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***Hard Core Cartels and Avoidance of Investigation in the Presence of an
Antitrust Authority***

by

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Hard Core Cartels and Avoidance of Investigation in the Presence of an Antitrust Authority

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Abstract

Hard Core Cartels aim to design, being aware of the presence of an antitrust authority, market practices granting avoidance of antitrust investigations. We show, in a dynamic game, that they can reach this goal and get extra-normal profits. However, the bulk of this opportunity does not lay, here, in limiting price changes across periods (as in Harrington [2004b]), but rather in sending a signal to the authority which has a twofold effect: (1) it does make evident that cartel's members are currently not engaged in an "excessive" degree of collusion, (2) it credibly shows that this moderate collusive activity has a persistence effect, i.e. it will be maintained also in future periods. We also show that antitrust remedies (e.g. behavioral constraints or injunction reliefs) are more powerful, in limiting the collusive activity, than fines. Last, we show that social welfare is higher if *Hard Core Cartels* have limited information about the type of authority (i.e. tough or accommodating) they are facing.

JEL classification: D43, L13, L41.

Keywords: Hard Core Cartels, antitrust authority, antitrust remedies and fines, persistence of collusion.

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1 Introduction

The OECD [2003] has recently released a report on “Hard Core Cartels” confirming that the harm they cause on welfare is very significant and amounts to the equivalent of billions of dollars each year world-wide.¹ This evidence asks, on the one hand, for a tougher antitrust policy; on the other hand, for a deeper understanding of their business strategies explicitly adopted to avoid an antitrust litigation.

Notwithstanding the relevance of these issues, only recently the economic literature has begun to focus the attention on how a cartel, being fully aware that an antitrust authority is active and may review its market conduct, might avoid an investigation. More precisely, researchers have deeply analyzed the process of cartel’s formation (either explicit or tacit) and its internal stability; but they have generally omitted the analysis of a strategic game with two players: a cartel and a public agency (henceforth the authority), where the latter has some devices which may hinder the formation and the development of cartels. We tackle this problem and analyze, by modeling a dynamic game, three distinct issues: (1) the cartel’s sequence of moves adopted to reduce (or even rule out completely) the probability of intervention by the authority. (2) Which instruments, among those available, have a stronger effect on the cartel’s behavior; (3) the role played by informational asymmetries when they favor the authority, i.e. when the cartel does not know which type of authority (tough or accommodating) is in charge of the policy.

We show that the cartel may choose a sequence of moves (e.g. a price sequence) granting avoidance of the authority’s intervention but compatible with extra-normal profits. The sequence, to be effective, must include a signal sent to the authority which has a twofold effect: (1) it does make evident that cartel’s members are currently not engaged in an “excessive” degree of collusion. (2) It credibly shows that such a moderate collusive activity will be maintained *also*

¹ “Hard core cartels” is the usual definition adopted by antitrust officers when they refer to (horizontal) price-fixing cartels. Examples are the lysine, vitamins. graphite electrodes cartels. Their main feature, for the purpose of this paper, is that their members are fully aware of illegality of their behavior, since they adopt strategies that aim to avoid antitrust disputes.

in the future, i.e. collusion has a persistent effect. We label the second effect as the “covenant” factor, i.e. an unwritten gentlemen agreement between cartel’s members.² The persistent effect of collusion is a crucial factor to achieve the no-intervention equilibrium since the authority can levy remedies in the event of collusion yielding social benefits³ only in future periods, i.e. not at the time where the illegal activity is performed and the social damaged is generated. Hence the authority has a strong incentive to intervene today in order to get an higher (discounted) social benefit in future. Only a credible covenant that the cartel will hold next periods the same behavior observed and tolerated today may persuade her from immediate investigation.

Moreover, we show that antitrust laws do increase the social welfare with respect to the *laissez faire* case. The laws have a deterrence power because they avoid that an “excessive” degree of collusion will be performed over time. However, among the traditional instruments available to fight price-fixing (see the discussion in Section 2), we will point out that deterrence is mainly due to behavioral constraints (henceforth BC’s), while fines have only a limited deterrence power. Indeed monetary sanctions are the object of an intense debate within the profession (see Harrington [2004a]) and antitrust practitioners (OECD [2003]), because they are usually much lower than collusive gains (hence they do not represent a credible threat to deter collusion) and they are very difficult to compute optimally (usually it is not possible to identify precisely the starting point and hence the duration of the collusive period, the “but for price” level, etc.). BC’s, known in laws discipline as “antitrust remedies” or “injunction reliefs”, have been less investigated in the literature, even though there exists some empirical evidence (Bizjak and Coles [1995]) displaying that they have a much stronger impact than antitrust fines.⁴ BC’s have both deterrence and desistance power

²Hence no visible proof of conspiracy is given to the authority.

³An increase in the intensity of competition within the industry due to, e.g. the cartel’s breakdown or, more generally, a reduction in the cartel’s conspiracy activity.

⁴They study the implications for shareholders of antitrust litigation in the US and show that the threat of monetary fines (being imposed by the authority or being computed on the basis of damages) has little power to explain the likelihood of settlement, *while the central concern of defendants is the potential prohibition of profitable business practices through injunction reliefs.*

(see Motta and Polo [2003]); desistance arises because the authority, once that firms have been proven guilty, is able to increase the intensity of competition (at least temporarily) by imposing restrictions and remedies on firms' behavior.⁵

Last, we highlight that society benefits of a situation where informational asymmetries favors the authority. More precisely, we show that if there exists a sufficiently high chance of facing a tough authority, the cartel loses some profit levels in comparison with the perfect information case. Moreover, under these circumstances, it may even not be possible to avoid antitrust disputes.

The model presented here has strong links with a collection of recent papers by Harrington [2004a, 2004b, 2005]. He analyzes the impact of antitrust laws on the cartel's behavior in a dynamic model and shows [2005] that the cartel's price path is gradually increasing up to the steady-state level, and that the latter is decreasing in the damage multiple and the probability of detection, while it is independent of the level of fixed fines.⁶ In another paper (Harrington [2004b]) he takes into account the cartel's internal stability (i.e. the incentive compatibility constraints ensuring that cheating is not optimal) and shows that antitrust laws may have perverse effects on cartel's pricing. More specifically, the cartel may price above the steady-state level in the absence of antitrust laws because the latter loose the incentive compatibility constraints associated with collusion. He also achieves an equilibrium where the cartel gets extra-normal profits without incurring in the authority's intervention.

The main intuition underlying Harrington's work on this topic is that cartels anticipate that the detection probability is function of price changes (rather than

⁵Usually guilty firms have to submit reports on their "modified" market strategies and are monitored for a certain period by antitrust officers.

⁶Hence if fines are the only penalty, the antitrust policy has no impact on the cartel's steady-state price. Harrington [2004a] investigates how the "before and after" approach for calculating damages (in the US antitrust practice the damages due to collusion depend upon (1) the observed collusive price, (2) the "but for" price, i.e. the price that would be charged without collusion, (3) the market demand and (4) the length of the collusive period) influences the firms' price path after the cartel has dissolved. He shows that firms price above the standard non-collusive level and this results in an overestimate of the but for price and so an underestimate of the collusive overcharge and antitrust damages.

price levels), and so they design a price sequence that minimize the probability of detection. This paper is an attempt to improve Harrington's works on three issues deserving more research efforts: First, Harrington's models do not consider a *strategic* antitrust agent (e.g. an antitrust authority) whose actions have an impact on the equilibrium. The antitrust side of the model is played by an "automata" identified by an "helicoptered-in" probability of detection. Second, in his papers detection is (exogenously) triggered by price changes: hence by smoothing the price path the cartel may achieve extra-normal profits without being investigated. However, this assumption leads to a peculiar situation where, if the cartel's steady-state price level is reached without detection, firms continue to price at such level forever without being inspected.⁷ Moreover, price levels do matter in the authorities' decisions.⁸ Third, there is an "*ad hoc*" assumption that once investigation is initiated the cartel breaks down, which is in contrast with the empirical evidence.⁹ No formal explanation of this effect is provided.

Other papers have weaker connections with this contribution. Block, Nold and Sidak [1981] study the effect of antitrust laws on prices in a static model where the probability of detection is increasing in the price levels; Salant [1987] anticipates the perverse effect of antitrust laws shown by Harrington [2004b] in a static model where consumers know that they will receive higher reward if cartel's members are induced to increase the degree of collusion; Besanko and Spulber [1989,1990], in a context where firms have private information about production costs, consider the optimal antitrust policy when, respectively, an authority can commit to an ex-ante probability of intervention and when consumers can get treble damage rewards. Souam [1998, 2001] extends Besanko and Spulber [1989] model by investigating the optimal policy when the antitrust authority has imperfect information over a continuous set of cartel's efficiency states. Cyrenne [1999] analyzes the impact of antitrust laws in the Green and Porter [1984] model of

⁷For instance, they will continue to price at the monopoly level without any suspicion of conspiracy by the authority.

⁸Many antitrust disputes start because the authority believes that price levels in a given sector are suspiciously high, rather than because she observe an anomalous price pattern.

⁹Bosch and Eckard [1991] report that of the 1300 firms indicted by the US Department of Justice over 1962–1980, 14% were recidivists.

tacit collusion with price wars.¹⁰ Other papers (Spagnolo [2000], Motta and Polo [2003]) have recently explored the impact of leniency programs on firms' incentives to collude. However they do not consider that the probability of detection depends upon the cartel's pricing behavior.¹¹

The paper is organized as follows: Section 2 reviews the set of instruments available to the authority in the event of horizontal price-fixing and discusses their implications for the modeling of the antitrust game in a dynamic context. Section 3 presents the model adopted in this paper, while Section 4 displays the perfect information case and shows the cartel's market behavior granting avoidance of antitrust intervention. The analysis is then extended to a situation where the cartel does not know the "type" of authority facing it (Section 5). Section 6 identifies the optimal level of BC's; Section 7 summarizes the paper and draws some policy implications, while all the proofs are reported in the Appendix at the end.

2 Fines and BC's: critical discussion and modeling implications

Antitrust authorities have two instruments to fight cartels: fines and remedies (BC's). The magnitude of the former varies according to the importance of the detected illegal behavior and is usually related to profits (especially in the US) or to sales (Euroland).¹² Antitrust fines are often considered, within the profession,

¹⁰Actually Cyrenne ends up to a quite trivial result: since penalties reduce the gains from collusion, a shorter punishment phase, with respect to the *laissez faire* case, is required to avoid deviation during the collusive phase. Hence antitrust laws increase the probability of collusion.

¹¹Motta and Polo have a probability of auditing and a separate probability of detection. The authority chooses between these two instruments given a budget constraint and being the first mover player in order to maximize social welfare. Hence both probabilities do not depend upon the degree of collusion observed in the market.

¹²Souam [1998, 2001] has studied these two fine regimes and has shown that fines related to sales are more efficient, in welfare terms, than fines linked with profits when rents achievable through collusion are not high. Achievable cartel profits are those obtained along the equilibrium path in a model where the public agency in charge of the policy has limited information about the cartel's productive efficiency. When the distance between the most efficient cartel's type and the less efficient one is small, the profit levels achievable through collusion are not high.

too small in comparison with the gains from collusion: the OECD states that, “... on a sample of 11 cartels discovered in different countries, the proportion of sanctions to gains ranged from 3% to 189%. Seven cartels had sanction much lower than the gains while none receives a fine as large as three times the gains, which is considered by many experts to be the optimum level.”¹³ Posner [1976, p. 32] estimates that, on average, fines in the US are 0.21 percent of annual sales involved in the conspiracy, McCutcheon [1997] that they are only about 0.6 percent of annual collusive profits. Harrington [2004a, 2005] shows that fines have no impact on the cartel’s steady-state price level. The OECD recommends to link sanctions to a multiple of the damages suffered by plaintiffs, as in the US private antitrust lawsuits, in order to increase the average fine level. However, damages are quite difficult to compute, so that their level tend always not to be sufficiently high to prevent collusion; moreover, sanctions have an upper bound related to firms limited liability.

Fines present some weaknesses on a theoretical ground too: when the authority objective function is social welfare, fines are a pure monetary transfer from colluding firms to consumers. In this case they do not represent for the authority an incentive to fight price-fixing *once* it has been observed.¹⁴ That is, monetary sanctions are not a *remedy* in the event of collusion; they have only a deterrence power and no desistance effect. Hence, for fines to be a credible threat, two alternative conditions must be meet: (1) the authority has to commit to an ex-ante announced policy (e.g. a 20% parallel price increase by all firms in an industry will surely trigger an investigation). (2) the authority must give different weights to consumer and producer surplus (e.g. her objective function is only consumer surplus).

BC’s are instead prohibitions to perform some illegal acts imposed to firms, and they are usually monitored by the authorities for some time after the final decision.¹⁵ These constraints have a direct effect on firms’ market decisions, since

¹³OECD [2003] p. 28.

¹⁴A fine does not modify the social welfare. Hence even if collusion is detected, the social welfare is the same if antitrust policy is implemented or not.

¹⁵For instance, the final decision might include the prohibition to exchange information about costs, or to issue price lists, or to impose vertical restraints to retailers in order to reduce

they modify, through agency’s monitoring of injunction reliefs, the cartel’s *modus operandi*. Hence BC’s are a remedy in the event of collusion since they increase the post–conviction intensity of competition. As it emerges from Bizjak and Coles [1995], they reduce the profitability of a conspiracy and act as a profit floor.¹⁶

The idea that BC’s have a desistance effect finds confirmation in a rich empirical literature on the consequences of antitrust policy against price–fixing. Feinberg [1980]¹⁷ and [1984]¹⁸ found that capital–adjusted price–cost margins in 1970 were significantly lower, *ceteris paribus*, for firms indicted for price–fixing between 1955 and 1970. The estimated effect on after–indictment prices is lowering the Lerner Index by two percentage points. The same results have been found by Block, Nold and Sidak [1981];¹⁹ they underscored that increases in the Antitrust Division’s inflation-adjusted budget have a significant negative effects on markups of white bread.²⁰ Choi and Philippatos [1983] have obtained the same result in a study on a sample of U.S. large firms indicted for violations of Section I of the Sherman Act between 1958–72, matched with a group of unindicted firms.²¹ Bizjak and Coles [1995], as mentioned before, emphasize that BC’s have

incentive to deviate between colluding firms.

¹⁶Alternatively, we might think that there exists a cost of collusion (e.g. the costs of cartel’s formation and maintenance), as in Bradburd and Over [1982] and in Alexander [1994], and that the antitrust activity increases this cost. Our results apply also to this alternative scenario.

¹⁷He analyzed a sample of U.S. 288 large manufacturing firms.

¹⁸He considers in this study the timing of Antitrust effects on pricing, and identifies two types of effects: the deterrent effects of Antitrust past indictments (firms think that the public agency will screen past offenders), and the firms’ strategic reaction (minimizing the probability of conviction and the expected penalties) to an ongoing investigation and indictment.

¹⁹They studied the effects of Antitrust indictments on prices in the U.S. white bread industry. They look at prices for white pan bread across 208 observations: 12 major cities for 12 years (1965–76) and other major cities for 8 years (1969–76).

²⁰In addition, Department of Justice’s price fixing prosecutions in the bread industry have negative effects on markups in the *region* (and the year) in which the case is filed, and a larger (what they call remedial) negative effect in the city in which the action occurs, *the year following the start of the case*. Their interpretation about these results is that, once discovered and prosecuted, colluding firms remedy by reducing their markups in the following period.

²¹They showed that indicted firms do suffer for a reduction in profits after the indictment, and that this result applies only if the firms are indicted for the first time. Once firms get used to the Antitrust process, they do not care too much about it, and its enforcement power is

strong power to explain the likelihood of settlement in US antitrust disputes.

