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Portfolio selection with options

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Abstract: - We describe an optimization model to evaluate the portfolio performance in the option's market. Hedgers, managers and investors, in agreement with Markovitz's theory, aimed at creating a portfolio made up by assets with negative correlation, so as to have a portfolio not linked to the economic cycle. The optimization portfolio problem with contingent claims allows to create wealth also in financial crisis without using short selling, since option returns show a strong negative correlation. The basic idea of this work is using only trading price options, in particular those written on principal stock Indexes, in order to create a diversified portfolio. Thus we propose an ex post analysis over a two-years period using different international portfolio strategies on the derivative market.

Key-Words: portfolio selection, call and put, performance strategy, liquidity constrains.

1 Introduction

The portfolio selection problem with contingent claims has been developed in several studies (see [1], [2], [3], [4]) that can be classified in three categories: a) portfolio selection among options, b) using classical option strategies, c) considering portfolio with options to hedge the global risk exposure. Blomvall and Lindberg [5], have shown that the efficient market hypothesis (see [6]) is not satisfied using call and put options in portfolio problems, in particular when liquidity constrains are not considered. They discuss a scenario generation approach in the Black-Scholes-Merton framework. However, it is well known that log returns are not Gaussian distributed (as in Black-Scholes-Merton model) (see [7]). Thus we could expect better results with other distributional assumptions. Moreover, Topaloglu, et al. 2011 [8] have shown how to select international hedged portfolios using option strategies in a

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stochastic optimization framework. The classical theory presents Black-Scholes-Merton as a pricing model and geometrics Brownian motion as an instrument to make choices simulating the future trend of the asset. These models present several critical points in their assumptions as also argued by the recent literature (see [9]).

In this paper we use trading prices of European options with long maturity in portfolio selection problems without assuming a fixed distributional assumption on the underlying.

To reduce the dimensionality of a large scale problem we propose a principal component analysis (PCA). Then we select the main factors that account of most of the variability of portfolio returns. Moreover we deal and discuss the problem of liquidity of several options traded in the market and we propose proper constrains in the portfolio optimization problem. Finally, we propose an ex-post analysis where we compare the ex-post wealth obtained maximizing weekly either the Rachev ratio (see [10]) or the classical Sharpe ratio (see [11]).

In section 2 we introduce the optimization problem and we describe the methodology used to select optimal portfolio of options. In section 3 we discuss the liquidity problem and we present the results of the ex-post analysis. In the last section we summarize the results.

2 The portfolio optimization problem

The optimization process have the objective to optimize the performance of portfolio of contingent claims. Therefore we want to find the optimal composition $x = [x_1, \dots, x_n]$ of percentages invested in each asset that solve the classical portfolio problem:

$$\begin{cases} \max_{x_1,\dots,x_n} PR \\ PR = f\left(z_{(k)}; p_k; x\right) \\ \sum_{i=1}^{n} x_i = 1;\ 0 \le x_i \le 0.2 \end{cases} \qquad (1)$$

where PR is a performance ratio applied to the gross returns[2] whose observation at time k are represented by the vector $z_{(k)} = [z_{1,k}, \dots, z_{n,k}]$ that is realized with probability p_k. In this paper we suppose that is not allowed short selling ($0 \le x_i$) and that is not possible to invest more than 20% of the global wealth in a specific option ($x_i \le 0.2$). As for the exponential weighted moving average model (EWMA, see [12]) we assign exponential probability to the historical observations:

[1] We define gross return $z_{i,k} = \frac{P_{i,k}}{P_{i,k-1}}$ where $P_{i,k}$ is the price of i-th asset at time k.

$$p_k = \lambda^k (1-\lambda) \qquad (2)$$

giving higher probability to the most recent observations and lower probability to the oldest (we assume $\lambda = 0.95$). Since the number of stocks exceeds the number of observations, to get a good approximation of the portfolio input statistical measures, it is necessary to find the right trade-off between the number of observations and a statistical approximation of the historical series. In particular, we use two techniques to reduce the dimensionality of large scale portfolio problems: pre-selection and Principal Component Analysis (PCA). With pre-selection, only a limited number of stocks is chosen before optimizing the portfolio. Since in the optimization problem (1) we use Sharpe ratio and Rachev ratio as measures of performance, for each optimization problem, we preselect the first *y* (20 or 200) options that present the highest Sharpe ratio or Rachev ratio. The Rachev ratio of a portfolio of gross returns x'z is defined as follows:

$$RR_{\alpha,\beta}(x'z) = \frac{CV@R_\beta(-x'z + r_b)}{CV@R_\alpha(x'z - r_b)} \qquad (3)$$

where r_b is a benchmark gross return that we assume equal to 1 when there is not (as in our empirical analysis),

$$CV@R_\beta(X) = \frac{1}{\beta}\int_0^\beta V@R_u(X)du \qquad (4)$$

is the conditional value at risk of random variable X and $V@R_u(X) = -F_X^{-1}(u) = -\inf\{x | P(X \leq x) \geq u\}$ is the value at risk of the random variable X. The conditional value at risk $CV@R_\beta(x'z - 1)$ is a coherent risk measure that is the opposite of the mean of the return portfolio losses below the percentile of its distribution. Rachev ratio allows to obtain options with fat tails in distribution function of return. Optimizing this performance the investors select titles with large returns and reduced losses. Using Rachev ratio, which selects assets with fat tails in a portfolio composed by options with high volatility the result is a higher degree of volatility of return.

