UNIVERSITY OF BERGAMO

DEPARTMENT OF MANAGEMENT, ECONOMICS AND QUANTITATIVE METHODS

Working paper n. 2/2013

ricerca_meq@unibg.it

Series Quantitative methods

Power Transmission Capacity Expansion in a Market Driven Environment with Application to the Italian Electricity Market

Paolo Pisciella, Marida Bertocchi, Maria Teresa Vespucci

Power Transmission Capacity Expansion in a Market Driven Environment with Application to the Italian Electricity Market

Paolo Pisciella*, Marida Bertocchi and Maria Teresa Vespucci
DMSIA, University of Bergamo
Paolo.Pisciella$\vee$Marida.Bertocchi$\vee$Maria-Teresa.Vespucci@unibg.it

Abstract

We introduce a model intended for the analysis of the upgrade of the national transmission grid which explicitly accounts for responses given by the power generating companies in terms of plant expansion. The problem is modeled as a bilevel program with mixed integer structure in both, upper and lower level. Upper level is defined by the transmission company problem which has to decide on how to upgrade the network so to avoid congestions. Lower level models the reactions of generating companies, which take a decision on new facilities and power output, and market operator which strikes a new balance between demand and supply providing new zonal prices.

1 Introduction

Emergence of liberalized system for power generation is expected to increase competition and lower prices levied to end users during the upcoming years. In many countries, the electricity sector has undergone substantial structural changes, due to a series of reforms which, during the last decade, have led to a process in the quest for efficiency and increase of social welfare. The reason why these reforms have been introduced may be mainly found on a list of key factors such as the poor performance of the state-run electricity sector in terms of high costs, inadequate expansion of access to electricity service for the load, and/or unreliable supply, the inability of the state sector to finance needed expenditures on new investment and/or maintenance, the need to remove subsidies to the sector in order to release resources for other pressing public expenditure needs and the desire to raise immediate revenue for the government through the sale of assets from the sector [2], [13], [19], [20].

Technological advances related to generation of electricity have also allowed for the developement of a whole new computing system used to meter dispatch power, which in turn has given rise to new industrial structures which state enterprises have been too slow to adapt. Namely, private companies started entering the power generation market, offering power at lower costs and laying the groundwork for the introduction of a competition pattern which, in turn, will pass a larger share of benefits on to consumers.

*Corresponding author, Phone: +393385035049

In this respect, the evolution of electricity industry from the Vertically Integrated Utilities which were the past standard structure for electricity generation and delivery to the nowadays deregulated structure is introducing several challenges in planning and operation of the electric energy systems. Particularly important are the aspects related to the coordination of generation systems with transmission lines. With no longer vertical structure ensuring automatic coordination of such activities it becomes necessary to develop mechanisms to align production and expansion decisions of the market actors covering these two roles.

In Italy, after the electricity market liberalization, most of the power generation system has been removed from the former monopolist and is now carried out by different GenCos (Generator Companies), even though competition is still really limited by an incumbent producer which alone holds about 50% of total generation capacity. The agency delegated to ensure the transmission system to operate in a secure way is the TransCo (Transmission Company), Terna [10], [18]. Finally, tariffs are defined by the GME (Gestore del Mercato Elettrico), which is the italian regulator, sometimes referred to as Market Operator (MO).

Investments on italian lines upgrade have been scarce during the past years, mainly due to low remunerative capacity of the transmission tariffs together with a consistent increase of number of actors. This situation has driven the electric system to face an augmented vulnerability, which has been highlighted by several incidents, such as the 2003 total black-out, when a tree flashover caused the tripping of a major tie-line between Italy and Switzerland [4], which brought to an unbalancing of transmission on the other lines which eventually began tripping one after another. Over the course of several minutes the entire Italian power system collapsed, causing the worst black-out ever recorded in the nation history.

These failures have called for a drastic redesign of the national transmission system, especially for what concerns the transmission expansion plans. Property and usage of the transmission grid, once up to different actors, have been unified according to a so-called TransCo model. Terna is nowadays the only company operating to secure the grid operations, while a plethora of different producers ensure power generations within the different market areas. The Italian market structure is composed of five areas (North, Center, South, Sicily and Sardinia), interconnected by different sets of transmission lines, as shown in figure 1.

On such a new kind of environment, when planning over the expansion of the transmission grid, it is important for the TransCo to take into account possible responses provided by GenCos. Not accounting for such reactions could most likely lead to potential over and under-investments beared by the TransCo across the different market areas. This would in turn lead to congestion and/or violation of the security standard, and ultimately to a rise in social costs. In this framework, a more integrated policy could be achieved by requiring the TransCo to account for responses given by GenCos with respect to changes in transmission capacity location in planned grid locations. This would result in a better exploitation of the existing transmission facilities [10].

We propose a model for analysis of transmission grid expansion planning with competitive generation capacity planning in electricity markets. The purpose of the model is providing a tool to define the optimal grid upgrading program in a market driven environment. Interplay between TransCo and several independent GenCos is treated as a leader-followers Stackelberg game. Such game is expressed as the following sequential decision pattern: the TransCo decides on the best possible upgrades of transmission lines and the GenCos modify their production plans and potential capacity expansion accordingly, reaching an equilibrium together with the Market Operator, which clears the market providing new zonal prices. Reaction of GenCos lead to an electricity production equilibrium that is properly taken into account within the TransCo decision problem, which receives

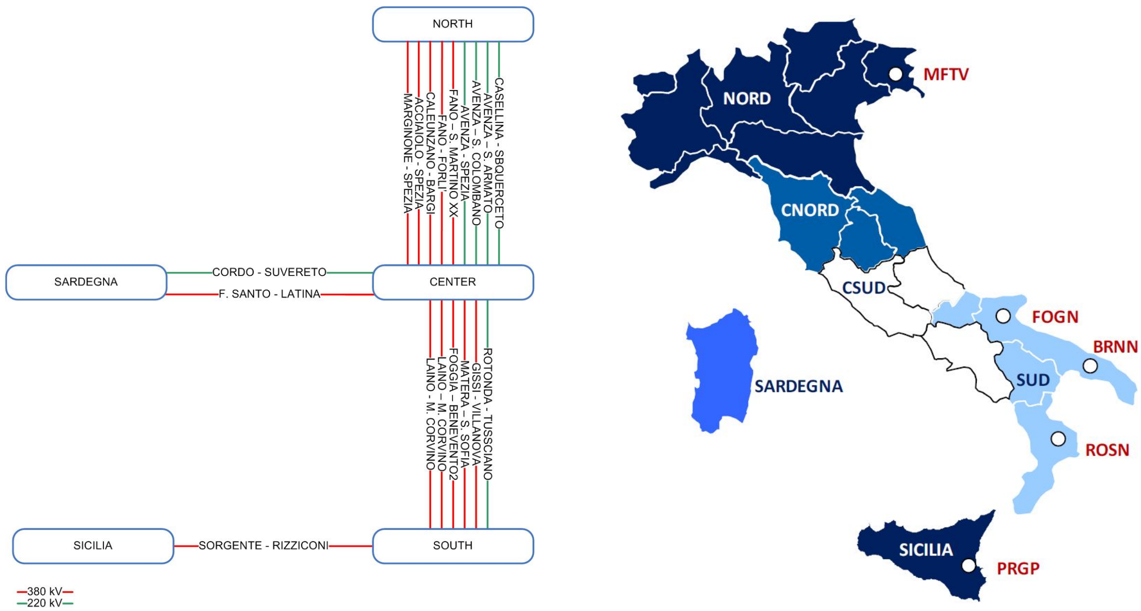

Figure 1: Italian national transmission grid structure and market zones

back new zonal prices used to meter the amount of social costs.

Problem of coordination of transmission expansion planning with competitive generation capacity planning in electricity markets has been addressed by some recent literature [14] [15], and usually solved by means of decomposition techniques aiming at computing shadow prices for zonal prices and introduce cuts to prevent GenCos and transmission companies expanding capacity in an uncoordinated manner. Here we seek the solution of the coordination problem using bilevel programming techniques. Our aim is to find a method to obtain a global optimal solution for the TransCo, given the responses provided by GenCos.

Zonal prices are obtained by including first order conditions of the market clearing conditions, into the TransCo problem. The resulting model is composed of two interdependent levels. The TransCo will take the role of leader by assuming the initial decision on how to upgrade the national transmission grid, while GenCos are considered as followers who react to the TransCo decision by reviewing their facility expansion plan and related power production. At the same level of the GenCos one finds the Market Operator, who takes care of clearing the market and setting the zonal price according to current load and bids submitted by GenCos.

Novelties introduced by our approach include:

1. Introduce a modeling framework that explicitly accounts for GenCos' investment decisions in a bi-level programming framework;

2. Elaborate on a solution scheme to obtain a Stackelberg-Nash equilibrium with mixed integer decision variables on both, upper and lower level.

Unfortunately we have not included the possibility of implementing strategic bidding as doing so would have dramatically increased the computational burden. As a consequence, bids defined by the GenCos will reflect the marginal production cost plus a predetermined bid up.

The model can be used as a tool to analyze consequences of evolution of demand on the national transmission grid, related profitability for the generation companies and shifts in social welfare. Further analyses can be conducted on a technical viewpoint, for example, by modelling the costs and benefits stemming by the introduction of a new technology.

In addition, the model could serve as a tool to investors to identify signals on the location of new generation and transmission facilities and help system planners and regulators to concur on the amounts and location of transmission capacity planning. Considerations about uncertainty related to demand and load growth are not made in the current work and shall be addressed in future research.

2 Literature Review

Due to its intrinsic complexity and multi-objective nature the problem of coordinating electricity transmission and generation has been tackled using many different techniques. The first attempts to solve such problem date back to the late sixties and were based on a centralized approach [6], [9]. Linear programming was mainly used with the aim of minimizing the pooled costs for the system. The models were focused on the analysis of possible outages stemming from over-utilization of the network. No interplay was considered amongst the actors involved in the planning activity. Interactive behaviour of electricity transmission and generation has gained more and more importance as game theoretic models have made their appearence within the realm of optimization theory. An example of application of such mixture of equilibrium concepts and optimization theory in analysis of electricity market equilibrium can be found in Hobbs [12].

