UNIVERSITY OF BERGAMO
ITALY
Faculty of Economics and Business Administration

PhD in Logistics and Supply Chain Management–XXV Cycle

in a collaboration with SDA Bocconi school of Management and Massachusetts Institute of Technology – Zaragoza Logistic Centre (MIT-ZLC)

Sourcing Decision and Inventory Management in Supply Chain

Supervisor:

Prof. Sergio Cavalieri

Department of Engineering, University of Bergamo, Dalmine BG, Italy

PhD Candidate:

Omid Jadidi

Matriculation Number: 1013216

March 2014

Table of Contents

List of Figures

CHAPTER 1

CHAPTER 2

CHAPTER 3

CHAPTER 4

CHAPTER 5

List of Tables

CHAPTER 1

CHAPTER 2

CHAPTER 3

CHAPTER 4

Notations

Chapter 1

$i = 1,2,\ldots,n$	index of suppliers
n	number of suppliers
X	vector of decision variables
x_i	(decision variable) number of units ordered from supplier i, $x_i \in X$
V_i	capacity of supplier i
C_i	purchasing price of the product from supplier i
q_i	expected defect rate of supplier i
F_i	percentage of items delivered late by supplier i
D	demand of the product
W_k	relative importance of objective k

Chapter 2

$i = 1,2,\ldots,n$	index of suppliers
n	number of suppliers
X_i	number of units ordered to supplier i
k	index for objectives, k=1, 2, …, K
V_i	capacity of supplier i
C_i	unit purchasing price from supplier i
q_i	expected defect rate of supplier i
F_i	percentage of items delivered late by supplier i
$f_k(X)$	objective k
D	demand

Chapter 3

$i = 1,2,\ldots,I$	index of suppliers
I	number of suppliers
$j = 1,2,\ldots,m_i$	index of price level (m_i indicates number of price levels offered by supplier i).
X_{ij}	order allocated to supplier i at price level j

p	selling price per unit at the market, determined exogenously
h	holding cost per unit
Sh	shortage cost per unit
C_{ij}	price per unit offered by supplier *i* at price level *j* ($C_{i,j+1} < C_{ij}$)
V_{ij}^{U}	upper bound of the order that can be allocated to supplier *i* at price level *j*
V_{ij}^{L}	lower bound of the order that can be allocated to supplier *i* at price level *j*
ξ	market demand (random variable)
$f(\xi)$	known probability density function of the demand
$F(\xi)$	known cumulative distribution function of the demand
Y_{ij}	binary variable: Y_{ij} =1 if order is placed to supplier *i* at price level *j*; otherwise Y_{ij} =0.

Chapter 4

$i = 1,2, \ldots, n$	index of suppliers
n	number of suppliers
C_i	wholesale price per unit of supplier *i*
V_i	production capacity of supplier *i*
y	stochastic demand of the buyer
θ_i	Supply unreliability level delivered by supplier *i*
Q^{θ}	total order quantity to the suppliers
$X_i \in [0, 1]$	percent of Q^{θ} assigned to supplier *i*
$q_i^{\theta} = Q^{\theta} X_i$	is the *i*th supplier's order.
b_i	buyback price per unit from supplier *i* for leftover inventory at the end of season

Acknowledgements

I am thankful to all people whose helps enable me to complete this research. In particular, I am deeply grateful to my supervisor Professor Srigio Cavalieri for his remarkable support, encouragement and guidance. I am also grateful to Professor Roberto Pinto for his useful participation in my research.

I would like to thank the faculty committee members of this PhD program, particularly Professor Enzo Baglieri, and to the administrative staffs.

I am really thankful to "Fondazione Italcementi" for the financial support to this PhD program.

I would also like to thank Professor Saeed Zolfaghari for hosting me at the department of mechanical and industrial engineering, Ryerson University, and for providing me precious suggestion and advice.

I am also thankful to MIT-ZLC and SDA Bocconi School of Management for their considerable contributions in our PhD research. A very special thank to Professor Youyi Feng for his unforgettable helps in our research during our stay at MIT-ZLC.

I am also grateful to Fatemeh Firouzi, Arvind Upadhyay, and Tommaso Lupo who shared with me joys of this doctoral program.

Above all, I would like to thank a lot to my devoted parents and family members for their long lasting supports and helps. Also, many tanks go to my Mather- and Father-in-Law for their valuable supports. I am really grateful to my lovely wife, Fatemeh Firouzi, who made my life and carrier so meaningful and wonderful.

Abstract

This study is dedicated to supplier selection problem (SSP) with two different aspects: (1) SSP with deterministic demand, and (2), SSP with stochastic demand. For both aspects, we assume that all suppliers were already pre-evaluated according to some criteria, such as financial strength and performance history, and now the buyer needs to further assess the suppliers for order allocation based on some quantitative criteria such as purchasing cost, etc.

SSP with deterministic demand: Such problems are usually modeled as a multi-objective optimization problem (MOOP) subject to suppliers' capacity, buyer's demand, etc. Number of objectives and constraints vary from one problem to another. For solving the MOOP, three different cases have been considered in the literature: Case 1) the Decision Makers (DMs) determine a goal for each objective and then try to drive achieved objectives towards their goals as close as possible (this case is known as the crisp MOOP); Case 2) the DMs determine the weights of objectives instead of the goals so that the objectives approach their possible ideal solution according to their weights (this case is known as the fuzzy MOOP); Case 3) the DMs determine multiple goals or an interval goal for each objective and then try to drive achieved objectives towards the goals (known as Multi-Choice Goal Programming (MCGP) model). In Case 1, goal programming is the most famous and wildly used approach in the literature. However, the approach is not able to make the achieved objectives consistent with their goals. In chapter 2 (as the first contribution), we develop a normalized goal programming approach to achieve some levels of consistency among different objectives. Then, the proposed approach is extended for solving the fuzzy MOOP of Case 2. Due to uncertainly/imprecision, Case 3 may be more applicable than Case 1 as the DMs can

determine an interval goal (or multiple goals) for every objective. In chapter 3 (as the second contribution), we propose an improved MCGP approach providing the DMs with more control over their preferences in comparison with the previous models.

SSP with stochastic demand: Such problems can be modeled by using probability distributions. The newsvendor model is one of the most famous models in this area that can be wildly used in reality because of the decline of product life cycle. In Chapter 4, we consider a multi-period SSP where a buyer procures an item (i.e., raw material) from a set of capacitated suppliers to meet the final product stochastic demand in order to maximize his/her expected profit. The suppliers may offer quantity discount as a competitive factor to induce the buyer to purchase more. We first model the problem by mixed integer nonlinear programming, and then propose an algorithm for solving the model (as the third contribution). In Chapter 5, we consider a single period newsvendor problem where a buyer purchases a single item from a set of capacitated suppliers. In this problem, we assume that both the demand and supply are uncertain. Wholesale prices offered by the suppliers and their supply uncertainties are considered as the competitive factors. In order to compensate the supply uncertainties, the suppliers may allow the buyer to return unsold products at the end of season (buyback policy). Therefore, the buyer has to take into account three criteria (suppliers' wholesale price, suppliers' unreliability level, and suppliers' buyback price) for evaluating the suppliers that contributes to the complexity of the problem. In this chapter, we develop an algorithm to solving such problem (as the forth contribution).

CHAPTER 1 - INTRODUCTION

1. Motivations and objectives of the thesis

Consumers' expectations for high quality products with short lead-time and low price are the factors that companies can utilize as competitive factors. Companies have to take the advantage of any opportunity to optimize their business processes, for handling and maintaining a competitive position. To do so, companies design an efficient supply chain management (SCM) system that allows them to work effectively with their supply chain partners. In this aspect, the purchasing function, affecting all areas of an organization, is taking an increasing importance (Aissaouia et al., 2007). Figure.1 illustrates that the major purchasing decision processes can be classified into six parts: (1) make or buy, (2) supplier selection, (3) contract negotiation, (4) design collaboration, (5) procurement, and (6) sourcing analysis (Aissaouia et al., 2007).

The literature shows that the cost of component parts and raw materials in manufacturing industries can equal up to 70% of the product cost (Ghodsypour and O'Brien, 1998). As a result, in a purchasing department, one of the most important tasks is the selection of the right suppliers as it can meaningfully decrease the cost of purchasing and improve corporate competitiveness (Willis et al., 1993; Dobler et al., 1990; Xia and Wu, 2007).

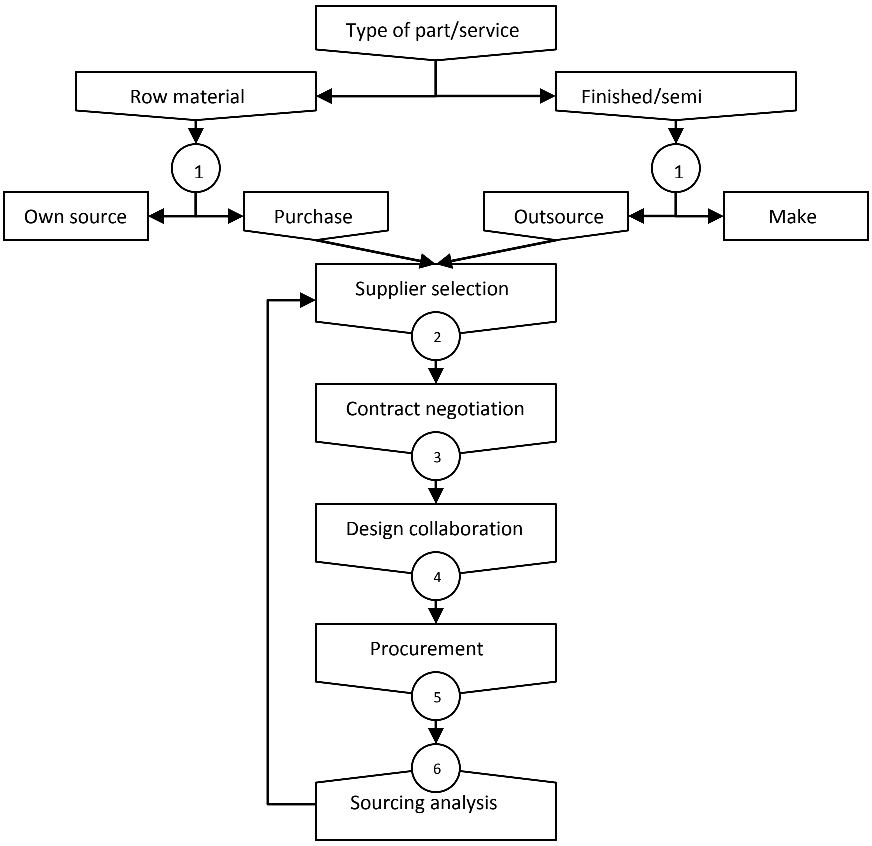


Fig. 1. Major Purchasing Processes (Aissaouia et al., 2007)

Buyers usually follow two different scenarios for SSP: single- sourcing and multiple-sourcing scenarios.

In the first scenario, almost all suppliers are able to meet the buyer's requirements so that the buyer selects the most appropriate one.

In multiple-sourcing scenario, due to suppliers' limitations on capacity, quality, etc, the buyer splits its order between multiple suppliers. Multiple-sourcing scenario is a practical way for ensuring the reliability of a buyer's supply stream (Aissaoui et al., 2007). Aissaoui et al. (2007) also concluded that mathematical programming is the most appropriate tool for modeling such problems.

Table 1.
Dickson's (1966) supplier selection criteria

Rank	Factor	Mean Rating	Evaluation
1	Quality	3.508	Extreme Importance
2	Delivery	3.417	
3	Performance History	2.998	
4	Warranties & Claims Policies	2.849	Considerable Importance
5	Production Facilities and Capacity	2.775	
6	Price	2.758	
7	Technical Capability	2.545	
8	Financial Position	2.514	
9	Procedural Compliance	2.488	
10	Communication System	2.426	
11	Reputation and Position in Industry	2.412	
12	Desire for Business	2.256	
13	Management and Organization	2.216	
14	Operating Controls	2.211	
15	Repair Service	2.187	
16	Attitude	2.120	Average Importance
17	Impression	2.054	
18	Packaging Ability	2.009	
19	Labor Relations Record	2.003	
20	Geographical Location	1.872	
21	Amount of Past Business	1.597	
22	Training Aids	1.537	
23	Reciprocal Arrangements	0.610	Slight Importance

By evaluating suppliers based on some criteria, competent suppliers are chosen for procurement process. Supplier selection problem (SSP) in its nature is a multi-criteria decision-making (MCDM) problem, as Dickson (1966) identified and ranked 23 criteria by sending a questionnaire to 273 purchasing agents and managers in the United States and Canada (see Table 1).

