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Minimax optimization of Tuned Mass Dampers under seismic excitation

Jonathan SALVI[1], Egidio RIZZI[1*]
[1]Università di Bergamo, Facoltà di Ingegneria (Dalmine), Dip.to di Progettazione e Tecnologie
viale G. Marconi 5, I-24044 Dalmine (BG), Italy
*Contact Author, email: egidio.rizzi@unibg.it

ABSTRACT: The present paper concerns the optimal tuning of the free parameters of passive Tuned Mass Damper (TMD) devices, added to benchmark frame structures taken from the literature and subjected to a given deterministic seismic excitation. The tuning procedure is achieved through a numerical optimization approach, namely a Minimax algorithm implemented in a MATLAB environment. Different objective functions have been considered, from both kinematic and energy response indicators of the primary structure. The optimization process is carried-out in time domain, whereby the dynamic response is evaluated numerically by a step-by-step integration based on Newmark's average acceleration method. In order to assess the efficiency of the proposed methodology and investigate the effectiveness of the so-conceived TMD, several numerical tests on both single- and multi-degree-of-freedom frame structures endowed with a TMD are performed. The salient numerical results are presented in plots and tables. Plots of the optimal TMD parameters (frequency ratio and damping ratio) as a function of mass ratio are reported and graphs showing the seismic response reduction in terms of top-floor displacement are provided. Tables gathering the optimal TMD parameters and the seismic response reduction at assumed given values of the mass ratio are outlined. Comparisons to results obtained from well-known tuning formulas are provided. The achieved output demonstrates the reliability of the proposed method and shows that, in principle, the design of an optimal TMD device for a specific seismic event is possible. This should have important implications in the framework of adaptive, semi-active and active TMD devices.

KEY WORDS: Structural control, Tuned Mass Damper, Tuning, Minimax optimization, Seismic input, Frame structures.

1 INTRODUCTION

The study presented in this paper is framed on an on-going research project at the University of Bergamo [1-4]. Specifically, this work concerns the optimal tuning of the free parameters of passive Tuned Mass Damper (TMD) devices, added to benchmark frame structures taken from the literature and subjected to a given deterministic seismic excitation. Best tuning is achieved through a numerical optimization approach based on a Minimax algorithm.

A TMD is a device that is composed of a secondary mass attached to a primary structure by a spring element and a viscous damper. Despite the original TMD concept was primarily developed in naval and mechanical engineering [5], such devices are used as well in civil engineering applications, in order to reduce, or possibly control, by the most appropriate tuning of the TMD parameters (frequency, damping and mass ratios), the effects of undesired structural vibrations due to different external actions.

In the literature, one observes that there appear two main approaches of best tuning of the TMD parameters, namely those that are dependent [5-9] or independent [10-11] of the applied actions. Since the fixed-points theory by Den Hartog, many tuning methods for the optimal parameters have been developed, mostly based either on analytical procedures, along the line of Den Hartog's work itself [5], or on numerical optimization methods [6-9], applied to structures subjected to ideal loadings, such as harmonic or gaussian white noise force or base excitation. Despite the bulk of literature on the tuning of TMDs, the efficiency of these devices in reducing the dynamic response of civil engineering constructions to seismic events is still debated.

In this paper, a novel tuning technique is proposed, which is based on a Minimax optimization method as applied to given structural systems (composed by principal structure + TMD), subjected to a known seismic input. Thus, the obtained optimal parameters are specific to each considered case. The optimization process is carried-out in time domain, whereby the dynamic response is evaluated numerically by performing a step-by-step integration based on Newmark's average acceleration method.

This procedure is applied to SDOF and MDOF structural systems (shear-type buildings) taken from the literature [9-11], subjected to El Centro (Imperial Valley, 1940) seismic ground motion. These structures are characterized by different numbers of storeys and values of inherent damping. Seismic response quantities are evaluated for each system in terms of different kinematic indicators, such as displacement x_S, velocity $\dot{x}_S$ and acceleration $\ddot{x}_S$ of the top floor, and also of energy indexes, such as kinetic energy T_S of the primary structure. Furthermore, response quantities are interpreted in terms of both maximum and average (RMS) values. Plots showing the trends of the optimal TMD parameters and the response reduction during the seismic event in the presence of a TMD are illustrated. The results obtained from the numerical tests are also summarized in table form, where the reduction of the seismic response with added TMD is quantified.

The achieved outcomes show, for the assumed seismic input, the considerable reduction of the structural response, for all the evaluated quantities. Despite the TMD appears obviously more efficient for very low structural damping ratios, considerable cutting of the seismic response has been observed also for typical values of structural damping that occur in civil engineering constructions. Moreover, in order to explore the differences with respect to more traditional tuning techniques, the resulting parameters, and relevant outcomes, have been compared to those that could be obtained by traditional Den Hartog's tuning formulas [5].

2 STATEMENT OF THE OPTIMAL TUNING PROCEDURE

2.1 *TMD for SDOF and MDOF structures*

A linear structural system composed of a SDOF structure, equipped with a TMD located on top of it and subjected to a seismic ground acceleration is sketched in Fig. 1.

