UNIVERSITA' DEGLI STUDI DI BERGAMO
DIPARTIMENTO DI INGEGNERIA GESTIONALE E DELL'INFORMAZIONE°
QUADERNI DEL DIPARTIMENTO†

Department of Management and Information Technology

Working Paper

Series "*Mathematics and Statistics*"

n. 3/MS – 2004

Statistical Modelling and Uncertainty Reduction of Monitoring Data in Geomechanics

by

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† Il Dipartimento ottempera agli obblighi previsti dall'art. 1 del D.L.L. 31.8.1945, n. 660 e successive modificazioni.

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Statistical modelling and uncertainty reduction of monitoring data in geomechanics

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December 2004

Keywords: robust regression, dynamic adjustment, heteroskedasticity, data validation, engineering evaluation.

ABSTRACT: The analysis of field data often shows unwanted trends, quasi periodical variations at the daily and/or yearly frequency and other fluctuations and noises which are not related to the phenomenon being monitored. For field instruments based on long hydraulic circuit (e.g. hydraulic settlement gages) or rods and wire gauges (e.g. extensometers), the temperature may have significant effects on the uncertainty of the measuring system. In this paper, we show how to reduce temperature drifts and the highly non Gaussian uncertainty by means of an appropriated computerized post-processing of the field data based on robust statistical methods. To do this routinely, we propose an easy-to-use software which helps the user in the data analysis and in the choice of the appropriate modelling technique.

1 Introduction

Geotechnical and structural monitoring of complex structures or critical geological situations have become normal practice.

Monitoring means a continuous measurement of significant physical parameters that describe the behaviour of rock and soil masses or of a structure and/or of their interaction. These phenomena cannot be predicted in details by means of a deterministic approach due to the complexity of the constitutive laws of the materials considered and to the high number of unknowns and the uncertainty about the nature and evolution of the boundary and environmental conditions which can affect the measured parameters as well as modification of mechanical and electrical characteristics of the instruments during a long term period of observation.

Another significant factor is that instruments are normally installed in open field and cover a wide area increasing the complexity of the systems enhancing the effects of environmental and boundary conditions.

A key point in the monitoring activity it has to be pointed out that quite often the history or, in other words, the variation of the values of a specific parameter is more significant than the knowledge of the absolute value itself. A single value very seldom shows the true behaviour of a structure or a soil mass because the structure or the soil, not being homogeneous and having complex interaction among different parts, cannot be represented by the values collected in a single point, because data collected in a single point cannot be representative of the real behaviour of the whole monitored phenomenon.

Among all the others conditions, the temperature can have significant effects on the measuring systems. These effects can be easily compensated when affecting the sensing element only. However for many instrumentation systems the sensing element is not the main source of unexpected values. Instruments using, for example, long hydraulic circuits or rods and wire - as for hydraulic settlement gages or extensometers - can show significant variations due to temperature cycles, both over a daily and seasonal period. Moreover, the response of the measuring system to temperature variations has a certain inertia and delay which could be explained by stochastic dynamic models.

A different approach is then needed since the solution cannot actually be found at the instrument or system level. It is necessary to operate at the data processing level developing a tool able to combine the variation of a parameter with the co-variation of other parameters. The aim of such a tool is to help to reduce noise, enhancing the true physical phenomenon understanding, eliminating trends and unwanted spurious co-variations and, vice versa, highlighting important dynamics of the system under observation.

A statistical approach has been selected because, in principle, all the observed physical phenomena and the correlated constitutive laws are, as mentioned, not deterministic phenomena and because in engineering practice most of the parameters are calculated on a statistical basis - such as modula, strength limits, elasto-plastic coefficients, etc.. – and therefore the use of a statistical approach also for monitoring data is recommended.

In section 2, a statistical monitoring model is introduced allowing to discuss the different sources of uncertainty and biases in the general framework of non Gaussian stochastic processes.

