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A risk averse stochastic optimization model for power generation capacity expansion

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Abstract

We present a mathematical model for maximizing the benefit of a price-taker power producer who has to decide the power generation capacity expansion planning in a long time horizon under uncertainty of the main parameters. These parameters are the variable production costs of the power plants already owned by the producer as well as of the candidate plants of the new technologies among which to choose; the market electricity price along the horizon, as well as the price of green certificates and CO_2 emission permits; the potential market share that can be at hand for the power producer. These uncertainties are represented in a two stage scenario tree, so the model is a two stage stochastic integer optimization one, subject to technical constraints, market opportunities and budgetarial constraints, whose first stage variables represent the number of new power plants for each candidate technology to be added to the existing generation mix (whose construction has to start in) every year of the planning horizon. The second stage variables (i.e., scenario dependent) are the continuous operation variables of all power plants in the generation mix along the time horizon. We start presenting the maximization of the net present value of the expected profit over the scenarios along the time horizon (i.e., considering the so named risk neutral strategy). Alternatively, we consider different risk averse strategies (i.e., Conditional Value at Risk, Shortfall Probability, Expected Shortage and First- and Second-order Stochastic

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Dominance constraint integer-recourse strategies). By using a pilot case we report the main results of considering the six strategies under different hypotheses of the available budget, analysing the impact of each risk averse strategy on the expected profit. For that purpose we use a state-of-the-art MIP solver, concluding that

1. the technical advantage of replacing the risk neutral with the risk averse strategies needs a substantial increase in the computing requirements, but it strongly reduces the risk of non-wanted scenarios at a price of a relatively small reduction on the expected profit;
2. the risk averse strategies considered provide consistent solutions, since for all of them the optimal generation mix mainly consists of conventional thermal power plants, for low risk aversion, which are replaced by renewable energy sources plants, as risk aversion increases;
3. it is mandatory to replace the plain use of the solver with ad-hoc decomposition algorithms that have the additional feature of tackling cross-scenario constraints.

Keywords: price-taker producer, generation expansion, long term planning, competitive electricity market, green certificates, CO_2 emission permits, Renewable Energy Source, uncertainty, scenario tree, two stage stochastic mixed integer optimization, risk neutral and risk averse strategies

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1. Introduction

The incremental selection of power generation capacity is of great importance for energy planners. In this paper we deal with the case of the single power producer's generator (GENCO) point of view, i.e. we want to determine the optimal mix of different technologies for electricity generation, ranging from coal, nuclear and combined cycle gas turbine to hydroelectric, wind and photovoltaic, taking into account the existing plants, the cost of investment in new plants, the maintenance cost, the purchase and sales of CO_2 emission trading certificates and green certificates to satisfy regulatory requirements over a long term planning horizon consisting of a set I of years (generally, 30 or more years). Uncertainty of prices (fuels, electricity, CO_2 emission permits and Green Certificates) should be taken into account: see [5]. However, in practice, the producer generally uses a standard business tool to compute the convenience of investments in new power generation technology, so named Levelized Cost of Electricity (LCoE), to find the technology that gives the lowest price of electricity or, equivalently, the net present value of the investments equals to zero. In this work, we assume that the producer is a price-taker, i.e. he cannot influence the price of electricity and we propose various models for maximizing the net present value of the profit in order to find an optimal trade-off between the expected profit (should it be the unique term in the objective function to maximize, it is so named risk neutral strategy) and the risk of getting a negative impact on the solution's profit due to a not-wanted scenario to occur. So this weighted mixture of expected profit maximization and risk minimization undoubtedly may be perceived as a more general model than LCoE. See some approaches in [2, 5, 6, 8, 9, 10, 13, 15], among others.

Our approach is closely related to [15] but we differ from it in the following items, that are the main contributions of this work:

1. Consideration of fixed and variable costs in the objective function.
2. Design of a risk-averse two-stage stochastic mixed-integer optimization model, by using different risk measures, such as Conditional Value at Risk (*CVaR*), Shortfall Probability (*SP*), Expected Shortage (*ES*), First-order Stochastic Dominance constraints (*FSD*) and Second-order Stochastic Dominance constraints (*SSD*), as an extension of our previous work, see [23].
3. A computational comparison of the risk measures strategies, so as to determine the reduction of expected net profit due to risk minimization: all the risk averse strategies considered increase the preference for production from renewable energy sources, as risk aversion increases: provide consistent solutions, since for all of them the optimal generation mix mainly consists of conventional thermal power plants, for low risk aversion, which are replaced by renewable energy sources plants, as risk aversion increases;
4. Given the problem dimensions, the plain use of even state-of-the-art MIP solvers need an unaffordable computing requirement (in memory and elapsed time), so ad-hoc decomposition algorithms are needed for practical applications and, in fact, it is a piece of our future work.

The remainder of the paper is organized as follows. Section 2 presents the main concepts and notation to be used in our two-stage mixed-integer linear stochastic programming approach, including the risk aversion strategies to be used. Section 3 specializes the risk aversion strategies presented in Section 2 to the generation capacity expansion problem subject of this work. Section 4 presents the case study. Section 5 reports the computational experience on using the risk aversion strategies considered in this work to analysing the hedging effect against negative impact on the profit in case of non-wanted scenarios in the generation capacity expansion. Section 6 concludes and outlines our future research work.

2. Risk-neutral two-stage stochastic mixed integer model for the generation capacity expansion problem

This work presents a decision support model for a power producer who wants to determine the optimal planning for investment in power generation capacity in a long term horizon. The power producer operates in a liberalized electricity market, where rules are issued by the Regulatory Authorities with the aim of promoting the development of power production systems with reduced CO_2 emissions. Indeed, CO_2 emission allowances have to be bought by the power producer as a payment for the emitted CO_2. Moreover, the Green Certificate scheme supports power production from Renewable Energy Sources (RES), i.e. by geothermal, wind, biomass and hydro power plants, and penalizes production from conventional power plants, i.e. CCGT, coal and nuclear power plants. Indeed, every year a prescribed ratio is required between the electricity produced from RES and the total electricity produced. In case the actual ratio, attained in a given year, is less than the prescribed one, the power produces must buy Green Certificates, in order to satisfy the related constraint. On the contrary, when the yearly attained ratio is greater than the prescribed one, the power produces can sell Green Certificates in the market.

The power producer aims at maximizing his own profit over the planning horizon. Revenues from sale of electricity depend both on the electricity market price and on the amount of electricity sold. The latter is bounded above by the power producer's market share; it also depends on the number of operating hours per year characterizing each power plant in the production system.

Costs greatly differ among the production technologies. Investment costs depend both on the plant rated power and on the investment costs per power unit. Typically, for thermal power plants rated powers are higher and unit investment costs are lower than for RES power plants. Variable generation costs for conventional power plants highly depend on fuel pricing.

Revenues and costs associated to the Green Certificate scheme depend on the Green Certificate price, as well as on the yearly ratio between production from RES and total annual production of the producer. Finally, costs for emitting CO_2 depend on the price of the emission allowances, as well as on the amount of CO_2 emitted, that greatly varies among the production technologies.

We notice that the evolution of prices along the planning horizon is not known at the time when the investment decisions have to be done. The model presented in this paper takes into account the risk associated with the capacity expansion problem due to the uncertainty of prices, as well as the uncertainty of market share. Uncertainty is included in the model by means of scenarios that represent different hypotheses on the future evolution of the market share and of the prices of electricity, Green Certificates, CO_2 emission allowances and fuels. The proposed decision support model determines the evolution of the production system along the planning horizon, taking into account all scenarios representing the uncertainty. Since construction time and industrial life greatly vary among power plants of different technology, the model determines the number of power plants for each technology, whose construction is to be started in every year of the planning period. Each new power plant is then availble for production when its construction is over and its industrial life is not ended. The annual electricity production of each power plant in the production system, and the corresponding Green Certificates and CO_2 emission allowances, are determined for each year of the planning horizon under each scenario into consideration.

In order to introduce the risk neutral stochastic mixed integer linear programming model, let the following notation be defined:

Sets.