The second performance, the Sharpe ratio, is characterized by the main elements of mean variance approach. Therefore it is defined as the ratio between expected value of gross return and their volatility which defines the profile of an investor who prefers titles with higher return in historical data and low standard deviation. The Sharpe ratio is measured as follows:

$$SR(x'z) = \frac{E(x'z-\mathrm{r_b})}{\sigma_{x'z}} \qquad (5)$$

2.1 Pre-selection and principal component analysis (PCA)

We compute two different strategies of pre-selection: in the first one we assume to select only 20 options and in the second one 200. Process is repeated every 5 trading day using a moving window of data.

Optimization process using 20 preselected option not present a difficult solution and maximization of Sharpe and Rachev ratio is obtained in less computation time. If we consider an institutional investor who can takes an advantage of large scale, pre-selection with 200 contingent claims respect a truthful image of problem. Therefore, we perform a PCA of the returns of the preselected 200 stock returns in order to identify what are the few factors $\mathrm{f_i}$ with the highest variability. For each optimization problem, we apply a PCA to the correlation matrix of the 200 preselected stocks to identify the first 12 components that explain the majority of the global variance. Subsequently, each series z_i $(i = 1, \dots ,200)$ can be represented as a linear combination of 12 factors plus a small uncorrelated noise. Using a factor model, we approximate the preselected returns z_i as follow:

$$\begin{gathered} z_{i,k} = \alpha_i + \sum_{j=1}^{12} \beta_{i,j} f_{j,k} + \epsilon_{i,k} \\ i = 1, \dots ,200 \end{gathered} \qquad (6)$$

Where $z_{i,k}$ and $\epsilon_{i,k}$ are, respectively, gross returns and errors in the approximation of i-th asset at time t_k and α_i; $\beta_{i,j}$ are the coefficients of the factor model. The randomness of the portfolio problem depends now on only 12 factors.

The implementation of the theoretical work explained before is developed with MatLab software and time series data were download from the Thompson Reuters Datastream. In this paper we summarize of the algorithm containing the filter presented before. To solve the optimization problems we use the fmincon function of the program.

The results of the optimization process cannot be considered as a global optimum as the fmincon function does not present characteristics of completeness and exhaustiveness in the research of every possible solutions. Moreover, in order to complete the optimization process with Rachev ratio, we introduce an heuristic to determine the global optimum in the cases of inefficient solution given by fmincon function (see [13]).

We optimize the portfolio every 5 trading days from 24 June 2010 . The pricing does not depends on the limits of Black-Scholes-Merton formula and future choices are not affected by simulation with the geometric Brownian motion. Moreover we compare the results obtained either recalibrating the portfolio daily with the same optimal portfolio composition obtained weekly or recalibrating the portfolio weekly (every 5 trading days). We consider a time series data of 125 observations of returns and a strategy that last for 374 days. Optimization process is repeated every 5 trading days for 74 times. During the analysis we used gross return and we start with an initial wealth of one (i.e., $W_{t_0} = \sum_{i=1}^{n} x_i = 1$). At the *k-th* optimization (k = 0, 1, 2,..., 74), the following three steps are performed to compute the ex-post final wealth:

1) Preselect the first x assets (where x is equal to 20 or 200) with the highest Rachev ratio or Sharpe ratio. When the number of preselected assets is 200, we apply the PCA component to the correlation matrix of the preselected stocks. Then we regress the returns on the first 12 principal components to approximate the variability of the preselected returns.
2) Determine the optimal portfolio x_k^* that has the proportions invested in each of the preselected stocks for the period $[t_k; t_{k+1}]$.
3) During the period $[t_k; t_{k+1}]$ (where $t_{k+1} = t_k + 5$) we have adopted two strategies: first one to recalibrate the portfolio daily by maintaining the proportions invested in each asset that are equal to those in the optimal portfolio x_k^* and the second to maintain the composition of portfolio until next optimization process, thus for 5 days. The ex-post final wealth is given by:

$$W_{t_{k+1}} = W_{t_k} \prod_{t=1}^{5} x_k^{*'} \, z_{t_{k+1}}^{(ex-post)} \qquad (7)$$

