81	10138	79.90	0
Nuclear $[Kg]$	2100	950171^*	2.21	0

Table 1: The fuels considered and their characteristics. *Notice that this isn't precisely a LHV but the energy released by 1 Kg of uranium

3 Fuel prices statistics

From Figure 1 we can guess some characteristics of these fuel prices: *positive correlation*, *no normality*, *no-stationarity* and *no-trend-stationarity*.

To verify these empiric hypothesis we report the statistical results obtained with software PCGive. We report the correlation matrix and the results for the normality test

$$
\Sigma = \begin{pmatrix} & gas & coal & CO2 & biooil & nuclear \\ gas & 1.00000 & 0.58199 & 0.25988 & 0.57585 & 0.17899 \\ coal & 0.58199 & 1.00000 & 0.20282 & 0.79733 & 0.10152 \\ C02 & 0.25988 & 0.20282 & 1.00000 & 0.55621 & 0.28884 \\ biooil & 0.57585 & 0.79733 & 0.55621 & 1.00000 & 0.24869 \\ nuclear & 0.17899 & 0.10152 & 0.28884 & 0.24869 & 1.00000 \end{pmatrix} \quad (1)
$$

In figure 3 we can observe the dynamic of the logarithmic returns.The return are also stationary. In table 3 we report the results of the normality test on the returns which is refused on prices (see 3) table while is accepted on returns.

Fuel oil	χ^2	Probability	Result
Gas	13.373	0.0012	refused
Coal	15.745	0.0004	refused
CO_2	0.89956	0.6378	accepted
Biooil	44.886	0.0000	refused
Nuclear	38.261	0.0000	refused

Table 2: Normality test on prices.

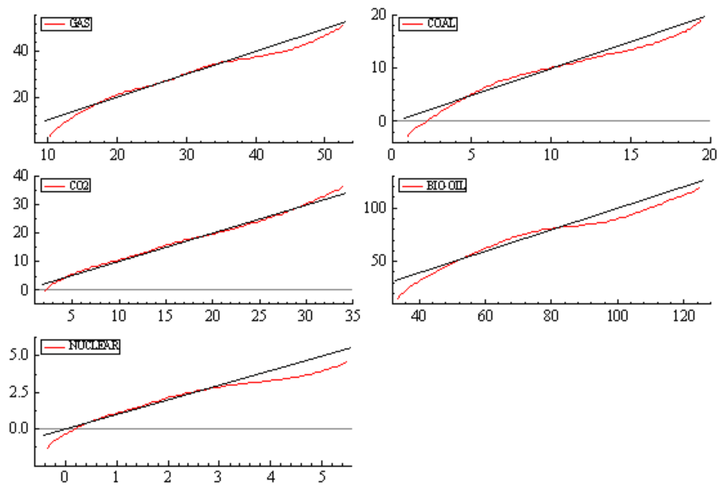

Figure 2: Distribution of the five fuel prices against normal.

Fuel oil return	$\hat{Chi2}$	Probability	Result
returnGas	5.5653	0.0619	accepted
returnCoal	4.0490	0.1321	accepted
returnCO_2	0.85189	0.6532	accepted
returnBiooil	5.7733	0.0558	accepted
returnNuclear	5.2058	0.0741	accepted

Table 3: Normality test on returns

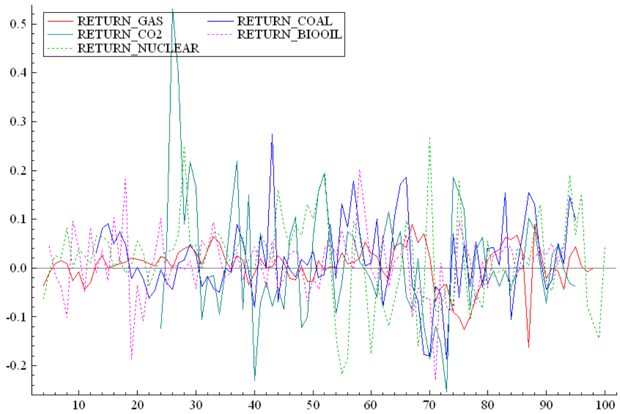

Figure 3: Behaviors of the five fuel returns considered from April 2003 to May 2011.