To do this, we first note that the object of the monitoring activity is the dynamics of a physical system, for example a building or a landslide or the wall of an excavation. Here, we do not consider detailed mathematical modelling of such a physical system as, for example, using FEM approach; rather, we consider an approach of modelling the measuring system which put together observed data and physical and engineering knowledge.
Generally speaking, if a detailed model of the physical system is available, then both approaches may be integrated using, for example, Data Assimilation techniques; see e.g. Sneddon (2000) or Thompson et Al. (2000).
In section 3, considering field applications based on hydraulic measuring systems and load measuring systems, we illustrate adjustment techniques for observed thermal effects as well as smoothing techniques for uncertainty reduction. It is also clear from these examples that a significant number of data, both of the observed parameter and of the covariant, is needed. This leads us to use instrumentation systems based on an automatic data acquisition system, and a consistent historical data base.
In section 4, we introduce the software package FieldStat which has been developed to made easy the statistical analysis also for the untrained user and, in section 5, some conclusions are drawn.

2 Modelling uncertainty

In order to discuss some techniques for assessing and reducing the environmental biases and uncertainty of a measuring system, we introduce a general statistical model allowing us to describe the sensitivity to thermal effects and the uncertainty under various operating conditions.
The need for a statistical approach arises, as mentioned before, because uncertainties and unknowns are naturally described by random variables and random processes. Moreover, since constants and relationships between relevant quantities are empirically estimated through observed data, a statistical approach allows us to assess also the uncertainty arising from estimation. Since the available physical knowledge and evidence are both used in our approach, we consider this as a grey-box modelling approach instead of purely black-box; see e.g. Ljung (1999).
In particular, the dynamics of gauge readings y_t at time *t*, is related to temperature and/or other environmental readings, denoted by u_t, by the equation:

$$y_t = \mu_t + g(u_t) + \pi_t + \zeta_t \,. \tag{1}$$

In equation (1) the four components have the following interpretation:

- μ_t is the "signal" or the "state" of the physical system and is supposed to be a "smooth" function;
- the function $g(\ )$ is the temperature drift function and may have a known shape, e.g. linear and contemporaneous, as in subsection 2.2, or may include lagged values, as in subsection 2.3;
- the quantity π_t describes the effect of unmeasured environmental and/or anthropic variations which have quasi-periodical cycles, e.g. daily or yearly;
- the error ζ_t is described by an unobserved stationary stochastic process; applications below suggest not to restrict to the Gaussian case.

In the ideal case, all environmental factors are observed and enclosed in the vector u_t and the function $g(\;)$ is completely known, then $\pi_t = 0$ and the error ζ_t, being a standard measurement error, is a purely random error, i.e. Gaussian and white noise.
In our case, since the measuring system is complex and we have only partial observation of environmental factors, we use a simplified explanation of environmental effects given by $g(u)$. Moreover, the measuring system may have certain inertia, delays and non symmetric dynamics which are described, in general terms, by the stochastic properties of ζ_t.
A question which arise is whether the unobserved stochastic process π_t is related only to the measuring system or also to the physical system itself. In principle, both may fluctuate with temperature. For example some small dilatations are possible on the wall of a building being monitored. In our monitoring setup we consider that these fluctuations are not of concern with respect of stability and safety which are the main motivations of monitoring. Hence the quantity of interest is μ_t or its variation $\partial\mu_t / \partial t$.

2.1 Uncertainty decomposition

Equation (1) allows for a detailed analysis of the accuracy of a measuring system. As a matter of fact, denoting, as usual, the standard deviation with σ and assuming that the four components of the right hand side of equation (1) are uncorrelated, we have the following decomposition of the variance σ_y^2 or total field uncertainty of the measuring system:

$$\sigma_y^2 = \sigma_\mu^2 + \sigma_{g(u)}^2 + \sigma_\pi^2 + \sigma_\zeta^2 . \tag{2}$$