If we recalibrate the portfolio daily. Or:

$$W_{t_{k+1}} = W_{t_k} x_k^{*'} \prod_{t=1}^{5} z_{t_{k+1}}^{(ex-post)} \qquad (8)$$

If we maintain the same composition of portfolio for 5 days. Where:

$$z_{t_{k+1}}^{(ex-post)} = \left[z_{1,t_k+t}^{(ex-post)}, \dots, z_{n,t_k+t}^{(ex-post)} \right]' \qquad (9)$$

It is a vector of observed daily gross returns for the period $[t_k + t - 1; t_k + t]$. These returns are given as follow:

$$z_{i,t_k+t}^{(ex-post)} = \frac{p_{i,t_k+t}^{(ex-post)}}{p_{i,t_k+t-1}^{(ex-post)}} \qquad (10)$$

Where $p_{i,t_k+t}^{(ex-post)}$ is the price of i-th asset observed at time $t_{k+1} + t$.

The three steps are repeated for all the optimization problems for all available observations. To evaluate the impact of pre-selection and the portfolio strategies, we show and compare the ex-post wealth in every different cases.

3 An ex-post empirical analysis with contingent claims

The proposed analysis focuses on options written on international stock Indexes in a period between June 2010 and December 2011. Using software Thompson Reuters Datastream we create a dataset of 935 European call put options with the same maturity, December 2011, and made up by a time series data of 500 days between 31 December 2009 and 30 November 2011. Thus we consider an historical horizon for the first optimization process of 125 returns observations and we invest in dollar currency.

The 16 international stock Indexes are: Austrian Traded Index, Cac 40 Index, Dax Index, Dow Jones Industrial Average Index, Euro Stoxx 50 Index, Euro Stoxx Banks, Euro Stoxx Media, Ftse 100 Index, Ftse Mib Index, Hang Seng Index, Ibex 35 Index, Nikkei 225 Index, S&P 500 Index, Stoxx Europe 50 Index, Stoxx Europe 600 Banks, Swiss Market Index.

In this section we introduce liquidity constrains to create a model based on real transactions reducing possibility to invest in illiquid markets. Then we analyze and discuss the development of a strategies with 20 or 200 options preselected considering the algorithm of the previous section.

3.1 Liquidity constraints

Before optimization and pre-selection processes we introduce 4 liquidity constraints to realize a portfolio selection characterized by an high degree of liquidity for which we can guarantee the transactions. An empirical analysis on dataset of volume and price shows time series data with unreal price and returns without volume of transactions. This result can be explained not only as a mistake of dataset but also with the behavior of market makers which defines a large spread between bid-ask price not to create the condition to allow a transaction when the option is deep out of the money. In this work the price considered is the closing price that is the mid between bid and ask price. Thus liquidity of option selection model is not guarantee. To solve the problem we introduce 4 liquidity constraints through time series data of volume transactions:

1) *Minimum liquidity*. We remove options which do not show historical transactions and whose present unitary gross returns for several days. This constraint allows to avoid taking into account options with a time series data characterized by rare and higher increase of volatility.
2) *Volume*. We require that is possible to purchase and sell only the options that presents in the last trading days some volume transactions.
3) *Jumps*, we introduce this constraint to remove time series that in the past have presented big jumps with null transaction volume.
4) *Range of strike price,* which select only options with a strike price in a range of 25% up or down the underlying Index. It's a static constraint apply when we start the strategy to do not distort optimization process with options without liquidity because deep out of the money.

Liquidity constraints reduce the dimensionality of optimization problem and selection but guarantee possibility to negotiate the options presented in the portfolio of investor.

3.2 Results with 20 options preselected

In ex-post portfolio selection using contingent claims we evaluate and observe the optimal strategies consequence of 20 options preselected. The optimization process before presented could underline the inefficiency of reference market. We considered that a private investor cannot control a large number of titles and would like to invest on a few options. With this approach the problem of optimization becomes easier and quicker to be solved. The analysis of wealth with a daily recalibration of portfolio's composition shows many interesting results.

Figure 1 compares the ex-post wealth obtained maximizing the Sharpe ratio and Rachev ratio strategies with a daily or weekly recalibration of the optimal portfolio's composition. We observe that the final wealth of the different strategies adopted depends on the performance in the last period. This characteristic is a consequence of the options's volatility that increase in the last months when increase volume of transactions. Moreover, the preselected dataset is numerous.

In the first part of every strategies the wealth is stable and there are no substantial differences between the strategies. While in the central part we observe that the volatility increases in thanks to gross returns obtained with Rachev performance strategy where after an increment at the beginning follow a decrease with a loss of 50%. Moreover, in this first two parts the frequency of recalibration is not an important factor which may influence the strategies.

