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Abstract

We propose a TransCo model for coordinating transmission expansion planning with competitive generation capacity planning in electricity markets. Our purpose is to provide a tool to simulate the equilibrium interplay regarding strategic decisions of a set of power producers and a single transmission operator. The solution represents an iterative process for defining the optimal transmission expansion program together with a correct guess of the power plants expansion program for each GenCo involved. The composition of new investments in power plants guessed by the TransCo must coincide with the optimal expansion plan defined by each GenCo. We illustrate the methodology by means of an example depicting a zonal electricity market with two zones.

1 Introduction

After liberalization of Electricity Markets, private companies started entering the power generation market, offering power at lower costs and laying the groundwork for the introduction of a competition pattern which, in turn, will pass a larger share of benefits on to consumers. Such a widening of competition within the electricity market gives rise to the problem of coordination of generation systems with transmission lines. With no longer vertical structure ensuring automatic coordination of such activities it becomes necessary to develop mechanisms to align production and expansion decisions of the market actors covering these two roles.

When planning over the expansion of the transmission grid, it is important for the TransCo to take into account possible responses provided by GenCos. Not accounting for such reactions could most likely lead to potential over and under-investments beared by the TransCo across the different market areas. This would in turn lead to congestion and/or violation of the security standard, and ultimately to a rise in social costs. In this framework, a more integrated policy could be achieved by requiring the TransCo to account for responses given by GenCos with respect to changes in transmission capacity location in planned grid locations. This would result in a better exploitation of the existing transmission facilities [10].

We propose a model for analysis of transmission grid expansion planning with competitive generation capacity planning in electricity markets. The purpose of the model is providing a tool to define the optimal grid upgrading program in a market driven environment. Interplay between

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TransCo and several independent GenCos is treated as a leader-followers Stackelberg game. Such game is expressed as the following sequential decision pattern: the TransCo decides on the best possible upgrades of transmission lines and the GenCos modify their production plans and potential capacity expansion accordingly, reaching an equilibrium together with the Market Operator, which clears the market providing new zonal prices. Reaction of GenCos lead to an electricity production equilibrium that is properly taken into account within the TransCo decision problem, which receives back new zonal prices used to meter the amount of social costs.

Due to its intrinsic complexity and multi-objective nature the problem of coordinating electricity transmission and generation has been tackled using many different techniques. The first attempts to solve such problem date back to the late sixties and were based on a centralized approach [6], [9]. Linear programming was mainly used with the aim of minimizing the pooled costs for the system. A centralized approach is also taken by [1].

Interactive behaviour of electricity transmission and generation has gained more and more importance as game theoretic models have made their appearence within the realm of optimization theory. An example of application of such mixture of equilibrium concepts and optimization theory in analysis of electricity market equilibrium can be found in Hobbs [12]. Recent approaches encompass the use of bilevel programming [3], [7], [8],[17],[11] where the transmission planner takes its decision taking the benefits of the first mover advantage, while generating companies have to decide accordingly, together with the market operator.

A rather different approach is taken by [15], [16], which consider a coordination model where the Independent System Operator (ISO), which takes care of system security sends economical signals to TransCos and GenCos to incentivate a coordinated expansion of transmission and generation. Incentives are computed via Benders decomposition and Lagrangean Relaxation techniques.

The remainder of the article is organized as follows. In section 2 we introduce the decision problems up to each player involved in the production and delivery of electric power. Section 3 describes a reformulation of the problem providing lower bounds for the TransCo problem. In section 4 we introduce an algorithm to provide steadily larger lower bounds for the relaxation introduced in section 3, until a feasible solution for the original problem is found. Finally we provide a numerical analysis of the problem based on a two-zones system in section 5 and we draw conclusion in section 6.

2 General model

The model introduced in this article aims at capturing the strategies stemming by a sequencial game between three players: the Transmission Company, a group of Generating Companies and the Market Operator. The model is structured in two different, interrelated decision levels which represent the sequencial nature of the decisions up to each player. Namely, the TransCo will take its decision on transmission structure as first mover, then each GenCo will decide on power production levels and potential new investments accordingly to the choice made by the TransCo. Bids provided by each GenCo are collected and sorted by a Market Operator, which takes care of clearing the market.

We assume that electricity shall be delivered to companies and consumers spread over different market zones. Each zone has different properties in terms of load and possible power production levels, which in turn depend on the amount of existing power plants and candidate areas for the installment of new power plants. The modeling framework considered in this paper is displayed in figure 1. The two-levels Stackelberg game involves only the network planner as the upper level

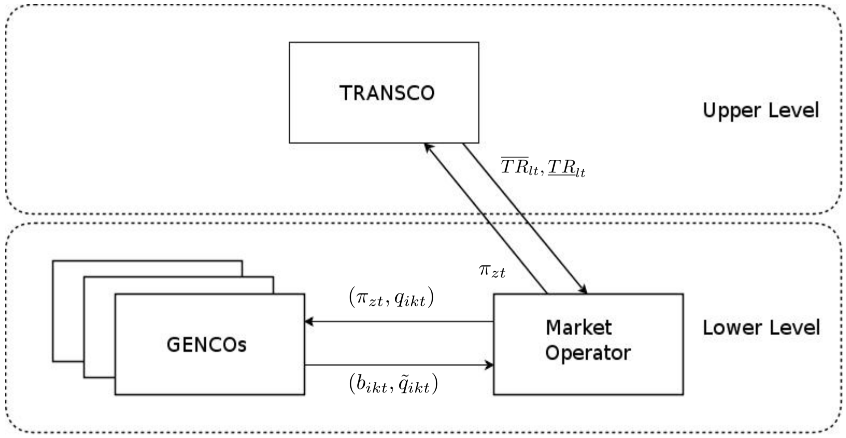


Figure 1: Interdependancies between transmission company, generator companies and market operator