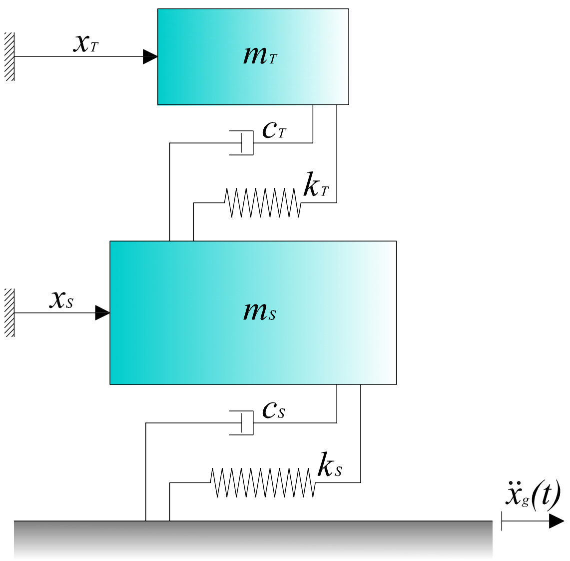

Figure 1. Linear structural system composed of a primary structure (subscript S) and a TMD (subscript T) added on top of it, subjected to base acceleration.

The primary structure is characterized by a mass m_S, a linear elastic stiffness k_S and a linear viscous damping coefficient c_S. The natural frequency ω_S and damping ratio ζ_S of the primary structure are defined as usual, i.e. respectively:

$$\omega_S = \sqrt{\frac{k_S}{m_S}}, \quad \zeta_S = \frac{c_S}{2\sqrt{k_S m_S}} \tag{1}$$

Conversely, the parameters of the attached TMD device are an added secondary mass m_T, a stiffness k_T of an added mutual elastic spring and a damping coefficient c_T of an added relative viscous damper. As above, the TMD angular frequency ω_T and damping ratio ζ_T are respectively:

$$\omega_T = \sqrt{\frac{k_T}{m_T}}, \quad \zeta_T = \frac{c_T}{2\sqrt{k_T\, m_T}} \tag{2}$$

The main free TMD parameters, useful to achieve the most appropriate tuning of the added device on the selected properties of the primary structure, are defined as follows, in terms of mass ratio μ and frequency ratio f of the structure + TMD system:

$$\mu = \frac{m_T}{m_S}, \; f = \frac{\omega_T}{\omega_S} \tag{3}$$

and of TMD damping ratio ζ_T, as already given in Eq. (2).

In case the given structural system is an MDOF shear-type building with n storeys (corresponding to n horizontal degrees of freedom), the characteristics parameters above are defined as follows:

$$\mu = \frac{m_T}{\boldsymbol{\phi}_{S,1}{}^T \boldsymbol{M}_S \boldsymbol{\phi}_{S,1}}, \; f = \frac{\omega_T}{\omega_{S,1}} \tag{4}$$

where $\boldsymbol{M}_S$ is the $n\times n$ diagonal mass matrix of the frame structure, $\omega_{S,1}$ is the angular frequency of its first mode of vibration and $\boldsymbol{\phi}_{S,1}$ is the corresponding first mode shape, normalized so that to have a unit component at the level of the top storey [12]. As above, the TMD damping ratio ζ_T is still given as in Eq. (2). Further details on the treatment of the equations of motion for both SDOF and MDOF systems with added TMD are provided in [4].

2.2 *Minimax algorithm*

The tuning procedure proposed here is based on a Minimax algorithm, implemented in a MATLAB environment by the existing *fminimax* function. The goal of this algorithm is the minimization of the worst case, in terms of maximum value, of a set of multivariable functions, starting at an initial estimation, possibly limited by lower and upper bounds on the variable values.

In the present context, the objective function is a specific quantity representative of the dynamic response of the structural system, which obviously depends, given the fixed primary structure parameters, on the free TMD parameters. The approach adopted in this work corresponds to the usual modus operandi that takes a given mass ratio μ and searches for the optimal values of the two remaining free TMD parameters, namely frequency ratio f and damping ratio ζ_T. The objective function that is finally adopted here in the numerical tests that follow is represented by average (RMS) quantities of the structural response. This is a consequence of evidences experienced in [3], where RMS quantities have emerged as better objective functions than maximum quantities since: a) the Minimax algorithm converges more rapidly, consistently and without great difficulties; b) the optimal so-conceived TMD assures a higher efficiency in reducing the global dynamic response in all the time window of analysis. The optimization routine is looped with a time solver based on classical Newmark's average acceleration method. A complete flowchart of the numerical algorithm and details on the various characteristic parameters, such as the tolerances on the objective function and on the TMD tuning variables are reported in [4], which represents a companion, more comprehensive investigation, reporting further and complementary results of the present.

3 NUMERICAL TESTS

In order to explore the reliability of the proposed method, in this section various numerical tests are presented, which regard benchmark frame structures (shear-type buildings) taken from the literature [9-11], subjected to El Centro seismic base acceleration recorded on 19 May 1940, as shown in Fig. 2.