During the summer of 2011 the volatility of wealth increases since the maturity of options is closed and also the volatility on the market increases with credit risk crisis. More and more options is been included in the process of pre-selection and we get the best results using strategies which presented a daily recalibration in the composition of portfolio. Thus, this methodology presents more advantage in a portfolio selection. Optimization process is efficient and keeping the same composition every day, portfolio perform the imperfection of reference market. Moreover the strategy that maximizes Sharpe ratio presents an ex-post wealth that dominates the one obtained maximizing Rachev ratio.

3.3 Results with 200 options preselected

The second ex-post portfolio selection using contingent claims has the objective to evaluate an investment with an institutional investor's point of view. In this case we can obtain advantages of large scale. In particular we increase the number of preselected options with the consequences to diversifying portfolio composition, but the complexity of optimization problem become relevant. Introducing a PCA in the algorithm we simplify the statistical approximation. Figure 2 reports the ex-post wealth obtained optimizing Sharpe and Rachev performance strategies. We observe that also in this case the ex-post wealth obtained maximizing the Sharpe ratio dominates the one obtained with the Rachev performance. Increasing dimensionality of pre-selection both strategies show an improvement of wealth compared to the previous analysis. In particular Sharpe ratio already double the initial wealth during the spring of 2011, while the strategies with Rachev ratio remain stable in the same period of time but they do not shown losses. Optimization process is more efficient with the introduction of PCA but also the diversification reduces the losses of wealth.

In the last 5 months of ex-post analysis we observe that the volatility increases. That is this phenomena is independent from the number of preselected options and appears as a characteristic of reference market. During this phase the best results have been achieved through the maximization of Sharpe ratio with a daily recalibration of portfolio's composition strategy. Moreover, we observe that in condition of high volatility, using this strategy we reduce the losses and maximize addictive shifts. The wealth of investor in a portfolio selection results maximizes but the rolling of portfolio could become more expensive for the transaction costs.

The table 1 shows the final wealth for Sharpe ratio and Rachev ratio in the different pre-selections.

4 Conclusion

In conclusion, this ex-post analysis shows the imperfection of contingent claim markets between June 2010 and December 2011. The portfolio composed with European options written on principal stock Indexes performed high returns which supports this conclusion. In particular the analysis use trading price options with liquidity constrains in optimization portfolio selection. In this paper we showed that a strategy with a daily recalibration of the portfolio's composition gives the best results in terms of performance. Moreover, daily recalibration reduces the losses with market presents high volatility of the price and it maximizes the additive shifts. In particular, we observe that optimizing Sharpe type strategies we can get more than 2,5 times the initial wealth.

However the model presents some limits that we mention and which can explain partially the results achieved. First of all we do not consider the transaction costs which can be relevant in term returns when we recalibrate the composition of portfolio every day. Moreover, horizon selection is bounded: only 2 years in times of big crisis of financial markets and sovereign states. There are also options with a low level of transaction in a short period that belong to markets with high degree of speculation. Finally, the dataset used is make up of options with the same maturity (December 2011). We also observe that there are more transactions and higher volatility when we solve the problem closed to the option maturities.

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Tables and Figures

Table 1

Final Wealth	Recalibration	
	Daily	Weekly
Sharpe 20	2,7986	1,2625
Rachev 20	1,4067	0,3288
Sharpe 200	2,6552	0,6990
Rachev 200	1,5245	0,8097

Fig.1

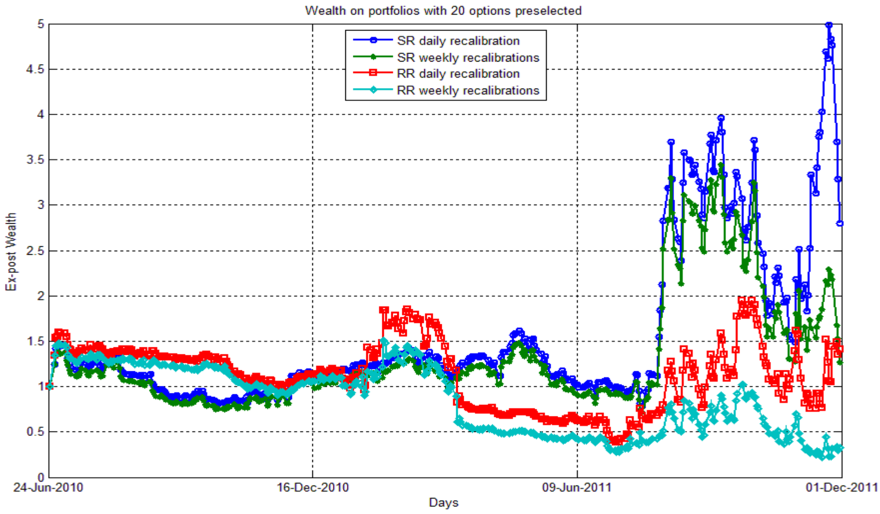

Fig.2