Finally we investigate the presence of unit root (see Dickey and Fuller (1979)[3]). Difference stationary and trend stationary models of the same time series may imply very different predictions. Deciding which model to use is therefore tremendously important for applied forecasters. Rather than employing one or the other model by default, one may use a unit root test as a diagnostic tool to guide the decision. In fact, one of the early motivations for unit root tests was precisely to help determine whether to use forecasting models in differences or levels in particular applications. If the series y is stationary (or trend stationary), then it has a tendency to return to a constant (or deterministically trending) mean. Therefore large values will tend to be followed by smaller values (negative changes), and small values by larger values (positive changes). Accordingly, the level of the series will be a significant predictor of next period's change, and will have a negative coefficient. If, on the other hand, the series is integrated, then positive changes and negative changes will occur with probabilities that do not depend on the current level of the series; The simplest no stationary autoregressive process is a random walk in which where you are now does not affect which way you will go next because the expected value of the process in t is equal to the expected value at time t-1 in fact

$$y_t = \phi \cdot y_{t-1} + \epsilon_t \qquad with \qquad E[\epsilon_t] = 0 \tag{2}$$

The null Hypothese $H_0 : \phi = 1$ means the process is not stationary against the hypothese $H_1 : \phi < 1$ means the process is stationary. With the sofware PCGive we have found for trace test the results showed in Figure (4) and (5) which imply our data are not stationary nor trend stationary but are difference stationary.

```
GAS: ADF tests (T=68, Constant; 5%=-2.90 1%=-3.53)
D-lag    t-adf       beta Y_1    sigma   t-DY_lag  t-prob      AIC   F-prob
  2     -2.675       0.91947     1.308      1.382  0.1719    0.5942
  1     -2.426       0.92800     1.317      5.164  0.0000    0.5942  0.1719
  0     -1.757       0.93870     1.552                       0.9085  0.0000

COAL: ADF tests (T=68, Constant; 5%=-2.90 1%=-3.53)
D-lag    t-adf       beta Y_1    sigma   t-DY_lag  t-prob      AIC   F-prob
  2     -1.761       0.94111    0.7913     0.5026  0.6169   -0.4110
  1     -1.702       0.94478    0.7868      3.582  0.0007   -0.4365  0.6169
  0     -1.048       0.96354    0.8544                      -0.2857  0.0028

CO2: ADF tests (T=68, Constant; 5%=-2.90 1%=-3.53)
D-lag    t-adf       beta Y_1    sigma   t-DY_lag  t-prob      AIC   F-prob
  2     -3.293*      0.82543     1.946      1.231  0.2227     1.389
  1     -3.075*      0.84119     1.954      2.462  0.0165     1.383  0.2227
  0     -2.661       0.85879     2.027                        1.442  0.0273

BIOOLIO: ADF tests (T=68, Constant; 5%=-2.90 1%=-3.53)
D-lag    t-adf       beta Y_1    sigma   t-DY_lag  t-prob      AIC   F-prob
  2     -1.975       0.92496     4.376      1.490  0.1412     3.009
  1     -1.738       0.93422     4.417      1.955  0.0548     3.014  0.1412
  0     -1.445       0.94473     4.510                        3.042  0.0539

NUCLEARE: ADF tests (T=68, Constant; 5%=-2.90 1%=-3.53)
D-lag    t-adf       beta Y_1    sigma   t-DY_lag  t-prob      AIC   F-prob
  2     -1.869       0.94275    0.1997     -2.556  0.0130    -3.164
  1     -2.428       0.92460    0.2081      5.045  0.0000    -3.097  0.0130
  0     -1.648       0.94042    0.2436                       -2.796  0.0000
GAS: ADF tests (T=68, Constant+Trend; 5%=-3.48 1%=-4.10)
D-lag    t-adf       beta Y_1    sigma   t-DY_lag  t-prob      AIC   F-prob
  2     -2.502       0.91766     1.318      1.374  0.1744    0.6233
  1     -2.209       0.92928     1.327      4.998  0.0000    0.6234  0.1744
  0     -1.321       0.95097     1.553                       0.9235  0.0000

COAL: ADF tests (T=68, Constant+Trend; 5%=-3.48 1%=-4.10)
D-lag    t-adf       beta Y_1    sigma   t-DY_lag  t-prob      AIC   F-prob
  2     -2.592       0.88147    0.7763     0.8732  0.3859   -0.4358
  1     -2.448       0.89311    0.7749      3.807  0.0003   -0.4531  0.3859
  0     -1.590       0.92516    0.8515                      -0.2784  0.0011

CO2: ADF tests (T=68, Constant+Trend; 5%=-3.48 1%=-4.10)
D-lag    t-adf       beta Y_1    sigma   t-DY_lag  t-prob      AIC   F-prob
  2     -4.065*      0.77946     1.872      1.166  0.2478     1.325
  1     -3.895*      0.79283     1.877      2.221  0.0299     1.317  0.2478
  0     -3.617*      0.80259     1.933                        1.362  0.0492

BIOOLIO: ADF tests (T=68, Constant+Trend; 5%=-3.48 1%=-4.10)
D-lag    t-adf       beta Y_1    sigma   t-DY_lag  t-prob      AIC   F-prob
  2     -2.004       0.92011     4.404      1.518  0.1341     3.036
  1     -1.735       0.93137     4.449      1.956  0.0548     3.042  0.1341
  0     -1.416       0.94349     4.544                        3.071  0.0518

NUCLEARE: ADF tests (T=68, Constant+Trend; 5%=-3.48 1%=-4.10)
D-lag    t-adf       beta Y_1    sigma   t-DY_lag  t-prob      AIC   F-prob
  2     -1.819       0.94410    0.2001     -2.625  0.0109    -3.147
  1     -2.399       0.92511    0.2091      4.875  0.0000    -3.072  0.0109
  0     -1.637       0.94094    0.2430                       -2.786  0.0000
```