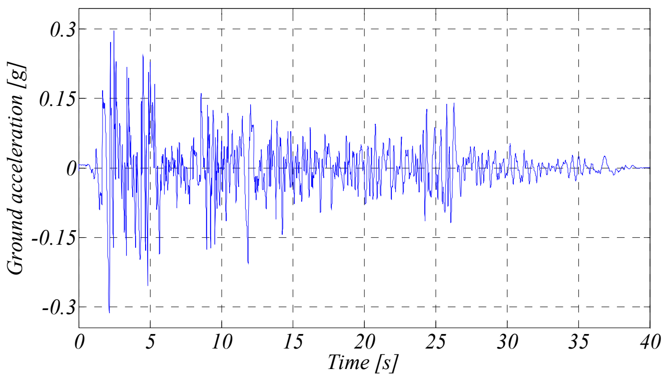

Figure 2. Accelerogram of the assumed seismic input at Imperial Valley (El Centro station), 19 May 1940.

The optimal TMD parameters are determined as a function of given mass ratio μ, taken in a range suitable for civil engineering applications, i.e.:

$$\mu = [0.005 : 0.005 : 0.1] \tag{5}$$

The optimal evaluation of the variable parameters starts with the following initial estimation, that takes into account: a) the main principle of resonance between primary structure and TMD; b) the reference value of the TMD damping ratio usually indicated in the literature:

$$f^{0} = 1, \quad \zeta_T{}^{0} = 0.1 \tag{6}$$

The variable parameters are also subjected to the following given lower and upper bounds in the Minimax algorithm, that allow for a wide search interval:

$$0.5 < f < 1.5, \quad 0.001 < \zeta_T < 1.0 \tag{7}$$

Besides the evaluation of the optimal TMD parameters, the efficiency of the so-conceived TMD has been estimated by the computation of the dynamic response of the structural system (composed of primary structure and TMD) for a given value of the mass ratio, namely $\mu = 0.05$.

3.1 Numerical tests on a SDOF frame structure

First, the optimization procedure has been applied to the Single-Degree-Of-Freedom system proposed by Leung et al. [9], with structural parameters reported in Table 1. Besides with the proposed method, the optimal TMD parameters and dynamic response for this test have been also computed with traditional Den Hartog's tuning formulas [5]:

$$f = \frac{1}{1+\mu}, \quad \zeta_T = \sqrt{\frac{3}{8}\,\frac{\mu}{1+\mu}} \tag{8}$$

Table 1. Structural parameters, natural frequency and period of the single-storey building (ζ_S=5%), taken from [9].

Storey/ Mode	Mass [$\times 10^3$ kg]	Stiffness [$\times 10^6$ N/m]	Frequency [Hz]	Period [s]
1	*1000*	*25*	*0.79578*	*1.25664*

The optimization process has been performed first by assuming as objective function the RMS displacement of the primary structure. The trends of the optimal TMD parameters, are represented in Figs. 3 and 4, respectively for f and ζ_T. Numerical results obtained for $\mu = 0.05$ in terms of tuning parameters (frequency ratio f and damping ratio ζ_T) and dynamic response represented by kinematic and energy indicators are summarized in Table 2. The dynamic response of the system, composed of primary structure + TMD ($\mu = 0.05$), in terms of displacement and kinetic energy of the primary structure is also shown, respectively in Figs. 5 and 6. These outcomes lead to the considerations pointed-out below.

The frequency ratio always appears to be above Den Hartog's tuning and takes values larger than 1 for $\mu < 0.03$, then decreases on average with an almost-linear trend (Fig. 3). The damping ratio, complementary to the frequency ratio, is always below Den Hartog's tuning and with opposite curvature trend for $\mu < 0.03$, then takes values with an almost-linear increasing trend (Fig. 4).

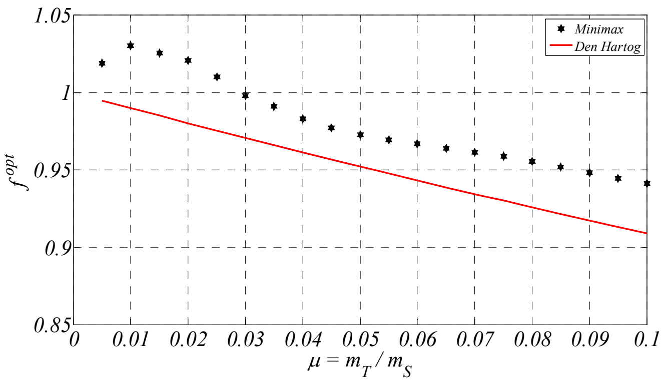

Figure 3. Optimal TMD frequency ratio f as a function of mass ratio μ for the single-storey building (ζ_S=5%), obtained at seismic input (Fig. 2) with the proposed method, with comparison to classical Den Hartog's tuning.