J^T	set of candidate technologies for thermal power production
J^R	set of candidate technologies for power production from RES
J	set of all candidate technologies (i.e., $J = J^T \cup J^R$)
K^T	set of thermal power plants owned by the power producer at the beginning of the planning horizon (i.e. year 0)
K^R	set of power plants using RES owned by the power producer in year 0
K	set of all power plants owned in year 0 (i.e., $K = K^T \cup K^R$)
I	set of years in the planning horizon
Ω	set of scenarios

Deterministic parameters.

S_j $[years]$	construction time of a power plant of candidate technology $j \in J$
L_j^J $[years]$	industrial life of a power plant of candidate technology $j \in J$
$\overline{Z}_j$ $[-]$	number of sites ready for constructing a power plant of candidate technology $j \in J$
P_j^J $[MW]$	rated power of a power plant of candidate technology $j \in J$
H_j^J $[h]$	operating hours per year of a power plant of candidate technology $j \in J$

ν_j^J	$[-]$	percentage of loss of a power plant of technology $j \in J$
$\overline{E}_{j,i}^J$	$[GWh]$	maximum energy that can be produced by a power plant of technology $j \in J$ in year $i \in I$
θ_j^J	$[t/GWh]$	CO_2 emission rate of a thermal power plant of candidate technology $j \in J^T$
I_j	$[MEUR/MW]$	investment cost of a power plant of candidate technology $j \in J$
R_j	$[kEUR]$	annualized investment cost of a power plant of candidate technology $j \in J$
f_j^J	$[kEUR]$	fixed production cost of a power plant of technology $j \in J$
v_j^J	$[kEUR]$	variable production unit cost of a RES power plant of candidate technology $j \in J^R$
L_k^K	$[years]$	residual life of power plant $k \in K$ owned by the power producer in year 0
P_k^K	$[MW]$	rated power of power plant $k \in K$
H_k^K	$[h]$	operating hours per year of power plant $k \in K$
ν_k^K	$[-]$	percentage of loss of power plant $k \in K$
$\overline{E}_{k,i}^K$	$[GWh]$	maximum energy that can be produced by power plant $k \in K$ in year $i \in I$
θ_k^K	$[t/GWh]$	CO_2 emission rate of thermal power plant $k \in K^T$
f_k^K	$[kEUR]$	fixed production cost of power plant $k \in K$
v_k^K	$[kEUR]$	variable production unit cost of RES power plant $k \in K^R$
β_i	$[-]$	ratio "electricity from RES / total electricity produced" to be attained in year $i \in I$
B	$[MEUR]$	budget available
r	$[-]$	interest rate

Uncertain parameters under scenario $\omega \in \Omega$.

$v_{j,\omega}^J$	$[kEUR]$	variable production unit cost of a thermal power plant of candidate technology $j \in J^T$, where the fuel cost is the main component
$v_{k,\omega}^K$	$[kEUR]$	variable production unit cost of thermal power plant $k \in K^T$, where the fuel cost is the main component
$\pi_{i,\omega}^E$	$[kEUR/GWh]$	market electricity unit price in year $i \in I$
$\pi_{i,\omega}^{GC}$	$[kEUR/GWh]$	Green Certificate unit price in year $i \in I$
$\pi_{i,\omega}^{CO_2}$	$[kEUR/t]$	CO_2 emission permit unit price in year $i \in I$
$\overline{M}_{i,\omega}$	$[GWh]$	market share in year $i \in I$

p_ω	$[-]$	probability (or weight) assigned by the modeler

Decision variables.

First-stage variables related to the power system evolution:

$w_{j,i}$	$[-]$	number of power plants of candidate technology $j \in J$ whose construction starts in year $i \in I$
$W_{j,i}$	$[-]$	number of power plants of candidate technology $j \in J$ available for production in year $i \in I$

Second-stage (i.e. dependent on scenario $\omega \in \Omega$) operation variables :

$E^J_{j,i,\omega}$	$[GWh]$	electricity produced by all new power plants of technology $j \in J$ in year $i \in I$
$E^K_{k,i,\omega}$	$[GWh]$	electricity produced in year $i \in I$ by power plant $k \in K$ (already owned by the power producer in year 0)
$G_{i,\omega}$	$[GWh]$	green certificates sold ($G_{i,\omega} > 0$) or bought ($G_{i,\omega} < 0$) in year $i \in I$
$Q_{i,\omega}$	$[t]$	CO_2 produced in year $i \in I$.

The two-stage stochastic mixed-integer model determines the optimal generation expansion plan, which is given by the values of the variables

$$w_{j,i} \in \mathbb{Z}_+ \qquad \forall\ j \in J\ ,\ \ i \in I\ , \tag{1}$$

that represent the number of new power plants of technology $j \in J$ whose construction is to start in year i along the planning horizon.

The optimal generation expansion plan must satisfy the following constraints. For every candidate technology j the total number of new power plants constructed along the planning horizon is bounded above by the number $\overline{Z}_j$ of sites ready for construction of new power plants of that technology, i.e. sites for which all the necessary administrative permits have been released,

$$\sum_{i \in I} w_{j,i} \leq \overline{Z}_j \qquad \forall\ j \in J\ . \tag{2}$$

Notice that the upper bound $\overline{Z}_{j,i}$ to $w_{j,i}$ could alternatively be considered.

The new power plants of technology j available for production in year i are those for which both the construction is over and the industrial life is not ended, i.e. the number $W_{j,i}$ of new power plants of technology j available for production in year i can be expressed

$$W_{j,i} = \sum_{i-(S_j+L_j^J)+1 \leq l \leq i-S_j} w_{j,l} \qquad \forall\ j \in J\ ,\ \ i \in I\ . \tag{3}$$

The sum of the present annual debt repayments, corresponding to the number of new power plants of each technology j available for production in every year i, is required not to exceed the available budget

$$\sum_{i \in I} \frac{1}{(1+r)^i} \left(\sum_{j \in J} R_j W_{j,i} \right) \leq B\ , \tag{4}$$

where the term $R_j W_{j,i}$ represents the annual debt repayment (i.e. the annualized investment cost) for investment in new power plants of technology j and R_j is computed by the usual formula for the periodic payment in an annuity along L_j^J years, i.e.

$$R_j = \frac{I_j P_j^J \cdot r \cdot 1000}{1 - (\frac{1}{1+r})^{L_j^J}}\ . \tag{5}$$

The annual electricity production obtained by all new power plants of technology j is nonnegative and bounded above by the number of new plants available for production in year i times the maximum annual production $\overline{E}_{j,i}^J$ of a plant plant of technology j, such that

$$0 \leq E_{j,i,\omega}^J \leq \overline{E}_{j,i}^J \cdot W_{j,i} \qquad \forall\ j \in J\ ,\ \ i \in I\ ,\ \ \omega \in \Omega\ , \tag{6}$$

where the maximum annual production $\overline{E}_{j,i}^J$ of a power plant of technology j is defined as

$$\overline{E}_{j,i}^J = \frac{1}{1000} P_j^J H_j^J (1 - \nu_j^J)\ . \tag{7}$$

The annual electricity production of power plant k is nonnegative and bounded above by the maximum annual production in year i

$$0 \leq E_{k,i,\omega}^K \leq \overline{E}_{k,i}^K \qquad \forall\ k \in K\ ,\ \ i \in I\ ,\ \ \omega \in \Omega\ , \tag{8}$$

where

$$\overline{E}^K_{k,i} = \begin{cases} \frac{1}{1000} \cdot P^K_k \cdot H^K_k \cdot (1 - \nu^K_k) & \text{if} \quad i \leq L^K_k \\ \\ 0 & \text{if} \quad i > L^K_k \ . \end{cases} \tag{9}$$

The parameters H^J_j and H^K_k take into account possible plant breakdown and maintenance. Notice that for some technologies a lower bound to the annual electricity production could be imposed, if it is selected, in order to take into account technical limitations.