Recent literature on transmission-generation expansion may be classified according to many different aspects. First and foremost, models can be classified as heuristic approaches or exact solution methods. Other considerations may be made on the granularity used to model the actors, whether each player is considered as an active competitor or as an element of a competitive fringe. In the first case the actor has the power of influencing the prices through a fine tuning of its output, while in the latter case all actors are considered as price takers. Hierarchic structure of the model is another prominent aspect to be considered. Recent approaches encompass the use of bilevel programming [3] where the transmission planner takes its decision taking the benefits of the first mover advantage, while generating companies have to decide accordingly, together with the market operator. Yet an important point is given by the approach for modeling the equilibrium between actors. This can be done in several ways, leading to diverse considerations on the stability of the equilibrium and the possibility to use an exact solution method. Many authors use a Linear Complementarity Problem (LCP) approach (see e.g. [14], [17]), i.e. they bundle Karush-Kuhn-Tucker conditions of the players involved in the market game and solve the resulting joint system. Conversely, other authors have modeled market equilibrium using best response functions [21], i.e. starting from an initial point, they looped over the different actors solving the i-th actor problem while keeping the other actors' decisions as parameters. If convergence criteria are met, this would eventually lead to a stable Nash equilibrium.

Finally, it is important to distinguish how generation expansion is treated by the various authors. Many models, particularly the ones where equilibrium amongst actors is modeled through a LCP, do not consider the expansion planning problem of the generation side. This is due to complexity linked in modeling binary decisions within a Karush-Kuhn-Tucker framework. Interplay among generating companies is therefore defined as a Cournot-Nash Game, where players compete through a wise choice of output quantities.

A recent work based on a bilevel programming approach is the one of Garcés et al. [7], which considers the transmission expansion problem as a Stackelberg game with the TransCo assuming the role of the leader and deciding over the transmission network upgrade, while the GenCos are considered as operating under perfect competition. This implies that the lower level problem of the game is defined by the market clearing condition and no expansion planning is considered for the GenCos. A similar approach is taken on a subsequent article by Garcés et al. [8] which add considerations on load shedding costs. Both works assume minimization of investment costs as the leader's objective.

Strategic behaviour of the GenCos in influencing market prices is considered in [17]. Here the approach is still based on a bilevel program, even though the two levels do not define decision problems up to two different decision makers, but rather two different decision stages - namely investment and operational stage - of a centralized system. The objective of such a system is to maximize the difference between a total welfare function and the system investment costs.

A centralized approach is also taken by [1], which considers a Mixed Integer Linear Programming problem to obtain a pool based decision for transmission network expansion to be evaluated with appropriate physical and economic metrics. GenCos are not assumed to make any expansion plan.

A hierarchical approach is taken by [11]. They consider the interaction between TransCo, GenCos and Market Operator on three different levels. On the top level they consider the TransCo, which decides on transmission expansion while taking into account the response obtained by the equilibrium output of the GenCos. In addition, each GenCo problem is defined as a bilevel problem having as lower level the market operator market clearing conditions. Each GenCo submits a bid composed of the maximum output level and a bid-price and receives the Locational Marginal Price by the lower level. The equilibrium between GenCos is obtained by means of a best response function approach. These equilibria are computed with respect to different network expansion choices. The best expansion choice is then selected via an heuristic method. Namely, the TransCo starts with a dummy and uneconomical solution containing all the candidate lines. Then lines are eliminated one by one starting from the least efficient one in terms of social welfare. The procedure iterates until budget constraints are satisfied.

A rather different approach is taken by [15], which consider a coordination model where the Independent System Operator (ISO), which takes care of system security sends economical signals to TransCos and GenCos to incentivate a coordinated expansion of transmission and generation. In this framework, ISO obtains the expansion plan of GenCos and TransCos and performs a security check on the network. Should any infeasibility be detected, the ISO generates a Bender's cut which is then included into the GenCos and TransCos problems via Lagrangean Relaxation. Locational Marginal Prices and Flowgate Marginal Prices are updated at each iteration with new dual variables of the load constraint and fed back to the GenCos and TransCos and the process starts anew. The last iteration is reached when there is no change in the Social Welfare function.

The authors also solve a stochastic version [16] of the model with scenarios defined by possible outages deriving by malfunctioning of plants or lines.

A similar approach to the one of Hobbs is taken by [14]. The authors consider the problems of TransCo, GenCos and Market Operator as the solution of a Linear Complementarity Problem. The LCP is solved eliminating the constraints containing integer variables related to generation and tranmission expansion. The LCP is then reformulated into a quadratic optimization problem and the previously eliminated constraints are incorporated back into the problem.

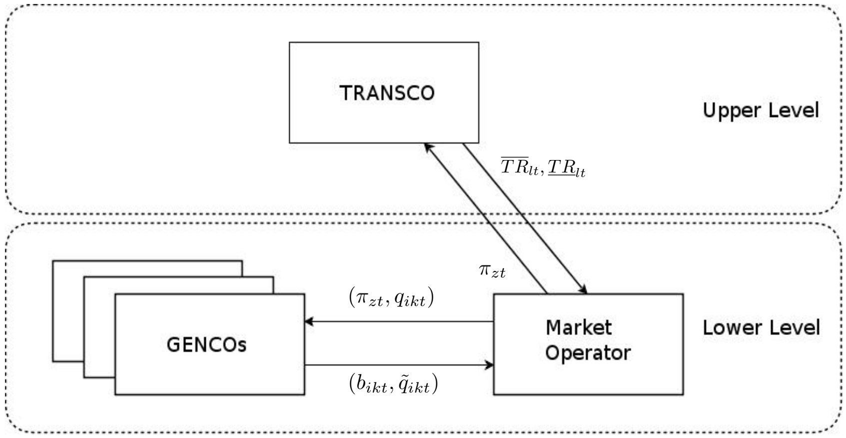

Figure 2: Interdependancies between transmission company, generator companies and market operator

3 General model

The model introduced in this article aims at capturing the strategies stemming by a sequencial game between three players: the Transmission Company, a group of Generating Companies and the Market Operator. The model is structured in two different, interrelated decision levels which represent the sequencial nature of the decisions up to each player. Namely, the TransCo will take its decision on transmission structure as first mover, then each GenCo will decide on power production levels and potential new investments accordingly to the choice made by the TransCo. Bids provided by each GenCo are collected and sorted by a Market Operator, which takes care of clearing the market.

We assume that electricity shall be delivered to companies and consumers spread over different market zones, denoted by index z. Each zone has different properties in terms of load and possible power production levels, which in turn depend on the amount of existing power plants and candidate areas for the installment of new power plants. The modeling framework considered in this paper is displayed in figure 2. The two-levels Stackelberg game involves only the network planner as the upper level player and a group of power generating companies, whose bid-ask mechanism with the consumers is mediated by a Market Operator, as the lower level. The TransCo aims at lowering the total social cost, defined as the sum of covered load times the zonal price payed to purchase energy over the different zones and time periods, plus investment costs for installment of new power transmission lines. Such objective is pursued by allowing more efficient GenCos selling energy in different areas. TransCo takes a decision on installation of new power transmission lines which, together with the existing transmission lines, will define new upper and lower bounds ($\overline{TR}_{lt}$, $\underline{TR}_{lt}$) on the line power flow streaming over a intra-zonal connection line l at time t. These bounds are used by the Market Operator to define how much power can be transferred between zones in order to accept bids that are supposed to cover the load at a given time t in all the considered areas at the least possible social cost. Bids are sent to the Market Operator by the GenCos in form of a pair (b_{ikt}, $\tilde{q}_{ikt}$) defining the price bid at time t from generator k belonging to GenCo i and the related quantity respectively. Such bids are defined on the ground of projected profitability of the energy production. This latter depends on installed capacity and decisions on installations of new power plants which concur as decision variables for the GenCos. Market Operator that, given the power flow bounds defined by the TransCo will define the zonal prices π_{zt} for zone z at time t and the accepted quantites from each power generator. GenCos aim at maximizing their profit, by deciding how much power to supply and whether to open new power plants. We assume that GenCos do not influence zonal prices through strategic bidding, but bids simply reflect a mark-up on marginal costs to each producer. This reflects quite well the Italian structure, where except for the dominant player and former monopolist ENEL, each GenCo plays a limited role in the delivery of electricity.

As aforementioned, the modeling framework is composed of three decision problems where the TransCo problem and the GenCo problems together with the Market Operator problem are sequentially interrelated. The model is complicated by the presence of binary variables both on the TransCo problem and on the GenCo problems. A dedicated algorithm has been developed in order to take care of binary decision variables featured in the equilibrium problem between GenCos and Market Operator. We tackle the problem by solving a relaxation of the TransCo problem in which all of the investments based binary decision variables are controlled by the TransCo, while first order conditions are imposed for the continuous part of the GenCo and Market Operator problems. This returns a lower bound for the TransCo problem which is normally not consistent with the response given by each GenCo in terms of binary variables. Therefore the solution is tested for consistency with the solution provided by each GenCo in order to check whether the binary decision variables up to the GenCos obtained by solving the relaxation are the same as the ones provided by the solution of each GenCo problem. If this is not the case, the algorithm creates appropriate cuts in order to prevent the TransCo to choose the same integer solution for the variables up to the GenCos in subsequent iterations.

Let us look more closely how each actor's problem is defined

MARKET OPERATOR PROBLEM

The main task of the Market Operator (MO) is the one of matching energy demand and supply at each time point. As a consequence of such match, MO will determine hourly zonal prices. Let us introduce the following notation

Sets:

- T: set of periods t, in every period we consider the hour of maximum load demand;
- I: set of oligopolistic producers i;
- Z: set of zones z;
- L^E: set of existing transmission links l;
- L^C: set of candidate transmission links l;
- K_{iz}^E: set of existing technologies k belonging to producer i in zone z;
- K_{iz}^C: set of candidate technologies k of producer i in zone z.