Table 1 shows that the importance of these criteria is different that, in practice, it may however change from one industry to another. The existence of many criteria with different importance, contribute to the added complexity of SSP (Wang and Yang, 2009), because purchasing managers should be able to effectively incorporate their preferences in decision making process. In the literature, for considering multiple criteria for evaluating suppliers in the multiple-sourcing scenario, multi-objective mathematical programming is usually used to model the SSP when the demand is deterministic. One very famous approach to solve the multi-objective problem is determining a goal for every objective and then trying to drive the achieved objectives towards their goals. Another very new approach is determining an interval goal, instead of a single goal, for every objective and then trying to drive the achieved objectives towards the lower bound of the interval goal for minimization objectives (or towards the upper bound of the interval goal for maximization objectives).

In addition, buyers' information on market demand may be considered as the most significant cause of uncertainty in reality (Tajbakhsh, 2007). In this case, probability distributions, which generally depend on previous data, can be used to model the uncertain demand. To deal with this problem, purchasing managers integrate SSP with inventory management so that they should simultaneously obtain the optimal inventory level, the appropriate set of suppliers, and the suppliers' order quantity. In this case, if the demand is greater than the inventory level, the unmet demand is lost; if the demand is less than the inventory level, the buyer has unsold products. As a result, the demand uncertainty is another aspect that increases the complexity of SSP. In the literature, studies dealing with SSP and inventory management consider only a single objective that is maximizing buyers' expected profit (or minimizing buyers' expected cost). They have assumed that the suppliers satisfy other buyers' requirements such as quality, delivery, etc, and the buyer only need to consider the suppliers' wholesale price in their evaluation. However, the suppliers may allow the buyer to return unsold product at the end of the period with a price. In addition, they may have different level of reliability on quality and/or delivery, etc. As a result, the buyer has to take into account suppliers' buyback price and unreliability level on quality and/or delivery, in addition to the wholesale price. To the best of our knowledge, no study has considered such complicated problem.

This thesis is dedicated to SSP and aims to effectively incorporate buyers' preferences and suppliers' conditions in suppliers' evaluation when (1) the demand is deterministic, and (2) the demand is stochastic. Within this perspective, the objectives of this thesis are as follows.

Objectives for deterministic demand problem

1. To model a **multi-objective SSP that allows the decision makers to determine a single goal for every objective**. The main objective is to **effectively incorporate**

the decision makers' preferences in suppliers' evaluation by making the achieved objectives consistent with their goals.

2. To model a **multi-objective SSP letting the decision makers to determine an interval goal for every objective.** The main objective is to effectively incorporate the decision makers' preferences in suppliers' evaluation by providing more control on both the inside and outside of the interval goals.

Objectives for stochastic demand problem

3. To develop a **multi-period multi-supplier newsvendor problem** (dynamic programming) **where the capacitated suppliers may offer quantity discount as a competitive factor**. The main objective is to concurrently obtain the optimal inventory level and suppliers' order quantity for each period by proposing a sophisticated algorithm.

4. To extend a **single-period multi-supplier newsvendor problem where the capacitated suppliers may be unreliable in terms of quality and/or delivery**, and in order to compensate their unreliability, they may allow the buyer to **return unsold products at the end of the single period (buyback).** The main objective is to concurrently obtain the optimal inventory level and suppliers' order quantity by considering multiple criteria (suppliers' wholesale price, suppliers' unreliability level, and suppliers' buyback rate) by means of proposing an algorithm.

2. Scope

This study deals with a two echelon supply chain with one buyer and multiple suppliers. For order allocation, the buyer evaluates the suppliers whose capacities are restricted so that the buyer should divide its order among the suppliers. We also take into account two types of demand: deterministic (or known) and stochastic (or uncertain) demands. In addition, we employ mathematical programming to model the problems.

3. Research methodology

Methodology used for the first and second objectives:

1. For the first and the second objectives that are on SSP with known demand, we consider a problem where the capacitated suppliers should be evaluated based on multiple criteria (wholesale price, rejects, and delivery) for order allocation. We employ multi-objective mixed integer linear programming (MOMILP) model to formulate the problem under consideration. Then, we develop two approaches for solving the MOMILP problem when (1) there is a single goal for every objective and (2) there is an interval goal for every objective.

Methodology used for the third objective:

2. For the third objective that is about the integration of SSP and inventory management under the stochastic demand, we consider a problem where the capacitated suppliers should be evaluated based on a single criterion (discounted wholesale price) for order allocation. We use mixed integer nonlinear programming (MINLP) model to formulate the problem that maximizes buyer's expected profit. The problem is multi-period dynamic programming and is solved recursively: first

the optimum solution (inventory level and the suppliers' order) of the last period is obtained; then the second last period is solved; until the first period. We then propose an algorithm for solving the problem.

Methodology used for the fourth objective:

3. For the fourth objective dealing with the integration of SSP and inventory management under stochastic demand, we consider a problem where the capacitated suppliers should be evaluated based on multiple criteria: (1) the suppliers may offer different wholesale prices, (2) they may have different level of uncertainty (or unreliability) on quality and/or delivery, (3) they may allow the buyer to return unsold products at the end of the season (they may have different buyback price). We first employ binomial random yield model to consider the uncertainty of suppliers on quality and/or delivery. We then use mixed integer nonlinear programming (MINLP) model to formulate the single period problem that maximizes buyer's expected profit. At the end, we propose an algorithm to solve the problem.

4. Contributions

The contributions of this research to the extent literature are as follows.

Contribution regarding to the first objective:

1. Goal programming approach is the widely used technique for solving multi-objective problems when a single goal is determined for each objective. However, this technique cannot guarantee that the achieved objectives to be consistent with their goals (i.e., the solution may not be consistent with decision makers'

preferences). To overcome this weakness, we develop a new normalized goal programming approach.

Contribution regarding to the second objective:

2. Multi-choice goal programming approaches are the new proposed techniques for solving multi-objective problems when an interval goal is determined for each objective. However, this technique cannot guarantee that the decision makers have control on both the inside and the outside of the interval goal. As a result of this, the decision makers' preferences may not be effectively incorporated in the model. To prevail over this limitation, we develop a new multi-choice goal programming approach.

Contribution regarding to the third objective:

3. On the literature, only few articles have integrated SSP with inventory management, especially when the suppliers have limitation on minimum and maximum order quantity or when the suppliers offer discount on the wholesale price. In addition, to the best of our knowledge, no study has considered multi-period supplier selection problem with stochastic demand. In order to fill this gap in the literature, we thus develop an algorithm maximizing the buyers' expected profit by simultaneously calculating the optimum inventory level and the suppliers' order quantity for the multi-period SSP.

Contribution regarding to the fourth objective:

4. In the literature, studies that have integrated SSP with inventory management, mostly evaluate the suppliers based only on one criterion (supplier's wholesale price). However, the suppliers may differ with each other on the other aspects: for example they may different buyback price, or they may have different level of unreliability on quality and/or delivery. In order to be able to evaluate the suppliers based on multiple criteria, we proposed an algorithm that maximizes the buyer expected profit on a single period problem.

5. Outline

We organize the thesis as follows.

Chapter 2 proposes the new normalized goal programming approach for solving multi-objectives SSP problems. In order to compare the effectiveness of the proposed method, a comparative analysis is presented which includes Weighted Goal Programming, Compromise Programming, TOPSIS, Weighted Objectives, Min–max Goal Programming and Weighted Max–min models. Chapter 3 proposes the new multi-choice goal programming approach and compares the new method with the existing multi-choice goal programming approaches.

Chapter 4 develops the algorithm for multi-period SSP with stochastic demand when the suppliers offer quantity discount.

Chapter 5 proposes the algorithm for single-period SSP with stochastic demand that enables the purchasing managers to evaluate the suppliers based on multiple criteria.

In chapter 6, we finally conclude the thesis and highlight its findings and limitations. In addition, we provide recommendation for future research.

References

Aissaoui, N., Haouari, M., Hassini, E., 2007. Supplier selection and order lot sizing modeling: A review. European Journal of Operational Research 34 (12), 3537-3540.

Dickson, G.W., 1966. An analysis of vendor selection: systems and decisions. Journal of Purchasing 2 (1), 5-17.

Dobler, D.W., Lee, L., Burt, N., 1990. Purchasing and materials management: text and cases, McGraw-Hill, New York.

Ghodsypour, S.H., O'Brien, C., 1998. A decision support system for supplier selection using an integrated analytical hierarchy process and linear programming. International Journal of Production Economics 56–57, 199-212.

Tajbakhsh, M.M., Inventory Management and sourcing decision in the presence of upstream uncertainty, 2007, Thesis.

Wang, T.Y., Yang, Y.-H., 2009. A fuzzy model for supplier selection in quantity discount environments. Expert Systems with Applications 36 (10), 12179-12187.

Willis, H.T., Huston, R.C., Pohlkamp, F., 1993. Evaluation measures of just in time supplier performance. Production and Inventory Management 34 (2), 1-5.

Xia, W., Wu, Z., 2007. Supplier selection with multiple criteria in volume discount environments. Omega 35 (5), 494-504.

CHAPTER 2 - A NEW NORMALIZED GOAL PROGRAMMING MODEL FOR MULTI-OBJECTIVE PROBLEMS: A CASE OF SUPPLIER SELECTION AND ORDER ALLOCATION

The main aim of this chapter is to develop a technique for solving multi-objective models so that the achieved objectives are consistent with their goals. We model the problem of supplier selection as a multi-objective optimization problem (MOOP) where minimization of price, rejects and lead-time are considered as three objectives. We here consider two different cases: 1) the crisp MOOP in which the goals of objectives are predetermined; and 2) the fuzzy MOOP in which the weights of objectives are predetermined. In both cases, the aim is to achieve some levels of consistency among different objectives. To do so, a Normalized Goal Programming approach is developed and tested for both cases. In order to compare the effectiveness of the proposed method, a comparative analysis is presented which includes Weighted Goal Programming, Compromise Programming, TOPSIS, Weighted Objectives, Min–max Goal Programming and Weighted Max–min models. An illustrative example reveals that our proposed model is able to achieve the desirable consistency among all objectives.

1. Introduction

In a purchasing department, one of the most important tasks is the selection of the right suppliers as it can meaning fully decrease the cost of purchasing and improve corporate competitiveness (Willis et al., 1993; Dobler et al., 1990; Xia and Wu, 2007). The literature shows that the cost of component parts and raw materials in manufacturing industries can equal up to 70% of the product cost (Ghodsypour and O'Brien, 1998).As carefully discussed by Aissaoui et al. (2007), purchasing decisions have six stages: "(1) 'make or buy', (2) supplier selection, (3) contract negotiation, (4) design collaboration, (5) procurement, and (6) sourcing analysis". Stages 2, 5 and 6 are entirely the responsibility of purchasing departments (Aissaoui et al., 2007). In Stage 2, a set of suppliers are pre-evaluated and selected according to some criteria. For instance, only those suppliers may be pre-approved who have access to the needed technology for producing a product that meets the buyer's requirements. After Stage 2, the question that how much and who (from the set of pre-approved suppliers) should provide the buyer with the products arises. The literature shows that in order to answer this question, the problem can be formulated as a mathematical programming model to further assess the suppliers according to some important factors such as price, quality, delivery, market demand, and suppliers' capacity. Decision makers (DMs) who may come from different roles (such as senior managers, production managers, and purchase managers), usually gather to evaluate suppliers (Demirtas and Ustun, 2008; Jolai et al, 2011). Studies that answer this question (or address lot sizing) fall under Stage 5, so does our study in this chapter.

The supplier selection in its nature is a multi-criteria decision-making (MCDM) problem since some conflicting criteria have influence on evaluation and selection of suppliers (Dickson, 1966; Aissaoui et al., 2007). By sending a questionnaire to 273 purchasing agents and managers in the United States and Canada, Dickson (1966) identified and ranked 23 criteria for supplier selection problems (SSP). The top six criteria were respectively quality, delivery, performance history, warranty policy, production facilities and capacity, and price. The existence of various criteria with different importance contributes to the added complexity of SSP (Wang and Yang, 2009). However in practice, the importance of those criteria may change from one industry to another. In the studies that have employed mathematical programming for SSP, price, defects and lead-time are widely used as the top three criteria influencing supplier selection (Roa and Kiser, 1980; Weber and Current, 1993; Ghodsypour and O'Brien, 1998; Kumar et al., 2004, 2006; Wadhwa and Ravindran (2007); Amid et al., 2006, 2009, 2011). The reason for choosing these three criteria from the top six list presented by Dickson (1966) is mainly because they are readily quantifiable. Other criteria that appear in the top six list include performance history and warranty policy, which are primarily qualitative measures. In this chapter, we also consider price, defects and lead-time as the objective of our model while the capacity of production facilities is considered as a constraint.