The electricity generated in year i cannot exceed the power producer's market share under scenario ω

$$\sum_{j \in J} E^J_{j,i,\omega} + \sum_{k \in K} E^K_{k,i,\omega} \leq \overline{M}_{i,\omega} \qquad \forall \ i \in I \ , \ \ \omega \in \Omega \ . \tag{10}$$

The amount of electricity $G_{i,\omega}$ for which the corresponding Green Certificates are bought, if $G_{i,\omega} \leq 0$, or sold, if $G_{i,\omega} \geq 0$ in year i under scenario ω can be expressed

$$G_{i,\omega} = \sum_{j \in J^R} E^J_{j,i,\omega} + \sum_{k \in K^R} E^K_{k,i,\omega} - \beta_i \left(\sum_{j \in J} E^J_{j,i,\omega} + \sum_{k \in K} E^K_{k,i,\omega} \right) \qquad \forall \ i \in I \ , \ \ \omega \in \Omega \ , \tag{11}$$

where β_i is the ratio, required in year i, between the electricity produced from RES and the total electricity produced.

The amount $Q_{i,\omega}$ of CO_2 emissions the power producer must pay for in year i under scenario ω can be expressed

$$Q_{i,\omega} = \sum_{j \in J^T} \theta^J_j \cdot E^J_{j,i,\omega} + \sum_{k \in K^T} \theta^K_k \cdot E^K_{k,i,\omega} \qquad \forall \ i \in I \ , \ \ \omega \in \Omega \ , \tag{12}$$

where θ^K_k and θ^J_j are the CO_2 emission rates of thermal power plant $k \in K^T$ and thermal power plant of candidate technology $j \in J^T$, respectively.

The annual profit F_ω under scenario $\omega \in \Omega$ can be expressed

$$F_\omega = \sum_{i \in I} \frac{1}{(1+r)^i} \cdot \left[\pi^E_{i,\omega} \left(\sum_{j \in J} E^J_{j,i,\omega} + \sum_{k \in K} E^K_{k,i,\omega} \right) + \pi^{GC}_{i,\omega} G_{i,\omega} - \pi^{CO_2}_{i,\omega} Q_{i,\omega} + \right.$$
$$- \sum_{j \in J^T} v^J_{j,\omega} E^J_{j,i,\omega} - \sum_{j \in J^R} v^J_j E^J_{j,i,\omega} - \sum_{j \in J} \left(f^J_j + R_j \right) W_{j,i} \quad +$$
$$\left. - \sum_{k \in K^T} v^K_{k,\omega} E^K_{k,i,\omega} - \sum_{k \in K^R} v^K_k E^K_{k,i,\omega} - \sum_{k \in K} f^K_k \right] , \tag{13}$$

where, for year i under scenario ω, $\pi^E_{i,\omega}$ is the electricity market price, $\pi^{GC}_{i,\omega}$ is the Green Certificate price and $\pi^{CO_2}_{i,\omega}$ is the CO_2 emission permit price. In order to take into account explicitly the uncertainty in fuel prices, different scenarios are considered for the variable production cost $v^K_{k,\omega}$ of thermal power plant $k \in K^T$ and for the variable production cost $v^J_{j,\omega}$ of a thermal power plant of candidate technology $j \in J^T$. The variable production cost v^K_k of RES power plant $k \in K^R$ and the variable production cost v^J_j of a RES power plant of candidate technology $j \in J^R$ are assumed to be known with certainty. The parameters f^K_k and f^J_j represent the fixed production costs of power plant $k \in K$ and of a power plant of technology $j \in J$, respectively.

In the risk neutral approach the expected profit over scenarios ω

$$\sum_{\omega \in \Omega} p_\omega \cdot F_\omega \tag{14}$$

is maximized subject to constraints (1)-(4), (6), (8) and (10)-(12).

3. Risk aversion strategies in stochastic modeling for power generation capacity expansion

The risk neutral approach determines the values of the decision variables that maximize, over all scenarios, the expected profit along the planning horizon. It does not take into account the variability of the objective function value over the scenarios and, then, the possibility of realizing in some scenarios a very low profit. In the literature several approches have been introduced for measuring the profit risk. These measures can be divided into three groups, on the basis of the information the user must assign

1. a confidence level: Value-at-Risk and Conditional-Value-at-Risk;
2. a profit threshold: Shortfall Probability and Expected Shortage;
3. a benchmark: First-order and Second-order Stochastic Dominance.

In this section we consider five of the above mentioned risk measures and how they can be used to reduce the risk of having low profits in some scenarios.

3.1. Risk aversion strategy 1: Conditional Value at Risk CVaR

Given the confidence level α, the Conditional Value at Risk (*CVaR*) of profit, see [14, 19, 20, 22], is the defined as

$$\max_{V} \left[V - \frac{1}{\alpha} \cdot \left(\sum_{\omega \in \Omega} p_\omega d_\omega \right) \right] \tag{15}$$

The auxiliary variables $V\ [kEUR]$ and $d_\omega\ [kEUR]$, $\omega \in \Omega$, used for computing the *CVaR*, are defined by constraints

$$d_\omega \geq V - F_\omega \qquad \text{with} \qquad d_\omega \geq 0\ , \quad \forall \omega \in \Omega\ , \tag{16}$$

and V in the optimal solution representing the Value at Risk (*VaR*). In order to use the *CVaR* of profit as a risk aversion strategy, the objective function can be expressed

$$\max \left\{ (1-\rho) \left(\sum_{\omega \in \Omega} p_\omega F_\omega \right) + \rho \left[V - \frac{1}{\alpha} \left(\sum_{\omega \in \Omega} p_\omega d_\omega \right) \right] \right\} \tag{17}$$

where $\rho \in [0,1]$ is the *CVaR* risk averse weight factor. In a different context, see [1, 3, 7, 18].

3.2. Risk aversion strategy 2: Shortfall Probability (SP)

Given a profit threshold ϕ, the shortfall probability, see [21], is the probability of the scenario to occur having a profit smaller than ϕ, i.e.

$$\sum_{\omega \in \Omega} p_\omega \mu_\omega \tag{18}$$

where μ_ω is a 0-1 variable defined by the constraint

$$\phi - F_\omega \leq M_\omega \mu_\omega, \qquad \forall \omega \in \Omega\ , \tag{19}$$

where variable μ_ω takes value 1 if $F_\omega < \phi$, i.e. if ω is a non-wanted scenario, and M_ω is a small enough constant such that it does not prevent any feasible solution to the problem, being this parameter crucial for the computational effort required to obtain the optimal solution. The profit risk can be hedged by simultaneously pursuing expected profit maximization and shortfall probability minimization: this is done by maximizing the objective function, where the weighting factor $\rho \in [0, 1]$,

$$(1-\rho)\left(\sum_{\omega\in\Omega} p_\omega F_\omega\right) - \rho\left(\sum_{\omega\in\Omega} p_\omega \mu_\omega\right) \tag{20}$$

subject to constraints (1)-(4), (6), (8), (10)-(12) and (19). In a different context, see [1].

3.3. Risk aversion strategy 3: Expected Shortage (ES)

Given a profit threshold ϕ, the expected shortage, see [9], is given by

$$\sum_{\omega\in\Omega} p_\omega d_\omega \tag{21}$$

where d_ω satisfies constraint

$$\phi - F_\omega \leq d_\omega \qquad \text{with} \qquad d_\omega \geq 0\,, \quad \forall \omega \in \Omega\,. \tag{22}$$

The profit risk can be hedged by simultaneously pursuing expected profit maximization and expected shortage minimization: this is done by maximizing the objective function, with $\rho \in [0, 1]$,

$$(1-\rho)\left(\sum_{\omega\in\Omega} p_\omega F_\omega\right) - \rho\left(\sum_{\omega\in\Omega} p_\omega d_\omega\right) \tag{23}$$

subject to constraints (1)-(4), (6), (8), (10)-(12) and (22).