Parameters:

- b_{ikt}: price of sell bid of plant k belonging to GenCo i in period t;

- A_{zl}: incidence matrix of the system;
- C_{zt}: load in zone z in period t;
- $\tilde{q}_{ikt}$: MW offered by generator k belonging to GenCo i at time t;
- $\overline{TR}_{lt}$: maximum capacity of transmission link l in period t;
- $\underline{TR}_{lt}$: minimum capacity of transmission link l in period t.

Decision variables:

- q_{ikt}: accepted bid in MW for technology k of producer i in period t;
- TR_{lt}: power flow on transmission link l in period t;

The Market Operator must enforce the load coverage at every time point. Electricity can be supplied within the area where it is used or transported from another area. Load coverage constraint is given by

$$\sum_{i\in I}\sum_{k\in K_{iz}^E\cup K_{iz}^C} q_{ikt} + \sum_{l\in L^E\cup L^C} A_{zl}TR_{lt} = C_{zt} \qquad z\in Z, t\in T$$

where A_{zl} represents the incidence matrix describing the cross areas connections. Such modeling framework for transmission lines suits well the italian grid structure because of its trunk based topology (see e.g. [22]). Constraints on power allocation and flow bounds must be fulfilled too. These are the following

$$q_{ikt} \leq \tilde{q}_{ikt} \qquad i\in I, \quad z\in Z, \quad k\in K_{iz}^E\cup K_{iz}^C, \quad t\in T$$

and

$$\underline{TR}_{lt} \leq TR_{lt} \leq \overline{TR}_{lt} \qquad l\in L^E\cup L^C, \quad t\in T$$

Overall market clearing conditions for the perfect competitive system considered for a group of similar producers is therefore given by the solution of the problem[1]

$$\min_{q_{ikt},TR_{lt}} \sum_{i\in I}\sum_{t\in T}\sum_{z\in Z}\sum_{k\in K_{iz}^E\cup K_{iz}^C} b_{ikt}q_{ikt} \tag{1}$$

subject to

$$\sum_{i\in I}\sum_{k\in K_{iz}^E\cup K_{iz}^C} q_{ikt} + \sum_{l\in L^E\cup L^C} A_{zl}TR_{lt} = C_{zt} \quad :\pi_{zt} \qquad z\in Z, t\in T \tag{2}$$

$$q_{ikt} \leq \tilde{q}_{ikt} \quad :\lambda_{ikt} \qquad i\in I, \quad z\in Z, \quad k\in K_{iz}^E\cup K_{iz}^C, \quad t\in T \tag{3}$$

$$\underline{TR}_{lt} \leq TR_{lt} \leq \overline{TR}_{lt} \quad :\eta_{lt}^+,\eta_{lt}^- \qquad l\in L^E\cup L^C, \quad t\in T \tag{4}$$

$$q_{ikt} \geq 0 \tag{5}$$

Solution of the introduced problem operates a sorting through the accepted bids, starting from

[1] We report the dual variables of each constraint right after the constraint itself preceded by a colon.

cheapest one and ending with the most expensive.

GENCO PROBLEM

At the same level as the Market Operator one finds the set of GenCos. These actors aim at maximizing their own profit by submitting bids $(b_{ikt}, \tilde{q}_{ikt})$ to the Market Operator and defining their optimal expansion plan according to the structure of the grid. The problem of the i-th GenCo involves the following notation

Parameters:

- π_{zt}: zonal price in area z at time t ;
- δ: discount factor;
- c_{ik}: generation cost of technology k for producer i;
- f^G_{ik}: investment cost of technology k for producer i;
- Γ^C_{ik}: capacity of candidate technology k of producer i;
- Γ^E_{ik}: capacity of existing technology k of producer i;
- q_{ikt}: accepted bid in MW for technology k of producer i in period t;

Decision variables:

- Y_{ik}: binary variable set to 1 if producer i activates technology k;
- $\tilde{q}_{ikt}$: MW offered by generator k belonging to GenCo i at time t;

Each GenCo has a capacity limit both for existing generator and for candidate ones. Such limit can be expressed by the constraints

$$\tilde{q}_{ikt} \leq \Gamma^E_{ik} \qquad z \in Z, \quad k \in K^E_{iz}, \quad t \in T$$

and

$$\tilde{q}_{ikt} \leq \Gamma^C_{ik} Y_{ik} \qquad z \in Z, \quad k \in K^C_{iz}, \quad t \in T$$

respectively.

Finally, the quantity that is sold by each GenCo cannot be larger than the quantity accepted by the Market Operator, i.e.

$$\tilde{q}_{ikt} \leq q_{ikt} \qquad z \in Z, \quad k \in K^E_{iz} \cup K^C_{iz}, \quad t \in T.$$

The decision problem up to each GenCo is therefore the following

$$\max_{\tilde{q}_{ikt}, b_{ikt}, Y_{ik}} \sum_{t \in T} \delta^{-t} \sum_{z \in Z} \left(\pi_{zt} \sum_{k \in K^E_{iz} \cup K^C_{iz}} \tilde{q}_{ikt} - \sum_{k \in K^E_{iz} \cup K^C_{iz}} c_{ik} \tilde{q}_{ikt} \right) - \sum_{z \in Z} \sum_{k \in K^C_{iz}} f^G_{ik} Y_{ik} \tag{6}$$

subject to

$$\tilde{q}_{ikt} \leq \Gamma^C_{ik} Y_{ik} \quad : \mu^{\Gamma C}_{ikt} \qquad z \in Z, \quad k \in K^C_{iz}, \quad t \in T \tag{7}$$

$$\tilde{q}_{ikt} \leq \Gamma^E_{ik} \quad : \mu^{\Gamma C}_{ikt} \qquad z \in Z, \quad k \in K^E_{iz}, \quad t \in T \tag{8}$$

$$\tilde{q}_{ikt} \leq q_{ikt} \quad : \mu_{ikt}^{\tilde{q}+} \qquad z \in Z, \quad k \in K_{iz}^{E} \cup K_{iz}^{C}, \quad t \in T \tag{9}$$

$$\tilde{q}_{ikt} \geq 0, \quad Y_{ik} \in \{0,1\} \tag{10}$$

In this problem, π_{zt} represents electricity price in zone z in period t and is defined as the shadow price of load covering constraint (2).

The solution of each GenCo problem changes accordingly to the bid level that can be accepted by the Market Operator in each market zone. This latter in turn depends on the possibility of transferring electricity between different zones. This mechanism establishes a hierarchical relation between the decision up to each GenCo in terms of MW to provide to the market and potential new installments and the decision taken by the TransCo in terms of transmission capacity over the transmission lines. According to this relation we will refer to the GenCo problem as $GP_i\left(\overline{TR}_{lt}, \underline{TR}_{lt}\right)$ in order to stress out the aforementioned relation.

TRANSCO PROBLEM

TransCo aims at minimizing the sum of social costs given the expansion investment budget. High social costs derive by high bids settled by GenCos which can be reduced by removing congestions over the transmission grid. We assume that only upgrade of existing lines is considered by the TransCo. The problem is formalized as follows.

Parameters:

- C_{zt}: load in zone z in period t;
- $\overline{\Lambda}_l^E, \underline{\Lambda}_l^E$: upper and lower bound on existing line capacity;
- $\overline{\Lambda}_l^C, \underline{\Lambda}_l^C$: upper and lower bound on candidate line capacity with voltage w;
- f_{wl}^T: investment cost for opening line l with voltage w;
- B: total budget for lines expansion.

Decision variables:

- π_{zt}: zonal price in area z at time t;
- X_{wl}: binary variable set to 1 if line l of type w is built;
- $\overline{TR}_{lt}, \underline{TR}_{lt}$: overall upper and lower bound line capacity;

$$\min_{\pi_{zt}, \overline{TR}_{lt}, \underline{TR}_{lt}, X_{wl}} \sum_{t \in T} \sum_{z \in Z} C_{zt} \pi_{zt} \tag{11}$$

subject to

$$\overline{TR}_{lt} = \overline{\Lambda}_l^E + \sum_{w \in W} \overline{\Lambda}_{wl}^C X_{wl} \qquad l \in L^E \cap L^C, \quad t \in T \tag{12}$$

$$\underline{TR}_{lt} = \underline{\Lambda}_l^E + \sum_{w \in W} \underline{\Lambda}_{wl}^C X_{wl} \qquad l \in L^E \cap L^C, \quad t \in T \tag{13}$$

$$\sum_{w \in W} \sum_{l \in L^C} f_{wl}^T X_{wl} \leq B \tag{14}$$

$$\pi_{zt} \in \Omega\left(\overline{TR}_{lt}, \underline{TR}_{lt}\right) \qquad z \in Z, \quad t \in T \tag{15}$$

$$\overline{TR}_{lt}, \underline{TR}_{lt} \geq 0, \quad X_{wl} \in \{0,1\} \tag{16}$$

where the first two constraints define the upper and lower bounds on power flows in a given line l and at a given time t. These bounds are given by the sum of the existing bounds $\overline{\Lambda}_l^E$ plus a potential bound upgrade, added to the existing bound if a candidate line is opened. The TransCo can choose among different types of transmission lines, depending on their voltage. W is the set of possible types of available transmission lines. The third constraint represents the upper bound on the capital expenses for lines expansion. Finally, $\Omega\left(\overline{TR}_{lt}, \underline{TR}_{lt}\right)$ represents the space of joint solutions of problems (1)-(5) and (6)-(10) parametrized by $\overline{TR}_{lt}$ and $\underline{TR}_{lt}$. Such set contains the possible equilibria, defined in terms of decision variables $\tilde{q}_{ikt}, b_{ikt}, Y_{ik}$ for the involved GenCos, decision variables q_{ikt}, TR_{lt} for the market operator and dual variables for both the aforementioned problems. Dual variables for problems (1)-(5) and (6)-(10) are the following. Namely we define π_{zt} and λ_{ikt} as the dual variables for constraints (2)-(3) respectively and η_{lt}^+ and η_{lt}^- as the positive and negative part of the dual variable for constraint (4) and $\mu_{ikt}^{\Gamma C}, \mu_{ikt}^{\Gamma E}$ and $\mu_{ikt}^{\tilde{q}+}$ as dual variables for constraints (7), (8) and (9). All of the aforementioned variables belong to the set $\Omega\left(\overline{TR}_{lt}, \underline{TR}_{lt}\right)$, which is paramatrized by the choice of upper and lower bounds for power transmission decided by the TransCo. Variables f_{ik}^G and f_{wl}^T have been divided by 8750, which represent the yearly operating hours for a plant. This is in order to harmonize the definition of load and offered quantity which are represented under a hourly time horizon with the investment costs, which must consider the whole energy sold in a time horizon expressed in years in order to be a valuable measure to be used for assessment of investment decisions.