The first implication of including BC's in the set of the authority's instruments (an issue not yet considered, to the best of my knowledge, in the literature, with the exception of Motta and Polo [2003]) is that the cartel/authority interaction has to be modeled as a dynamic game, since BC's show their effects *after* firms have been sanctioned. The second implication is that, while keeping social welfare as the authority's objective function, it is possible to relax the ex-ante commitment hypothesis.²² By doing so we can model, more realistically, the above interaction as a signaling game, where the authority's intervention is triggered when she believes, after receiving a signal from the market, that the probability of an existing price-fixing cartel is high. The same approach has been recently adopted by Cyrenne [1999], Harrington [2004a, 2004b, 2005], and Martini and Rovesti [2004]; moreover, it is supported by the empirical evidence provided by Hay and Kelley [1974]²³, the OECD [2003], Levenstein and Suslow [2001] and the Italian antitrust experience.²⁴

For the above considerations we choose to model the cartel/authority game as a two-stage/two-period signaling model where in each period firms are the first players to move (deciding whether to form a conspiracy to raise price or not) and then the authority selects her action. This choice warrants further discussion since it does not model the strategic interaction as an infinite horizon game.²⁵

much lower. It seems that the only effect is in this case the monetary fine.

²²Besanko and Spulber [1989], Souam [1998, 2001] and Martin [2000] adopt this hypothesis in their contributions.

²³They find that in the US detection was attributed to a complaint by a customer or a local, state or federal agency in 13 of 49 price-fixing case.

²⁴The well known Nasdaq case was initiated by academics (see Christie and Schultz [1994]). The recent graphite electrodes case began with a complaint from a steel manufacturer, the famous vitamins case by a firm engaged in the conspiracy calling for a leniency program (see Levenstein and Suslow [2001]). In Italy, among the 31 illegal agreements convicted by the Italian authority over 1997–1999, 17 (54,8%) were initiated by private agents. The unique announcement made ex-ante usually consists in what it is declared in the antitrust laws, where agreements made to fix prices are considered illegal (Grillo [2002]). Threshold intervention prices or commitments to a probability of investigation are not announced.

²⁵The results presented in this paper can be reproduced also in a dynamic model where at the first period players have to select their strategies and from the second period a continuation

I believe that it provides a reasonable compromise between tractability and the minimal structure to model dynamic interaction. Indeed a two-stage/two-period model entails the minimum dynamics to analyze the impact of fines *and* BC's on the cartel's behavior.²⁶ Hence the model presented below does not consider the usual (supergame style) individual incentive not to cheat the tacit agreement: it focuses on the impact of antitrust laws on the firms' joint decision to form a cartel and at what level to increase prices.²⁷

3 The model

We consider a two-stage/two-period model where an industry composed by N risk neutral firms produces an homogeneous good, with market demand $p = 1 - q$ ($q = \sum_{i=1}^N q_i$),²⁸ the latter is the same in both periods and is common knowledge. Firms have a common cost function $C_i(q_i) = \theta q_i$ ($i = 1, \dots, N$). As in Besanko and Spulber [1989] and in Souam [1998, 2001], firms compete *à la* Bertrand and decide whether to collude or not both at $t = 1$ and at $t = 2$. If they do not collude, as in a standard finitely repeated game, they replicate in each period the Bertrand equilibrium of the static game, i.e. $p^c = \theta$, $q^c = 1 - \theta$, $q_i^c = \frac{1-\theta}{N}$, and make normal profits.

We assume that firms have to pay a cost if their output is different among the two periods. The aggregate cost of changing production levels is equal to Ω . Such a cost may be due to (1) adjustment costs, (2) collusive costs of changing decision²⁹ (as in Alexander [1994]) or (3) a combination of these two factors.

game takes place, conditional to the first stage decisions.

²⁶Since BC's increase the costs of collusion *after* the conviction, two periods are the minimum requirement to analyze their impact on the cartel's strategy. By contrast, the role of fines may be studied also in a static model, since they have only a deterrent effect.

²⁷The analysis of the impact of antitrust laws in supergame oligopoly model must include other weapons (e.g. leniency programs) and it is left to future research.

²⁸The assumption of linear demand is to simplify the analysis. The results are valid for any specification of market demand yielding a concave profit function and social welfare function.

²⁹If the cartel decides to change the price, members have to meet, spend time to reach a new agreement and so on.

Hence at $t = 2$, cartel's costs are:

$$\begin{cases} \theta q_1 & \text{if } q_2 = q_1 \\ \theta q_2 + \Omega & \text{if } q_2 \neq q_1 \end{cases}$$

If cartels are legal firms maximize industry profits $\pi = (1 - q)q - \theta q$; hence $\frac{d\pi}{dq} = 1 - 2q - \theta = 0$. Solving the latter for q we get the monopoly output $q^m = \frac{1-\theta}{2}$, with q_t (aggregate output at period t , $t = 1, 2$) equal to $q^m \forall t$. We label q^m as the “fully collusive output”. In each period cartel's profits are $\pi(q^m) = \frac{(1-\theta)^2}{4}$, and total profits are $(1 + \delta)\pi(q^m)$, where δ is the discount factor, which is assumed to be the same for the cartel and the authority.

If instead cartels are forbidden, the sequence of events is the following: at $t = 1$ the industry decides whether to collude or not (i.e. it chooses the aggregate output q_1); if a cartel is formed the authority, observing q_1 and knowing market demand, chooses whether to investigate (action $\{i\}$) or not (action $\{ni\}$). If the authority investigates the cartel is convicted to pay a fine $A(q_1) = m[1 - q - \theta]q$ ($m > 1$), i.e. a multiple of its profits, to compensate the damage suffered by consumers.³⁰ Moreover, BC's are imposed to the members, so that the cartel faces a prohibition to adopt profitable business practices and, consequently, suffers of a reduction in its profit possibilities. We assume that BC's operates as a profit floor, so that the maximum feasible post-conviction profits are equal to $\alpha\pi(q^m)$, with $0 \leq \alpha < 1$.³¹ An investigation involves a fixed cost K , which is paid by the authority. At $t = 2$, both the firms and the authority observe the outcome of the first period and decide which actions to perform in the last two-stage period. First firms decide q_2 , then the authority chooses between $\{i\}$ and $\{ni\}$.

The authority's objective function is social welfare $W(q) = \int_0^q p(t)dt - \theta q$. If Bertrand competition takes on at both periods, social welfare is at its maximum, i.e. $(1 + \delta)W(q^c) = (1 + \delta)\frac{(1-\theta)^2}{2}$; if cartels are legal social welfare is instead

³⁰The analysis can be extended to a regime of fine proportional to cartel's sales, as in Souam [2000]; in this case $A'(q) = \phi(1 - q)q$ ($\phi > 0$).

³¹A desistance effect due to BC's may also be obtained by assuming that there exist positive costs of collusion, so that cartel's costs are equal to: $\theta q + c(q^c - q)$. The higher the degree of collusion the lower is q and the higher are the costs of organization, control and maintenance of collusion. BC's increase these costs, e.g. they become equal to: $\theta q + d(q^c - q)$, with $d > c$. Our results also hold under this alternative framework.

$(1 + \delta)W(q^m) = (1 + \delta)\frac{3}{8}(1 - \theta)^2$. In this game a strategy for the cartel is the choice of a pair of output levels $\{q_1, q_2\}$, a strategy for the authority is, in each period, the choice of a single action within the set $\{i, ni\}$. Before computing the optimal policy under perfect information, we state two Lemmas which simplify the analysis.

Lemma 1 *At $t = 2$ the authority's best reply to every cartel's decision is $\{ni\}$.*

Proof: see Appendix.

Lemma 1 shows that in the last two-stage period of the game the authority will never investigate. The unique remedy in the event of collusion at $t = 2$ is the fine (BC's are useless because they have no longer a desistance effect). However, since sanctions are a pure monetary transfer from producers to consumers, social welfare is not modified by the investigation; hence the authority has no *ex-post* incentive to investigate even if collusion is detected.³²

Lemma 2 *If at $t = 1$ $q_1 = q^c$ the authority chooses $\{ni\}$.*

Proof: see Appendix.

The authority knows market demand and so she can spot a competitive output; in this case any investigation is useless, since no fines and BC's can be imposed. Hence producing the competitive output at $t = 1$ and then q^m at $t = 2$ is a permanent option available to the cartel, whose profits amount to, in this case, $\delta\pi(q^m)$. We label it as the "Late Monopoly solution". Clearly, this solution emerges because the dynamic game is truncated at the second period. However a steady-state collusive solution where the cartel is not prosecuted by the authority emerges also in a infinite horizon game (Harrington [2004b]). The only difference

³²Clearly, Lemma 1 holds in the general case where the authority's objective function is social welfare. If instead the authority maximizes consumers' surplus or places different weights on consumers' and firms' surplus (with an higher weight on the former), she may find optimal to intervene also at $t = 2$. Section 5 explores the case where firms have imperfect information about the authority's objective function. The results presented in this Section are sufficiently general, since they have been obtained in a framework where the authority's enforcement power is at its minimum.

here is that at the final period the cartel might get the monopoly profit (and not a lower level as in Harrington [2004b]); however a limitation of the collusive profit under what we have called as the late monopoly solution will not change significantly the results.

4 Perfect information

The aim of this Section is to show the optimal policy in case of perfect information. The extensive form of the game (given Lemmas 1–2) is shown in Figure 1. The cartel makes the first move: at C_1 it chooses between Bertrand competition (q^c) and collusion: in the latter case $q_1 \in]0, q^c[$. Then the authority selects her intervention decision: at A_1 she chooses $\{ni\}$ (by Lemma 1), while at A_2 we still need to investigate what is her optimal option. This ends up stage 1. At the beginning of stage 2 the cartel observes the history of the game and then takes its output decision at the various nodes, anticipating that no investigations will be performed at $t = 2$ (by Lemma 2). At C_2 collusion prevails and q^m is chosen. This is the late monopoly solution path. At C_3 choosing q^m is not always the best option: there exists a trade-off between the profit increase due to an higher degree of collusion and the adjustment costs. Hence also the option $q_2 = q_1$ is considered at C_3 . At C_4 the cartel has been convicted at $t = 1$. Given that BC's have been imposed, it may choose between collusion (but with less profitable practices) and q^c . Since the authority will not investigate at $t = 2$, the choice to collude will prevail as long as $\pi(q_2) - \Omega \geq 0$. We need to identify, to solve the game shown in Figure 1, a Subgame Perfect Equilibrium.

We first compute, by working backwards, the cartel's output level at C_4 . In case of collusion the cartel will set its output in order to achieve exactly the profit floor. We label this output level as the “desistance effect output”, and denote it as \hat{q} , where

$$(1 - \hat{q})\hat{q} - \theta\hat{q} = \frac{\alpha(1 - \theta)^2}{4}$$

Solving the above for \hat{q} we get:

$$\hat{q} = \frac{(1 - \theta)(1 + \sqrt{1 - \alpha})}{2} \tag{1}$$

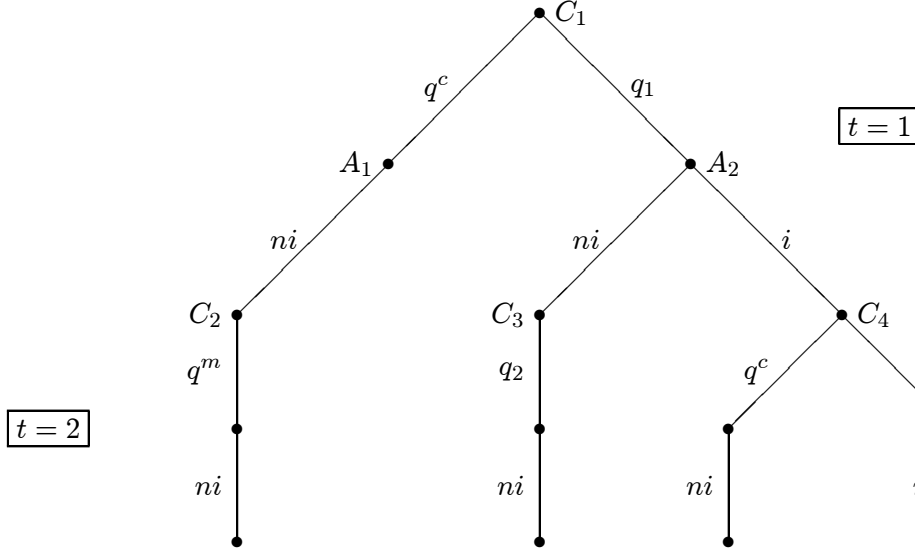


Figure 1: The extensive form of the game with perfect information

with $q^m < \hat{q} < q^c$. We assume that $\pi(\hat{q}) - \Omega \geq 0$, i.e. $\Omega < \frac{\alpha(1-\theta)^2}{4}$. Welfare in this case is:

$$W(\hat{q}) = \frac{(1-\theta)^2}{8} [2(1 + \sqrt{1-\alpha}) + \alpha]$$

Next, we have to identify the cartel decision at node C_3 (Figure 1). Cartel's profits are the following (note that if $q_2 \neq q_1$, by Lemma 1 the Nash equilibrium at C_3 yields $q_2 = q^m$):

$$\pi = \begin{cases} \frac{(1-\theta)^2}{4} - \Omega & \text{if } q_2 \neq q_1 \\ (1-q_1)q_1 - \theta q_1 & \text{if } q_2 = q_1 \end{cases} \quad (2)$$

Changing the collusive output level will then be the cartel's optimal decision if the following condition is satisfied:

$$\underbrace{\frac{(1-\theta)^2}{4} - \Omega}_{q_2=q^m} > \underbrace{(1-q_1)q_1 - \theta q_1}_{q_2 \neq q_1} \quad (3)$$

The r.h.s. of inequality (3) represents the ‘‘covenant’’, i.e. a strategy where the cartel is credibly engaged in keeping its output fixed at $t = 2$. Figure 2 plots condition (3). By inspection the cartel's best reply at C_3 is the following:

$$q_2^* = \begin{cases} q^m & \text{if } 0 \leq q_1 < q_1^2 \text{ or } q_1^1 < q_1 \leq q^c \\ q_1 & \text{otherwise} \end{cases} \quad (4)$$

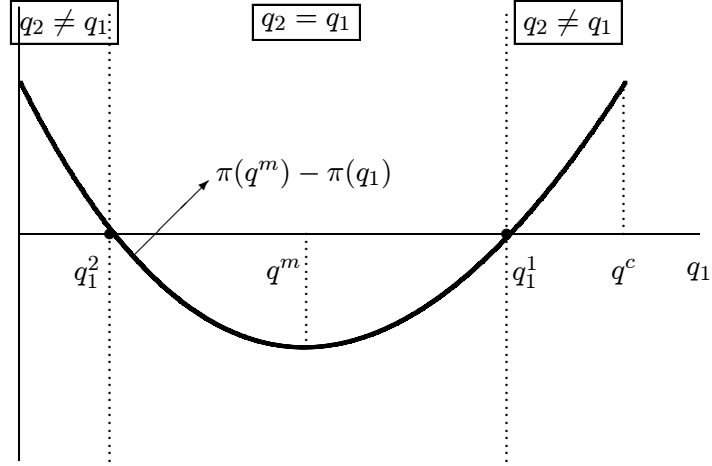


Figure 2: Profits differential at $t = 2$ if $q_2 \neq q_1$

where

$$q_1^1 = \frac{1-\theta}{2} + \Omega^{1/2} \quad \text{and} \quad q_1^2 = \frac{1-\theta}{2} - \Omega^{1/2} \quad (5)$$

Note that if $\Omega \uparrow$ then $q_1^1 \rightarrow q^c$ and $q_1^2 \rightarrow 0$; the higher the adjustment costs, the larger is the interval where the cartel finds optimal to keep the production level fixed at both periods. Meanwhile, the closer is the first-period output to q^c , the higher is the incentive to change the degree of collusion at $t = 2$, setting the monopoly level. The intuition is that the lower is the first-period profit the higher is the incentive to produce q^m at $t = 2$, since the increase in profit outweighs the adjustment costs.