The first variance is zero if the physical system is stationary and the second is often the largest component which can be eliminated by appropriate temperature monitoring and the techniques of sections 2.2 and 2.3. The third one is related to the unobserved environmental effects and can be eliminated by the smoothing techniques of section 2.4. The last term is the residual uncertainty of the measuring system and is the common objective of the rest of this paper. Its reduction is not considered in details here but, in principle, it can be reduced in a dynamical sense by iteratively considering the present uncertainty conditionally on the past observations.
The assumptions of uncorrelation in equation (1), which are used in equation (2), deserve some comments. First of all they require correct model specification and estimation and this may be checked using standard statistical tools. Moreover the "signal" μ_t, after removing environmental and anthropic components, is stationary and, by definition is uncorrelated with the other quantities.
The major source of correlation may arise between the temperature effect $g(\;)$ and the unmeasured variation π_t which may be both periodical especially when they are only partially observed. Whenever we do not deepen this topic here we note that orthogonalization is possible.
With this model setup in mind, we will briefly discuss three important particular cases in the next three subsections which are related to the applications of the section 3 below.

2.2 Static adjustment

In this section, we discuss the modelling of the observed temperature effect on the gauge readings y_t when the system under monitoring is supposed stationary, i.e. $\mu_t = const$ with the notation of equation (1).
The simplest idea is to suppose the above effect linear and immediate, i.e.:

$$y_t = \alpha + \beta u_t + e_t \tag{3}$$

whit the error $e_t = \pi_t + \zeta_t$ as in equation (1).
Due to the above mentioned non Gaussianity, estimation of coefficients α and β is performed using robust techniques based on iteratively weighted least-squares coupled with a Cochran-Orcutt type correction for autocorrelation (see e.g. Holland and Welsh, 1977, Huber, 1981, and Fassò and Perri, 2001).

2.3 Dynamical adjustment

Due to partial observation, inertia and delays the model (3) is often simplistic. A more complete model allows for a transfer function (see e.g. Lijung, 1999, Box, Jenkins and Reinsel, 1994). In this case we consider a so called Box-Jenkins (BJ) model given by

$$y_t = \alpha + b_0 u_t + b_1 u_{t-1} + \ldots + e_t \tag{4}$$

where

$$e_t + c_1 e_{t-1} + \ldots = \varepsilon_t + d_1 \varepsilon_{t-1} + \ldots$$

and, extending the standard BJ model we allow for Gaussian heteroskedastic innovations, i.e. we suppose that the ε_t are Gaussian with zero mean and conditional variance h_t given by a function of previous innovations

$$h_t = v_0 + v_1 \varepsilon_{t-1}^2 + \ldots$$

In other words, the uncertainty of innovation ε_t depends on the squared size of the previous innovations.

2.4 Smoothing

In this subsection, we discuss methods for reducing the measurements uncertainty and variations

after eliminating possible observed environmental effects by adjusted readings $y_t - g(u_t)$ as in the previous subsection 2.1. In terms of the model of equation (1), we are then interested in filtering the stochastic components $\pi_t + \zeta_t$ and, to do this, we use local polynomial smoothing (see e.g. Fan and Gijbels, 1996).

The basic idea of this approach is based on a local approximation of the smooth function μ_t, so that at every time point t_0, we can approximate μ_t, for t close to t_0, using a first order series expansion giving the following linear function of time:

$$\mu(t) = a(t_0) + b(t_0)(t - t_0). \tag{5}$$

The coefficient function $a(t)$ and $b(t)$ represent the local level and local first derivative respectively. If μ_t is the quantity of interest it is then given by $\mu_t = a(t)$ where $a(t)$ and $b(t)$ are estimated for every point t in an appropriate grid by repeatedly minimizing a square sum weighted by the rate between a kernel function, which provides smoothing, and a skedastic function similar to subsection 2.1, which provides robustness against heavily tailed errors (see Fassò and Locatelli, 2002). These estimates will be acronimized by LLS for Local (robustified) Least Squares.

If on the other hand, the derivative $\partial\mu_t / \partial t$ is of interest then its optimal estimation is given by $\partial\mu_t / \partial t = b(t)$ where $b(t)$ is as above but in equation (5) a second order expansion is used.