Figure 4: No stationarity nor trend stationarity.

DGAS: ADF tests (T=67, Constant; 5%=-2.90 1%=-3.53)

D-lag	t-adf	beta Y_1	sigma	t-DY_lag	t-prob	AIC	F-prob
2	-2.712	0.63463	1.372	-1.289	0.2020	0.6902	
1	-3.527*	0.56388	1.379	-0.8111	0.4203	0.6864	0.2020
0	-4.530**	0.51431	1.375			0.6667	0.3190

DCOAL: ADF tests (T=67, Constant; 5%=-2.90 1%=-3.53)

D-lag	t-adf	beta Y_1	sigma	t-DY_lag	t-prob	AIC	F-prob
2	-4.878**	0.23963	0.7930	1.930	0.0581	-0.4060	
1	-4.412**	0.38444	0.8097	-0.1184	0.9062	-0.3784	0.0581
0	-5.442**	0.37521	0.8036			-0.4080	0.1625

DCO2: ADF tests (T=67, Constant; 5%=-2.90 1%=-3.53)

D-lag	t-adf	beta Y_1	sigma	t-DY_lag	t-prob	AIC	F-prob
2	-4.264**	0.25106	2.059	0.3439	0.7321	1.503	
1	-4.777**	0.28167	2.045	-0.5620	0.5761	1.475	0.7321
0	-6.566**	0.22928	2.034			1.450	0.8072

DBIOOLIO: ADF tests (T=67, Constant; 5%=-2.90 1%=-3.53)

D-lag	t-adf	beta Y_1	sigma	t-DY_lag	t-prob	AIC	F-prob
2	-3.323*	0.40639	4.506	-1.014	0.3146	3.069	
1	-4.356**	0.31880	4.507	-1.148	0.2553	3.055	0.3146
0	-6.520**	0.20579	4.518			3.046	0.3162

DNUCLEARE: ADF tests (T=67, Constant; 5%=-2.90 1%=-3.53)

D-lag	t-adf	beta Y_1	sigma	t-DY_lag	t-prob	AIC	F-prob
2	-4.582**	0.33078	0.2067	-0.1134	0.9101	-3.095	
1	-5.779**	0.32108	0.2051	2.992	0.0039	-3.125	0.9101
0	-4.668**	0.49738	0.2173			-3.023	0.0161

Figure 5: Difference stationarity.