Taking into account the cartel's best reply at C_3 shown in (4), we can now investigate the authority's decision at A_2 (Figure 1), the unique node where the antitrust policy can be implemented under perfect information. If she chooses action $\{i\}$ welfare is $W(q_1) - K + \delta W(\hat{q})$, while if she selects $\{ni\}$ welfare is function of q_1 , as described in (4), i.e.

$$\begin{cases} W(q_1) + \delta W(q^m) & \text{if } 0 \leq q_1 < q_1^2 \text{ or } q_1^1 < q_1 \leq q^c \\ \underbrace{(1 + \delta)W(q_1)}_{\text{"covenant welfare"}} & \text{otherwise} \end{cases} \quad (6)$$

Expression (6) shows that we have two possibilities at node A_2 ; if $0 \leq q_1 < q_1^2$ or $q_1^1 < q_1 \leq q^c$ investigation is a best reply if the following condition holds:

$$\delta[W(\hat{q}) - W(q^m)] > K \Rightarrow \boxed{\text{knife-edge condition}} \quad (7)$$

If instead $q_1^2 \leq q_1 \leq q_1^1$ $\{i\}$ is a best reply if

$$\delta[W(\hat{q}) - W(q_1)] > K \Rightarrow \boxed{\text{strategic condition}} \quad (8)$$

Clearly, (7)–(8) require that $\delta \neq 0$. We label (8) as strategic condition because it depends upon the cartel’s behavior at $t = 1$; hence $\{ni\}$ might be the authority’s best reply if the cartel produces a level of output at the first period such that inequality (8) does not hold, with $q_2 = q_1$ (the “covenant”). We can instead assume that inequality (7) is always fulfilled; if the opposite is true the authority has never an incentive to investigate at $t = 1$.³³ To see why, consider that $\{i\}$ at $t = 1$ yields $W(q_1) - K + \delta W(\hat{q})$, while $\{ni\}$ gives $W(q_1) + \delta W(q^m)$. Then if $\delta[W(\hat{q}) - W(q^m)] \leq K$ at $t = 1$, $\{ni\}$ always dominates $\{i\}$. In this case antitrust policy is a trivial matter. Hence assuming that (7) always holds implies that if the authority observes $0 \leq q_1 < q_1^2$ or $q_1^1 < q_1 \leq q^c$ then $\{i\}$ is always a best reply.³⁴

We have now to identify the q_1 -range where expression (8) is not fulfilled, i.e. where the no-intervention equilibrium might arise. Rewriting it as

$$\delta \left[W(\hat{q}) - (1 - \theta)q_1 + \frac{1}{2}q_1^2 \right] > K$$

³³This assumption rules out the possibility that $q_1 = q^m$; in this case $\{i\}$ dominates $\{ni\}$ if $\delta[W(\hat{q}) - W(q^m)] > K$, which is the same condition presented in expression (7).

³⁴It may seem odd that the authority investigates also if she observes at $t = 1$ an output very close to the competitive level. This is due to her objective function, i.e. overall welfare and not only current welfare. She knows that if cartel’s output at $t = 1$ is close to q^c , given that $\{ni\}$ will prevail at $t = 2$, it will then be limited to q^m at $t = 2$, with a consistent welfare reduction, given that the cartel’s adjustment costs are too small with respect to the profit increase. If instead at $t = 1$ cartel’s output is sufficiently low, the authority might consider the no-intervention option, since the cartel will not choose q^m at $t = 2$: the adjustment costs offset the profit increase.

and solving it for q_1 we get:

$$q_1 = (1 - \theta) \pm \frac{\sqrt{\delta^2(1 - \theta)^2[2(1 - \sqrt{1 - \alpha}) - \alpha] + 8\delta K}}{2\delta} \quad (9)$$

Since no output greater than q^c will be produced we have:

$$\bar{q} = \underbrace{1 - \theta}_{q^c} - \frac{\sqrt{\delta^2(1 - \theta)^2[2(1 - \sqrt{1 - \alpha}) - \alpha] + 8\delta K}}{2\delta} \quad (10)$$

where \bar{q} is defined as the “covenant collusive output”. It is quite easy to show that $\frac{\partial \bar{q}}{\partial \alpha} < 0$, so that an increase in BC’s (i.e. a reduction in α) leads to a tougher profit floor and to an increase in the covenant collusive output. Note that inequality (8) is fulfilled if $0 \leq q_1 < \bar{q}$. However since (8) is true for $q_1^2 \leq q_1 \leq q_1^1$, we must have, for the covenant to be credible, that $q_1^2 \leq \bar{q} \leq q_1^1$.³⁵ Suppose for the moment that there exists an output \bar{q} such that $q_1^2 \leq \bar{q} \leq q_1^1$ (a formal proof will be provided later): hence the authority’s best reply at A_2 is:

$$\begin{cases} \{i\} & \text{if } 0 \leq q_1 < \bar{q} \quad \text{or} \quad q_1^1 < q_1 \leq q^c \\ \{ni\} & \text{if } \bar{q} \leq q_1 \leq q_1^1 \end{cases} \quad (11)$$

Expression (11) highlights an interesting feature of the authority’s behavior at $t = 1$: the intervention decision has a non-monotonic relation with q_1 . It would have been reasonable to expect that the probability of intervention increases as the observed output decreases. We have instead shown that the authority will investigate if the observed output is high or low, while she chooses not to intervene when the output is at an intermediate level. This non-monotonic relation is due to the trade-off between BC’s desistance effect and the “covenant” persistence effect (the same degree of collusion observed today will persist also in the future). If q_1 is very high or too low (and so q^m will prevail at $t = 2$) the authority chooses the desistance effect (she imposes BC’s and so avoids q^m at $t = 2$); if q_1 is at an intermediate level she opts for the persistence effect, i.e. the “covenant” (the investigation costs would not cover the discounted net gains from investigation).

³⁵By inspection of (10) $\frac{\partial \bar{q}}{\partial K} < 0$, and so there exists a level of antitrust’s costs \bar{K} such that $q_1^2 \leq \bar{q} \leq q_1^1$. Hence if $K \geq \bar{K}$ we know that the authority’s best reply at A_2 is $\{ni\}$.

The last decision to analyze is the cartel's choice at C_1 . Three different options are feasible: (1) $\{q_1 = q^c, q_2 = q^m\}$, (2) $\{0 < q_1 < \bar{q}, q_2 = \hat{q}\}$ or $\{q_1^1 < q_1 < q^c, q_2 = \hat{q}\}$ and (3) $\{q_1 = q_2 = \bar{q}\}$.³⁶ Option (1) is the late monopoly solution and yields:

$$\delta[\pi(q^m) - \Omega] \quad (12)$$

i.e. the cartel sacrifices some profits at $t = 1$ to enjoy discounted monopoly profits (net of adjustment costs) later. Under option (2) cartel's overall profits are:

$$\underbrace{(1 - m)}_{<0} \pi(q_1) + \delta[\pi(\hat{q}) - \Omega] \quad (13)$$

while under option (3) (the ‘‘covenant strategy’’) overall profits are:

$$(1 + \delta)\pi(\bar{q}) \quad \Rightarrow \quad \boxed{\text{covenant profits}} \quad (14)$$

We can now state the following Lemma:

Lemma 3 *Producing $\{0 < q_1 < \bar{q}\}$ or $\{q_1^1 < q_1 < q^c\}$ at $t = 1$ (i.e. option (2)) yields positive aggregate profits only if:*

$$0 < q_1 < \frac{1 - \theta}{2} - \frac{\sqrt{(m - 1)^2(1 - \theta)^2 - 4\delta(m - 1)[\pi(\hat{q}) - \Omega]}}{2(m - 1)}$$

or

$$\frac{1 - \theta}{2} + \frac{\sqrt{(m - 1)^2(1 - \theta)^2 - 4\delta(m - 1)[\pi(\hat{q}) - \Omega]}}{2(m - 1)} < q_1 < 1 - \theta$$

with

$$m > \frac{4(1 + \delta)\Omega - (1 - \theta)^2 - 4\delta\pi(\hat{q})}{4\Omega - (1 - \theta)^2} \quad (15)$$

³⁶Note that in the interval $\bar{q} \leq q_1 \leq q_1^1$ cartel's profits are decreasing in q_1 , so that \bar{q} maximizes them.

Proof: see Appendix.

Lemma 3 shows that colluding at $t = 1$ and being investigated by the authority might be profitable. The intuition is the following: even if m is high the effective sanction paid by the cartel is small if at $t = 1$ the collusive output is close to the competitive level (or to 0). Moreover, future net profits are positive (i.e. $\pi(\hat{q}) - \Omega > 0$) and larger than the first period losses. Condition (15) implies that the sanction has to be sufficiently high so that $q_1^1 < \frac{1-\theta}{2} + \frac{\sqrt{(m-1)^2(1-\theta)^2 - 4\delta(m-1)[\pi(\hat{q}) - \Omega]}}{2(m-1)}$. However option (2) is dominated by the other two strategies available at C_1 . Indeed option (1) is better than (2) if $\delta[\pi(q^m) - \Omega] > (1-m)\pi(q_1) + \delta[\pi(\hat{q}) - \Omega]$ (with $0 < q_1 < q_1^2$ or $q_1^1 < q_1 < q^c$), that is if:

$$\underbrace{\delta[\pi(q^m) - \pi(\hat{q})]}_{>0} > \underbrace{(1-m)\pi(q_1)}_{<0}$$

which is always true. Option (3) is better than (2) if $(1+\delta)\pi(\bar{q}) > (1-m)\pi(q_1) + \delta[\pi(\hat{q}) - \Omega]$, i.e. if:

$$\underbrace{\pi(\bar{q}) + \delta[\pi(\bar{q}) - \pi(\hat{q})]}_{>0} > \underbrace{(1-m)\pi(q_1)}_{<0} - \delta\Omega$$

which, again, it is always verified (provided that there exists an output \bar{q} such that $q_1^2 \leq \bar{q} \leq q_1^1$). Hence the only two options that the cartel will consider at C_1 are $\{q_1 = q^c, q_2 = q^m\}$ (late monopoly) and $\{q_1 = q_2 = \bar{q}\}$ (the covenant). We can now establish the equilibrium under perfect information.

Proposition 1 *The authority never investigates in case of perfect information.*

There exist two equilibria:

(1) *the covenant equilibrium, i.e. $\{q_1 = q_2 = \bar{q}\}$, $\{ni, ni\}$, if*

$$\Omega \geq \frac{\left[\delta(1-\theta) - \sqrt{\delta^2(1-\theta)^2 [2(1-\sqrt{1-\alpha}) - \alpha] + 8\delta K} \right]^2}{4\delta^2} \quad (16)$$

(2) *the late monopoly solution, i.e. $\{q_1 = q^c, q_2 = q^m\}$, $\{ni, ni\}$, if condition (16) does not hold.*

Proof: See Appendix.

Proposition 1 shows that in case of perfect information a little degree of collusion is always tolerated even if the authority can impose injunction reliefs that yield a desistance effect. Under the covenant equilibrium (1) the cartel produces an output level that induces the authority not to investigate also at the first period, since this output yields a persistence effect (the same degree of collusion will be maintained at $t = 2$ when antitrust policy has less enforcement power). The intuition is simple: trying to induce the authority not to investigate is better than the late monopoly solution because the covenant equilibrium yields strictly positive short-run gains (choosing the competitive output at $t = 1$ gives normal profit) while it may produce long-run gross losses (i.e. not considering the adjustment costs) only if the discount factor is sufficiently high. However it yields also strictly positive long-run net gains (i.e. taking into account of Ω), so that, if feasible, it always dominates the alternative strategy.

This result confirms Harrington's [2004b] intuition that the cartel may choose *ad hoc* strategies to avoid the possibility of antitrust investigation. However the bulk of this opportunity does not lay, in this case, in limiting price changes across periods, but rather in committing itself to a low degree of collusion *forever*, i.e. in sending the signal that no price changes from the "tolerated" level will ever be observed. Last, Proposition 1 highlights that antitrust policy is effective against collusion even if it is implemented in a discretionary way (i.e. social welfare is higher than in case of *laissez faire*). Its effectiveness is mainly due to BC's: their desistance effect is a key factor affecting the cartel's optimal behavior. This confirms some empirical findings (e.g. Bizjak and Coles [1995]) that BC's are the main concern of defendants in antitrust disputes.

In this Section we have assumed that the policy parameters are exogenous, especially the impact of BC's (the level of α). We have highlighted here that BC's have an impact on the equilibrium, since their level influences the covenant collusive output, and so, in principle, they may cause the covenant equilibrium to fail. Hence it is interesting to analyze which is the level of BC's that maximizes welfare. The latter under the covenant equilibrium is $W^{COV} = (1 + \delta)W(\bar{q})$, while in case of late monopoly is equal to $W^{LM} = W(q^c) + \delta W(q^m)$. The optimal level

of BC's will be analyzed in Section 6.

4.1 An example: calibration of the model

In this Section we provide a calibration of the model, which may be useful to understand the economic intuition of the covenant equilibrium and the impact of some variables, e.g. adjustment and policy costs, that influence the equilibrium outcomes. Table 1 provides a summary of the parameters adopted in this examples and of output, profit and welfare in case of Bertrand equilibrium and monopoly equilibrium (i.e. the “fully collusive output”). We assume that adjustment costs are about 7% of unit costs, that investigation costs are about 16% of the monopoly profit, that BC's limit profits to 20% of the monopoly profit and that both the cartel and the authority have an high discount factor ($\delta = 0.95$).

| | | | | |
|----------------|------------------|---------------------|------------------|--------------------|
| $\theta = 0.3$ | $\Omega = 0.023$ | $K = 0.02$ | $\alpha = 0.2$ | $\delta = 0.95$ |
| $q^c = 0.7$ | $q^m = 0.35$ | $\pi(q^m) = 0.1225$ | $W(q^c) = 0.245$ | $W(q^m) = 0.18375$ |

Table 1: Model's calibration under perfect information

It follows, by substituting the relevant parameters in (1), (5) and (10), that $\hat{q} = 0.6630$, $q_1^1 = 0.5017$, $q_1^2 = 0.1983$, $\bar{q} = 0.4915$, with $\pi(\hat{q}) = 0.0245$, $W(\hat{q}) = 0.2443$, $\pi(\bar{q}) = 0.1025$, $W(\bar{q}) = 0.2233$. Given q_1^1 and q_1^2 , we have that $q_2 = q^m = 0.35$ if $0 < q_1 < 0.1983$ or if $0.5017 < q_1 < 0.7$, while $q_2 = q_1$ if $0.1983 \leq q_1 \leq 0.5017$. The knife-edge condition (7) is fulfilled in this example, since $0.95(0.2443 - 0.18375) = 0.0575 > 0.02$, as well as the strategic condition (8) since $0.95(0.2443 - 0.2233) = 0.021 < 0.02$. Hence from (11) we have that the antitrust policy at $t = 1$ is:

$$\begin{cases} \{i\} & \text{if } 0 \leq q_1 < 0.4915 \quad \text{or} \quad 0.5017 < q_1 \leq 0.7 \\ \{ni\} & \text{if } 0.4915 \leq q_1 \leq 0.5017 \end{cases}$$

Last, if at C_1 the cartel chooses option (1) (late monopoly solution) gets $0.95(0.1225 - 0.023) = 0.094525$, while option (3) (the covenant equilibrium) yields $1.95(0.1025) = 0.1998$, that clearly dominates the former. Note that $W^{COV} = (1 + 0.95)0.223 = 0.435$ while $W^{LM} = 0.245 + 0.95(0.18375) = 0.420$, so that $W^{COV} > W^{LM}$.