4 Model Reformulation

In the remainder of the article, we introduce an iterative procedure in which the TransCo takes control of all of the binary variables at each iteration, then a check is performed, to ensure whether the solution found by the TransCo coincides with the solution of each GenCo with respect to the binary variables. Shall this not happen, a cut and a penalty are introduced into the TransCo problem to prevent the same solution to be chosen in a subsequent iteration.

As we have mentioned earlier in the article, we assume that GenCos do not perform any sort of strategic bidding. The consequence is that bids will reflect marginal costs c_{ik} of the production from a given plant k plus a GenCo specific bid up χ_i. This implies that the structure of the offer curve is completely determined by the marginal costs up to each power plant of each GenCo and there will be no interplay to determine an equilibrium on offer prices. On the other hand the equilibrium on quantities stems from the level of bids accepted by the Market Operator. Namely the MO will carry on accepting bids with steadily increasing prices until the load is covered. Nevertheless capacity of each GenCo needs to be considered when the Market Operator defines a candidate value for the amount of power produced by each GenCo. This is true because GenCos could find beneficial expand their capacity for a given load request over time, allowing a larger amount of energy to be accepted by the Market Operator for load covering.

In principle, if GenCos have no possibility to expand capacity, it would be possible to compute the Stackelberg equilibrium simply considering the TransCo problem as upper level and the Market Operator problem as lower level. The reason for this is that bid prices are already given and profit function for each GenCo is increasing with respect to quantity produced. This implies that GenCos selected by the Market Operator will be willing to offer power as long as their capacity limit is not hit.

When considerations about GenCos capacity constraints are included in the modeling framework one needs to assure that Market Operator takes care of such constraints when setting up the electricity exchanges. This can be done by explicitly inserting capacity expansion constraints (7)-(8) of GenCos into the MO problem. For a given arrangement of candidate power plants for each GenCo the MO will know which are the bounds on power generation. Such bounds are iteratively updated in the following manner.

1. Market Operator problem is explicitly considered in the TransCo decision model by means of its Karush-Kuhn-Tucker conditions considering decisions on new installments of power plants as parameters.

2. TransCo solves a mathematical program with complementarity constraints with mixed integer structure due to the presence of binary variables defining the state of new power plants. Such problem is a relaxation of the original bilevel problem in the sense that it provides a lower bound for the minimization of the social costs and the solution of the original bilevel problem is feasible for such problem. The solution of such problem returns a tentative decision on the candidate power plants to be opened and bounds on the quantities accepted by the Market Operator, besides the value for the transmission bounds on each line.

3. GenCos solve their decision problems with bounds on the power accepted by the MO. The unit margin for the profit of GenCos whose bids are accepted by the MO is positive, therefore GenCos will increase the production until they hit an upper bound. Such upper bound can be their maximum production capacity or the maximum accepted quantity by the MO. The result of the decision problem up to the GenCos is their actual decisions on capacity expansion and power generated and injected into the grid.

4. Solutions on candidate power plants provided by TransCo and GenCos are compared. If the decisions coincide then the candidate solution is optimal, otherwise a procedure will prevent the TransCo choosing the same solution in a subsequent iteration of the problem by means of a cut and an additive penalty term in the objective function.

The problem solved by the Market Operator is therefore given by (1)-(5) with the inclusion of constraints (7) and (8) with $\tilde{q}_{ikt}$ replaced by q_{ikt}. We call such problem $MO(\overline{TR}_{lt}, \underline{TR}_{lt})$ in order to stress out its dependancy on the upper and lower bounds on power transmission provided by TransCo. In such a problem Y_{ik} is a parameter defined by the TransCo as a tentative value[2] and corrected or confirmed by GenCos after the solution of their decision problem.

Problem up to the generic i-th GenCo is given by (6)-(10) We stress out that the GenCos are willing to provide as much power as they possibly can as their profit increases with power supplied. We call such problem $GP_i(\overline{TR}_{lt}, \underline{TR}_{lt})$ for the i-th Genco.

[2]The decision on tentative values for candidate power plants is up to the TransCo. This is true because the Market Operator problem is integrated into the TransCo problem through its first order conditions.

Finally the problem to be solved is the TransCo problem, defined as

$$
\begin{aligned}
\min_{\pi_{zt},\overline{TR}_{lt},\underline{TR}_{lt},X_{wl}} \quad & \sum_{t\in T}\sum_{z\in Z} C_{zt}\pi_{zt} \\
\text{s.t.} \quad & \overline{TR}_{lt} = \overline{\Lambda}_l^E + \sum_{w\in W} \overline{\Lambda}_{wl}^C X_{wl} \qquad l\in L^E\cap L^C, \quad t\in T \\
& \underline{TR}_{lt} = \underline{\Lambda}_l^E + \sum_{w\in W} \underline{\Lambda}_{wl}^C X_{wl} \qquad l\in L^E\cap L^C, \quad t\in T \\
& \sum_{w\in W}\sum_{l\in L^C} f_{wl}^T X_{wl} \le B \\
& \pi_{zt} \in \Omega\left(\overline{TR}_{lt}, \underline{TR}_{lt}\right) \qquad z\in Z, \quad t\in T \\
& \overline{TR}_{lt}, \underline{TR}_{lt} \ge 0, \quad X_l \in \{0,1\}
\end{aligned}
\tag{17}
$$

which defines a mathematical problem with bilevel structure entailing one leader (TransCo) and multiple followers (GenCos and Market Operator). We have included a constraint on the total budget B that the TransCo is allowed to use for line expansion. As explained earlier, the lack of interplay in the definition of the price bids allows us to consider only the MO problem as the lower level. Accordingly, if we define $dualMO\left(\overline{TR}_{lt}, \underline{TR}_{lt}\right)$ as the dual problem for the Market Operator problem parametrized by the upper and lower bounds on transmission capacity we have that $\Omega\left(\overline{TR}_{lt}, \underline{TR}_{lt}\right) = \left\{\pi_{zt} \mid \left(\pi_{zt}, \sigma_{ikt}^C, \sigma_{ikt}^E, \eta_{lt}^+, \eta_{lt}^-, \lambda_{ikt}\right) \text{ is optimal solution to } dualMO\left(\overline{TR}_{lt}, \underline{TR}_{lt}\right)\right\}$.

In order to provide a viable relaxation for the TransCo problem we define the KKT conditions of problem $MO(\overline{TR}_{lt}, \underline{TR}_{lt})$ and plug it as replacement of set $\Omega\left(\overline{TR}_{lt}, \underline{TR}_{lt}\right)$. Karush-Kuhn-Tucker conditions for problem $MO(\overline{TR}_{lt}, \underline{TR}_{lt})$ are the following

$$
\begin{aligned}
& \sum_{z\in Z} A_{zl}\pi_{zt} - \eta_{lt}^+ + \eta_{lt}^- = 0 \qquad l\in L^E\cup L^C, \quad t\in T \\
& \sum_{i\in I}\sum_{k\in K_{iz}^E\cup K_{iz}^C} q_{ikt} + \sum_{l\in L^E\cup L^C} A_{zl} TR_{lt} = C_{zt} \qquad z\in Z, \quad t\in T \\
& 0 \le b_{ikt} + \sigma_{ikt}^C + \sigma_{ikt}^E - \pi_{zt} \perp q_{ikt} \ge 0 \qquad i\in I, \quad z\in Z, \quad k\in K_{iz}^E\cup K_{iz}^C, \quad t\in T \\
& 0 \le \Gamma_{ik}^C Y_{ik} - q_{ikt} \perp \sigma_{ikt}^C \ge 0 \qquad i\in I, \quad z\in Z, \quad k\in K_{iz}^C, \quad t\in T \\
& 0 \le \Gamma_{ik}^E - q_{ikt} \perp \sigma_{ikt}^E \ge 0 \qquad i\in I, \quad z\in Z, \quad k\in K_{iz}^E, \quad t\in T \\
& 0 \le \overline{TR}_{lt} - TR_{lt} \perp \eta_{lt}^+ \ge 0 \qquad l\in L^E\cup L^C, \quad t\in T \\
& 0 \le TR_{lt} - \underline{TR}_{lt} \perp \eta_{lt}^- \ge 0 \qquad l\in L^E\cup L^C, \quad t\in T \\
& \pi_{zt}, TR_{lt} \in \Re
\end{aligned}
\tag{18}
$$

