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The Two-Echelon Location Routing Problems: State of the Art, Models and Algorithms

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Non esistono condizioni ideali in cui scrivere, studiare, lavorare o riflettere, ma è solo la volontà, la passione e la testardaggine a spingere un uomo a perseguire il proprio progetto.

Konrad Lorenz.

Ai miei genitori che mi hanno sempre sostenuto e continuano a farlo nei momenti più difficili.

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Abstract

The Two-Echelon Routing Problem constitutes a class of combinatorial optimization problems with applications in many different fields such as city logistic planning, intermodal transportation, postal and parcel delivery distribution systems and so on. This thesis contains:

- a survey of the main contributions related to the two-echelon routing, focusing on the Two-Echelon Location Routing Problem, on the Two-Echelon Vehicle Problem and on the Truck and Trailer Routing Problem;
- an analysis of a particular version of the Two-Echelon Location Routing Problem with single source, referred as Single Vehicle Two-Echelon Location Routing Problem, with two mathematical formulations and a heuristic algorithm;
- some computational experiments aiming at test the effectiveness of the formulations and the heuristic algorithm.

The thesis is divided in three parts:

- a literature review of the Two-Echelon Location Routing Problem, of the Two-Echelon Vehicle Routing Problem and of the Truck and Trailer Routing Problem. The Two-Echelon Location Routing Problem refers to a class of problem directly derived from the well-known Location Routing Problem in which two sets of facilities must be located and two levels of routing have to be considered. The Two-Echelon Vehicle Routing Problem is derived from the Vehicle Routing Problem. In this class of problems the distribution of freight is managed through a set of satellites, where operation of consolidation, and/or transhipment and/or storage are performed. The Truck and Trailer Routing Problem models a distribution system in which a vehicle fleet, composed by truck units and trailer units, is used to serve the demand of a set of customers, some of which are accessible only by a truck without trailer;

- the description of the Single Vehicle Two-Echelon Location Routing Problem with emphasis on its structure and its components, plus the introduction of two integer programming formulations to determine the optimal location and the optimal number of a set of capacitated facilities, the assignment of customers to these facilities and the related routes. Four sets of instances are adapted from the literature and the results of proposed formulation are reported, comparing their performances in terms of solution quality and computation times.

- the introduction of a heuristic algorithm to solve the problem. Five variants of the algorithm are developed, differing from each other for the assignment procedures of the customers to the satellites. The results provided by such algorithm are compared with the optimal solutions, or in some cases with the best lower bounds provided by CPLEX solving the two integer models.

Keywords: Operations Research, Combinatorial Optimization, Heuristic, Location, Two-Echelon, Transportation.

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Introduction

According to the definition provided by the Supply Chain Management Professionals [58], logistic management is that part of supply chain management that plans, implements, and controls the efficient, effective forward and reverse flow and storage of goods, services and related information between the point of origin and the point of consumption in order to meet customers' requirements. This definition includes inbound, outbound, internal and external movements, and return of materials for environmental purposes. In the last decades the importance of the logistic management has been growing in several economic and social areas. For instance, in the industrial sector, logistics helps the enterprises to optimise their production and distribution process, leading them to become more efficient and competitive.

The element usually characterizing a logistic chain is the transportation system, which joins the separate activities, providing better logistic efficiency, reducing operation cost and promoting service quality. About one third to two thirds of the expenses of enterprises' logistics costs are spent on transportation, see [35].

Not surprisingly, operations research has been applied for developing optimization methods for large-scale transportation and logistic problems. The approaches often require the development of new models and algorithms, and their implementation in real operating environments. Usually in the transportation problems a set of vehicles has to satisfy the demand for goods (or services) by a set of customers. Focusing on the way the freight reaches the final destination, two main shipping strategies can be found in outbound logistics:

1. direct shipping, which consists of delivering freight directly from the manufacturing plants (or origins) to the final destinations;

2. multi-echelon distribution, in which the freight, before reaching the final destinations, has to pass through other facilities called satellites or storage areas, where consolidation, transhipment and storage operations are usually performed.

As concerns the multi-echelon distribution, in recent years companies have been changing and improving their distribution strategies, to cope with the changing demand, preferring the activity of consolidation to the activity of storage. As González Feliu reports in [27] the following are several real life applications of the multi-echelon transportation system:

- postal and parcel delivery distribution system, in which the freight is transhipped or consolidated at some intermediate platforms;

- press distribution, in which national and regional platforms are used to distribute the products to the retailers through a system of consolidation platforms, where these products are repackaged;

- grocery distribution, home delivery service and e-commerce, activity that are developing intermediary reception points;

- logistics system for urban freight transportation (i.e. city logistic) in which some urban consolidation centers, located in the periphery, are used to receive and tranship the freight in eco-friendly vehicles directed to the city;

- multimodal transportation, in which the freight is delivered by two or more means of transport, with no alteration during the operations of cross-docking.

This thesis focuses on a particular case of multi-echelon transportation systems, the so called two-echelon transportation system. In such system only two levels of facilities are involved: the origins (or depots), where the freight is initially located, and the intermediate facilities (or satellites) where the freight is transhipped and/or consolidated before reaching the final destinations (customers). The links between the origins and the intermediate facilities constitute the so called first echelon, whereas the links between the intermediate facilities and the final destinations constitute the so called second echelon. Two fleets of homogeneous vehicles are available, one for each level.

We survey the main contributions in the OR literature related to such transportation system, in order to provide an organized framework that can be useful for researchers and transport practitioners. The first chapter has been divided into three separate sections in order to provide a simple and clear account of the two-echelon transportation system. The first section concerns the Two-Echelon Location Routing Problem, the second the Two-Echelon Vehicle Problem and the third the Truck and Trailer Routing Problem. In this research work we are proposing two mathematical formulation and a heuristic algorithm to address the Single Vehicle Two-Echelon Location Routing Problem, a particular case of the Two-Echelon Location Routing Problem with a single source and a single vehicle available at each facility (both depot and satellites). In this problem some freights, available at a central depot, have to be delivered to a set of customers through a set of intermediate satellites which must be located. A large uncapacitated vehicle starts from the central depot, performs a primary tour among the selected satellites and then returns to the origin. Then a set of smaller capacitated vehicles (one for each satellite) performs some secondary tours delivering the freights to the customers. The goal of the problem is to minimize the total costs, i.e., the routing costs and the fixed opening costs associated with the satellites, respecting the capacity constraints and satisfying the customer demands.

The two mathematical formulation has been implemented using ILOG Concert Technology 2.3 and CPLEX 12.1. Their performance is evaluated solving four sets of instances adapted from those used for Two-Echelon Location Routing Problem with Single Source [44].

The proposed heuristic algorithm is characterized by a constructive phase and an improvement phase. In the improvement phase the problem is divided into three subproblems:

- the assignment of the customers to the satellites;
- the construction of the primary tour;
- the construction of the secondary tours.

In the improvement phase, five different local search procedures are applied.

Numerical results, reported in Chapter 3, show the effectiveness of the algorithm which provides good quality solutions in very short time.

This thesis comprises four chapter: the first is devoted to the survey on the most important problems related to the two-echelon transportation system; the second chapter presents a description of the Single Vehicle Two-Echelon Location Routing Problem and the two mathematical formulations used to solve the problem using CPLEX; the third part describes the heuristic algorithm and five different variants. The last chapter presents global conclusion and future research perspectives.

Chapter 1

Literature review

Distribution refers to the transportation of freight from the origin stage (e.g., the supplier or the production plant) to the destination stage (e.g., the wholesaler, the retailer or the final customer) in a supply chain. Freight transportation occurs between each pair of stages in the supply chain. Each pair of stages represents one level of the distribution network and is usually referred to as an *echelon*. Freight transportation is a key driver for strategic, tactical and operational decisions of many companies since it affects considerably both the product costs and the customer experience directly. Chopra and Meindl [11] highlight that distribution-related costs make up about 10.5% of the US economy and about 20% of the cost of manufacturing. It is therefore not surprising that such topic has attracted considerable efforts from the operational research community aimed at developing innovative and effective optimization models and solution algorithms capable to provide the necessary quantitative tools to support the decision makers.

Freight transportation from its origin to its destination can be broadly categorized into two classes according to the presence or not of one or more intermediate stages. Direct shipping happens when the freight is delivered directly from its origin to its destination, i.e., the network comprises only one echelon. Conversely, *indirect shipping* happens when the freight is moved through some intermediate stages (e.g., cross-docks or distribution centers) before reaching its destination, i.e., the network comprises more than one echelon. In a multi-echelon distribution system the freight is delivered, usually compulsorily, by means of indirect shipments. Two-echelon distribution systems are a special case of multi-echelon systems where the network is composed of only two echelons. That is, after leaving its origin, the freight is first delivered to an intermediate facility where storage, merging and/or consolidation operations are performed. The freight is then moved from the intermediate facility towards its destination. We define as *two-echelon routing problems* a wide class of problems that study how to optimally route freights in two-echelon distribution systems taking into consideration strategic and/or tactical planning issues.

Area covered. Two-echelon routing problems can be classified according to the type of decisions that are involved. Particularly, we consider:

- *strategic planning decisions*: they are long-term decisions that a company takes once every some years to change its strategic organization. They include decisions concerning the infrastructure of the network (e.g., the number and the location of the facilities to open) and the type and quantity of equipment to install in each facility;

- *tactical planning decisions*: they are decisions with a mid-term horizon and include, among others, the routing of freight through the network, the allocation of customers to the intermediate facilities, and the type of service to be used.

In recent years, a considerable number of papers focusing on two-echelon routing problems have been published in the literature. Some of them tackle variants of the same basic problem, while others propose different solution methods for the same problem. This chapter aims at providing a classification and a systematic overview of the foremost contributions on two-echelon routing problems. We survey the foremost contributions in the operational research literature dealing with two-echelon routing problems where strategic and/or tactical planning decisions are taken into consideration. Operational planning issues (e.g., product planning, implementation and adjustment of schedules for services) are not covered.

Terminology and classification used. We consider the following three classes of two-echelon routing problems.

We refer to the Two-Echelon Location Routing Problem (*2E-LRP*, hereafter) when the problem definition involves both strategic (typically the location of facilities) and tactical (typically the routing of freight and the allocation of customers to the intermediate facilities) planning decisions. Specifically, in the 2E-LRP goods available at different origins (called *depots* or, sometimes, platforms) have to be delivered to the respective *destinations* moving mandatorily through a set of intermediate facilities called *satellites*. An opening cost is associated with each depot and each satellite. The depots, as well as the satellites, to be open have to be selected from a set of possible depot (satellites) locations.

We refer to the Two-Echelon Vehicle Routing Problem (*2E-VRP*, from now on) when the problem definition involves only tactical planning decisions.

Problem Class	Location Decisions	Opening Costs for Depots, Satellites and Trailer Points	1^{st} Echelon Fleet	2^{nd} Echelon Fleet
2E-LRP	$\surd$	$\surd$	Homogeneous Vehicles	Homogeneous Vehicles
2E-VRP			Homogeneous Vehicles	Homogeneous Vehicles
TTRP			ν_K Trucks and $\nu_L \leq \nu_K$ Trailers	ν_K Trucks

Table 1.1: A summary of the main characteristics of the three classes of problems surveyed.

Particularly, in the 2E-VRP only one depot is usually considered, the set of satellites to use is given, and no cost is associated with the use of a satellite.

Finally, we consider the Truck and Trailer Routing Problem (*TTRP*, henceforth). In the TTRP freight transportation is managed by means of a set of trucks and trailers. A subset of customers can be served by a complete vehicle (i.e., a truck pulling a trailer) or by a truck alone, whereas the remaining customers can only be visited by the truck alone.

The main characteristics of the three families of problems surveyed are summarized in Table 1.

Applications. Due to the several real-life problems that can be modeled as two-echelon distribution systems, an ever increasing number of examples of design and implementation of this type of distribution system appear in the literature. We mention, among other applications, city logistic, multimodal transportation, postal and parcel delivery, press and grocery distribution.

City logistic is probably the most frequently cited application. Crainic *et al.* [19] claim that "city logistic aims to reduce the nuisances associated to freight transportation in urban areas while supporting their economic and social development". Indeed, freight transportation in urban areas is one of the main reasons of congestion, disorder, pollution emissions and noises. Implementing a two-echelon distribution system could be an effective response to these problems. In more details, in such systems each satellite corresponds to a facility located, usually, in the outskirts of the city where large trucks are allowed to arrive and where freights headed to different destinations are unloaded, sorted and consolidated. Freights are then loaded onto smaller and environment-friendly (also called eco-friendly) vehicles that are allowed to travel in the city center and so they can serve the final customers. Several papers cited in this chapter are related to this particular problem of urban management.

Although multimodal transport is not as cited as city logistic, it represents a relevant application of freight distribution systems involving two or more echelons. As a matter of fact, in recent years, the number of intermodal logistic centers in central and south-west European countries increased significantly (e.g., see [32]). As an example we mention the ship-road multimodal distribution system (e.g., see [31]) where the freight travels from the supplier to a satellite by ship (i.e., the first echelon) and then it is loaded onto a truck that delivers the freight to its final destination (i.e., the second echelon).

Surveys for related problems and structure of the chapter. Among the related problems we mention the Location Routing Problem (LRP) and the Vehicle Routing Problem (VRP). For an overview on the LRP we refer the interested reader to the paper by Nagy and Salhi [42], whereas the survey by Laporte [36] provides a summary of the most important studies on the VRP. Two-echelon freight transport optimization problems are analyzed in González Feliu [27] that aims at identifying its main concepts and issues. Papers dealing with the presence of intermediate facilities in distribution networks are surveyed in [28]. In the latter survey the focus is mostly put on the role of the intermediate facilities in service network design problems within tactical planning decisions, i.e., strategic planning decisions are not taken into consideration and the focus is put more on the service network design aspects than on the routing one. The reader should be aware that the scope of the latter survey is partially overlapping with that of this chapter, in particular in what concerns the 2E-VRP that is covered in both research works. Multi-echelon issues that either are not explicitly related to the routing of freight or consider the routing of freight but assuming the presence of more than two echelons, and therefore not included in this survey, are considered, among others, in Pirkul and Jayaraman [51], Tragantalerngsak *et al.* [59], Marín and Pelegrín [41], Crainic *et al.* [20], Ambrosino and Scutellá [1], and Hamidi *et al.* [29].

The structure of the chapter is as follows. Section 1.1 is devoted to the description of the 2E-LRP and to review the related literature. The 2E-VRP is considered in Section 1.2, while the TTRP is surveyed in Section 1.3.

1.1 The Two-Echelon Location Routing Problems

In this section, we first provide a brief introductory description of the 2E-LRP and then review the most important papers tackling the problem.

1.1.1 Problem Description

Let us consider a two-echelon distribution network composed of three disjoint sets of vertices corresponding to the depots (i.e., the origins), the satellites (i.e., the intermediate facilities), and the customers (i.e., the destinations), respectively. Hence, the distribution network can be decomposed into two echelons.

The first echelon comprises the links between the depots and the satellites, and those connecting pairs of satellites. The second echelon connects the satellites to the customers, and includes also the links between pairs of customers. Some freights that are available at one or more depots have to be delivered to some customers passing through the satellites, compulsorily. Freight transportation is performed by two different fleets of vehicles, one at each echelon. Vehicles are usually assumed to be capacitated and homogeneous within the same echelon. Vehicles belonging to the first echelon are referred to as *primary vehicles*, whereas those in the second echelon are called *secondary vehicles*. A fixed opening cost is associated with each depot and each satellite. Additionally, a fixed cost of usage is usually associated with each vehicle. A capacity limit for each depot as well as each satellite, representing the maximum amount of freight that can be handled in the facility, is also considered. An usual assumption is that each open satellite has to be visited by exactly one primary vehicle that routes from an open depot. Additionally, each customer has to be generally served by exactly one secondary vehicle that routes from an open satellite.

The 2E-LRP aims at finding the optimal set of location sites for the depots and the satellites as well as the optimal set of vehicle routes that satisfy the customer demands and do not violate the capacity requirements, while minimizing the total cost of the system. The total cost is given by the sum of the fixed opening costs, related to the facilities open, the usage cost of the vehicles routed, if present, and the routing costs.

The main characteristic that differentiates the 2E-LRP from the 2E-VRP, analyzed in Section 1.2, is that in the 2E-LRP location decisions are involved, i.e., not all the facilities of first and second level (i.e., depots and satellites, respectively) have to be necessarily open.

More formally, we assume the following definition of the 2E-LRP. We consider a weighted undirected graph $G = (N, E)$, where $N = \{V_d \bigcup V_s \bigcup V_c\}$ is the set of vertices and E is the set of edges (i, j), $i \neq j$, connecting vertex $i \in N$ and vertex $j \in N$. Particularly, $V_d = \{1, \dots, d\}$ represents the set of d potential locations where a depot can be open, $V_s = \{d + 1, \dots, d + m\}$ is the set of m possible satellites locations and $V_c = \{d + m + 1, \dots, d + m + n\}$ is the customer set. A nonnegative traveling cost c_{ij} is associated with each edge $(i, j) \in E$. An opening cost o_i and a capacity w_i, $i \in V_d \bigcup V_s$, are given for each depot and each satellite. Each customer $j \in V_c$ has a known and deterministic demand r_j to be served by exactly one vehicle. A fleet of homogeneous primary vehicles with a capacity q_1 is shared by the depots, while a set of homogeneous secondary vehicles with a capacity q_2 is shared by the satellites. A fixed cost f_1 is paid for each primary vehicle routed, whereas for

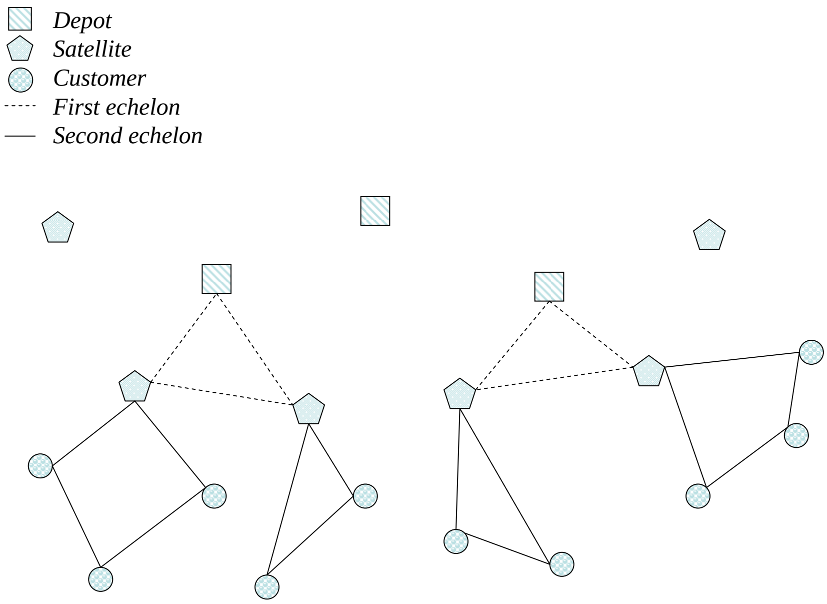

Figure 1.1: An example of a 2E-LRP feasible solution.

the use of each secondary vehicle a fixed cost f_2 is paid.

The 2E-LRP is $\mathcal{NP}$-hard because it is a generalization of other well-known $\mathcal{NP}$-hard problems (see [44]): namely, the two-echelon facility location problem, the 2E-VRP (surveyed in Section 1.2), and the aforementioned LRP.

Figure 1.1 shows an example of a 2E-LRP feasible solution. The squares represent the depots, the pentagons are the satellites, and the circles are the customers. The routes belonging to the first echelon are represented as dashed lines, whereas the routes belonging to the second echelon are depicted as solid lines.

1.1.2 Literature Review

As the 2E-LRP has been introduced relatively recently, the related literature is rather limited. A general overview of the main characteristics of the papers included in this section is reported in Table 1.2, including some details on the solution method proposed (heuristic and/or exact), on the optimization model introduced, and some specific characteristics, if any, of the problem studied.

The 2E-LRP, as defined above, has been formally introduced in Boccia *et al.* [8] where the authors study the problem of designing a two-echelon freight distribution system in which the location of two types of facilities (depots and satellites), the vehicle fleet size and the routes belonging to the two echelons

Reference	**Solution Algorithm**	**Optimization Model**	**Specific Characteristics**
Boccia *et al.* [8]	Heuristic		
Crainic *et al.* [22]	Solver	MILP	
Schwengerer *et al.* [57]	Heuristic		
Contardo *et al.* [13]	Exact and Heuristic	MILP	
Nikbakhsh and Zegordi [46]	Heuristic	MILP	Soft Time Windows, No Routes on 1^{st} Ech.
Dalfard *et al.* [23]	Heuristic	NLMIP	Veh. Fleet Capacity, Max Route Length, No Routes on 1^{st} Ech.
Single Depot			
Jacobsen and Madsen [33]	Heuristic		Delivery Time, Tour Length
Nguyen *et al.* [45]	Heuristic	ILP	
Nguyen *et al.* [44]	Heuristic	MILP	

Table 1.2: A summary of the papers on the 2E-LRP.

have to be simultaneously optimized. The authors implement a Tabu Search (TS) algorithm based on algorithms originally designed for the LRP: namely, the nested approach proposed by Nagy and Salhi [43] and the two-phase iterative approach introduced by Tuzun and Burke [60]. In few words, the basic idea of the the algorithm is, firstly, to decompose the original problem into two LRP and, secondly, decompose each resulting LRP into a capacitated facility location problem and a Multi-Depot Vehicle Routing Problem (MDVRP). An initial feasible solution is computed by means of a simple heuristic that aims at minimizing the number of facilities to open. Subsequently, the four subproblems resulting from the decomposition are solved and the solutions are combined together to obtain a global feasible solution. The TS consists of two main phases: a location phase, in which the number and the location of the facilities are determined, and a routing phase, in which the routing component is considered and possibly improved. A bottom-up approach is used. Specifically, a second echelon solution is firstly built and then, given that solution, a first echelon solution is computed and optimized. Computational experiments are given for small (up to 4 depots, 10 satellites and 25 customers) and large-scale (up to 5 depots, 20 satellites and 200 customers) instances. The computational results show the effectiveness of the method both in terms of quality of the solutions found and computing times, although the heuristic requires an important tuning phase to perform well.

Crainic *et al.* [22] propose three Mixed Integer Linear Programming (MILP) formulations for the 2E-LRP. The first, using three-index variables, and the third, adopting one-index variables, formulations are inspired to classical VRP formulations available in the literature, whereas the second, using two-index variables, derives from the MDVRP literature. The authors develop an instance generator with the scope of reproducing a schematic representation of a multi-level urban area and to test the effectiveness of the proposed formulations. Computational experiments are conducted solving two of the optimization models (the third formulation is not considered in the experiments) by means of the XPRESS solver and the formulations are compared in terms of computing times, lower bounds provided and quality of the solutions. The computational results show that the three-index formulation provides better lower bounds and outperforms the two-index model solving medium-scale instances.