4 Vectorial Autoregressive Econometric model of the forward prices curves

If two or more series are individually integrated (in the time series sense) but some linear combination of them has a lower order of integration, then the series are said to be cointegrated(see Engle & Granger 1987 [4]). A common example is where the individual series are first-order integrated (I(1)) but some (cointegrating) vector of coefficients exists to form a stationary linear combination of them. For instance, a stock market index and the price of its associated futures contract move through time, each roughly following a random walk. Testing the hypothesis that there is a statistically significant connection between the futures price and the spot price could now be done by testing for the existence of a cointegrated combination of the two series. (If such a combination has a low order of integration - in particular if it is I(0), this can signify an equilibrium relationship between the original series, which are said to be cointegrated.)

Before the 1980s many economists used linear regressions on non-stationary time series data, which Nobel laureate Clive Granger and others showed to be a dangerous approach that could produce spurious correlation. In his 1987 paper Robert Engle formalized the cointegrating vector approach, and coined the term.

We have checked the price series cointegration by using Johansen's procedure (see [5]) based on trace test where the null hypothesis is the number of cointegration vectors r and then the secon step is to estimate the regression model in which we have decided to work with logaritm of prices in order to lower prices volatility. With Granger casuality test we have verified that no price series x_t is Granger-caused where we say that x Granger-causes y if lags of x explain y, so we have used an endogenous vectorial autoregressive $VAR(p)$: $y_t = c + A_1 y_{t-1} + \ldots + A_p y_{t-p} + \epsilon_t$

We have found that the VAR(p) with $p = 7$ is well posed with normal residual and with a good forecasting ability as we can see in the Figure (6) and (7).

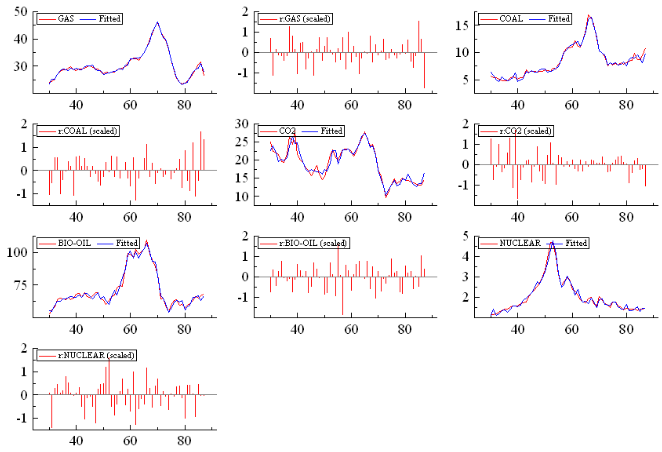

Figure 6: Comparison between fitted value and simulate value with VAR(7) model of the fuel prices time serie.