Following such approach we can formulate the Integer Leader Relaxation as the following mathematical program with complementarity constraints and mixed integer structure

$$
\begin{aligned}
\min \quad & \sum_{t\in T}\sum_{z\in Z} C_{zt}\pi_{zt} \\
\text{s.t.} \quad & \overline{TR}_{lt} = \overline{\Lambda}^E_l + \sum_{w\in W}\overline{\Lambda}^C_{wl}X_{wl} \qquad l\in L^E\cap L^C, \quad t\in T \\
& \underline{TR}_{lt} = \underline{\Lambda}^E_l + \sum_{w\in W}\underline{\Lambda}^C_{wl}X_{wl} \qquad l\in L^E\cap L^C, \quad t\in T \\
& \sum_{w\in W}\sum_{l\in L^C} f^T_{wl}X_{wl} \leq B \\
& \sum_{z\in Z} A_{zl}\pi_{zt} - \eta^+_{lt} + \eta^-_{lt} = 0 \qquad l\in L^E\cup L^C, \quad t\in T \\
& \sum_{i\in I}\sum_{k\in K^E_{iz}\cup K^C_{iz}} q_{ikt} + \sum_{l\in L^E\cup L^C} A_{zl}TR_{lt} = C_{zt} \qquad z\in Z, \quad t\in T \\
& 0\leq b_{ikt} + \sigma^C_{ikt} + \sigma^E_{ikt} - \pi_{zt} \perp q_{ikt} \geq 0 \qquad i\in I, \quad z\in Z, \quad k\in K^E_{iz}\cup K^C_{iz}, \quad t\in T \\
& 0\leq \Gamma^C_{ik}Y_{ik} - q_{ikt} \perp \sigma^C_{ikt} \geq 0 \qquad i\in I, \quad z\in Z, \quad k\in K^C_{iz}, \quad t\in T \\
& 0\leq \Gamma^E_{ik} - q_{ikt} \perp \sigma^E_{ikt} \geq 0 \qquad i\in I, \quad z\in Z, \quad k\in K^E_{iz}, \quad t\in T \\
& 0\leq \overline{TR}_{lt} - TR_{lt} \perp \eta^+_{lt} \geq 0 \qquad l\in L^E\cup L^C, \quad t\in T \\
& 0\leq TR_{lt} - \underline{TR}_{lt} \perp \eta^-_{lt} \geq 0 \qquad l\in L^E\cup L^C, \quad t\in T \\
& \overline{TR}_{lt}, \underline{TR}_{lt} \geq 0, \quad \pi_{zt}, TR_{lt} \in \Re \\
& X_{wl}, Y_{ik} \in \{0,1\}
\end{aligned} \tag{19}
$$

When this problem is solved one is left with optimal values q^*_{ikt} and Y^*_{ik}, besides the incumbent value for the zonal prices π^*_{zt} and the candidate value for the i-th GenCo profit v^*_i. Such values can be used to perform a comparison with the GenCo problems. Namely one has to solve problem $GP_i(\overline{TR}_{lt}, \underline{TR}_{lt})$ with $\pi_{zt} = \pi^*_{zt}$. Once one obtains the solution of problem $GP_i(\overline{TR}_{lt}, \underline{TR}_{lt})$ it will be possible to compare the decision on opening of candidate power plants stemming from $GP_i(\overline{TR}_{lt}, \underline{TR}_{lt})$ which we denote by $\bar{Y}_{ik}$ and the candidate value provided by the solution of (19), which we denote by Y^*_{ik}. In addition one will compare the candidate profit value obtained by solving (19) and denoted by $\bar{v}_i$ with the actual optimal profit v^*_i. If the comparison returns that $\bar{Y}_{ik} \neq Y^*_{ik}$ or $\mid v^*_i - \bar{v}_i \mid > \epsilon$ with $\epsilon > 0$ and small, then a cut and penalty shall be inserted in problem (19). The details of such procedure shall be explained in some more detail in the remainder of the article.

5 Solution Approach

What one can expect is that the solution of (19) will not be the solution of the original problem. In fact, if we define the i-th GenCo problem with all other players in equilibrium as $\mathrm{GP}_i(\overline{TR}_{lt}, \underline{TR}_{lt})$ and given a solution (X^*_l, Y^*_{ik}) of the (ILR), this coincides with the solution of (11)-(16) iff

$$
Y^*_{ik} = \arg\max \mathrm{GP}_i(\overline{TR}_{lt}, \underline{TR}_{lt}) \qquad i\in \mathcal{I}, \quad z\in Z, \quad k\in K^C_{iz} \tag{20}
$$

which can be obtained by fixing all the players except i-th's decision variables to the ones supplied by the solution of the ILR and solving problem $\mathrm{GP}_i(\overline{TR}_{lt}, \underline{TR}_{lt})$ for the i-th GenCo. If solution of i-th GenCo problem does not coincide with the one found with the TransCo problem, it is necessary to devise a procedure to prevent the TransCo choosing the same integer solution when solving the Integer Leader Relaxation. The purpose is to require the TransCo to pick the second best integer solution of the ILR. Generally speaking, we want to force the TransCo to choose progressively worse optimal solutions of the ILR until we find the best solution satisfying the equilibrium problem between the GenCos (20). This is done by inserting an appropriate cut and a penalty term for the incumbent optimal ILR solution

Namely, at generic iteration n we define the following auxiliary variable

$$\tilde{Y}_{ik}^{(n)} = \begin{cases} Y_{ik} & \text{if } Y^*_{ik[n-1]} = 1 \\ 1 - Y_{ik} & \text{if } Y^*_{ik[n-1]} = 0 \end{cases}$$

where $Y^*_{ik[n-1]}$ defines the integer values of the incumbent optimal solution of the ILR problem, taken among all of the $\left|K^C\right|$ variables controlled by the upper level decision maker through the ILR problem. Such an auxiliary variable is then included in the term

$$\frac{\sum_{i \in \mathcal{I}} \sum_{k \in K_i^C} \tilde{Y}_{ik}^{(n)}}{|K^C|}$$

which sums up to unit when the solution taken is the same as the incumbent optimal ILR solution. In order to avoid such solution to be picked in a new iteration we define a binary variable for iteration n, denoted by u_n, which turns to unit and enables a penalty term in the leader objective funtion when the ILR integer solution is chosen. Formally we need to build a condition such that u_n turns to unit when $\frac{\sum_{i \in \mathcal{I}} \sum_{k \in K_i^C} \tilde{Y}_{ik}^{(n)}}{|K^C|} = 1$. Such a cut is defined as

$$1 + u_n \geq \frac{\sum_{i \in \mathcal{I}} \sum_{k \in K_i^C} \tilde{Y}_{ik}^{(n)}}{|K^C|} + \epsilon \tag{21}$$

where $\epsilon < \frac{1}{|K^C|}$.

When the RHS of (21) is larger than one, binary variable u_n takes value 1 to satisfy the constraint. At the same time, a penalty term Mu_n is added to the ILR objective function, with M being a large positive scalar.

In order to be used as a constraint to penalize the ILR problem an explicit reformulation of (21) in terms of the involved binary decision variables is needed. By multiplying each side of (21) by $\left|K^C\right|$ and rearranging the result we obtain

$$\sum_{i \in \mathcal{I}} \sum_{k \in K_i^C} \tilde{Y}_{ik}^{(n)} - \left|K^C\right| u_n \leq \left|K^C\right| (1 - \epsilon) \tag{22}$$

Posing $\epsilon = \frac{1}{|K^C|} - \frac{1}{|K^C|^2}$ we can express the cut as

$$\sum_{i \in \mathcal{I}} \sum_{k \in K_i^C} \tilde{Y}_{ik}^{(n)} - \left|K^C\right| u_n \leq \frac{\left|K^C\right|^2 - \left|K^C\right| + 1}{|K^C|} \tag{23}$$

and defining $\tilde{Y}_{ik}^{(n)} = a_{ik}^{(n)} Y_{ik} + b_{ik}^{(n)}$ with

$$a_{ik}^{(n)} = \begin{cases} 1 & \text{if } Y^*_{ik[n-1]} = 1 \\ -1 & \text{if } Y^*_{ik[n-1]} = 0 \end{cases}$$
$$b_{ik}^{(n)} = \begin{cases} 0 & \text{if } Y^*_{ik[n-1]} = 1 \\ 1 & \text{if } Y^*_{ik[n-1]} = 0 \end{cases} \tag{24}$$

we finally obtain the reformulation

$$\sum_{i \in \mathcal{I}} \sum_{k \in K_i^C} a_{ik}^{(n)} Y_{ik} - \left|K^C\right| u_n \leq \frac{\left|K^C\right|^2 - \left|K^C\right| + 1}{|K^C|} - \sum_{i \in \mathcal{I}} \sum_{k \in K_i^C} b_{ik}^{(n)} \tag{25}$$

At iteration n we will have the TransCo solving the Penalized Integer Leader Relaxation problem (PILR) defined as the ILR problem with the addition, at each iteration n, of an additional cut of type (25) and with the following modification of the objective function

$$\sum_{t \in T} \sum_{z \in Z} C_{zt} \pi_{zt} + M \sum_{r=1}^{n} u_r$$

After problem PILR is solved, the solution is tested with problem (20), and if the solution coincides with the one obtained solving the PILR the algorithm stops. Otherwise, a new cut and penalty term are included in the PILR and the process starts anew. Optimal solution is reached because the leader will explore all the feasible integer solutions as each optimal solution is fathomed by the cut and penalty and a second best solution has to be chosen. Since the optimal solution of the original problem (11)-(16) lies within the feasible set, it has to be eventually picked up as candidate by the algorithm. One problem that can arise with the described approach is anyway linked to degenerate solutions of the ILR and PILR problems. Namely such problem could return a result which is not the same found by the GenCos in terms of profitability and choice of binary variables only because of degeneracy, i.e. there exists another solution with the same objective function for the TransCo sharing the same values for profit and choice of binary variables with all of the GenCos. This can generate sub-optimal solutions for the problem (11)-(16).

6 Progressive Penalization Algorithm

The idea of the algorithm is to obtain candidate solutions as lower bounds resulting by the solution of the PILR problem, and test whether such solutions are feasible by solving the Followers Problem (FP). The test consists in checking if either the optimal function of each follower is the same as the value obtained by solving the PILR or the optimal choice of each follower, in terms of new installed plants, coincides with the ones obtained by solving the PILR problem. If the integer solution of the FP coincides with the corresponding values obtained solving the PILR problem or the objective function value is the same as the optimal value of each follower, the result is optimal for problem

(11)-(16). Otherwise a cut and a penalization are included in the constraints and the objective function of the PILR respectively to prevent the incumbent optimal solution of the PILR to be chosen again in a new iteration. The algorithm is thus described in the table below, where symbols $\overline{v}_{i[n]}$ and $v^*_{i[n]}$ denote the optimal profit of the i-th GenCo at iteration n and the candidate profit for GenCo i computed by the TransCo at iteration n, while ϵ is a small positive real number.