Schwengerer *et al.* [57] present a Variable Neighborhood Search (VNS) for the 2E-LRP drawing on a VNS algorithm proposed in [52] for the LRP. The algorithm uses seven different general neighborhood structures parameterized with several size perturbations leading to a total of 21 different specific neighborhood structures. Two local search methods are applied to intensify the search. Computational results are given for three sets of instances: two sets introduced in Nguyen *et al.* [45], and the third generated according to Boccia *et al.* [8]. The computational results show that the proposed VNS is competitive with other approaches for the same problem previously proposed in the literature.

As far as the solution method is considered, all the papers on the 2E-LRP surveyed here propose a heuristic algorithm with the only exception of the exact method designed in Contardo *et al.* [13]. The authors propose a two-index vehicle flow formulation and develop a Branch and Cut (B&C) algorithm able to solve small and medium-scale instances to optimality within reasonable computing times. The mathematical formulation is a MILP model that is strengthened by means of valid inequalities derived from the papers on the LRP by Belenguer *et al.* [6] and by Contardo *et al.* [12]. Furthermore, the authors introduce also an Adaptive Large Neighborhood Search (ALNS) algorithm for the 2E-LRP. Both the exact and the heuristic algorithms are based on the idea of decomposing the 2E-LRP into two LRP, one at each echelon. This allows to apply algorithms proposed for the LRP at each echelon and then combine the partial solutions to achieve a global feasible solution. The exact and the heuristic algorithms are tested on five sets of instances, amounting to a total of 147 instances. The first and second sets are taken from [45]. These instances consider only 1 depot, and up to 10 satellites and 200 customers. The last three sets of instances are taken from Nguyen *et al.* [44]. The number of depots in these instances ranges from 2 to 5, the number of satellites from 3 to 20 and the number of customers from 8 to 200. They differ in the location of satellites and depots. The ALNS has been able to find the best-known solution for 133 instances out of 147, while the B&C solved to optimality 75 instances out 147. Additionally, the comparison of the two methods shows that the lower bounds obtained by the B&C lie on average no further than 3.06% below the solution values found by the ALNS.

Nikbakhsh and Zegordi [46] address a variant of the 2E-LRP where soft time windows are associated with the customers and the routing on the first echelon is neglected. The variant is called by the authors the 2E-LRP with soft Time Window constraints (2E-LRPTW). The authors propose a four-index MILP formulation and a heuristic composed by a construction phase followed by an improvement phase. In the construction phase an initial solution is created by means of a location-first, allocation-routing-second algorithm, and then improved with an Or-opt heuristic (see [47]). Then, in the improvement phase, the final solution is computed by exploring six neighborhoods of the initial solution and using an Or-opt heuristic to possibly improve the routes. A lower bound for the 2ELRPTW is computed based on an objective function decomposition. To validate the heuristic the authors generate randomly 21 instances. The computational results show that the larger the instance size, the larger the gap between the lower bound and the heuristic solution found.

Dalfard *et al.* [23] design two heuristics, namely hybrid genetic and Simulated Annealing (SA) algorithms, to solve another variant of the 2E-LRP where vehicle fleet capacity (i.e., a maximum number of vehicles assignable to a facility is considered) and maximum route length constraints are taken into consideration. Similar to [46], the routing of the vehicles among the facilities belonging to the first echelon is neglected. The authors propose a Non-Linear Mixed Integer Programming (NLMIP) model based on a flow commodity formulation. To assess the performance of the two heuristics they generate randomly 20 instances. The former instances are also solved with the LINGO solver. The size of the instances tested is up to 10 depots, 50 satellites and 100 customers. The computational experiments show that both algorithms outperform LINGO. Indeed, on the one hand, as far as the small-scale instances are considered, both heuristics find slightly sub-optimal solutions in quite short computing times. On the other hand, when the large-scale instances are taken into account, LINGO is not able to find any feasible solution within the time limit (the solver is stopped after 20 hours of computing time), whereas both heuristics find feasible solutions in reasonable computing times given the size of instances solved.

Each of the aforementioned papers addresses the 2E-LRP when several depots are taken into consideration. Conversely, Jacobsen and Madsen [33] tackle a variant of the 2E-LRP where a single depot is present. Hence, in this case, decisions on the location of the depot are clearly not involved. The authors consider a real-life problem concerning newspaper distribution in Denmark. Particularly, newspapers that are available at a printing office (i.e., the only origin) have to be delivered to some Sale Points (SP) through some Transfer

Points (TP). The delivery has to adhere to a set of constraints regarding vehicle capacities, length of the tours and delivery times. The decisions to be made are: the locations of the TPs, the connections of the TPs with the printing office to form primary tours, the connections of the SPs with the TPs to form secondary tours and the sequencing of primary tours, that gives the order in which the primary vehicles leave. As the size of the instances considered is large (the number of SPs is around 4500), the authors do not attempt to compute optimal solutions but design three different heuristics to solve the problem. Specifically, the authors provide a comparison of three different algorithms:

1. a tour construction method with implicit TP location;
2. an alternate location-allocation procedure for the TP locations followed by saving procedures for the routing of primary and secondary tours;
3. a saving procedure for the construction of secondary tours, a drop procedure for the location and a saving procedure for the primary tours.

Nguyen *et al.* [45] formulate the 2E-LRP with single depot as an Integer Linear Programming (ILP) model with two-index decision variables. They present four constructive heuristics and a Greedy Randomized Adaptive Search Procedure (GRASP) complemented by a learning process and a path relinking procedure to solve the problem. Path relinking is a procedure that aims at improving the performance of a metaheuristic exploring the trajectory between two solutions. Starting from one solution, it converts, step-by-step, the first solution into a second one obtaining a pool of intermediate solutions to be verified. The GRASP uses three greedy randomized heuristics to generate trial solutions and two Variable Neighborhood Descent (VND) procedures to improve them. Computational experiments are given for three sets of instances involving up to 10 satellites and 200 customers. Computational results show that the GRASP with learning process and path relinking outperforms the other heuristics proposed in their paper. Additional experiments are conducted on instances for the LRP and indicate that the GRASP is competitive with other heuristics specifically designed for the the latter problem. Furthermore, the authors compute a lower bound solving with CPLEX a relaxation of the proposed ILP model obtained removing a set of constraints of exponential size and including some simple cuts. The resulting lower bound is, on average, equal to 80% of the solution value achieved by their metaheuristic. The same authors propose in [44] a new MILP formulation for the 2E-LRP with single depot and a multi-start Iterated Local Search (ILS) algorithm. The algorithm includes some special features. The first feature is a multi-start procedure consisting in restarting the search from another initial solution instead of restarting from

Reference	Solution Algorithm	$\lvert V_d\rvert_{max}$	$\lvert V_s\rvert_{max}$	$\lvert V_c\rvert_{max}$
Boccia *et al.* [8]	TS	5	20	200
Crainic *et al.* [22]	XPRESS	3	10	25
Schwengerer *et al.* [57]	VNS	5	10	200
Contardo *et al.* [13]	B&C, ALNS	5	20	200
Nikbakhsh and Zegordi [46]	Two-Phase Heuristic	10	50	100
Dalfard *et al.* [23]	Hybrid Genetic, SA	10	50	100
Single Depot				
Jacobsen and Madsen [33]	Problem Specific Heuristics	1	42	4510
Nguyen et al [45]	GRASP with Learning Process and Path Relinking	1	10	200
Nguyen *et al.* [44]	Multi-Start ILS with Tabu Lists and Path Relinking	1	100	200

Table 1.3: A summary of solution algorithms and maximum size instances in each paper on the 2E-LRP.

the same solution. The second one consists in using cyclically three heuristics to provide each ILS execution with a feasible solution. The third feature is that a child (i.e., a successive solution) can be accepted only if its gap from the best-known solution does not exceed a given threshold. The forth feature is the coexistence of two improvement procedures. The first procedure involves low complexity neighborhoods and is always performed; the second one is used depending on the gap between the child and the best-known solution and involves more complex moves, as for instance the switching of the status open/closed of a satellite. The fifth feature is similar to a tabu list and is based on storing recent visited solution. Finally, the sixth feature is based on the alternation between two search spaces: the 2E-LRP solution space and the space generated by the solutions of a Traveling Salesman Problem (TSP) considering the main depot and the customers as vertices of a TSP instance. The algorithm has been tested on benchmark instances for the 2E-LRP with single depot, as well as on instances for the 2E-LRP involving up to 5 depots and on instances for the LRP. Computational results show that the algorithm outperforms other heuristics previously proposed for the 2E-LRP.

A summary of the solution methods proposed in each paper on the 2E-LRP along with the maximum size instance solved is reported in Table 1.3. $|V_d|_{max}$ denotes the maximum number of depots, $|V_s|_{max}$ denotes the maximum number of satellites and $|V_c|_{max}$ denotes the maximum number of customers.

1.2 The Two-Echelon Vehicle Routing Problem

In this section, we first provide a brief introductory description of the 2E-VRP and then review the most important papers tackling the problem.

1.2.1 Problem Description

Consider a two-level distribution network where the delivery from a central (and unique) depot to the customers is managed by routing and consolidating the freight through some intermediate facilities. The first echelon comprises one depot, where the freight originates, a set of intermediate facilities, where the freight is firstly delivered by a fleet of primary vehicles, and all the links connecting the depot with the intermediate facilities, on the one hand, and each intermediate facility with the other intermediate facilities, on the other hand. In the literature, the depot is sometimes called consolidation center, whereas the intermediate facilities are referred to as distribution centers or, when such distribution centers are smaller than a depot and/or have only short-term inventory holding capacity, they are called satellite platforms, or, simply, satellites (see [50]). To the sake of simplicity, henceforth we refer to an intermediate facility shortly as satellite. The second echelon involves the delivery of freight from the satellites to the customers by a fleet of secondary vehicles. Direct shipments from the depot to the customers are not allowed. Each customer has a known and deterministic demand and is usually served by exactly one secondary vehicle, i.e., split delivery is not allowed on the second echelon. Conversely, each satellite can usually be served by one or more primary vehicles, so that the aggregated freight delivered to each satellite can be split into two or more primary vehicles. Each primary vehicle can deliver the freight of one or more customers, as well as can serve more than one satellite in the same route. Capacity constraints on the vehicles and the satellites are usually imposed.

The 2E-VRP aims at finding the optimal set of vehicle routes at both levels such that the demand of all customers is satisfied, the satellites and vehicles capacity constraints are not violated, while the total traveling cost is minimized.

As mentioned above, the 2E-VRP differs from the 2E-LRP, surveyed in Section 1.1, as no location decisions have to be taken and all the freight originates at the unique depot.

More formally, we assume the following definition of the 2E-VRP. We consider a weighted undirected graph $G = (N, E)$, where $N = \{\{0\} \bigcup V_s \bigcup V_c\}$ is the set of vertices and E is the set of edges (i, j), $i \neq j$, linking vertex $i \in N$ with vertex $j \in N$. Particularly, vertex 0 represents the depot, $V_s = \{1, \dots, m\}$ is the set of m satellites and $V_c = \{m + 1, \dots, m + n\}$ is the customer set. If both endpoints of edge $(i, j) \in E$ belong to set V_s, or one endpoint is the depot and the other one is in set V_s, edge (i, j) pertains to the first echelon. Conversely, if both vertices i and j belong to set V_c, or one is in set V_s and the other one belongs to V_c, edge (i, j) pertains to the second echelon. A non-

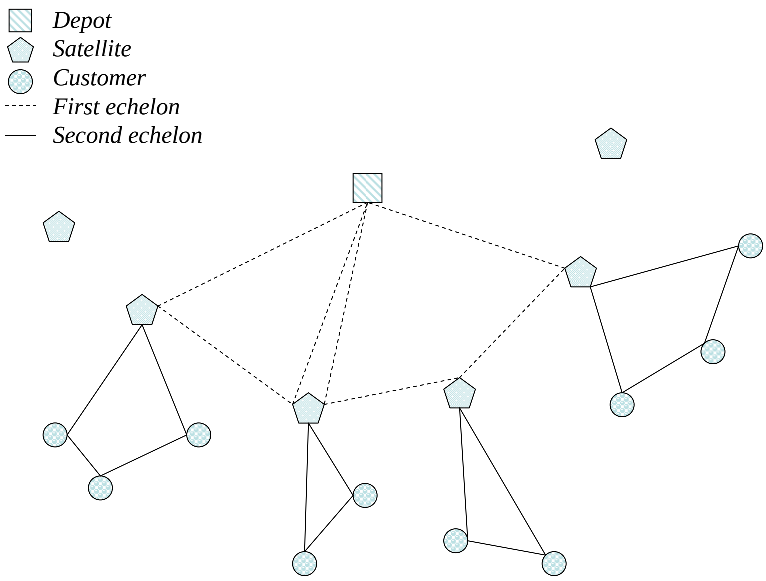

Figure 1.2: An example of a 2E-VRP feasible solution.

negative traveling cost c_{ij} is associated with each edge $(i, j) \in E$. A fleet of ν^1 homogeneous and capacitated primary vehicles is located at the depot. Each primary vehicle has a capacity q_1 and starts and ends its route at the depot after visiting one or more satellites. A fleet of ν_l^2 homogeneous and capacitated secondary vehicles is available at each satellite $l \in V_s$ to deliver the freight from satellite l to the customers. Usually, at most $\nu^2 < \sum_{l \in V_s} \nu_l^2$ secondary vehicles can be used. Each secondary vehicle has a capacity q_2. Each customer $j \in V_c$ demands r_j units of goods from the depot that cannot be split among different secondary vehicles. Each satellite $l \in V_s$ usually has a capacity w_l that limits the total customer demands that can be managed in the satellite. Moreover, a handling cost h_l for loading/unloading operations is sometimes paid for each unit of freight managed in satellite $l \in V_s$.

The 2E-VRP is proved to be $\mathcal{NP}$-hard via a reduction from the VRP (see [50]), which is a special case of 2E-VRP arising when just one satellite is considered. Finally, note that if an assignment of the customers to each satellite is given, the 2E-VRP reduces to 1 + $|V_s|$ VRPs, i.e., 1 for the first echelon and $|V_s|$ for the second echelon.

Figure 1.2 shows an example of a 2E-VRP feasible solution. The square represents the depot, the pentagons are the satellites, and the circles are the customers. The routes belonging to the first echelon are represented as dashed lines, whereas the routes belonging to the second echelon are depicted as solid

Reference	Solution Algorithm	Optimization Model
Perboli *et al.* [50]	Exact and Heuristic	MILP
Crainic *et al.* [14]	Heuristic	
Perboli *et al.* [49]	Exact	
Crainic *et al.* [16]	Heuristic	
Hemmelmayr *et al.* [30]	Heuristic	
Crainic *et al.* [15]	Heuristic	
Jepsen *et al.* [34]	Exact	MILP
Baldacci *et al.* [3]	Exact	MILP

Table 1.4: A summary of the papers on the 2E-VRP.

lines.

1.2.2 Literature Review

A general overview of the main characteristics of the papers on the 2E-VRP is reported in Table 1.4.

The 2E-VRP has been firstly formalized in Perboli *et al.* [50] where the authors propose a MILP formulation along with two families of valid inequalities and two matheuristics. The optimization model is inspired to the literature on multi-commodity network design problems and uses the flow of freight on each arc as main decision variable. The first family of valid inequalities is derived from the subtour elimination constraints proposed for the TSP, while the second family is based on the flow decision variables. The authors mention that other cuts have been derived from the VRP literature in Perboli *et al.* [49] but that, after performing some preliminary experiments, they verified that the improvement after their introduction was quite marginal with respect to the additional computational effort and, therefore, they decide to not consider them. Matheuristics, sometimes also called math-based heuristics, are optimization algorithms that combine elements of mathematical programming with elements of metaheuristics. The matheuristics proposed in [50] are based on information retrieved from the optimal solution of the linear relaxation of the proposed model. Computational experiments are given for four sets of instances: three sets are built from benchmark instances for the VRP, while the fourth set is taken from Crainic *et al.* [17]. The authors also design the following B&C algorithm to solve the 2E-VRP. The two matheuristics are applied at the root node of the B&C tree only, and the best integer solution found is the initial solution of the algorithm. Then, the proposed valid inequalities are introduced when violated. As the authors found, in preliminary experiments, that the effectiveness of the valid inequalities derived from the subtour elimination constraints involving more than 3 vertices is negligible, they do not use a separation algorithm for them but adopt instead a direct inspection of the constraints up to cardinality equal to 3 to identify the violated inequalities. The computational results show that the use of the proposed valid inequalities often helps to improve the initial solutions, the lower bounds and computing times.

Crainic *et al.* [14] introduce and compare two heuristics for the 2E-VRP based on separating the first from the second level routing problems and applying an iterative procedure where the two resulting sub-problems are solved sequentially. Particularly, the general idea of both heuristics is to solve the 2E-VRP by means of a two-phase approach. In the first phase, a feasible solution for the second level routing problem is computed giving a customers-to-satellite assignment configuration. Subsequently, given that assignment configuration, a feasible solution for the first level routing problem is computed. The first level sub-problem is treated as a VRP in which each satellite is managed like a customer whose demand is given by the sum of the demands of the customers assigned to it. The feasible solution resulting from the first phase is possibly improved in the second phase by some improvement procedures focusing directly on the routes. The two heuristics proposed mainly differ in the approach used to tackle the second level routing problem in the first phase. The first approach is based on the use of a clustering technique to decompose the second level routing problem into a set of independent VRPs. The clustering procedure assigns each customer to the nearest satellite on the basis of the euclidean distance and considering the capacity restrictions, i.e., vehicle capacities and fleet size constraints. The resulting independent VRPs are solved by means of the commercial solver ILOG Dispatcher. Given the second level solution, a first level solution is computed consequently. An initial feasible solution for the 2E-VRP is then obtained combining the solutions at the two levels. A pseudo-greedy multi-start procedure is used to attempt to improve the initial clustering solution. On the other hand, the second approach is based on the idea of treating the second level routing problem as a MDVRP. Three improvement procedures are presented for the second phase. Specifically:

- a split-large-route heuristic: it aims to avoid routes with excessively long distances between two consecutive customers by increasing the number of routes;
- an add heuristic: it moves one customer from its current route to another route;
- an exchange heuristic: it swaps two customers within two different routes.

Computational results include a comparison of the performance of both heuristics as well as an analysis of the impact of different customers-satellites distributions and satellite location patters on algorithmic efficiency and solution quality.

As mentioned above, Perboli *et al.* [49] propose several valid inequalities derived from the VRP literature to strengthen the flow-based formulation introduced in [50]. Additionally, the presence of the network flows in the mathematical formulation allows the authors to define some valid inequalities based on the interaction between routing and arc activation variables. Other classes of valid inequalities are derived from considering connectivity and feasibility property of any feasible solution of routing problems. To validate the proposed inequalities the authors implement a B&C.

The idea of separating the first and the second level routing problems is also used in Crainic *et al.* [16] where a family of multi-start heuristics in which the two sub-problems are sequentially solved is proposed. The main steps of the heuristics are the following. The algorithms begin assigning each customer to a satellite according to a distance-based greedy rule. Then, an initial solution is computed by solving the resulting first and second level VRPs. Subsequently, a local search algorithm based on changing one customer-to-satellite assignment at a time is applied attempting to improve the initial solution. Finally, a multi-start procedure is run until a maximum number of iterations has been performed in order to avoid being trapped in local optima. Generally speaking, given the best solution found, the multi-start strategy perturbs the customer-to-satellite assignments according to some rules that take into account the reassignment costs. If the new solution is infeasible, then a procedure to recover the feasibility is applied. Conversely, if the solution is feasible and promising, i.e., its value is better or within a given threshold from the value of the best solution found, the aforementioned local search algorithm is applied. The heuristics differ on the rule used to generate perturbed solutions and on the strategies used to recover feasibility. The performance of the proposed heuristics is compared with that of the matheuristics introduced in [50] and validated using the lower bounds reported in [49]. Computational results show that the tested heuristics are in general quite fast, and that the best heuristic proposed improves some of the best results previously reported in the literature.

Hemmelmayr *et al.* [30] present an ALNS algorithm for the 2E-VRP as well as for the LRP. Indeed, the authors point out that the LRP can be seen as a special case of the 2E-VRP in which the vehicle routing is performed only at the second echelon. To model the LRP as a special case of the 2E-VRP the authors suggest to introduce a dummy vertex representing the 2E-VRP depot. The set of potential facilities in the LRP instance corresponds to the set of satellites in a 2E-VRP instance. Then, the cost of each edge connecting the 2E-VRP depot with each satellite is set equal to the opening cost of the corresponding potential facility in the LRP instance. The main difference of this special case with respect to the standard 2E-VRP is that the first level consists only of single customer routes. The main idea of the ALNS algorithm is to remove, at each iteration, a subset of customers from the current solution by means of a destroy operator and, then, re-insert the customers in other positions using a repair operator. Each operator is associated with a score and is selected randomly from a probability distribution function built on its past success. In other words, an operator that has found several improving solutions has a higher score than other operators, and thus a higher probability to be chosen. The destroy operators proposed in the paper are divided into two classes: those that change the configuration of the current solution by closing or opening a satellite, and those that affect a more restricted area of the search space, for instance removing a small number of customers and keeping the current satellite configuration unchanged. The destroy operators of the first class are used whenever a given number of iterations have been performed without any improvement. Every time that one of these operators is used, a local search is performed on the new solution. It is worth pointing out that the search is not restricted to only the feasible solutions. Indeed, violations of the constraints on the vehicle capacity, the number of vehicles available and the capacity limits at the satellites are allowed. A weighted penalty term is included in the objective function to consider those violations. Computational results are given for three sets of instances for the 2E-VRP introduced in [50] and [17], three sets of instances for the LRP tested in [53], [60] and [5], respectively, and a new set of instances obtained adapting the LRP instances introduced in [53] to the 2E-VRP. Computational results show that the proposed ALNS heuristic improves several best-known solutions previously published in the literature for the 2E-VRP, while competitive results are obtained for the LRP instances.

Crainic *et al.* [15] propose a hybrid heuristic to solve the 2E-VRP that combines a GRASP algorithm with a path relinking procedure. Similarly to other approaches previously proposed by the same authors (see [14] and [16]), the 2E-VRP is tackled decomposing it into two sub-problems. The hybrid algorithm consists of four main phases: a GRASP is used to generate solutions in the first phase; a feasibility search phase is applied if the solution is infeasible; a local search phase that aims at improving the solution and, finally, a path relinking phase. Particularly, the algorithm begins computing an initial customer-to-satellite assignment by means of the clustering heuristic presented in [16]. The corresponding solution for the 2E-VRP is obtained solving the resulting VRPs, given the customer-to-satellite assignment, by means of the hybrid heuristic proposed in Perboli *et al.* [48]. At each iteration, a new assignment is computed by means of the GRASP procedure, and the corresponding 2E-VRP solution is evaluated. If the solution is infeasible a repair procedure, referred to as feasibility search, is performed to recover the feasibility. Conversely, if the solution is feasible and promising (see reference [16] above), an intensification step composed by a local search and a path relinking heuristic is applied, otherwise it is discarded. Computational experiments are given for a set of instances originally introduced in [17] that comprises 50 customers and 5 satellites. The performance of the GRASP heuristic with path relinking is compared with the multi-start heuristic introduced in [16], the matheuristics proposed in [50], and the B&C designed in [49]. The GRASP with path relinking outperforms both the multi-start and the matheuristics, and finds slightly sub-optimal solutions with considerable savings in terms of computing time with respect to the B&C proposed in [49].