3.4. Risk aversion strategy 4: First-order Stochastic Dominance (FSD)

A benchmark is given by assigning a set P of profiles (ϕ^p, τ^p), $p \in P$, where ϕ^p is the threshold to be satisfied by the profit at each scenario and τ^p is the upper bound of its failure probability. The profit risk is hedged by maximizing the expected value of profit (14), while satisfying constraints (1)-(4), (6), (8) and

(10)-(12), as well as the so named first-order stochastic dominance constraints, see [4, 17],

$$\sum_{\omega \in \Omega} p_\omega \mu^p_\omega \leq \tau^p \qquad \forall p \in P \;, \tag{24}$$

where the 0-1 variables μ^p_ω, by which the shortfall probability with respect to threshold ϕ^p is computed, are defined by constraints

$$\phi^p - F_\omega \leq M_\omega \mu^p_\omega \qquad \forall \; p \in P \;, \; \omega \in \Omega \;, \tag{25}$$

with

$$\mu^p_\omega \in \{0,1\} \qquad \forall \; p \in P \;, \; \omega \in \Omega \;. \tag{26}$$

Notice that the constraints (24) belong to the cross scenario constraint type and, then, they introduce an additional difficulty in the decomposition algorithms, see in [11] a multistage algorithm for dealing with this type of constraints.

3.5. Risk aversion strategy 5: Second-order Stochastic Dominance (SSD)

A benchmark is assigned by defining a set of profiles (ϕ^p, e^p), $p \in P$, where ϕ^p denotes the threshold to be satisfied by the profit at each scenario and e^p denotes the upper bound of the expected shortage over the scenarios. The profit risk is hedged by maximizing the expected value of profit (14), while satisfying constraints (1)-(4), (6), (8) and (10)-(12), as well as the so named Second-order Stochastic Dominance constraints, see [16],

$$\sum_{\omega \in \Omega} p_\omega \cdot d^p_\omega \leq e^p \qquad \forall p \in P \;, \tag{27}$$

where

$$\phi^p - F_\omega \leq d^p_\omega \qquad \forall \; p \in P \;, \; \omega \in \Omega \;, \tag{28}$$

with

$$d^p_\omega \geq 0 \qquad \forall \; p \in P \;, \; \omega \in \Omega \;. \tag{29}$$

This measure is closely related to the risk measure introduced in [13], where the threshold $p \in P$ is considered as the benchmark. In a different context see [3]. Notice that (27) is also a cross scenario constraint type, but the related variables are continuous ones. It is interesting to point out that the risk aversion strategies *FSD* and *SSD* have the important advantage that the expected profit maximization is hedged against a set of non-wanted scenarios, such that the hedging is represented by the requirement of forcing the scenario profit to be not smaller than a set of thresholds, with a bound on failure probability for each of them in strategy *FSD* and an upper bound on the expected shortage in strategy *SSD*. The price to be paid is the increase of the number of constraints and variables (being 0-1 variables in strategy *FSD*). Another disadvantage for specialized decomposition algorithms is the cross scenario constraints (24) and (27), as it was pointed out above. In any case, in this work we only analyze the results obtained by the model with the above presented risk measures by plain use of a MIP solver. For large-scale instances, decomposition approaches must be used, like the exact multistage Branch-and-Fix Coordination (*BFC-MS*) method [10], currently being expanded to allow constraint types (24) and (27), see [11], multistage metaheuristics, like Stochastic Dynamic Programming [6, 12], for very large scale instances, and Lagrangian heuristic approaches, see [16].

4. Case study

The stochastic model introduced in Section 2, jointly with the risk aversion strategies presented in Section 3, have been implemented in GAMS 23.2.1 and the CPLEX 12.1.0 solver has been used for computing the optimal solution. The model validation has been performed on a testbed of realistic instances. Here we report the results obtained for a power producer who owns in year 0 the medium-size generation system described in Table 1, where rated power, residual life and efficiency are reported for every power plant of the system. The generation system owned initially, with total capacity of 8629 MW, is biased toward the *CCGT* technology and is therefore highly dependent on the gas price. .

Table 1: Power plants owned by the power producer in year 0.

power plant k	type	rated power P_k^K $[MW]$	residual life $\hat{L}_k^K$ $[years]$	efficiency η_k^K $[-]$
1	*CCGT*	840	19	0.52
2	*CCGT*	425	18	0.52
3	*CCGT*	1090	16	0.50
4	*CCGT*	532	15	0.51
5	*CCGT*	75	9	0.52
6	*CCGT*	227	7	0.50
7	Coal	600	8	0.40
8	Coal	500	3	0.39
9	Coal	402	12	0.41

A planning horizon of 50 years is considered. Since in this paper we plainly use the MIP solver, we limit the computational time by considering only fuel price uncertainty. The electricity price and the Green Certificate price are 106 $€/MWh$ and 73.3 $€/MWh$ respectively, the CO_2 emission allowance price along the horizon is depicted in Fig. 1 and for the annual market share a 2% increase per year is assumed, which implies that the maximum amount of energy that can be sold to the market in 50 years is $4229\ TWh$.

Figure 1: Scenario for CO_2 price (EUA).

Table 2 reports measure unit, lower heating value, CO_2 emission rate and cost for each fuel type. In Table 3, for each technology available for new power plants the following data are reported: investment cost, rated power, operating hours per year, construction time, industrial life and efficiency.

Table 2: Thermal plant fuels: prices in year 0 and characteristics.

Fuel	Measure Unit $[m.u.]$	Fuel cost in year 0 $[€/m.u.]$	Lower heating value $[kWh/m.u.]$	Fuel cost in year 0 $[€/MWh]$	Fuel CO_2 emission rate $[t/GWh]$
Coal	$[t]$	115	8141	14.13	338
Gas	$[Nm^3]$	0.29	9.58	30.27	200
Nuclear	$[kg]$	2100	950171	2.21	0

Tables 4 and 5 provide six alternative values of the ratio ”estimated price over price in year 0” that are considered, together with the associated probabilities, for coal and gas, respectively. Analogously, Table 6 provides three alternative values and the associated probabilities for nuclear fuel. By combining all alternatives, 108 independent scenarios of fuel prices are obtained. As the number of scenarios is increased, the new 0-1 variables, necessary for modeling the risk measures shortfall probability and first-order stochastic dominance, substantially increase the computing time and decomposition algorithms must be used for obtaining the solution in an affordable computing time. The price $\pi_{i,\omega}^{CO_2}$ of the CO_2 emission allowances is assumed to increase along the years of the planning horizon. We consider only one scenario, see Fig. 1.

The ratio β_i ”electricity from RES / total electricity produced” is set according to the 20-20-20 European target from year 2011 to year 2020 (i.e. from year 1 to year 10 of the planning period); after 2020 a further increase is assumed as shown

Table 3: Characteristics of candidate technologies.

Technology	investment cost I_j $[M€/MW]$	rated power P_j^J $[MW]$	operating hours per year H_j^J $[h]$	construction time S_j $[years]$	industrial life L_j^J $[years]$	efficiency η_j^J $[-]$
Coal	1	600	7446	4	25	0.44
CCGT	0.47	800	7446	2	25	0.56
Nuclear	3.2	1200	7884	7	40	0.34
Biomass	3	20	6000	1	15	0.35
Wind	1.55	100	2215	1	20	–
Geothermal	3.5	40	7500	3	20	–
Mini hydro	3	1	3500	1	40	–
Photovoltaic	2.5	1	1500	1	25	–
Waste to energy	5	50	6100	1	20	0.25
Wind offshore	2.9	100	2500	2	20	–

Table 4: Alternative values of "estimated price over price in year 0" ratio (coal price in year 0: 115 $€/t$ (12.3 $€/MWh$)

ratio	0.786	0.9	1.00	1.246	1.6571	2.0
probability	0.035	0.1	0.565	0.26	0.035	0.005

Table 5: Alternative values of "estimated price over price in year 0" ratio (gas price in year 0: 0.3 $€/Nm^3$ (31.3 $€/MWh$)

ratio	0.786	0.9	1.00	1.246	1.6571	2.0
probability	0.035	0.1	0.565	0.26	0.035	0.005

Table 6: Alternative values of "estimated price over price in year 0" ratio (nuclear fuel price in year 0: 2100 $€/kg$ (2.21 $€/MWh$)

ratio	0.8313	1.00	1.28
probability	0.08	0.72	0.2

in Fig. 2. The GenCo can satisfy the imposed β_i ratio by either producing from RES or buying Green Certificates. However, according to Decree 3 March 2011, in Italy this incentive will be substituted by a *feed-in tariff* (or a *feed-in premium*), to be applied to renewable energy sold in the Italian electricity market.