5 Limited information on authority's type

The aim of this Section is to explore the impact on the equilibria identified in Proposition 1 of limited information about the type of authority (i.e. “tough” or “accommodating”) the cartel is facing. In particular, we model a tough authority as one with negligible investigation costs (i.e. $K = 0$).³⁷ This implies that a tough authority will always investigate. On the contrary, the authority may incur in positive policy costs ($K > 0$); this type must balance her preference towards a more competitive behavior with her non-negligible policy costs; under these circumstances a covenant equilibrium may arise and so the authority has an accommodating behavior. We define type A as the tough authority ($K = 0$) and type B as the accommodating one ($K > 0$). Moreover we assume that condition (16) holds, so that, when facing a type B authority, the covenant equilibrium ($q_1 = q_2 = \bar{q}$) arises.

We assume that Nature selects the authority's type and reveals it to her, while the cartel has only a (common knowledge) probability distribution over the two possible types, with $\eta = Prob(K = 0)$. Given these assumptions and looking for a Perfect Bayesian Equilibrium (PBE),³⁸ we first highlight that at the last stage of the game the two authority types hold a *separating* behavior: type A chooses $\{i\}$ for $0 < q_2 \leq q^c$, while type B selects $\{ni\}$ for $0 < q_2 \leq q^c$ (by Lemma 1). By moving backwards, the cartel's behavior at $t = 2$ depends upon both its production level at $t = 1$ and the authority's decision once q_1 is observed.

Figure 3 presents the feasible cartel's decisions at $t = 1$ and the corresponding behavior of the two authority types. If $q_1 \in]0, \bar{q}[$ or if $q_1 \in]q_1^1, q^c[$ both types selects $\{i\}$ at $t = 1$ (*pooling* behavior). Hence firms at $t = 2$ still do not know whether they face a tough authority or not. By applying Bayes Rule the posterior

³⁷It is the same to assume that the authority's objective function is given only by consumer surplus.

³⁸It is well known that a PBE satisfies three conditions: (1) for each type i ($i = A, B$) the authority's investigation strategy is a best response to the cartel's strategy, (2) for each authority's moves the cartel's strategy maximizes, given its beliefs about the authority type, its overall profits and (3) for each authority's moves on the equilibrium path, the cartel's belief are consistent with Bayes Rule and with the authority's equilibrium strategy.

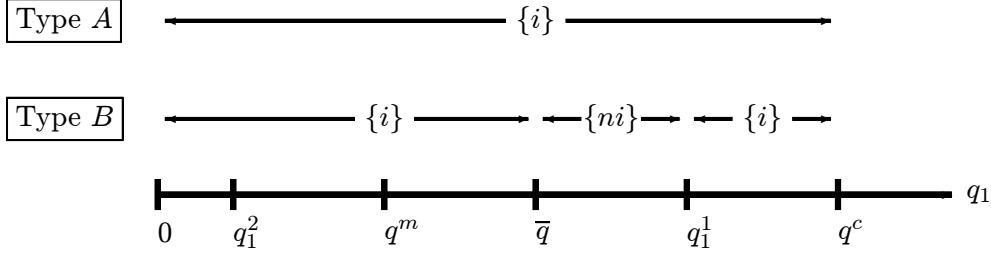


Figure 3: Authority's decisions at $t = 1$: *pooling* and *separating* behaviors

beliefs about the two types coincide with the priors.³⁹ The cartel has, in this case, two options available at $t = 2$: (1) $q_2 = q^c$, (2) $q_2 = \hat{q}$.⁴⁰ Option (1) implies that no collusion takes place at $t = 2$ and firms make normal profits; if $q_2 = \hat{q}$ the cartel will face type A with probability η and so it will be investigated, while with probability $1 - \eta$ it will observe $\{ni\}$ at $t = 2$ (it faces type B). Hence we can state the following Lemma.

Lemma 4 *Choosing $q_2 = \hat{q}$ after $q_1 \in]0, \bar{q}[$ or after $q_1 \in]q_1^1, q^c[$ is profitable only if*

$$\eta \leq \frac{1}{m} \tag{17}$$

Proof: See Appendix.

If (17) holds the cartel's overall profits if $q_1 \in]0, \bar{q}[$ or $q_1 \in]q_1^1, q^c[$ are equal to:

$$\frac{\eta(1 - m)\pi(q_1) + (1 - \eta)(1 - m)\pi(q_1) + \delta(1 - \eta m)[\pi(\hat{q}) - \Omega]}{1}$$

³⁹If we define ϱ_i as the probability that a type i ($i = A, B$) authority chooses $\{i\}$ after $q_1 \in]0, \bar{q}[$ or after $q_1 \in]q_1^1, q^c[$, and $\varpi(A|\{i\})$ as the posterior probability that the authority is of type A after having observed the previous moves, we get, by Bayes Rule,

$$\varpi(A|\{i\}) = \frac{\varrho_A \eta}{\varrho_A \eta + \varrho_B (1 - \eta)} = \frac{(1)\eta}{(1)\eta + (1)(1 - \eta)} = \eta.$$

⁴⁰BC's are imposed at $t = 1$, so the only feasible collusive output at $t = 2$ is \hat{q} .

Rearranging the above expression we get:

$$(1 - m)\pi(q_1) + \delta(1 - \eta m)[\pi(\hat{q}) - \Omega] \quad (18)$$

By applying again the same procedure shown in Lemma 3 it is possible to demonstrate that (18) yields positive overall profits if the following conditions hold:

$$0 < q_1 < \frac{1 - \theta}{2} - \frac{\sqrt{(m - 1)^2(1 - \theta)^2 - 4\delta(m - 1)(1 - \eta m)[\pi(\hat{q}) - \Omega]}}{2(m - 1)}$$

or

$$\frac{1 - \theta}{2} + \frac{\sqrt{(m - 1)^2(1 - \theta)^2 - 4\delta(m - 1)(1 - \eta m)[\pi(\hat{q}) - \Omega]}}{2(m - 1)} < q_1 < 1 - \theta$$

with

$$m > \frac{4(1 + \delta)\Omega - (1 - \theta)^2 - 4\delta\pi(\hat{q})}{4(1 + \delta\eta)\Omega - (1 - \theta)^2 - 4\delta\eta\pi(\hat{q})}$$

If instead (17) is not true the cartel never selects an output $q_1 \in]0, \bar{q}[$ or $q_1 \in]q_1^1, q^c[$ since this strategy yields $(1 - m)\pi(q_1) \ll 0$.

Figure 3 shows that the cartel may choose $q_1 = \bar{q}$ and that the two authority types will respond differently to this choice: type A investigates, while type B selects $\{ni\}$ (*separating* behavior). In this case the behavior of the two types reveals to the cartel whether the authority is tough or not and so the cartel will set $q_2 = q^c$ if $\{i\}$ is observed at $t = 1$ and $q_2 = \bar{q}$ if $\{ni\}$ is implemented at the first stage. The cartel's overall profits are:

$$\eta[(1 - m)\pi(\bar{q}) + 0] + (1 - \eta)(1 + \delta)\pi(\bar{q})$$

which can be written as:

$$[1 + \delta - \eta(m + \delta)]\pi(\bar{q}) \quad (19)$$

Clearly $q_1 = \bar{q}$ is profitable if

$$\eta < \frac{1 + \delta}{m + \delta} = \eta_1 \quad (20)$$

Moreover, Figure 3 points out that also choosing $q_1 \in]\bar{q}, q_1^1]$ induces a *separating* behavior by the two types (type A 's reply is $\{i\}$ while type B responds with $\{ni\}$) and yields the following overall profits (after rearranging them):

$$[1 + \delta - \eta(m + \delta)]\pi(q_1). \quad (21)$$

The latter is, on the one hand, positive only if $\eta < \eta_1$; on the other hand, it is always dominated by $q_1 = \bar{q}$ since $\pi(\bar{q}) \gg \pi(q_1)$.⁴¹ The last option available to the cartel is $q_1 = q^c$ (see Figure 3). Again the two types have a *separating* behavior and so the cartel's overall profits are:

$$\eta(0) + (1 - \eta)\delta[\pi(q^m) - \Omega] \quad (22)$$

To sum up, the cartel's moves at $t = 2$ are the following ones:

$$\left\{ \begin{array}{l} q_2 = q^m \quad \text{if } q_1 = q^c \text{ and } \{ni\} \\ q_2 = \bar{q} \quad \text{if } q_1 = \bar{q} \text{ and } \{ni\} \\ q_2 = q_1 \quad \text{if } q_1 \in]\bar{q}, q_1^1] \text{ and } \{ni\} \\ q_2 = \hat{q} \quad \text{if } q_1 \in]0, \bar{q}[\text{ or } q_1 \in]q_1^1, q^c[\text{ and } \eta \leq \frac{1}{m} \\ \\ q_2 = q^c \quad \text{if } \left\{ \begin{array}{l} q_1 \in]0, \bar{q}[\text{ or } q_1 \in]q_1^1, q^c[\text{ and } \eta > \frac{1}{m} \\ q_1 = \bar{q} \text{ and } \{i\} \\ q_1 \in]\bar{q}, q_1^1] \text{ and } \{i\} \\ q_1 = q^c \text{ and } \{i\} \end{array} \right. \end{array} \right.$$

We are now in the position to state the PBE if asymmetric information favors the authority.

Proposition 2 *In case of limited information on authority's type there exist two fully separating PBE:*

(i) if $\eta \leq \eta_2$, where

$$\eta_2 = \frac{\delta[\pi(q^m) - \Omega] - (1 + \delta)\pi(\bar{q})}{\delta[\pi(q^m) - \Omega] - (m + \delta)\pi(\bar{q})}; \quad (23)$$

⁴¹By comparing (19) and (21) it is evident that the former dominates the latter if $\pi(\bar{q}) > \pi(q_1)$. But this is always true since $\pi(\cdot)$ is a concave function and $q^m < \bar{q} < q_1$, given that $q_1 \in]\bar{q}, q_1^1]$.

then $q_1 = \bar{q}$,

$$q_2 = \begin{cases} \bar{q} & \text{if } \{ni\} \text{ at } t=1 \\ q^c & \text{if } \{i\} \text{ at } t=1 \end{cases}$$

type *A* selects $\{i\}$ at both periods, type *B* chooses $\{ni\}$ at both periods;

(ii) if $\eta > \eta_2$, then $q_1 = q^c$,

$$q_2 = \begin{cases} q^m & \text{if } \{ni\} \text{ at } t=1 \\ q^c & \text{if } \{i\} \text{ at } t=1 \end{cases}$$

type *A* selects $\{i\}$ at both periods, type *B* chooses $\{ni\}$ at both periods.

Proof: See Appendix.

Proposition 2 points out that the covenant equilibrium may be implemented in this limited information context only if the probability that the cartel is facing a tough authority is sufficiently low and if the cartel observes that the authority does not investigate at \bar{q} . On the contrary, an investigation once that \bar{q} is observed reveals to the cartel that it is facing the type *A* authority, and so any credible covenant will be ineffective. Under these circumstances the cartel's best reply at $t = 2$ is to unfasten the agreement and produce q^c . We label PBE^i as the “covenant with intervention” equilibrium, since even if the probability of facing a tough authority is low, we might observe an antitrust investigation along the equilibrium path.

Moreover Proposition 2 highlights that also the late monopoly equilibrium is not always available here, since the tough authority always investigates. This implies that delaying the formation of the cartel at $t = 2$ is a feasible choice only if no investigation is observed at $t = 1$ in presence of a non-cooperative equilibrium outcome. We label this equilibrium as PBE^{ii} , and in this case, if the authority is tough, a price-fixing conspiracy is never realized.

The above Proposition shows that the optimal cartel's strategy at $t = 1$ has a “bang-bang” property, as shown in Figure 4: if the probability of facing a tough authority is small, the cartel will try to induce the covenant equilibrium. If instead the opposite is true, a first-best outcome is reached at the first period, and, moreover, it may be possible to achieve complete deterrence. This result

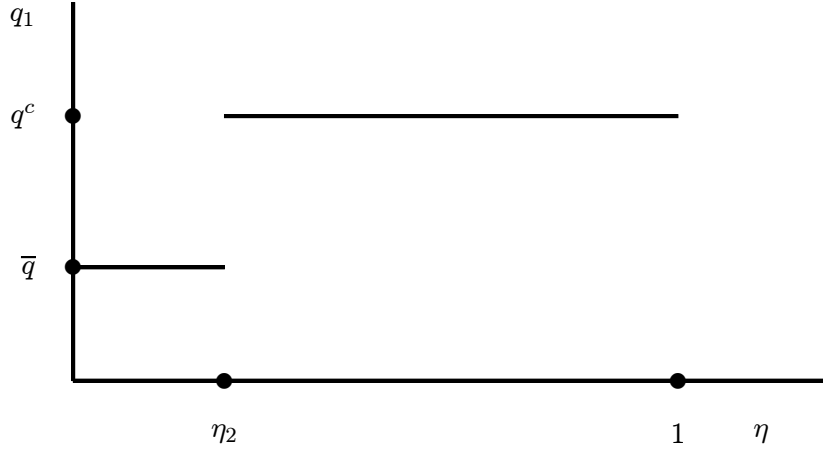


Figure 4: Cartel's output at $t = 1$ with limited information

underscores that the authority can benefit from the presence of uncertainty about her opportunity costs in fighting collusion: a sufficiently high chance that the authority has strong preferences towards marginal costs pricing will force the cartel to lose some profits, since in case of perfect information the outcome would always have been $q_1 = q_2 = \bar{q}$, i.e. a little degree of collusion would be tolerated.

If PBE^i arises, cartel's profits are lower than under perfect information, while welfare is equal to: $W^i = [1 + \delta(1 - \eta)]W(\bar{q}) + \eta\delta W(q^c)$, which is always greater than that obtained in case of perfect information, since:

$$W^i = [1 + \delta(1 - \eta)]W(\bar{q}) + \eta\delta W(q^c) > (1 + \delta)W(\bar{q}) = W^{COV}$$

given that $W(q^c) \gg W(\bar{q})$. Furthermore under PBE^{ii} cartel's profits are again lower than those arising in case of perfect information, while welfare is equal to: $W^{ii} = (1 + \delta\eta)W(q^c) + \delta(1 - \eta)W(q^m)$. The following Lemma states the ranking between W^{ii} and W^{COV} .

Lemma 5 $W^{ii} \geq W^{COV}$ if (i) $W^{COV} < W^{LM}$; otherwise if (ii) $\eta \leq \eta_3$, where

$$\eta_3 = \frac{(1 + \delta)W(\bar{q}) - [W(q^c) + \delta W(q^m)]}{\delta[W(q^c) - W(q^m)]} \quad (24)$$

Proof: See Appendix.

Hence under both the feasible equilibria arising with limited information welfare is greater than that prevailing under the perfect information covenant equilibrium. This implies that society always benefits if asymmetric information favors the authority.

5.1 Model's calibration under limited information on authority policy costs

In this Section we extend the example discussed in Section 4.1 to the limited information case. In case of perfect information we have: $q_1 = q_2 = 0.491$, $\pi = 0.1998$, $W = 0.435$ (see Table 1 and Section 4.1). From (20) and from (23) we have that $\eta_1 = 0.494$ and $\eta_2 = 0.339$. Hence by Proposition 2 we have that PBE^i arises if $0 \leq \eta \leq 0.339$ and that PBE^{ii} prevails if $0.339 < \eta \leq 1$. Under PBE^i we have: $q_1 = 0.491$ while

$$q_2 = \begin{cases} 0.491 & \text{if } \{ni\} \text{ at } t = 1 \\ 0.7 & \text{otherwise} \end{cases}$$

Under PBE^{ii} we have instead: $q_1 = 0.7$ and

$$q_2 = \begin{cases} 0.35 & \text{if } \{ni\} \text{ at } t = 1 \\ 0.7 & \text{otherwise} \end{cases}$$

| | Perfect information | Limited Information |
|-------|---------------------|---|
| π | 0.1998 | 0.1998–0.062 ($\eta = 0$)–($\eta = 0.339$) |
| | | 0.062–0 ($\eta = 0.34$)–($\eta = 1$) |
| W | 0.435 | 0.435–0.439 ($\eta = 0$)–($\eta = 0.339$) |
| | | 0.439–0.478 ($\eta = 0.34$)–($\eta = 1$) |

Table 2: Model's calibration under limited information on authority's type

Table 2 shows how cartel's overall profits and welfare vary according to η and to the prevailing equilibrium, and it compares them with the perfect information

case. It is evident that as long as $\eta \neq 0$ cartel's profits are lower than those arising with perfect information, while welfare is higher.