Of course other methods can be used for smoothing. The simplest approach is based on moving averages with a windows say of one day. It will be apparent from the next section that, due to non-Gaussianity, this approach is not reliable. We can also use moving medians and this will be shown to be an appropriate techniques alternative to LLS. Other techniques may be for example based on Kalman filtering but, once again, non Gaussian data prevent from standard implementation and robust filtering has to be used.

3 Case studies

3.1 Ara Pacis

The Ara Pacis is a historical monument in Rome dating back to the first century B.C., which has been monitored for measuring settlements of foundations and slabs during the recovery activities being carried out in the area.

The monitoring system consisted in a hydraulic settlement system with one reference vessel and a number of measuring points where vertical movements were measured in terms of height of a liquid head by means of pressure transducers. The hydraulic circuits were affected by temperature changes which caused fluctuation of the actual values, shadowing the real behaviour of the structure and the overall trend of the measurements.

By an off-line analysis of raw data on a statistical basis, using FieldStat software, it was possible to eliminate the thermal effect obtaining useful values.

To see this, Figure 1a represents the raw data which are level measurements (*mm*) taken every six hours from 26-Sep-2003 to 04-Feb-2004. The marked downward trend amounting to about 9 *mm* in 4 months is mainly due to the approximately linear relation with the measured temperature shown by Figure 1b.

Using the robust method of section 2.2, we get the following estimated thermal effect

$$g(u) = -11.62(0.074) + 0.926(0.0064)u$$

where, in brackets, the estimated standard deviations are reported.

In Figure 1c, the adjusted values $y_t - g(u_t)$ show that the thermal effect has been successfully removed and

$$y_t - g(u_t) = \mu^0 + \pi_t + \zeta_t$$

is now a stationary stochastic process with μ^0 which can be assumed constant over time and standard deviation $\sigma_{y-g} = 0.184$ which gives a relevant reduction of the total uncertainty of y amounting to $\sigma_y = 2.83$. Then the percentage of explained variance from the temperature is $R^2 = 97\%$ and the total field uncertainty in terms of σ is reduced to 6.5%.

The autocorrelation function of the erratic component $\pi_t + \zeta_t$ depicted in Figure 1d suggests a significant deviation from the simple white noise stochastic process and motivates the assumptions of equation (1).

3.2 Excavation monitoring

This case study is related to a measuring system for monitoring settlements of existing structures during the excavation for the foundations of an adjacent new multistorey building. The instrumentation which was used was a hydraulic settlement system consisting of a number of vessels interconnected by means of one hydraulic line (tube). The vessels were installed on walls of the building with different exposition to solar radiation. As a consequence, the hydraulic circuit was affected by temperature gradients and the measuring gauge inside the vessels were working at different temperatures.