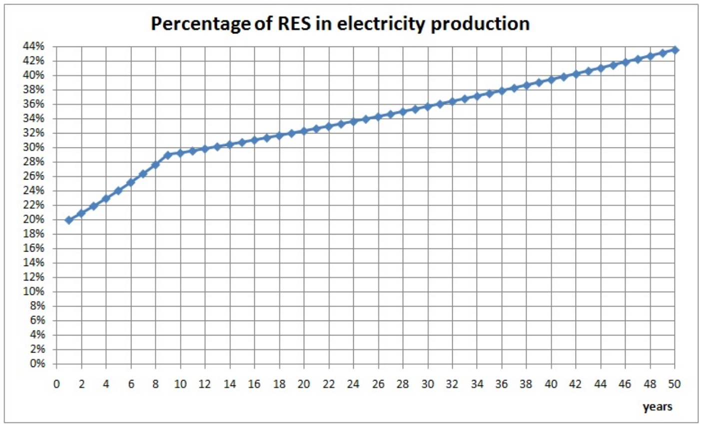

Figure 2: Assumed scenario for ratio β_i ”electricity from RES / total electricity produced”.

5. Numerical results

The HW/SW platform used for the numerical experiments has the following characteristics: Intel Core i7-2670QM, 2.20 GHz, 8 GB RAM and the Microsoft Windows 7 Home Premium 64 bit. Only 4 parallel threads (4 cores out of 8) are used in CPLEX. In this section we study the influence on investments in new power plants by using different risk measures. In Table 7 the main characteristics of the risk measures under consideration are shown: two of them are characterized by a single threshold which is a parameter of the model, two of them are characterized by selecting a set of thresholds (benchmarks), while *CVaR* is defined by setting a value of the α-percentile. Moreover, two of them use additional 0-1 variables, while the other ones use continuous variables. The following functions of

Table 7: Classification of risk measures used.

	0-1 variables	continuous variables
α-percentile	–	*CVaR*
profit threshold	shortfall probability	expected shortage
Benchmark	*FSD*	*SSD*

the set cardinalities give the model dimensions in the risk neutral problem: the number of constraints is $m_{RN} = |T| \left(3 \left|J^T\right| + 3 \left|J^R\right| + 4\right) + \left|J^T\right| + \left|J^R\right| + 2$; the number of continuous variables is $nc_{RN} = |T| \left(3 \left|J^T\right| + 3 \left|J^R\right| + \left|K^T\right| + \left|K^R\right| + 3\right)$ and the number of discrete variables is $nd_{RN} = |T| \left(\left|J^T\right| + \left|J^R\right|\right)$. The number of constraints increases to $m_{RN} + |\Omega|$, for the models with risk aversion strategies $CVaR$, SP and ES, and to $m_{RN} + |P|\,(1 + |\Omega|)$, for the models with risk aversion strategies FSD and SSD. The number of continuous variables increases to $nc_{RN} + |\Omega|$, for the models with risk aversion strategies $CVaR$ and ES, and to $nc_{RN} + |P|\,|\Omega|$, for the models with risk aversion strategy SSD. The models with risk aversion strategies FSD and SP also require $|P|\,|\Omega|$ and $|\Omega|$ 0/1-variables respectively. Table 8 shows the model dimensions in our case study for each risk measure under consideration. The row headings are as follows: m, number of constraints; nc, number of continuous variables; nd, number of discrete variables; $n0/1$, number of 0-1 variables; nel, number of nonzero elements in the constraint matrix; and $den\,[\%]$, constraint matrix density. In the case study the range of the integer variables, i.e. the maximum number of new power plants of each technology available in the planning horizon are 20 CCGT plants, 30 coal plants, 15 nuclear plants and 60 wind power plants.

Table 8: Problem dimensions in the case study.

	m	nc	nd	$n0/1$	nel	$den(\%)$
risk neutral	1863	2451	550	550	16028	0.0035
SP	2430	3018	1117	1117	641996	0.0875
ES	2430	3018	550	550	641996	0.0875
CVaR	2430	3019	550	550	642564	0.0876
FSD	4703	5286	3385	3385	3145868	0.1265
SSD	4703	5286	550	550	3145868	0.1265

Initially we ran the model without existing plants, without any risk averse measure (i.e. maximizing the expected profit, using the risk neutral strategy) and with only one technology for each run, in order to evalute the Net Present Value of the investment in a single technology for a fixed budget of 3.84 G€ but with different electricity market prices, see Fig. 3.

In the figure we can observe that if the market price is fixed at 110 €$/MWh$, the most convienent technology is the CCGT, followed by Geothermal, Coal, Mini hydro, Wind, nuclear and then the other renewable technologies, which have low NPVs. As the electricity price decreses, the optimal solution changes: with a price of 100 €$/MWh$ the first technology is the Geothermal, followed by Mini hydro, Coal, CCGT, Wind, nuclear and the other renewable technologies. In the rest of the case study it is supposed that the technologies available to the power producer for new investment are the three thermal technologies and the wind power technology, see Fig. 4.

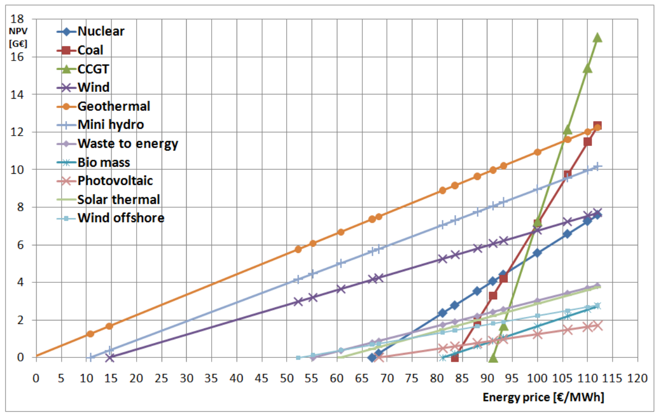

Figure 3: Net present value of optimal expected profit when investment is in a single technology.

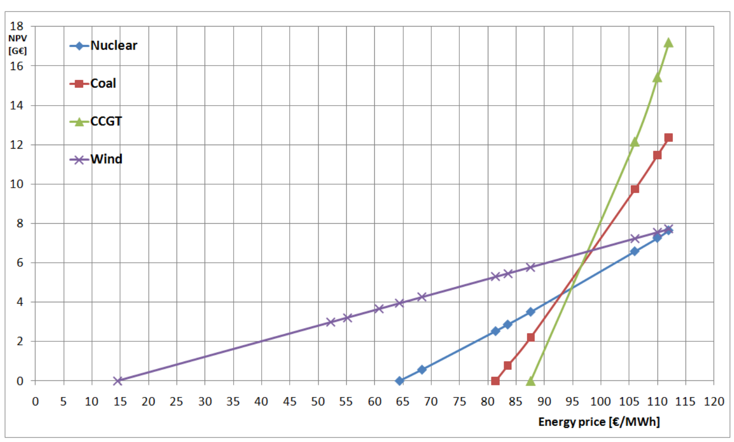

Figure 4: Net present value of optimal expected profit when investment is in a single technology (only the available technologies are shown)

The optimal technology mix obtained with different values of the initial budget and market electricity price are shown in Table 9, where all the results for profit are in G€. In order to avoit being biased, we assume that the generation capacity of the power plants owned by the producer are set to zero, in this way the resulting mixes are not affected by the existing installed capacity.

Table 9: Risk neutral mix depending on budget value and electricity price.