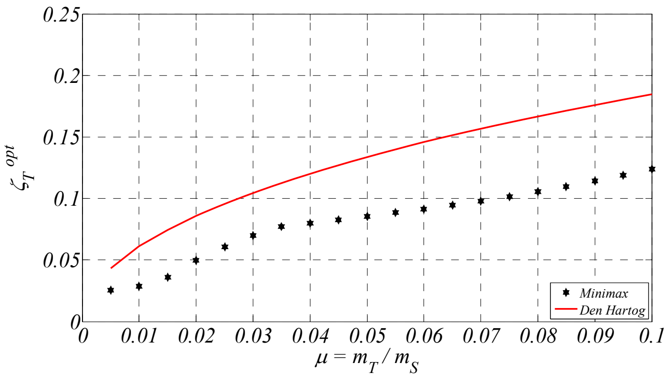

Figure 4. Same as Fig. 3. Optimal TMD damping ratio ζ_T.

Table 2 below indicates that the response reduction in terms of RMS quantities is higher than that on maximum values, although the latter is also noticeable. The optimal TMD designed with the proposed method is a bit more efficient than that obtained from classical Den Hartog's tuning. Nevertheless, the latter is also proven to be effective (despite being based on an undamped primary structure, subjected to harmonic loading).

Table 2. Optimal TMD parameters and seismic response of a single-storey building (ζ_S=5%) + TMD system (μ=5%), with comparison to Den Hartog's tuning. RMS displacement of the structure taken as objective function.

Optimal parameters of the TMD						
Parameter		Den Hartog		Minimax		
f		0.95238		0.97305		
ζ_T		0.13363		0.08561		
Dynamic response of the main structure						
Displacement/ Kinetic energy	No TMD Value	Den Hartog Value	Den Hartog Δ (%)	Minimax Value	Minimax Δ (%)	
$x_S{}^{max}$ [m]	0.11460	0.08642	-24.6	0.08045	-29.8	
$x_S{}^{RMS}$ [m]	0.03186	0.02133	-33.0	0.02084	-34.6	
$T_S{}^{max}$ [N m]	178460	135921	-23.8	133152	-25.4	
$T_S{}^{RMS}$ [N m]	27241.2	15829.0	-41.9	14931.3	-45.2	

Both following Figs. 5 and 6 show that: a) a considerable reduction of the seismic response of the primary structure has been obtained (on both peak and average responses); b) during the first 5 seconds or so from the beginning of the seismic event, the TMD does not sort any appreciable effect. This observation should have important implications for seismic input that may produce significant effects during the early instants of shaking.

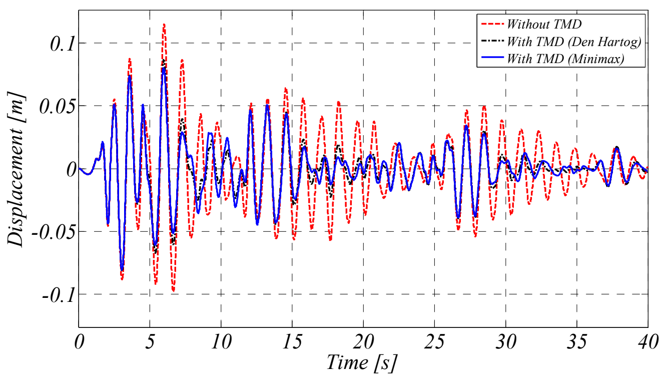

Figure 5. Kinematic response in terms of displacement of the single-storey building (ζ_S=5%), with and without TMD (μ=5%).

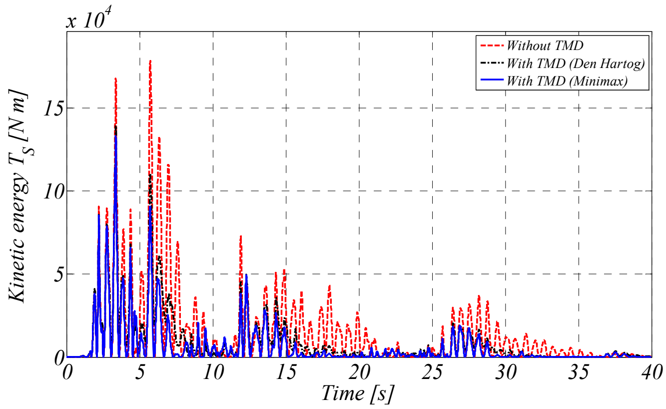

Figure 6. Same as Fig. 5. Energy response in terms of kinetic energy.

Second, the optimization process has been performed as well by using as objective function the kinetic energy T_S of the primary structure. This is conceived to reduce the global energy involved in the dynamic behavior of the structure. The obtained numerical results are reported in Table 3, which could be compared to those in Table 2. It can be noticed that the so-conceived TMD further increases f and decreases ζ_T and allows for additional reduction of the dynamic response (except for the RMS displacement index, which was the previously assumed objective function), especially on the targeted index $T_S{}^{RMS}$, with cut that now really approaches the value of 50%.

Table 3. Same as Table 2. RMS kinetic energy of the primary structure taken as objective function.