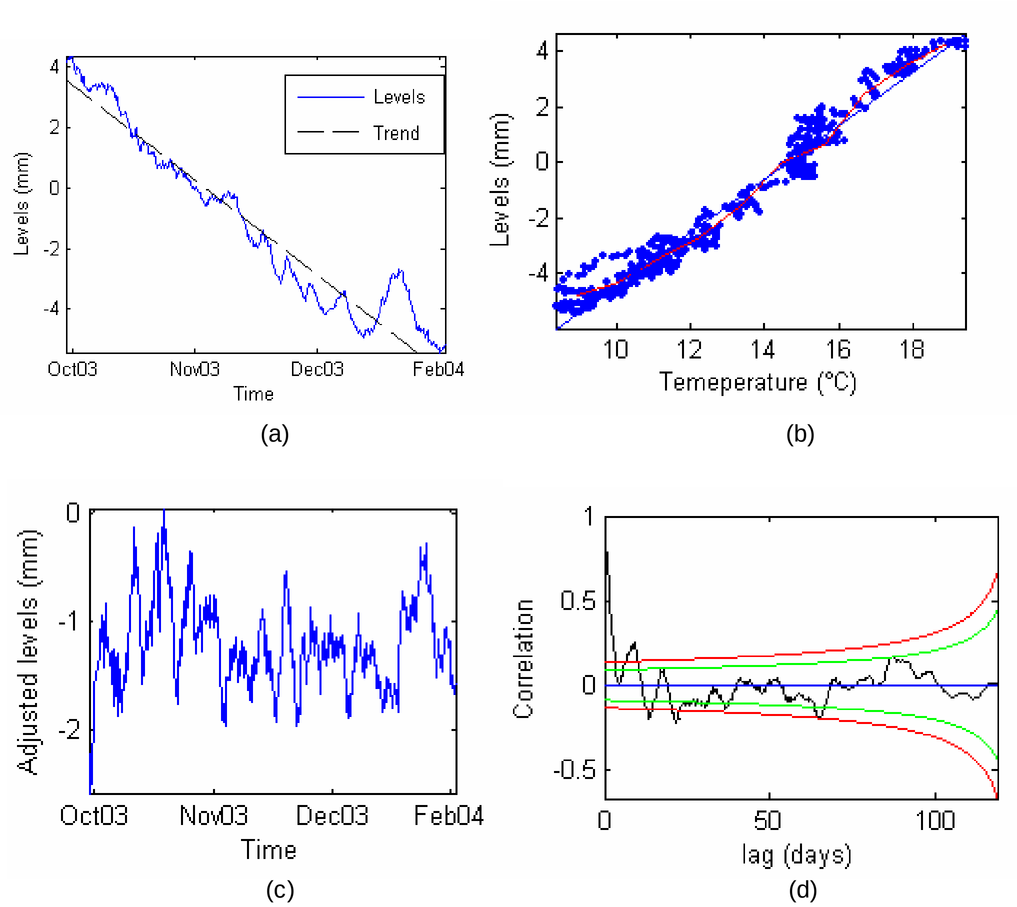
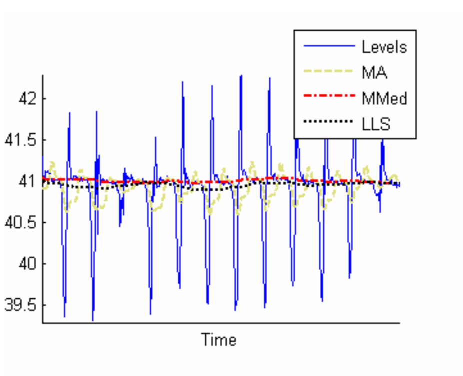
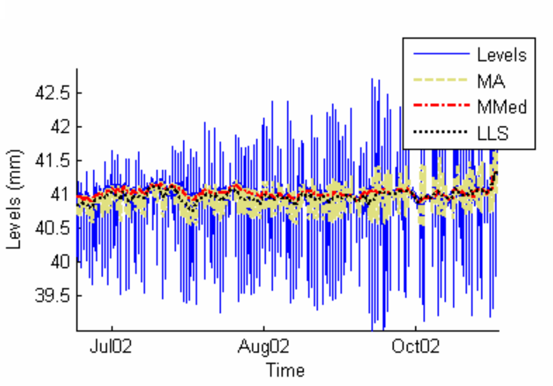


Figure 1. (a) Level measurements. (b) Temperature versus Level measurements. (c) Adjusted levels. (d) Autocorrelation function of the adjusted levels.

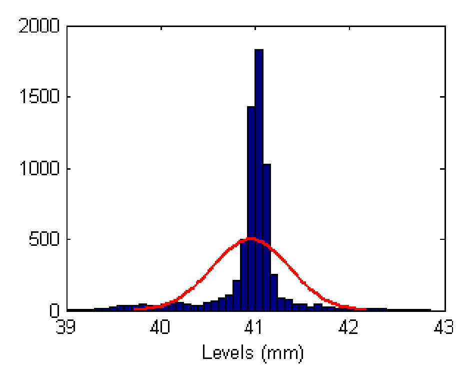
The raw data showed significant oscillation probably due to temperature change and other anthropic effects both daily and during the year. Moreover long term drift due to some leakage in the hydraulic circuit occurred. These effects made it almost impossible to set any reasonable threshold values for the alarm activation due to the on going activity.
The Figure 2b represents the raw data taken every 30 minutes from 21-Jun-2002 to 06-Nov-2002. The relation with measured temperature reported in Figure 2c does not show a clear variation in the mean level but suggests heteroskedasticity, i.e. varying uncertainty. Hence the linear model of section 2.2 used in the previous case study does not work here. As a matter of fact an uncertainty decomposition for heteroskedastic models is considered in Fassò et Al. (2003). We postpone further discussion of this issue to the next subsection and, here, we consider the smoothing approach of section 2.3, i.e. we try to reduce the high frequency uncertainty without using the measured temperature.
Figure 2a, depicting 13 days data, shows the marked but non constant daily periodicity. Together with Figure 2b it also shows that both the Moving Median and the LLS smoothing eliminate these fluctuations and reduce the total field uncertainty to 15-17% (Table 1).