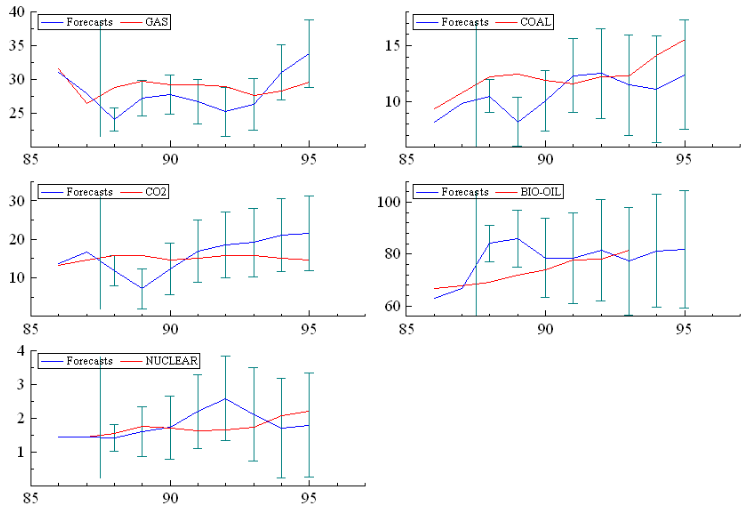

Figure 7: Forecating analysis of endogenous model VAR(7) for the fuel prices time series.

5 Scenario generation on the VAR model's errors

From VAR model's construction we have obtained that every fuel price serie is described by the equation $y_t = c + A_1 y_{t-1} + \ldots + A_7 y_{t-7} + \epsilon_t$ where the errors ϵ_t are indipendent and identically distributed with normal distribution and so can be described by the following brownian motion

$$d\epsilon_t = \mu\epsilon_t dt + \sigma\epsilon_t dZ_t \tag{3}$$

with $dZ_t \in N(0, dt)$.

In equation (3) the risk factor correlation is hidden in dZ_t, to explicit it we have used Choleski decomposition method that says that we can decompose the symmetric and positive definite matrix of variance and covariance as $\Sigma = C^T C$ where C is a lower triangular matrix with strictly positive diagonal entries, and C^T denotes the conjugate transpose of C.

The Cholesky decomposition is unique: given a Hermitian, positive-definite matrix Σ, there is only one lower triangular matrix C with strictly positive diagonal entries such that $\Sigma = C^T C$. The converse holds trivially: if Σ can be written as $C^T C$ for some invertible C, lower triangular or otherwise, then Σ is Hermitian and positive definite.

The Cholesky decomposition is commonly used in the Monte Carlo method for simulating systems with multiple correlated variables: the correlation matrix is decomposed, to give the lower-triangular C. Applying this to a vector of uncorrelated samples, y_t, produces a sample vector Cy_t with the covariance properties of the system being modeled.

So we have analyzed VAR errors obtaining descriptive statistics shown in table below then we have obtained Choleski matrix (4) and finally we generated 100 scenarios on errors over 30 years as reported in figure 8.

descriptive statistics	Gas	Coal	CO_2	biooil	nuclear
mean	0	0	0	0	0
standard deviation	0.68826	0.49073	1.2579	2.3026	0.12676

$$\mathcal{C} = \begin{pmatrix} & gas & coal & CO2 & biooil & nuclear \\ gas & 1.00000 & 0.58199 & 0.25988 & 0.57585 & 0.17899 \\ coal & 0.58199 & 1.00000 & 0.20282 & 0.79733 & 0.10152 \\ C02 & 0.25988 & 0.20282 & 1.00000 & 0.55621 & 0.28884 \\ biooil & 0.57585 & 0.79733 & 0.55621 & 1.00000 & 0.24869 \\ nuclear & 0.17899 & 0.10152 & 0.28884 & 0.24869 & 1.00000 \end{pmatrix} \tag{4}$$

6 Montecarlo scenarios

In this section we face with problem of scenarios generation of long time period forecasting fuel prices. A Monte Carlo simulation was implemented and the set of variables considered are gas, coal, nuclear, bio-oil and CO_2. Instead of constructing scenarios on the VAR model's errors we work directly on these prices which can be considered as correlated random variables with a lognormal distribution with correlation matrix (1). After discretization over time of the stochastic equation, the dynamic of these variables over a time horizon T can be obtained as (5)