player and a group of power generating companies, whose bid-ask mechanism with the consumers is mediated by a Market Operator, as the lower level. The TransCo aims at lowering the total social cost, defined as the sum of covered load times the zonal price payed to purchase energy over the different zones and time periods. Such objective is pursued by allowing more efficient GenCos selling energy in different areas. TransCo takes a decision on installation of new power transmission lines which, together with the existing transmission lines, will define new upper and lower bounds on the line power flow streaming over a intra-zonal connection line l at time t. These bounds are used by the Market Operator to define how much power can be transferred between zones in order to accept bids that are supposed to cover the load at a given time t in all the considered areas at the least possible social cost. Bids are sent to the Market Operator by the GenCos in form of a pair (b_{ikt}, $\tilde{q}_{ikt}$) defining the price bid at time t from generator k belonging to GenCo i and the related quantity respectively. Such bids are defined on the ground of projected profitability of the energy production. This latter depends on installed capacity and decisions on installations of new power plants which concur as decision variables for the GenCos. Market Operator, given the power flow bounds defined by the TransCo, will define the zonal prices π_{zt} for zone z at time t and the accepted quantities from each power generator. GenCos aim at maximizing their profit, by deciding how much power to supply and whether to open new power plants.

A dedicated algorithm has been developed in order to take care of binary decision variables featured in the equilibrium problem between GenCos and Market Operator. We tackle the problem by solving a relaxation of the TransCo problem in which all of the investments based binary decision variables are controlled by the TransCo, while first order conditions are imposed for the continuous part of the GenCo and Market Operator problems. This returns a lower bound for the TransCo problem which is normally not consistent with the response given by each GenCo in terms of binary variables. Therefore the solution is tested for consistency with the solution provided by each GenCo in order to check whether the binary decision variables up to the GenCos obtained by solving the relaxation are the same as the ones provided by the solution of each GenCo problem. Should this not happen, the algorithm creates appropriate cuts in order to prevent the TransCo to choose the same integer solution for the variables up to the GenCos in subsequent iterations.

The main task of the Market Operator (MO) is the one of matching energy demand and supply at each time point. As a consequence of such match, MO will determine hourly zonal prices. Let us introduce the following notation
Let us denote by T the set of periods t, I the set of oligopolistic producers, Z the set of zones, L^E, L^C the set of existing and candidate transmission links, K^E_{iz} and K^c_{iz} the set of existing and candidate technologies belonging to producer $i \in I$ in zone $z \in Z$.

Let b_{ikt} denote price of sell bid of plant $k \in K$ belonging to GenCo $i \in I$ in period $t \in T$, A_{zl} the incidence matrix of the system, C_{zt} the load in zone $z \in Z$ in period $t \in T$, $\tilde{q}_{ikt}$ the power in MW offered by generator $k \in K$ belonging to GenCo $i \in I$ at time $t \in T$, $\overline{TR}_{lt}$ and $\underline{TR}_{lt}$ the maximum and minimum capacity of transmission link $l \in L$ in period $t \in T$.

Let q_{ikt} be accepted bid in MW for technology $k \in K$ of producer $i \in I$ in period $t \in T$ and TR_{lt} denote the power flow on transmission link $l \in L$ in period $t \in T$;

The Market Operator must enforce the clearing conditions for the perfect competitive system considered for a group of similar producers, which is given by the solution of the problem[1]

$$\min_{q_{ikt}, TR_{lt}} \sum_{i \in I} \sum_{t \in T} \sum_{z \in Z} \sum_{k \in K^E_{iz} \cup K^C_{iz}} b_{ikt} q_{ikt} \tag{1}$$

subject to

$$\sum_{i \in I} \sum_{k \in K^E_{iz} \cup K^C_{iz}} q_{ikt} + \sum_{l \in L^E \cup L^C} A_{zl} TR_{lt} = C_{zt} \quad : \pi_{zt} \qquad z \in Z, t \in T \tag{2}$$

$$q_{ikt} \le \tilde{q}_{ikt} \quad : \lambda_{ikt} \qquad i \in I, \quad z \in Z, \quad k \in K^E_{iz} \cup K^C_{iz}, \quad t \in T \tag{3}$$

$$\underline{TR}_{lt} \le TR_{lt} \le \overline{TR}_{lt} \quad : \eta^+_{lt}, \eta^-_{lt} \qquad l \in L^E \cup L^C, \quad t \in T \tag{4}$$

$$q_{ikt} \ge 0 \tag{5}$$

Solution of the introduced problem operates a sorting through the accepted bids, starting from the cheapest one and ending with the most expensive.

At the same level as the Market Operator one finds the set of GenCos. These actors aim at maximizing their own profit by submitting bids $(b_{ikt}, \tilde{q}_{ikt})$ to the Market Operator and defining their optimal expansion plan according to the structure of the grid. The problem of the i-th GenCo involves the following notation

Let π_{zt} be zonal price in area $z \in Z$ at time $t \in T$, δ denote the discount factor, c_{ik} the generation cost of technology $k \in K$ for producer $i \in I$, f^G_{ik} the investment cost of technology k for producer i, Γ^C_{ik} and Γ^E_{ik} denote the capacity of candidate technology $k \in K$ of producer $i \in I$ and q_{ikt} define accepted bid in MW for technology $k \in K$ of producer $i \in I$ in period $t \in T$;

Let us define Y_{ik} as the binary variable set to 1 if producer $i \in I$ activates technology $k \in K$, $\tilde{q}_{ikt}$ as the power in MW offered by generator $k \in K$ belonging to GenCo $i \in I$ at time $t \in T$ and b_{ikt} as the price of sell bid of technology $k \in K$ belonging to GenCo $i \in I$ in period $t \in T$

The decision problem up to each GenCo is the following

$$\max_{\tilde{q}_{ikt}, b_{ikt}, Y_{ik}} \sum_{t \in T} \delta^{-t} \sum_{z \in Z} \left(\pi_{zt} \sum_{k \in K^E_{iz} \cup K^C_{iz}} \tilde{q}_{ikt} - \sum_{k \in K^E_{iz} \cup K^C_{iz}} c_{ik} \tilde{q}_{ikt} \right) - \sum_{z \in Z} \sum_{k \in K^C_{iz}} f^G_{ik} Y_{ik} \tag{6}$$