There exist two kinds of SSP: single- and multiple-sourcing scenarios. In the first scenario, almost all suppliers are capable of meeting the buyer's needs, and therefore, the buyer needs to select the best supplier. In the second kind, limitations on quality, capacity, price, delivery, etc. force the buyer to purchase the same item from more than one supplier. Applying multiple-sourcing scenario is a practical way for ensuring the reliability of a manufacturer's supply stream (Aissaoui et al., 2007). In multiple-sourcing scenario, a buyer needs to make a decision on how much should be purchased from which supplier. DMs or managers usually make a decision analytically or intuitively (Simon, 1987; Sadler-Smith and Shefy, 2004). Sadler-Smith and Shefy (2004) discussed that analytical decision making is more advisable for well-structured and routine situations, while intuitive decision making is wiser in loosely structured situations. SSP is typically quantitative and deliberative and according to Simon (1987) this kind of problem is greatly impacted by analytical decision making. Aissaoui et al. (2007) also concluded that mathematical programming is the most appropriate tool that can be used to model such problems. Therefore, the structured multi-sourcing SSP that we study here can be modeled as a Multi-Objective Optimization Problem (MOOP).

For solving MOOP, two different cases can be considered. In the first case, DMs first determine the precise goal value for each objective function that can be considered as a crisp goal (crisp MOOP), and then techniques such as Weighted Goal Programming (WGP) is used to solve the problem. In the second case, the DMs first determine a weight for each goal that can be considered as a fuzzy goal (fuzzy MOOP), and then techniques such as min–max GP (MGP) also known as fuzzy GP can be used to solve the problem.

1.1. Crisp MOOP

There are some techniques used for solving MOOP in which the goal of each objective is precisely determined. Ustun and Demirtas (2008a) proposed an integrated multi-period Multi-Objective (MO) model for SSP and order allocation. To solve the MO problem, they used ε-constraint method, a reservation level driven Tchebycheff procedure (RLTP) and Preemptive GP, and then compared the results of these three techniques. In ε-constraint method, DMs select one of the objectives as a single-objective and put other objectives in constraints. That is, the DMs do not determine any goal. In RLTP, the weighted distance of an achieved objective from its reference, which is almost equal to the positive ideal solution (PIS), is minimized. For maximization problems, the achieved objective should also be greater than reservation levels that are adjusted by the DMs from one iteration to another. Even if the weighted distances become normalized and equal weights are assigned to them, the method cannot guarantee that the achieved objectives have a proportional distance from their references. By preemptive goal programming, after determining the goal of each objective, DMs place the goals into different priority levels, so that the goals of a higher priority level is satisfied first. Clearly, since the objectives are not considered simultaneously, the consistency between the achieved objectives and their goals cannot be guaranteed. In another study, Ustun and Demirtas (2008b) defined an additive achievement function by combining MGP and Archimedean GP (AGP, also known as WGP) to solve the MO problem. An integrated MO mixed integer linear programming model was also proposed by Jolai et al. (2011) for SSP and order allocation. They also used WGP to solve their model. The WGP method simultaneously tries to minimize the objectives' weighted deviations from their goals. Similar to RLTP, there may be at least one objective whose weighted deviations are dominating others. Therefore, the model is biased towards this objective and neglects others. As a result, WGP cannot guarantee the achieved objectives to be consistent with their weights.

1.2. Fuzzy MOOP

In reality, the input data and information related to suppliers and the market is not always precisely known to the buyers, and thus, researchers often employ fuzzy set theory as the best tool for handling SSP in an uncertain environment (Kumar et al., 2004, 2006; Amid et al., 2006, 2009, 2011). Fuzzy set theory was first introduced by Zadeh (1965). In contrast to the classical set theory, in which the membership of an element to a set is a binary (0, 1) term, the fuzzy set theory allows this membership to be from a real interval [0, 1]. In other words, in the classical set theory, an element is either a member of a set or a non-member. In the fuzzy set theory, however, an element can be considered a member with a certain degree of membership, while also being a non-member. For example, a buyer may consider a batch of products with a zero percent defective rate as a member of a perfect set with a membership value of 1. In addition, if the batch contains 5% or more defective items, the buyer would no longer consider it as a member of the perfect set (i.e., the membership value is zero). If the membership function for this example is linear as shown in the following figure, a batch with 1% defective rate can have a membership value of 0.8.

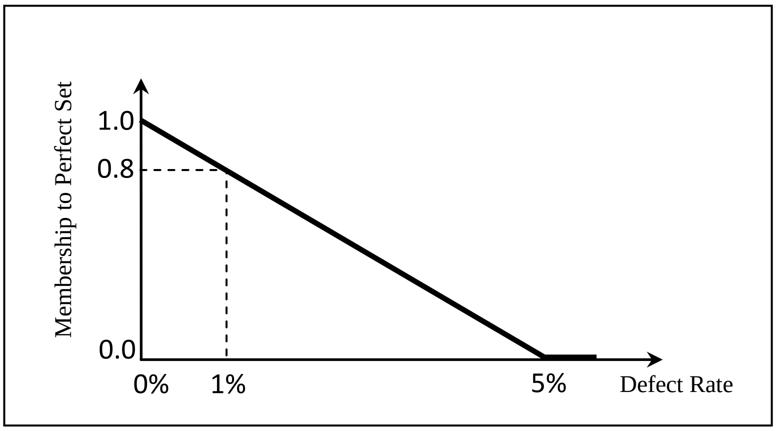


Fig. 1. An example of a linear membership in fuzzy set theory.

Kumar et al. (2004, 2006) formulated a fuzzy mixed integer goal programming for multiple sourcing SSP including three fuzzy goals: cost, quality and delivery subject to buyer's demand, suppliers' capacity, etc. They used Zimmermann's (1978) max–min technique to solve the multi-objective problem. However, the technique of Zimmermann was not able to consider the weight of the three objectives. In real situations, objectives (or criteria) have various weights related to strategies of the purchasing department (Wang et al., 2004; Amid et al., 2006, 2009, 2011). To cope with the problem, Amid et al. (2006, 2009) formulated a fuzzy MO model for SSP including three fuzzy goals: cost, quality and delivery subject to capacity restriction and market demand. In order to be capable of taking into account the objectives' weight, they used the additive model of Tiwari et al. (1987) for solving the MO problem. Lin (2004) argued that by using the additive model of Tiwari et al., the achievement levels of objective functions are not necessarily consistent with their weights because an objective with a higher weight receives a higher achieved level than others. Lin (2004) subsequently proposed a weighted max–min model (WMM) for solving fuzzy MO problems. This approach was later applied by Amid et al. (2011) to a fuzzy MO SSP with three fuzzy goals: cost, quality and delivery subject to capacity and demand requirement constraints.

Other techniques that are applicable when the objectives have different weights are fuzzy or min–max GP (MGP), weighted objectives (WO), compromise programming (CP) (see Wadhwa and Ravindran, 2007 for WO and CP) and TOPSIS (Technique for Order Preference by Similarity to Ideal Solution) (see Abo-Sinna et al., 2008). The CP approach attempts to bring the solution close to the PIS. On the other hand, TOPSIS, which was developed for solving multi-attribute decision making problems (MADM) by Hwang and Yoon (1981), has another concept: the solution should be close to the PIS and far from the negative ideal solution (NIS). In fact, for constructing the TOPSIS method, we only need to extend CP to consider NIS. Although TOPSIS has more computational complexity than other MO optimization techniques such as WO, MGP, etc., it has been recently utilized to solve MO problems in some studies (Abo-Sinna, 2000; Abo-Sinna and Amer, 2005; Abo-Sinna and Abou-El-Enien, 2006; Abo-Sinna et al., 2008). However, these studies do not justify using such a method for solving MO problems. Hence, the question is whether using TOPSIS for solving MO problems can also result in the achievement levels of objective functions to be consistent with their weights. This question is answered in a numerical example later.

Similar to the model of Tiwari et al. (1987), MGP, WO, CP and TOPSIS also focus on the objective with a higher weight and neglect other objectives. As a result, the achievement levels of objective functions are not necessarily consistent with their weights. In the following section, we elaborate the concept of the consistency that is the focus of this chapter.

1.3. The consistency concept

In Crisp MOOPs, the solution is consistent if the achieved objectives have a proportional distance from their goals. In other words, when the actual achieved objective of one criterion is obtained, say in the middle of its PIS and goal, other objectives should do so; otherwise, the solution is not consistent. By the following example, this concept is better illustrated. Consider f_k^+ as the best value (or PIS) and f_k^- as the worst value (or NIS) for the k^{th} minimization objective k=1,2,3. Assume that f_1^+=50, f_1^-=80, f_1^*=60, f_2^+=300, f_2^-=400, f_2^*=370, f_3^+=150, f_3^-=300, and f_3^*=150. In addition, assume that after solving the MOOP, the actual achieved objectives are obtained as f_1 =65, f_2 =377.5 and f_3 =187.5. Here, we see that the ratio of $(f_k - f_k^*)/(f_k^- - f_k^*)$ for the three objectives are the same and equal to 0.25. That is, the distance of achieved objectives from their goals is proportional. Therefore, we can say that the actual achieved objectives are consistent with their goals. This concept is also illustrated in Figure. 2.

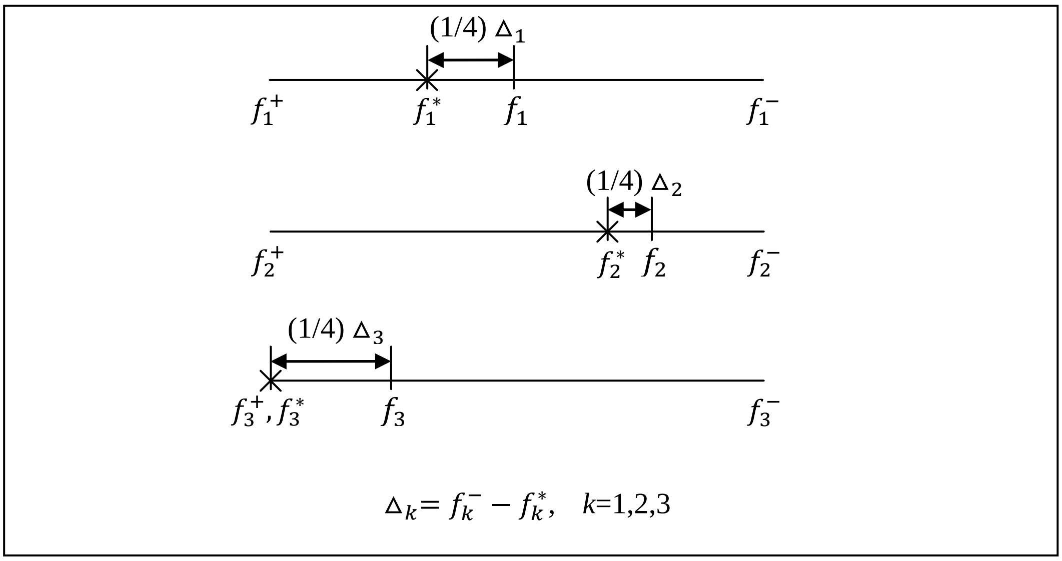


Fig. 2. The concept of consistency.

The techniques used for solving crisp MOOPs in the literature are not able to guarantee the consistency between the achieved objectives and their goals. To address this issue, we first need to choose an appropriate technique. One of the well-known multi-objective techniques is Goal programming (GP) that for several reasons such as robustness, mathematical flexibility, and the possibility of introducing many system constraints, has been the most widely used technique for solving MO problems (Dhahri and Chabchoub, 2007; Chang, 2007; Jolai et al., 2011). In the GP approach, DMs require to specify the most desirable value or goal for each objective as the aspiration level. For finding the optimal solution, they subsequently minimize deviations from aspiration levels. There are three basic approaches to GP: 1) WGP, 2) lexiographic or preemptive GP, and 3) MGP. As discussed earlier, the first two approaches are used to solve crisp MOOPs and the last one is used for fuzzy MOOPs. Furthermore, WGP considers all objectives simultaneously in comparison to preemptive GP. Therefore, we adopt WGP in this chapter as a means to overcome the inconsistency issue.