Optimal parameters of the TMD					
Parameter		Den Hartog		Minimax	
f		0.95238		1.01753	
ζ_T		0.13363		0.06006	
Dynamic response of the main structure					
Displacement/ Kinetic energy	No TMD Value	Den Hartog Value	Den Hartog Δ (%)	Minimax Value	Minimax Δ (%)
$x_S{}^{max}$ [m]	0.11460	0.08642	-24.6	0.07933	-30.8
$x_S{}^{RMS}$ [m]	0.03186	0.02133	-33.0	0.02168	-31.9
$T_S{}^{max}$ [N m]	178460	135921	-23.8	128008	-28.3
$T_S{}^{RMS}$ [N m]	27241.2	15829.0	-41.9	14093.8	-48.6

Despite this further gain in structural performance, in the following just the RMS displacement will be taken as objective function, mainly because of the higher computational cost that turns-out to be required for the use of the RMS kinetic energy index. A simple example can be reported for the cases above: on a standard lap-top computer, the optimization process lasts about 3 minutes by assuming as target the RMS displacement vs. about 20 minutes for the RMS kinetic energy objective function. Higher differences in CPU time would also be expected for MDOF structures.

About the achieved values on tuned parameters and relevant response, it is worthwhile to report that the best performance accompanies itself to a high relative displacement between the TMD and the primary structure. This is a consequence of two basic facts: a) the optimization process refers only to the primary structure, i.e. it does not consider the dynamic response of the TMD itself; b) the obtained TMD damping ratio always takes rather low values ($\zeta_T < 10\%$), with respect to those usually reported in the literature (see e.g. [10-11]), thus it is not sufficient to damp-down efficiently the TMD absolute and relative displacements.

In order to investigate this issue, which might have important practical implications, some further numerical tests have been carried-out, for given ranges of both mass ratio and TMD damping ratio, in a single-parameter optimization process that regards only the TMD frequency ratio. The outcomes of these experiments are basically the following: an increase of the TMD damping ratio (a value of about $\zeta_T = 20\%$-40% is sufficient) allows to reduce significantly the TMD kinematic response but implies a corresponding increase of the primary structure response. Perhaps, this could be due to the fact that a higher TMD damping ratio does not allow to take full advantage of the opposite motion between primary structure and TMD, on which the TMD principle is actually based (see Den Hartog's tuning concept [5], which has been initially conceived in the absence of TMD damping). Indeed, an increasing ζ_T strengthens the mutual link between TMD and primary structure and, for high values of ζ_T, a sort of dynamic connection between the two parts of the structural system occurs. Moreover, consistent attempts to reduce both TMD and primary structure responses might require the use of multi-objective optimization approaches, as it has been initially attempted in [3]. Further results on all these topics will be reported in [4].

Additional numerical results with the proposed method are also obtained by further taking additional kinematic response quantities as objective function, i.e. RMS floor velocity and acceleration, as summarized in Table 4. This interesting comparison takes inspiration from similar numerical tests and relevant results provided in [3].

Table 4. Comparison of results obtained by taking different kinematic indicators as objective function, i.e. RMS displacement, velocity and acceleration of the structure.

	Objective function		
	x_S^{RMS} [m]	$\dot{x}_S^{RMS}$ [m]	$\ddot{x}_S^{RMS}$ [m]
f	0.97305	0.98755	1.00576
ζ_T	0.08561	0.08425	0.08731
x_S^{max} [m]	0.08045	0.08015	0.07984
$\dot{x}_S^{max}$ [m]	0.51605	0.51335	0.51041
$\ddot{x}_S^{max}$ [m]	4.21104	4.20763	4.20266
x_S^{RMS} [m]	0.02084	0.02091	0.02112
$\dot{x}_S^{RMS}$ [m]	0.11077	0.11052	0.11087
$\ddot{x}_S^{RMS}$ [m]	0.77738	0.77463	0.77344

One may observe that both TMD parameters and response reductions are approximately the same for the tuning on structural displacement or velocity, while are slightly different for the tuning on system acceleration. However, differences appear quite marginal. In any case, the important issue concerning these attempts is that one may take as well the RMS velocity and acceleration as suitable effective objective functions towards the optimization process.

3.2 *Numerical tests on MDOF frame structures*

In the following, the proposed method has been further applied to MDOF frame structures. The structural systems assumed in these tests are frame structures taken from [10] and [11]: a three-, a six- and a ten-storey building, respectively with inherent damping ratios referred to the fundamental mode of vibration of $\zeta_S = 0$, 0.05 and 0.02. The structural parameters are resumed in Tables 5-7. The objective function assumed in the optimization processes in all cases is the RMS displacement of the top storey. A TMD is added on top, with an assumed mass ratio of $\mu = 5\%$, as defined in Eq. (4).

Table 5. Structural parameters, natural frequencies and periods of the three-storey building (ζ_S=0), taken from [11].

Storey/ Mode	Mass [$\times 10^3$ kg]	Stiffness [$\times 10^6$ N/m]	Frequency [Hz]	Period [s]
1	100	41	1.40443	0.71203
2	100	38	3.86190	0.25894
3	100	36	5.56665	0.17964

Table 6. Structural parameters, natural frequencies and periods of the six-storey building (ζ_S=5%), taken from [11].