[1] We report the dual variables of each constraint right after the constraint itself preceded by a colon.

subject to

$$\tilde{q}_{ikt} \leq \Gamma^C_{ik} Y_{ik} \quad : \mu^{\Gamma C}_{ikt} \qquad z \in Z, \quad k \in K^C_{iz}, \quad t \in T \tag{7}$$

$$\tilde{q}_{ikt} \leq \Gamma^E_{ik} \quad : \mu^{\Gamma C}_{ikt} \qquad z \in Z, \quad k \in K^E_{iz}, \quad t \in T \tag{8}$$

$$\tilde{q}_{ikt} \leq q_{ikt} \quad : \mu^{\tilde{q}+}_{ikt} \qquad z \in Z, \quad k \in K^E_{iz} \cup K^C_{iz}, \quad t \in T \tag{9}$$

$$\tilde{q}_{ikt}, b_{ikt} \geq 0, \quad Y_{ik} \in \{0,1\} \tag{10}$$

In this problem, π_{zt} represents electricity price in zone z in period t and is defined as the shadow price of load covering constraint (2). The last constraint defines an upper bound on the power accepted by the Market Operator.

The solution of each GenCo problem changes accordingly to the bid level that can be accepted by the Market Operator in each market zone and accordingly to the decision taken by the TransCo in terms of transmission capacity over the transmission lines. We will refer to the GenCo problem as $GP_i\left(\overline{TR}_{lt}, \underline{TR}_{lt}\right)$ in order to stress out the aforementioned relation.

TransCo aims at minimizing the sum of social costs given the expansion investment budget. High social costs derive by high bids settled by GenCos which can be reduced by removing congestions over the transmission grid. We assume that only upgrade of existing lines is considered by the TransCo. The problem is formalized as follows.

Let us define the parameters C_{zt} as load in zone $z \in Z$ in period $t \in T$, $\overline{\Lambda}^E_l, \underline{\Lambda}^E_l$ as upper and lower bound on existing line capacity, $\overline{\Lambda}^C_l, \underline{\Lambda}^C_l$ as upper and lower bound on candidate line capacity with voltage $w \in W$, f^T_{wl} as investment cost for opening line $l \in L$ with voltage $w \in W$ and B as the total budget for lines expansion.

Decision variables for the TransCo are π_{zt} which denotes the zonal price in area $z \in Z$ at time $t \in T$, X_{wl} which denotes a binary variable set to 1 if line $l \in L$ of type $w \in W$ is built and $\overline{TR}_{lt}, \underline{TR}_{lt}$, denoting the overall upper and lower bound line capacity. TransCo problem can be formulated as

$$\min_{\pi_{zt}, \overline{TR}_{lt}, \underline{TR}_{lt}, X_{wl}} \sum_{t \in T} \sum_{z \in Z} C_{zt} \pi_{zt} \tag{11}$$

subject to

$$\overline{TR}_{lt} = \overline{\Lambda}^E_l + \sum_{w \in W} \overline{\Lambda}^C_{wl} X_{wl} \qquad l \in L^E \cap L^C, \quad t \in T \tag{12}$$

$$\underline{TR}_{lt} = \underline{\Lambda}^E_l + \sum_{w \in W} \underline{\Lambda}^C_{wl} X_{wl} \qquad l \in L^E \cap L^C, \quad t \in T \tag{13}$$

$$\sum_{w \in W} \sum_{l \in L^C} f^T_{wl} X_{wl} \leq B \tag{14}$$

$$\pi_{zt} \in \Omega\left(\overline{TR}_{lt}, \underline{TR}_{lt}\right) \qquad z \in Z, \quad t \in T \tag{15}$$

$$\overline{TR}_{lt}, \underline{TR}_{lt} \geq 0, \quad X_{wl} \in \{0,1\} \tag{16}$$

where the first two constraints define the upper and lower bounds on power flows in a given line l and at a given time t. These bounds are given by the sum of the existing bounds $\overline{\Lambda}^E_l$ plus a potential bound upgrade, added to the existing bound if a candidate line is opened. The TransCo can choose among different types of transmission lines, depending on their voltage. W is the set of possible types of available transmission lines. The third constraint represents the upper bound on the capital expenses for lines expansion. Finally, $\Omega\left(\overline{TR}_{lt}, \underline{TR}_{lt}\right)$ represents the space of joint solutions of problems (1)-(5) and (6)-(10) parametrized by $\overline{TR}_{lt}$ and $\underline{TR}_{lt}$. Such set contains the possible equilibria, defined in terms of decision variables $\tilde{q}_{ikt}, b_{ikt}, Y_{ik}$ for the involved GenCos, decision variables q_{ikt}, TR_{lt} for the market operator and dual variables for both the aforementioned problems. Dual variables for problems (1)-(5) and (6)-(10) are the following. Namely we define π_{zt} and λ_{ikt} as the dual variables for constraints (2)-(3) respectively and η_{lt}^{+} and η_{lt}^{-} as the positive and negative part of the dual variable for constraint (4) and $\mu_{ikt}^{\Gamma C}, \mu_{ikt}^{\Gamma E}$ and $\mu_{ikt}^{\tilde{q}+}$ as dual variables for constraints (7), (8) and (9). All of the aforementioned variables belong to the set $\Omega\left(\overline{TR}_{lt}, \underline{TR}_{lt}\right)$, which is paramatrized by the choice of upper and lower bounds for power transmission decided by the TransCo. Variables f_{ik}^{G} and f_{wl}^{T} have been divided by 8750, which represent the yearly operating hours for a plant. This is in order to harmonize the definition of load and offered quantity which are represented under a hourly time horizon with the investment costs, which must consider the whole energy sold in a time horizon expressed in years in order to be a valuable measure to be used for assessment of investment decisions.