1.4. The purpose of the study

In this article, we model a single-item SSP as a MOOP including three goals: cost, rejects and lead-time subject to suppliers' capacity and buyer's demand. We consider two different cases: *Case* 1) the crisp MOOP in which the goals or the aspiration levels of objectives are predetermined; *Case* 2) the fuzzy MOOP in which the weight of objectives, instead of the aspiration levels, are predetermined. The first part of the aim of this study is that the actual achieved objective, f_k, in Case 1 should be consistent with its goal, f_k^*. Recall the consistency example in previous subsection. When the actual achieved objectives are obtained as f_1=65, f_2=377.5 and f_3=187.5, we see that the ratio of $(f_k - f_k^*)/(f_k^- - f_k^*)$ for the three objectives are the same and equal to 0.25. Therefore, the first part of our aim is satisfied. The second part of the aim is that for minimization objectives, it is more desirable if some of the achieved objectives can be better than their consistency without damaging the consistency of other objectives. That is, if for example, f_3 approaches to 150 (i.e., $f_3 \leq 187.5$, thus better than its consistency) without disturbing the consistency of other objectives, a better solution is achieved.

The aforementioned aim is also applicable to Case 2 so that the actual achieved levels of the objectives are consistent with the weights, or, it is more desirable if some can be better than their consistency without damaging the consistency of other objectives.

Due to the widespread use of GP, in this study, we focus on WGP as a means to address the inconsistency issue in both crisp and fuzzy cases. Therefore, a Normalized GP (NGP) is first proposed for solving the problem of Case1 in order to satisfy the first part of the aim. Then, the method is relaxed (R-NGP) to allow the fulfillment of the second part. Finally, NGP and R-NGP are extended to solve the fuzzy problem of Case 2. In Case1, WGP and in Case 2, CP, TOPSIS, WO, MGP and WMM are used as comparators.

The rest of this chapter is organized as follows: In the subsequent section, the MO model for the SSP is presented. In Section 3, the NGP method is proposed and developed for solving the MO model. Section 4 provides a numerical example and compares the solution of the NGP with other aforementioned methods. In Section 5, feasibility of the NGP approach is discussed. Finally, conclusions are drawn in Section 5.

2. Multi-objective model for supplier selection and order allocation

The literature shows that price, rejects and lead-time are the most commonly used ordering decisions (Roa and Kiser, 1980; Weber and Current, 1993; Ghodsypour and O'Brien, 1998; Kumar et al., 2004, 2006; Amid et al., 2006, 2009, 2011). In this study, these three criteria are used in the single-item MO supplier selection model. It is assumed that the demand is known, and a set of approved suppliers with limited production capacity can satisfy the demand.

The notations used to formulate the problem under consideration can be stated as follows:

n	number of suppliers
X	vector of decision variables
x_i	(decision variable) number of units ordered from supplier i, $x_i \in X$
V_i	capacity of supplier i

C_i purchasing price of the product from supplier i

q_i expected defect rate of supplier i

F_i percentage of items delivered late by supplier i

D demand of the product

W_k relative importance of objective k

The MO model for procuring an item from multiple suppliers is formulated as follows:

$$\text{Min } f_1(X) = \sum_{i=1}^{n} C_i x_i \tag{1}$$

$$\text{Min } f_2(X) = \sum_{i=1}^{n} q_i x_i \tag{2}$$

$$\text{Min } f_3(X) = \sum_{i=1}^{n} F_i x_i \tag{3}$$

Subject to:

$$\sum_{i=1}^{n} x_i = D \tag{4}$$

$$x_i \leq V_i \qquad i = 1, 2, \ldots, n \tag{5}$$

$$x_i \geq 0 \qquad i = 1, 2, \ldots, n. \tag{6}$$

Eq. (1), Eq. (2) and Eq. (3) minimize total purchasing cost, the number of rejected items and the number of units that are late, respectively. The demand is satisfied by constraint (4). Constraint (5) ensures that the order quantity assigned to supplier i does not exceed its capacity.

As mentioned in the introduction, one of the most common techniques to GP is Weighted Goal Programming (WGP). WGP approach requires the DMs to determine the most desirable value or goal (f_k^*) for each objective as the aspiration level, and then attempts to minimize the deviations from goals. Since our proposed approach is based on WGP, we first introduce the above MO model using WGP as follows:

$$\text{Min } \sum_{k=1}^{3} W_k (d_k^- + d_k^+) \tag{7a}$$

Subject to:

$$f_k + d_k^- - d_k^+ = f_k^* \qquad k=1,2,3 \tag{7b}$$

$d_k^- . d_k^+ = 0, d_k^-, d_k^+ \geq 0$ $\quad k=1,2,3$ (7c)

(4), (5) and (6).

where d_k^- and d_k^+ are negative and positive goal deviations, respectively.

3. NGP approach

The idea of normalized goal programming (NGP) is not very new. Many researchers have addressed the need for normalization when goal programming is used. Tamiz et al. (1998) reviewed some techniques to deal with incommensurability. These techniques attempt to transfer different units of the deviational variables to a common unit in order to eliminate an unintentional bias towards a larger magnitude objective. Iskander (2012) takes a different normalization approach in which the achieved objectives are proportional to the weights initially decided by the DMs. These normalization approaches cannot guarantee that the achieved objectives are consistent with their goals. Lin (2004) focused on fuzzy MO problems and argued that, "When the DMs provides relative weights for fuzzy goals with corresponding membership functions, the ratio of the achieved levels should be as close to the ratio of the objective weights as possible to reflect their relative importance." In this study, we provide a different version of normalization and we argue that for deterministic GP, when the aspiration level for each objective is provided by DMs, the achieved objectives should be consistent with their aspiration levels as much as possible.

3.1. The R-NGP approach

In order for the achieved objectives, f_k, to be consistent with their goals, we need to eliminate the scale effect of different objectives. Thus, we normalize the deviations using $[(f_k^* - f_k^+)y_2 + (f_k^- - f_k^*)y_1 + (d_k^- - d_k^+)]/[(f_k^* - f_k^+)y_2 + (f_k^- - f_k^*)y_1] = \lambda$, in which $y_1 = 0$, $y_2 = 1$ if $\lambda > 1$; $y_1 = 1$, $y_2 = 0$ if $\lambda < 1$; and either y_1 or y_2 can equal to one if $\lambda = 1$, *k*=1,2,3 ($\lambda \in [0, 2]$). In addition, $f_k^+ = \{\min_{X \in S} f_k(X)\} \forall k$ is called the PIS or the best value and $f_k^- = \{\max_{X \in S} f_k(X)\} \forall k$ is called the NIS or the worst value for the k^{th} minimization objective, in which *S* is the feasible set.

Now, the R-NGP for Case 1 can be developed as follows:

Max λ (8a)

Subject to:

$f_k + d_k^- - d_k^+ \leq f_k^*$, *k*=1,2,3, (8b)

$[(f_k^* - f_k^+)y_2 + (f_k^- - f_k^*)y_1 + (d_k^- - d_k^+)]/[(f_k^* - f_k^+)y_2 + (f_k^- - f_k^*)y_1] = \lambda$, *k*=1,2,3, (8c)

$d_k^- . d_k^+ = 0$, $d_k^-, d_k^+ \geq 0$, *k*=1,2,3, (8d)

$\lambda - 1 \leq y_2 \leq \lambda$, (8e)

$y_1 + y_2 = 1$, (8f)

$y_1, y_2 \in \{0, 1\}$, (8g)

(4), (5) and (6).

To differentiate this model from model (7), this model is called R-NGP as the equality constraint of (7b) is relaxed to the inequality constraint of (8b). Also note that, if needed, DMs can present their preferences among the goals through assigning relative weights to each objective by multiplying W_k by $(d_k^- - d_k^+)$, *k*=1,2,3, in Eq. (8c).

To evaluate the consistency of the solution, we introduce here a consistency ratio as $R_k^+ = y_1(f_k - f_k^*)/(f_k^- - f_k^*)$ and $R_k^- = y_2(f_k^* - f_k)/(f_k^* - f_k^+)$. The consistency is achieved if for $\lambda < 1$, $R_1^+ = R_2^+ = R_3^+ > 0$ and $R_k^- = 0$, $k = 1,2,3$, and for $\lambda > 1$, $R_1^- = R_2^- = R_3^- > 0$ and $R_k^+ = 0, k = 1,2,3$. For $\lambda = 1$, either R_k^+ or R_k^-, $k = 1,2,3$, is equal to zero, depending on y_1 and y_2. When the consistency ratios of the three objectives are equal, the distances of achieved objectives from their goals are proportional.

For the R-NGP model, we prove the following proposition.

Proposition 1. *Equation* (8c) *normalizes the deviations* (d_k^-, d_k^+) *so that the actual achieved objectives obtained by R-NGP are consistent with their goals or better*.

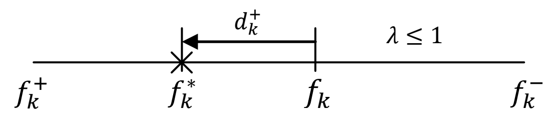
Proof. To prove proposition 1, we consider three cases:

If $\lambda = 1$, then $y_2 = 0$, $y_1 = 1$ and (8c) becomes $[(f_k^- - f_k^*) + (d_k^- - d_k^+)]/[(f_k^- - f_k^*)] = 1$. (Note that in this case, either binary variable can be one that has no impact on the solution.) Therefore, $d_k^- = d_k^+ = 0$ for all k, and (according to (7b)) all achieved objectives are equal to their goals.

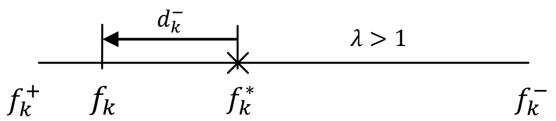
If $\lambda < 1$ (assume $\lambda = 0.5$), then $y_2 = 0$, $y_1 = 1$ and (8c) becomes $[(f_k^- - f_k^*) + (d_k^- - d_k^+)]/[(f_k^- - f_k^*)] = 0.5$. Then $(d_k^- - d_k^+) = 0.5(f_k^- - f_k^*) - (f_k^- - f_k^*)$. Since the right hand side is negative, for all k, we have $d_k^- = 0$ and $d_k^+ = (f_k^- - f_k^*) - 0.5(f_k^- - f_k^*) \geq 0$. That is, for all k the achieved objective is in the middle of f_k^- and f_k^*. In an extreme case where $\lambda = 0$, the achieved value of all objective functions is in the farthest point of f_k^*, i.e., f_k^-. In other words, for $\lambda < 1$, $f_k = f_k^* + (\Delta_k^- - \lambda\Delta_k^-)$, $k = 1,2,3$ where $\Delta_k^- = f_k^- - f_k^*$.

Similarly, if $\lambda > 1$ (assume $\lambda = 1.5$), the achieved value of all objective functions is in the middle of f_k^+ and f_k^*. In other words, for $\lambda > 1$, $f_k = f_k^* - (\lambda\Delta_k^+ - \Delta_k^+)$, $k = 1,2,3$ where $\Delta_k^+ = f_k^* - f_k^+$.

By considering these three cases, it is demonstrated that the deviations are normalized since the distance of all achieved objectives from their goals are proportional to each other, and thus, consistent with their goals. For minimization objectives, as is the case here, buyers are more concerned with minimizing positive goal deviations (d_k^+) and maximizing negative ones (d_k^-) in order to make achieved objectives closer to their PIS (f_k^+). In addition, a less achieved objective from its consistency is more desirable for minimization objectives. For this reason, we maximize λ and we change the equality constraint of (7b) to the inequality constraint of (8b) (see Figure. 3). □



For $\lambda \leq 1$, as λ increases, d_k^+ reduces and the achieved objective approaches its goal.



For $\lambda > 1$, as λ increases, d_k^- increases and the achieved objective approaches its PIS.

Fig. 3. The effect of increased λ on achieved objective.