Jepsen *et al.* [34] show, by means of an example, that the optimization model proposed in [50] for the 2E-VRP may not provide correct upper bounds when more than two satellites are selected in the solution. Hence, the authors introduced a new MILP formulation for the 2E-VRP that overcomes the limitations of the formulation introduced in [50]. The mathematical formulation is based on the observation that if the assignment of customers to satellite is given, then the 2E-VRP can be decomposed into two sub-problems: a Split Delivery Vehicle Routing Problem (SDVRP, see Archetti and Speranza [2] for a survey) on the first echelon, and one VRP for each satellite that is in use on the second echelon. Hence, in the proposed formulation, the routing in the first echelon is modeled as a SDVRP, whereas the modeling of the second level routing is based on a one-commodity flow formulation for the MD-VRP. However, the authors highlight that the proposed optimization model is highly symmetric and its linear relaxation tends to provide poor lower bounds. Therefore, they propose an alternative MILP formulation that turns out to be a relaxation for the 2E-VRP but provides better lower bounds and eliminates the symmetries. The new formulation is based on the relaxation for the SD-VRP suggested by Belenguer *et al.* [7] in the first echelon, whereas a modified version of the optimization model for the LRP introduced in Contardo *et al.* [12] is used in the second echelon. Even if the proposed optimization model has four sets of constraints of exponential size, the authors show that three out of the four sets can be optimally separated in polynomial time and that the constraints in the fourth set are $\mathcal{NP}$-hard to separate. Additionally, since the proposed relaxation provides lower bounds for the 2E-VRP but not necessarily feasible solutions, the authors devise a feasibility test and a specialized branching scheme to obtain feasible integer solutions. A B&C algorithm is

Reference	Solution Algorithm	$\lvert V_s \rvert_{max}$	$\lvert V_c \rvert_{max}$
Perboli *et al.* [50]	B&C, Matheuristics	5	50
Crainic *et al.* [14]	Two-Phase Heuristics	5	150
Perboli *et al.* [49]	B&C	5	50
Crainic *et al.* [16]	Multi-Start Heuristics	5	50
Hemmelmayr et.al. [30]	ALNS	10	200
Crainic *et al.* [15]	GRASP with Path Relinking	5	50
Jepsen *et al.* [34]	B&C	5	50
Baldacci *et al.* [3]	Problem Specific Exact Algorithm	6	100

Table 1.5: A summary of solution algorithms and maximum size instances in each paper on the 2E-VRP.

developed to solve the 2E-VRP using the specialized branching rule. Computational experiments are given for the instances tested in [50]. The experiments are conducted setting a computational time limit equal to 10,000 seconds. The computational results show that the proposed B&C algorithm outperforms the B&C tested in [50]. Indeed, the proposed exact method solves 47 instances out of 93 to optimality within the time limit.

In a recent paper, Baldacci *et al.* [3] introduce a new mathematical formulation for the 2E-VRP that is used to derive both continuous and integer relaxations. The authors present a new bounding procedure based on dynamic programming, a dual ascent method, and an exact algorithm that decomposes the 2E-VRP into a limited set of MDVRPs with side constraints. Then, the optimal solution for the 2E-VRP is obtained by solving the set of MDVRPs generated. The proposed exact method consists of three main steps:

1. the set of first level routes is enumerated, and a lower and an upper bounds on the 2E-VRP are computed by means of a bounding procedure;

2. the set of all possible subsets of first level routes that could be used in any optimal 2E-VRP solution (denoted as $\mathcal{P}$) is generated. Bounding functions and dominance criteria are used to limit the size of set $\mathcal{P}$;

3. for each subset of first level routes $M \in \mathcal{P}$ a MDVRP with side constraints is obtained fixing to 1 all the binary variables related to the first level routes in set M and, conversely, to 0 all those in $\mathcal{M} \backslash M$. The resulting MDVRP with side constraints is solved by means of an extension of the algorithm proposed in Baldacci and Mingozzi [4].

The exact algorithm is tested on 207 instances, taken both from the literature and newly generated, with up to 6 satellites and 100 customers. The new exact algorithm solves to optimality 144 out of the 153 instances from the literature and closed 97 of theme for the first time. The comparison with previous exact algorithms shows that they are outperformed by the new exact method in terms of both size of the instances solved, number of problems solved to optimality and computing times.

A summary of the solution methods proposed in each paper on the 2E-VRP along with the maximum size instance solved is reported in Table 1.5.

1.3 The Truck and Trailer Routing Problem

In this section, we first provide a brief introductory description of the TTRP and then review the most important papers tackling the problem.

1.3.1 Problem Description

In the TTRP a set of trucks and trucks pulling a trailer have to serve a set of customers with the following restrictions. The set of customers is divided into two subsets: one subset comprises the customers that have be served by a truck alone (without any trailer), while the other subset includes the customers that can be served either by a truck pulling a trailer or by a truck alone. The rationale of separating the customer set is related to the presence of real-life logistic constraints. Indeed, some customers might be located in inaccessible areas for a truck pulling a trailer, but they can be reached by the truck alone. Some examples are customers located in countryside, in mountain areas or in city centers where it is often forbidden to drive large vehicles.

The TTRP aims at finding the optimal set of vehicles routes serving each customer by a compatible vehicle, while minimizing the total routing cost, respecting the capacity of the allocated vehicles and using a number of trucks and trailers not greater than the number of vehicles available.

More formally, we assume the following definition of the TTRP. We consider a weighted undirected graph $G = (N, E)$, where $N = \{\{0\} \bigcup V_c\}$ is the set of vertices and E is the set of edges (i, j), $i \neq j$, connecting vertex $i \in N$ with vertex $j \in N$. The depot corresponds to vertex 0, whereas $V_c = \{1, 2, \dots, n\}$ is the customer set. A nonnegative traveling cost c_{ij} is associated with each edge $(i, j) \in E$. A fleet of ν_K trucks and ν_L trailers, with $\nu_L \leq \nu_K$, is available at the depot to serve the customers. Each *truck alone* has a capacity q_K, whereas the capacity of each trailer is q_L. Hence, a *complete vehicle* (i.e., a truck pulling a trailer) has a capacity equal to $q_K + q_L$. In its essence, the trailer is used as a "mobile depot" that increases the capacity of its truck. A known and deterministic demand r_j is associated with each customer $j \in V_c$. Customer set V_c is partitioned as follows: set V_c^K comprises those customers, referred to as *truck customers*, that can only be served by the truck alone, while set V_c^L

contains those customers,called *vehicle customers*, that can be served by the complete vehicle or by the truck alone. Three types of routes can be identified in a feasible solution of the TTRP: pure truck routes, pure vehicle routes, and complete vehicle routes. A *pure truck route* visits customers in V_c^K and V_c^L by a truck alone. A *pure vehicle routes* visits customers in V_c^L by a complete vehicle and without any sub-tour. Finally, a *complete vehicle route* consists of a main tour, starting and ending at the depot and traveled by a complete vehicle, and one or more sub-tours, traveled by a truck alone. Each sub-tour starts and ends at a vehicle customer location visited in the main tour where the trailer is temporarily parked. Particularly, at a customer in V_c^L visited in the main tour the trailer is unhooked from the truck. Then, the truck alone serves some customers in V_c^K and returns to the customer where the trailer is parked. The trailer is hooked to the truck that can continue its main tour. For every route there might be a restriction on total route length. Finally, it is usually allowed to transfer loads between a truck and its trailer but a cost h per units of load moved is sometimes paid.

We point out that the nature of the TTRP is that of a two-echelon routing problem since in a complete vehicle route the first level routing is represented by the main tour traveled by the complete vehicle among the vehicle customers, whereas the second level routing is represented by the sub-tours among truck customers starting and ending at a vehicle customer location.

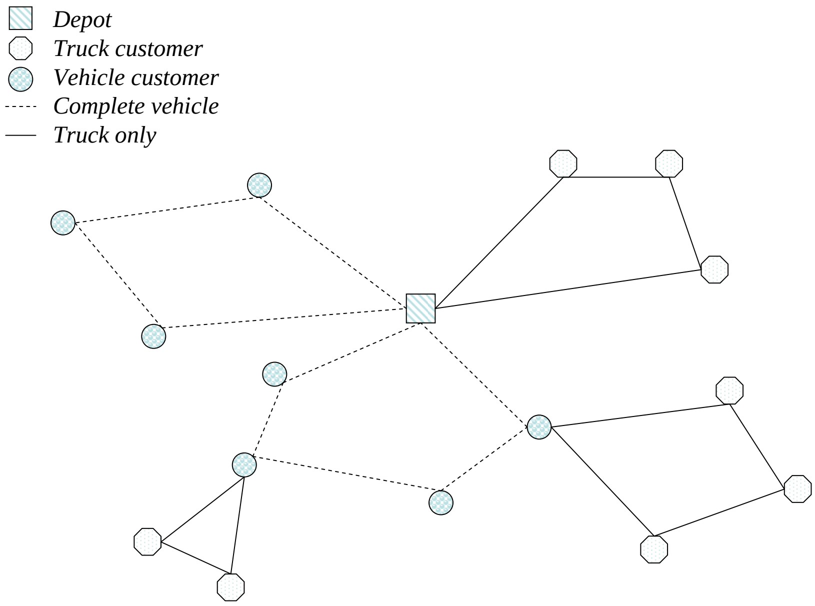

Figure 1.3: An example of a TTRP feasible solution.

Reference	Solution Algorithm	Optimization Model	Specific Characteristics
Chao [10]	Heuristic		
Scheuerer [56]	Heuristic		
Caramia and Guerriero [9]	Heuristic		
Lin *et al.* [38]	Heuristic		
Villegas *et al.* [62]	Heuristic		
Villegas *et al.* [63]	Heuristic	ILP	
Lin *et al.* [39]	Heuristic		No Fleet Size Constraints
Villegas *et al.* [61]	Heuristic	ILP	Single Vehicle, No Vehicle Customers, Trailer Points
Lin *et al.* [40]	Heuristic		Hard Time Windows
Drexl [25]	Exact and Heuristic	MILP	Trailer Points, Hard Time Windows, Heterogeneous Fleet, Fixed Cost for Vehicles
Derigs *et al.* [24]	Heuristic		Hard Time Windows, With and Without Load Transfer

Table 1.6: A summary of the papers on the TTRP.

As the TTRP can be seen as an extension of the VRP, then the TTRP is an $\mathcal{NP}$-hard problem. Indeed, the TTRP reduces to the classical VRP if there are only truck customers (see [10]), i.e., $V_c^L = \emptyset$ and $V_c = V_c^K$. Conversely, if only vehicle customers are present, i.e., $V_c^K = \emptyset$ and $V_c = V_c^L$, the problem could still be solved as a VRP (with an heterogeneous fleet of vehicles if $\nu_L < \nu_K$), as there is no need for uncoupling the trailers (see [56]).

In Figure 1.3 an example of a TTRP feasible solution is depicted. The square represents the depot, the octagons are the truck customers, and the circles are the vehicle customers. The edges traveled by the complete vehicle are represented as dashed lines, whereas those traveled by the truck alone are represented as solid lines.

1.3.2 Literature Review

A general overview of the main characteristics of the papers on the TTRP is reported in Table 1.6.

The TTRP has been formally introduced in Chao [10] where the author also proposes the following heuristic to solve it. The algorithm consists of a procedure computing an initial feasible solution for the TTRP followed by an improvement phase based on a TS heuristic. The procedure computing the initial solution is composed of three steps: a relaxed generalized assignment step, followed by a route construction step, and then a descent improvement step. In the first step, the procedure allocates the customers to a route type solving a relaxed generalized assignment problem. Its solution assigns one route type (i.e., either pure truck, or pure vehicle, or complete vehicle routes) to each customer, but allows the presence of infeasible configurations. The feasibility, if needed, is recovered in the remaining steps in which a penalty function is used. In the second step, the three types of routes are treated as TSPs routes that are constructed by means of a cheapest insertion heuristic. In the descent improvement step, customers are moved from one route to another trying to obtain a feasible solution from an infeasible one. The improvement phase is based on a TS algorithm combined with a deviation concept from deterministic annealing to improve upon the initial solution generated. Computational results are given for 21 instances adapted from the VRP literature comprising up to 17 trucks, 9 trailers and a total of 199 customers, partitioned in different ways among truck customers and vehicle customers.

Scheuerer [56] designs two constructive heuristics and a TS algorithm to solve the TTRP. The first constructive heuristic, called T-Cluster, is a cluster-based sequential insertion procedure where routes are constructed one-by-one up to full vehicle utilization. The second constructive heuristic, called T-Sweep, is based on the classical sweep algorithm, i.e., feasible routes are constructed by rotating a ray centered at the depot and customers are gradually added to the current route. The TS algorithm works as follows. The best solution found by the T-Cluster heuristic is used as the initial solution for the TS. A shifting penalty approach is incorporated into the TS to allow the visit of intermediate infeasible solutions during the search (e.g., solutions where one or more vehicle capacity constraints are violated). At each iteration, random sampling is used to reduce the number of moves to evaluate. Furthermore, to intensify the search in promising regions of the solution space, every time that the TS performs a certain number of iterations without improvements, the search is restarted from the current best solution. Computational results for the 21 instances introduced in [10] show that the TS proposed in [56] outperforms the algorithm designed by Chao [10]. The author also reports some sensitivity analysis of the TS algorithm changing different parameter settings.

Caramia and Guerriero [9] propose an approach based on the solution of two mathematical formulations and a local search procedure. The two formulations, solved sequentially, model two different sub-problems: the Customer-route Assignment Problem (CAP) and the Route Definition Problem (RDP). The goal of the CAP is to minimize the number of vehicles used to serve the customers, whereas the RDP aims at minimizing the total route length given the set of customers assigned to each vehicle in the CAP. Since the solution of the CAP can return infeasible solutions, a local search procedure is implemented in order to recover feasibility. The mathematical formulations and the local search work iteratively, embedded into a multiple restarting mechanism able to diversify the search. Differently from previous approaches, and in particular from Chao [10], the CAP is not followed by a route construction heuristic; rather they use RDP to accomplish such a task. The heuristic is tested on the set of instances introduced in Chao [10]. The computations results show that this approach is competitive with the previous methods available in the literature and produces, on average, a smaller total tour length.

A SA algorithm to solve the TTRP is introduced in Lin *et al.* [38]. The authors propose a standard SA procedure with a random neighborhood structure that features three types of moves, namely insertion, swap, and change of vehicle type used for the vehicle customers (e.g., change from the truck alone to the complete vehicle). Computational results are given for the 21 instances introduced in [10] and show the effectiveness of the proposed solution algorithm. Indeed, the SA found 11 new best solutions with respect to the results reported in [10] and in [56].

Villegas *et al.* [62] propose a route-first, cluster-second procedure embedded within a hybrid metaheuristic based on GRASP, VNS (hybrid GRASP/VNS, hereafter) and path relinking to solve the standard TTRP. The VNS component plays the role of an improving mechanism for feasible solutions, and of a repairing operator for infeasible solutions since, during the search, the hybrid algorithm is allowed to explore infeasible solutions. The authors explore different hybridization alternatives for the path relinking component. Specifically, path relinking is tested as a post-optimization procedure, as an intensification mechanism, and within evolutionary path relinking. Computational experiments are given for the 21 instances introduced in [10] and show that the proposed hybrid GRASP/VNS with evolutionary path relinking performs better than previously published heuristics for the TTRP. The computational results also show that the GRASP/VNS with path relinking used as a post-optimization mechanism is, on average, significantly faster than the GRASP/VNS with evolutionary path relinking achieving slightly worse solutions.

The same authors propose in [63] a matheuristic to solve the standard TTRP. Particularly, the authors combine a set partitioning formulation for the TTRP with a hybrid metaheuristic. The resulting matheuristic follows an iterative two-phase approach. In the first phase, a GRASP algorithm is used to populate a pool of routes with a subset of all possible routes, and an ILS procedure is used to improve the quality of the routes found. Then, in the second phase, the set partitioning formulation is solved on the subset of routes previously identified to obtain a feasible solution. Two variants are proposed: the first variant considers a large pool of routes, while the second one contemplates a small pool of routes. The matheuristic is tested on the benchmarks instances proposed by Chao [10] and those originally proposed for the RTTRP by Lin *et al.* [39] (see below for the details) including the data about the available fleet size. The first variant finds, slightly worse solutions than the best-known ones (the average error is 0.22 %) with computing times that are comparable to the previous methods. The second variant spends one third of the running time taken by the large variant and finds solutions with an average gap which is smaller than 0.5% compared to the best-known solutions in the literature. Finally, a set of computational experiments tested on instances for the RTTRP shows that this method outperforms the algorithm introduced in [39] both in terms of solutions quality and computing times.

A relaxation of the TTRP is studied in Lin *et al.* [39]. The authors begin their study pointing out that in the standard TTRP there are limitations on the number of trucks and trailers available at the depot and that no fixed costs are associated with the use of those vehicles. Thus, their study aims at determining if it is possible to construct better routes by utilizing more vehicles than those available or allowing that a vehicle takes on multiple trips. A second goal of their research is that of determining a better fleet mix in terms of optimal number of trucks and optimal number of trailers. To these aims, they relax the fleet size constraint in the TTRP and call the resulting problem the Relaxed TTRP (RTTRP). A SA is developed to solve the RTTRP. To evaluate the impact of relaxing the fleet size constraint, the authors compare the heuristic solutions found by their SA for the RTTRP with the best-known solutions for the TTRP available in the literature for the 21 instances introduced in Chao [10]. The authors highlight that, for this set of instances, the objective function value of an RTTRP solution found by the SA is, on average, 1.33% smaller than the value of the best-known solution to the TTRP. Computational experiments are also given for 36 further instances generated by the authors converting benchmark instances taken from the VRP literature. For this second set of instances, the RTTRP solutions obtained by the proposed SA heuristic are compared with the TTRP solutions obtained running the SA heuristic described in Lin *et al.* [38]. For this second set of instances, the value of a feasible solution for the RTTRP is, on average, 5% smaller than the value of a feasible solution for the TTRP. More importantly, the authors highlight that for most test problems the number of trucks used in a RTTRP solution is less than or equal to that selected in the corresponding TTRP solution. Conversely, the number of trailers selected in a RTTRP solution is larger than or equal to that used in the corresponding TTRP solution. Consequently, in RTTRP solutions, the ratio trailer/truck is usually closer to one than in TTRP solutions, which the authors claim to be more consistent with the common practice in the truckload industry.

A variant of the TTRP that deserve particular notice is the Single TTRP with Satellites Depots (STTRPSD) introduced in Villegas *et al.* [61]. Instead of considering a fleet of vehicles as in the TTRP, in the STTRPSD only one complete vehicle is available to serve the set of customers. Additionally, there are only customers to be served only by the truck alone, i.e., $V_c = V_c^K$. A set of *trailer points* (sometimes also called parking locations or satellite depots) is available, where it is possible to unhook the trailer and transfer loads between the truck and the trailer. The STTRSD differs from the TTRP since the trailer points do not correspond to customer locations. The authors mention that the STTRPSD reduces to the MDVRP when all distances between parking locations are null. The authors mention that, among others applications, milk collection can be modeled as a STTRPSD. Indeed, in milk collection customers (i.e., farms) are usually visited by a single tanker with a removable tank trailer. Trailer points are in general parking locations located on main roads, whereas farms are often located in areas that can be reached only driving on narrow streets that are inaccessible for the vehicle with the trailer. The authors point out that farms are usually clustered based on their geographical location, and each cluster is then assigned to one vehicle. In the STTRPSD the following two levels of routing are involved. The first level route is performed by the complete vehicle, it starts and ends at the depot and links the selected trailer points. Each second level route starts and ends at one selected trailer point, and is performed by the truck alone that visits the subset of customers assigned. Thus, the total load in a second level route must not exceed the truck capacity. The authors propose an ILP formulation and two metaheuristics for the STTRPSD. The two metaheuristics are a GRASP algorithm hybridized with a VND procedure (hybrid GRASP/VND hereafter) and a multi-start evolutionary local search, respectively. Both methods are tested on 32 randomly generated instances comprising up to 200 customers and 20 trailer points. The computational experiments show that the multi-start evolutionary local search is more accurate, faster, and scales better as the number of customers increases than the hybrid GRASP/VND algorithm. To provide some further insights on the overall performance of the multi-start evolutionary local search, the authors perform computational experiments on some benchmark instances for the MDVRP. The computational results show that the multi-start evolutionary local search is competitive with respect to state-of-the-art heuristics for the MDVRP.

The TTRP with Time Windows (TTRPTW) is introduced in Lin *et al.* [40]. Drawing on the SA algorithm proposed in [38] for the TTRP, the authors design a SA heuristic to solve the TTRPTW that can also be used to tackle the VRP with Time Window (VRPTW). Two sets of computational experiments are performed. In the first set, some benchmark instances for the VRPTW are solved with the proposed SA heuristic to validate its effectiveness in comparison with the best-known solutions available in the literature. In the second set of

Reference	Solution Algorithm	$\lvert V_c^K\rvert_{max}$	$\lvert V_c^L\rvert_{max}$
Chao [10]	TS	49	150
		99	100
		149	50
Scheuerer [56]	Constructive Heuristics, TS	Inst. in [10]	
Caramia and Guerriero [9]	Matheuristic with Local Search	Inst. in [10]	
Lin *et al.* [38]	SA	Inst. in [10]	
Villegas *et al.* [62]	Hybrid GRASP/VNS with (Evolutionary) Path Relinking	Inst. in [10]	
Villegas *et al.* [63]	Matheuristic with GRASP and ILS	Inst. in [10] and [39]	
Lin *et al.* [39]	SA	37	113
		75	75
		113	37
Villegas *et al.* [61]	Hybrid GRASP/VND, Multi-Start Evolutionary Local Search	200	0
Lin *et al.* [40]	SA	50	150
		100	100
		150	50
Drexl [25]	Branch and Price, Heuristic Variants	Inst. in [10]	
Derigs *et al.* [24]	Hybrid Local Search and Large Neighborhood Search	Inst. in [10] and [40]	

Table 1.7: A summary of solution algorithms and maximum size instances in each paper on the TTRP.

experiments, some benchmark instances for the VRPTW are used to generate 54 instances for the TTRPTW that are solved with the proposed SA approach.