3 Model Reformulation

As no strategic bidding is considered in the model, bids will reflect marginal costs c_{ik} of the production from a given plant k plus a GenCo specific bid up χ_i. This implies that the structure of the offer curve is completely determined by the marginal costs up to each power plant of each GenCo and there will be no interplay to determine an equilibrium on offer prices. Under this assumption, GenCos could find beneficial expand their capacity for a given load request over time, allowing a larger amount of energy to be accepted by the Market Operator for load covering. This implies that GenCos selected by the Market Operator will be willing to offer power as long as their capacity limit is not hit. Capacity of each GenCo needs to be considered when the Market Operator defines a candidate value for the amount of power produced by each GenCo. In this respect, one needs to assure that Market Operator takes care of such constraints when setting up the electricity exchanges. This can be done by explicitly inserting capacity expansion constraints (7)-(8) of GenCos into the MO problem. For a given arrangement of candidate power plants for each GenCo the MO will know which are the bounds on power generation. Such bounds are iteratively updated after getting the optimal response from each GenCo, in case the two solutions do not coincide.

1. Market Operator problem is explicitly considered in the TransCo decision model by means of its Karush-Kuhn-Tucker conditions considering decisions on new installments of power plants as parameters.

2. TransCo solves a mathematical program with complementarity constraints with mixed integer structure due to the presence of binary variables defining the state of new power plants. Such problem is a relaxation of the original bilevel problem in the sense that it provides a lower bound for the minimization of the social costs and the solution of the original bilevel problem is feasible for such problem. The solution of such problem returns a tentative decision on the candidate power plants to be opened and bounds on the quantities accepted by the Market Operator, besides the value for the transmission bounds on each line.

3. GenCos solve their decision problems with bounds on the power accepted by the MO. The unit margin for the profit of GenCos whose bids are accepted by the MO is positive, therefore GenCos will increase the production until they hit an upper bound. Such upper bound can be their maximum production capacity or the maximum accepted quantity by the MO. The result of the decision problem up to the GenCos is their actual decisions on capacity expansion and power generated and injected into the grid.

4. Solutions on candidate power plants provided by TransCo and GenCos are compared. If the decisions coincide then the candidate solution is optimal, otherwise a procedure will prevent the TransCo choosing the same solution in a subsequent iteration of the problem by means of a cut and an additive penalty term in the objective function.

The problem solved by the Market Operator is therefore given by (1)-(5) with the inclusion of constraints (7) and (8) with $\tilde{q}_{ikt}$ replaced by q_{ikt}. We call such problem $MO(\overline{TR}_{lt}, \underline{TR}_{lt})$ in order to stress out its dependancy on the upper and lower bounds on power transmission provided by TransCo. In such a problem Y_{ik} is a parameter defined by the TransCo as a tentative value[2] and corrected or confirmed by GenCos after the solution of their decision problem.

Problem up to the generic i-th GenCo is given by (6)-(10) We stress out that the GenCos are willing to provide as much power as they possibly can as their profit increases with power supplied. We call such problem $GP_i(\overline{TR}_{lt}, \underline{TR}_{lt})$ for the i-th Genco.

Finally the problem to be solved is the TransCo problem, defined as (11)-(16), which defines a mathematical problem with bilevel structure entailing one leader (TransCo) and multiple followers (GenCos and Market Operator). As explained earlier, the lack of interplay in the definition of the price bids allows us to consider only the MO problem as the lower level. Accordingly, if we define $dualMO\left(\overline{TR}_{lt}, \underline{TR}_{lt}\right)$ as the dual problem for the Market Operator problem $MO(\overline{TR}_{lt}, \underline{TR}_{lt})$ parametrized by the upper and lower bounds on transmission capacity and on the value of binary variables Y_{ik} we have that $\Omega\left(\overline{TR}_{lt}, \underline{TR}_{lt}\right) = \left\{\pi_{zt} \mid \left(\pi_{zt}, \sigma^C_{ikt}, \sigma^E_{ikt}, \eta^+_{lt}, \eta^-_{lt}, \lambda_{ikt}\right)\right.$ is optimal solution to $\left. dualMO\left(\overline{TR}_{lt}, \underline{TR}_{lt}\right)\right\}$.

Following such approach we can formulate the Integer Leader Relaxation (ILR) as the following mathematical program with complementarity constraints and mixed integer structure where

[2]The decision on tentative values for candidate power plants is up to the TransCo. This is true because the Market Operator problem is integrated into the TransCo problem through its first order conditions.

responses of the MO are replaced by the related Karush-Kuhn-Tucker conditions

$$
\begin{aligned}
\min \quad & \sum_{t\in T}\sum_{z\in Z} C_{zt}\pi_{zt} \\
\text{s.t.} \quad & \overline{TR}_{lt} = \overline{\Lambda}_l^E + \sum_{w\in W} \overline{\Lambda}_{wl}^C X_{wl} \qquad l \in L^E \cap L^C, \quad t \in T \\
& \underline{TR}_{lt} = \underline{\Lambda}_l^E + \sum_{w\in W} \underline{\Lambda}_{wl}^C X_{wl} \qquad l \in L^E \cap L^C, \quad t \in T \\
& \sum_{w\in W}\sum_{l\in L^C} f_{wl}^T X_{wl} \le B \\
& \sum_{z\in Z} A_{zl}\pi_{zt} - \eta_{lt}^+ + \eta_{lt}^- = 0 \qquad l \in L^E \cup L^C, \quad t \in T \\
& \sum_{i\in I}\sum_{k\in K_{iz}^E\cup K_{iz}^C} q_{ikt} + \sum_{l\in L^E\cup L^C} A_{zl} TR_{lt} = C_{zt} \qquad z \in Z, \quad t \in T \\
& 0 \le b_{ikt} + \sigma_{ikt}^C + \sigma_{ikt}^E - \pi_{zt} \perp q_{ikt} \ge 0 \qquad i \in I, \quad z \in Z, \quad k \in K_{iz}^E \cup K_{iz}^C, \quad t \in T \\
& 0 \le \Gamma_{ik}^C Y_{ik} - q_{ikt} \perp \sigma_{ikt}^C \ge 0 \qquad i \in I, \quad z \in Z, \quad k \in K_{iz}^C, \quad t \in T \\
& 0 \le \Gamma_{ik}^E - q_{ikt} \perp \sigma_{ikt}^E \ge 0 \qquad i \in I, \quad z \in Z, \quad k \in K_{iz}^E, \quad t \in T \\
& 0 \le \overline{TR}_{lt} - TR_{lt} \perp \eta_{lt}^+ \ge 0 \qquad l \in L^E \cup L^C, \quad t \in T \\
& 0 \le TR_{lt} - \underline{TR}_{lt} \perp \eta_{lt}^- \ge 0 \qquad l \in L^E \cup L^C, \quad t \in T \\
& \overline{TR}_{lt}, \underline{TR}_{lt} \ge 0, \quad \pi_{zt}, TR_{lt} \in \Re \\
& X_{wl}, Y_{ik} \in \{0,1\}
\end{aligned}
\tag{17}
$$