6 Optimal level of BC's

The aim of this Section is to identify the level of BC's that maximizes welfare both in case of perfect and asymmetric information. The previous analysis has highlighted that (1) the cartel may induce a solution where the authority never investigates by making a credible covenant (i.e. the observed collusive solution is persistent) and that (2) the authority can improve the efficiency if she has an informational rent. However in the previous Sections the level of BC's was exogenous: now we relax this assumption and compute endogenously their optimal level. This implies a change in the timeline of the game: the authority makes the first move by choosing the profit floor in case of conviction, and strictly commits to it. The remaining of the game is as before. First we compute the optimal level of α in case of perfect information.

6.1 The perfect information case

From (10) we have that

$$W(\bar{q}) = \frac{(1-\theta)^2}{8} [2(1 + \sqrt{1-\alpha}) + \alpha] - \frac{K}{\delta}$$

so that under the covenant equilibrium welfare is:

$$W^{COV} = (1 + \delta) \left\{ \frac{(1-\theta)^2}{8} [2(1 + \sqrt{1-\alpha}) + \alpha] - \frac{K}{\delta} \right\} \quad (25)$$

In absence of BC's, i.e. $\alpha = 1$, welfare under the covenant equilibrium is:

$$W^{COV}|_{\alpha=1} = (1 + \delta) \left[\frac{3}{8}(1-\theta)^2 - \frac{K}{\delta} \right] \quad (26)$$

while in case of maximum BC's, i.e. $\alpha = 0$, we have that:

$$W^{COV}|_{\alpha=0} = (1 + \delta) \left[\frac{(1-\theta)^2}{2} - \frac{K}{\delta} \right] \quad (27)$$

It is easy to show that $W^{COV}|_{\alpha=0} \gg W^{COV}|_{\alpha=1}$, while

$$\frac{\partial W^{COV}}{\partial \alpha} = (1 + \delta) \frac{(1 - \theta)^2}{2} \left(1 - \frac{1}{\sqrt{1 - \alpha}} \right) < 0$$

and

$$\frac{\partial^2 W^{COV}}{\partial \alpha^2} = -\frac{(1 + \delta)(1 - \theta)^2}{1 + \alpha^{3/2}} < 0$$

so that welfare under the covenant equilibrium is decreasing in α and its maximum is at $\alpha = 0$ (corner solution). However, since by condition (16) in Proposition 1, BC's have an impact on \bar{q} and so on the existence of a covenant equilibrium, it is now useful to rewrite (16), in order to highlight the role of the policy parameters (instead of the cartel's adjustment costs):

$$K \geq \frac{\delta\{[(1 - \theta) - 2\Omega^{1/2}]^2 - (1 - \theta)^2[2(1 - \sqrt{1 - \alpha}) - \alpha]\}}{8} \equiv \Lambda \quad (28)$$

Expression (28) points out that there exists a trade-off between the two relevant policy parameters (i.e. K and α), so that a tougher level of BC's (a reduction in α) must not yield that $K < \Lambda$ if the authority also wants a covenant equilibrium to prevail. Otherwise, since an output $\bar{q} \leq q_1^1$ no longer exists, the cartel will select the late monopoly solution. Note that:

$$\Lambda(\alpha = 0) = \frac{\delta[(1 - \theta) - 2\Omega^{1/2}]^2}{8} \equiv \chi \quad (29)$$

and that

$$\Lambda(\alpha = 1) = \frac{\delta\{[(1 - \theta) - 2\Omega^{1/2}]^2 - (1 - \theta)^2\}}{8} \equiv \Upsilon \quad (30)$$

with

$$\frac{\partial \Lambda}{\partial \alpha} = -\frac{\delta(1 - \theta)^2}{8} \left(\frac{1}{\sqrt{1 - \alpha}} - 1 \right) < 0$$

$$\frac{\partial^2 \Lambda}{\partial \alpha^2} = -\frac{\delta(1 - \theta)^2}{16} \frac{1}{(1 - \alpha)^{3/2}} - 1 < 0$$

Moreover, $\chi > \Upsilon$, since it is easy to show that $\chi \gg 0$ while $\Upsilon \ll 0$. Hence condition (28) is a decreasing function of α , i.e. the higher the BC's level (the lower is α) the higher must be the policy costs K . Furthermore, if $\alpha = 1$, since $\Upsilon \ll 0$, we have that $K \geq \Upsilon$ is always satisfied. To identify the α -range where a covenant equilibrium exists, we set $A \equiv 1 - \theta - 2\Omega^{1/2}$ and rewrite inequality (28) as follow:

$$\frac{8K}{\delta} \geq A^2 - 2(1 - \theta)^2 + 2(1 - \theta)^2\sqrt{1 - \alpha} + (1 - \theta)^2\alpha$$

The following Lemma identifies the interval where BC's induce a covenant equilibrium.

Lemma 6 *A covenant equilibrium as function of BC's exists $\forall \alpha \in [0, 1]$ if $K \geq \chi$; otherwise it exists if $\alpha' \leq \alpha \leq 1$, where*

$$\alpha' \equiv 1 - \frac{(1 - \theta - \sqrt{H})^2}{(1 - \theta)^2} \quad (31)$$

and $H \equiv A^2 - \frac{8K}{\delta}$.

Proof: See Appendix.

Proposition 1 points out that we can also have a late monopoly solution: welfare in this case is independent of α , since an investigation is never a credible threat along the equilibrium path, and it is equal to:

$$W^{LM} = \frac{(1 - \theta)^2}{2} + \delta \frac{3}{8}(1 - \theta)^2 = \frac{(1 - \theta)^2}{8} (4 + 3\delta) \quad (32)$$

We can now state when the covenant equilibrium dominates the late monopoly solution in welfare terms.

Proposition 3 *Welfare is higher under the covenant equilibrium than under the late monopoly solution if (1) policy costs are sufficiently small, (2) the discount factor is sufficiently high and (3) BC's are not too soft, i.e. iff*

$$0 < K \leq \frac{(1 - \theta)^2}{16} \equiv \Phi, \quad (33)$$

$$\delta_2 \leq \delta \leq 1 \quad (34)$$

where

$$\delta_2 \equiv \frac{4K + 2\sqrt{2K[2K + (1 - \theta)^2]}}{(1 - \theta)^2} \quad (35)$$

and if

$$0 \leq \alpha \leq \bar{\alpha} \quad (36)$$

where

$$\bar{\alpha} \equiv 1 - (1 - \sqrt{1 - Z})^2 \quad (37)$$

and

$$Z \equiv \frac{8K}{\delta(1 - \theta)^2} + \frac{4 + 3\delta}{1 + \delta} - 3. \quad (38)$$

Proof: See Appendix.

Proposition 3 points out that in case of perfect information welfare is greater under the covenant equilibrium than under the late monopoly solution if policy costs are sufficiently small, the authority is sufficiently patient and BC's are not too soft. The condition regarding the level of the policy costs is more stringent: if (33) is not satisfied then the late monopoly solution is better than the covenant equilibrium independently of the discount factor and of the BC's levels. The intuition is the following: an increase in K leads to a reduction in the covenant collusive output, so that the covenant equilibrium becomes more costly for society. On the contrary, policy costs have no effect on the late monopoly solution, and so the higher is K the more likely is that the authority prefers a competitive solution at the first period and full collusion in the future rather than a costly limited degree of collusion.

However, Proposition 3 illustrates the ranking between the two possible equilibria but understates that the prevailing equilibrium is chosen by the cartel, having seen the ex-ante announced level of BC's. In other words, the late monopoly

solution may dominates the covenant equilibrium in welfare terms but the latter is the prevailing one (and vice versa), because the cartel chooses it since it is more profitable. We have in this case a third–best solution.⁴² Hence it is interesting to analyze whether the authority can commit herself to a level of α that modifies the decentralized equilibrium and that restores the second–best welfare (i.e. the ranking identified in Proposition 3). To get this result it is useful to highlight the relation between χ and Φ , and between $\bar{\alpha}$ and α' .

Lemma 7 $\chi < \Phi$ iff

$$\delta < \frac{(1 - \theta)^2}{2[(1 - \theta) - 2\Omega^{1/2}]^2} \quad (39)$$

Moreover, $\bar{\alpha} < \alpha'$ if $K \geq \chi$; otherwise iff $K < \Gamma$ and

$$\delta > \frac{A^2}{(1 - \theta)^2 - A^2} \quad (40)$$

where

$$\Gamma \equiv \frac{\delta[\delta(1 - \theta)^2 - (1 + \delta)A^2]}{7(1 + \delta)} \quad (41)$$

Proof: See Appendix.

We can now state the optimal policy for BC's in case of perfect information.

Proposition 4 *In case of perfect information BC's are always effective since they modify the decentralized equilibrium. The optimal level of BC's is: $\alpha^* = \alpha'$ if (39) holds and*

$$\{K < \chi \leq \Phi \text{ (or } K \leq \Phi < \chi), \delta_2 \leq \delta \leq 1\}; \quad (42)$$

otherwise $\alpha^ = 0$. Last, a third best welfare is achieved (i.e. a second best welfare cannot be restored) either when $W^{COV} < W^{LM}$ but the covenant equilibrium exists independently of α or when $W^{COV} \geq W^{LM}$ but $\bar{\alpha} \ll \alpha'$.*

⁴²First–best is achieved at marginal cost pricing, while second–best is identified by Proposition 3.

Proof: See Appendix.

Proposition 4 shows that even if the authority can precommit to a BC's level, she is not always able to achieve a second best outcome pricing, even if the adoption of BC's always allow to change the decentralized equilibrium, i.e. BC's have indeed a direct effect on the cartel's decisions. A second best welfare can be achieved either when $W^{LM} > W^{COV}$ independently of the BC's level and setting them at a tough level the authority destroy the covenant equilibrium (that will instead prevail if α is sufficiently high); or when $W^{COV} \geq W^{LM}$ if α is sufficiently low and so the authority reaches this goal by fixing the toughest level of BC's compatible with the existence of a covenant equilibrium (i.e. either $\alpha^* = 0$ or $\alpha^* = \alpha'$). Antitrust policy achieves a third best welfare either if $W^{COV} < W^{LM}$ but it is not possible to destroy the covenant equilibrium or if $W^{COV} \geq W^{LM}$ but the cartel requires, to implement a covenant equilibrium, a level of BC's sufficiently low, that the authority is not willing to grant. Now we analyze the optimal level of BC's in case of limited information about the type of authority facing the cartel.

6.2 The case with limited information on authority's policy costs

If the cartel does not know whether the authority is tough or accommodating the timing of the game is the following: First the authority announces the level of α and commits to it, then the Nature selects the authority's type and communicates it to the authority, then the game proceeds as in Section 5. We assume that BC's are regarded by the authority, independently of her type, as a general antitrust guideline, and so there exists a unique level of α . The two types have instead a different attitude toward investigation, as explained in Section 5.

The PBE^i (see Proposition 2) yields the following expected welfare: $W^i = [1 + \delta(1 - \eta)]W(\bar{q}) + \eta\delta W(q^c)$. The latter can be written as

$$W^i = [1 + \delta(1 - \eta)] \left\{ \frac{(1 - \theta)^2}{8} [2(1 + \sqrt{1 - \alpha} + \alpha)] - \frac{K}{\delta} \right\} \quad (43)$$

which depends upon α . Moreover, in absence of BC's welfare is:

$$W^i \Big|_{\alpha=1} = [1 + \delta(1 - \eta)] \left[\frac{3}{8}(1 - \theta)^2 - \frac{K}{\delta} \right] \quad (44)$$

If instead BC's are very tough welfare under PBE(i) is equal to:

$$W^i|_{\alpha=0} = [1 + \delta(1 - \eta)] \left[\frac{1}{2}(1 - \theta)^2 - \frac{K}{\delta} \right] \quad (45)$$

As in the perfect information case, it is easy to show that $W^i|_{\alpha=0} \gg W^i|_{\alpha=1}$ and that $\frac{\partial W^i}{\partial \alpha} < 0$, $\frac{\partial^2 W^i}{\partial \alpha^2} < 0$. Again, expression (28) represents the condition for existence of PBE^i as function of the policy parameters K and α , and the thresholds χ and Υ play a crucial role in the identification of the optimal policy; last, Lemma 6 still hold. The PBE^{ii} (again from Proposition 2) gives instead rise to $W^{ii} = (1 + \delta\eta)W(q^c) + \delta(1 - \eta)W(q^m)$, which can be written as:

$$W^{ii} = \frac{(1 - \theta)^2}{8} (4 + 3\delta + \delta\eta) \quad (46)$$

which is, in this case, independent of α . Tedious computations allow to set that $W^{ii} \gg W^i|_{\alpha=1}$ since their comparison yields the following, always fulfilled, inequality:

$$\frac{(1 - \theta)^2}{8} (1 + 3\delta\eta) > -[1 + \delta(1 - \eta)] \frac{K}{\delta}$$

If we compare instead $W^i|_{\alpha=0}$ and W^{ii} we get that the former is greater than the latter if the following inequality is satisfied:

$$\frac{\delta(1 - \theta)^2(5\eta - 1)}{8} < -\frac{1 + \delta(1 - \eta)}{\delta} K \quad (47)$$

Since the l.h.s. of the above inequality is positive or equal to 0 if $\eta > \frac{1}{5}$, while the r.h.s. is negative, we can state in the following Proposition the ranking between the two equilibria.

Proposition 5 *If the cartel has limited information about the type of authority is facing, $W^{ii} > W^i \forall \alpha \in [0, 1]$ if $\eta \geq \frac{1}{5}$. If instead $\eta < \frac{1}{5}$ then $W^i \geq W^{ii}$ if (1) policy costs are sufficiently small, (2) the discount factor is sufficiently high and (3) BC's are not too soft, i.e. iff*

$$K \leq \frac{(1 - \theta)^2(1 - 5\eta)}{8(2 + \eta^2 - 2\eta)} \equiv \Sigma \quad (48)$$

$$\delta_2^{LI} \leq \delta \leq 1 \quad (49)$$

where

$$\delta_2^{LI} \equiv \frac{4(1-\eta)K + 2\sqrt{2K[2K(1-\eta)^2 + (1-\theta)^2(1-5\eta)]}}{(1-\theta)^2(1-5\eta)} \quad (50)$$

and if

$$0 \leq \alpha \leq \bar{\alpha}^{LI} \quad (51)$$

where

$$\bar{\alpha}^{LI} \equiv 1 - (1 - \sqrt{1 - J}) \quad (52)$$

and

$$J \equiv \frac{8K}{\delta(1-\theta)^2} + \frac{4 + 3\delta + \delta\eta}{1 + \delta(1-\eta)} - 3 \quad (53)$$

Proof: See Appendix.

The above Proposition points out that the probability of facing a tough authority plays a key role (in setting the ranking between the covenant equilibrium and the late monopoly solution) when the cartel has limited information about the type of authority in charge of antitrust policy. If this probability is sufficiently high then the expected welfare under PBE^{ii} is always greater than under PBE^i . The intuition is that a high probability of facing a tough authority increases the probability that the cartel will never be engaged in price-fixing and so it raises up the expected welfare under PBE^{ii} . On the contrary, an equilibrium where the covenant output is sometimes observed along the equilibrium path might dominates W^{ii} only if η is small, and if, as in the perfect information case, policy costs are small, the discount factor is high and BC's are sufficiently tough.