Electricity price €/MWh	Technology	Budget 2 $G€$	Budget 3.84 $G€$	Budget 7.68 $G€$	Budget 14.5 $G€$	Budget 30 $G€$
110	CCGT	6	11	20	14	4
	Nuclear	-	-	-	-	4
	Coal	-	-	5	14	20
	Wind	-	-	1	60	60
	Profits [$G€$]	10.464	20.471	30.459	42.668	63.936
106	CCGT	5	11	7	1	7
	Nuclear	-	-	-	-	5
	Coal	-	-	10	16	9
	Wind	-	-	-	60	60
	Profits [$G€$]	8.484	16.617	24.769	36.730	57.281
100	CCGT	5	9	2	-	1
	Nuclear	-	-	-	-	5
	Coal	-	1	12	14	7
	Wind	-	-	1	59	60
	Profits [$G€$]	5.717	11.013	18.584	30.078	48.506
95	CCGT	-	-	-	-	-
	Nuclear	-	-	-	-	5
	Coal	3	6	13	10	6
	Wind	2	3	1	58	60
	Profits [$G€$]	3.655	7.049	14.185	25.224	41.844

In case of high electricity price and low initial budget, CCGT plants are preferred as they are characterized by relatively high production per unit of investment cost. As the budget increases, we can buy more and more plants and we reach the market share so it results that the best decision consists of investing in technologies that are more capital intensive like Coal, Wind and Nuclear; for further budget increase the wind power plants are preferred and for very high budget, nuclear plants come to place. We can observe that nuclear plants appear only when the maximum number of wind plants (60, in this case) is reached. If the electricity market price is low, the optimal mix consists mainly of coal and wind plants due to their lower variable costs. Notice that for certain budgets (i.e.2 $G€$ and 30 $G€$) there is in some simulation of the electricity price a different number of plants of the same type; it is due to the fact that they are built in different years.

5.1. Numerical results with risk aversion strategy 1: Conditional Value at Risk (CVaR)

Let us consider a market electricity price of 106 $€/MWh$ and two values for the initial budget, in order to show its influence on the technology mix. The confidence level α used to compute the *CVaR* is 5%.
For the case corresponding to the budget $B = 3.84$ $G€$, the profit distributions are shown in Fig. 5.

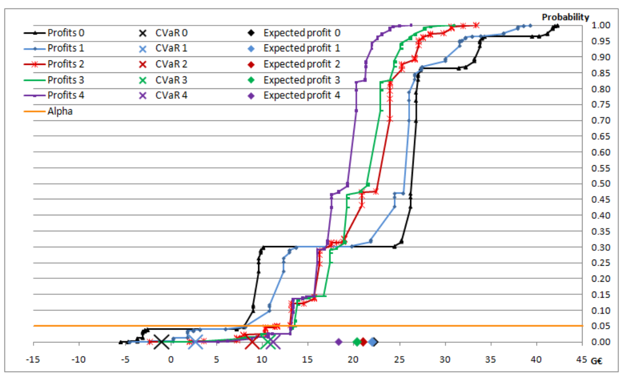

Figure 5: Profit distribution for different risk aversion parameters using CVaR with α equal to 5%, budget = 3.840 G€ and a market electricity price of 106 €/MWh.

In Table 10 we can observe a regular change in the technology mix as the risk aversion parameter increases. Notice the difference between the case with $\rho = 0$ (risk neutral) and the case with $\rho = 0.2$ (risk averse): the espected profit decreases by only 5% (1.154 $G€$), while the *CVaR* has grown by 9.926 $G€$. By further increasing the risk aversion parameter ($\rho = 0.3$ and $\rho = 0.8$), the corresponding optimal technology mixes have a small growth of the *CVaR* but have a big decrease in the expected profit.

For the case with a higher budget, say 14.5 $G€$, the profit distributions are shown in Fig. 6. The nuclear technology appears in the mix (see Table 11) only with high risk aversion ($\rho \geq 0.5$) and it represents a low percentage of the new capacity mix (see Fig. 7).

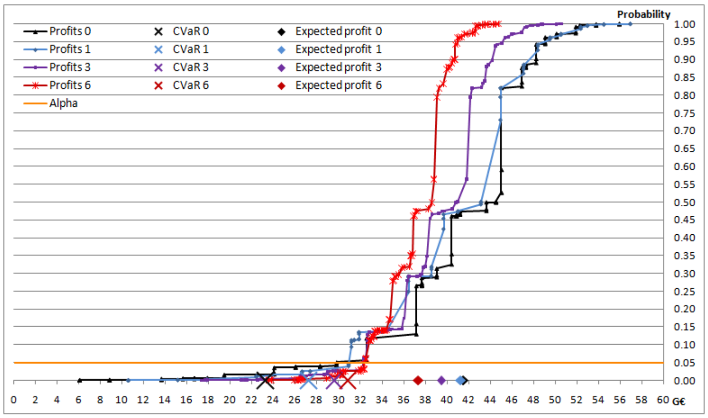

Figure 6: Profit distribution for different risk aversion parameters using CVaR with α equal to 5%, budget = 14.5 G€ and a market electricity price of 106 € /MWh.

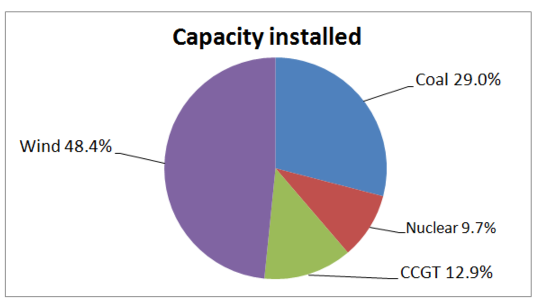

Figure 7: Percentage of new capacity installed for the case of Budget = 14500 € , CVaR and ρ=0.5.

Table 10: Technology mix obtained with *CVaR* and different risk aversion parameters - Budget: 3.84 $G€$.

Case	0	1	2	3	4
ρ	0	0.1	0.2	0.3	0.8
CCGT	13	9	5	1	0
coal	0	2	3	6	4
wind	1	0	0	1	10
expected profit [$G€$]	22.173	21.990	21.019	20.367	18.357
CVaR	-1.026	2.682	8.910	10.585	11.164
Z_{LP} [$G€$]	22.468	20.227	19.153	17.783	12.816
Z_S [$G€$]	22.171	20.061	18.619	17.456	12.609
Z_{MIP} [$G€$]	22.170	20.059	18.597	17.432	12.603
T_{LP} [s]	5.84	8.35	8.03	9.11	8.52
T_S [s]	0.08	0.08	0.09	0.19	0.08
T_{MIP} [s]	153.80	148.17	49.24	59.80	23.67

With a budget lower than 14.5 $G€$ the nuclear technology is not included in the optimal mix, due to the rated power of nuclear plants. Moreover it can be seen in Fig. 6 that for electricity prices less than 112 €MWh the *LCoE* associated to the nuclear technology is less than the *LCoE* associated to the wind power technology.

Table 11: Technology mix obtained with *CVaR* and different risk aversion parameters - Budget $B = 14.5$ $G€$.

Case	0	1	2	3	4	5	6
ρ	0	0.2	0.4	0.5	0.6	0.8	0.95
CCGT	17	10	8	6	5	4	3
nuclear	0	0	0	1	1	1	1
coal	1	6	5	2	1	0	0
wind	60	60	60	60	60	59	59
Expected profit	41.484	41.243	40.627	39.496	38.785	37.904	37.347
CVaR	23.234	27.227	28.439	29.584	30.205	30.768	30.824

5.2. Numerical results with risk aversion strategy 2: Shortfall Probability (SP)

Let us consider a market electricity price of 106 €MWh and an initial budget $B = 3.84$ $G€$ and assume the profit threshold $\phi = 10$ $G€$ has been assigned by the power produced. The risk neutral profit distribution obtained is depicted in Fig. 8.

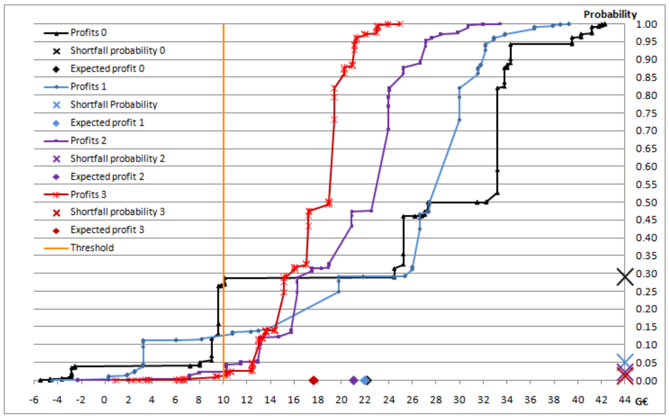

Figure 8: Profit distribution for different risk aversion parameters using the Shortfall Probability with ϕ equal to 10 G€ and budget = 3.840 G€ .