When this problem is solved one is left with optimal values q_{ikt}^* and Y_{ik}^*, besides the incumbent value for the zonal prices π_{zt}^* and the candidate value for the i-th GenCo profit v_i^*. Such values can be used to perform a comparison with the GenCo problems. Namely one has to solve problem $GP_i(\overline{TR}_{lt}, \underline{TR}_{lt})$ with $\pi_{zt} = \pi_{zt}^*$. Once one obtains the solution of problem $GP_i(\overline{TR}_{lt}, \underline{TR}_{lt})$ it will be possible to compare the decision on opening of candidate power plants stemming from $GP_i(\overline{TR}_{lt}, \underline{TR}_{lt})$ which we denote by $\bar{Y}_{ik}$ and the candidate value provided by the solution of (17), which we denote by Y_{ik}^*. In addition one will compare the candidate profit value obtained by solving (17) and denoted by $\bar{v}_i$ with the actual optimal profit v_i^*. If the comparison returns that $\bar{Y}_{ik} \ne Y_{ik}^*$ or $| v_i^* - \bar{v}_i | > \epsilon$ with $\epsilon > 0$ and small, then a cut and penalty shall be inserted in problem (17). The details of such procedure shall be explained in some more detail in the remainder of the article.

4 Solution Approach

What one can expect is that the solution of (17) will not be the solution of the original problem (11)-(16). In fact, if we define the i-th GenCo problem with all other players in equilibrium as $\text{GP}_i(\overline{TR}_{lt}, \underline{TR}_{lt})$ and given a solution (X_l^*, Y_{ik}^*) of the (ILR), this coincides with the solution of (11)-(16) iff

$$
Y_{ik}^* = \arg\max \text{GP}_i(\overline{TR}_{lt}, \underline{TR}_{lt}) \qquad i \in \mathcal{I}, \quad z \in Z, \quad k \in K_{iz}^C \tag{18}
$$

which can be obtained by fixing all the players except i-th's decision variables to the ones supplied by the solution of the ILR and solving problem $\mathrm{GP}_i(\overline{TR}_{lt}, \underline{TR}_{lt})$ for the i-th GenCo. If solution of i-th GenCo problem does not coincide with the one found with the TransCo problem, it is necessary to devise a procedure to prevent the TransCo choosing the same integer solution when solving the Integer Leader Relaxation. The purpose is to require the TransCo to pick the second best integer solution of the ILR. Generally speaking, we want to force the TransCo to choose progressively worse optimal solutions of the ILR until we find the best solution satisfying the equilibrium problem between the GenCos. This is done by inserting an appropriate cut and a penalty term for the incumbent optimal ILR solution.

If at iteration n we denote by $Y^*_{ik[n-1]}$ the optimal installation plan for GenCo i obtained in the previous iteration and

$$a^{(n)}_{ik} = \begin{cases} 1 & \text{if } Y^*_{ik[n-1]} = 1 \\ -1 & \text{if } Y^*_{ik[n-1]} = 0 \end{cases}$$
$$b^{(n)}_{ik} = \begin{cases} 0 & \text{if } Y^*_{ik[n-1]} = 1 \\ 1 & \text{if } Y^*_{ik[n-1]} = 0 \end{cases} \tag{19}$$

we can express the cut as

$$\sum_{i\in\mathcal{I}} \sum_{k\in K^C_i} a^{(n)}_{ik} Y_{ik} - \left|K^C\right| u_n \le \frac{\left|K^C\right|^2 - \left|K^C\right| + 1}{\left|K^C\right|} - \sum_{i\in\mathcal{I}} \sum_{k\in K^C_i} b^{(n)}_{ik} \tag{20}$$

with $\left|K^C\right|$ denoting the total number of candidate plants and u_n defined as a binary variable.

At iteration n we will have the TransCo solving the Penalized Integer Leader Relaxation problem (PILR) defined as the ILR problem with the addition, at each iteration n, of an additional cut of type (20) and with the following modification of the objective function

$$\sum_{t\in T} \sum_{z\in Z} C_{zt}\pi_{zt} + M \sum_{r=1}^{n} u_r$$

After problem PILR is solved, the solution is tested with problem (18), and if the solution coincides with the one obtained solving the PILR the algorithm stops. Otherwise, a new cut and penalty term are included in the PILR and the process starts anew. Optimal solution is reached because the leader will explore all the feasible integer solutions as each optimal solution is fathomed by the cut and penalty and a second best solution has to be chosen. Since the optimal solution of the original problem (11)-(16) lies within the feasible set, it has to be eventually picked up as candidate by the algorithm. One problem that can arise with the described approach is anyway linked to degenerate solutions of the ILR and PILR problems. Namely such problem could return a result which is not the same found by the GenCos in terms of profitability and choice of binary variables only because of degeneracy, i.e. there exists another solution with the same objective function for the TransCo sharing the same values for profit and choice of binary variables with the optimal solutions of all the GenCos. This can generate sub-optimal solutions for the problem (11)-(16).