Storey/ Mode	Mass [$\times 10^3$ kg]	Stiffness [$\times 10^6$ N/m]	Frequency [Hz]	Period [s]
1	8000	10000	1.23357	0.81066
2	8000	9000	3.26467	0.30631
3	8000	8000	5.23216	0.19113
4	8000	7500	6.77099	0.14769
5	8000	5500	8.33027	0.12004
6	8000	4500	9.76431	0.10241

Table 7. Structural parameters, natural frequencies and periods of the ten-storey building (ζ_S=2%), taken from [10].

Storey/ Mode	Mass [$\times 10^3$ kg]	Stiffness [$\times 10^6$ N/m]	Frequency [Hz]	Period [s]
1	179	62.47	0.50037	1.99853
2	170	59.26	1.32631	0.75397
3	161	56.14	2.15121	0.46485
4	152	53.02	2.93387	0.34085
5	143	49.91	3.65320	0.27373
6	134	46.79	4.29197	0.23299
7	125	43.67	4.83557	0.20680
8	116	40.55	5.27169	0.18969
9	107	37.43	5.59050	0.17887
10	98	34.31	5.78653	0.17282

Results of the numerical tests are organized as follows. First, the trends of the optimal TMD parameters are depicted in Fig. 7 (frequency ratio) and Fig. 8 (damping ratio). In Tables 8-10 the optimal TMD parameters and relevant seismic response of the structural systems are summarized. The seismic response in terms of top-floor displacement is reported in Figs. 9-11, for both cases with and without TMD.

From the trends of the frequency ratio f, represented in Fig. 7 for the various MDOF buildings, it appears that f takes higher values (larger than 1 for $\mu < 0.04$) in the case of the three-storey building (that has no inherent damping), with respect to the other two shear-type buildings (with damping). As a general trend, it appears that, as the inherent damping ratio increases, the tuned frequency ratio decreases. Rather flat dependencies on μ are also displayed by the two cases with higher structural damping. Also, the two ideal curves $f^{opt}(\mu)$ of the three- and six-storey buildings intersect at a common point with μ near 5.5%.

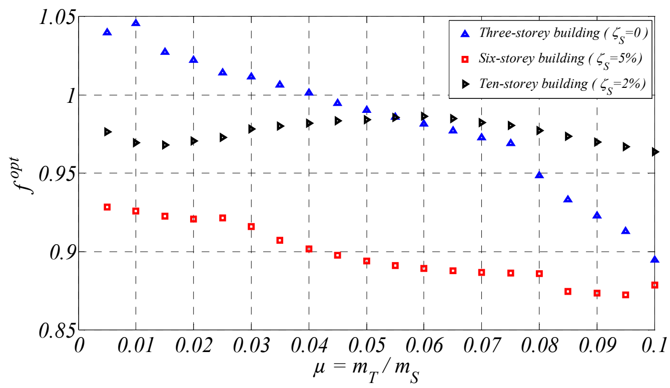

Figure 7. Optimal frequency ratio f as a function of mass ratio μ for the various multi-storey frame buildings.

Similar increasing trends of $\zeta_T^{\,opt}(\mu)$ as those obtained in Fig. 4 for the single-storey building are experienced in Fig. 8 for the various multi-storey buildings. The relative positioning of the three ideal curves $\zeta_T(\mu)$ seems to be basically ruled by the value of inherent structural damping (i.e. the higher ζ_S, the lower $\zeta_T^{\,opt}$). Indeed, the six-storey building ($\zeta_S = 5\%$) results in the lower values of $\zeta_T^{\,opt}$, while larger values are those relative to the ten-storey building ($\zeta_S = 2\%$) and, most of all, to the three-storey building with zero inherent damping. Hence, it appears that the tuned TMD damping ratio attempts to replace possibly-missing inherent damping.

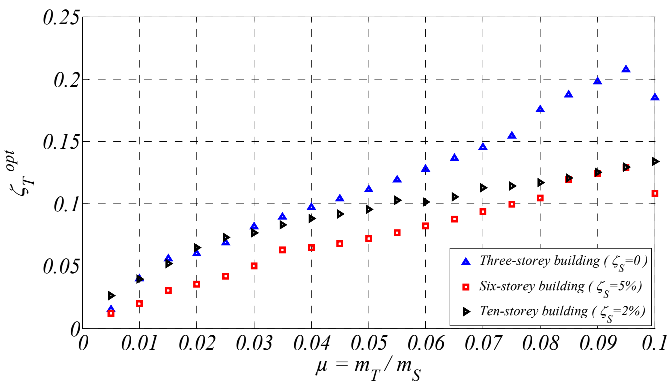

Figure 8. Same as Fig. 7. Optimal damping ratio ζ_T.