Drexl [25] introduces the Generalized TTRP (GTTRP) as a generalization of the TTRP in the following senses. Trailers can be parked and loads can be transferred between truck and trailer at vehicle customers locations, as in the TTRP, but also at trailer points, as in the STTRPSD. Additionally, in the GTTRP fixed costs for using the vehicles are considered, all customers and trailer points have hard time windows associated with them, and the fleet is composed of heterogeneous vehicles. The author propose a MILP formulation for the GTTRP based on binary arc-flow variables, and design a Branch and Price algorithm, based on a path-flow reformulation of the MILP model, as well as some heuristic variants of the exact algorithm. Computational experiments are given for randomly generated instances structured to resemble real-world situations (the motivating application is also here milk collection) and on the benchmark instances for the TTRP introduced in [10]. The computational results show that the instances of realistic structure and size can be solved in short computing times with high solution quality with a heuristic column generation approach. Conversely, the results on the benchmark instances for the TTRP are not so successful.

Derigs *et al.* [24] study the following two variants of the TTRP: the TTRPTW (also studied in [40] described above), and the TTRP with and without the option of load transfer between the truck and the trailer. The authors present a hybrid algorithm which combines local search and large neighborhood search moves guided by two simple metaheuristic control strategies. The authors claim that the approach is quite flexible and that it can be applied to different variants of the TTRP after small modifications. Computational results are given for the standard TTRP solving the benchmark instances introduced in [10] along with a comparison of the performance of the proposed heuristic with the hybrid heuristic designed in [62]. Further computational experiments are provided on the benchmark instances for the TTRPTW introduced in [40]. Finally, computational results are also given for the two variants of the standard TTRP and the TTRPTW where load transfers are not allowed. The computational results show that the method proposed is competitive with other heuristics previously proposed for the TTRP, whereas it outperforms the previous methods when time windows are taken into consideration. Finally, the authors claim that the benchmark instances for the TTRP available in the literature are not exploiting the complexity of the problem because they do not consider costs and consumption of time in several situation. For instance, no costs are considered to transfer loads between the truck and the trailer, and also to hook and unhook the trailer en-route. Additionally, the authors claim that the benchmark instances for the TTRPTW available in the literature, which have been constructed from benchmark instances for the VRPTW by clustering the customers into the two classes of truck and vehicle customers, are not appropriate since some instances do not offer the opportunity to perform sub-tours, therefore being almost equivalent to VRPTW instances.

A summary of the solution methods proposed in each paper on the TTRP along with the maximum size instance solved is reported in Table 1.7.

Chapter 2

The Single Vehicle 2E-LRP

2.1 Introduction.

One of the most important and natural applications of the multi-echelon transportation system is intermodal transportation. It can be defined as a chain made up of several coordinated transportation modes that interact in intermodal terminals in order to ensure the delivery of freight from the origin to the final destinations. Distances play an important role as the longer the distance, the more likely an intermodal transport chain will be used. Distances over 500 km (longer than one day of trucking) usually require intermodal transportation, see [54]. This is usually suitable for intermediate and finished goods in load units of less than 25 tons. The most common intermodal transportation units are represented by containers, large metal boxes of standard dimensions and measured in twenty-foot equivalent units. In this way, the freight, cram in the containers, moves from mode to mode without reloading the shipper container.

There are five basic modes of transportation service: ship, barge rail, truck, air and pipeline. It is well known that the most expensive mode is air transport, whereas the ship represents the cheapest mode when long distance are involved (more than 750 Km). There is, in fact, a strict relationship between transport costs, distance and the mode of transportation chosen. Road transport is, usually, convenient when short distances are involved (until 500 Km) whereas the railway is convenient when average distances (between 500 and 750 Km) are involved. The basic modes can generate several combinations of transportation. For example air-truck transportation, rail-truck transportation and ship-truck transportation. It is important to underline that the intermodal transportation has a considerable importance both in Italy and Europe, as a tool capable to balance the percentage of freights transported by the different modes. In fact, the development of the intermodality allows to relieve congestion in the traffic areas, and consequently relieve pollution, improve the service transport quality and, at the same time, improve the competitiveness of the underdeveloped areas. In Europe rail intermodal services are becoming well-established among the major ports, such as Rotterdam, southern Germany, and between Hamburg and Eastern Europe, see [54]. The limits of intermodality are represented by factors of space, time, form, pattern of the network, the number of nodes and linkages, and the type and characteristics of the vehicles and terminals. Italy, for its natural geographic configuration, facilitates the use of this kind of transportation, even for the national north-south transportation. Despite this, the ship-truck combination is available only on about thirty seaports on a total of one hundred and thirty seaports active in Italy. The seaport terminals play the role of exchange hubs where containers are moved from ships to trucks. The location of sites where the modal exchange takes place is another important element when evaluating the competitiveness of intermodal transport.

In this context Infante *et al.* [31] proposed a heuristic algorithm to tackle the ship-truck Intermodal Transportation Problem (ITP). Their study deals with the selection of seaport terminals where containers move to/from the hinterland by truck; the origin/destination points in the hinterland are special logistic centers (for example, distribution centers or manufacturing/assembly plants) located in different places. Specifically, their ship-truck ITP can be described as follows: a ship starts from the initial seaport terminal carrying containers that must be delivered to certain logistic centers. The ship then arrives at some intermediate seaport terminals where it unloads the containers that have to be transported by truck to the logistic centers, and loads containers coming from the logistics centers, directed to the initial seaport terminal, to which the ship returns at the end of its tour. The objective is to minimize the total cost of transportation. The following factors are known to the transport operators: the initial seaport terminal, the set of intermediate seaport terminals, the set of logistic centers, the number of containers to deliver/pickup at each logistic center, the transport costs between seaport terminals (independent of the number of containers transported by the ship), and the cost for moving one container from/to each intermediate seaport terminal to/from each logistic center.

In the following we will analyze and study a similar problem, different only in some aspects compared to the one proposed in [31]. Originally our problem was conceived as an intermodal problem, then after analyzing thoroughly the structure we decided to present it as a particular version of 2E-LRP, called *The Single Vehicle 2E-LRP*. In the first version we considered the distribution of freight using the ship-truck combination mode. It can be described in the following way. Given a primary seaport, where the freight is available, a set of intermediate seaports to be located and a set of customers with a known demand, the freight has to be delivered to the customers passing through the intermediate seaports. In particular, a ship, where the freight is loaded at the primary seaport, has to perform a tour among the selected intermediate seaports (first echelon) and a set of trucks (one for each selected seaports) has to deliver the freight to the customers (second echelon). Every customer has to be served by one vehicle that starts and ends its route at the same intermediate seaport. The goal of the problem is to select a subset of seaports through which the freight is delivered, minimizing the total cost given by the fixed costs associated with the selection of each intermediate seaport and the total routing costs. In the next section we present the problem in its definitive version contextualizing it in the 2E-LRPs area.

2.2 Description and Mathematical Formulations

In the following we introduce the basilar assumptions necessary to define and describe the problem in the 2E-LRP contest.

2.2.1 Problem Introduction

The principal components involved in our problem are a *depot*, a set of *satellites* and a set of *customers*. They interact among themselves by means of an uncapacitated vehicle performing deliveries in the first echelon, and by means of a set of smaller vehicles performing deliveries in the second echelon.

1. *Depot or primary facility*: large capacitated facility generally located far from the customers. At this facility the freight is loaded on a large uncapacitated vehicle which performs the distribution among satellites and then returns to the depot. The vehicle performs only one tour, therefore it does not travel back to the depot before having served all the satellites. In the ship-truck intermodal application aforementioned the depot is represented by the primary seaport.

2. *Satellites or secondary facilities*: small facilities, closer to the customers compared to the depot, used to tranship or consolidate the freight. At each secondary facility a small capacitated vehicle is available. Each vehicle performs a tour among a subset of customers, delivers the freight and travels back to the satellite. In the aforementioned ship-truck intermodal application the satellites correspond to the intermediate seaport.

3. *Customers*: end-points of the distribution, usually located in inaccessible places for big vehicles (for example city centers, country sides, mountains and so on). Each customer is served by only one vehicle belonging to the second echelon.

In this work we do not consider other parameters, constraints and relationships that could be considered in a two-echelon distribution system. In the following we provide a list of possible features characterizing a two-echelon distribution system:

- freight typology. The freight flow could be constituted by a heterogeneous type of goods;
- time constraints. Two types of time constraints could be considered: classical time windows constraints involving satellites and customers, and synchronization constraints requiring the synchronization between fleets of first and second echelon;
- time dependency. A distribution system could consider single period (static problem) or multi-period horizon (dynamic problem);
- data uncertainty. In some case part of input data could not be deterministic, but stochastic.

In this thesis we consider a problem with:

- a freight constituted by a single commodity, required in deterministic quantity by customers, in a single planning period;
- a central depot where the freight is available and from which it cannot be delivered directly to the customers. In particular, the first echelon route starts at the central depot, pass through the open satellites and returns to the starting point. The second echelon routes start from an open satellite, pass through a subset of customers and return to the satellite;
- a set of customers that have to be served by secondary vehicles belonging to open satellites;
- vehicles belonging to the second echelon with different capacities, larger than the demand of each customer;
- no time windows and synchronization constraints.

Thus, in the problem one can take the following decisions:

- location decisions: define number and location of satellites;
- allocation decision: assign each customer to an open satellite;
- routing decisions.

2.2.2 Problem Settings

The Single Vehicle 2E-LRP can be stated as follows. A central depot, 0, a set $P = \{1, 2, \ldots, m\}$ of m intermediate satellites, and a set $J = \{m+1, m+2, \ldots, m+n\}$ of n customers are given. A vehicle with capacity w_p is available at each satellite $p \in P$, and each customer $j \in J$ requires a quantity q_j of goods, that is available at central depot 0. A vehicle, with a capacity w^1 large enough to transport the total demand of goods required by the customers, is available at the central depot 0. This vehicle starts from the central depot, stops at some intermediate satellite where a quantity of goods is unloaded to be transported by smaller vehicles to the customers, and then returns to the central depot. Each customer is served only by one satellite and each secondary vehicle starts from a selected satellite, visits the customers to which it must deliver the goods, and ends its route at the starting satellite. The objective is to minimize the total cost.

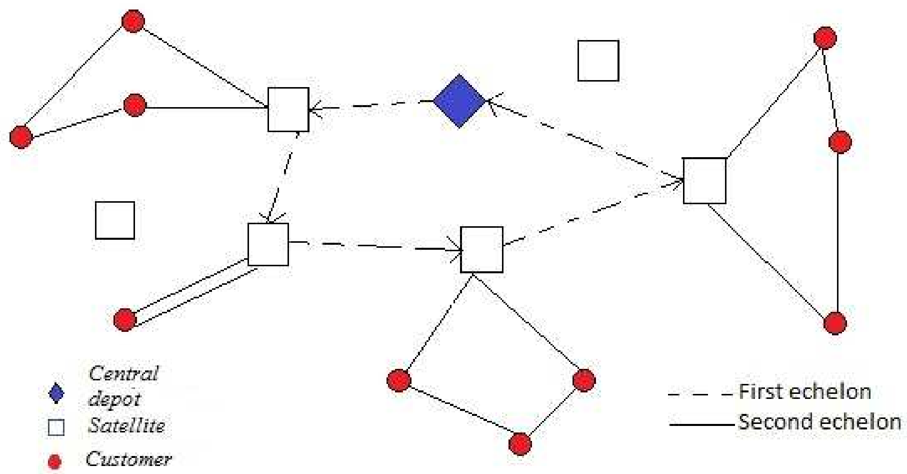

Figure 2.1: Scheme of the problem.

A general scheme of the problem is illustrated in the Figure 2.1. The blue rhombus represents the central depot, the white squares represent the satellites and the red circles represent the customers. The first echelon, i.e. the primary tour, is marked with the dashed arrows, whereas the second echelon (routes connecting customers and satellites) is depicted with solid line. As shown in the figure, a single second echelon route starts from every open satellite. The open satellites are included both in the first and second echelons, whereas the closed satellites are included neither in the first nor in second echelon.

2.2.3 Flow Based Formulation

In the following an integer programming formulation inspired by the one proposed in [50] for the 2E-VRP is presented. The problem is represented on a graph $G = (V, A)$, where $V = \{0\} \cup P \cup J$ is the set of nodes (central depot, satellites and customers) and $A = \{(i, j) : \ i \ and \ j \ \in V\}$ is the set of arcs. F_p is the fixed cost to open the satellite $p \in P$, and c_{ij} represents the travel cost between the nodes $i \in V$ and $j \in V$. It is assumed that $c_{ji} = c_{ij}, \ \forall \ j$ and $\ i \in V$, and $c_{jj} = 0, \ j \in V$ and that the triangular inequality holds. Let y_p be the binary variable taking value 1 if satellite p is open, 0 otherwise. Let z_{ij} be the binary variable taking value 1 if the uncapacitated vehicle uses arc (i, j) belonging to the first echelon, 0 otherwise. Let x^p_{ij} be the binary variable taking value 1 if vehicle p uses arc (i, j) belonging to the second echelon, 0 otherwise. Let v_{pj} be the binary variable taking value 1 if customer j is assigned to satellite p, 0 otherwise.

We define the flow of freight passing through each satellite p:
$Q_p = \sum_{j \in J} q_j v_{pj}$.

Let Q^1_{ij} and Q^2_{ijp} be the flow of freight passing through arc (i, j) belonging to the first echelon and the flow of freight passing through arc (i, j) and satellite p belonging to the second echelon respectively. These quantities are constrained to be not negative.

This formulation will be referred as flow based formulation because it is based on the flow of freight passing through each selected arc.

The problem can be formulated as follows:

$$Min \sum_{i \in P \cup \{0\}} \sum_{j \in P \cup \{0\}, i \neq j} c_{ij} z_{ij} + \sum_{p \in P} \sum_{i \in J \cup \{p\}} \sum_{j \in J \cup \{p\}, i \neq j} c_{ij} x^p_{ij} + \sum_{p \in P} F_p y_p \qquad (2.1)$$

$$y_0 = 1 \qquad (2.2)$$

$$\sum_{p \in P \setminus \{0\}} z_{0p} = 1 \qquad (2.3)$$

$$\sum_{j\in P\setminus\{i\}} z_{ij} = \sum_{j\in P\setminus\{i\}} z_{ji} \quad \forall i \in P \tag{2.4}$$

$$\sum_{j\in P} z_{pj} = y_p \quad \forall p \in P \tag{2.5}$$

$$\sum_{j\in P} z_{jp} = y_p \quad \forall p \in P \tag{2.6}$$

$$\sum_{j\in J} x^p_{pj} = y_p \quad \forall p \in P \setminus \{0\} \tag{2.7}$$

$$\sum_{j\in J} x^p_{jp} = y_p \quad \forall p \in P \setminus \{0\} \tag{2.8}$$

$$\sum_{j\in J} x^p_{pj} = \sum_{j\in J} x^p_{jp} \quad \forall p \in P \tag{2.9}$$

$$\sum_{i\in P\setminus\{j\}} Q^1_{ij} - \sum_{i\in P\setminus\{j\}} Q^1_{ji} = \begin{cases} Q_j & \text{if } j \text{ is not the depot,} \\ \sum_{i\in J} -q_i & \text{otherwise} \end{cases} \quad \forall j \in P \tag{2.10}$$

$$Q^1_{ij} \leq w^1 z_{ij} \qquad \forall i,j \in P, i \neq j \tag{2.11}$$

$$\sum_{i\in J\cup\{p\}, i\neq j} Q^2_{ijp} - \sum_{i\in J\cup P} Q^2_{jip} = \begin{cases} v_{pj}q_j & \text{if } j \text{ is not a satellite,} \\ -Q_j & \text{otherwise} \end{cases} \quad \forall j \in P \cup J \tag{2.12}$$

$$Q^2_{ijp} \leq w_p x^p_{ij} \qquad \forall i,j \in P, i \neq j \tag{2.13}$$

$$\sum_{i\in P} Q^1_{i0} = 0 \tag{2.14}$$

$$\sum_{j\in J} Q^2_{jpp} = 0 \qquad \forall p \in P \tag{2.15}$$

$$x^p_{ij} \leq v_{pj} \qquad \forall i \in P \cup J, \forall j \in J, \forall p \in P \tag{2.16}$$

$$x^p_{ji} \leq v_{pj} \qquad \forall i \in P \cup J, \forall j \in J, \forall p \in P \tag{2.17}$$

$$\sum_{i \in P \cup J} x_{ij}^{p} = v_{pj} \qquad \forall p \in P, \forall j \in J \tag{2.18}$$

$$\sum_{i \in P \cup J} x_{ji}^{p} = v_{pj} \qquad \forall p \in P, \forall j \in J \tag{2.19}$$

$$\sum_{p \in P} v_{pj} = 1 \qquad \forall j \in J \tag{2.20}$$

$$x_{pj}^{p} \leq y_{p} \qquad \forall p \in P, \forall j \in J \tag{2.21}$$

$$x_{ij}^{p} \in \{0,1\} \qquad \forall (i,j) \in A \quad \forall p \in P \tag{2.22}$$

$$z_{ij} \in \{0,1\} \qquad \forall (i,j) \in A \tag{2.23}$$

$$y_{p} \in \{0,1\} \qquad \forall p \in P \cup \{0\} \tag{2.24}$$

$$Q_{ij}^{1} \geq 0 \qquad \forall (i,j) \in A \tag{2.25}$$

$$Q_{ijp}^{2} \geq 0 \qquad \forall (i,j) \in A, \quad \forall p \in P \tag{2.26}$$

The objective function comprises three terms, the first one represents the length of the first echelon trip, the second one the total length traveled by the vehicles belonging to the second echelon and the third one is the sum of the fixed costs of opening the satellites. Constraints (2.2) assures that the central depot is always open. Constraints (2.3) assures that only one vehicle starts a tour from the central depot. Constraints (2.4) with (2.5) and (2.6) guarantee that, in the first echelon, exactly one arc enters and one arc exits for each node associated with open satellite. Constraints (2.7), (2.8), and (2.9) are the corresponding constraints for the second echelon. Constraints (2.10) and (2.12) indicate that the flow balance for each node corresponds to the demand of the node, with exception for the central depot in the first echelon (in which the flow balance is equal to the total demand of the customers) and for the satellites in the second echelon (where the balance flow of each satellite is equal to the total demand of the customers assigned to this satellite). As Perboli *et al.* explain in [50], these constraints forbid also the presence of subtours. Constraints (2.11) and (2.13) are capacity constraints. Constraints (2.14) guarantee that the entering flow in the central depot is equal to zero, as well as the constraints (2.15) guarantee that, in the second echelon, the entering flow in each open satellite is zero. Constraints (2.16) and (2.17) state that for a generic customer j the entering arc in node j and the exiting arc from j belong to an open satellite. Constraints (2.18) and (2.19) are degree constraints on the customer nodes. Constraints (2.20) assure that each customer can be served only by one satellite. Constraints (2.21) state that the generic second echelon arc can assume value 1 only if the respective satellite is open. Finally, (2.22)-(2.26) specify the domains of the variables.

The flow based formulation can be used to solve both symmetric and asymmetric problems. It requires a large number of variables, but a smaller number of constraints compared to the formulation containing Sub-tour Elimination Constraints (SECs).

Flow Constraints

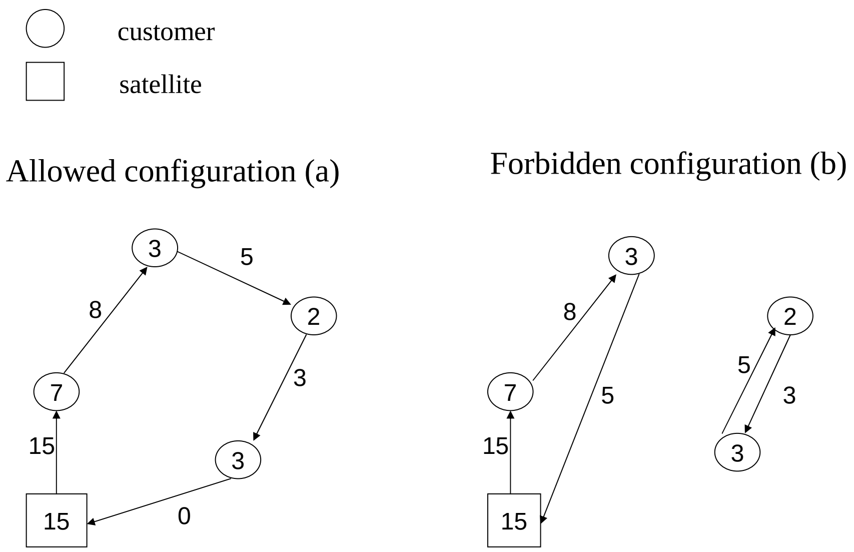

Figure 2.2: Allowed and forbidden configurations for the flow based formulation.

In Figure 2.2 an example of a feasible configuration (a) and an example of an infeasible configuration (b) for a vehicle tour are depicted. In both configurations the capacities of the vehicles are equal to 15, the amount of freight required by each satellite is given in the squares, the binary variables corresponding to the arcs shown all take value 1 and the amount of freight passing through each arc is reported on the side of the arc. As the constraints (2.12) impose, for each customer the difference between the entering flow of freight and the exit flow is equal to the customer demand. Configurations of type (b), in which sub-tours can be present, are prevented by the flow constraints combined with the constraints (2.15) which impose that the entering flow of freight in each satellite has to be equal to zero (whereas in the configuration (b) is 5).

2.2.4 Symmetric Formulation

We present a second model, which is defined on an undirected graph and will be referred to as the symmetric model. This formulation is similar to the one presented in Belenguer *et al.* [6] for the Capacitated Location Routing Problem.