The idea of the algorithm is to obtain candidate solutions as lower bounds resulting by the solution of the PILR problem, and test whether such solutions are feasible by solving the Followers

Problem (FP). The test consists in checking if either the optimal function of each follower is the same as the value obtained by solving the PILR or the optimal choice of each follower, in terms of new installed plants, coincides with the ones obtained by solving the PILR problem. If the integer solution of the FP coincides with the corresponding values obtained solving the PILR problem or the objective function value is the same as the optimal value of each follower, the result is optimal for problem (11)-(16). Otherwise a cut and a penalization are included in the constraints and the objective function of the PILR respectively to prevent the incumbent optimal solution of the PILR to be chosen again in a new iteration. The algorithm is thus described in the table below, where symbols $\overline{v}_{i[n]}$ and $v^*_{i[n]}$ denote the optimal profit of the i-th GenCo at iteration n and the candidate profit for GenCo i computed by the TransCo at iteration n, while ϵ is a small positive real number.

Algorithm 1 Progressive Penalization Algorithm

```
(Step 1)
n ← 0
solve ILR → (TR̄*_lt, TR*_lt, q*_ikt, Y*_ik[n], X*_wl, v*_i[n])
if ILR is infeasible then
  STOP. The problem is infeasible
else
  (Step 2)
  for i = 1 → |I| do
    solve GP_i(TR̄*_lt, TR*_lt) → (Ȳ_ik[n], v̄_i[n])
  end for
  if Ȳ_ik[n] = Y*_ik[n] or | v̄_i[n] − v*_i[n] |< ε then
    (TR̄*_lt, TR*_lt, X*_wl) is the optimal solution of (11)-(16)
  else
    (Step 3)
    while Ȳ_ik[n] ≠ Y*_ik[n] or | v̄_i[n] − v*_i[n] |≥ ε do
      n ← n + 1
      solve PILR → (TR̄*_lt, TR*_lt, q*_ikt, Y*_ik[n], X*_wl, v*_i[n])
      if PILR is infeasible then
        STOP. The problem is infeasible
      else
        for i = 1 → |I| do
          solve GP_i(TR̄*_lt, TR*_lt) → (Ȳ_ik[n], v̄_i[n])
        end for
      end if
    end while
    (TR̄*_lt, TR*_lt, X*_wl) is the optimal solution of (11)-(16)
  end if
end if
```

The algorithm is composed of a master problem, defined by the PILR, and a sub problem, defined by the FP, exchanging information about feasibility (and therefore overall optimality) of the incumbent solution. At generic iteration n, PILR proposes an upper bound for the solution to the problem, together with the candidate values for incumbent choice for new power plant installments up to each GenCo and related optimal profit value $(Y^*_{ik[n]}, v^*_{i[n]})$. Such solution is feasible only if

GenCos involved actually confirm the incumbent values when they solve their decision problem. Solution of problem GP_i for each GenCo will provide the vector $(\bar{Y}_{ik[n]}, \overline{v}_{i[n]})$, where the first element denotes the optimal value for the variables defining installments of new power plants for GenCo i when TransCo picks solution $(\overline{TR}^*_{lt}, \underline{TR}^*_{lt})$, while $\overline{v}_{i[n]}$ denotes the optimal profit for GenCo i when TransCo picks solution $(\overline{TR}^*_{lt}, \underline{TR}^*_{lt})$. Vectors $(\bar{Y}_{ik[n]}, \overline{v}_{i[n]})$ and $(Y^*_{ik[n]}, v^*_{i[n]})$ are therefore compared and if either $\bar{Y}_{ik[n]}$ and $Y^*_{ik[n]}$ coincide, or $\overline{v}_{i[n]}$ and $v^*_{i[n]}$ coincide then the problem is feasible and incumbent solution is optimal. If feasibility requirements are not met, a new cut and a penalty term are introduced into the PILR to force the next iteration to avoid picking the previous candidate solution $Y^*_{ik[n]}$.

Step 1 computes the lowest possible bound, Step 2 checks whether the solution obtained in Step 1 is feasible for the overall problem problem (11)-(16) and while the program is not feasible a new iteration will add a cut and a new penalty term into the PILR problem. This is done by defining a loop in Step 3. The algorithm terminates when the first feasible solution is found or concludes that no feasible solution exists.

The algorithm attains a global optimum for the original Stackelberg-Nash problem up to degenerate cases for the PILR problem. Namely, a degenerate solution delivers the same objective function value against many possible combinations of decision variables. Therefore one could obtain an output profit computed by the TransCo for each GenCo i, v^*_i, which is lower than the profit computed by each GenCo by solving problem (6)-(10) or such that the binary decision variables do not correspond to the choice that each GenCo would pick under optimality even when there actually exists another vector of solutions which corresponds to the optimal one for each GenCo and delivering the same optimal objective function for the TransCo. When this happens, the algorithm will not recognize it as a feasible solution and it will keep generating cuts reaching a sub-optimal solution.

5 Numerical analyses

In order to carry out economical and technical analyses we have tested the model using a small scale example. This has lead to an easier study of optimal responses of all the actors involved in the provision system to environmental changes, such as an increase of load in a given area or the introduction of a new generation technology. The second stage of the analysis considers the model applied to a representation of the Italian market. At this stage, we assume to have a fictional system composed of two zones, which we refer to as North and South, two generating companies, GenCo1 and GenCo2 each having, besides existing plants, four candidate plants uniformally distributed over the different zones. GenCo1 only has one candidate plant, located in the North area, while GenCo2 has three candidate plants, one located in the North, and two located in the South. Finally we consider one existing transmission line which should be eventually upgraded in case of a load increase. We also assume that four types of candidate lines are available, which are different by costs and capacity. Type1 line has the least transmission capacity and is less expensive, while the opposite holds for type4 line.