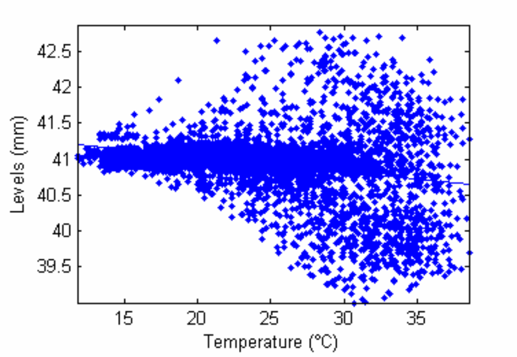
(a)



(b)



(c)



(d)

Figure 2. (a) Level measurements for 13 days with Moving Averages (MA), Moving Medians (MMed) and Local Linear Smoothing (LLS). (b) Level measurements taken every 30 minutes from 21-Jun-02 to -Nov-2002. (c) Histogram of the level measurements and the Normal

distribution. (d) Temperature versus Level measurements scatterplot.

Note that, due to the heavy tail dynamics, the moving average with the same one day windows have a weaker compensation effect. As can be expected the LLS smoother is a slightly more sensitive to asymmetric peaks than the moving median, giving the dotted LLS lines slightly lower then the dot-dashed ones in Figures 2a and b; they behave essentially the same for symmetric dynamics.

Table 1: Variances e % of Standard Deviations of Excavation monitoring

Uncertainty Decomposition		
	Variance	Std %
Raw Data	0.1670	100.0%
Moving Average	0.0267	39.9%
Moving Median	0.0039	15.3%
LLS	0.0047	16.8%

3.3 Load monitoring

A 20 m deep excavation in a sliding area presents a number of steel struts as temporary support for the concrete walls; the force acting on them had to be measured in order to evaluate both the effects of the excavation and of the landslide movements on the reinforced concrete walls.
For measuring the forces in the struts, electric load cells were installed between the struts and the retaining walls. See Figure 3.