3.2. The fuzzy R-NGP approach

The problem (1)–(6) with fuzzy goals maybe presented as follows (Zimmermann, 1978):

$$\text{Min } \tilde{f}_1(X) = \sum_{i=1}^{n} C_i x_i \lesssim f_1^0 \tag{1a}$$

$$\text{Min } \tilde{f}_2(X) = \sum_{i=1}^{n} q_i x_i \lesssim f_2^0 \tag{2a}$$

$$\text{Min } \tilde{f}_3(X) = \sum_{i=1}^{n} F_i x_i \lesssim f_3^0 \tag{3a}$$

Subject to:

$$\sum_{i=1}^{n} x_i = D \tag{4}$$

$$x_i \leq V_i \qquad i = 1, 2, \ldots, n \tag{5}$$

$$x_i \geq 0 \qquad i = 1, 2, \ldots, n. \tag{6}$$

In the above objective functions, the fuzzified version of $\leq$ is denoted by the symbol $\lesssim$, and f_k^0, k=1,2,3, is the fuzzy goal or aspiration level for k^{th} objective that is determined by the DMs. Due to the conflict of the three fuzzy objectives, they may not be achieved concurrently according to the constraints of the model, and the DMs might hence define a membership function $\mu_{f_k}(X)$ for every objective to determine the achieved levels (Amid et al. 2006, 2009, 2011) as follows:

$$\mu_{f_k}(X) = \begin{cases} 1 & f_k \leq f_k^+ \\ (f_k^- - f_k)/(f_k^- - f_k^+) & f_k^+ \leq f_k \leq f_k^- \\ 0 & f_k \geq f_k^- \end{cases} \tag{9}$$

Now, we easily extend R-NGP for solving fuzzy MO problems, where the weight of objectives is predetermined (Case 2). The actual achieved level of each objective is obtained using Eq. (9) (Lin, 2004). Therefore, if $\mu_{f_k^*}(X) = W_k$, in which W_k is the weight of each objectiveand $\mu_{f_k^*}(X) = (f_k^- - f_k^*)/(f_k^- - f_k^+)$, the aspiration level for objective k can be equivalent to $f_k^* = f_k^- - [W_k(f_k^- - f_k^+)]$. As a result, the Fuzzy R-NGP model is constructed as follows:

$$\text{Max } \lambda \tag{10a}$$

Subject to:

$$f_k + d_k^- - d_k^+ \leq f_k^- - [W_k(f_k^- - f_k^+)], \qquad k=1,2,3, \tag{10b}$$

$$[(f_k^* - f_k^+)y_2 + (f_k^- - f_k^*)y_1 + (d_k^- - d_k^+)]/[(f_k^* - f_k^+)y_2 + (f_k^- - f_k^*)y_1] = \lambda \quad , \quad k=1,2,3, \tag{10c}$$

$$d_k^-.d_k^+ = 0, d_k^-, d_k^+ \geq 0, \qquad k=1,2,3, \tag{10d}$$

$$\lambda - 1 \leq y_2 \leq \lambda, \tag{10e}$$

$$y_1 + y_2 = 1, \tag{10f}$$

$$y_1, y_2 \in \{0, 1\}, \tag{10g}$$

(4), (5) and (6).

where$\sum_{k=1}^{3} W_k$=1.

4. An illustrative example

In this section, the following numerical example is used to illustrate how the proposed NGP can be employed to solve both crisp and fuzzy MOOPs. The example is divided into two cases. In the first case, we consider a problem in which the goals or aspiration levels for each objective are predetermined (crisp MOOP). The problem is solved by WGP, NGP and R-NGP approaches for comparison. In the second case, a fuzzy MOOP in which the objectives have different levels of importance (instead of their goals) are considered. CP, TOPSIS, WO, MGP and WMM are also used to compare the solutions in Case 2.

In this example, three suppliers are going to meet a predicted buyer's demand of 5,000 units. Table 1 provides the suppliers' information: prices (C_i in \$), defect rate (q_i), late delivery rate (F_i) and capacity (V_i).

Table 1.
Data for the numerical example.

Supplier	Capacity (units)	Price (\$)	Defect Rate (%)	Late Delivery (%)
S_1	2,500	6.5	0.10	0.45
S_2	2,500	5.5	0.30	0.40
S_3	2,500	6	0.20	0.60

For the given data, the following crisp formulation is obtained:

$$\text{Min } f_1(X) = 6.5x_1 + 5.5x_2 + 6x_3$$
$$\text{Min } f_2(X) = 0.001x_1 + 0.003x_2 + 0.002x_3$$
$$\text{Min } f_3(X) = 0.0045x_1 + 0.004x_2 + 0.006x_3$$

Subject to

$$x_1 + x_2 + x_3 = 5{,}000$$
$$x_i \leq 2{,}500 \qquad i = 1, 2, 3$$
$$x_i \geq 0 \qquad i = 1, 2, 3.$$

Before we apply cases 1 and 2, we need to obtain PIS (or lower bound) and NIS (or upper bound) for all three objectives by solving each objective function as a single objective:

PIS: f^+ = (28,750, 7.5, 21.25)

NIS: f^-= (31,250, 12.5, 26.25)

Case1: The NGP approach for crisp MOOP

This case considers models (7) and (8), WGP and NGP, respectively. The goals of cost, quality and delivery are assumed to be f_1^*=29,500, f_2^*=9 and f_3^*=22, respectively. For WGP, it is assumed that $W_1 = W_2 = W_3$ =1/3. To solve the problem, GAMS or Solver is employed and the optimal solutions of the three methods are compared in Table 2.

Table 2.
Solution to Case 1.

	NGP	R-NGP	WGP
f_1	30,000	30,000	29,500
f_2	10.00	10.00	11.00
f_3	23.21	21.25	22.75
X_1	1,938.77	2,500.00	1,500.00
X_2	1,938.77	2,500.00	2,500.00
X_3	1,122.45	0.00	1,000.00
R_1^+	0.29	0.29	0.00
R_2^+	0.29	0.29	0.57
R_3^+	0.29	-0.18	0.18

By considering the consistency ratio, R_k^+, Table 2 shows that in NGP, the actual achieved objectives are consistent with their goals. In addition, the value of the third objective improved from 23.28 in NGP, to 21.25 in R-NGP, without disturbing the consistency of objectives 1 and 2 (i.e. $R_1^+ = R_2^+ = 0.29$). Hence, it is demonstrated that the R-NGP is able to preserve the second part of our aim. However, WGP fully achieved the first objective, but not the other two objectives. Therefore, there is no consistency among the objectives. The reason is that WGP only minimizes the deviations of an objective that its $(d_k^- + d_k^+)$ prevails others.

Case 2: The fuzzy NGP approach for MOOP

To demonstrate that NGP is applicable for fuzzy situations, we compare the solution of NGP with those of CP, TOPSIS, MGP, WO and WMM. First, it is assumed that the weights of cost, quality and delivery are W_1=0.6, W_2=0.3 and W_3=0.1, respectively. Based on fuzzy R-NGP model, the problem is formulated as follows:

Max λ

Subject to

$6.5x_1 + 5.5x_2 + 6x_3 + d_1^- - d_1^+ \leq f_1^- - [0.6\ (f_1^- - f_1^+)]$

$0.001x_1 + 0.003x_2 + 0.002x_3 + d_2^- - d_2^+ \leq f_2^- - [0.3\ (f_2^- - f_2^+)]$

$0.0045x_1 + 0.004x_2 + 0.006x_3 + d_3^- - d_3^+ \leq f_3^- - [0.1\ (f_3^- - f_3^+)]$

$[(f_k^* - f_k^+)y_2 + (f_k^- - f_k^*)y_1 + (d_k^- - d_k^+)]/[(f_k^* - f_k^+)y_2 + (f_k^- - f_k^*)y_1] = \lambda$ *k*=1,2,3

$d_k^- . d_k^+ = 0,\ d_k^-, d_k^+ \geq 0$ *k*=1,2,3

$\lambda - 1 \leq y_2 \leq \lambda$

$y_1 + y_2 = 1$

$y_1, y_2 \in \{0, 1\}$

$x_1 + x_2 + x_3 = 5{,}000$

$x_i \leq 2{,}500$ *i*=1,2,3

$x_i \geq 0$ *i*=1,2,3.

Using GAMS or Solver, the optimal solution of fuzzy NGP is calculated as x_1=909, x_2=1,591, x_3=2,500, μ_{f_1}=0.64, μ_{f_2}=0.36 and μ_{f_3}=0.20. Also, the optimal solution of fuzzy R-NGP is calculated as x_1=1,818, x_2=2,500, x_3=682, μ_{f_1}=0.64, μ_{f_2}=0.36 and μ_{f_3}=0.81.

For the latter, it can be seen again that the actual achieved level of objectives 1 and 2 are consistent with their relative importance (W_k), and μ_{f_3} is better than its consistency. Therefore, the solution of fuzzy R-NGP satisfies the second part of our aim.

The solutions obtained by the other aforementioned approaches are compared in Table 3.

Table 3.
Comparison of solutions obtained by different approaches (W_1=0.6, W_2=0.3, W_3=0.1).

	Fuzzy NGP	Fuzzy R-NGP	WMM	MGP	WO	TOPSIS(p=2)	CP(p=2)
f_1	29,659	29,659	29,583	28,750	28,750	28,955	29,286
f_2	10.68	10.68	10.83	12.50	12.50	12.09	11.43
f_3	25.45	22.27	25.42	25.00	25.00	24.39	23.39
μ_{f_1}	0.64	0.64	0.67	1.00	1.00	0.92	0.79
μ_{f_2}	0.36	0.36	0.33	0.00	0.00	0.08	0.21
μ_{f_3}	0.20	0.81	0.17	0.25	0.25	0.37	0.57

Table 3 shows that the solutions obtained by CP, TOPSIS, MGP and WO may not be acceptable because there is no consistency between weights and the achieved levels of objectives. Similar to the argument of Lin (2004), we can say that these methods only minimize the heaviest-weight objective, and others may be neglected. This is evident in the above example where for all methods the first objective achieved a higher level, while the second objective despite of having a higher weight than the third one, achieved a lower level. However, the solution of WMM and NGP can be acceptable since the achieved levels of objectives are consistent with their weights. Furthermore, fuzzy R-NGP improves the third objective without disturbing the consistency of the other two objectives. Therefore, the solution fulfills the second part of our main aim.

The above methods are also compared via three other different combinations of weights as presented in Table 4:

Table 4.
Numerical example solutions using different combinations of weight values.

	Fuzzy NGP	Fuzzy R-NGP	WMM	MGP	WO	TOPSIS	CP
	The actual achieved level of objectives, μ_{f_k}						
W_1=0.6	0.636	0.636	0.667	1.000	1.000	0.918	0.786
W_2=0.3	0.364	0.364	0.333	0.000	0.000	0.082	0.214
W_3=0.1	0.199	0.805	0.167	0.250	0.250	0.374	0.571
W_1=0.3	0.500	0.500	0.500	0.500	0.500	0.500	0.500
W_2=0.3	0.500	0.500	0.500	0.500	0.500	0.500	0.500
W_3=0.3	0.500	0.500	0.500	1.000	1.000	1.000	1.000
W_1=0.3	0.417	0.417	0.375	0.500	0.500	0.340	0.340
W_2=0.5	0.583	0.583	0.625	0.500	0.500	0.660	0.660
W_3=0.2	0.333	0.841	0.250	1.000	1.000	0.680	0.680
W_1=0.1	0.182	0.200	0.111	0.000	0.000	0.017	0.043
W_2=0.8	0.818	0.800	0.889	1.000	1.000	0.983	0.957
W_3=0.1	0.182	0.306	0.222	0.000	0.000	0.033	0.087

The summary of comparison of the two cases is presented as follows:

Case 1, crisp model:

- Unlike NGP, WGP was not able to make the achieved objectives consistent with their goals. In addition, R-NGP showed its ability to achieve the second part of the aim.

Case 2, fuzzy model:

- Despite the computational complexity of TOPSIS and CP for solving the fuzzy MOOP, their solutions may be less desirable when the consistency of objectives is important. The same argument can be made for MGP and WO methods, despite the fact that they do not have the complexity of TOPSIS and CP.

- Among comparators, WMM is the only method that can compete with fuzzy NGP and fuzzy R-NGP in maintaining consistency of objectives. However, WMM has not been applied to crisp models.

5. Discussion on the solution feasibility

5.1. Feasibility of the NGP approach

In the WGP approach, model (7), the deviations can get any value resulting in a feasible solution. However, in our new model, NGP, all deviations are simultaneously restricted to be proportional to λ. For example, when $\lambda < 1$, $d_k^+ = \Delta_k^- - \lambda\Delta_k^-$ and d_k^-=0 for all $k = 1,2,3$, where $\Delta_k^-= f_k^- - f_k^*$. Such restrictions may cause the solution of NGP to be infeasible in some cases such as below.

Suppose that there are three objectives in which the first two objectives are completely in agreement with each other while they are completely in conflict with the third one. In addition, assume that the DMs set f_1^*=f_1^+ and f_k^*=f_k^-, k=2,3. The NGP cannot find a feasible solution for this case as explained below.

