Evaluating Different Periodic Seasonal Time Series Model for Efficient Short-Term Wind Speed Prediction

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Abstract. *The importance of wind speed predictions, which is the source for wind power, is in the focus of interest. Accurate predictions are crucial for the energy production. Developing short-term wind forecasts helps to increase the productivity of wind energy. Moreover, the energy supply can be optimized, by increasing the accuracy of wind speed predictions, particularly the feed-in of wind power. The wind speed forecasting approaches presented here use 10-minute data collected at several stations in Germany. An overview of different periodic and seasonal time series models are given. The seasonality that is modelled by some periodic base function is combined with a long memory process and heteroscedasticity. Therefore, an ARFIMA(p,d,q)-APARCH(P,Q) process is comprised and applied to the correlated residuals. In contrast to the classical Fourier functions, cubic B-splines are used to model the periodicity. Furthermore, a common time series model is provided and applied to the wind speed. The feature is a time-saving approach for modelling and predicting. Hence, we introduce an iteratively reweighted least squares and lasso method. The most important findings are forecasting enhancements up to six hours and a simple and fast estimation and prediction method.*

Keywords. *Wind Speed; Forecasting; Periodic Autoregressive Model; GARCH Model; Iteratively Reweighted Least Squares*

1 Introduction

In recent years, renewable energy has a growing interest in Europe. Resources like Fossil fuels and conventional energy seem to be outdated. Furthermore, it is observable that national policy is setting incentives to increase the renewable energy production. The [10] remarks that one of the most emergent renewable energy sources is the wind power, whereat a growth rate of about 30% per year within the last ten years is achieved. Moreover, [10] finds in Germany an increase from 2011 to 2012 of installed capacity of about 1.1 GW. Considering competitive electricity markets, the importance of proper wind predictions have a huge importance. Furthermore, they are very helpful for planning purposes with the aim of smoothing the overall energy supply.

This article focuses on Germany and provides different and new methods to predict wind speed whichever is the essential part of wind energy. The production of wind power, as [2, 6] point out, strongly depends on the respective types of wind turbines. Moreover, it depends on the specific conditions at the respective locations of wind parks, the efficiency of the rotor and the capacity. Therefore, modelling wind speed instead of energy is a common approach to provide a more general predictor. Statistical approaches which use time series models are appropriate for short-term (up to six hours) forecasts (see [9]). [5] propose a long-memory autoregressive moving average model to model the wind. Thenceforth, different time series models are used for wind speed and wind power forecasting. [4] analyse 15-minute data and investigate a heteroscedastic behaviour within the conditional variance of the wind speed. [1] use an ARMA model with Fourier regressors within the mean and variance. An extension proposed by [8] is an ARFIMA-GARCH model with seasonal explanatory variables. They compare these models to calibrated and smoothed ensemble-based wind forecasts, but for hourly data.

Here, high frequency data is considered to forecast the wind speed up to six hours. Our aim is to take these ideas into account and provide an overview of different wind speed prediction models. The well established models are used as benchmarks. Their in-sample performance is evaluated by accuracy measures, diagnostic checking and the computing time that is required. Time-saving models have a huge relevance whenever short-term forecasts are conducted. Moreover, we cover several model extensions as for example periodic and seasonal explanatory variables within the mean and the variance. The periodicity is modelled either by Fourier series or by peridic B-splines. [7], for example, use a periodic seasonal ARFIMA-GARCH model with regressors to explain the daily electricity spot prices. A similar approach is used for the investigated wind data, but instead of an ARFIMA-GARCH approach we use an ARFIMA-APARCH model with periodic regressors (X). The asymmetric power GARCH is a useful extension for covering the asymmetry within the variance and the heteroscedasticity. A big drawback of this model is related to the estimation which is very slow. The entire periodic ARFIMAX-APARCH process is determined by quasi maximum likelihood (QML). Though, what if it is possible to derive a simple and less computationally intensive approach for wind speed modelling? Therefore, we present an alternative and not so challenging model. The improvement of this model, beside the computing time, is the iteratively reweighted least squares and lasso estimation. The resulting model can be summarised as periodic seasonal ARX-TARCHX model. The estimation of this approach is done pretty fast. This model needs many predictor variables to cover the structure of the underlying process and thus many observations are needed. The observed dataset contains per station nearly $210,000$ observations. Finally, all models are evaluated regarding their forecasting performance.

2 Wind Speed Data

The wind speed data set which is analysed in this study is collected at Manschnow ($52°33'N$ $14°32'E$), Lindenberg ($52°13'N$ $14°07'E$), Angermünde ($53°02'N$ $14°00'E$) and Grünow ($53°19'N$ $13°56'E$). These stations are situated in Eastern Germany in a rural plain region which is preferable for wind parks. The data is collected $18m$, $98m$, $54m$ and $56m$ above mean sea level and provided by the “Deutscher Wetterdienst” (DWD) with an anemometer. The data reaches from January 2008 to December 2011. For model fitting, a time frame of two years is used and the remaining months (January 2011 to December 2011) are used for out-of-sample forecasts. The wind speed is measured in m/s in a 10-minute interval. Clearly, the wind speed data has several periodic components. Therefore, we analyse the spectral density to identify significant frequencies and map them to the time domain. Finally, diurnal and annual seasons are of major relevance.