Algorithm 1 Progressive Penalization Algorithm

(Step 1)
$n \leftarrow 0$
solve $ILR \rightarrow (\overline{TR}^*_{lt}, \underline{TR}^*_{lt}, q^*_{ikt}, Y^*_{ik[n]}, X^*_{wl}, v^*_{i[n]})$
if ILR is infeasible **then**
  STOP. The problem is infeasible
else
  (Step 2)
  for $i = 1 \rightarrow |\mathcal{I}|$ **do**
    solve $\text{GP}_i(\overline{TR}^*_{lt}, \underline{TR}^*_{lt}) \rightarrow (\bar{Y}_{ik[n]}, \overline{v}_{i[n]})$
  end for
  if $\bar{Y}_{ik[n]} = Y^*_{ik[n]}$ **or** $| \overline{v}_{i[n]} - v^*_{i[n]} | < \epsilon$ **then**
    $(\overline{TR}^*_{lt}, \underline{TR}^*_{lt}, X^*_{wl})$ is the optimal solution of (11)-(16)
  else
    (Step 3)
    while $\bar{Y}_{ik[n]} \neq Y^*_{ik[n]}$ **or** $| \overline{v}_{i[n]} - v^*_{i[n]} | \geq \epsilon$ **do**
      $n \leftarrow n + 1$
      solve $PILR \rightarrow (\overline{TR}^*_{lt}, \underline{TR}^*_{lt}, q^*_{ikt}, Y^*_{ik[n]}, X^*_{wl}, v^*_{i[n]})$
      if PILR is infeasible **then**
        STOP. The problem is infeasible
      else
        for $i = 1 \rightarrow |\mathcal{I}|$ **do**
          solve $\text{GP}_i(\overline{TR}^*_{lt}, \underline{TR}^*_{lt}) \rightarrow (\bar{Y}_{ik[n]}, \overline{v}_{i[n]})$
        end for
      end if
    end while
    $(\overline{TR}^*_{lt}, \underline{TR}^*_{lt}, X^*_{wl})$ is the optimal solution of (11)-(16)
  end if
end if

The algorithm is composed of a master problem, defined by the PILR, and a sub problem, defined by the FP, exchanging information about feasibility (and therefore overall optimality) of the incumbent solution. At generic iteration n, PILR proposes an upper bound for the solution to the problem, together with the candidate values for incumbent choice for new power plant installments up to each GenCo and related optimal profit value $(Y^*_{ik[n]}, v^*_{i[n]})$. Such solution is feasible only if GenCos involved actually confirm the incumbent values when they solve their decision problem. Solution of problem GP_i for each GenCo will provide the vector $(\bar{Y}_{ik[n]}, \overline{v}_{i[n]})$, where the first element denotes the optimal value for the variables defining installments of new power plants for GenCo i when TransCo picks solution $(\overline{TR}^*_{lt}, \underline{TR}^*_{lt})$, while $\overline{v}_{i[n]}$ denotes the optimal profit for GenCo i when

TransCo picks solution $(\overline{TR}^*_{lt}, \underline{TR}^*_{lt})$. Vectors $(\bar{Y}_{ik[n]}, \overline{v}_{i[n]})$ and $(Y^*_{ik[n]}, v^*_{i[n]})$ are therefore compared and if either $\bar{Y}_{ik[n]}$ and $Y^*_{ik[n]}$ coincide, or $\overline{v}_{i[n]}$ and $v^*_{i[n]}$ coincide then the problem is feasible and incumbent solution is optimal. If feasibility requirements are not met, a new cut and a penalty term are introduced into the PILR to force the next iteration to avoid picking the previous candidate solution $Y^*_{ik[n]}$.

Step 1 computes the highest possible bound, Step 2 checks whether the solution obtained in Step 1 is feasible for the overall problem problem (11)-(16) and while the program is not feasible a new iteration will add a cut and a new penalty term into the PILR problem. This is done by defining a loop in Step 3. The algorithm terminates when the first feasible solution is found or concludes that no feasible solution exists.

The algorithm attains a global optimum for the original Stackelberg-Nash problem up to degenerate cases for the PILR problem. Namely, a degenerate solution delivers the same objective function value against many possible combinations of decision variables. Therefore one could obtain an output profit computed by the TransCo for each GenCo i, v^*_i, which is lower than the profit computed by each GenCo by solving problem (6)-(10) or such that the binary decision variables do not correspond to the choice that each GenCo would pick under optimality even when there actually exists another vector of solutions which corresponds to the optimal one for each GenCo and delivering the same optimal objective function for the TransCo. When this happens, the algorithm will not recognize it as a feasible solution and it will keep generating cuts reaching a sub-optimal solution.

7 Numerical analyses

In order to carry out economical and technical analyses we have tested the model using a small scale example. This has lead to an easier study of optimal responses of all the actors involved in the provision system to environmental changes, such as an increase of load in a given area or the introduction of a new generation technology. The second stage of the analysis considers the model applied to a representation of the Italian market. At this stage, we assume to have a fictional system composed of two zones, which we refer to as North and South, two generating companies, GenCo1 and GenCo2 each having, besides existing plants, four candidate plants uniformally distributed over the different zones. GenCo1 only has one candidate plant, located in the North area, while GenCo2 has three candidate plants, one located in the North, and two located in the South. Finally we consider one existing transmission line which should be eventually upgraded in case of a load increase. We also assume that four types of candidate lines are available, which are different by costs and capacity. Type1 line has the least transmission capacity and is less expensive, while the opposite holds for type4 line.

All computations have been carried out using GAMS/CPLEX 12.1.0 using four parallel threads on a Intel Core i7 machine running at 2.67 GHz with 8 GB of RAM. Computational time is heavily influenced by the data, ranging between little over one minute for cases where GenCos respond to transmission investments by opening new plants to several minutes, when no GenCo finds it profitable to open a new power plant.

A representation of the investments made by each player in the system is shown in figures 3 and 4. These figures provide a simplified graphical description of the national power delivery system. Here we depict each area by means of a rectangle containing information about load, power production and locational marginal price within the area. Right below the slots for such

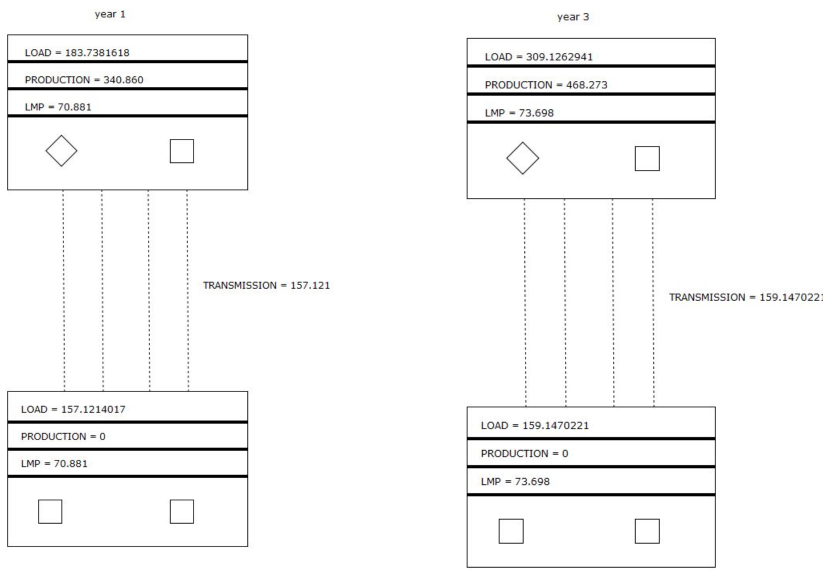

Figure 3: Investments with low load scenario, small and large load respectively

information there can be found a blank area containing two types of elements, namely a diamond shape and a square which represent the candidate plant of GenCo1 and GenCo2 respectively. The two aforementioned areas are connected by existing transmission lines, which are not represented in the figures, while we have denoted by dotted lines four candidate transmission links with different capacity levels. Alongside such connecting lines one can find information about the amount of electricity transferred between the zones. One has plus sign if transmission is from North to South, while the other way around holds when electricity is transferred from South to North. Depending on the environmental factors such as load and technology, which represent the data for our model, both TransCo and GenCo can decide to review and upgrade their investment decisions by opening new transmission lines and, possibly, new power plants. Should this happen, we will represent the opening of a new line by replacing the dotted line with a thicker continuous line. Similarly, we will represent the opening of a new power plant by filling the related element with red color.

In the remainder of the article we study the effects of a load increase on the amount and allocation of the investments. We are primarily interested in the behavior of the Transmission Operator, but responses of GenCos also need to be considered as they play a relevant role in the expansion decision process of the TransCo. For each zone, we report information on the load level, the production level, the locational marginal prices (LMP) and possible transmission grid and power plant expansions.