1. Assume that $\lambda \leq 1$, then $d_k^+ \geq 0$ and $d_k^- = 0$, k=1,2,3. However, since f_k^*=f_k^-, k=2,3, then d_k^+=0 as well (i.e., $d_k^+, d_k^- = 0$, k=2,3).
   1.1. If $\lambda = 1$, then d_k^+=0 for all k and, as a result, we have f_1=f_1^*=f_1^+ and f_k=f_k^*=f_k^-, k=2,3. If the first objective equals f_1^+, the second objective should also be equal to f_2^+since they are both in agreement with each other. However, according to the NGP model, the second objective must be equal to f_2^-since $d_2^+, d_2^- = 0$. As a result, the model becomes infeasible.
   1.2. If $\lambda = 0$, then $d_k^+ = f_k^- - f_k^*$ (because $d_k^+ = \Delta_k^- - \lambda\Delta_k^-$) for all k and, as a result, we have f_k=$f_k^- - f_k^* + f_k^*$=f_k^-, k=1,2,3. Again, if the first and the second objectives move towards their worst solutions, the third objective should do the opposite, which is impossible (because according to the NGP model f_k=f_k^-, k=1,2,3). Therefore, here the model is again infeasible. By similar argument, we can conclude that there is no feasible solution when $0 < \lambda < 1$.

2. Assume that $\lambda > 1$, then $d_k^- \geq 0$ and $d_k^+ = 0$, k=1,2,3. Since f_1^*=f_1^+ then d_1^-=0 as well (i.e., $d_1^+, d_1^- = 0$). Since $d_2^- \geq 0$, f_2 can approach to f_2^+. On the other hand, as d_2^- increases, we expect to see d_3^- also increases, due to the consistency rule, resulting in f_3 to approach $f_3^* = f_3^+$. However, this is not possible since the third objective is in conflict with the first and the second objectives, hence, another infeasible solution.

The above infeasibility has been addressed in the R-NGP approach by replacing the equality of Eq. (7b) with "≤" in (8b). The following numerical example is intended to clarify the above discussion.

5.2. Numerical example

Table 5 shows the data of three suppliers, where the first and the second objectives are in agreement with each other while they are in conflict with the third objective. The demand is 5,000 units.

Table 5.
Data for suppliers.

Supplier	Capacity	Price ($)	Defect Rate (%)	Late Delivery (%)
S_1	2,500	6.5	0.30	0.40
S_2	2,500	6.0	0.20	0.45
S_3	2,500	5.5	0.10	0.60

Here, we need to obtain PIS and NIS for all three objectives by solving each objective function as a single objective:

$$\text{PIS: } f^{+} = (28{,}750, 7.5, 21.25)$$

$$\text{NIS: } f^{-} = (31{,}250, 12.5, 26.25)$$

If the DM sets the goals such that $f_1^{*}=f_1^{+}$, $f_2^{*}=f_2^{-}$ and $f_3^{*}=f_3^{-}$, solving the problem by NGP results in an infeasible solution, while the solution of R-NGP is feasible as follows:

x_1=0 x_2=2,500 x_3=2,500

f_1=28,750 f_2=7.5 f_3=26.25

λ=1

As can be seen, the first and the third objectives are consistent and equal to their goals and the second objective is better than its consistency.

In the above problem, if the DM sets the goals such that f_1^*=f_1^+, f_2^*=f_2^- and f_3^*=f_3^+, by a similar discussion we can conclude that the solution of NGP is infeasible. Using Solver for R-NGP, we can obtain a feasible solution as follows:

x_1=1,250 x_2=2,500 x_3=1,250

f_1=30,000 f_2=10 f_3=23.75

λ=0.5

As can be seen, the first and the third objectives are consistent with their goals and the second objective is better than its consistency.

6. Concluding remarks

In this article, the supplier selection problem was modeled as a MOOP with three minimization objective functions: price, rejects and lead-time. Then, we proposed the NGP method to solve the problem in two different cases: Case 1) a crisp MOOP in which the aspiration levels (or goals) of objectives are predetermined; Case 2) a fuzzy MOOP in which the weights of objectives, instead of the goals, are predetermined.

It was the main aim of our model that, for minimization objectives, the actual achieved objectives in Case 1 be consistent with their goals (first part), or preferably better than their consistency if this could be achieved without disturbing the consistency of other objectives (second part). The aim was also applicable to Case 2.

The NGP was first developed for solving the crisp problem of Case1. Then, the method was relaxed (R-NGP) for fulfilling the second part. In real problems, the input data and information related to suppliers and market is not always firmly known to the buyers, resulting in goals and sometimes their relative importance not to be known precisely. To overcome the difficulty of imprecise goals in such uncertain environments, we employed fuzzy set theory to extend our proposed approach (fuzzy MOOP of Case 2). This can be seen as a strength of this study since DMs can implement our proposed approach in any MOOP, in which the goals can be expressed in two ways: precise and imprecise. Since we did not discuss the imprecise relative importance of goals here, this remains as an avenue for future research.

In Case1, WGP and in Case 2, CP, TOPSIS, WO, MGP and WMM were used to compare solutions. An illustrative example was used to present the developed models. The result revealed that, while comparators CP, TOPSIS, WO and MGP were not able to guarantee the consistency, in both cases NGP and R-NGP were able to make achieve objectives consistent with their goals. Moreover, R-NGP was able to maintain the second part of the aim. This can be seen as another strength of this study as we can effectively incorporate DMs' preference in the decision making process.

It is worthwhile to mention that while the fuzzy MOOP of Case 2 makes the approach one step closer to reality, further adjustments in the model may be needed when we apply this approach to real world problems. For example, mathematical programming normally considers tangible criteria only, while SSP may be affected by both tangible and intangible factors in reality. In order to take into consideration both factors, AHP or ANP can be integrated with mathematical programming, as reviewed in the Introduction section. Demirtas and Ustun's (2008) study is a very good example for practical SSP, where our NGP approach can be employed to solve their MO problem. Setting meetings for DMs to express their preferences and judgments on the decision making process can be considered as another adjustment.

References

Abo-Sinna M.A., Abou-El-Enien, T.H.M., 2006. An interactive algorithm for large scale multiple objective programming problems with fuzzy parameters through TOPSIS approach. Applied Mathematics and Computation 177 (2), 515-527.

Abo-Sinna, M.A., 2000.Extensions of the TOPSIS for multi-objective dynamic programming problems under fuzziness. Advances in Modelling and Analysis 43 (4), 1-24.

Abo-Sinna, M.A., Amer, A.H., 2005.Extensions of TOPSIS for multi-objective large-scale nonlinear programming problems, Applied Mathematics and Computation 162 (1), 243-256.

Abo-Sinna, M.A., Amer, A.H., Ibrahim, A. S., 2008. Extensions of TOPSIS for large scale multi-objective non-linear programming problems with block angular structure. Applied Mathematical Modelling 32 (3), 292-302.

Aissaoui, N., Haouari, M., Hassini, E., 2007. Supplier selection and order lot sizing modeling: A review. European Journal of Operational Research 34 (12), 3537-3540.

Amid, A., Ghodsypour, S.H., O'Brien, C., 2006. Fuzzy multiobjective linear model for the supplier selection in a supply Chain. International Journal of Production Economics 104 (2), 394-407.

Amid, A., Ghodsypour, S.H., O'Brien, C., 2009. A weighted additive fuzzy multi-objective model for the supplier selection problem under price breaks in a supply Chain. International Journal of Production Economics 121 (2), 323-332.

Amid, A., Ghodsypour, S.H., O'Brien, C., 2011. A weighted max–min model for fuzzy multi-objective supplier selection in a supply chain. International Journal of Production Economics 131 (1), 139-145.

Chang, C.T., 2007b. Binary fuzzy goal programming. European Journal of Operational Research 180 (1), 29-37.

Demirtas, E.A., Ustun, O., 2008. An integrated multiobjective decision making process for supplier selection and order allocation. Omega 36 (1), 76-90.

Demirtas, E.A., Ustun, O., 2009. Analytic network process and multi-period goal programming integration in purchasing decisions. Computers and Industrial Engineering 56 (2), 677-690.

Dhahri, I., Chabchoub, H., 2007. Nonlinear goal programming models quantifying the bullwhip effect in supply chain based on ARIMA parameters. European Journal of Operational Research 177 (3), 1800-1810.

Dickson, G.W., 1966. An analysis of vendor selection: systems and decisions. Journal of Purchasing 2 (1), 5-17.

Dobler, D.W., Lee, L., Burt, N., 1990. Purchasing and materials management: text and cases, McGraw-Hill, New York.

Ghodsypour, S.H., O'Brien, C., 1998. A decision support system for supplier selection using an integrated analytical hierarchy process and linear programming. International Journal of Production Economics 56–57, 199-212.

Hwang, C.L., Yoon, K.P., 1981. Multiple Attributes Decision Making: Methods and Applications. Springer, Berlin Heidelberg.

Iskander, M.G., 2012. A Suggested Approach for Solving Weighted Goal Programming Problem. American Journal of Computational and Applied Mathematics 2(2), 55-57

Jolai, F., Yazdian, S.A., Shahanaghi, K., Khojasteh, M.A., 2011. Integrating fuzzy TOPSIS and multi-period goal programming for purchasing multiple products from multiple suppliers. Journal of Purchasing & Supply Management 17 (1), 42-53.

Kumar, M., Vart, P., Shankar, R., 2004.A fuzzy goal programming approach for supplier selection problem in a supply chain. Computer and Industrial Engineering 46 (1), 69-85.

Kumar, M., Vart, P., Shankar, R., 2006.A fuzzy programming approach for vendor selection problem in a supply chain. International Journal of Production Economics 101 (2), 273-285.

Lin, C.C., 2004. A weighted max–min model for fuzzy goal programming. Fuzzy Sets and Systems 142 (3), 407-420.

Roa, C.P., Kiser, G.E., 1980. Educational buyers’ perceptions of vendor attributes. Journal of Purchasing and Materials Management 16(4), 25-30.

Sadler-Smith, E., Shefy, E., 2004. The Intuitive Executive: Understanding and Applying 'Gut Feel' in Decision-Making. Academy of Management Executive 18(4), 76-91.

Simon, H. A., 1987. Making Management Decisions: the Role of Intuition and Emotion. Academy of Management Executive 1(1), 57-64.

Tamiz, M., Jones, D., Romero, C., 1998. Goal programming for decision making: An overview of the current state-of-the-art. European Journal of Operational Research 111(3), 569-581.

Tiwari, R.N., Dharmahr, S., Rao, J.R., 1987. Fuzzy goal programming-an additive model. Fuzzy Sets and Systems 24(1), 27-34.

Ustun, O., Demirtas, E.A., 2008a, An integrated multi-objective decision-making process for multi-period lot-sizing with supplier selection. Omega 36 (4), 509-521.

Ustun, O., Demirtas, E.A., 2008b, Multi-period lot-sizing with supplier selection using achievement scalarizing functions. Computers and Industrial Engineering 54 (4), 918-931.

Wadhwa, V., Ravindran, A.R., 2007.Vendor selection in outsourcing. Computers and Operations Research 34 (12), 3725-3737.

Wang, G., Hang, S.H., Dismukes, J.P., 2004. Product-driven supply chain selection using integrated multi-criteria decision making methodology. International Journal of Production Economics 91(1),1-15.

Wang, T.Y., Yang, Y.-H., 2009.A fuzzy model for supplier selection in quantity discount environments. Expert Systems with Applications 36 (10), 12179-12187.

Weber, C.A., Current, J.R., 1993.A multiobjective approach to vender selection. European Journal of Operational Research 68 (2), 173-184.

Willis, H.T., Huston, R.C., Pohlkamp, F., 1993. Evaluation measures of just in time supplier performance. Production and Inventory Management 34 (2), 1-5.

Xia, W., Wu, Z., 2007.Supplier selection with multiple criteria in volume discount environments. Omega 35 (5), 494-504

Zadeh, L. A., 1965. Fuzzy sets. Information and Control 8 (3), 338-353.

Zimmermann, H.J., 1978. Fuzzy programming and linear programming with multiple objective functions. Fuzzy Sets and Systems 1, 45-55.

CHAPTER 3 – AN IMPROVED MULTI-CHOICE GOAL PROGRAMMING APPROACH FOR SUPPLIER SELECTION PROBLEMS

The main goal of this chapter is to propose a technique for solving multi-objective models so that decision makers can determine an interval goal for each objective (instead of a single goal) and can have more control on both the inside and the outside of the interval. In this study, a supplier selection problem is first modeled as a multi-objective optimization problem with three minimization objectives: price, rejects and lead-time. In reality, the objectives may have different relative weights. In addition, due to uncertainly/imprecision, it may be easier for decision makers to determine an interval goal for every objective, instead of a single goal. Also, the decision makers may have other preferences such as the purchasing cost not significantly exceeding the budget. For this purpose, a new Multi-Choice Goal Programming (MCGP) approach is proposed. One of the main advantages of the proposed model is that it provides decision makers with more control over their preferences. Finally, an illustrative example demonstrates the effectiveness of our proposed model.