Let $G = (V, E)$ be an undirected graph, where $V = \{0\} \cup P \cup J$ represents the set of nodes and E represents the set of edges. $E' = \{(i, j) \in E,\ i, j \in \{0\} \cup P\}$ is a subset of E referring to the edges belonging to the first echelon, whereas the subset $E \setminus E'$ represents the edges belonging to the second echelon. $\delta(i)$ refers to the edges with one endpoint in the generic node i, $E'(U)$ and $E \setminus E'(U)$ are the subsets of edges with both endpoints in the subset of nodes U and belonging to set E' and set $E \setminus E'(U)$ respectively. As in the previous model, q_j and w_p denote the demand of the customer j and the capacity of the satellites p, and y_p represents the binary variable associated with the use of satellite p. y_p in this case is not a proper variable because it can be obtained in the following way:

$2y_p = 2\bar{z}_{0p} + z(\delta(p)) \qquad \forall p \in P \setminus \{0\};$

$2y_p = 2\bar{x}^p(\delta(p)) + x^p(\delta(p)) \qquad \forall p \in P;$

We also define the following variables:

$z_{ij} = 1$ iff the uncapacitated vehicle uses edge $(i, j) \in E'$ only once;

$\bar{z}_{0j} = 1$ iff the uncapacitated vehicle uses edge $(0, j) \in E'$ twice;

$x^p_{ij} = 1$ iff the vehicle p (i.e. located at satellite p) uses the edge $(i, j)i, j \in E \setminus E'$ once;

$\bar{x}^p_{pj} = 1$ iff the vehicle uses edge $(p, j)p \in P,\ j \in J$ twice;

Thus, the problem can be formulated as follows:

$$Min \sum_{(i,j)\in E'} c_{ij} z_{ij} + 2\sum_{j\in P} c_{0j}\bar{z}_{0j} + \sum_{p\in P}\sum_{(i,j)\in E\setminus E'} c_{ij} x^p_{ij} + 2\sum_{p\in P}\sum_{j\in J} c_{pj}\bar{x}^p_{pj} + \sum_{p\in P} F_p y_p \tag{2.27}$$

$$y_0 = 1 \tag{2.28}$$

$$2\bar{z}_{0j} + z(\delta(j)) = 2y_j \qquad \forall j \in P \setminus \{0\} \tag{2.29}$$

$$z(E'(U)) \leq |U| - 1 \qquad \forall U \subset P \setminus \{0\} \quad 3 \leq |U| \leq |P| - 1 \tag{2.30}$$

$$\sum_{p \in P} (2\bar{x}^p_{pj} + x^p(\delta(j))) = 2 \qquad \forall j \in J \tag{2.31}$$

$$\sum_{p \in P} (\bar{x}^p_{pj} + x^p_{pj}) \leq 1 \qquad \forall j \in J \tag{2.32}$$

$$\sum_{p \in I_1} x^p_{jk} + \sum_{p \in I_2} x^p_{kl} \leq 1 \quad \forall j, k, l \in J \quad \forall I_1 \subset P, \quad I_2 = P \setminus I_1 \tag{2.33}$$

$$x^p(E \setminus E'(U)) \leq |U| - 1 \qquad \forall U \subset J : q(U) \leq w_p, \quad \forall p \in P \tag{2.34}$$

$$2\bar{x}^p(\delta(p)) + x(\delta(p)) = 2y_p \qquad \forall p \in P \tag{2.35}$$

$$\sum_{j \in J} q_j x^p_{pj} + \sum_{i \in J} \sum_{j \in J} q_j x^p_{ij} \leq w_p \qquad \forall p \in P \tag{2.36}$$

$$\bar{z}_{ij} \in \{0, 1\} \qquad \forall (i, j) \in E \tag{2.37}$$

$$z_{ij} \in \{0, 1\} \qquad \forall (i, j) \in E \tag{2.38}$$

$$\bar{x}^p_{ij} \in \{0, 1\} \qquad \forall (i, j) \in E \quad \forall p \in P \tag{2.39}$$

$$x^p_{ij} \in \{0, 1\} \qquad \forall (i, j) \in E \quad \forall p \in P \tag{2.40}$$

$$y_p \in \{0, 1\} \qquad \forall p \in P \cup \{0\} \tag{2.41}$$

The objective function represents the total cost which is defined like in the previous formulation as the sum of routing costs and fixed opening costs. As in the previous formulation, constraint (2.28) imposes that the central depot is open. Constraints (2.29) concern the first echelon and impose, for each node representing a satellite, that the degree is equal to 2 if the satellite j

is selected, 0 otherwise. Constraints (2.30) are the SECs for the first echelon. Constraints (2.31) are the degree constraints for the node representing the customers, whereas constraints (2.32) force each customer to use either one variable $\bar{x}^p_{pj}$ or one variable x^p_{pj}, and oblige each customer to be served at maximum by one satellite. Constraints (2.33) serve to prevent that a node, belonging to the set of the customers, is included in the routes of two different satellites. It is important to notice that this number of constraints is exponential. Constraints (2.34) are the SECs for the second echelon. Constraints (2.35) concern the second echelon and are degree constraints for the nodes representing the selected satellites. Constraints (2.36) are capacity constraints. Constraints (2.37)-(2.41) specify the domains of the variables.

The symmetric formulation is very flexible and can be extended easily to take into account other possible feature of the problem, as time windows, maximum length constraints and so on. On the other hand it is difficult to solve because involves a large number of constraints and variables.

Exponential Constraints

In this formulation three sets of exponential constraints are used:

1. the SECs on the first echelon (2.30);
2. the SECs on the second echelon (2.34);
3. the constraints used to prevent the formation of tours starting from a satellite and ending to another one (2.33).

The SECs on the first echelon (2.30) are inserted into the formulation in the classic way, imposing that for each subset of satellites $U \in P$ the number of edges with both endpoints in U must be less than or equal to the cardinality of U minus 1. The SECs on the second echelon (2.34) are structured in a slightly different way. We do not know in advance the number of customers assigned to each satellite and consequently we do not know the maximum cardinality of U for which we have to implement the SECs constraints. In order to remedy to this problem we consider all the possible subsets U with cardinality comprised between 2 and $|J|$. The most important feature is the fact that we consider only the edges connecting the customers, excluding the edges connecting the satellites with the customers. In Figure 2.3 a feasible configuration containing only one tour (c), and an infeasible configuration (d) containing two subtours are depicted. In (c), the tour visits a satellite and four customers. The number of edges connecting only the customers among themselves is equal to three. For each subset U of customers the SECs inequalities are verified. For

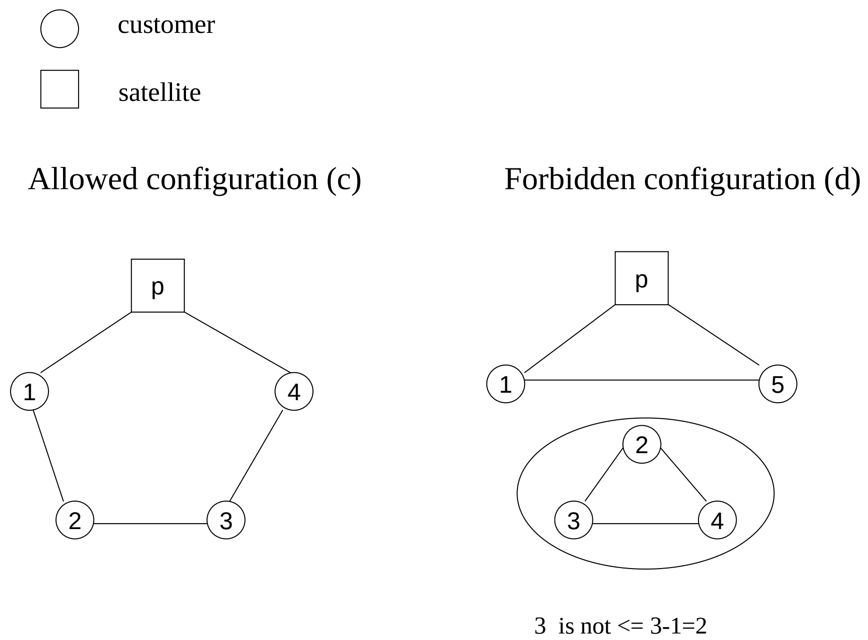

Figure 2.3: Allowed and forbidden configurations with SECs for the symmetric formulation.

instance when $U = \{1, 2, 3, 4\}$ the following inequality holds: $3 = \sum_{(i,j)\in J} x_{ij}^p \leq |U| - 1 = 3$. In (d) an example of an infeasible solution is depicted. This solution is infeasible because there are two sub-tours: $(p-1-5)$ and $(2-3-4)$. Let us consider the sub-tour of length 3. If we take $U = \{2, 3, 4\}$ we can verify that the number of edges connecting the customers among themselves is equal to three, as well as the cardinality of U. Consequently the sub-tour constraint $\sum_{(i,j)\in J} x_{ij}^p \leq |U| - 1$ is violated for a such U, in fact we have $3 \leq 2$.

The use of SECs avoid the presence of sub-tours, but we have to consider another set of constraints (2.33) in order to avoid the presence of tours starting from a satellite and ending to another one. In figure 2.4 an example of tour including two satellites is illustrated. Two edges belonging to two different satellites are incident to vertex k. The first edge, to which the variable x_{jk}^1 is associated, connects j and k and belongs to satellite 1; the second edge, to which the variable x_{jk}^2 is associated, connects k and l and belongs to satellite 2. Obviously this solution is infeasible and impossible to employ in a real life application. The constraints (2.33) avoid the presence of similar solutions. Given a partition of the set of satellites in two subset, I_1 and I_2, they state

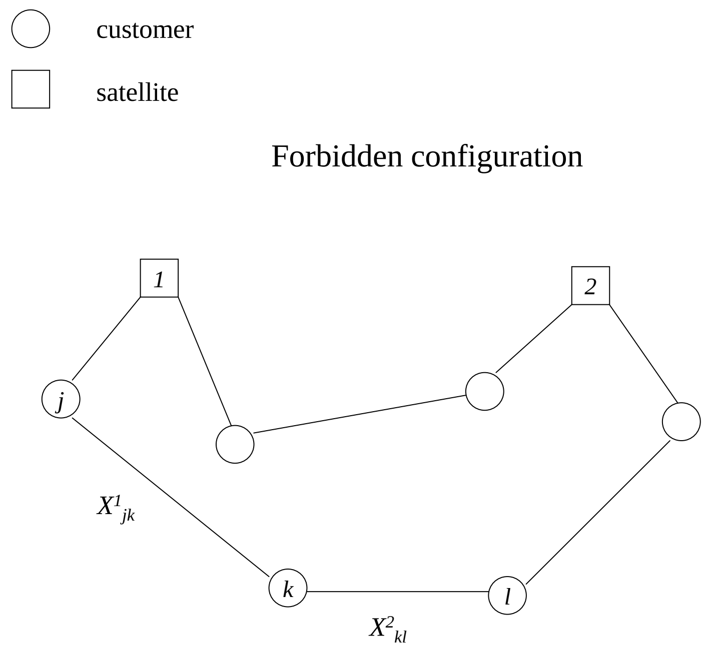

Figure 2.4: Forbidden configuration.

that for a triple of nodes j, k, l, the sum on the satellites belonging to I_1, of the edges going from j to k, plus the sum over the satellites belonging to I_2 of the edges going from k to l, has to be less than or equal to one. This must hold for all the possible partitions of the set P into two subsets. In other words they avoid routes in which for a generic node k we have an incident vertex to k belonging to a satellite and another one belonging to a different satellite.

2.3 Computational Results

In this section the performance of the two formulations in terms of solution quality and computational efficiency is analyzed. The 2E-LRP problems are weel known in literature, but the Single Vehicle 2E-LRP is introduced in this thesis for the first time. Four sets of instances adapted from a set in literature called Nguyen (introduced for the 2E-LRP by Nguyen *et al.* [45] and available at http://prodhonc.free.fr/) are used in this chapter. Since the size of the Nguyen set ranges from 5 satellites and 25 customers to 10 satellites and 200 customers respectively, in order to obtain instances with smaller sizes, we considered only the first n satellites and m customers, where in each instance n and m are the sizes of satellites and customers respectively. Moreover, for every set, we considered the satellite capacities of the Nguyen set as capacities of our vehicles. We do not use the capacities of the second fleet vehicles of the Nguyen set because they are too small and they would make infeasible our instances. The first three sets contain small (3 satellites and 6 customers), medium (4 satellites and 8 customers) and large (5 satellites and 10 customers) instances. They are called *Cu_s*, *Cu_m* and *Cu_l* respectively. They are built adapting the first 24 instances contained in the Nguyen set. The fourth set, composed by the first 18 instances contained in the Nguyen set, covers up to 19 nodes (4 satellites and 15 customers) and are called *Cu*. It is composed by instances of different sizes. The tests reported in this section have been performed on an Intel(R) Xeon (R) CPU W3680, 3.33GHz, 12 GB RAM personal computer. The formulations have been implemented in C++ and solved by means of the CPLEX 12.1 solver, setting a computing time limit of 7200 seconds. The results of the computational experiments are summarized in Tables 2.1-2.4. Each table contains the instance name, the number of satellites and the number of customers in columns 1, 2 and 3. Columns 4, 5, 6 and 7 refer to the symmetric formulation. Columns 4 and 5 contain the lower bounds and the percentage gaps compared with the upper bounds. Columns 6 and 7 contain the percentage gap of the upper bounds compared with the best lower bounds provided among the two formulations, and the computing time expressed in seconds. Finally columns 8, 9 and 10 are referred to the flow based formulation. Column 8 contains the lower bounds, column 9 contains the percentage gaps between the upper bounds and the lower bounds and column 10 contains the computing times in seconds. The instances marked with an asterisk are not optimally solved by the symmetric formulation because of the computing time limit, although the upper bounds provided by the symmetric formulation coincide, unless some very slight variations, with the upper bounds provided by the flow based formulation. The evaluation of the gap for a generic instance is computed with the following expression:

$$GAP(\%) = \frac{upper\ bound - lower\ bound}{lower\ bound} \cdot 100$$

On the first set (small instances) both formulations perform well, solving optimally all the instances. The computing times are very short, all less than 0.5 seconds.

Also all the instances belonging to the second set (medium instances) are optimally solved. Since the size of this set is larger than the previous one,

Table 2.1: Comparison of the two models, small instances

Instance details			Symmetric formulation				Flow based formulation		
Name	satellites	customers	LB	U. B. GAP w.r.t L.B.	GAP w.r.t. BEST L.B	CPU time	LB	U. B. GAP w.r.t L.B.	CPU time
Cu_s_1	3	6	9937.5	0.00%	0.00%	0.343	9937.5	0.00%	0.436
Cu_s_2	3	6	11820.8	0.00%	0.00%	0.327	11820.8	0.00%	0.297
Cu_s_3	3	6	14560.1	0.00%	0.00%	0.343	14560.1	0.00%	0.202
Cu_s_4	3	6	11982.8	0.00%	0.00%	0.327	11982.8	0.00%	0.343
Cu_s_5	3	6	11173.8	0.00%	0.00%	0.390	11173.8	0.00%	0.436
Cu_s_6	3	6	9650.8	0.00%	0.00%	0.296	9650.8	0.00%	0.327
Cu_s_7	3	6	17440.8	0.00%	0.00%	0.280	17440.8	0.00%	0.358
Cu_s_8	3	6	11392.8	0.00%	0.00%	0.343	11392.8	0.00%	0.483
Cu_s_9	3	6	15156.8	0.00%	0.00%	0.327	15156.8	0.00%	0.390
Cu_s_10	3	6	18578.3	0.00%	0.00%	0.343	18578.3	0.00%	0.405
Cu_s_11	3	6	14779.6	0.00%	0.00%	0.265	14779.6	0.00%	0.390
Cu_s_12	3	6	14232	0.00%	0.00%	0.296	14232	0.00%	0.375
Cu_s_13	3	6	9902.7	0.00%	0.00%	0.374	9902.7	0.00%	0.327
Cu_s_14	3	6	7016	0.00%	0.00%	0.265	7016	0.00%	0.265
Cu_s_15	3	6	11649.3	0.00%	0.00%	0.343	11649.3	0.00%	0.249
Cu_s_16	3	6	7134.9	0.00%	0.00%	0.327	7134.9	0.00%	0.312
Cu_s_17	3	6	10794.2	0.00%	0.00%	0.296	10794.2	0.00%	0.312
Cu_s_18	3	6	13344.9	0.00%	0.00%	0.343	13344.9	0.00%	0.343
Cu_s_19	3	6	17562.1	0.00%	0.00%	0.265	17562.1	0.00%	0.343
Cu_s_20	3	6	9749.9	0.00%	0.00%	0.381	17616.9	0.00%	0.218
Cu_s_21	3	6	7929.3	0.00%	0.00%	0.39	7929.3	0.00%	0.249
Cu_s_22	3	6	11511.4	0.00%	0.00%	0.546	11511.4	0.00%	0.421
Cu_s_23	3	6	19155.3	0.00%	0.00%	0.405	19155.3	0.00%	0.218
Cu_s_24	3	6	8568.7	0.00%	0.00%	0.327	8568.7	0.00%	0.202

the number of constraints in the symmetric formulation grows faster than the number of constraints in the flow formulation. This fact is reflected by computing times. The symmetric formulation is rather slower than the other one, even if the worse computing time is about 39 seconds (no too long). The flow based formulation is very fast, presenting computing times no larger than 2.2 seconds.

The third set further underlines the differences between the two formulations both in terms of computing times and solution quality. The symmetric formulation could not optimally solve 7 instances out of a total of 24 because of the computing time limit. However, also for these seven instances, the formulation provides good upper bounds that coincide with the optimal solutions. The flow based formulation solves optimally all the instances with computing times very short (max 13 seconds).

On the fourth set we can notice a considerable difference between the upper and lower bounds provided by the symmetric formulation. The gap is zero up to 15 nodes, whereas it increases dramatically when the number of nodes is larger than or equal to 16. Given the complexity of the formulation and in particular the number of integer variables and constraints involved, it is not surprising that the solver does not provide reasonable lower bounds in

Table 2.2: Comparison of the two models, medium instances

Instance detalis			Asymmetric formulation				Flow based formulation		
Name	satellites	customers	LB	U. B. GAP w.r.t L.B.	GAP w.r.t. BEST L.B	CPU time	LB	U. B. GAP w.r.t L.B.	CPU time
Cu_m_1	4	8	15328.5	0.00%	0.00%	31.138	15328.5	0.00%	1.341
Cu_m_2	4	8	16820.6	0.00%	0.00%	30.591	16820.6	0.00%	0.936
Cu_m_3	4	8	20286.1	0.00%	0.00%	16.036	20286.1	0.00%	0.811
Cu_m_4	4	8	19488	0.00%	0.00%	36.457	19488	0.00%	2.293
Cu_m_5	4	8	11355.2	0.00%	0.00%	23.088	11355.2	0.00%	1.185
Cu_m_6	4	8	9819.4	0.00%	0.00%	18.298	9819.4	0.00%	0.593
Cu_m_7	4	8	20945.1	0.00%	0.00%	23.524	20945.1	0.00%	1.575
Cu_m_8	4	8	17177	0.00%	0.00%	31.450	17177	0.00%	1.950
Cu_m_9	4	8	22231.9	0.00%	0.00%	23.353	22231.9	0.00%	0.686
Cu_m_10	4	8	18743.5	0.00%	0.00%	19.016	18743.5	0.00%	1.123
Cu_m_11	4	8	24194.3	0.00%	0.00%	29.671	24194.3	0.00%	2.184
Cu_m_12	4	8	12001.3	0.00%	0.00%	6.084	12001.3	0.00%	0.687
Cu_m_13	4	8	10206.3	0.00%	0.00%	23.166	10206.3	0.00%	0.717
Cu_m_14	4	8	11548.3	0.00%	0.00%	26.067	11548.3	0.00%	0.624
Cu_m_15	4	8	11778.3	0.00%	0.00%	10.670	11778.3	0.00%	0.234
Cu_m_16	4	8	11815	0.00%	0.00%	37.596	11815	0.00%	2.434
Cu_m_17	4	8	18848.2	0.00%	0.00%	13.369	18848.2	0.00%	1.092
Cu_m_18	4	8	13656.2	0.00%	0.00%	20.046	13656.2	0.00%	0.562
Cu_m_19	4	8	23468.3	0.00%	0.00%	9.11	23468.3	0.00%	1.185
Cu_m_20	4	8	17616.9	0.00%	0.00%	9.11	17616.9	0.00%	0.561
Cu_m_21	4	8	17637.1	0.00%	0.00%	38.719	17637.1	0.00%	0.53
Cu_m_22	4	8	11511.4	0.00%	0.00%	19.387	11511.4	0.00%	0.826
Cu_m_23	4	8	19774.4	0.00%	0.00%	26.41	19774.4	0.00%	0.546
Cu_m_24	4	8	15460	0.00%	0.00%	38.422	15460	0.00%	0.514

Table 2.3: Comparison of the two models, large instances

Instance detalis			Asymmetric formulation				Flow based formulation		
Name	satellites	customers	LB	U. B. GAP w.r.t L.B.	GAP w.r.t. BEST L.B	CPU time	LB	U. B. GAP w.r.t L.B.	CPU time
Cu_l_1	5	10	4250	0.00%	0.00%	397.110	4250	0.00%	0.280
Cu_l_2	5	10	17519.7	0.00%	0.00%	2805.340	17519.7	0.00%	12.027
Cu_l_3	5	10	24361.8	0.00%	0.00%	4369.610	24361.8	0.00%	1.591
Cu_l_4	5	10	19958.1	0.00%	0.00%	3436.830	19958.1	0.00%	13.010
Cu_l_5*	5	10	994.902	1707.94%	0.00%	7200.300	17987.2	0.00%	5.242
Cu_l_6*	5	10	1503.89	815.54%	0.00%	7200.210	13768.7	0.00%	2.402
Cu_l_7	5	10	24789.7	0.00%	0.00%	4038.490	24789.7	0.00%	1.404
Cu_l_8	5	10	16909.9	0.00%	0.00%	5476.940	16909.9	0.00%	9.453
Cu_l_9	5	10	30196.7	0.00%	0.00%	4710.380	30196.7	0.00%	11.341
Cu_l_10	5	10	25640.5	0.00%	0.00%	2189.230	25640.5	0.00%	11.529
Cu_l_11	5	10	24416.9	0.00%	0.00%	2827.930	24416.9	0.00%	1.435
Cu_l_12	5	10	17954.9	0.3%	0.00%	6293.480	17954.9	0.00%	1.887
Cu_l_13	5	10	12038.3	0.0%	0.00%	7047.720	18975.2	0.00%	4.102
Cu_l_14*	5	10	1417.38	1119.9%	0.00%	7200.230	17290.2	0.00%	5.584
Cu_l_15*	5	10	2154.68	780.7%	0.00%	7200.230	18975.2	0.00%	5.444
Cu_l_16*	5	10	2449	387.7%	0.00%	7200.190	11943.9	0.00%	2.434
Cu_l_17	5	10	26716.7	0.0%	0.00%	2993.100	26716.7	0.00%	3.962
Cu_l_18	5	10	17873.6	0.0%	0.00%	7135.380	17873.6	0.00%	5.679
Cu_l_19	5	10	31636.8	0.0%	0.00%	2330.68	31636.8	0.00%	5.818
Cu_l_20	5	10	17616.9	0.0%	0.00%	6978.32	17616.9	0.00%	3.963
Cu_l_21*	5	10	875.756	1938.08%	0.00%	7200.23	17848.6	0.00%	0.951
Cu_l_22	5	10	12128.4	0.00%	0.00%	4356.06	12128.4	0.00%	2.683
Cu_l_23	5	10	19801.2	0.00%	0.00%	5619.76	19801.2	0.00%	0.858
Cu_l_24*	5	10	1967.47	599.10%	0.00%	7200.23	13754.5	0.00%	2.106

Table 2.4: Comparison of the two models, C_u set

Instance details			Symmetric formulation				Flow based formulation		
Name	satellites	customers	LB	U. B. GAP w.r.t L.B.	GAP w.r.t. BEST L.B	CPU time	LB	U. B. GAP w.r.t L.B.	CPU time
Cu_1	3	5	6416.07	0.00%	0.00%	0.27	6416.07	0.00%	0.186
Cu_2	3	7	12411.2	0.00%	0.00%	0.42	12411.2	0.00%	0.362
Cu_3	3	9	15474.3	0.00%	0.00%	10.19	15474.8	0.00%	0.482
Cu_4	3	11	14799.2	0.00%	0.00%	498.96	14799.2	0.00%	0.851
Cu_5*	3	13	1797.48	702.68%	0.00%	7200.18	14428.3	0.00%	0.870
Cu_6*	3	15	2002.41	565.97%	1.08%	7200.55	13192.3	1.08%	4.346
Cu_7	4	5	13174.9	0.00%	0.00%	0.13	13174.9	0.00%	0.277
Cu_8	4	7	11097.1	0.00%	0.00%	0.93	11097.1	0.00%	0.251
Cu_9	4	9	15269.2	0.00%	0.00%	50.86	15268	0.00%	0.639
Cu_10	4	11	13439.8	0.00%	0.00%	1774.91	13439.9	0.00%	1.052
Cu_11*	4	13	2297.65	561.30%	0.10%	7200.22	15179.7	0.10%	2.101
Cu_12*	4	15	2236.23	1021.83%	0.27%	7200.47	25018	0.3%	14.432
Cu_13	5	5	10072.6	0.00%	0.00%	0.19	10072.6	0.0%	0.416
Cu_14	5	7	11494.9	0.00%	0.00%	4.57	11494.9	0.0%	0.595
Cu_15	5	8	11778.9	0.00%	0.00%	14.40	11778.9	0.0%	0.299
Cu_16	5	9	11859.1	0.00%	0.00%	329.61	11859.1	0.0%	1.128
Cu_17	5	10	19107.1	0.00%	0.00%	901.61	19107.1	0.0%	1.339
Cu_18*	5	11	1134.87	1941.60%	0.00%	7200.14	23169.5	0.0%	9.745

2 hours. Despite this fact the asymmetric formulation optimally solves 11 instance out of a total of 18 within the computing time limit. The flow based formulation, instead, optimally solves all the benchmark instances, with very short computing times (at most 14 seconds).