From figure 3 we can notice that a small load both in the North and in the South areas allows the entire demand to be covered by the GenCos offering power at the least price, which in our case happen to be in the North. Existing transmission is enough to ensure coverage for both the areas. Lack of congestion over the transmission lines between the two areas make the locational marginal prices to be the same in the two areas. This is equivalent to having only one area in

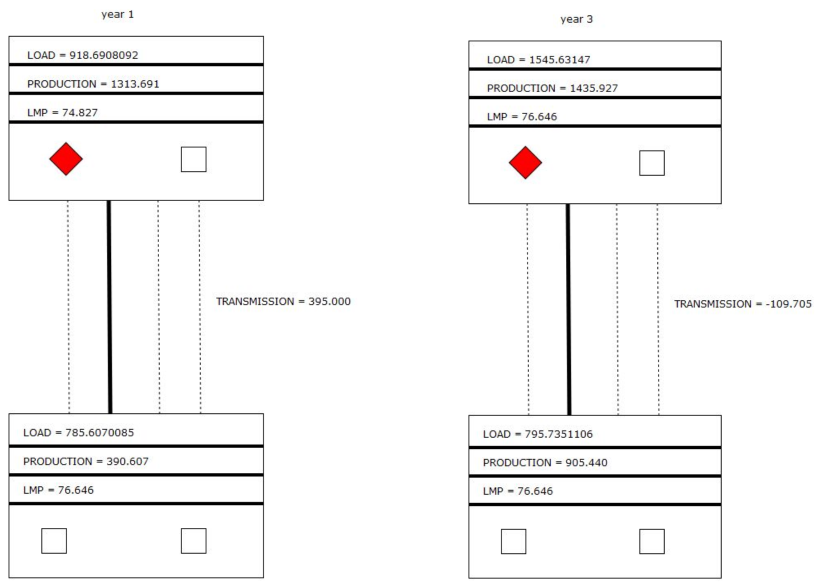

Figure 4: Investments with high load scenario, small and large load respectively

our system. Figure 4 shows a different situation. Here we apply our model to a system with a larger load level. In this case, the TransCo will prevent consumers in South area to pay too much for buying electricity by opening a new type2 line between North and South. This increases the potential market for GenCo1 which will respond by opening a new power plant in the North. This is beneficial to GenCo1 since such power plant has lower marginal costs than the ones operating in the south, which in turn will allow GenCo1 to sell a lager amount of power, by serving the South area. Yet, GenCo1 will not manage to cover the entire demand from South area, which will therefore be covered by plants operating in that area, which offer energy at a higher price. In the second diagram of figure 4 we are presented with the opposite situation. Here the North buys energy from the South. This happens since the load in the North increases to an extent that cannot be covered any longer solely by the power plants in the very same area. As a consequence, power plant in the South will increase the production and sell it to the consumers in the North. Locational marginal price will be the same in both markets.

Let us extend the ongoing experiment to analyze the results of a progressive expansion of load on the performance measures of the actors involved in the power generation and delivery. We have taken in consideration a reference load scenario and then increased the load by the same percentage in every year of the multiperiod model. Figure 5 shows the somewhat anticipated effect that a load growth has on social costs, defined as the total costs community have to bear to buy electricity. As the load grows, higher bids are progressively accepted by the Market Operator in order to cover the demand, with the results that both prices and demand will rise, leading on an increase in social costs.

Nevertheless the pattern of the Locational marginal prices can be quite interesting. As shown

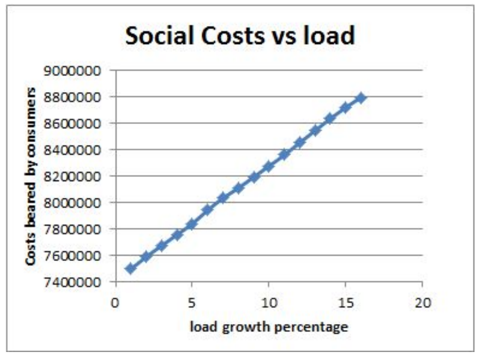

Figure 5: Social costs with respect of larger load levels

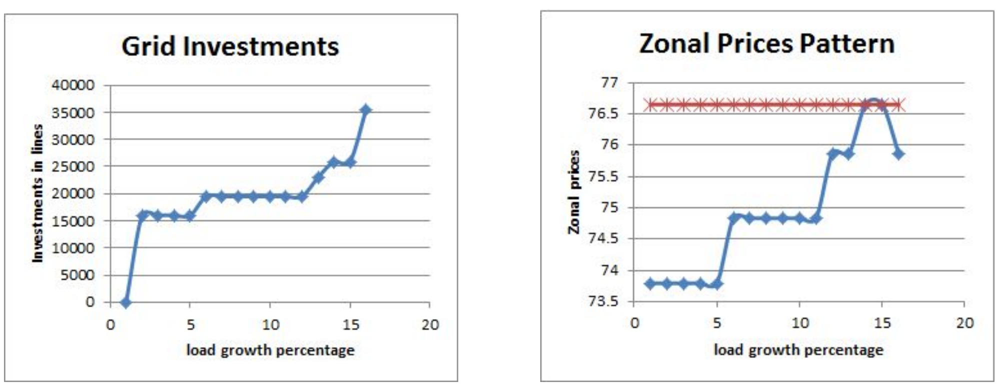

Figure 6: Grid Upgrading Investments and Locational Marginal Price vs load growth

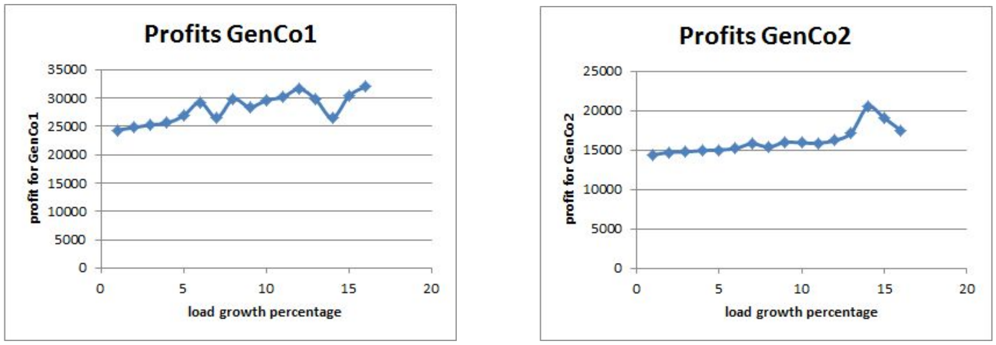

Figure 7: Profits of GenCo1 and GenCo2 respectively vs load growth

in right figure 6 such price increases as the aforementioned effect of new bids being accepted, but suddenly it starts decreasing. This happens because, as the load increases the TransCo will be required to expand the grid, eventually obtaining a network design in which there is enough line capacity to allow the GenCos to sell an indefinite amount of power cross areas. This has the effect of aligning the zonal prices of the two areas creating a unique market. Still, a further increase in load and related load will call for a different network setting which may prevent a GenCo with lower costs, which in our example is located in the North area, placing a full bid in the South area, as doing so would increase the zonal price in the North, leading to higher overall social costs.

The effect on GenCos on an increase in load is a steadily growing profit level as a consequence of the combined effect of higher bids being accepted in each area and the growing amount of power consumers. Nevertheless, equilibria outcomes obtained when lines are uncongestioned can be misleading when considering the profits pattern as the one shown in figures 7. In fact, with no congestion on the lines, zonal prices will be the same in each area, and for each production allocation between the GenCo involved the optimal social cost for the consumer will be the same. Therefore the system profit sharing pattern among GenCos will be non influential to grid upgrade purposes. This is the reason of the behavior of the profits in the rightmost part of each chart in figure 7. In other words there can be several equilibria leading to the same optimal objective function for the TransCo and different profit levels for the GenCos involved.

Let us consider now the italian case by defining a system composed of five areas, each with its own load and production capacity. We assume that existing lines cover part of the cross areas transmission request. The TransCo can add up to two additional lines between each market area to increase existing transmission capacity. We consider nine of the largest italian GenCos, with three of them deciding whether to expand their capacity by installing new power plants. The total number of candidate power plants is eight. Bounds on existing and candidate transmission lines are displayed on tables 1 and 2. We investigate on the state of the system as a consequence og changes in the amount of budget available to the TransCo. Figures 8, 11 describe the reallocation of investments in new lines in correspondance of two different budget constraints, respectively Budget 1 and Budget 2, consisting in an increase of available funds compared with the former case. One can clearly notice that the TransCo moves the investments from the line connecting Sardinia to Center to the more expensive line connecting South to Sicily. As a consequence Sicily will be able to purchase energy at a lower cost from GenCos in the South. At the same time, the Center area will obtain a lower amount of power from the South and will increase its demand for the energy from the North region. As energy flows from North to Center increase a GenCo will find beneficial to open a new power plant in the North.

Table 1: Bounds on Transmission Capacity (MW)

From \ To		North	Center	South	Sardinia	Sicily
North	Existing	-	2700	-	-	-
	Type1	-	100	-	-	-
	Type2	-	225	-	-	-
Center	Existing	1700	-	2300	300	-
	Type1	170	-	320	125	-
	Type2	485	-	400	319	-
South	Existing	-	2200	-	-	300
	Type1	-	200	-	-	220
	Type2	-	240	-	-	340
Sicily	Existing	-	-	600	-	-
	Type1	-	-	135	-	-
	Type2	-	-	220	-	-
Sardinia	Existing	-	300	-	-	-
	Type1	-	120	-	-	-
	Type2	-	190	-	-	-

Table 2: Investment Costs (M €)

From \ To		North	Center	South	Sardinia	Sicily
North	Type1	-	680	-	-	-
	Type2	-	1940	-	-	-
Center	Type1	680	-	1280	500	-
	Type2	1940	-	1600	1276	-
South	Type1	-	1280	-	-	880
	Type2	-	1600	-	-	1360
Sicily	Type1	-	-	880	-	-
	Type2	-	-	1360	-	-
Sardinia	Type1	-	500	-	-	-
	Type2	-	1276	-	-	-

8 Conclusion

We have developed a model to analyze and support the decisions on transmission grid upgrades within a market environment. In particular we have taken explicit account of possible reaction of GenCos and Market Operator to possible grid upgrade decisions both under an operational and a investment viewpoint. The problem has been modeled as a Mathematical Program with Equilibrium Constraints featuring mixed integer decision variables on both upper level and lower equilibrium problem. This mixed integer structure has been taken care by devising a dedicated algorithm which leads to the computation of a global optimum for the TransCo while featuring a Nash Equilibrium among the followers, i.e. GenCos and Market Operator. The model has been analyzed using a two zones-one line case considering the possible outcomes deriving by a load increase. Results show how the TransCo should upgrade the lines accordingly to different load levels together with some interesting behaviour shown by zonal prices in correspondance of different levels of line congestions.

There are several possible directions for future research. One of them is to include strategic bidding into the framework. This will seriously increase the computational burden as the modeling approach will entail a three level hierarchical formulation where GenCos and Market Operator are respectively in the intermediate and lower level. As a consequence, new solution approaches must be considered. Another possible developement could be to include the effects of risk attitude on decisions of each player both on upper and lower levels. Finally it would be beneficial to consider possible collaborative patterns between TransCo and GenCos, where a vector of side payments could complement a coordinated effort to provide electricity under the optimization of a system performance.