1. Introduction

Today companies need to take advantage of any opportunity to increase their abilities for competing with their rivals. They should fulfill the expectations of customers for acquiring a high quality and low price product with a short lead-time delivery. It is also notable that for most industries up to 70% of the product cost comes from raw materials and component parts (Ghodsypour and O'Brien, 2001). In such environments, suppliers play a very important role for companies. When suppliers can provide companies with low price and high quality raw materials (or component parts) in a right time, the companies may also do so for their customers. As a result, different criteria such as price, quality and delivery should be considered at the time of evaluating suppliers (Dickson, 1966; Weber et al., 1991). Depending on the companies' strategy on purchasing, the supplier selection criteria may have different priorities (Wang et al., 2004).

Our study assumes that buyers have pre-evaluated all suppliers according to some criteria (such as financial strength, performance history, technical capability, geographical location, etc.) and now they need to further assess the pre-approved suppliers for order allocation based on some quantitative criteria such as price, quality and lead time. The order allocation exercise may result in either a *single sourcing scenario* if the best supplier has enough capacity to fulfill the buyer's demand, or a *multi-sourcing scenario* when the capacity limitation becomes an issue. The most suitable tool for Decision Makers (DMs) to formulate multi-supplier selection problem is mathematical programming (Aissaouia et al., 2005). Therefore, supplier selection problem (SSP) can be modeled as a multi-objective optimization problem subject to some constraints such as suppliers' capacity, buyer's demand, etc.

For multi-objective problems, the ideal solution for the DMs is to have the optimal objective values for each and every objective. However, this may not happen in reality due to conflicts among objectives. Popular approaches for solving multi-objective problems in the literature can be categorized into two main groups: (1) fuzzy goal programming, and (2) general goal programming approaches. In the first group, the

DMs allow the objectives to take any value between their minimum and maximum possible values, and then try to come as close to their best point as possible: For minimization objectives, the minimum and maximum possible values can be called respectively the positive ideal solution (PIS) and the negative ideal solution (NIS). Kumar et al. (2004) formulated a mixed integer goal programming for SSPs including three objectives: cost, quality and delivery, subject to some constraints. They adopted the max–min approach proposed by Zimmermann (1978) to solve the multi-objective model. Wadhwa and Ravindran (2007) modeled the SSP as a multi-objective programming problem, in which price, lead-time and rejects were considered as three conflicting criteria. They presented and compared several multi-objective optimization methods, including weighted objective method, goal programming (GP) method, and compromise programming, for solving their multi-objective problem. By weighted objective and compromise programming, the DMs do not need to determine a specific goal for the objectives. Amid et al. (2006, 2009) formulated a multi-objective model for SSPs including three goals: cost, quality and delivery under the influence of capacity and demand requirement constraints. They adopted a weighted additive method, proposed by Tiwari et al. (1987), to solve their model. In another study, Amid et al. (2011) used a weighted max–min approach, proposed by Lin (2004), for solving a multi-objective SSP with three goals: cost, quality and delivery subject to suppliers' capacity and market demand. Amin and Zhang (2012) developed an integrated multi-objective model for SSP and order allocation, and then employed the compromise programming approach for solving the multi-objective model. Shaw et al. (2012) proposed an integration of fuzzy-AHP and fuzzy multi-objective linear programming for SSP and order allocation, in which purchasing costs, rejects and lead time were considered as some of the objectives. Similar to Amid et al. (2006, 2009), they used the model of Tiwari et al. (1987) to solve their multi-objective model. Lin (2012) developed an integrated fuzzy multi-objective linear programming model for SSP and order allocation, and then proposed a two-phase approach, based on Zimmermann (1978) and Chen and Chou (1996) to solve the fuzzy multi-objective model. Nazari-Shirkouhi et al. (2013) developed a fuzzy goal programming approach for solving a fuzzy multi-objective multi-product SSP with multi-price level, in which cost, quality and delivery were their three objectives.

In the second group, the general goal programming approach, the DM determines a specific goal for every objective and then tries to achieve the goal as much as possible. Ustun and Demirtas (2008) proposed an integrated multi-period multi-objective model for SSP and order allocation, in which ε-constraint method, a reservation level driven Tchebycheff procedure (RLTP) and preemptive goal programming were used to solve the multi-objective model. In another study, Ustun and Demirtas (2008) defined an additive achievement function by combining min–max goal programming (MGP) and weighted goal programming (WGP) for their multi-objective problem. Demirtas and Ustun (2009) also employed WGP for solving their multi-objective SSP and order allocation. Jolai et al. (2011) proposed an integrated multi-objective mixed integer linear programming model for SSP and order allocation, and used WGP for solving their model. Jadidi et al. (2014) proposed a new goal programming approach for both deterministic and fuzzy multi-objective models that guarantees the achieved objectives to be consistent with their goals. They also applied the proposed model to multi-objective SSP. In real situations, however, the DMs may not always have precise data and information related to their criteria. Therefore, it may be difficult for them to specify an exact goal for every objective. Thus, the general goal programming approach becomes less favorable unless the DMs are allowed to choose more than one goal for each objective. This can be done either by choosing multi-goals for each objective or by specifying a range of values instead of a single goal. Chang (2007) proposed a new technique so-called multi-choice goal programming (MCGP) approach enabling DMs to determine multiple goals for every objective. In the original MCGP model Chang (2007), multiplicative terms of binary variables were used to express multiple discrete goals that resulted in increased complexity of the model. To overcome the complexity, Chang (2008) revised the original MCGP approach and instead of multiple discrete goals, proposed a range for each goal. Subsequently, Liao and Kao (2010, 2011) used the revised MCGP approach for SSP. However, Chang (2011) argued that the revised MCGP model is not able to consider the DMs' preference value, and consequently added general utility functions to this approach in order to maximize the DMs' expected utility.

In the approaches of Chang (2008, 2011), an interval for each goal is defined by an upper and lower bounds. Then, a continuous decision variable is considered within the interval as an aspiration level. The MCGP models aim at driving (1) the aspiration levels towards their lower bounds for minimization objectives (or upper bounds for maximization objectives), and (2) the achieved objectives towards their aspiration levels. The mechanism of Chang (2008, 2011) models is somewhat similar to the general goal programming approach: derive the achieved objective towards the aspiration level as much as possible. However, the DMs may be concerned with making the achieved objectives closer to their PIS values (Jadidi et al., 2014). The DMs may also prefer that if an achieved objective cannot stay within the interval, it would remain in a close proximity of the interval limits. In situations where the goal interval is selected at its minimum position (i.e., the lower bound is at PIS), the upper bound may be defined as a critical point from which the achieved objective should not significantly exceed. In this study, we try to look at the MCGP from this angle that gives the DMs more control on both the inside and the outside of the interval goal. If an objective can stay within the interval, it can be driven towards the lower bound (PIS); at the same time, if another objective stays outside the interval (due to conflicting objectives), it should be kept not too far from its upper bound. The previous MCGP approaches have not been designed for such conditions, and here we try to address this kind of problems. The new MCGP approach of this study is an extension of the weighted additive method (a fuzzy multi-objective method) proposed by Tiwari et al. (1987).

We present the rest of the research as follows. Section 2 formulates the SSP by multi-objective mathematical programming and then introduces the model by the goal programming and the MCGP approaches (Chang 2007, 2008, 2011). The new MCGP is proposed in section 3, followed by an illustrative example in section 4. Finally, concluding remarks are presented in section 5.

2. Multi-objective supplier selection model

As shown by the literature, the most important criteria for SSP are purchasing cost, rejects and lead-time. Here, we model a single item SSP in which a set of approved suppliers having limitation on their production capacity should satisfy the buyer's expectations on these three criteria and the demand, which is assumed to be known. The notations of the multi-objective problem are presented as follows:

k	index for objectives, k=1, 2, …, K
n	number of suppliers
X_i	number of units ordered to supplier i
V_i	capacity of supplier i
C_i	unit purchasing price from supplier i
q_i	expected defect rate of supplier i
F_i	percentage of items delivered late by supplier i
$f_k(X)$	objective k
D	demand

The multi-objective SSP is formulated as follows:

<u>*Model 1:*</u>

$$\text{Min } f_1(X) = \sum_{i=1}^{n} C_i X_i \tag{1.1}$$

$$\text{Min } f_2(X) = \sum_{i=1}^{n} q_i X_i \tag{1.2}$$

$$\text{Min } f_3(X) = \sum_{i=1}^{n} F_i X_i \tag{1.3}$$

Subject to:

$$\sum_{i=1}^{n} X_i = D \tag{1.4}$$

$$X_i \leq V_i \qquad i = 1, 2, \ldots, n \tag{1.5}$$

$$X_i \geq 0 \qquad i = 1, 2, \ldots, n. \tag{1.6}$$

Eq. (1.1), Eq. (1.2) and Eq. (1.3) minimize the three criteria: purchasing cost, rejects and late deliveries, respectively. Constraints (1.4) and (1.5) consider the buyer's demand and the suppliers' capacity, respectively.

2.1. The Weighted Goal Programming (WGP) approach

Since the previous MCGP approaches (Chang 2007, 2008, 2011) were developed based on WGP, the above multi-objective model is first introduced using WGP. In WGP, proposed by Charnes and Cooper (1961), DMs first determine aspiration level (f_k^*) for every objective, and then try to minimize deviations between aspiration levels and their achievements as follows:

Model 2:

$$\text{Min. } \sum_{k=1}^{3} w_k(d_k^- + d_k^+) \quad (2.1)$$

Subject to:

$$f_k(X) + d_k^- - d_k^+ = f_k^* \qquad k=1,2,3 \quad (2.2)$$

$$d_k^- . d_k^+ = 0 \qquad k=1,2,3 \quad (2.3)$$

$$d_k^-, d_k^+ \geq 0 \qquad k=1,2,3 \quad (2.4)$$

and the constraints of (1.4), (1.5) and (1.6).

where d_k^- and d_k^+ are negative and positive goal deviations, respectively, and w_k is the relative importance of the k^{th} objective.

2.2. The original MCGP approach

Chang (2007) argued that due to uncertainly/imprecision, the DMs may prefer to set multiple goals for every objective. Since the above WGP approach has not been designed for this purpose, Chang (2007) proposed the MCGP approach as follows:

Model 3:

$$\text{Min. } \sum_{k=1}^{3} w_k(d_k^- + d_k^+) \quad (3.1)$$

Subject to:

$$f_k(X) + d_k^- - d_k^+ = \sum_{j=1}^{J} f_{kj}^* S_{kj}(B) \qquad k=1,2,3 \quad (3.2)$$

$$d_k^- . d_k^+ = 0 \qquad k=1,2,3 \quad (3.3)$$

$$d_k^-, d_k^+ \geq 0 \qquad k=1,2,3 \quad (3.4)$$

$$S_{kj}(B) \in R_k(x) \qquad k=1,2,3 \quad (3.5)$$

and the constraints of (1.4), (1.5) and (1.6).

where f_{kj}^* ($k = 1,2,3$ and $j = 1,2,\dots,J$) is the j^{th} goal of the k^{th} objective, $f_{k,j-1}^* \leq f_{kj}^* \leq f_{k,j+1}^*$, and $S_{kj}(B)$ represents a function of binary serial numbers that is defined according to the number of goals for each objective and based on resource limitations $R_k(x)$. The main role of $S_{kj}(B)$ is to ensure that each objective chooses only one of the multiple goals. Interested readers are referred to Chang [24] for further discussions on $S_{kj}(B)$.

2.3. The revised MCGP approach

Chang (2008) discussed that in Chang's (2007) model, the multiplicative terms of binary variables that are used to express multiple goals increase the complexity of the model. To address this issue, Chang (2008) proposed a revised MCGP approach as follows:

Model 4:

$$\text{Min.} \sum_{k=1}^{3}[w_k^d(d_k^- + d_k^+) + w_k^e(e_k^- + e_k^+)] \qquad (4.1)$$

Subject to:

$$f_k(X) + d_k^- - d_k^+ = y_k \qquad k=1,2,3 \qquad (4.2)$$

$$y_k + e_k^- - e_k^+ = f_{k,min} \qquad k=1,2,3 \qquad (4.3)$$

$$f_{k,min} \leq y_k \leq f_{k,max} \qquad k=1,2,3 \qquad (4.4)$$

$$d_k^- . d_k^+ = 0 \qquad k=1,2,3 \qquad (4.5)$$

$$e_k^- . e_k^+ = 0 \qquad k=1,2,3 \qquad (4.6)$$

$$d_k^-, d_k^+, e_k^-, e_k^+ \geq 0 \qquad k=1,2,3 \qquad (4.7)$$

and the constraints of (1.4), (1.5) and (1.6).

where $f_{k,min}$ and $f_{k,max}$ are the minimum and maximum acceptable goals (upper and lower bounds) for k^{th} objective, respectively, y_k is the continuous variable representing aspiration level for objective k, d_k^+ and d_k^- are respectively the positive and negative deviations of $f_k(X)$ from y_k, e_k^+ and e_k^- are respectively the positive and negative deviations of y_k from $f_{k,min}$, and w_k^d and w_k^e are the relative importance related to (d_k^+, d_k^-) and (e_k^+, e_k^-), respectively.