As in the previous case, to compute the optimal BC's policy, we need to identify the relationships between χ and Σ and between α' and $\bar{\alpha}^{LI}$, shown in the following Lemma.

Lemma 8 $\chi \leq \Sigma$ iff

$$\delta \leq \frac{1 - 5\eta}{2 + \eta^2 - 2\eta} \frac{(1 - \theta)^2}{A^2} \quad (54)$$

while $\bar{\alpha}^{LI} < \alpha'$ if $K \geq \chi$; otherwise iff $K \leq \Gamma^{LI}$ and

$$\delta > \frac{A^2}{(1 - \theta)^2(1 - 5\eta) - (1 - \eta)A^2} \quad (55)$$

where

$$\Gamma^{LI} \equiv \frac{\delta\{\delta(1 - 5\eta) - [1 + \delta(1 - \eta)]\}A^2}{7[1 + \delta(1 - \eta)]} \quad (56)$$

Proof: See Appendix.

Now we can state the optimal level of BC's under limited information.

Proposition 6 *In case of limited information about the authority's type, BC's are neutral in welfare terms when $\eta > \eta_2$, since a PBE(*i*) does not exist. If instead $\eta \leq \eta_2$ then BC's are effective in modifying the decentralized equilibrium. The optimal level of BC's is: $\alpha^* = \alpha'$ if $\eta \leq \eta_2 < \frac{1}{5}$ and*

$$\{\Gamma^{LI} \leq K < \chi \leq \Sigma \text{ and (55) holds}\}; \quad (57)$$

otherwise $\alpha^ = 0$. Last a third best welfare is achieved (i.e. a second best outcome cannot be restored), as in the perfect information case, either when $W^i < W^{ii}$ but PBE(*i*) exists independently of α , or when $W^i \geq W^{ii}$ but $\bar{\alpha} \ll \alpha'$; furthermore it is achieved when $\frac{1}{5} \leq \eta \leq \eta_2$ and $K \geq \chi$ (while a second best outcome is restored if $\frac{1}{5} \leq \eta \leq \eta_2$ and $K < \chi$).*

Proof: See Appendix.

Proposition 6 points out that the main difference in the optimal BC's level between the perfect and limited information cases is that in some circumstances under the latter regime BC's are welfare neutral. This happens when the probability of facing a tough authority is high, so that a covenant equilibrium under limited information does not exist. Hence the welfare gains due to the authority's informational advantage make redundant the adoption of BC's.

6.3 Model’s calibration: optimal level of BC’s under perfect information

Under the values of the parameters shown in Table 1 but keeping α as a variable, we have that: $\Lambda = -0.098 + 0.116\sqrt{1-\alpha} + 0.058\alpha$, $\chi = 0.018$, $\Upsilon = -0.039$, $W^{LM} = 0.042$, $A = 0.4$ and $H = -0.011$. Hence since $K = 0.02$ and so $K > \chi$, we have, by Lemma 6, that a covenant equilibrium exists $\forall \alpha \in [0, 1]$. Furthermore, we have that $\Phi = 0.031$ and $\delta_2 = 0.758$, so that, since $K < \Phi$, (33) holds. Moreover, since $\delta = 0.95$, $\delta > \delta_2$ and (34) holds as well. Last, $\bar{\alpha} = 0.614$, so that $W^{COV} \geq W^{LM}$ if $0 \leq \alpha \leq 0.614$. Hence by Proposition 4 (note that $\chi < \Phi$ so that (39) holds, but $\chi < K < \Phi$) $\alpha^* = 0$. Under these circumstances the authority can implement a second best welfare (if $\alpha > 0.614$ a late monopoly solution would arise and a third best welfare would be achieved) and $W^i = 0.437 > 0.435 = W^{COV}(\alpha = 0.023)$.

7 Summary and conclusions

In this paper we have analyzed how a *Hard Core Cartel* can design a market conduct avoiding the appearance of an agreement and, consequently, of incurring in antitrust sanctions and remedies. We have shown that the cartel achieves this goal by choosing a sequence of prices making evident that it is not engaged in an excessive degree of collusion and that the latter will also persist in future periods. We have labeled this conduct as “covenant”, i.e. a cartel’s informal obligation displaying to the antitrust authority that the current “low” collusive activity has a persistent effect. This result confirms Harrington’s [2004b] intuition that the cartel may choose *ad hoc* strategies to avoid the possibility of antitrust investigation. However, the bulk of this opportunity does not lay, as in Harrington [2004b], in limiting price changes across periods, but rather in sending the signal that no price changes will ever be observed. Moreover, we have investigated a framework where the cartel has limited information about the type of authority in charge of antitrust policy, i.e. whether the authority is “tough” (e.g. her objective function is only consumer surplus) or “accommodating”. We have shown that a sufficiently high chance of facing a tough authority will force the cartel to lose some profit

levels in comparison with the perfect information case, and will induce an equilibrium path where it is not always possible to avoid the antitrust intervention. This implies that society benefits if asymmetric information favors the authority. Last, we have also considered the possibility that the authority can precommit to an ex-ante announced level of behavioral constraints (i.e. antitrust remedies or injunction reliefs). We have shown that in this case the authority can modify the decentralized equilibrium, i.e. the degree of collusion among the cartel's members. This confirms some empirical evidence (Bizjak and Coles [1995]) that behavioral constraints are the main concern of defendants in antitrust disputes. Under these circumstances the optimal policy covers that behavioral constraints should be set at their maximum level (i.e. post-conviction collusive profits become negligible) unless such a tight level would forbid the cartel to implement the collusive scheme that the authority is willing to tolerate.

We can also draw some interesting implications from the model presented here. First, the effectiveness of behavioral constraints in setting the prevailing degree of collusion sheds light, as policy implication, on some important feature of the European approach to antitrust disputes, in comparison with the US system. The former pays more attention to behavioral remedies (e.g. the recent imposition to Microsoft to unbundle *Windows Media Player* from the Windows operating system) than to fines (which usually are not sure, swift and substantial). Second, since we have shown that investigations are more likely if there exists uncertainty about the type of authority in charge of antitrust policy, an empirical prediction that might be tested is that antitrust disputes should be more frequent when a “new” authority is appointed (members of antitrust authority are subject to a turnover). Last, another testable prediction produced by the model is that investigations should be triggered by sudden and substantial price changes, while no price variations over time should not lead to antitrust disputes.

8 Appendix

Proof of Lemma 1: Suppose that $q_2 < q^c$, i.e. the authority observes a collusive output. If she chooses $\{i\}$ social welfare is $W(q_2) - K$; under the alternative action social welfare is $W(q_2)$; then $\{ni\}$ always dominates $\{i\}$.

□

Proof of Lemma 2: If the authority selects $\{i\}$ at $t = 1$ when $q_1 = q^c$ social welfare is $W(q^c) - K + \delta W(q^m)$ (at $t = 2$ by Lemma 1 the authority will not investigate and so $q_2 = q^m$). If she chooses $\{ni\}$ social welfare is $W(q^c) + \delta W(q^m)$, $\rightarrow \{ni\}$ always dominates $\{i\}$ when $q_1 = q^c$.

□

Proof of Lemma 3: From (13) we require that $(1 - m)(1 - \theta)q_1 - (1 - m)q_1^2 + \delta[\pi(\hat{q}) - \Omega] > 0$. It is straightforward to show that it is fulfilled if:

$$q_1 < \frac{1 - \theta}{2} - \frac{\sqrt{(m - 1)^2(1 - \theta)^2 - 4\delta(m - 1)[\pi(\hat{q}) - \Omega]}}{2(m - 1)}$$

or

$$q_1 > \frac{1 - \theta}{2} + \frac{\sqrt{(m - 1)^2(1 - \theta)^2 - 4\delta(m - 1)[\pi(\hat{q}) - \Omega]}}{2(m - 1)}$$

But we require also that $0 < q_1 < q_1^2$ or that $q_1^1 < q_1 < q^c$, with $q_1^2 < \frac{1 - \theta}{2} < q_1^1$. Hence we need that:

$$0 < \frac{1 - \theta}{2} - \frac{\sqrt{(m - 1)^2(1 - \theta)^2 - 4\delta(m - 1)[\pi(\hat{q}) - \Omega]}}{2(m - 1)} \tag{A.1}$$

$$\frac{1 - \theta}{2} - \frac{\sqrt{(m - 1)^2(1 - \theta)^2 - 4\delta(m - 1)[\pi(\hat{q}) - \Omega]}}{2(m - 1)} < \frac{1 - \theta}{2} - \Omega^{1/2} \tag{A.2}$$

$$\frac{1 - \theta}{2} - \frac{\sqrt{(m - 1)^2(1 - \theta)^2 - 4\delta(m - 1)[\pi(\hat{q}) - \Omega]}}{2(m - 1)} < 1 - \theta \tag{A.3}$$

$$\frac{1-\theta}{2} + \Omega^{1/2} < \frac{1-\theta}{2} - \frac{\sqrt{(m-1)^2(1-\theta)^2 - 4\delta(m-1)[\pi(\hat{q}) - \Omega]}}{2(m-1)} \quad (\text{A.4})$$

Inequalities (A.1) and (A.3) are fulfilled only if $m > 1$, while (A.2) and (A.4) are verified if the following condition is true:

$$\underbrace{[4\Omega - (1-\theta)^2]}_G m < \underbrace{4\Omega(1+\delta) - (1-\theta)^2 - 4\delta[\pi(\hat{q})]}_N \quad (\text{A.5})$$

Note that $G \geq 0$ would require: $\Omega \geq \frac{(1-\theta)^2}{4} = \pi(q^m)$, which implies that adjustment costs are greater than monopoly profits. By excluding this unrealistic hypothesis we have that $G \ll 0$. Moreover, $N \geq 0$ yields:

$$\Omega \geq \frac{(1-\theta)^2}{4(1+\delta)} + \frac{\delta}{1+\delta} \pi(\hat{q})$$

and since $\pi(\hat{q}) = \alpha \frac{(1-\theta)^2}{4}$, we can write that, to have $N \geq 0$, we need:

$$\Omega \geq \frac{1+\alpha\delta}{1+\delta} \underbrace{\frac{(1-\theta)^2}{4}}_{\pi(q^m)} \quad (\text{A.6})$$

Now, we know that (by assumption)

$$\Omega < \alpha \frac{(1-\theta)^2}{4}, \quad (\text{A.7})$$

Hence (A.6)–(A.7) must be simultaneously satisfied. But this is not the case if we show that

$$\alpha \frac{(1-\theta)^2}{4} < \frac{1+\alpha\delta}{1+\delta} \frac{(1-\theta)^2}{4}$$

Rearranging it we get that the above inequality is true if: $\alpha < 1$, which always holds since, by assumption, $0 < \alpha < 1$. Hence $N \ll 0$. Given that $G < 0$ and $N < 0$, it is possible to rewrite condition (A.5) as

$$m > \frac{\overbrace{4(1+\delta)\Omega - (1-\theta)^2 - 4\delta\pi(\hat{q})}^{<0}}{\underbrace{4\Omega - (1-\theta)^2}_{<0}}$$

so that in order to exist an output q_1 yielding positive profits but also satisfying $0 < q_1 < q_1^2$ and $q_1^1 < q_1 < q^c$, we need the following conditions to be simultaneously fulfilled:

$$m > 1, \quad m > \frac{4(1 + \delta)\Omega - (1 - \theta)^2 - 4\delta\pi(\hat{q})}{4\Omega - (1 - \theta)^2} \equiv \frac{N}{G}$$

Next, we show that $1 < \frac{N}{G}$. The latter implies that

$$1 < \frac{\overbrace{4(1 + \delta)\Omega - (1 - \theta)^2 - 4\delta\pi(\hat{q})}^{<0}}{\underbrace{4\Omega - (1 - \theta)^2}_{<0}}$$

i.e. $4\Omega - (1 - \theta)^2 > 4(1 + \delta)\Omega - (1 - \theta)^2 - 4\delta\pi(\hat{q})$, which can be written as

$$0 > \delta \underbrace{[\Omega - \pi(\hat{q})]}_{<0}$$

that is always true. Hence we require that (15) is fulfilled.

□

Proof of Proposition 1: (1) The cartel at C_1 compares option (1) (i.e. $\{q_1 = q^c, q_2 = q^m\}$) and (3) (i.e. $\{q_1 = q_2 = \bar{q}\}$). For $\{q_1 = q^c, q_2 = q^m\}$ to dominate $\{q_1 = q_2 = \bar{q}\}$ we need that: $\delta[\pi(q^m\theta) - \Omega] > (1 + \delta)\pi(\bar{q})$, which can be rewritten as:

$$\delta \underbrace{[\pi(q^m\theta) - \pi(\bar{q}) - \Omega]}_L > (1 + \delta)\pi(\bar{q})$$

However note that, by definition of \bar{q} and being $q_1^2 \leq \bar{q} \leq q_1^1$, we have that $L \ll 0$ (from (4)). Hence the above condition is never fulfilled and so $\{q_1 = q_2 = \bar{q}\}$ strictly dominates $\{q_1 = q^c, q_2 = q^m\}$. However it is necessary to investigate whether there exists an output \bar{q} such that $q_1^2 \leq \bar{q} \leq q_1^1$. First we show that $q_1^2 \leq \bar{q}$. From (5) and from (10) we require:

$$\frac{1 - \theta}{2} - \Omega^{1/2} \leq 1 - \theta - \frac{\sqrt{\delta^2(1 - \theta)^2[2(1 - \sqrt{1 - \alpha}) - \alpha] + 8\delta K}}{2\delta}$$

i.e.

$$-\Omega^{1/2} \leq \frac{1-\theta}{2} - \frac{\sqrt{\delta^2(1-\theta)^2[2(1-\sqrt{1-\alpha})-\alpha] + 8\delta K}}{2\delta}$$

which may be rewritten as condition (16). Second, we require:

$$1-\theta - \frac{\sqrt{\delta^2(1-\theta)^2[2(1-\sqrt{1-\alpha})-\alpha] + 8\delta K}}{2\delta} \leq \frac{1-\theta}{2} + \Omega^{1/2}$$

i.e.

$$\frac{1-\theta}{2} - \frac{\sqrt{\delta^2(1-\theta)^2[2(1-\sqrt{1-\alpha})-\alpha] + 8\delta K}}{2\delta} \leq \Omega^{1/2}$$

which is, again, condition (16). This proves equilibrium (1).

(2) If (16) does not hold an output $q_1^2 \leq \bar{q} \leq q_1^1$ no longer exists. Hence the only option available at C_1 is $\{q_1 = q^c, q_2 = q^m\}$.

□

Proof of Lemma 4: If $q_2 = \hat{q}$ after $q_1 \in]0, \bar{q}[$ or after $q_1 \in]q_1^1, q^c[$ the cartel's expected profits are: $\eta(1-m)[\pi(\hat{q}) - \Omega] + (1-\eta)[\pi(\hat{q}) - \Omega]$, which can be written as $(1-\eta m)[\pi(\hat{q}) - \Omega]$. The latter is positive only if (17) holds.