$$S(t+1) = S(t)\left[1 + \mu\Delta t + \sigma\epsilon\Delta t\right] \tag{5}$$

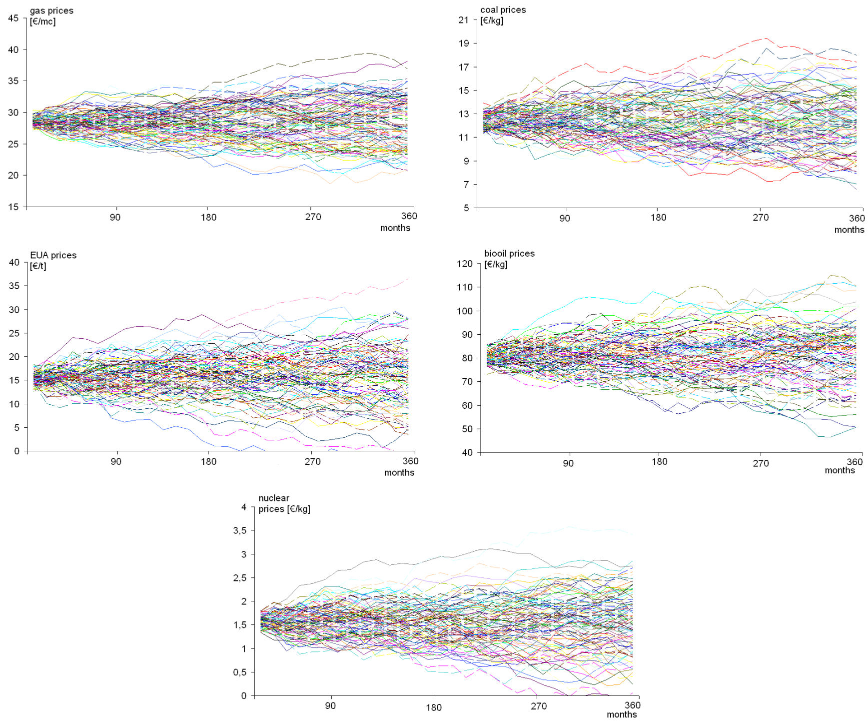

Figure 8: Simulate value of the fuel prices time series with scenario generation on the VAR model's errors

in which μ is the annual percentage drift, σ is the annual percentage volatility, ϵ are extracted from a multivariate normal distribution with correlation matrix (1) and Δt is the discretization interval (in this work Δt is the number of mouth in one year).

By analyzing historical prices, it can be computed the deterministic component of the price in each equation as the mean of monthly prices yields from april 2003 to May 2011. Table 6 shows the calibration of the scenario variables parameters adopted for all Monte Carlo replications. The price in instant t_0 represents the current average price of the correspondent variables that will be the starting point for the Brownian motion generated for each scenario. The annual volatility has been evaluated processing historical data of each variables. In particular, the volatility was calculated as: $\sigma = \sigma\sqrt{T}$, where T is the time interval of the observation of the historical series and σd is the standard deviation of the monthly historical series of the returns.

In Figure (9) we show 100 scenarios from every variable while in Figure (10) we show one scenario of the five variables in a logaritmic scale.

By increasing the number of Monte Carlo trials and so obtaining a large number of possible scenarios, the model will define a probability distribution.

scenario variables	deterministic component	Annual volatility	Price in istant t_0
Gas	0.005810587	0.047882283	27.66922293
Coal	0.014966595	0.083379959	12.33180778
CO_2	0.016609074	0.128397656	15.66758413
Biooil	0.008468842	0.05904478	81.61448564
Nuclear	0.017209791	0.095053344	1.741946199

Table 4: Parameters of scenario variables.