According to the results, the flow based formulation dominates the symmetric formulation. The results of the latter formulation are not satisfactory when the number of nodes increases. It should be interesting to remove the constraints whose number is exponential and introduce cuts within the framework of a Branch and Cut algorithm.

Chapter 3

Heuristic Algorithm

3.1 Description

The problem described in the previous chapter is $\mathcal{NP}$-hard and is solved using a two-phase heuristic algorithm. In the first phase of the algorithm, referred to as *Constructive Phase*, a feasible solution is built by decomposing the problem into sub-problems and by solving each of them in order to generate an initial solution. An obvious way to decompose the problem is to divide it into three sub-problems. The first sub-problem is based on customer-to-satellite assignment decisions, that can be seen as the Variable Cost and Size Bin Packing Problem (*VCSBPP*), see [18], where all customers (items) must be assigned (loaded) to heterogeneous satellites (bins) that can be selected among several types, differing in capacities and fixed costs. The total assignment cost is computed as the sum of the fixed costs of the open satellites. The objective of the problem is to select satellites to pack all the customer requests at minimum total assignment cost. The second sub-problem consists in building the primary tour, joining the central depot and the open satellites. The third sub-problem consists in building the vehicle routes, each of them joining an open satellite and its assigned customers. Building the primary tour and the vehicle tours corresponds to solving several Travelling Salesman Problem (TSP), one for each tour. In the second sub-problem the central depot can be seen as the starting city of the salesman, whereas the satellites can be seen as the cities that the salesman has to visit. In the third sub-problem each satellites can be seen as a starting city of a salesman, whereas the customers assigned to each satellites represent the cities. In the second phase of the algorithm, referred to as *Improvement Phase*, the current solution is improved iteratively by using several local search procedures. A basic two-phase algorithm will be formally described, and then five different variants will be designed, each of one differing from the others on the calculation of the estimate of the cost to assign a customer to a satellite. Summarizing the problem consists of:

- assigning each customer $j \in J$ to a satellite. The subset $J_s \subseteq J$ of customers assigned to each satellite s must satisfy the capacity constraint, i.e. $Q_s \leq w_s$, the union of the customers assigned to each satellites must be equal to set J and each customer j must be served only by a single satellite $\cup_{p \in P} J_p = J$, and $J_p \cap J_s = \emptyset$. Let $S = \{p \in P \;\; : \;\; J_p \neq \emptyset\}$ be the set of open satellites, and $P \backslash S$ the set of closed satellites;
- designing a vehicle tour $\mathcal{T}_s$ connecting $\{s\} \cup J_s$, for each $s \in S$ (the tours of the closed satellites are empty), and
- designing a primary tour $\mathcal{T}$ for the uncapacitated vehicle connecting the satellites in $\{0\} \cup S$,

so that the total cost, including the fixed costs of the open satellites, the primary tour cost, and the vehicle tour costs, is minimized.

Let $\sigma(S, \mathcal{T}, J_p's, \mathcal{T}_p's, Q_p's)$ be a feasible solution of the problem, including the set S of open satellites, the primary tour $\mathcal{T}$, and for each $p \in S$ the subset J_p of customers assigned to each satellite p, the vehicle tour $\mathcal{T}_p$ and the quantity Q_p of goods loaded in the vehicle at satellite p. The total fixed cost is $f(S) = \sum_{s \in S} F_s$, the cost of primary tour $\mathcal{T}$ is represented by $f(\mathcal{T}) = \sum_{(i,j) \in \mathcal{T}} c_{ij}$, (the notation $(i,j) \in \mathcal{T}$ indicates that $j \in \{0\} \cup S$ immediately follows the satellite $i \in \{0\} \cup S$ in the tour $\mathcal{T}$), and the cost of each vehicle tour $\mathcal{T}_s$ is $f(\mathcal{T}_s) = \sum_{(i,j) \in \mathcal{T}_s} c_{ij}$ (again, $(i,j) \in \mathcal{T}_s$ indicates that $j \in \{s\} \cup J_s$ immediately follows $i \in \{s\} \cup J_s$ in the tour $\mathcal{T}_s$).

The *total cost* of the solution σ is given by

$$f(\sigma) = f(S) + f(\mathcal{T}) + \sum_{s \in S} f(\mathcal{T}_s). \tag{3.1}$$

In the following we report a general outline of the algorithm with a very short description of the local search procedures.

- Constructive phase

Step 1. Rank the customers in non increasing order of their demand.

Step 2. For each customer $j = 1, 2..., n$

- ∗ Implement a procedure called *assign* to evaluate the insertion cost of j in vehicle tour $\mathcal{T}_s$;
- ∗ Implement a procedure called *insert* to insert j in the tour $\mathcal{T}_s$;

- *if* s is selected for the first time then
    · Use *insert* to insert s in the primary tour $\mathcal{T}$.

- Improvement phase

Step 3. Use procedure RC (Removing a Customer) to remove a customer from a route and insert it into another one with sufficient capacity;

Step 4. Use procedure STC (Swap two Customers) to swap two customers belonging to two different tours;

Step 5. Use procedure EOS (Eliminate an Open Satellite) to eliminate an open satellite and reassign its customers to another satellite;

Step 6. Use procedure STOS (Swap Two Open Satellites) to swap the customers belonging to two different satellites.

Step 7. Use procedure SOCS (Swap an Open with a Closed Satellite) to swap an open satellite with a closed satellite with sufficient capacity.

3.1.1 Constructive Phase

Once the customers are ranked in non-increasing order of their requests, a feasible solution of the problem is built in the following way. At each iteration a partial solution is available. When the customer $j \in J$ is considered, it is assigned to the satellite $s \in P$ at which corresponds the minimum assignment cost Δ_{jp}. Then customer j is inserted into the vehicle tour $\mathcal{T}_s$ and the satellite s is inserted into the primary tour $\mathcal{T}$ if necessary (i.e if s is selected for the first time).

At the end of the iterative process, a feasible solution $\sigma(S, \mathcal{T}, J_p's, \mathcal{T}_p's, Q_p's)$ is obtained.

Constructive Phase

***Step* 0.** Consider the set J sorted in non-increasing order of customer demands $q_j's$, and the set P. Set the primary tour $\mathcal{T} = \emptyset$, the set of open satellite $S = \emptyset$ and, for each satellite $p \in P$, set $J_p = \emptyset$, $\mathcal{T}_p = \emptyset$, and $Q_p = 0$.

***Step* 1.** For each customer $j = 1, 2, ..., n$:

- for each $p \in P$ compute an estimate of the assignment cost, Δ_{jp};
- select the satellite $s = arg\ min_{p \in P}\{\Delta_{jp}\}$;
- insert j in $\mathcal{T}_s$, set $Q_s = Q_s + q_j$ and $J_s = J_s \cup \{j\}$;

- if $s \notin S$ then insert s in $\mathcal{T}$ and set $S = S \cup \{s\}$.

***Step* 2.** Return $\sigma(S, \mathcal{T}, J'_p s, \mathcal{T}'_p s, Q'_p s)$ and $f(\sigma) = f(S) + f(\mathcal{T}) + \sum_{s \in S} f(\mathcal{T}_s)$.

3.1.2 Improvement Phase

The local improvement procedure helps us to improve a given solution by exploring solutions in its neighborhood. We can classify them as:

a intra-route local improvement heuristics (equivalently, a tour improvement);

b inter-route local improvement heuristics (equivalently, an improvement through the change of the assignment of the customers).

The intra-route local improvement heuristics focus on the improvement of tours without involving changes in the assignment of customers to satellites. For example any local exchange heuristic proposed for a TSP belongs to this category (λ-Opt exchange operator, or Or-Opt operator and so on). In this algorithm we use the 2-Opt exchange operator which replaces a set of two edges in a route by another set of two edges.

Any local exchange heuristic involving two tours for the classical VRP belongs to the inter-route category. In the following we describe accurately five local search procedures which aim at moving one or more customers from a vehicle tour to another one. Obviously, in this case, the assignment of customers to satellites is changed if an improvement in the solution value is achieved.

The first improvement procedure, referred to as Eliminating an Open Satellite (EOS), iteratively eliminates the satellite $p \in S$ if $\sum_{j \in S \setminus \{p\}} (w_j - Q_j) > Q_p$, and re-assigns each customer $j \in J_p$ to the remaining open satellites if the capacity constraints are not violated. If a new feasible solution is obtained and an advantage has been identified then the new solution becomes the incumbent one. In this case the number of open satellites decreases.

Procedure EOS

***Step* 0.** Consider the current solution $\sigma(S, \mathcal{T}, J'_p s, \mathcal{T}'_p s, Q'_p s)$ and its cost $f(\sigma) = f(S) + f(\mathcal{T}) + \sum_{s \in S} f(\mathcal{T}_s)$.

***Step* 1.** Set $\mathcal{S} = \{p : p \in S \textit{ and } \sum_{j \in S \setminus \{p\}} (w_j - Q_j) > Q_p\}$.

***Step* 2.** If $\mathcal{S} = \emptyset$ go to Step 3 else choose an open satellite p from $\mathcal{S}$:

- set $S' = S \backslash \{p\}$, $\mathcal{T}'_p = \emptyset$, $J'_p = \emptyset$, $Q'_p = 0$. For $k \in S'$, set $\mathcal{T}'_k = \mathcal{T}_k$, $J'_k = J_k$ and $Q'_k = Q_k$. Let $\mathcal{T}'$ be the primary tour obtained by removing p from $\mathcal{T}$;
- for each $j \in J_p$:
  a. set $S'' = \{k : k \in S' \; and \; Q_k + p_j \leq w_k\}$, if $S'' = \emptyset$ go to Step 2,
  b. for each $k \in S''$ compute an estimate of the assignment cost, Δ_{jk},
  c. select the satellite $s = arg \; min_{k \in S''}\{\Delta_{jk}\}$,
  d. insert j in $\mathcal{T}'_s$, and set $Q'_s = Q'_s + q_j$ and $J'_s = J'_s \cup \{j\}$;

  -if $f(\sigma') < f(\sigma)$ then update σ by setting $S = S'$, $\mathcal{T} = \mathcal{T}'$, $\mathcal{T}_p = \emptyset$, $J_p = \emptyset$, $Q_p = 0$, and for $k \in S$, set $\mathcal{T}_k = \mathcal{T}'_k$, $Q_k = Q'_k$ and $J_k = J'_k$, and go to Step 1 else go to Step 2.

***Step* 3.** Return $\sigma(S, \mathcal{T}, J'_p s, \mathcal{T}'_p s, Q'_p s)$ and $f(\sigma) = f(S) + f(\mathcal{T}) + \sum_{s \in S} f(\mathcal{T}_s)$.

The second procedure, referred as Swapping Two Open Satellites ($STOS$), iteratively swaps two open satellites and reverses their vehicle routes if the capacity constraints are not violated. If an advantage has been identified, then the new solution is the incumbent. In this case the number of open satellites does not change.

Procedure STOS

***Step* 0.** Consider the current solution $\sigma(S, \mathcal{T}, J'_p s, \mathcal{T}'_p s, Q'_p s)$ and $f(\sigma) = f(S) + f(\mathcal{T}) + \sum_{s \in S} f(\mathcal{T}_s)$.

***Step* 1.** For each $p \in S$:

- for each $s \in S \backslash \{p\}$ such that $Q_p \leq w_s$ and $Q_s \leq w_p$:
  a. let $\mathcal{T}'_p$ be the vehicle tour obtained by removing p from $\mathcal{T}_p$,
  b. let $\mathcal{T}'_s$ be the vehicle tour obtained by removing s from $\mathcal{T}_s$,
  c. insert s in $\mathcal{T}'_p$,
  d. insert p in $\mathcal{T}'_s$,
  e. if $f(\mathcal{T}'_s) + f(\mathcal{T}'_p) < f(\mathcal{T}_s) + f(\mathcal{T}_p)$ then set $\mathcal{T}_s = \mathcal{T}'_p$, $\mathcal{T}_p = \mathcal{T}'_s$, $Q_s = Q_p$, $Q_p = Q_s$, $J_s = J_p$, and $J_p = J_s$ and repeat Step 1.

***Step* 2.** Return $\sigma(S, \mathcal{T}, J'_p s, \mathcal{T}'_p s, Q'_p s)$ and $f(\sigma) = f(S) + f(\mathcal{T}) + \sum_{s \in S} f(\mathcal{T}_s)$.

The third procedure, referred as Swapping an Open with a Closed Satellite ($SOCS$), iteratively considers an open satellite p, reverses the vehicle tour $\mathcal{T}_s$ to a closed satellite if the capacity constraint is verified. Once all closed satellite have been examined, the closed satellite that yields the smallest cost is considered to be open and the open satellite p is closed if an advantage has been identified. In this case the number of open satellites does not change.

Procedure SOCS

***Step* 0.** Consider the current solution $\sigma(S, \mathcal{T}, J_p's, \mathcal{T}_p's, Q_p's)$ and $f(\sigma) = f(S) + f(\mathcal{T}) + \sum_{s \in S} f(\mathcal{T}_s)$.

***Step* 1.** For each $p \in S$:

- set $CS = f(\mathcal{T}_p) + f(\mathcal{T}) + F_p$;
- for each $s \in P \backslash S$ such that $Q_p \leq w_s$:
  a. let $\mathcal{T}'_p$ be the vehicle tour obtained by removing p from $\mathcal{T}_p$,
  b. let $\mathcal{T}'$ be the primary tour obtained by removing p from $\mathcal{T}$,
  c. insert s in $\mathcal{T}'_p$,
  d. insert s in $\mathcal{T}'$,
  e. if $f(\mathcal{T}'_p) + f(\mathcal{T}') + F_s < CS$ then set $s^* = s$, $CS = f(\mathcal{T}'_p) + f(\mathcal{T}') + F_s$, $\mathcal{T}'_{s^*} = \mathcal{T}'_p$ and $\mathcal{T}^* = \mathcal{T}'$;
- if $(CS < f(\mathcal{T}_p) + f(\mathcal{T}) + F_p)$ then set $\mathcal{T} = \mathcal{T}^*$, $S = S \cup \{s^*\} \backslash \{p\}$, $Q_{s^*} = Q_p$, $\mathcal{T}_{s^*} = \mathcal{T}'_{s^*}$, $J_{s^*} = J_p$, $Q_p = 0$, $\mathcal{T}_p = \emptyset$, and $J_p = \emptyset$ and repeat Step 1.

***Step* 2.** Return $\sigma(S, \mathcal{T}, J_p's, \mathcal{T}_p's, Q_p's)$ and $f(\sigma) = f(S) + f(\mathcal{T}) + \sum_{s \in S} f(\mathcal{T}_s)$.

The fourth procedure, referred as Removing a Customer (RC), iteratively removes a customer from an open tour and re-assigns it to another open tour with a sufficient remaining capacity. If an advantage has been identified, then the new solution is the incumbent.

Procedure RC

***Step* 0.** Consider the current solution $\sigma(S, \mathcal{T}, J_p's, \mathcal{T}_p's, Q_p's)$ and $f(\sigma) = f(S) + f(\mathcal{T}) + \sum_{s \in S} f(\mathcal{T}_s)$.

***Step* 1.** For each $p \in S$:

- for each $j \in J_p$:

a. let $\mathcal{T}'_p$ be the vehicle tour obtained removing j from $\mathcal{T}_p$, and set $Q'_p = Q_p - q_j$ and $J'_p = J_p \setminus \{j\}$,
b. if $\mathcal{T}'_p = \emptyset$ then let $\mathcal{T}'$ be the primary tour obtained by removing p from $\mathcal{T}$ and set $S' = S \setminus \{p\}$ else set $\mathcal{T}' = \mathcal{T}$ and $S' = S$,
c. for each $k \in S$ compute an estimate of the assignment cost, Δ_{jk},
d. select the satellite $s = arg\ min_{k \in S}\{\Delta_{jk}\}$,
e. insert j in $\mathcal{T}'_s$, and set $Q'_s = Q_s + q_j$ and $J'_s = J_s \cup \{j\}$,
f. if $f(\mathcal{T}'_p)+f(\mathcal{T}'_s)+f(\mathcal{T}')+f(S') < f(\mathcal{T}_p)+f(\mathcal{T}_s)+f(\mathcal{T})+f(S)$ then set $Q_s = Q_s + q_j$, $Q_p = Q_p - q_j$, $J_s = J_s \cup \{j\}$, and $J_p = J_p \setminus \{j\}$, compute $f(\sigma) = f(S) + f(\mathcal{T}) + \sum_{s \in S} f(\mathcal{T}_s)$ and return to Step 1.

***Step* 2.** Return $\sigma(S, \mathcal{T}, J'_p s, \mathcal{T}'_p s, Q'_p s)$ and $f(\sigma) = f(S) + f(\mathcal{T}) + \sum_{s \in S} f(\mathcal{T}_s)$.

The fifth procedure, referred as Swapping Two Customers (STC), iteratively swaps two customers from different open vehicle tours if their capacity constraints are verified. If an advantage has been identified the new solution becomes the incumbent.

Procedure STC

***Step* 0.** Consider the current solution $\sigma(S, \mathcal{T}, J'_p s, \mathcal{T}'_p s, Q'_p s)$ and $f(\sigma) = f(S) + f(\mathcal{T}) + \sum_{s \in S} f(\mathcal{T}_s)$.

***Step* 1.** For each $p \in S$:

- for each $j \in J_p$:
  - for each $s \in S \setminus \{p\}$:
    - for each $i \in J_s$ such that $Q_s + q_j - q_i \leq w_s$ and $Q_p - q_j + q_i \leq w_p$:

      a. let $\mathcal{T}'_p$ be the primary tour obtained removing j from $\mathcal{T}_p$, set $Q'_p = Q'_p - q_j$ and $J'_p = J'_p \setminus \{j\}$,
      b. let $\mathcal{T}'_s$ be the primary tour obtained removing i from $\mathcal{T}_s$, set $Q'_s = Q'_s - q_i$ and $J'_s = J'_s \setminus \{i\}$,
      c. insert j in $\mathcal{T}'_s$, set $Q'_s = Q'_s + q_j$ and $J'_s = J'_s \cup \{j\}$,
      d. insert i in $\mathcal{T}'_p$, set $Q'_p = Q'_p + q_i$ and $J'_p = J'_p \cup \{i\}$,
      e. if $f(\mathcal{T}'_p) + f(\mathcal{T}'_s) < f(\mathcal{T}_p) + f(\mathcal{T}_s)$ then set $f(\mathcal{T}_p) = f(\mathcal{T}'_p)$, $f(\mathcal{T}_s) = f(\mathcal{T}'_s)$, $Q_s = Q'_s$, $J_s = J'_s$, $\mathcal{T}_s = \mathcal{T}'_s$, $Q_p = Q'_p$, $J_p = J'_p$, and $\mathcal{T}_p = \mathcal{T}'_p$, compute $f(\sigma) = f(S) + f(\mathcal{T}) + \sum_{s \in S} f(\mathcal{T}_s)$ and repeat Step 1.

***Step* 2.** Return $\sigma(S, \mathcal{T}, J'_p s, \mathcal{T}'_p s, Q'_p s)$ and $f(\sigma) = f(S) + f(\mathcal{T}) + \sum_{s \in S} f(\mathcal{T}_s)$.

Great importance is given to the order in which the improvement procedures are performed. Based on pilot experiments, the improvement Phase is organized as follows:

Improvement Phase: Apply *RC*$\rightarrow$ *STC*$\rightarrow$ *EOS* $\rightarrow$ *STOS*$\rightarrow$ *SOCS*.

3.1.3 Implementing the Basic Algorithm

The rule used to insert a center (customer or satellite) into a tour (primary or vehicle tour), the rule to remove a center from a tour, and the rule to estimate the cost to assign a customer to a satellite are critical to the success of the heuristic algorithm. In the implementation of the heuristic, the procedure *Insert* is used to insert a center in a tour, and the procedure *Remove* is used to remove a center from a tour. Moreover by specifying how to estimate the cost to assign a customer to a satellite, five different variants (A1, A1B, A2, A2B and A3) are designed in this chapter. The first variant to estimate the cost to assign a customer to a satellite uses the procedure *Assign1*, the second uses the procedure *Assign1B*, the third variant uses *Assign2*, the fourth uses *Assign2B* and the fifth uses *Assign3*.

Inserting a center into a tour

The following procedure is used during the algorithm to insert the center e into the tour $\mathcal{X}$, see [55]. In what follows, once a visit direction on the current tour $\mathcal{X}$ has been chosen, k_+ and k_- represent, respectively, the center that follows and that precedes the center k in $\mathcal{X}$.

Procedure Insert

- If $\mathcal{X} = \emptyset$ then $\mathcal{X} = \{x, e, x\}$, where x represents a satellite, in particular $x = 0$ if $\mathcal{X} = \mathcal{T}$ or $x = s$ if $\mathcal{X} = \mathcal{T}_s$. The insertion cost is $ic(e, \mathcal{X}) = 2c_{xe}$.
- else
    - delete arc $(u, u_+) = argmin(c_{ie} + c_{ei_+} - c_{ii_+})$ and insert arcs (u, e) and (e, u_+).