□

Proof of Proposition 2: Since we have already shown that (19) always dominates (21), we need to compare the following remaining options available to the cartel at $t = 1$: (i) $q_1 \in]0, \bar{q}[$ or $q_1 \in]q_1^1, q^c[$, which yields as overall profits (18), (ii) $q_1 = \bar{q}$, so that it gets (19), (iii) $q_1 = q^c$, that leads to (22). Comparing (22) and (18) implies that the former is higher than the latter if:

$$(1-\eta)\delta[\pi(q^m) - \Omega] \geq (1-m)\pi(q_1) + (1-\eta m)\delta[\pi(\hat{q}) - \Omega]$$

By rearranging it we get:

$$(1-\eta)\delta[\pi(q^m) - \Omega] - (1-\eta m)\delta[\pi(\hat{q}) - \Omega] \geq \underbrace{(1-m)\pi(q_1)}_{<0}$$

Hence a sufficient condition to prove that (22) dominates (18) is:

$$(1 - \eta)\delta[\pi(q^m) - \Omega] > (1 - \eta m)\delta[\pi(\hat{q}) - \Omega]$$

which can be written as:

$$\underbrace{\pi(q^m) - \pi(\hat{q})}_{>0} - \underbrace{\eta(1 - m)}_{<0} \underbrace{[\pi(q^m) - \Omega]}_{>0} > 0$$

which is always true. Hence $q_1 = q^c$ always dominates $q_1 \in]0, \bar{q}[$ or $q_1 \in]q_1^1, q^c[$. We need now to check when $q_1 = q^c$ dominates $q_1 = \bar{q}$. We have to investigate, from (22) and (19), when

$$[1 + \delta - \eta(m + \delta)]\pi(\bar{q}) < (1 - \eta)\delta[\pi(q^m) - \Omega]$$

which can be written as

$$\eta\{\delta[\pi(q^m) - \Omega] - (m + \delta)\pi(\bar{q})\} < \delta[\pi(q^m) - \Omega] - (1 + \delta)\pi(\bar{q})$$

Note that if there exists an output $q_1 = \bar{q}$ such that $q_1^2 \leq \bar{q} \leq q_1^1$ then, by Proposition 1, $\delta[\pi(q^m) - \Omega] < (1 + \delta)\pi(\bar{q})$, and so the r.h.s. of the above inequality is always negative. If we investigate the l.h.s., it is easy to see that it is negative as well, since $\delta[\pi(q^m) - \Omega] < (1 + \delta)\pi(\bar{q}) < (m + \delta)\pi(\bar{q})$, by Proposition 1 and $m > 1$. Hence we need to show when:

$$\eta \underbrace{\{\delta[\pi(q^m) - \Omega] - (m + \delta)\pi(\bar{q})\}}_{<0} < \underbrace{\delta[\pi(q^m) - \Omega] - (1 + \delta)\pi(\bar{q})}_{<0}$$

The above is fulfilled if

$$\eta > \frac{\delta[\pi(q^m) - \Omega] - (1 + \delta)\pi(\bar{q})}{\delta[\pi(q^m) - \Omega] - (m + \delta)\pi(\bar{q})}$$

Hence $q_1 = q^c$ dominates $q_1 = \bar{q}$ if the above inequality is true, while $q_1 = \bar{q}$ dominates $q_1 = q^c$ if condition (23) holds. Note that $\eta_1 \gg \eta_2$ since, starting from

$$\frac{1 + \delta}{m + \delta} > \frac{\delta[\pi(q^m) - \Omega] - (1 + \delta)\pi(\bar{q})}{\delta[\pi(q^m) - \Omega] - (m + \delta)\pi(\bar{q})}$$

we obtain $(1+\delta) < (m+\delta)$, which is always fulfilled. Hence we have: if $\eta_1 \leq \eta \leq 1$ then $\{q_1 = q^c\}$ dominates $\{q_1 = \bar{q}\}$ because the latter strategy yields negative profits, if $\eta_2 < \eta < \eta_1$ then $\{q_1 = q^c\}$ dominates $\{q_1 = \bar{q}\}$ because the latter strategy yields lower overall profits than the former, while if $0 \leq \eta \leq \eta_2$ then $\{q_1 = \bar{q}\}$ dominates $\{q_1 = q^c\}$. This implies that if (23) holds then the cartel finds profitable to select $q_1 = \bar{q}$; it will then wait and observe the authority's response. If the latter is no investigation, the cartel is sure of facing type B and so will maintain collusion at $t = 2$; if the authority investigates it means that the cartel is facing type A and so it will select $q_2 = q^c$. This proves equilibrium (i). If instead (23) is not satisfied, then the cartel chooses $q_1 = q^c$ at $t = 1$; it will then observe the authority's response. If no investigation is implemented, then $q_2 = q^m$, while in the opposite case no collusion will take place also at $t = 2$. This proves equilibrium (ii).

□

Proof of Lemma 5: $W^{ii} \geq W^{COV}$ when

$$(1 + \eta\delta)W(q^c) + (1 - \eta)\delta W(q^m) \geq (1 + \delta)W(\bar{q})$$

which can be written as: $\eta\delta[W(q^c) - W(q^m)] \geq \delta[W(\bar{q}) - W(q^m)] - [W(q^c) - W(\bar{q})]$. Solving it for η we get $\eta \geq \eta_3$, where η_3 is defined in (24). This inequality is always fulfilled if $\eta_3 < 0$, i.e. if $\delta[W(\bar{q}) - W(q^m)] - [W(q^c) - W(\bar{q})] < 0$. But this implies that $W^{COV} = (1 + \delta)W(\bar{q}) < W(q^c) + \delta W(q^m) = W^{LM}$. This proves (i). If instead $\eta_3 \geq 0$ we require $\eta \geq \eta_3$. Note that $\eta_3 \ll 1$, since the latter implies $-\delta[W(q^c) + W(q^m)] - [W(q^c) - W(\bar{q})] \leq 0$, which is always true. This proves (ii).

□

Proof of Lemma 6: In $\frac{8K}{\delta} \geq A^2 - 2(1 - \theta)^2 + 2(1 - \theta)^2\sqrt{1 - \alpha} + (1 - \theta)^2\alpha$ we impose that $t \equiv \sqrt{1 - \alpha}$, so that it becomes, after substituting and rearranging, equal to: $(1 - \theta)^2(t - 1)^2 \geq H$, where $H = A^2 - \frac{8K}{\delta}$. Now, if $H \leq 0$ the inequality is always verified; this is true when $K \geq \chi$. If instead $H > 0$ (i.e. $K < \chi$), we

get two roots: $t'_1 = 1 - \frac{\sqrt{H}}{1-\theta}$ and $t'_2 = 1 + \frac{\sqrt{H}}{1-\theta}$. However $t'_2 \gg 1$ and so it has to be ruled out. Hence $t \leq t_1$. Moreover we require that $t'_1 > 0$, and the latter is true for $K \geq \Upsilon$. Since $\Upsilon \ll 0$ the latter restriction is always verified. Hence we get (31), while it is easy to show that $0 \leq \alpha' \leq 1$.

□

Proof of Proposition 3: From (25)–(27) we know that $W^{COV}|_{\alpha=0} \gg W^{COV}|_{\alpha=1}$ and that W^{COV} is a decreasing function of α . If we compare (26) and (32) we get that $W^{LM} \gg W^{COV}|_{\alpha=1}$. By checking instead $W^{COV}|_{\alpha=0} \geq W^{LM}$, we get, after rearranging:

$$8(1 + \delta)K - \delta^2(1 - \theta)^2 \leq 0 \quad (\text{A.8})$$

Solving it for δ we have

$$\delta_1 = \frac{4K - 2\sqrt{2K[2K + (1 - \theta)^2]}}{(1 - \theta)^2}$$

and (35), so that (A.8) is fulfilled if $\delta \leq \delta_1$ or $\delta \geq \delta_2$ (clearly $\delta_1 < \delta_2$). Furthermore, $\delta_1 \ll 0$ since $8(1 - \theta)^2 K \gg 0$, while $\delta_2 \gg 0$, given that $4K + 2\sqrt{2K[2K + (1 - \theta)^2]} \gg 0$. It is important to check whether $\delta_2 \leq 1$, since $0 < \delta \leq 1$ by definition. This condition implies that:

$$2\sqrt{2K[2K + (1 - \theta)^2]} \leq (1 - \theta)^2 - 4K$$

which is never fulfilled if $K > \frac{(1-\theta)^2}{4}$. Hence a first identified condition to have that $W^{COV}|_{\alpha=0} \geq W^{LM}$ is that

$$K \leq \frac{(1 - \theta)^2}{4} \quad (\text{A.9})$$

Moreover, we require, to have $\delta_2 \leq 1$, that $0 < K \leq \Phi$, and since the latter condition is more stringent than condition (A.9) this proves (33). Last, if (33) holds then $\delta_2 \leq 1$, but we need that the discount factor is higher (or equal) than this threshold level to have that $W^{COV}|_{\alpha=0} \geq W^{LM}$, and this proves (34). Hence

if (33)–(34) hold then $W^{COV}|_{\alpha=0} \geq W^{LM}$. But since $W^{COV}|_{\alpha=1} \ll W^{LM}$ and W^{COV} is a decreasing function of α , there exists a level of BC's, defined as $\bar{\alpha}$, such that $W^{COV} \geq W^{LM}$ iff $0 \leq \alpha \leq \bar{\alpha}$, where $\bar{\alpha}$ is identified by solving the following inequality:

$$(1 + \delta) \left\{ \frac{(1 - \theta)^2}{8} [2 + 2\sqrt{1 - \alpha} + \alpha] - \frac{K}{\delta} \right\} \geq \frac{(1 - \theta)^2}{8} (4 + 3\delta)$$

Rearranging it we can write

$$2 + 2\sqrt{1 - \alpha} \geq \frac{8K}{\delta(1 - \theta)^2} + \frac{4 + 3\delta}{1 + \delta}$$

By setting $\sqrt{1 - \alpha} = t$ it is possible to rewrite the above inequality as follow:

$$t^2 - 2t + \underbrace{\frac{8K}{\delta(-\theta)^2} + \frac{4 + 3\delta}{1 + \delta}}_Z - 3 \leq 0$$

and solving it for t we get: $t_1 = 1 - \sqrt{1 - Z}$, $t_2 = 1 + \sqrt{1 - Z}$. t_2 is ruled out since $0 < t < 1$ given that $0 \leq \alpha \leq 1$. To have a solution we require that $1 \geq Z$, and this is fulfilled if (33) holds. Then from t_1 it is easy to get (37). Furthermore, note that $0 \leq \bar{\alpha} \leq 1$.

□

Proof of Lemma 7: From (30) and (33) we know that $\Upsilon < \Phi$ given that $\Upsilon \ll 0$. Next we compare (29) and (33), and we obtain that $\chi < \phi$ if

$$\chi = \frac{\delta[(1 - \theta) - 2\Omega^{1/2}]^2}{8} < \frac{(1 - \theta)^2}{16} = \Phi$$

which can be written as (39). Moreover, we want to show when $\bar{\alpha} < \alpha'$. The latter is true when

$$(1 - \sqrt{1 - Z})^2 < \frac{(1 - \theta - \sqrt{H})^2}{(1 - \theta)^2}$$

Rearranging it we get: $H < (1 - \theta)^2(1 - Z)$. This inequality is always fulfilled if $H \leq 0$. But, as shown in the proof of Lemma 6, $H \leq 0$ if $K \geq \chi$. Hence if

the latter inequality is true then $\bar{\alpha} < \alpha'$ always. If instead $H > 0$ (i.e. if $K < \chi$), then $H < (1 - \theta)^2(1 - Z)$ implies, after substituting for Z ,

$$4 - \frac{4 + 3\delta}{1 + \delta} > \frac{8K}{\delta(1 - \theta)^2} + \frac{H}{(1 - \theta)^2}$$

which can be written, after substituting for H , as

$$\frac{\delta^2(1 - \theta)^2}{1 + \delta} > 8K + \delta A^2 - K$$

and solving for K we get (41). However, since $K \gg 0$, we need to show when $\delta(1 - \theta)^2 - (1 + \delta)A^2 > 0$. Solving it for δ we obtain (40).

□

Proof of Proposition 4: First, we assume that (39) holds. Hence we have to explore 5 cases.

(i): $\chi \leq K \leq \Phi$ and $\delta_2 \leq \delta \leq 1$. This implies that a covenant equilibrium exists $\forall \alpha \in [0, 1]$, and that $W^{COV} \geq W^{LM}$ if $0 \leq \alpha \leq \bar{\alpha}$. Since W^{COV} is decreasing in α , the authority sets $\alpha^* = 0$. She modifies the decentralized equilibrium because chooses the highest level of BC's, and gets a second best welfare.

(ii): $\chi \leq K \leq \Phi$ and $\delta < \delta_2$. In this case a covenant equilibrium exists $\forall \alpha \in [0, 1]$ but $W^{COV} < W^{LM} \forall \alpha \in [0, 1]$. Then $\alpha^* = 0$ but a second best welfare cannot be restored (the authority would prefer the late monopoly solution but it is not possible to use BC's to destroy the covenant equilibrium).

(iii): $K < \chi \leq \Phi$ and $\delta_2 \leq \delta \leq 1$. The covenant equilibrium exists $\forall \alpha \in [\alpha', 1]$, while $W^{COV} \geq W^{LM} \forall \alpha \in [0, \bar{\alpha}]$. Hence, by Lemma 7, it becomes relevant the relation between $\bar{\alpha}$ and α' . This means that we have two sub-cases: (a) $K < \Gamma$ and $\delta > \frac{A^2}{(1-\theta)^2 - A^2}$. Under these circumstances $\bar{\alpha} < \alpha'$, so that the α -range where $W^{COV} \geq W^{LM}$ does not overlap with the α -range where a covenant equilibrium exists. So, $\alpha^* = 0$, but a late monopoly solution prevails not a covenant equilibrium (the latter is the second best). (b) Either $K \geq \Gamma$ or $\delta \leq \frac{A^2}{(1-\theta)^2 - A^2}$, so that $\bar{\alpha} \geq \alpha'$. The two relevant intervals overlap and $\alpha^* = \alpha'$ ($\alpha^* = 0$ cannot be imposed since a covenant equilibrium does not exist for such a tough level of BC's).

(iv): $K < \chi \leq \Phi$ and $\delta < \delta_2$. This implies that a covenant equilibrium exists $\forall \alpha \in [\alpha', 1]$, while $W^{COV} < W^{LM} \forall \alpha \in [0, 1]$. Then $\alpha^* = 0$ and second best welfare is restored.

(v): $\chi \leq \Phi < K$ and so $\delta_2 > 1$. In this case a covenant equilibrium exists $\forall \alpha \in [0, 1]$ but $W^{COV} < W^{LM} \forall \alpha \in [0, 1]$. Hence $\alpha^* = 0$, but, as in case (ii), it is not possible to restore second best welfare through BC's.

Next, we assume that (39) does not hold, so that $\Phi < \chi$. We have to explore 4 cases here.

(i): $\Phi < K < \chi$ and so $\delta_2 > 1$. The covenant equilibrium exists $\forall \alpha \in [\alpha', 1]$ but $W^{COV} < W^{LM} \forall \alpha \in [0, 1]$. Hence $\alpha^* = 0$, the covenant equilibrium is destroyed through BC's and second best welfare is restored.

(ii): $\Phi < \chi \leq K$ and so $\delta_2 > 1$. In this case a covenant equilibrium exists $\forall \alpha \in [0, 1]$ but $W^{COV} < W^{LM} \forall \alpha \in [0, 1]$. Then $\alpha^* = 0$ but it is not possible to destroy the covenant equilibrium, and so second best cannot be restored.

(iii): $K \leq \Phi < \chi$ and $\delta_2 \leq \delta \leq 1$. This implies that covenant equilibrium exists $\forall \alpha \in [\alpha', 1]$, while $W^{COV} \geq W^{LM} \forall \alpha \in [0, \bar{\alpha}]$. Hence, by Lemma 7, it becomes relevant the relation between $\bar{\alpha}$ and α' . This means that we have two sub-cases:

(a) $K < \Gamma$ and $\delta > \frac{A^2}{(1-\theta)^2 - A^2}$. In this case $\bar{\alpha} < \alpha'$ and so the two intervals do not overlap. Consequently, $\alpha^* = 0$, a late monopoly solution arises even if the authority prefers the covenant equilibrium. (b) Either $K \geq \Gamma$ or $\delta \leq \frac{A^2}{(1-\theta)^2 - A^2}$, so that $\bar{\alpha} \geq \alpha'$. The two relevant intervals overlap and $\alpha^* = \alpha'$ ($\alpha^* = 0$ cannot be imposed since a covenant equilibrium does not exist for such a tough level of BC's). A second best welfare is restored.