From Tables 8-10 it can be appreciated that the addition of a TMD allows for a significant reduction of the seismic response. In all three cases the response quantity that is less reduced by the presence of the TMD is the maximum displacement of the top storey. As expected, the comparative efficiency of the TMD decreases as the inherent damping ratio increases. Indeed, a response reduction has been obtained in Table 8 from about 50% to 90% for the three-storey building (ζ_S=0), while reductions to 20% to 50% are achieved in Table 9 for the six-storey building (ζ_S=5%) and to 20% to 60% in Table 10 for the ten-storey building (ζ_S=2%). Notice again the resulting high value of ζ_T in Table 8 for the case of the frame with zero inherent damping: the best tuning attempts to supply damping to the structural system through the added TMD device. Moreover, it seems that the further gain in passing from maximum to RMS response indicator quantities is lower at increasing inherent damping ratio.

Table 8. Optimal TMD parameters and seismic response of the three-storey building (ζ_S=0) + TMD system (μ=5%). RMS displacement of the structure taken as objective function.

Optimal parameters of the TMD			
f		0.99058	
ζ_T		0.11166	
Dynamic response of the main structure (top floor)			
Displacement/ Kinetic energy	No TMD Value	Minimax Value	Δ (%)
$x_S^{\,max}$ [m]	0.18362	0.09186	-50.0
$x_S^{\,RMS}$ [m]	0.10382	0.02067	-80.1
$T_S^{\,max}$ [N m]	225191	49418.1	-78.1
$T_S^{\,RMS}$ [N m]	100715	7524.94	-92.5

Table 9. Same as Table 8. Six-storey building (ζ_S=5%).

Optimal parameters of the TMD			
f		0.89416	
ζ_T		0.07192	
Dynamic response of the main structure (top floor)			
Displacement/ Kinetic energy	No TMD Value	Minimax Value	Δ (%)
$x_S^{\,max}$ [m]	0.11518	0.09044	-21.5
$x_S^{\,RMS}$ [m]	0.02736	0.01803	-34.1
$T_S^{\,max}$ [N m]	8029253	6013624	-25.1
$T_S^{\,RMS}$ [N m]	1207838	609186	-49.6

Table 10. Same as Table 8. Ten-storey building (ζ_S=2%).

Optimal parameters of the TMD			
f		0.98439	
ζ_T		0.09567	
Dynamic response of the main structure (top floor)			
Displacement/ Kinetic energy	No TMD Value	Minimax Value	Minimax Δ (%)
x_S^{max} [m]	0.33490	0.27193	-18.8
x_S^{RMS} [m]	0.12002	0.05796	-51.7
T_S^{max} [N m]	453774	212384	-53.2
T_S^{RMS} [N m]	79343.8	29389.0	-63.0

Further proof on the influence of the inherent damping can be noticed by observing the dynamic response of the three structures, with and without TMD (Figs. 9-11). As expected, the greater response reduction has been obtained for the undamped three-storey building with $\zeta_S = 0$, while in comparison the TMD appears less effective for the six-storey building with larger ζ_S. As remarked previously, the TMD begins to operate in all cases after 3-6 seconds from the beginning of the seismic event. This phase of TMD inactivity lasts much longer as higher is the fundamental period of the structure, fact that results immediately by confronting data in Tables 5-7 and output in Figs. 9-11.

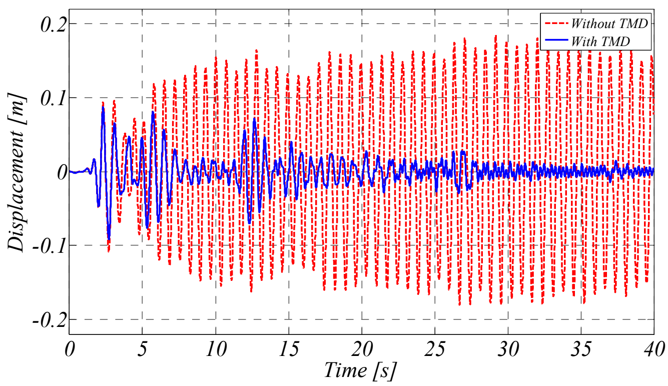

Figure 9. Kinematic response in terms of displacement of the top floor of the three-storey building (ζ_S=0), with and without TMD (μ=5%).

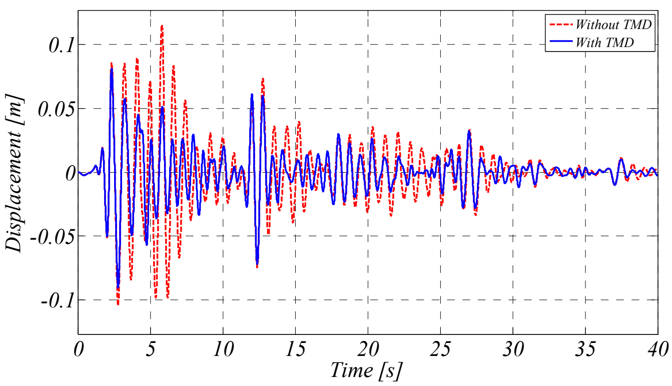

Figure 10. Same as Fig. 9. Six-storey building (ζ_S=5%).