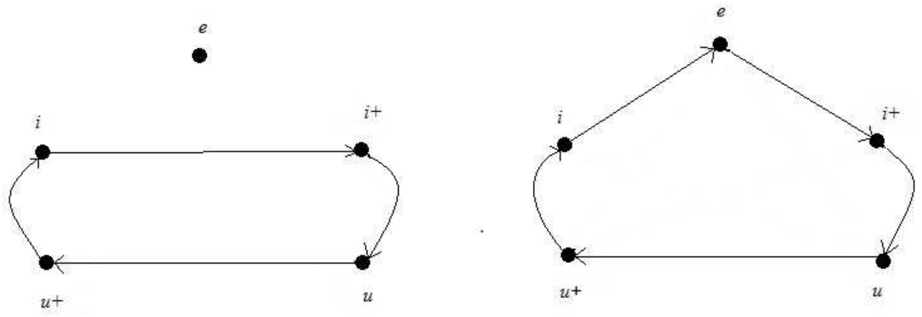

Figure 3.1: Insertion of e between the customers i and i_+.

Removing a center from a tour

The following procedure is used to remove a center e from a given tour $\mathcal{X}$. When e is removed from $\mathcal{X}$, its incident arcs are deleted. Then, one at a time the remaining arcs are deleted from $\mathcal{X}$ and the two resulting chains are reconnected to form a new tour, as in the two arc interchange algorithm (see [37]). Once all arcs have been examined, the reconnection that yields the shortest tour is considered to be the new current tour. The cost of removing the city e from tour $\mathcal{X}$ is equal to the sum of the travel costs of the arcs that are inserted minus the sum of the travel costs of the arcs that are deleted from $\mathcal{X}$ in order to obtain the tour.

Procedure Remove

- If $\mathcal{X}$ consists of only the two arcs incident to e then set $\mathcal{X} = \emptyset$.
- If $\mathcal{X}$ consists of more than two arcs, then remove the arcs (e_-, e) and (e, e_+) and arc $(l^*, l^*_+) = arg\, min_{l \in \mathcal{X}, l \neq s}\{c_{e_-l} + c_{e_+l_+} - c_{e_-e} - c_{e_+e} - c_{ll_+}\}$. Introduce arcs (e_-, l), (e_+, l_+), and invert the orientation of the path between the nodes l_+ and e_+.

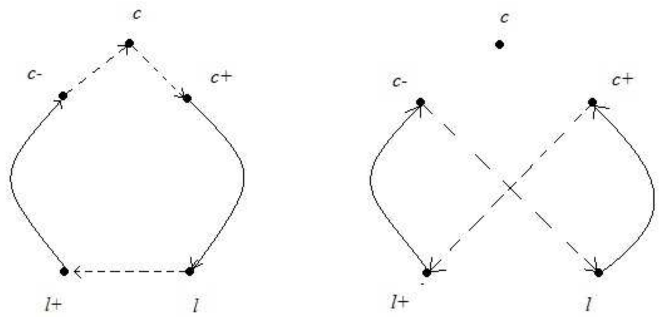

Figure 3.2: Removing a center e from a tour

Estimating the assignment cost

Five different procedures, *Assign1*, *Assign1B*, *Assign2*,*Assign2B* and *Assign3*, are used to estimate the cost to assign customer $j \in J$ to satellite $p \in P$.

Assign1 estimates the assignment cost Δ_{jp}, of customer $j \in J$ to satellite $p \in P$ by considering only the ratio F_p/w_p, that represents the fixed cost per unit of freight, and by ignoring the insertion cost of customer j into the vehicle tour $\mathcal{T}_p$, and the insertion cost of p into the primary tour $\mathcal{T}$.

Procedure Assign1

$$\Delta_{jp} = \begin{cases} F_p/w_p & \mathcal{T}_p = \emptyset, \text{ and } Q_p + q_j \leq w_p \\ 0 & \mathcal{T}_p \neq \emptyset, \text{ and } Q_p + q_j \leq w_p \\ \infty & \text{if } Q_p + q_j > w_p \end{cases}$$

Assign2 estimates the assignment cost Δ_{jp}, of customer $j \in J$ to each satellite $p \in P$ by considering the ratio F_p/w_p, the insertion cost $ic(i, \mathcal{T}_p)$ of customer j into the vehicle tour $\mathcal{T}_p$, and the insertion cost $ic(p, \mathcal{T})$ of p into the primary tour $\mathcal{T}$ if p is not used.

Procedure Assign2

$$\Delta_{jp} = \begin{cases} F_p/w_p + ic(p,\mathcal{T}) + ic(j,\mathcal{T}_p), & \text{if } \mathcal{T}_p = \emptyset, \text{ and } Q_p + q_j \leq w_p \\ ic(j,\mathcal{T}_p) & \text{if } \mathcal{T}_p \neq \emptyset, \text{ and } Q_p + q_j \leq w_p \\ \infty, & \text{if } Q_p + q_j > w_p. \end{cases}$$

Assign3 estimates the assignment cost Δ_{jp}, of customer $j \in J$ to each satellite $p \in P$ by considering the insertion cost $ic(p,\mathcal{T})$ of p in the primary tour $\mathcal{T}$ and the insertion cost $ic(j,\mathcal{T}_p)$ of customer j in the vehicle tour $\mathcal{T}_s$ if p is not used, and by ignoring the fixed cost.

Procedure Assign3

$$\Delta_{jp} = \begin{cases} ic(p,\mathcal{T}) + ic(j,\mathcal{T}_p) & \text{if } \mathcal{T}_p = \emptyset, \text{ and } Q_p + q_j \leq w_p \\ ic(j,\mathcal{T}_p) & \text{if } \mathcal{T}_p \neq \emptyset, \text{ and } Q_p + q_j \leq w_p \\ \infty, & \text{if } Q_p + q_j > w_p. \end{cases}$$

Procedure $A1B$ and $A2B$ differ from $A1$ and $A2$ respectively because they consider only the fixed cost F_p instead of the ratio F_p/w_p.

3.2 Computational Results

The heuristic described in previous section has been implemented in Fortran and run on an Intel Core (TM) $i5-2400$ CPU, 3.10 GHr, 6 GB RAM personal computer. The mathematical formulation has been implemented in C++ using ILOG Concert Technology 2.3 and CPLEX 12.1 and run on an Intel(R) Xeon (R) CPU W3680, 3.33GHz, 12 GB RAM personal computer. The goals of the experiments are:

- to obtain optimal solutions for small instances using the flow formulation;
- to test the effectiveness of the heuristic algorithm;
- to compare the different assignment procedures of the heuristic algorithm.

We tested our methods adapting the instances used by Nguyen *et al* in [45], containing only one depot at the first level (as in our problem). The first set, called "Prodhon", contains 30 instances with 20-200 customers and 5-10 satellites. The second set, called "Nguyen", contains 24 instances with 25-200 customers and 5-10 satellites. From these instances we built three sets, by changing only the vehicle capacities. The first one, called Ca, is obtained taking the first four instances of the Prodhon set and the first four instances of the Nguyen set (i.e., the instances with the smaller number of nodes). We used the capacity of the second echelon fleet of vehicles in Prodhon and Nguyen as the capacity of our vehicles. In order to differentiate the instance set, we created 16 more instances by decreasing and then increasing the vehicle capacities of each instance by 5%, obtaining in total $8 \cdot 3 = 24$ different instances. On set Ca we tested 5 variants of our algorithm and compared the results provided by them with the optimal solutions (or in some case with the upper bounds) provided by CPLEX. The second set, called Na, is made by adapting the Nguyen set. Since the capacities of the second echelon fleet in the Nguyen set led to infeasibility and the capacities of the first echelon fleet and of the satellites resulted not binding, we built the capacities of our second echelon vehicles multiplying the capacity of the satellites in the Nguyen set for a ratio. This ratio is obtained from the Nguyen set dividing the sum of the second echelon vehicle capacities by the sum of the satellites capacities. Also in this case, in order to differentiate the instances we multiplied the capacity obtained in this way by 5, 6 and 7, obtaining $24 \cdot 3 = 72$ instances. The third set, called Pa, is obtained considering the Prodhon set and it is made in the same way as Na (for the same reasons). The set contains $30 \cdot 3 = 90$ instances. Set Na and Pa were used only to compare the variants of the algorithm.

3.2.1 Comparison with CPLEX

Table 3.1 presents the results obtained on instance set Ca. The sizes of the instances ranges from 20 to 25 customers and 5 satellites. The first column shows the name of the instance. Instances 1-8, 9-16 and 17-24 are with vehicle capacities multiplied by 1, 2 and 3 respectively. The second and the third columns show the size of the instances. The following five columns report the results achieved by the five variants of the heuristic. The last column reports the results of the model implemented in CPLEX. The relative errors committed by the five variants, compared with CPLEX, are shown in columns 4-8 in the left section, whereas the computing times expressed in seconds appear in the right section. The average relative errors, the average computing times, the maximum errors, the number of optimal solutions and the number of best solutions found by each variant of the heuristic are reported at the bottom of the columns. A few instances could not be optimally solved. These instances are marked with an asterisk and report the upper bound instead of the optimal value. In the column on the customer size, we can observe that the instances not optimally solved have 25 customers, whereas the instances with 20 customers are all optimally solved. Table 3.1 allows us to comment on the performance of the heuristic. We can observe that the best variant of the heuristic performs as good as the results of CPLEX out of 9 instances on 24 and does not fail more than 0.25% on average. The average error of the five variants is 1.79%, 1.79%, 0.78%, 1.79% and 0.68% respectively. The

Table 3.1: Relative errors

			A1		A1B		A2		A2B		A3		CPLEX	
Problem	m	n	e%	t	e%	t	e%	t	e%	t	e%	t	z	t
Ca_1	5	20	.05	.05	.05	.05	.10	.05	.03	.05	.07	.05	44354.30	23.15
Ca_2	5	20	.14	.03	.14	.03	.00	.03	.20	.03	.00	.03	27900.30	17.93
Ca_3	5	20	.00	.05	.00	.05	.00	.05	.00	.05	.00	.05	6731.90	538.96
Ca_4	5	20	.03	.03	.03	.03	.09	.03	.00	.03	.09	.03	23253.20	17.88
*Ca_5	5	25	1.13	.04	1.13	.04	.39	.04	1.59	.04	.39	.04	21710.00	7811.00
Ca_6	5	25	1.47	.03	1.47	.03	5.54	.03	5.54	.03	.00	.03	18630.20	532.58
Ca_7	5	25	6.20	.04	6.20	.04	1.90	.04	1.47	.04	1.90	.04	25483.50	6056.10
*Ca_8	5	25	.87	.03	.87	.03	1.56	.03	1.76	.03	1.56	.03	17895.00	8839.00
Ca_9	5	20	.10	.05	.10	.05	.10	.05	.06	.05	.06	.05	44354.00	701.00
Ca_10	5	20	.15	.03	.15	.03	.00	.03	.00	.03	.00	.03	27900.30	15.28
Ca_11	5	20	.00	.05	.00	.05	.01	.05	.19	.05	.00	.05	6731.90	313.14
Ca_12	5	20	.00	.03	.00	.03	.14	.03	.14	.03	.14	.03	23258.20	31.56
*Ca_13	5	25	-5.98	.04	-5.98	.04	-6.83	.04	-6.63	.04	-6.83	.04	23301.00	559.00
Ca_14	5	25	1.56	.03	1.56	.03	.78	.03	.78	.03	6.00	.03	18630.20	303.00
Ca_15	5	25	23.32	.05	23.32	.05	6.14	.05	4.26	.05	6.14	.05	26101.00	13252.00
Ca_16	5	25	1.93	.03	1.93	.03	2.38	.03	1.65	.03	2.38	.03	17949.10	3537.75
Ca_17	5	20	.07	.05	.07	.05	.00	.05	.00	.05	.06	.05	44344.60	557.27
Ca_18	5	20	.48	.02	.48	.02	.48	.02	.43	.02	.47	.02	15824.80	11.39
Ca_19	5	20	.02	.05	.02	.05	.09	.05	.09	.05	.02	.05	46727.30	638.92
Ca_20	5	20	.04	.02	.04	.02	.04	.02	.04	.02	.02	.04	14208.30	14.88
*Ca_21	5	25	2.29	.04	2.29	.04	1.82	.04	3.75	.04	1.82	.04	21295.00	2611.00
Ca_22	5	25	4.95	.03	4.95	.03	.78	.03	.78	.03	3.39	.03	18630.20	537.23
Ca_23	5	25	2.59	.04	2.59	.04	.00	.04	.00	.04	.00	.04	25357.40	4014.11
Ca_24	5	25	1.53	.03	1.53	.03	.95	.03	.15	.03	.95	.03	17804.00	860.76
average			1.79	0.4	1.79	0.4	0.78	0.4	1.79	0.4	0.68 .	0.4	27432.32	2158.12
max e%			23.32		23.32		6.14		4.26		6.14			
n. opt. sol.			3		3		5		5		6			
n. best sol.			4		4		6		5		7			

average computing time required by each variant of the heuristic is very short (0.4 seconds) compared with CPLEX (2158.12 seconds). Analyzing in deep the solutions of the instance presenting the larger gaps, we noticed that these gaps are due to the assignment of customers to the satellites, and consequently to the routes performed by each vehicle. Conversely the satellites open by the heuristic variants coincide with the satellites open by CPLEX. The same thing holds for the primary tours, that are identical both in the solutions provided by CPLEX and in the solutions provided by the five variants of the heuristic. However, in general, we can observe a satisfactory behaviour of the heuristic algorithm, which provides good solutions in very short times.

3.2.2 Comparison of the Five Variants

The way in which the assignment cost is estimated influences the performance of the heuristic algorithm. It is interesting to compare the different variants in order to establish which one performs better.

Table 3.2 is referred to the Na instances set and shows, for each variant, the relative error committed compared to the best variant, and the computing times. The results provided by the best variant are reported in the last column of the table. At the bottom of the table the average relative errors, the average computing times, the number of best solutions (i.e. the number of solutions for which a variant performs better if compared with the others) and the maximum error committed by each variant are reported.

The size of this set of instances ranges from 25 to 200 customers and from 5 to 10 satellites. Instances 1-24, 25-48 and 49-72 are instances with capacities multiplied by 5, 6 and 7 respectively. The results show that the procedure A3 slightly outperforms the other variants yielding 37 best solutions out of 72 instances and presenting the smallest average objective function value, the second shortest average computing time and the smallest average relative error. Also in this case we analyzed the solutions of the instances presenting a large gap. For example variants A1, A2 and A2B present a large gap on the instance Na_34 compared with the variants A3 and A2. This gap is due to the fact that in the solutions provided by A1, A1B and A2B one more satellite is selected compared with the unique satellite selected by procedures A2 and A3. In the instance Na_44, although the number of selected satellites is the same for each variant, we noticed a gap equal to about 12% between the variants A1, A1B and A3 compared with A2B (the best). In this case, the customer assignment changes variant by variant and the local search procedures cannot perform moves capable to obtain better results. In particular since the procedure $STOS$ cannot swap an open satellite with a closed satellite when the capacity of the latter one is not large enough, there are solutions in which the appropriate satellites are not used. Allowing the swap of two satellites without considering the capacity constraints, and trying to restore the feasibility with designed ad hoc procedures could represent a remedy to this issue. The same considerations can be extended to the other instances reporting considerable gaps between the best variant and the others ones.

Table 3.2: Na Inst.: results

			A1		**A1B**		**A2**		**A2B**		**A3**		A_{best}
Problem	m	n	s	t	s	t	s	t	s	t	s	t	s
Na_1	5	25	0.00	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.01	7319.6
Na_2	5	25	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	8321.3
Na_3	5	25	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.00	8337.8
Na_4	5	25	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	7664.4
Na_5	5	50	0.00	0.09	0.00	0.09	0.20	0.18	0.20	0.19	0.20	0.18	15567.9
Na_6	5	50	0.67	0.19	0.67	0.18	0.00	0.16	0.00	0.17	0.00	0.16	12699.8
Na_7	5	50	0.98	0.19	0.98	0.19	24.94	0.16	0.00	0.33	24.94	0.16	13666.8
Na_8	5	50	0.00	0.09	0.00	0.08	0.54	0.10	0.00	0.08	0.54	0.10	13466.8
Na_9	10	50	2.56	0.23	2.56	0.23	0.00	0.09	2.48	0.27	0.00	0.09	18735.5
Na_10	10	50	1.48	0.10	1.48	0.08	0.00	0.06	1.48	0.09	0.00	0.06	15093.7
Na_11	10	50	1.67	0.12	1.67	0.13	0.00	0.16	1.81	0.17	0.00	0.16	17076.5
Na_12	10	50	0.00	0.19	0.00	0.18	0.31	0.11	1.38	0.17	0.31	0.11	14181.4
Na_13	5	100	0.00	3.22	0.00	3.22	0.86	1.08	1.80	1.11	0.86	1.08	28624.3
Na_14	5	100	4.17	1.80	4.17	1.79	0.00	0.89	1.40	1.60	0.00	0.88	23750.5
Na_15	5	100	0.32	2.53	0.32	2.53	0.43	1.06	1.40	2.02	0.00	3.21	33184.1

Table 3.2: continue

			A1		**A1B**		**A2**		**A2B**		**A3**		A_{best}
Na_16	5	100	2.08	1.27	2.08	1.27	0.00	1.54	2.40	1.38	2.90	1.27	22048
Na_17	10	100	0.00	4.19	0.00	4.18	12.39	1.93	1.10	1.32	13.69	0.79	31635.4
Na_18	10	100	3.59	1.85	3.59	1.85	0.00	1.11	2.96	2.05	0.00	1.10	23982.6
Na_19	10	100	2.43	2.41	2.43	2.40	0.00	2.67	3.64	1.28	0.00	2.65	33456.8
Na_20	10	100	2.95	3.01	2.95	3.00	2.49	1.67	0.00	1.85	2.49	1.67	20843.6
Na_21	10	200	2.71	62.07	2.71	61.88	0.99	54.49	2.51	68.60	0.00	73.95	64026.7
Na_22	10	200	3.56	77.56	3.56	77.32	0.00	88.40	2.48	56.14	0.00	88.26	42233.71
Na_23	10	200	0.33	79.12	0.33	79.27	3.50	34.23	0.00	46.38	3.50	34.22	61944.7
Na_24	10	200	0.09	68.58	0.09	68.78	0.00	43.45	0.05	94.03	0.00	43.32	36322.9
Na_25	5	25	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	7319.6
Na_26	5	25	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.01	0.00	0.01	8321.3
Na_27	5	25	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.00	8337.8
Na_28	5	25	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	7664.4
Na_29	5	50	1.99	0.13	1.99	0.13	0.00	0.10	0.00	0.10	0.00	0.10	15194.3
Na_30	5	50	3.82	0.01	3.82	0.01	0.00	0.01	17.90	0.02	0.00	0.01	10332.21
Na_31	5	50	0.00	0.11	0.00	0.10	1.26	0.12	2.07	0.15	1.26	0.13	13467.6
Na_32	5	50	0.83	0.00	0.83	0.01	0.00	0.01	0.83	0.00	0.00	0.01	8959.9
Na_33	10	50	2.66	0.17	2.66	0.17	0.00	0.08	2.66	0.18	0.00	0.08	18578.8
Na_34	10	50	50.44	0.10	50.44	0.09	0.00	0.01	48.01	0.10	0.00	0.00	10117
Na_35	10	50	3.81	0.10	3.81	0.10	2.99	0.24	0.00	0.15	2.99	0.24	12819.3
Na_36	10	50	0.00	0.02	0.00	0.02	0.97	0.01	0.00	0.01	0.97	0.01	10566.4
Na_37	5	100	0.00	2.60	0.00	2.60	3.38	1.58	0.56	2.07	3.38	1.58	22355.7
Na_38	5	100	3.53	1.10	3.53	1.10	3.90	0.94	0.00	1.06	3.90	0.95	18040.9
Na_39	5	100	0.00	3.26	0.00	3.27	1.03	3.47	2.04	2.21	4.11	3.63	24479.8
Na_40	5	100	0.15	2.18	0.15	2.19	0.40	1.74	1.36	1.28	0.00	1.90	17199.4
Na_41	10	100	2.67	4.24	2.67	4.26	27.85	1.11	1.25	2.30	0.00	1.38	23210.5
Na_42	10	100	0.00	1.15	0.00	1.14	1.57	1.53	0.46	1.05	1.57	1.52	15633.6
Na_43	10	100	0.00	3.08	0.00	3.08	2.14	2.03	1.24	2.38	2.14	2.03	27349.5
Na_44	10	100	12.35	3.52	12.35	3.53	12.08	1.60	0.00	4.75	12.08	1.60	15269.4
Na_45	10	200	7.02	72.20	7.02	72.35	10.35	52.11	2.35	92.55	0.00	27.92	51186.9
Na_46	10	200	0.00	89.01	0.00	89.23	1.93	30.77	1.08	19.35	1.93	30.62	35498.5
Na_47	10	200	4.67	72.14	4.67	72.35	4.36	34.42	0.00	23.87	4.36	34.39	51079.81
Na_48	10	200	1.77	19.06	1.77	19.12	3.63	55.27	0.00	40.72	3.63	55.13	30126.4
Na_49	5	25	0.00	0.01	0.00	0.01	1.58	0.00	1.58	0.01	1.58	0.01	11439.8
Na_50	5	25	0.00	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.01	8321.3
Na_51	5	25	2.28	0.01	2.28	0.01	0.00	0.00	0.46	0.01	0.00	0.00	13193.3
Na_52	5	25	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.00	0.00	0.01	7664.4
Na_53	5	50	0.00	0.16	0.00	0.16	1.67	0.16	3.72	0.09	1.67	0.16	21781.8
Na_54	5	50	0.00	0.14	0.00	0.14	1.42	0.14	1.42	0.15	1.42	0.14	12784.5
Na_55	5	50	0.00	0.22	0.00	0.22	0.68	0.10	0.95	0.14	0.68	0.11	20907.3
Na_56	5	50	0.48	0.08	0.48	0.08	0.20	0.14	0.00	0.09	0.20	0.14	13829.7
Na_57	10	50	1.77	0.22	1.77	0.21	0.16	0.06	2.40	0.16	0.00	0.09	25716.3
Na_58	10	50	0.00	0.16	0.00	0.16	0.94	0.20	0.38	0.12	0.94	0.19	15106
Na_59	10	50	0.00	0.15	0.00	0.14	7.43	0.14	1.20	0.14	7.43	0.13	20840.8
Na_60	10	50	5.05	0.10	5.05	0.10	0.00	0.13	1.61	0.19	0.00	0.13	14244.1
Na_61	5	100	0.00	2.09	0.00	2.10	3.01	1.71	0.30	3.77	3.01	1.70	37492.4
Na_62	5	100	0.00	6.39	0.00	6.41	2.45	3.26	2.76	3.38	2.18	4.50	24845.2
Na_63	5	100	0.15	4.24	0.15	4.24	2.17	1.07	0.04	2.65	0.00	1.55	42244.81
Na_64	5	100	0.00	3.68	0.00	3.69	3.08	1.53	0.92	2.11	3.51	0.82	21906.7
Na_65	10	100	0.00	4.36	0.00	4.38	15.09	2.52	3.40	2.40	14.06	2.86	37778
Na_66	10	100	2.11	1.41	2.11	1.41	0.89	0.87	1.91	1.86	0.00	2.98	32030.99
Na_67	10	100	12.01	4.52	12.01	4.53	2.89	1.33	0.00	6.53	2.89	1.32	43043.91
Na_68	10	100	1.77	2.53	1.77	2.53	0.00	3.24	3.30	2.99	0.00	3.22	24713.29
Na_69	10	200	7.01	60.52	7.01	60.51	0.00	92.05	6.95	55.29	2.19	52.89	89070.41
Na_70	10	200	3.45	40.60	3.45	40.65	8.00	42.27	1.37	57.77	0.00	58.77	61561.01
Na_71	10	200	0.00	58.29	0.00	58.40	0.43	21.02	12.84	24.91	6.27	45.68	88886.21
Na_72	10	200	0.43	89.41	0.43	89.58	0.00	74.95	0.67	47.16	0.00	74.64	50781.09
average			2.25	11.98	2.25	11.99	2.45	9.97	2.18	9.52	1.94	9.29	
max e%			50.44		50.44		27.85		48.01		24.94		
n. best sol.			33		33		30		23		37		

Tables 3.3 gives some details on the nature of the solutions. In the first row, for each variant, the ratio between the capacities of the active vehicles and the total demands is reported. This ratio provides a measure of the average loading of each vehicle. A ratio equal to one means that the vehicle corresponding to the selected satellites have a capacity exactly equal to the customer demand. High values of this ratio mean that the capacity of the vehicle is hardly used to the limit. The table shows that there is no significant difference among the five variants. The ratio between set-up costs of the active vehicles and the total demand of the customers is reported in the second row. This ratio

gives a measure of the set-up cost for each unit of freight transported by the active vehicles. The total set-up cost for the active vehicles are reported in the last row of the table. We notice neither significant differences among the five variants, nor correspondence between low total set-up costs and good objective function values. This consideration suggests that, on this set of instances, the routing has an importance greater than the full loading of the vehicles. Finally, the last row shows, for each instance, the number of active vehicles (one for each selected satellites).