Table 12: Technology mix obtained with *SP* and different risk aversion parameters - Budget $B = 3.84$ $G€$.

Case	0	1	2	3
ρ	0	0.5	0.875	0.992
CCGT	13	9	3	0
coal	0	2	5	3
wind	1	0	0	14
Expected profit [$G€$]	22.173	21.990	21.028	17.650
shortfall probability	0.291	0.050	0.022	0.012
Z_{LP} [$G€$]	22.389	11.060	2.748	0.162
Z_S [$G€$]	22.177	9.591	2.438	0.022
Z_{MIP} [$G€$]	22.170	9.587	2.432	0.022
T_{LP} $[s]$	5.24	4.37	6.32	4.12
T_S $[s]$	0.09	0.09	0.08	0.06
T_{MIP} $[s]$	327.37	337.89	190.97	215.94

As expected, the increase of the risk aversion parameter ρ yields a decrease of both the expected profit and the shortfall probability as the technology mix changes. Indeed the gas technology is being decreased as the risk aversion parameter increases. At the same time the number of coal plants and wind power plants increases, see Table 12. The probability distributions depicted in Fig. 8 show 108 points corresponding to the profit values in the fuel price scenarios. Graphically, the shortfall probability is the probability of the intersection of the vertical segment (the threshold) with the distribution.

5.3. Numerical results with risk aversion strategy 3: Expected Shortage (ES)

For the case corresponding to the budget of 3.84 $G€$, we have defined the threshold profit on the basis of the risk neutral profit distributions, i.e. we fix $\phi = 10$ $G€$. In the risk neutral case the probability of having the profit under the threshold and the associated expected shortage has been 29.09% and 0.683 $G€$, respectively. Fig. 9 depicts the change in the profit distributions due to an increase of the risk aversion parameter ρ.

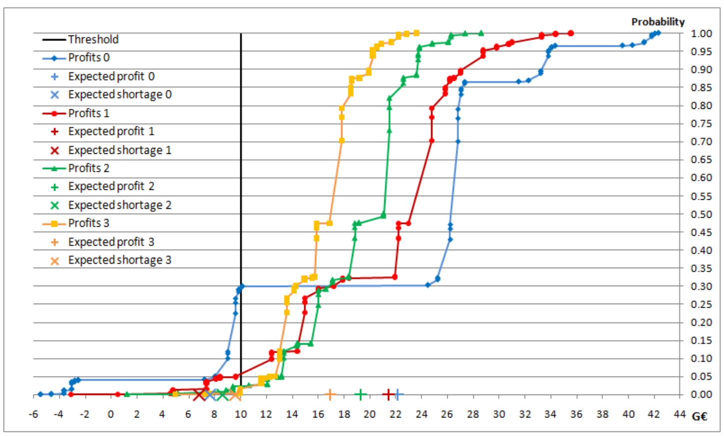

Figure 9: Profit distribution for different risk aversion parameters using the Expected shortage with ϕ equal to 10 G€ and budget = 3.840 G€.

Let us consider the case corresponding to the risk aversion parameter $\rho = 0.7629$. We can observe that the expected profit is slightly lower than in the risk neutral case and it is clear that the left tail is reduced, see Fig. 9,

Observe in Table 13 that when we increase the risk aversion parameter, the model finds new technology mix that decreases the expected shortage. This happens until the shortfall probability is very small, so that after this point the technology mix does not change anymore. As for the shortfall probility measure, a further increase of the threshold is the only way to find a new technology mix. So the selection of the threshold ϕ is sensitive since, as we can observe in Fig. 9, when choosing a threshold too high (i.e. 19 $G€$) then the profit cumulated distribution associated to $\rho = 0.9925$ is considered more risky than the risk neutral based profit.

Table 13: Technology mix obtained with *ES* and different risk aversion parameters - Budget $B = 3.84$ $G€$.

Case	0	1	2	3
ρ	0	0.7629	0.9899	0.9925
CCGT	13	5	1	0
coal	0	4	4	2
wind	1	0	8	18
Expected profit [$G€$]	22.172	21.449	19.291	16.943
Shortfall probability	0.291	0.051	0.022	0.022
expected shortage [$G€$]	0.683	0.162	0.032	0.009
T_{LP} [s]	4.88	5.72	4.54	7.82
T_S [s]	0.16	0.11	0.08	0.08
T_{MIP} [s]	195.02	92.29	24.43	25.07

5.4. *Numerical results with risk aversion strategy 4: First-order Stochastic Dominance (FSD)*

Let us consider a budget of 3.84 $G€$ and four benchmarks of profiles as shown in Table 14. The construction of a benchmark is not simple. We have constructed benchmark by considering the profit distribution of the risk neutral approach. To construct the other benchmarks we adopted an iterative procedure which selects the thresholds of any benchmark by inspecting the profit distribution of the benchmark whose model has been previously solved in order to decrease the uncertainty while penalizing the expected profit.

Table 14 shows the profile set (ϕ^p, τ^p), for $p = 1, 2, 3, 4$, for each benchmark under consideration, see also Fig. 10, where their thresholds are highlighted by dots. For Benchmark 0 we get the same mix of technologies of the risk neutral case, already considered in Table 12. This is due to the magnitude of the first three thresholds, that have low values and high probabilities, even allowing large losses. The expected profit is the highest among the various tested benchmarks, see Table 15 and Fig. 11.

The profit distributions related to the four benchmarks show a decreasing risk in Fig. 11. It is clear from the figure that a comprehensive risk analysis requires to

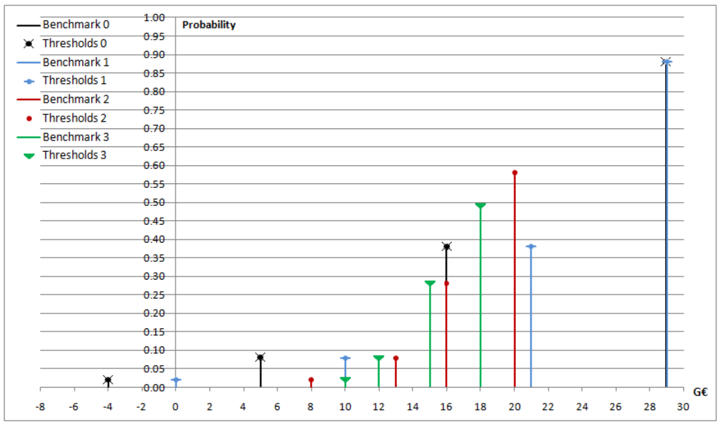

Figure 10: FSD: Benchmarks with their thresholds.

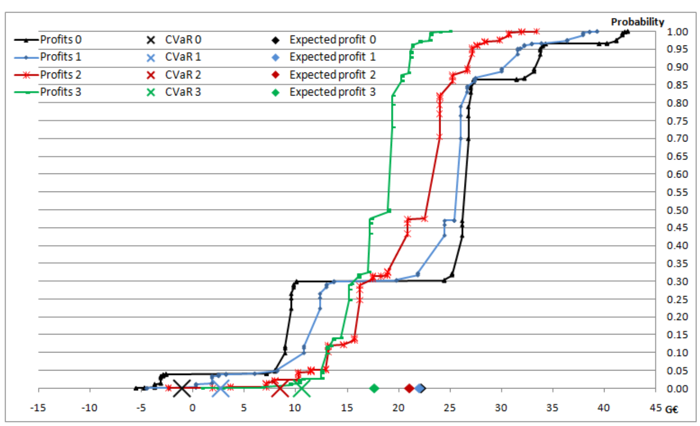

Figure 11: FSD: Profit distributions for the different benchmarks.