All computations have been carried out using GAMS/CPLEX 12.1.0 using four parallel threads on a Intel Core i7 machine running at 2.67 GHz with 8 GB of RAM. Computational time is heavily influenced by the data, ranging between little over one minute for cases where GenCos respond to transmission investments by opening new plants to several minutes, when no GenCo finds it profitable to open a new power plant.

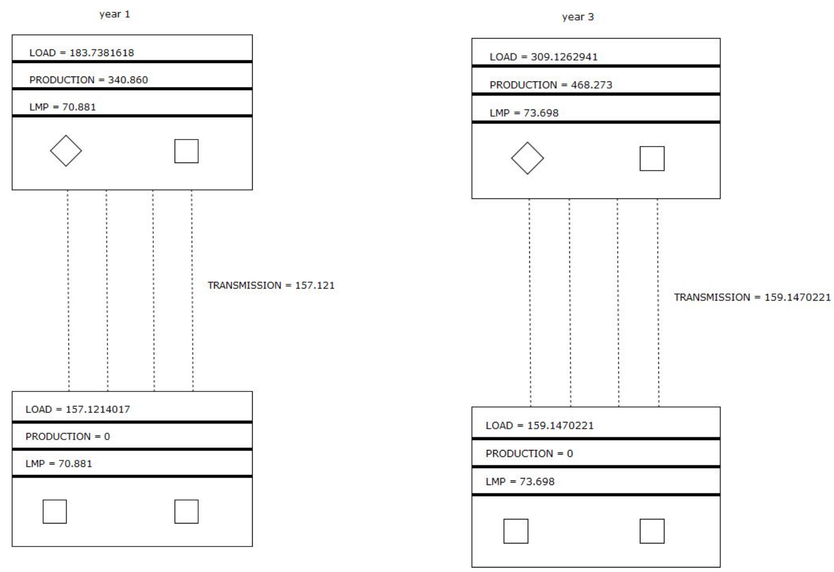


Figure 2: Investments with low load scenario, small and large load respectively

A representation of the investments made by each player in the system is shown in figures 2 and 3. These figures provide a simplified graphical description of the national power delivery system. Here we depict each area by means of a rectangle containing information about load, power production and locational marginal price within the area. Right below the slots for such information there can be found a blank area containing two types of elements, namely a diamond shape and a square which represent the candidate plant of GenCo1 and GenCo2 respectively. The two aforementioned areas are connected by existing transmission lines, which are not represented in the figures, while we have denoted by dotted lines four candidate transmission links with different capacity levels. Alongside such connecting lines one can find information about the amount of electricity transferred between the zones. One has plus sign if transmission is from North to South, while the other way around holds when electricity is transferred from South to North. Depending on the environmental factors such as load and technology, which represent the data for our model, both TransCo and GenCo can decide to review and upgrade their investment decisions by opening new transmission lines and, possibly, new power plants. Should this happen, we will represent the opening of a new line by replacing the dotted line with a thicker continuous line. Similarly, we will represent the opening of a new power plant by filling the related element with red color.

In the remainder of the article we study the effects of a load increase on the amount and allocation of the investments. We are primarily interested in the behavior of the Transmission Operator, but responses of GenCos also need to be considered as they play a relevant role in the expansion decision process of the TransCo. For each zone, we report information on the load level, the production level, the locational marginal prices (LMP) and possible transmission grid and power plant expansions.

From figure 2 we can notice that a small load both in the North and in the South areas allows

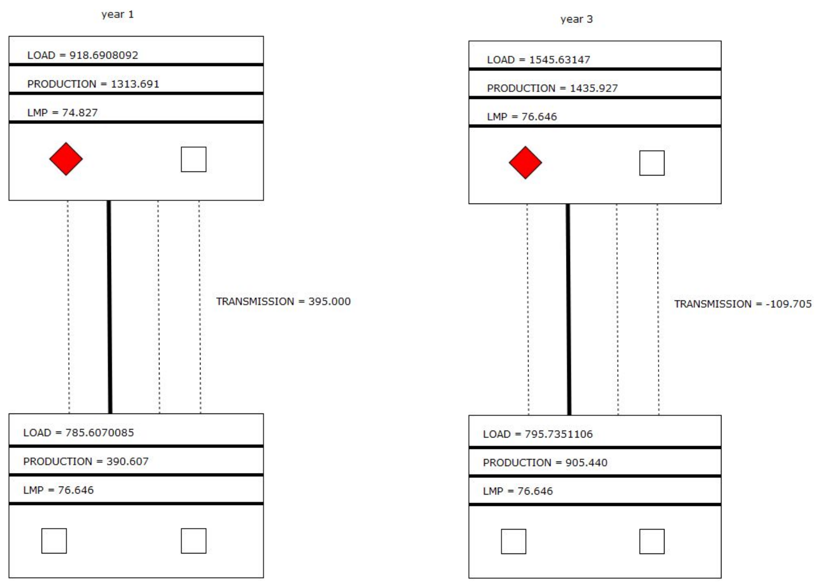


Figure 3: Investments with high load scenario, small and large load respectively

the entire demand to be covered by the GenCos offering power at the least price, which in our case happen to be in the North. Existing transmission is enough to ensure coverage for both the areas. Lack of congestion over the transmission lines between the two areas make the locational marginal prices to be the same in the two areas. This is equivalent to having only one area in our system. Figure 3 shows a different situation. Here we apply our model to a system with a larger load level. In this case, the TransCo will prevent consumers in South area to pay too much for buying electricity by opening a new type2 line between North and South. This increases the potential market for GenCo1 which will respond by opening a new power plant in the North. This is beneficial to GenCo1 since such power plant has lower marginal costs than the ones operating in the south, which in turn will allow GenCo1 to sell a lager amount of power, by serving the South area. Yet, GenCo1 will not manage to cover the entire demand from South area, which will therefore be covered by plants operating in that area, which offer energy at a higher price. In the second diagram of figure 3 we are presented with the opposite situation. Here the North buys energy from the South. This happens since the load in the North increases to an extent that cannot be covered any longer solely by the power plants in the very same area. As a consequence, power plant in the South will increase the production and sell it to the consumers in the North. Locational marginal price will be the same in both markets.