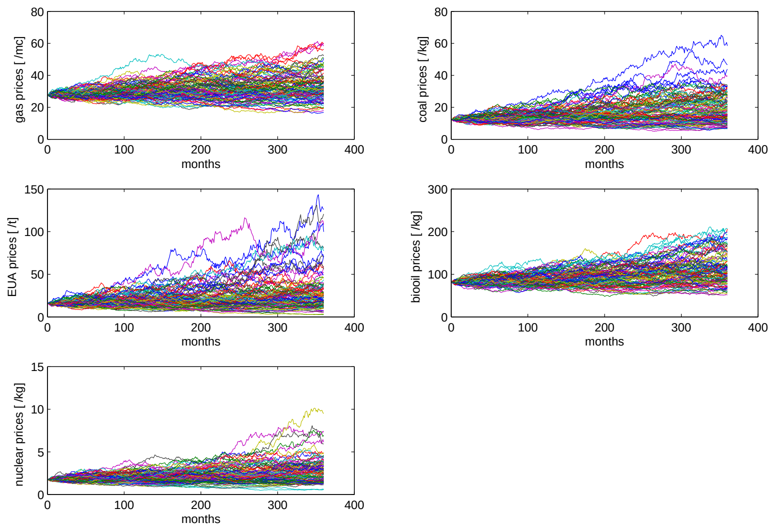

Figure 9: Simulate value of the fuel prices time series with montecarlo generation.

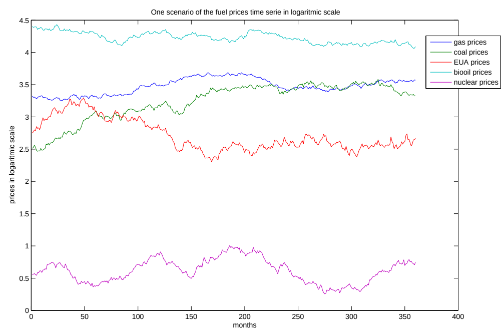

Figure 10: One scenario of the fuel prices time serie in logaritmic scale.

7 Conclusions and further works

In this section we show the results obtained comparing generation scenarios in the two different approaches described above. We have analyzed mean, standard deviation and kurtosis in year 5, 10, 20 and 30 as we can see in the two tables below.

year 5	coal	gas	CO2	biooil	nuclear
mean	12.926	28.171	16.172	84.381	1.818
standard deviation	2.532	2.413	4.066	10.317	0.328
curtosis	0.2045	-0.0465	0,8228	-0.0210	-0.1989
year 10	coal	gas	CO2	biooil	nuclear
mean	13.4367	29.2473	16.8172	85.7316	1.9701
standard deviation	3.5423	4.0115	7.1544	13.7881	0.5036
curtosis	-0.1284	-0.5668	4.2034	2.0070	-0.2760
year 20	coal	gas	CO2	biooil	nuclear
mean	15.210	31.031	20.0457	91.7199	2.2478
standard deviation	5.9124	7.0239	12.6444	23.2656	0.8586
curtosis	1.5656	0.5783	4.8606	0.7222	0.7235
year 30	coal	gas	CO2	biooil	nuclear
mean	17.6478	31.7154	22.2670	10.3239	2.6631
standard deviation	9.2004	7.9175	15.2450	32.9379	1.4167
curtosis	2.9992	-0.3677	2.7446	-0.1975	1.3890

Table 5: Statistics for the scenarios generated by Montecarlo simulation

year 5	coal	gas	CO2	biooil	nuclear
mean	12.4962	28.4093	15.5381	80.8744	1.5720
standard deviation	1.1277	1.6211	2.7121	5.4491	0.2922
curtosis	1.0151	0.4406	0.0312	-0.1058	-0.3044
year 10	coal	gas	CO2	biooil	nuclear
mean	13.3920	28.3634	15.9924	81.3603	1.5935
standard deviation	1.4941	2.0611	3.5521	7.1442	0.3699
curtosis	-0.4852	0.2294	0.2543	0.5454	0.6426
year 20	coal	gas	CO2	biooil	nuclear
mean	12.2455	28.6333	15.7684	81.1483	1.5116
standard deviation	2.1044	3.3925	5.5089	9.9350	0.5544
curtosis	0.0835	-0.3949	0.6328	-0.4907	0.5096
year 30	coal	gas	CO2	biooil	nuclear
mean	12.1064	28.1979	15.5810	81.8152	1.5207
standard deviation	2.4006	4.044	6.8702	12.2326	0.6792
curtosis	-0.3068	-0.8530	0.3973	0.1005	-0.0438

Table 6: Statistics for the scenarios generated by VAR simulation

From results obtained we can conclude that scenarios obtained with Montecarlo method are more diversified having an higher standard deviation and they don't follow a normal distribution as we can see in Figure (11). These are two important features to forecast long period time series.