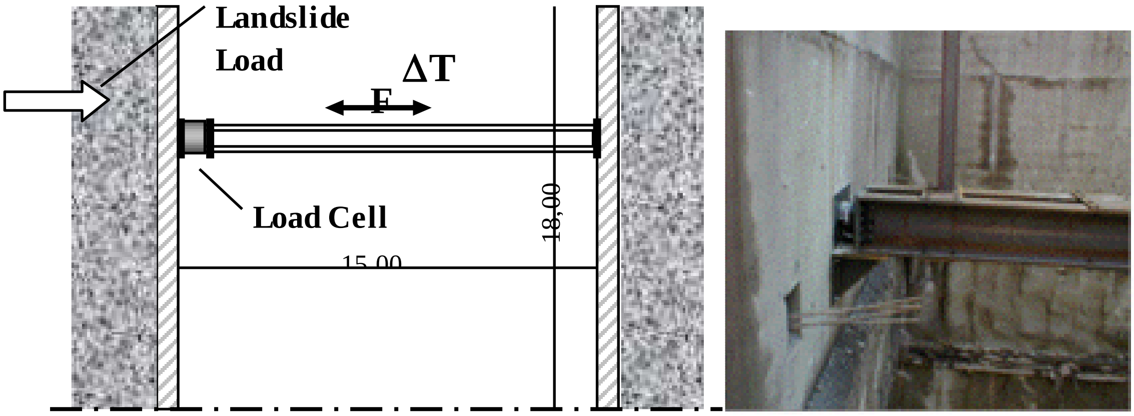


Figure 3. Outline and photography of measuring system for load monitoring

As expected, the force in the struts is highly influenced by the temperature which makes the struts to change their length and, therefore, as deformation is not allowed by the contrast on the retaining walls, the stresses into them change as a function of the temperature. The aim of the data analysis was to separate the two effects: apparent load due to temperature changes and real load due to landslide movements and consequent walls deformations.

In this case we have an example of variation of the physical state μ_t which is quite different from the apparent variation in y_t shown by Figure 4, where one year of hourly data for both load and temperature is divided by two vertical lines, the first on September 7 and the second on November 27.

From the measured load it is not clear what happened and spurious changes in load could be concluded. As a matter of fact, the relationship between load and temperature is better understood from Figure 5, which shows clearly that, whenever global linear correlation is close to zero, the relation is approximately linear before September 7 and the transient of Fall 2002 corresponds to a transient in the bidimensional data as already noted for other instrumentations by Ceccuci et Al. (2003).

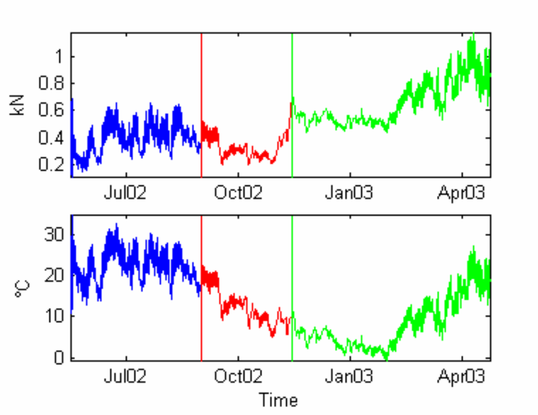


Figure 4: Load (kN) and Temperature (°C). Vertical lines on September 7 and November 27.

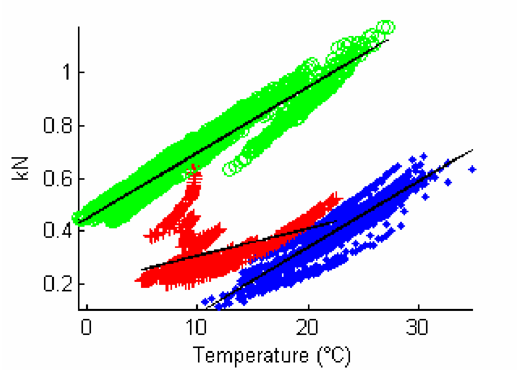


Figure 5: Load (kN) versus Temperature (°C). Black squares: before 9/7, grey stars between 9/7 and 11/27, light grey circles after 11/27.

Assuming that the system shift μ_t is supposed negligible until September 7, we used these data to get the following estimated dynamical relation between load and temperature

$$g(u_t) = -0.1575(0.015) + 0.0230(0.0003)u_{t-1} + 0.0037(0.0003)u_{t-2} + 0.0003(0.0003)u_{t-3} - 0.0020(0.0003)u_{t-4}$$

where, in brackets, the estimated standard deviations are reported. Let $e_t = y_t - g(u_t)$, then

$$\varepsilon_t + 1.019\ (0.09791)\ \varepsilon_{t-1} + 0.258\ (0.054)\ \varepsilon_{t-2}$$
$$= e_t - 0.2558\ (0.092)\ e_{t-1} - 0.4432\ (0.082)\ e_{t-2} - 0.2681\ (0.025)\ e_{t-12}$$

where the innovations ε_t have conditional variance h_t given by

$$h_t = 2.3\times 10^{-5} + 0.297\,(0.07)\,\varepsilon_{t-1}^2 + 0.327\,(0.08)\,\varepsilon_{t-12}^2$$