Let us extend the ongoing experiment to analyze the results of a progressive expansion of load on the performance measures of the actors involved in the power generation and delivery. We have taken in consideration a reference load scenario and then increased the load by the same percentage in every year of the multiperiod model. Figure 4 shows the somewhat anticipated effect that a load growth has on social costs, defined as the total costs community have to bear to buy electricity. As

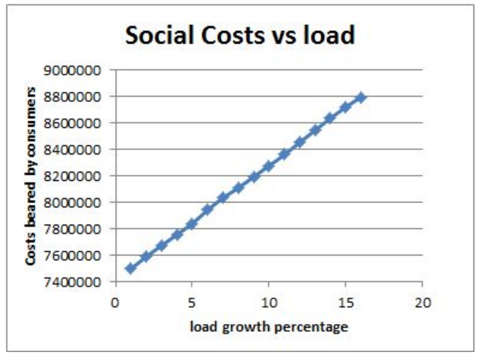
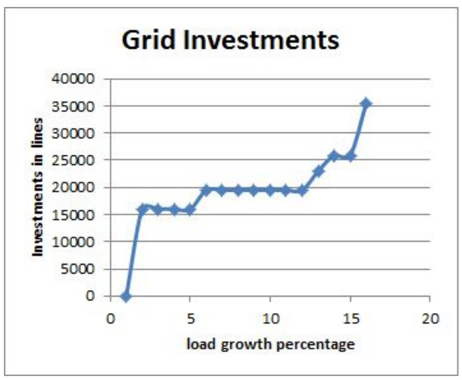


Figure 4: Social costs with respect of larger load levels



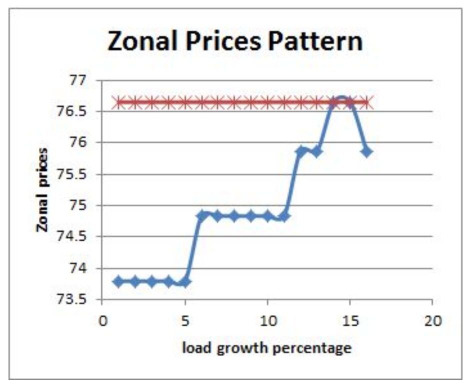


Figure 5: Grid Upgrading Investments and Locational Marginal Price vs load growth

the load grows, higher bids are progressively accepted by the Market Operator in order to cover the demand, with the results that both prices and demand will rise, leading on an increase in social costs.

Nevertheless the pattern of the Locational marginal prices can be quite interesting. As shown in right figure 5 such price increases as the aforementioned effect of new bids being accepted, but suddenly it starts decreasing. This happens because, as the load increases the TransCo will be required to expand the grid, eventually obtaining a network design in which there is enough line capacity to allow the GenCos to sell an indefinite amount of power cross areas. This has the effect of aligning the zonal prices of the two areas creating a unique market. Still, a further increase in load and related load will call for a different network setting which may prevent a GenCo with lower costs, which in our example is located in the North area, placing a full bid in the South area, as doing so would increase the zonal price in the North, leading to higher overall social costs.

The effect on GenCos on an increase in load is a steadily growing profit level as a consequence of the combined effect of higher bids being accepted in each area and the growing amount of power consumers. Nevertheless, equilibria outcomes obtained when lines are uncongestioned can be misleading when considering the profits pattern as the one shown in figures 6. In fact, with no

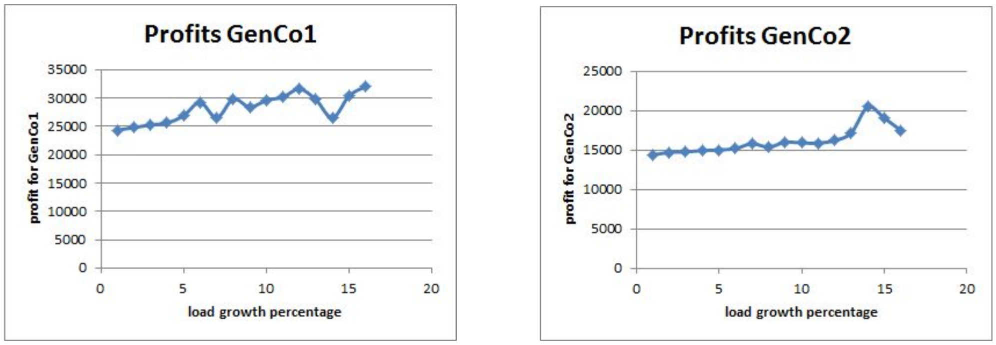


Figure 6: Profits of GenCo1 and GenCo2 respectively vs load growth

congestion on the lines, zonal prices will be the same in each area, and for each production allocation between the GenCo involved the optimal social cost for the consumer will be the same. Therefore the system profit sharing pattern among GenCos will be non influential to grid upgrade purposes. This is the reason of the behavior of the profits in the rightmost part of each chart in figure 6. In other words there can be several equilibria leading to the same optimal objective function for the TransCo and different profit levels for the GenCos involved.

6 Conclusion

We have developed a model to analyze and support the decisions on transmission grid upgrades within a market environment. In particular we have taken explicit account of possible reaction of GenCos and Market Operator to possible grid upgrade decisions both under an operational and a investment viewpoint. The problem has been modeled as a Mathematical Program with Equilibrium Constraints featuring mixed integer decision variables on both upper level and lower equilibrium problem. This mixed integer structure has been taken care by devising a dedicated algorithm which leads to the computation of a global optimum for the TransCo while featuring a Nash Equilibrium among the followers, i.e. GenCos and Market Operator. The model has been analyzed using a two zones-one line case considering the possible outcomes deriving by a load increase. Results show how the TransCo should upgrade the lines accordingly to different load levels together with some interesting behaviour shown by zonal prices in correspondance of different levels of line congestions.

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