Table 3.3: Na Inst.: Average statistics.

	A1	**A1B**	**A2**	**A2B**	**A3**
average capacity of active vehicle/total demand	1.22	1.22	1.21	1.23	1.22
average set-up costs of active vehicles/total demand	12.52	12.52	12.78	12.45	12.69
average number of active vehicles	3.04	3.04	3.01	3.07	3.04

Tables 3.4 and 3.5 are structured in the same way as Tables 3.2 and 3.3. They show the results obtained by considering the Pa instance set. The size of this set ranges from 20 to 200 customers and from 5 to 10 satellites. Table 3.4 shows that the variant $A2B$ finds the largest number of best solutions (35). A1, A1B, A2 and A3 find 33, 33, 34 and 32 best solution respectively, out of 90 instances. It is interesting to notice that the variant A2B improves the number of best solutions found on the Pa instance set compared with the Na instance set (where it finds only 23 best solutions out of 72). Although this variant finds the largest number of optimal solutions, it also obtains the largest average objective function value and the largest average relative error. The same considerations already done for the Na set, concerning the structure of the solutions and the gaps between the different variants, holds for the Pa set.

Table 3.4: Pa Inst.: results

			A1		**A1B**		**A2**		**A2B**		**A3**		A_{best}
Problem	m	n	s	t	s	t	s	t	s	t	s	t	s
Pa_1	5	20	0.09	0.00	0.09	0.01	0.09	0.01	0.00	0.01	0.09	0.00	6365.90
Pa_2	5	20	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	7216.00
Pa_3	5	20	0.00	0.00	0.00	0.01	0.07	0.00	0.07	0.00	0.00	0.01	6871.50
Pa_4	5	20	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.00	0.00	0.00	6108.90
Pa_5	5	50	0.01	0.17	0.01	0.16	0.00	0.18	0.27	0.07	0.00	0.18	15200.10
Pa_6	5	50	0.00	0.01	0.00	0.00	0.00	0.01	0.00	0.01	0.00	0.00	5407.80
Pa_7	5	50	0.02	0.15	0.02	0.15	0.02	0.15	0.00	0.17	0.10	0.13	18436.30
Pa_8	5	50	0.05	0.01	0.05	0.01	0.00	0.01	0.00	0.01	0.05	0.01	9313.90
Pa_9	5	50	0.00	0.00	0.00	0.01	0.00	0.00	0.00	0.01	0.00	0.01	6013.60
Pa_10	5	50	0.00	0.09	0.00	0.08	0.03	0.14	0.03	0.14	0.03	0.10	11104.70
Pa_11	5	50	0.18	0.13	0.18	0.13	0.00	0.09	0.06	0.13	0.12	0.11	11148.10
Pa_12	5	50	0.02	0.00	0.02	0.01	0.00	0.01	0.02	0.01	0.00	0.01	5517.30
Pa_13	5	100	0.01	7.01	0.01	7.03	0.04	3.13	0.02	2.42	0.00	2.80	179519.80
Pa_14	5	100	0.03	1.70	0.03	1.71	0.00	2.92	0.04	1.42	0.00	2.09	87703.00
Pa_15	5	100	0.00	2.89	0.00	2.90	0.03	4.12	0.04	2.71	0.00	1.44	199580.50
Pa_16	5	100	0.00	2.78	0.00	2.79	0.01	1.02	0.01	1.02	0.01	1.01	89009.89
Pa_17	5	100	0.01	2.47	0.01	2.48	0.00	0.85	0.03	2.43	0.01	2.07	182081.30

Table 3.4: continue

			A1		A1B		A2		A2B		A3		A_{best}
Pa_18	5	100	0.00	1.20	0.00	1.21	0.01	1.60	0.00	1.33	0.00	3.11	86751.30
Pa_19	10	100	0.02	2.96	0.02	2.96	0.02	5.18	0.04	4.00	0.00	1.38	196891.70
Pa_20	10	100	0.00	1.57	0.00	1.58	0.00	1.07	0.01	2.07	0.02	1.02	96384.01
Pa_21	10	100	0.89	2.93	0.89	2.93	0.90	7.23	0.00	6.11	1.94	1.85	189345.60
Pa_22	10	100	0.00	1.30	0.00	1.30	4.46	1.45	0.00	2.96	4.46	1.44	91102.82
Pa_23	10	100	0.00	4.29	0.00	4.29	0.00	5.94	0.01	4.68	2.30	1.95	179799.60
Pa_24	10	100	0.01	2.01	0.01	2.01	0.00	3.06	0.02	2.25	0.00	1.13	87491.31
Pa_25	10	200	0.03	74.02	0.03	74.19	0.03	44.23	0.03	36.19	0.00	20.44	746847.80
Pa_26	10	200	0.02	91.07	0.02	91.43	0.03	35.71	0.02	40.13	0.00	44.63	318418.00
Pa_27	10	200	0.21	92.25	0.21	92.27	0.21	107.78	0.00	107.82	0.21	80.13	864319.10
Pa_28	10	200	0.01	42.43	0.01	42.59	0.00	34.19	0.00	61.35	0.37	42.62	390804.80
Pa_29	10	200	0.01	45.60	0.01	45.55	0.00	72.93	0.01	48.71	1.86	37.88	764342.30
Pa_30	10	200	0.01	97.21	0.01	97.49	0.00	72.65	0.01	24.23	9.24	15.73	323065.60
Pa_31	5	20	0.09	0.00	0.09	0.00	0.09	0.00	0.00	0.00	0.09	0.00	6365.90
Pa_32	5	20	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.00	0.00	0.01	7216.00
Pa_33	5	20	0.00	0.00	0.00	0.00	0.07	0.00	0.07	0.00	0.00	0.00	6871.50
Pa_34	5	20	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	6108.90
Pa_35	5	50	0.05	0.10	0.05	0.10	0.00	0.09	0.00	0.09	0.00	0.09	15200.40
Pa_36	5	50	0.00	0.01	0.00	0.01	0.00	0.01	0.00	0.01	0.00	0.01	5407.80
Pa_37	5	50	0.01	0.08	0.01	0.08	0.00	0.11	0.00	0.10	0.00	0.11	18446.20
Pa_38	5	50	0.05	0.01	0.05	0.01	0.00	0.01	0.00	0.01	0.05	0.01	9313.90
Pa_39	5	50	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.00	0.00	0.00	6013.60
Pa_40	5	50	0.02	0.11	0.02	0.12	0.01	0.08	0.01	0.12	0.00	0.11	11104.40
Pa_41	5	50	0.00	0.09	0.00	0.09	0.00	0.08	0.01	0.08	0.00	0.08	11156.00
Pa_42	5	50	0.02	0.00	0.02	0.01	0.00	0.01	0.02	0.01	0.00	0.01	5517.30
Pa_43	5	100	3.94	3.36	3.94	3.36	0.01	2.60	0.02	2.50	0.00	3.68	179507.00
Pa_44	5	100	0.00	1.77	0.00	1.77	0.02	2.67	0.01	1.38	0.02	1.03	87707.67
Pa_45	5	100	0.00	4.05	0.00	4.04	0.02	1.69	0.02	1.70	0.00	2.11	199619.00
Pa_46	5	100	0.00	0.98	0.00	0.98	0.01	1.22	0.01	1.22	0.01	1.20	89012.39
Pa_47	5	100	0.01	3.03	0.01	3.04	0.00	2.57	0.00	1.68	0.01	1.82	182096.90
Pa_48	5	100	0.02	1.37	0.02	1.38	0.02	1.39	0.02	1.39	0.00	3.87	86756.22
Pa_49	10	100	0.01	2.56	0.01	2.56	0.01	1.11	0.01	1.83	0.00	2.17	196920.90
Pa_50	10	100	0.00	1.05	0.00	1.06	0.00	1.10	0.00	1.10	0.00	1.05	96392.31
Pa_51	10	100	0.00	4.74	0.00	4.74	7.23	2.41	29.17	1.86	0.00	1.53	146591.60
Pa_52	10	100	0.04	1.81	0.04	1.81	4.46	1.11	0.00	2.09	4.46	1.11	91097.63
Pa_53	10	100	29.13	1.80	29.13	1.79	0.00	3.73	28.41	5.29	9.50	1.75	139989.40
Pa_54	10	100	0.01	2.41	0.01	2.40	0.02	1.85	0.00	0.99	0.02	1.84	87497.88
Pa_55	10	200	0.03	57.78	0.03	57.83	0.03	65.51	0.04	27.78	0.00	24.11	634169.30
Pa_56	10	200	0.00	124.97	0.00	125.19	3.14	115.09	0.01	60.75	3.12	59.43	246903.50
Pa_57	10	200	0.02	60.17	0.02	60.04	0.01	34.00	0.01	28.57	0.00	40.11	741616.30
Pa_58	10	200	1.92	52.83	1.92	52.57	0.00	10.83	0.00	106.29	1.91	51.35	277121.90
Pa_59	10	200	0.00	44.36	0.00	44.14	1.77	25.21	0.00	98.59	0.66	53.07	646790.50
Pa_60	10	200	0.01	30.33	0.01	30.30	0.00	75.19	0.01	29.64	14.89	12.85	235318.30
Pa_61	5	20	0.09	0.00	0.09	0.00	0.09	0.00	0.00	0.00	0.09	0.00	6365.90
Pa_62	5	20	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	7216.00
Pa_63	5	20	0.00	0.00	0.00	0.00	0.07	0.00	0.07	0.00	0.00	0.00	6871.50
Pa_64	5	20	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	6108.90
Pa_65	5	50	0.00	0.23	0.00	0.22	22.83	0.20	22.99	0.08	22.83	0.20	20584.80
Pa_66	5	50	0.00	0.01	0.00	0.01	0.09	0.01	0.00	0.01	0.09	0.01	10113.80
Pa_67	5	50	0.17	0.11	0.17	0.11	0.08	0.20	0.19	0.09	0.00	0.21	29747.00
Pa_68	5	50	0.17	0.09	0.17	0.09	0.17	0.09	0.17	0.09	0.00	0.12	18407.40
Pa_69	5	50	0.74	0.08	0.74	0.08	0.74	0.08	0.00	0.03	0.74	0.08	11998.40
Pa_70	5	50	0.00	0.12	0.00	0.12	0.29	0.27	0.29	0.19	0.29	0.28	16877.20
Pa_71	5	50	0.11	0.13	0.11	0.13	0.07	0.11	0.06	0.16	0.00	0.20	21824.70
Pa_72	5	50	0.06	0.01	0.06	0.01	0.00	0.00	0.02	0.00	0.00	0.01	5517.30
Pa_73	5	100	0.03	3.07	0.03	3.07	0.00	3.49	0.01	3.03	0.02	2.03	232358.00
Pa_74	5	100	0.01	3.66	0.01	3.65	0.23	1.73	0.00	1.41	0.02	1.16	133484.20
Pa_75	5	100	0.03	2.65	0.03	2.64	0.03	1.93	0.00	3.22	0.03	1.93	257915.50
Pa_76	5	100	0.00	4.42	0.00	4.39	0.07	0.80	0.07	0.55	0.07	0.79	97179.91
Pa_77	5	100	0.04	2.39	0.04	2.37	0.00	4.47	0.00	4.46	0.00	0.77	236060.40
Pa_78	5	100	0.05	1.40	0.05	1.39	0.00	1.57	0.03	1.23	0.00	2.40	131365.00
Pa_79	10	100	0.00	5.62	0.00	5.59	2.43	6.02	0.02	5.46	1.13	1.35	250062.40
Pa_80	10	100	0.00	2.09	0.00	2.08	0.01	5.83	0.00	3.82	0.02	4.81	96383.70
Pa_81	10	100	0.81	6.52	0.81	6.49	1.07	3.35	0.00	8.46	3.43	3.02	239717.10
Pa_82	10	100	0.00	0.49	0.00	0.49	7.35	1.31	46.89	1.22	7.35	1.30	95236.63
Pa_83	10	100	0.02	5.05	0.02	5.02	1.86	1.91	0.00	3.46	0.02	2.34	228976.20
Pa_84	10	100	0.00	3.02	0.00	3.01	9.81	2.23	0.02	2.36	0.04	4.03	87465.40
Pa_85	10	200	1.24	75.11	1.24	74.62	1.25	60.46	0.00	74.63	1.23	27.65	860822.50
Pa_86	10	200	13.19	44.30	13.19	44.05	0.00	54.52	13.19	41.96	14.18	25.10	371847.90
Pa_87	10	200	0.00	61.80	0.00	61.77	0.00	81.52	0.00	98.88	0.00	38.43	988844.80
Pa_88	10	200	26.89	55.77	26.89	55.80	0.00	81.75	26.88	71.16	26.88	25.19	398714.00
Pa_89	10	200	0.14	81.06	0.14	80.96	0.14	104.86	0.00	56.84	0.00	62.29	883517.10
Pa_90	10	200	21.06	48.45	21.06	48.57	0.00	60.56	21.06	22.44	0.00	66.36	352931.30
average			1.13	14.26	1.13	14.26	.79	13.72	2.12	12.59	1.49	8.89	
max e%			26.89		26.89		22.83		46.89		26.88		
n.best sol.			33		33		34		35		32		

Table 3.5 does not show significant difference concerning the behaviour of

the five variants. The ratio between the capacity of the active vehicles and the total demand is almost the same in the five variants. A2B is also the variant with the largest total set-up cost per unit of freight and the greatest number of total active vehicles.

Table 3.5: Pa Inst: Average statistics.

	A1	**A1B**	**A2**	**A2B**	**A3**
average capacity of active vehicle/total demand	1.31	1.31	1.31	1.31	1.31
average set-up costs of active vehicles/total demand	82	82	81.30	82.55	82.11
average number of active vehicles	3.06	3.06	3.03	3.1	3.05

Finally, Table 3.6 provides, for each instance set, a comparison of the five variants. In the first column is reported the name of the instance set considered, whereas in column 2, 3, 4, 5 and 6 are reported the results achieved by the five different variants. For each of them the average total error is shown in the left section, and the CPU time (expressed in seconds) is shown in the right section. At the bottom of the table the average total error and the average total computing time of each variant is reported. Variants A1 and A1B provide almost identical results and are characterized by the worse total average computing times. A2 is the variant that provides the lowest average total error, whereas A2B provides the largest. The difference is due to the fact that A2 in the insertion procedure considers the fixed costs of the satellites divided by their capacity, whereas A2B considers only the fixed costs. A3 is the fastest variant, considering both the average total computing time and the average computing times for each set of instances, but does not perform well on the Pa instance set. Thus, in our numerical experiments, we do not notice significant difference among the five variants. The use of different instances generates different performance of the variants. We remark that although procedures A1 and A1B have deficiency of geographic information utilizations (they do not consider the insertion cost of the customers into the vehicle tour and the insertion of the satellite into the primary tour) it does not imply that an initial solution from A1 is inferior than one from A2, A2B or A3 (i.e. it is also possible that the initial solution from A1 could lead to better final solution than one from A2, A2B or A3 after the application of the main local search procedures).

At any rate, the computational results suggest that all the variants perform well, with very short computational times.

Table 3.6: Comparison among the five variants on the three sets of instances

	A1		A1B		A2		A2B		A3	
Instance set	e%	CPU (sec)	e%	CPU (sec)	e%	CPU (sec)	e%	CPU (sec)	e%	CPU (sec)
Ca	1.79	0.4	1.79	0.4	0.78	0.4	1.79	0.4	0.68	0.4
Na	2.25	11.98	2.25	11.99	2.45	9.97	2.18	9.52	2.94	9.29
Pa	1.13	14.26	1.13	14.26	0.79	13.72	2.12	12.59	1.49	8.89
Total	1.72	8.88	1.72	8.88	1.34	8.03	2.03	7.50	1.70	6.19

Chapter 4

Conclusions

The delivery of freight from its origin to its destination is often managed moving the load through one or more intermediate facilities where storing, merging and/or consolidation activities are performed. This type of distribution system is commonly called multi-echelon, where each echelon refers to one level of the distribution network. Multi-echelon distribution systems are often adopted by public administrations in their transportation and traffic planning strategies as well as by private companies to design their distribution networks. City logistic and multi-modal transportation systems are the most cited examples of multi-echelon distribution systems. Two-echelon distribution systems are a special case of multi-echelon systems where only two levels of the distribution network are taken into consideration. In the last years, two-echelon distribution systems have inspired an ever growing body of literature creating a new large family of combinatorial optimization problems that we called Two-Echelon Routing Problem. The study of these problems has been much motivating not only for the important help that specific optimization techniques can provide in real-life applications, but also because designing efficient route solutions can lead to reduction of air polluting emissions.

In this thesis, we have provided an extensive overview of the operations research literature on two-echelon routing problems, that is a class of problems that study the optimal routing of freights in two-echelon distribution systems. This research area is relatively new and is attracting an increasing attention both from practitioners and academics due to the relevant real-life applications that are related (among others, city logistics and multi-modal transportation) and the intellectual challenges that their study poses. We classified the literature on two-echelon routing problems into three classes: the two-echelon location routing problem, the two-echelon vehicle routing problem and the track and trailer routing problem. For each class, we have provided a general description of the problem, identified the main variants studied in literature and reviewed the exact and heuristic solution approaches of such problem. All the problems we considered have been introduced in the literature only recently, so this research area is still relatively unexplored. Most of the contributions cited in Chapter 1 focus on the basic problems and propose heuristic solution approaches. Hence, promising research directions are, on the one side, the study of more realistic variants of the basic problems and, on the other hand, the design of efficient exact solution algorithms. Additionally, only few papers deal with time-dependent variants of the basic problems. For instance, only some authors have studied variants of the problems with soft and/or hard time windows, or variants where a limit on the total duration of each route is given. Other variants that are worthwhile to investigate are the two-echelon location routing and the two-echelon vehicle routing problems with satellite synchronization constraints. In these variants, time constraints are considered on the arrival of the vehicles at a satellite such that once a first echelon vehicle has unloaded its load, it is immediately loaded onto a second echelon vehicle. An interesting variant of the two-echelon vehicle routing problem is its multi-depot version, i.e. more than one depot serve the satellites in the first echelon. As concerns the truck and trailer routing problem, more realistic versions of the problem should consider a cost (and/or the time spent) for transferring loads between a truck and its trailer, and also to hook and unhook the trailer. On the other hand, the development of exact solution methods is, at the moment, very limited and could also be a valid research area. Finally, the study of dynamic versions of the problems and stochastic models also seems to be a promising research directions.

We also studied the Single Vehicle 2E-LRP a particular version of the 2E-LRP with a single source. The problem concerns a two-echelon transport system with limited capacity, in which some freights, available at a central depot, have to be delivered to a set of customers through a set of intermediate satellites which must be located. We proposed a integer linear programming formulation (referred as symmetric formulation) and a mixed integer programming formulation (referred as flow based formulation) extending and/or adapting known LRP and $2E-VRP$ models. The proposed formulations assume as known the position of the depot and the of customers and are able to find the optimal or sub optimal locations for the satellites. The formulations have been tested on test instances of different size by CPLEX 12.1. The result obtained are good, especially as concerning the flow formulation that encourage us to go ahead with the work in this research field. On the other hand the development of specific valid inequalities for the symmetric formulation could lead to improve the performance of the formulation, allowing us to tackle problems of larger size.

Moreover, a heuristic algorithm with five different procedures used to estimate the assignment cost of customers to satellites have been presented and tested using 3 sets of instances adapted from the literature. On the first set, called Ca and containing up to 25 customers and 20 satellites, we compared the results of our heuristic with the results obtained by CPLEX using the flow based formulation. The model solved exactly 19 instances out of 24 and the best variant of our heuristic found 9 optimal solutions. The other two instance sets, called Na and Pa, have been used to compare the five variants of our heuristic. The results of the comparison have shown that the best variants do not lie more than 1.34 on average with an average computing time shorter than 8 seconds. Therefore, on all instance sets, the five variants performed in very short running times, even on the largest instances with 200 customers and 10 satellites. Future directions of research could consist in developing metaheuristics and a branch and cut algorithm able to solve medium instances.

Despite the good results achieved by our heuristic algorithm, it should be possible improve some of the methods presented in this thesis. It would be interesting to study valid inequalities to be included on a branch and cut algorithm for our problem and to be added to our formulations, and to identify strategies to further improve the performance of our heuristic. It would be useful to extend the method proposed in this thesis to tackle other and more general two-echelon location routing problems. Finally, natural extensions like the 2E-LRP with time windows, 2E-LRP with limit on the total duration of each route or heterogeneous trucks and trailers could be addressed in the future.

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