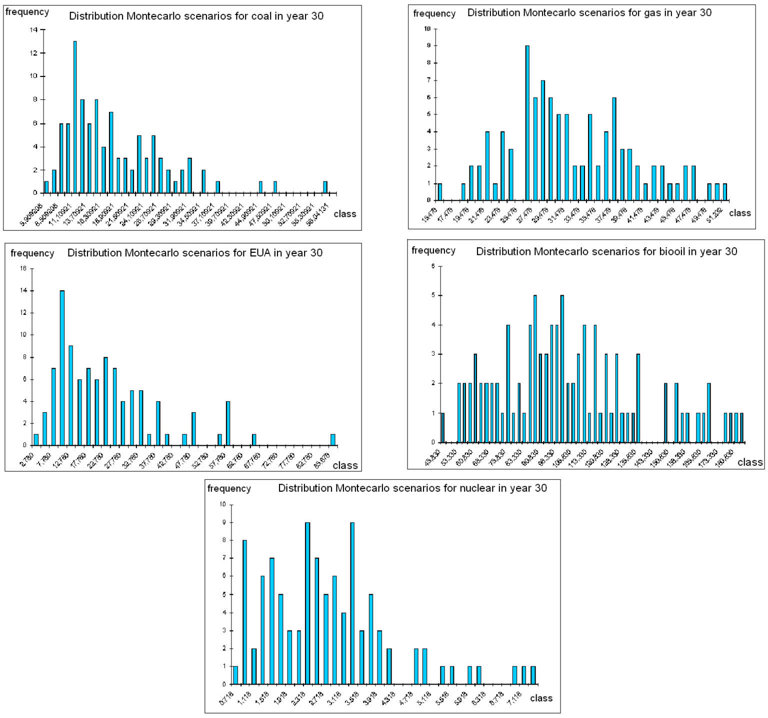

Figure 11: Distribution Montecarlo scenarios.

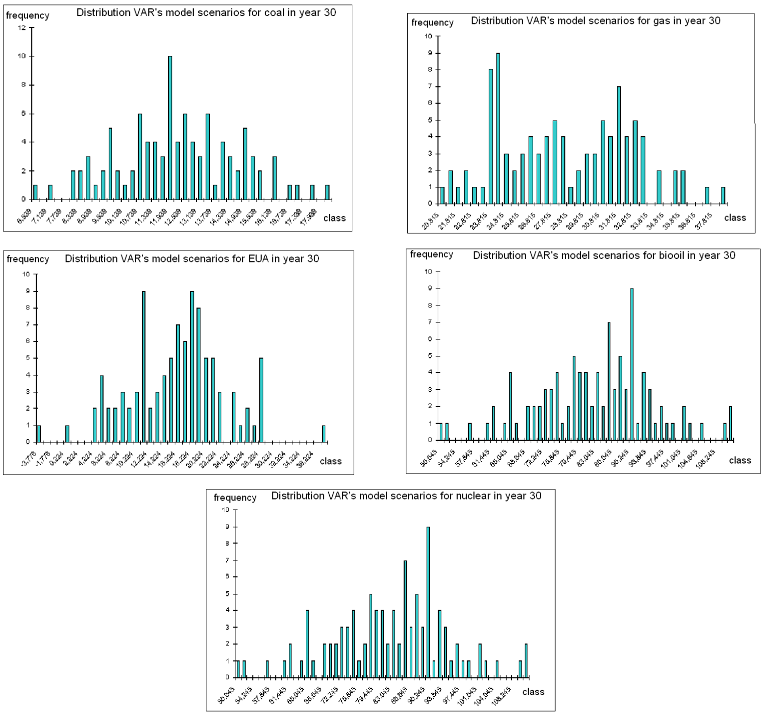

Figure 12: Distribution VAR's model scenarios.

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