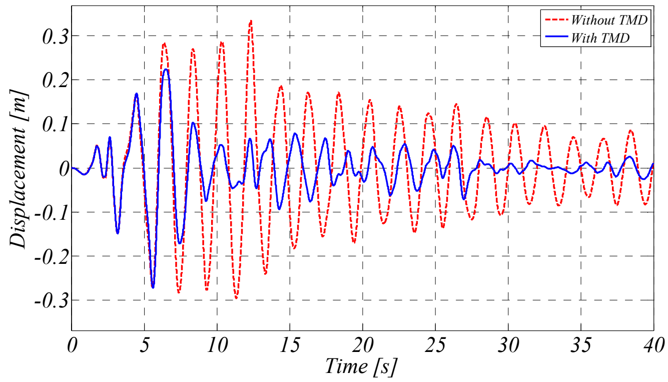

Figure 11. Same as Fig. 9. Ten-storey building (ζ_S=2%).

4 CONCLUSIONS

A novel numerical tuning procedure for obtaining the optimal TMD parameters at given seismic input has been presented. This method consists in a direct application of a Minimax optimization procedure to different specific cases of frame structures, for a single assumed seismic event, so that the tuning optimization turns-out the most efficient for each considered case. The Minimax method has been proven in the literature to provide an efficient and robust numerical method, that also ensures good accuracy together with fast convergence. The algorithm is applied here in a loop that links it to the numerical evaluation of the seismic response in time domain, which is achieved through the integration of the equations of motion by Newmark's average acceleration method.

In order to assess the efficiency of the proposed method, TMD tuning has been performed for several numerical tests carried-out on prototype both SDOF and MDOF frame structures, subjected to a single benchmark seismic event (El Centro, 1940). The average (RMS) displacement of the top storey has been taken as a common objective function for all cases, since this response quantity turns-out to be more efficient in the numerical method, by producing accurate and reliable results. It has been also demonstrated numerically that RMS velocity, acceleration and kinetic energy are as well suitable objective functions, which lead to results that are similar to those obtained by using the RMS displacement as objective function.

The salient results obtained from the numerical tests have been depicted qualitatively in plot form and summarized quantitatively in table form. The seismic response of the structural system has been represented by both kinematic and energy response indicators. The plots have pointed-out the trends of the optimal TMD parameters in the chosen range of mass ratios, which is suitable for civil engineering applications, and the seismic response for a given, typical value of mass ratio, in terms of the displacement of the top storey (and, for the SDOF structure only, also of the kinetic energy of the primary structure). Useful comparisons to Den Hartog's tuning formulas have been also provided in both plots and tables. The obtained results support the main considerations outlined below.

The optimal TMD parameters evaluated for the SDOF structure represent a sort of bilateral validation: on one hand, the so-obtained TMD allows to achieve a sensible reduction of seismic response, from 25% to 45%, with respect to the case without TMD, depending on the considered response quantity. On the other hand, the optimal parameters appear to be in line with those that could be evaluated by standard tuning formulas known in the literature.

The numerical tests performed on the MDOF structures demonstrate the effectiveness of the tuning achieved by the present optimization procedure. One may observe that, as the structural damping ratio increases, the value of the optimal TMD damping ratio decreases and, above all, the TMD efficiency in reducing the seismic response also decreases. Nevertheless, the so-conceived TMD is demonstrated to be quite efficient, since the structural response is heavily reduced (from up to 50% for the six-storey structure with $\zeta_S = 0.05$, to 60% for the ten-storey building with $\zeta_S = 0.02$, to even 90% for the three-storey building with zero damping), thus also in cases of structural damping ratios that are representative of real buildings.

The TMD dynamic response is characterized, on all kinematic indexes, by a much higher response, with respect to that of the main system. This might have significant technological implications in terms of device design, allowable stroke and so on. This is a consequence of two main facts: a) the optimization process has been applied here just on the response quantities of the primary structure; b) the optimal values obtained for the TMD damping ratio turn-out relatively small. This latter consideration implies that a real implemented TMD might need a higher TMD damping ratio, so that it may become possible to mitigate also its dynamic response. However, this leads to a diminution of the achieved gain in dynamic response reduction of the primary structure. Such open topic, which appears quite important, especially on the side of practical design and installation of TMD devices in constructions and mechanical systems, should be further investigated in future developments.

In conclusion, the proposed tuning procedure proofs to yield an efficient tuning of the free TMD parameters. In this sense, the choice of average response quantities as objective functions allows to optimize the TMD performance in the context of seismic engineering applications. Furthermore, the adopted approach shows that, in principle, the best tuning of a TMD device at given seismic input is possible.

An important topic, which should be discussed in future works, is the application of the proposed optimization method in the framework of a frequency domain analysis. Also, further analyses could consider the validity of the trends experienced here for other, bigger and different structures and, most important, for different seismic input. This would allow to assess further if general trends on the tuning parameters could be traced-down in useful terms, as compared to classical tuning formulas, for a most convenient and effective reduction of the dynamic response of civil engineering constructions through the use of TMD devices in the context of seismic engineering.

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