(iv): $K \leq \Phi < \chi$ and $\delta < \delta_2$. The covenant equilibrium exists $\forall \alpha \in [\alpha', 1]$ but $W^{COV} < W^{LM} \forall \alpha \in [0, 1]$. Hence $\alpha^* = 0$, the covenant equilibrium is destroyed through BC's and second best welfare is restored.

□

Proof of Proposition 5: From (47) it is clear that if $\eta \geq \frac{1}{5}$ then $W^{ii} > W^i \forall \alpha \in [0, 1]$. If instead $\eta < \frac{1}{5}$ (47) can be written as:

$$8[1 + \delta(1 - \eta)] - \delta^2(1 - \theta)^2(1 - 5\eta) < 0$$

and solving it for δ we get:

$$\delta_1^{LI} = \frac{4(1-\eta)K - 2\sqrt{2K[2K(1-\eta)^2 + (1-\theta)^2(1-5\eta)]}}{(1-\theta)^2(1-5\eta)}$$

and (50), with $\delta_1^{LI} < \delta_2^{LI}$. Moreover $\delta_1^{LI} \ll 0$ since rearranging the numerator we get

$$0 < 2K(1-\theta)^2 \underbrace{(1-5\eta)}_{>0}$$

while $\delta_2^{LI} \gg 0$. Last $\delta_2^{LI} \leq 1$ implies as necessary condition that

$$K \leq \frac{(1-\theta)^2(1-5\eta)}{4(1-\eta)}$$

and, if the above condition holds, it also requires that (48) is satisfied. This proves (1). Next, if (48) holds it is also necessary that (49) is satisfied; this proves (2). Last, we need to identify the α -range where $W^i \geq W^{ii}$, i.e. when

$$[1 + \delta(1-\eta)] \left\{ \frac{(1-\theta)^2}{8} [2(1 + \sqrt{1-\alpha}) + \alpha] - \frac{K}{\delta} \right\} \geq \frac{(1-\theta)^2}{8} (4 + 3\delta + \delta\eta)$$

which can be written, after rearranging, as

$$2 + 2\sqrt{1-\alpha} + \alpha \geq \frac{8K}{\delta(1-\theta)^2} + \frac{4 + 3\delta + \delta\eta}{1 + \delta(1-\eta)}$$

By imposing that $\sqrt{1-\alpha} \equiv t$ we can write, after substituting and rearranging, the above inequality as $t^2 - 2t + J \leq 0$. The latter has two roots: $t_1 = 1 - \sqrt{1-J}$ and $t_2 = 1 + \sqrt{1-J}$ (which has to be ruled out since $0 \leq t \leq 1$). Note that we need $1 \geq J$ and this is fulfilled when both (48)–(49) hold. Hence t_1 yields (52) and, consequently, (51). This proves (3). Note that $0 \leq \bar{\alpha}^{LI} \leq 1$.

□

Proof of Lemma 8: We want to show when the following inequality is satisfied:

$$\frac{\delta A^2}{8} \leq \frac{(1-\theta)^2(1-5\eta)}{8(2+\eta^2-2\eta)}$$

Solving it for δ we get (54). Next we want to identify when $\bar{\alpha}^{LI} < \alpha'$, i.e. when

$$(1 - \sqrt{1-J})^2 < \frac{(1-\theta - \sqrt{H})^2}{(1-\theta)^2}$$

which can be written as $H < (1-\theta)^2(1-J)$. Clearly, if $H \leq 0$ then $\bar{\alpha}^{LI} \leq \alpha' \forall \alpha \in [0, 1]$. The latter is true if $K \geq \chi$. If instead $H > 0$, then the above inequality can be written, after rearranging and substituting for H , as

$$\frac{\delta^2(1-\theta)^2(1-5\eta)}{1+\delta(1-\eta)} > 7K + \delta A^2$$

and solving it for K we get $K < \Gamma^{LI}$. However, since $K \gg 0$ we need to verify that Γ^{LI} is positive. The latter is true when (55) holds.

□

Proof of Proposition 6: We have to consider six cases (and within each of them several subcases):

(1): $\eta < \frac{1}{5} \leq \eta$. In this case PBE(i) does not exist $\forall \alpha \in [0, 1]$ and $W^i < W^{ii} \forall \alpha \in [0, 1]$. Hence $\alpha^* \in [0, 1]$ and BC's are irrelevant.

(2): $\frac{1}{5} \leq \eta_2 < \eta$. The analysis is similar to case (1).

(3): $\frac{1}{5} \leq \eta \leq \eta_2$. Under these circumstances PBE(i) exists $\forall \alpha \in [0, 1]$ if $K \geq \chi$, and if $\alpha' \leq \alpha \leq 1$ otherwise. Hence we have two subcases:

(i): $K \geq \chi$. This implies that PBE(i) exists $\forall \alpha \in [0, 1]$ and $W^i < W^{ii} \forall \alpha \in [0, 1]$. So $\alpha^* = 0$, PBE(i) cannot be destroyed and a third best welfare is achieved. However the authority sets the highest achievable level of welfare under PBE(i). d (ii): $K < \chi$, i.e. a situation where PBE(i) exist $\forall \alpha \in [\alpha', 1]$ and $W^i < W^{ii} \forall \alpha \in [0, 1]$. Consequently $\alpha^* = 0$, PBE(i) is destroyed and a second best welfare is reached.

(4): $\eta \leq \eta_2 < \frac{1}{5}$. Under these circumstances PBE(i) exists $\forall \alpha \in [0, 1]$ if $K \geq \chi$, and if $\alpha' \leq \alpha \leq 1$ otherwise. Moreover, $W^i \geq W^{ii}$ if $K \leq \Sigma, \delta_2^{LI} \leq \delta \leq 1, 0 \leq$

$\alpha \leq \bar{\alpha}^{LI}$. Since we know that $K \leq \Sigma$ depends upon condition (54), we have the following subcases:

(i): (54) holds. Hence we have the following subcases:

(a): $\chi \leq K \leq \Sigma$ and $\delta_2^{LI} \leq \delta \leq 1$. In this case PBE(*i*) exists $\forall \alpha \in [0, 1]$ while $W^i \geq W^{ii} \forall \alpha \in [0, \bar{\alpha}^{LI}]$. Then $\alpha^* = 0$ and a second best welfare is reached.

(b): $\chi \leq K \leq \Sigma$ and $\delta < \delta_2^{LI}$. This implies that PBE(*i*) exists $\forall \alpha \in [0, 1]$ and that $W^i < W^{ii} \forall \alpha \in [0, 1]$. Then $\alpha^* = 0$ and a second best welfare cannot be restored.

(c): $K < \chi \leq \Sigma$ and $\delta_2^{LI} \leq \delta \leq 1$. With these parameters' intervals PBE(*i*) exists $\forall \alpha \in [\alpha', 1]$ while $W^i \geq W^{ii} \forall \alpha \in [0, \bar{\alpha}^{LI}]$. Again we have two subcases:

(*): $K < \Gamma^{LI}$ and (55) holds. In this case $\bar{\alpha}^{LI} < \alpha'$ and so $\alpha^* = 0$: a third best welfare is achieved.

(+): $K \geq \Gamma^{LI}$ or (55) does not hold. Under these circumstances $\bar{\alpha}^{LI} \geq \alpha'$ and so $\alpha^* = \alpha'$; a second best welfare is restored.

(d): $K < \chi \leq \Sigma$ and $\delta < \delta_2^{LI}$. This implies that PBE(*i*) exists $\forall \alpha \in [\alpha', 1]$ while $W^i < W^{ii} \forall \alpha \in [0, 1]$. Then $\alpha^* = 0$ and a second best welfare is reached.

(e): $\chi \leq \Sigma < K$ so that $\delta_2^{LI} > 1$. With this K -interval PBE(*i*) exists $\forall \alpha \in [0, 1]$ and $W^i < W^{ii} \forall \alpha \in [0, 1]$. Then $\alpha^* = 0$ and a third best welfare is achieved.

(ii): (54) does not hold. The subcases to analyze are the following ones:

(a): $\Sigma < K < \chi$ so that $\delta_2^{LI} > 1$. Hence PBE(*i*) exists $\forall \alpha \in [\alpha', 1]$ but $W^i < W^{ii} \forall \alpha \in [0, 1]$. Consequently $\alpha^* = 0$ and PBE(*i*) is destroyed, getting a second best welfare.

(b): $\Sigma < \chi \leq K$ so that $\delta_2^{LI} > 1$. This implies that PBE(*i*) exists $\forall \alpha \in [0, 1]$ but $W^i < W^{ii} \forall \alpha \in [0, 1]$. Consequently $\alpha^* = 0$ since PBE(*i*) cannot be destroyed, and the authority limits the welfare losses in a third best outcome.

(c): $K \leq \Sigma < \chi$ and $\delta_2^{LI} \leq \delta \leq 1$. In this case PBE(*i*) exists $\forall \alpha \in [\alpha', 1]$ and $W^i \geq W^{ii} \forall \alpha \in [0, \bar{\alpha}^{LI}]$. Consequently we need to investigate two subcases:

(*): $K < \Gamma^{LI}$ and (55) holds. In this case $\bar{\alpha}^{LI} < \alpha'$ and so $\alpha^* = 0$: a third best welfare is achieved.

(+): $K \geq \Gamma^{LI}$ or (55) does not hold. Under these circumstances $\bar{\alpha}^{LI} \geq \alpha'$ and so $\alpha^* = \alpha'$; a second best welfare is restored.

(d): $K \leq \Sigma < \chi$ and $\delta < \delta_2^{LI}$. This implies that $\text{PBE}(i)$ exists $\forall \alpha \in [\alpha', 1]$ while $W^i < W^{ii} \forall \alpha \in [0, 1]$. Then $\alpha^* = 0$ and a second best welfare is reached.

(5): $\eta_2 < \eta < \frac{1}{5}$. Under these circumstances $\text{PBE}(i)$ does not exist $\forall \alpha \in [0, 1]$ and $W^i \geq W^{ii}$ if $K \leq \Sigma, \delta_2^{LI} \leq \delta \leq 1, 0 \leq \alpha \leq \bar{\alpha}^{LI}$. Hence we need to investigate the following subcases:

(i): $K \leq \Sigma$ and $\delta_2^{LI} \leq \delta \leq 1$. In this case $\text{PBE}(i)$ does not exist $\forall \alpha \in [0, 1]$ while $W^i \geq W^{ii} \forall \alpha \in [0, \bar{\alpha}^{LI}]$. Hence $\alpha^* = [0, 1]$ and BC's are irrelevant since a third best welfare is always achieved.

(ii): $K \leq \Sigma$ and $\delta < \delta_2^{LI}$. With these intervals $\text{PBE}(i)$ does not exist $\forall \alpha \in [0, 1]$ while $W^i < W^{ii} \forall \alpha \in [0, 1]$. Hence $\alpha^* = [0, 1]$ and BC's are irrelevant and a second best welfare is always achieved.

(iii): $K > \Sigma$ so that $\delta_2^{LI} > 1$. This subcase is similar to subcase (ii).

(6): $\eta < \frac{1}{5} < \eta_2$. Under these circumstances $\text{PBE}(i)$ exists $\forall \alpha \in [0, 1]$ if $K \geq \chi$ and $\forall \alpha \in [\alpha', 1]$ otherwise. Moreover, $W^i \geq W^{ii}$ if $K \leq \Sigma, \delta_2^{LI} \leq \delta \leq 1, 0 \leq \alpha \leq \bar{\alpha}^{LI}$. The analysis of all the possible subcases is similar to case (4).

□

References

- Alexander, B., 1994, The Impact of the National Industrial Recovery Act on Cartel Formation and Maintenance Costs, *Review of Economics and Statistics*, 76, 245–254.
- Besanko, D.,- Spulber, D., 1989, Antitrust Enforcement under Asymmetric Information, *Economic Journal*, 99, 408–425.
- —, —, 1990, Are Treble Damages Neutral? Sequential Equilibrium and Private Antitrust, *American Economic Review*, 80, 870–887.
- Bizjak, J.M.,- Coles, J.L., 1995, The Effect of Private Antitrust Litigation on the Stock–Market Valuation of the Firm, *American Economic Review*, 85, 436–461.
- Block, M.K.,- Nold, F.C. and J.G. Sidak, 1981, The Deterrent Effect of Antitrust Enforcement, *Journal of Political Economy*, 89, 429–445.
- Bosch, J.C.,- Eckard, E.W., 1991, The Profitability of Price–fixing: Evidence from Stock Market Reaction to Federal Indictments, *Review of Economics and Statistics*, 73, 309–317.
- Bradburd, R.M.,- Over, M.A., 1982, Organizational Costs, Sticky Equilibria and Critical Levels of Concentration, *Review of Economics and Statistics*, 64, 50–58.
- Choi, D.,- Philippatos, G.C., 1983, Financial Consequences of Antitrust Enforcement, *Review of Economics and Statistics*, 65, 501–6.
- Christie, W.G.,- Schultz, P.H., 1994, Why do Nasdaq Market Makers Avoid Odd–Eighth Quotes?, *Journal of Finance*, 49, 1813–1840.
- Cyrenne, P., 1999, On Antitrust Enforcement and the Deterrence of Collusive Behaviour, *Review of Industrial Organization*, 14, 257–272.
- Feinberg, R.M., 1980, Antitrust Enforcement and Subsequent Price Behavior, *Review of Economics and Statistics*, 62, 609–12.
- —, 1984, The Timing of Antitrust Effects on Pricing, *Applied Economics*, 16, 397–409.

- Green, E.J.,- Porter, R.H., 1984, Noncooperative Collusion under Imperfect Price Information, *Econometrica*, 52, 87–100.
- Grillo, M., 2002, Collusion and Facilitating Practices: A New Perspective in Antitrust Analysis, *European Journal of Law and Economics*, 14, 151–169.
- Harrington, J.E., 2004a, Post–Cartel Pricing during Litigation, *Journal of Industrial Economics*, 52, 517–534.
- —, 2004b, Cartel Pricing Dynamics in the Presence of an Antitrust Authority, *RAND Journal of Economics*, 35, 651–673.
- —, 2005, Optimal Cartel Pricing in the Presence of an Antitrust Authority, *International Economic Review*, 46, 145–169.
- Hay, G.,- Kelly, D., 1974, An Empirical Survey of Price Fixing Conspiracies, *Journal of Law and Economics*, 17, 13–38.
- Levenstein, M.,- Suslow, V., 2001, Private International Cartels and their Effect on Developing Countries, pdf copy, University of Massachusetts.
- Martin, S., 2000, Resource Allocation by a Competition Authority, in Einar Hope (ed.), *Foundations of competition policy analysis*, Routledge.
- Martini, G.,- Rovesti, C., 2004, Antitrust Policy and Price Collusion: Public Agencies vs Delegation, *Recherches Economiques de Louvain/Louvain Economic Review*, 70, 127–151.
- McCutcheon, B., 1997, Do Meetings in Smoke–Filled Rooms Facilitate Collusion?, *Journal of Political Economy*, 105, 330–350.
- Motta, M.,- Polo, M., 2003, Leniency Programs and Cartel Prosecution, *International Journal of Industrial Organization*, 21, 347–379.
- OECD, 2003, *Hard Core Cartels: Recent Progress and Challenges Ahead*, Paris.
- Posner, R.A., 1976, *Antitrust Law: An Economic Perspective*, Chicago, Chicago University Press.
- Salant, S. 1987, Treble Damage Awards in Private Lawsuits for Price Fixing, *Journal of Political Economy*, 95, 1326–1336.

- Souam, S. 1998, Dissuasion de la collusion et efficacité de deux systèmes d’amendes, *Revue Economique*, 49, 755–765.
- —, 2001, Optimal Antitrust Policy under Different Regimes of Fines, *International Journal of Industrial Organization*, 19, 1–26.
- Spagnolo, G., 2000, Optimal Leniency Programs, pdf copy, Stockholm School of Economics.