Using this model and exponential smoothing, we get the smoothed adjusted values

$$\tilde{y}_t = \lambda \tilde{y}_{t-1} + (1-\lambda)(y_t - g(u_t))$$

which are depicted in Figure 6, where we used the forgetting factor $\lambda = 0.95$. The horizontal lines reported there are the quantiles of $\tilde{y}_t$ at level the 0.005 and the 0.995 respectively, computed using the first two months of stationary data. These and similar quantities can be used to define the bands of "stationary uncertainty" and, using this approach we can apply "change detection algorithms" (e.g. Fassò, 1997 and 1999 and references therein) to geotechnical monitoring in order to get warning and/or alarming signals with a specified rate of false warning and/or alarms.

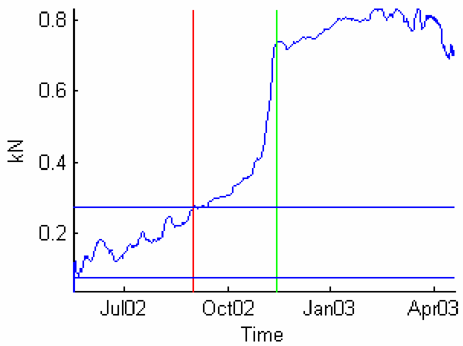


Figure 6: Smoothed-adjusted loads.

Figure 6 clearly shows that the first period, before September 7, assumed stationary for estimation purposes, actually, is characterized by a slow increase in the corrected load. After this point, the load increase becomes exponentially steep until November 27 and, after that, the state comes to a new stationary state.

In Table 2, we have the variance components from equation (2) based on the above estimated BJ model. From the second column, assuming approximate stationarity before 9/7, we can see that adjusting the loads by temperature gives a reduction in terms of variance of R^2=92% and, correspondingly the total field uncertainty in terms of standard deviation is reduced to 28.3%. If one had used the simpler model (3) with the same data, then he would get the less marked reductions of R^2=81.4% and to 43.2% respectively.

Table 2. Variances for load model

Source	< Sept. 7	All data
y	0.0127	0.038
g(u)	0.0102	0.038
y-g(u)	0.0010	0.077
e	2.3E-05	

4 Software

The FieldStat software package has been developed for the standard Windows environment using MatLab as mathematical and statistical computation engine. It has been provided by an user-friendly graphical interface and it can be easily used by most operators. It only requires a minimum knowledge of statistics since it is application oriented, meaning that the decisions the operator has to take are related to engineering and practical aspects more than theoretical ones. On the other hand, the advanced user may configure models and algorithms in an assisted environment. FieldStat is actually working as an off-line processing package; the development program includes the possibility to have FieldStat as subroutine of general purpose Data Acquisition packages both for off-line and on-line analysis.

5 Conclusions

Data from geomechanical monitoring need to be validated. Validation means to eliminate from a series of data all those effects due to instrumental uncertainties, instability, fluctuations, drift and, moreover, to boundary and environmental conditions which can shadow the behaviour of the monitored phenomena increasing the overall uncertainty of the measurements.

In order to cope with these effects, a computerized tool has been developed using a statistical model based on a physical and engineering approach. This tool enables to eliminate the effect of the co-variates (i.e. temperature) under three basic conditions:

- the availability of a statistically significant number of observations or measurements, as well as a consistent historic data base;
- a data acquisition frequency significant with respect to the period of the covariates;
- the availability of significant values of the covariates in terms of frequency and location .

The results which have been obtained in the first stage are encouraging; attention has been put on the analysis of the thermal effects on field measurements in order to separate these effects from the physical ones that are the most important for the engineers who have to evaluate the behaviour of a material or a structure.

The overall field uncertainty, in terms of σ, has decreased to 6.5%, 16% and 28% for the three cases considered; therefore, for the mentioned cases, it has been possible to improve the understanding of the true behaviour of the phenomena helping engineers to take decisions.

The software package enables to analyse the behaviour of the phenomena and also to evaluate the performances of the monitoring systems with the aim of optimising their configuration and specification.

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