3 Modelling approaches for the Wind Speed

The following regression model with periodic autoregressive fractionally integrated moving average and external regressors (ARFIMAX) and asymmetric power ARCH (A-PARCH) disturbances summarises several wind speed processes. This model is an effective approach for capturing the salient effects of the wind, that is

$$\phi(B)(1-B)^d(W_t-\mu_t) = \theta(B)\varepsilon_t, \qquad \varepsilon_t|F_{t-1} \overset{iid}{\sim} F, \qquad t=1,\dots,T, \tag{1}$$

$$\mu_t = \underbrace{\vartheta_{11}}_{\text{Intercept}} + \underbrace{\vartheta_{trend}\cdot t}_{\text{Trend}} + \underbrace{\sum_{i_1=1}^{k_1}\sum_{i_2=1}^{k_2} I_{i_1 i_2}\vartheta_{i_1 i_2} f^{s_1}_{i_1,i_2}(t) f^{s_2}_{i_1,i_2}(t)}_{\text{Seasonal}}. \tag{2}$$

The periodic explanatory variables f_i^s are either modelled by Fourier functions or B-splines. One example is given by

$$f^{s_1}_{i_1 i_2}(t) = \begin{cases} 1 & , i_1 = 1 \\ \cos\left(\frac{\pi i_1 t}{s_1}\right) & , i_1 = \text{even} \\ \sin\left(\frac{\pi (i_1-1) t}{s_1}\right) & , i_1 = \text{odd}, \backslash\{1\} \end{cases} \tag{3}$$

and

$$f^{s_2}_{i_1 i_2}(t) = \begin{cases} 1 & , i_2 = 1 \\ \cos\left(\frac{\pi i_2 t}{s_2}\right) & , i_2 = \text{even} \\ \sin\left(\frac{\pi (i_2-1) t}{s_2}\right) & , i_2 = \text{odd}, \backslash\{1\}, \end{cases} \tag{4}$$

where $d \in (-0.5, 0.5)$ is the differencing parameter, the polynomials ϕ and θ are given by $\phi(B) = 1 - \phi_1 B - \ldots - \phi_p B^p$ and $\theta(B) = 1 + \theta_1 B + \ldots + \theta_q B^q$. Assuming a constant trend component leads to an infinite increase or decrease in wind speed in the limit. Here, we observe only four years which is a short period of time. If we consider such short periods of time, there are linear increases or decreases within the data. Therefore, including a trend component into the model should be a reasonable assumption. Here, we analyse different indicator matrices $I_{i_1 i_2} \in \{0,1\}$ to derive different combinations of periodic basis functions. For example consider the following matrix with $k_1 = k_2 = 5$

$$\boldsymbol{I} = \begin{pmatrix} 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{pmatrix} = (I_{i_1 i_2}). \tag{5}$$

In addition, the A-PARCH model is provided

$$\varepsilon_t = \sigma_t \eta_t \qquad \sigma_t^\delta = \alpha_0 + \sum_{l=1}^{q} \alpha_l (|\varepsilon_{t-l}| - \gamma_l \varepsilon_{t-l})^\delta + \sum_{l=1}^{p} \beta_l \sigma_{t-l}^\delta, \tag{6}$$

with $\mu_t = E(W_t|F_{t-1})$ and $\sigma_t^2 = Var(W_t|F_{t-1})$. Moreover, δ is the power of the A-PARCH process, γ is the asymmetry parameter and $\{\eta_t\}$ are the residuals following some distribution F. It has to hold that $E(Z_t) = 0$, $E(Z_t^2) = 1$ and $E(Z_t^4) < \infty$.

In contrast to the established periodic regression model with ARFIMA–A-PARCH process we use a multiplicative periodic seasonal autoregressive model with external regressors and threshold autoregressive conditional heteroscedasticity (multiplicative periodic SARX-TARCHX model). Clearly, the

ARFIMAX–A-PARCH model has only a few parameter, but the seasonal part might increases the dimension of the model. Moreover, the QML estimation of the model involves some other problems. For instance, the correct model identification and the calculation time. Besides, the distributional assumption which contains some limitations. The multiplicative periodic and seasonal ARX model for the wind speed is given by

$$W_t = \vartheta_t + \sum_{j=1}^{m} \beta_j x_{tj} \sum_{i=1}^{p} \phi_i(t) W_{t-i} + \varepsilon_t, \tag{7}$$

Here ε_t follows a TARCHX process. Additionally, it is assumed that the autoregressive coefficients ϕ_t and the trend ϑ_t are seasonal and time varying.

Finally, several wind speed models which are described by (1) are compared with (7). Moreover, we introduce multivariate modelling approaches comparable to [6] which are based on periodic vector autoregressive models. The latter modelling approaches are calculated by iteratively reweighted least-squares or lasso. Therefore, we do not need a distributional assumption. The in-sample and the out-of-sample performance are evaluated. Moreover, we give some recommendation for preferable wind speed modelling and forecasting approaches.

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