Table 14: Benchmarks used for testing the FSD risk aversion strategy.

	p	ϕ^p [G€]	τ^p
Benchmark 0	1	-4	0.02
	2	5	0.08
	3	16	0.38
	4	29	0.88
Benchmark 1	1	0	0.02
	2	10	0.08
	3	21	0.38
	4	29	0.88
Benchmark 2	1	8	0.02
	2	13	0.08
	3	16	0.28
	4	20	0.58
Benchmark 3	1	10	0.02
	2	12	0.08
	3	15	0.28
	4	18	0.49

consider several benchmarks. Indeed, we can notice that for some of them the associated values of the expected profit and *CVaR* are more favourable than in other benchmarks. However, this analysis may require an unaffordable computing time by plain use of a MIP solver and, then, a decomposition algorithm is requered, see e.g. [6, 10, 11].

5.5. *Numerical results with risk aversion strategy 5: Second-order Stochastic Dominance (SSD)*

We consider the four benchmarks presented in Table 16. The budget is $B = 3.84$ *G*€ . The benchmarks are illustrated in Fig. 12 where their thresholds are highlighted by dots.

With respect to Benchmark 0, we get the same mix of technologies of the risk neutral case due to the magnitude of the thresholds that has been fixed to negative values, allowing large losses. The expected profit is the highest among the various tested benchmarks, see Table 17 and Fig. 13. In Benchmark 1, 2 and 3 we have changed the thresholds by increasing their values and decreasing their bound of the expected shortfall over undesired scenarios. This implies that the chosen mix of technologies is moving from CCGT technology to coal plants.

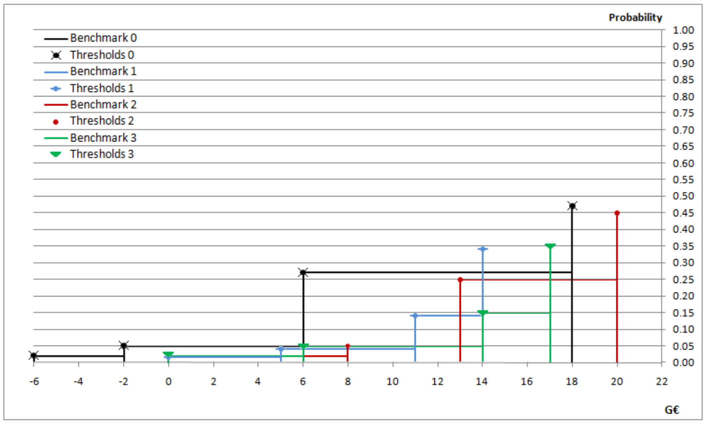

Figure 12: SSD: Benchmarks with their thresholds.

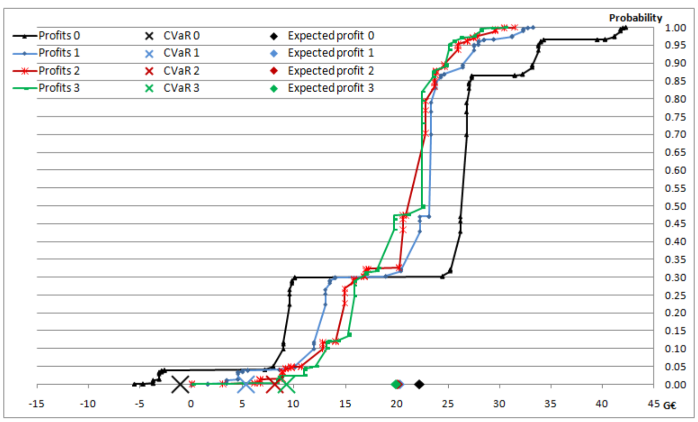

Figure 13: SSD: Profit distributions for the different benchmarks.

Table 15: Optimal technology mix obtained with the FSD risk aversion strategy using the benchmarks defined in Table 14.

Benchmark	0	1	2	3
CCGT	13	9	3	0
coal	0	2	5	3
wind	1	0	0	14
expected profit [G€]	22.173	21.990	21.028	17.650
VaR [G€]	8.016	8.174	11.490	12.472
CVaR [G€]	-1.026	2.682	8.463	10.583
Z_{LP} [G€]	22.427	22.381	22.156	22.472
Z_S [G€]	22.180	22.006	21.088	20.704
Z_{MIP} [G€]	22.173	21.990	21.082	20.704
T_{LP} [s]	8.00	7.50	9.05	9.10
T_S [s]	0.09	0.09	0.14	0.20
T_{MIP} [s]	274.72	402.49	4173.22	$>$50000

6. Conclusions and future work

In this paper we have presented a two-stage stochastic mixed-integer linear optimization model for power generation capacity expansion in a long term, where different types of technologies are considered. In our case study we have considered CCGT, coal, nuclear and wind, but the methodology can be used with any other technology. The uncertain parameters of the model are, in general, the fuel prices, electricity price, CO_2 price, Green Certificate price and the market share. In the case study we have only considered the fuel prices as stochastic parameters, since otherwise the plain use of the MIP solver of choice cannot handle the huge dimension of the problem. However, the modeling of risk measures under consideration is not affected. Traditional sophisticated approaches to the problem represent the uncertainty by two-stage scenario trees, instead of replacing the uncertain information with its expected value. This approach is so named a risk neutral strategy. Recognizing that this strategy usually outperforms the deterministic one, we can observe that it could provide bad results (i.e. small profit) for some scenarios to occur in case of high volatility of the uncertain parameters. In this work we extend the risk neutral model by considering several risk averse measures, such as *CVaR*, Shortfall Probability, Expected Shortage and First- and Second-order Stochastic Dominance constraint strategies. By using a realistic case of a price-

Table 16: Benchmarks used for testing the SSD risk aversion strategy.

	p	profit threshold ϕ^p ($G€$)	upper bound to expected shortage e^p
Benchmark 0	1	-6	0.02
	2	-2	0.05
	3	6	0.27
	4	18	0.47
Benchmark 1	1	0	0.02
	2	5	0.04
	3	11	0.14
	4	14	0.34
Benchmark 2	1	0	0.02
	2	8	0.05
	3	13	0.25
	4	20	0.45
Benchmark 3	1	0	0.02
	2	6	0.05
	3	14	0.15
	4	17	0.35

taker GenCo, we perform a comparison of the risk-averse strategies versus the risk neutral one under several hypotheses on the available budget, analyzing the expected profit decrease by reducing the risk of bad solutions for non-wanted scenarios. In any case, given the volatility of the uncertain information along a long term, a high number of scenarios is required and, as a consequence, the mixed integer Deterministic Equivalent Optimization model has very big dimensions (i.e., number of constraints and integer variables) even in the two-stage simplification approach that we have used in this paper. The risk averse strategies pay an additional computational price of requiring high potential computer capabilities, if the model optimization is performed by plain use of even state-of-the-art MIP solvers. Therefore, the model optimization must be performed by considering the more realistic approach based on the multi-stage environment modeling and using decomposition algorithms that exploit the characteristics of the stochastic model, being one of them the difficulty of tackling the non-anticipativity constraints in the continuous variables and the cross scenario constraints required by all risk averse strategies but *CVaR*, which is a subject of our future work. We are planning

Table 17: Optimal technology mix obtained with the SSD risk aversion strategy using the benchmarks defined in Table 12.

Benchmark	0	1	2	3
CCGT	13	5	3	2
coal	0	1	3	4
wind	1	13	8	6
expected profit [G€]	22.173	21.291	20.079	19.905
CVaR [G€]	-1.078	5.305	8.131	9.292
Z_{LP} [G€]	22.391	20.452	22.264	20.376
Z_S [G€]	22.188	20.292	20.085	19.906
Z_{MIP} [G€]	22.173	20.291	20.079	19.905
T_{LP} $[s]$	11.92	11.08	19.28	12.28
T_S $[s]$	0.11	0.11	0.16	0.19
T_{MIP} $[s]$	289.23	176.78	177.54	192.87

to consider all the uncertain parameters of our problem while using the ad-hoc decomposition algorithm. Notice that the two-stage approach of our problem is in fact a simplification of the multistage approach, since the non-anticipativity constraints of all the periods but the first one are implicitly relaxed. So, the multistage modelization and resolution of our problem is another future research that we are planning to work on.

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