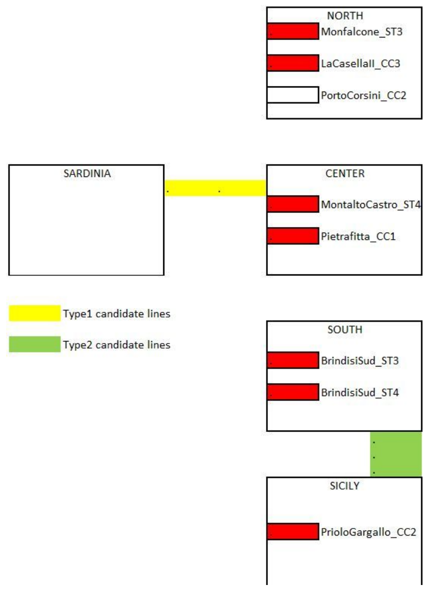

Figure 8: Investments Overview: Budget 1

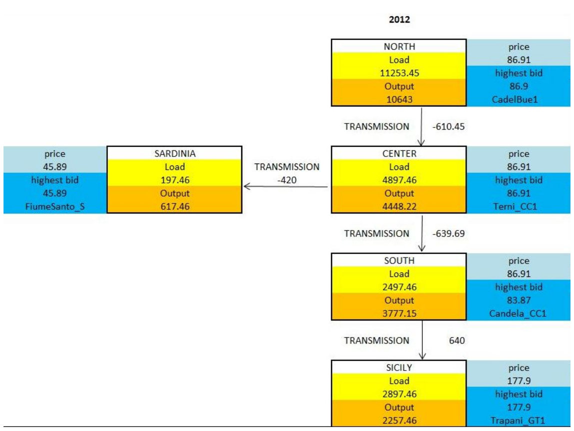

Figure 9: Generation and Transmission in year 2012

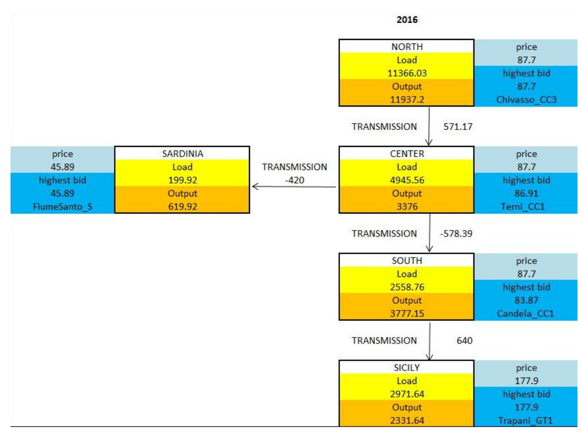

Figure 10: Generation and Transmission in year 2016

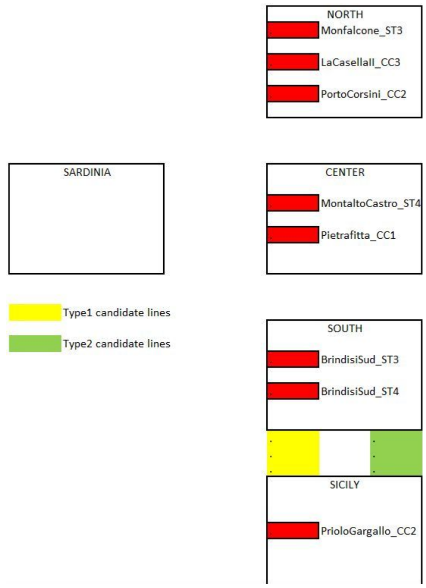

Figure 11: Investments Overview: Budget 2

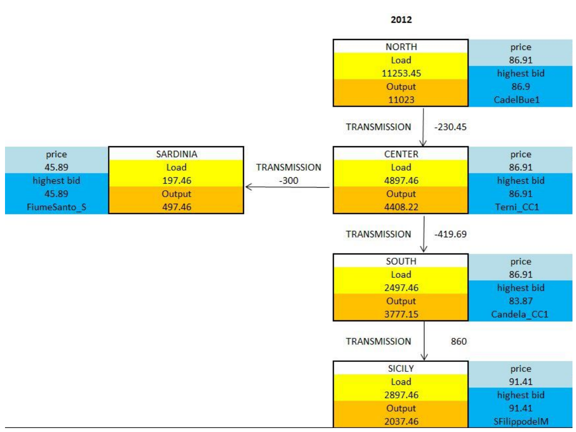

Figure 12: Generation and Transmission in year 2012

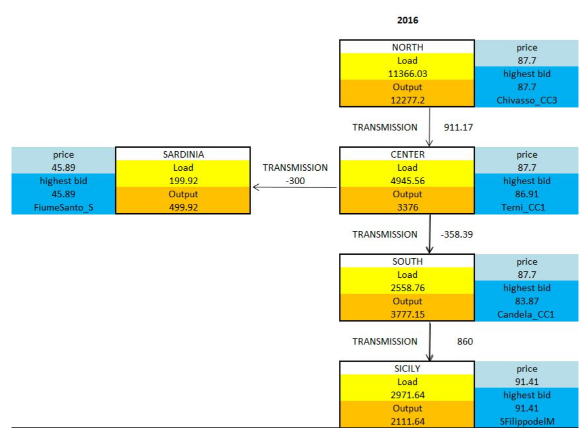

Figure 13: Generation and Transmission in year 2016

References

[1] J. A. Aguado, S. de la Torre, J. Contreras, A. J. Conejo, A. Martínez. Market-driven dynamic transmission expansion planning. *Electric Power Systems Research, Volume 82, Issue 1, Pages 88-94.* 2012

[2] R. W. Bacon. Privatization and Reform in the Global Electricity Supply Industry. *Annu. Rev. Energy Environ. 20:119-43,* 1995

[3] J. F. Bard. Practical Bilevel Optimization. *Nonconvex optimization and its applications (Vol. 30) Dordrecht: Kluwer Academic.* 1998

[4] Final Report of the Investigation Committee on the 28 September 2003 Blackout in Italy, *UCTE.* 2004.

[5] J. Fortuny-Amat and B. McCarl. A Representation and Economic Interpretation of a Two-Level Programming Problem. *The Journal of the Operational Research Society Vol. 32, No. 9, pp. 783-792.* 1981

[6] C. D. Galloway, L. L. Garver, L. K. Kirchmayer, and A. J. Wood. Generation-Transmission Expansion Planning. *Proc. Power Systems Computation Conf. pt. 5, Stockholm, Sweden.* 1966

[7] L. P. Garcés, A. J. Conejo, R. Garcia-Bertrand, R. Romero. A Bilevel Approach to Transmission Expansion Planning Within a Market Environment. *Power Systems, IEEE Transactions on , vol.24, no.3, pp.1513-1522.* 2009

[8] L. P. Garcés, R. Romero, J. M. Lopez-Lezama. Market-driven Security-constrained Transmission Network Expansion Planning. *Transmission and Distribution Conference and Exposition: Latin America (T&D-LA), 2010 IEEE/PES, pp.427-433.* 2010

[9] L. L. Garver. Transmission Network Estimation using Linear Programming. *IEEE Trans. Power Apparat. Syst. 89:1688-1697,* 1970

[10] C. Genesi, P. Marannino, I. Siviero, F. Zanellini, E. M. Carlini, P. P. Pericolo. Coordinated Transmission and Generation Planning to Increase the Electricity Market Efficiency. *XVI Power Systems Computation Conference (PSCC 2008), Glasgow, United Kingdom.* 2008

[11] M. R. Hesamzadeh, N. Hosseinzadeh, P. J. Wolfs. A leader-followers Model of Transmission Augmentation for Considering Strategic Behaviours of Generating Companies in Energy Markets. *International Journal of Electrical Power. Energy Systems, Volume 32, Issue 5, Pages 358-367.* 2010

[12] B. E. Hobbs. Linear Complementarity Models of Nash-Cournot Competition in Bilateral and POOLCO Power Markets. *Power Systems, IEEE Transactions on , vol.16, no.2, pp.194-202.* 2001

[13] Int. Energy Agency. Electricity Market Reform: An IEA Handbook. *Paris: OECD/IEA,* 1999

[14] S. K. K. Ng, J. Zhong, and C. W. Lee. A game-theoretic Study of the Strategic Interaction Between Generation and Transmission Expansion Planning. *Power Systems Conference and Exposition, IEEE, Ed. Seattle, USA: IEEE. p. 10.* 2009

[15] J. H. Roh, M. Shahidehpour, and Y. Fu. Market-based Coordination of Transmission and Generation Capacity Planning. *Power Systems, IEEE Transactions on, vol. 22, no. 4, pp. 1406-1419.* 2007.

[16] J. H. Roh, M. Shahidehpour, and L. Wu. Market-Based Generation and Transmission Planning With Uncertainties. *Power Systems, IEEE Transactions on , vol.24, no.3, pp.1587-1598* 2009

[17] J. Shan, S. M. Ryan. Capacity Expansion in the Integrated Supply Network for an Electricity Market. *Power Systems, IEEE Transactions on , vol.26, no.4, pp.2275-2284.* 2011

[18] TERNA, S.p.A. Development Plan of the National Electric Transmission Network. *www.terna.it.* 2007

[19] World Energy Counc. The Benefits and Deficiencies of Energy Sector Liberalisation, Vol. 1. *London: World Energy Counc.* 1998

[20] W. Patterson W. Transforming Electricity. *London: Earthscan.203,* 1999

[21] M. T. Vespucci, E. Allevi, A. Gnudi, M. Innorta. Cournot Equilibria in Oligopolistic Electricity Markets. *IMA J Management Math 21 (2): 183-193.* 2010

[22] M. T. Vespucci, M. Innorta, G. Cervigni. A Mixed Integer Linear Programming Model of a Zonal Electricity Market with a Dominant Producer. *Energy Econ, Available online 12 January 2012, ISSN 0140-9883, 10.1016/j.eneco.2011.11.021.* 2012