For SSP, deviations may have different units that cause unintentional bias among the objectives. Some techniques that aim at transferring different units of the deviations to a common unit in order to remove the incommensurability were reviewed by Tamiz et al. (1998). Here, we normalize the deviations of the Chang's (2008) model as follows:

$$\text{Min.} \sum_{k=1}^{3} [w_k^d \frac{(d_k^+ + d_k^-)}{(f_k^- - f_k^+)} + w_k^e \frac{(e_k^+ + e_k^-)}{(f_{k,max} - f_{k,min})}]$$

where $f_k^+ = \{\min_X f_k(X)\} \; \forall \, k$, and $f_k^- = \{\max_X f_k(X)\} \; \forall \, k$.

From here on, we refer to the normalized revised MCGP (Model 4) as NR-MCGP.

2.4. The MCGP approach considering utility function

Chang (2011) argued that the *NR-MCGP* model cannot consider the DMs' preference value, and therefore, added a general utility function to the revised approach in order to maximize the DMs' expected utility. Chang (2011) considered linear and S-shape utility functions. In this research, we only review the linear utility function. However, the discussion can be extended to the S-shape utility function as well. The new model of Chang (2011) is presented as follows:

Model 5:

$$\text{Min.} \sum_{k=1}^{3} [w_k^d (d_k^- + d_k^+) + w_k^\delta \delta_k^-] \tag{5.1}$$

Subject to:

$\lambda_k \leq \frac{f_{k,max}-y_k}{f_{k,max}-f_{k,min}}$ k=1,2,3 (5.2)

$f_k(X) + d_k^- - d_k^+ = y_k$ k=1,2,3 (5.3)

$\lambda_k + \delta_k^- = 1$ k=1,2,3 (5.4)

$f_{k,min} \leq y_k \leq f_{k,max}$ k=1,2,3 (5.5)

$d_k^-.d_k^+ = 0$ k=1,2,3 (5.6)

$d_k^-, d_k^+, \delta_k^-, \lambda_k \geq 0$ k=1,2,3 (5.7)

and the constraints of (1.4), (1.5) and (1.6).

where δ_k^- represents the normalized deviation of y_k from $f_{k,min}$, w_k^δ is the weight associated with δ_k^-, and λ_k is the utility value. Other variables are defined as before.

If needed, the objective function of Chang (2011) can be also normalized as follows:

$$\text{Min.} \sum_{k=1}^{3}[w_k^d \frac{(d_k^- + d_k^+)}{(f_k^- - f_k^+)} + w_k^\delta \delta_k^-]$$

where δ_k^- does not need to be normalized because $0 \leq \delta_k^- \leq 1\ \forall k$.

From here on, we refer to the above MCGP that considers the utility function (Model 5) as *MCGP-U*. In the following section, we propose a new version of the MCGP model that we call it the *New-MCGP*.

3. The *New-MCGP* approach

In addition to the significant improvement on the original MCGP, the *NR-MCGP* and *MCGP-U* models also contribute to the general goal programming approach by considering an interval goal instead of a single goal. By regulating w_k^d in both methods, the achieved objective, $f_k(X)$, is driven towards the aspiration level, y_k, that is bounded within the interval goal, $[f_{k,min}, f_{k,max}]$. At the same time, y_k is also driven towards $f_{k,min}$ by adjusting w_k^{δ} in *MCGP-U* (or w_k^{e} in *NR-MCGP*). However, it might be worthwhile to consider the MCGP from a different angle: setting the lower bound, $f_{k,min}$, at the PIS, f_k^{+}, (which may be the DMs' preference (Jadidi et al., 2014)) and considering the upper bound, $f_{k,max}$, as a critical point so that the achieved objective should not significantly exceed it. For example, some manufacturers such as Toyota and Honda, before selecting their suppliers, may determine the maximum price of components and raw materials that they can afford to pay for (Liker and Choi, 2004) (i.e., they predetermine the maximum purchasing cost). In this case, the predetermined cost can be considered as a critical point. In other words, if an objective stays within the interval goal, $[f_{k,min}, f_{k,max}]$, the lower bound, $f_{k,min}$, should be considered as a pivot point which magnetizes the objective towards itself; if at the same time another objective falls outside the interval goal (due to conflicting objectives), the upper bound, $f_{k,max}$, should be considered as a new pivot point which keeps the objective as close as possible to itself. The structure of the two previous methods does not support this type of conditions while it may happen in reality.

In order to address such conditions in SSP, we propose a *New-MCGP* approach inspired by the fuzzy model of Tiwari et al. (1987). This new approach will pay special attention to $f_{k,min}$ and $f_{k,max}$ as two pivot points that enables the DMs to have control on both the inside and the outside of the interval. Applying the original Tiwari et al. (1987) approach to Model 1 results in:

<u>*Model 6:*</u>

$$\text{Max.} \sum_{k=1}^{3} w_k \alpha_k \tag{6.1}$$

Subject to:

$$\frac{f_k^- - f_k(X)}{f_k^- - f_k^+} = \alpha_k \qquad k=1,2,3 \qquad (6.2)$$

$$0 \leq \alpha_k \leq 1 \qquad k=1,2,3 \qquad (6.3)$$

and the constraints of (1.4), (1.5) and (1.6).

where α_k is a continuous coefficient, $0 \leq \alpha_k \leq 1$, that represents the normalized distance of the achieved objective from f_k^-. Constraint (6.2) can be rewritten as:

$$f_k(X) = \alpha_k f_k^+ + (1 - \alpha_k) f_k^- \qquad k=1,2,3 \qquad (6.2a)$$

As the range for each objective is decided by the DMs, here we propose that the lower bound of the range, $f_{k,min}$, be set equal to f_k^+, while the upper bound, $f_{k,max}$, can be less than or equal to f_k^-. The rational for this suggestion is that in a minimization problem, the DMs would normally prefer the lowest value for the objective. As a result, the range $[f_k^+, f_k^-]$ is divided into two sub-ranges of $[f_{k,min}, f_{k,max}]$ and $[f_{k,max}, f_k^-]$ that we call them the *more desirable range* (MDR) and the *less desirable range* (LDR), respectively.

We also propose that α_k be the normalized distance of the achieved objective k from $f_{k,max}$ so that by maximizing this coefficient, we approach to $f_{k,min}$. Therefore, Eq. (6.2) can be written as:

$$\alpha_k = \frac{f_{k,max} - f_k(X)}{f_{k,max} - f_{k,min}} \qquad k=1,2,3$$

Realizing that by moving the upper limit of α_k to $f_{k,max}$ the range for each objective is tightened, we allow the achieved objective to take a value outside this tightened range subject to a penalty. We do so by introducing another variable, β_k, that represents the normalized distance of the achieved objective k from $f_{k,max}$ when it is greater than $f_{k,max}$. Thus:

$$\beta_k = \frac{f_k(X) - f_{k,max}}{f_k^- - f_{k,max}} \qquad k=1,2,3$$

As both α_k and β_k determine the position of a single objective k, only one of them can be non-zero. That is, $\alpha_k . \beta_k = 0$.

Figure 1 illustrates the relationship between α_k and β_k. If the achieved objective $f_k(X)$ falls within the MDR (e.g., point 1), then $\alpha_k > 0$. However, if it falls outside (e.g., point 2), then $\beta_k > 0$. While the aim is to obtain a value within the range and as close as possible to the lower bound $f_{k,min}$, the model allows the objective to take a value outside the range subject to a penalty in order to avoid infeasible solutions. Therefore, the objective of the new model will be to maximize α_k and to minimize β_k.

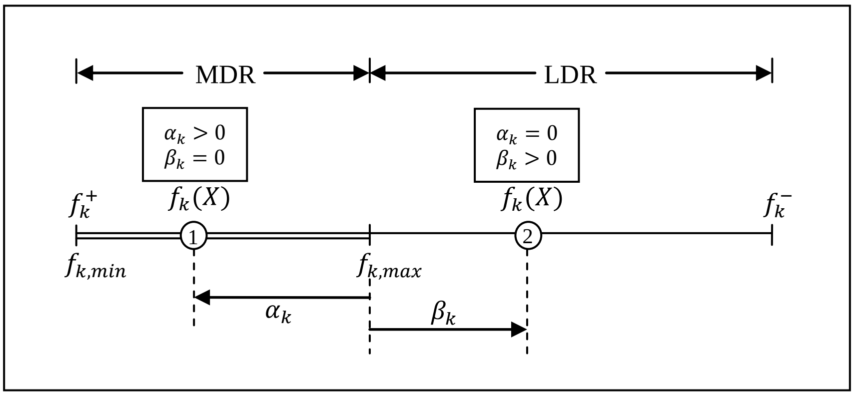


Fig. 1. The relationships among model parameters.

Using these new variables, we can re-write Eq. (6.2a) as follows:

$$f_k(X) = \alpha_k f_{k,min} + (1-\alpha_k) f_{k,max} + \beta_k (f_k^- - f_{k,max}) \qquad k=1,2,3 \qquad (6.2b)$$

Therefore, the *New-MCGP* approach for the multi-objective SSP of Model 1 is formulated as follows:

Model 7:

$$\text{Max.} \sum_{k=1}^{3} (w_k^\alpha \alpha_k - w_k^\beta \beta_k) \qquad (7.1)$$

Subject to:

$$f_k(X) = \alpha_k f_{k,min} + (1-\alpha_k) f_{k,max} + \beta_k (f_k^- - f_{k,max}) \qquad k=1,2,3 \qquad (7.2)$$

$$\alpha_k . \beta_k = 0 \qquad k=1,2,3 \qquad (7.3)$$

$$0 \le \alpha_k, \beta_k \le 1 \qquad k=1,2,3 \qquad (7.4)$$

and the constraints of (1.4), (1.5) and (1.6).

As illustrated in Figure 1, the *New-MCGP* approach enables DMs to have control on both the MDR and the LDR. This will increase the effectiveness of the *New-MCGP* by involving some certain DMs' preferences.

The *New-MCGP* approach guarantees a feasible solution as $f_k(X)$ moves between its minimum, f_k^+, and maximum, f_k^-, values. Furthermore, since $0 \le \alpha_k, \beta_k \le 1$, it can facilitate the DMs' preference modeling by eliminating the incommensurability caused by scale differences among objectives.

Discussion on $f_{k,max}$:

Since $f_{k,max}$ is a user-selected parameter, it is worthwhile to have some guidelines for choosing an appropriate value for this parameter. The purpose of $f_{k,min}$ and $f_{k,max}$ is to determine a focus area between f_k^+ and f_k^-. As we make $f_{k,min} = f_k^+$, $f_{k,max}$ divides the range of f_k^+ and f_k^- into two sections: (1) more desirable one (MDR) that is on the left side of $f_{k,max}$, and (2) less desirable one (LDR) that is between $f_{k,max}$ and f_k^-. One consideration in determining $f_{k,max}$ is whether the objectives are conflicting or not. In case of conflicting objectives, it is recommended that if one $f_{k,max}$ is close to f_k^-, the other $f_{k,max}$ be chosen relatively close to f_k^+. In other words, if the range of one objective is chosen tightly, the range for the other objective should be chosen wider to allow more movements for conflicting objectives. In the above example, the first and the second objectives are in conflict, and since the range of first objective was chosen more tightly, the range for the second objective was not as tight as the first one.

4. An illustrative example

The following numerical example is going to illustrate how the New-MCGP can solve multi-objective SSP. This example considers a situation in which six suppliers, whose information is presented in Table 1, should meet the buyer's demand of 16 units.

Table 1.
The data for the numerical example

Supplier i	Price, C_i	Rejection Rate, q_i (%)	Late Delivery Rate, F_i (%)	Capacity, V_i
S1	3	0.40	0.25	5
S2	3.5	0.35	0.30	4
S3	4	0.30	0.15	3.5
S4	4.5	0.25	0.20	6
S5	5	0.20	0.40	5.5
S6	6	0.15	0.35	5

Using the above data, we can obtain $\boldsymbol{f_1^+}$= 58.75, $\boldsymbol{f_2^+}$= 0.03225, $\boldsymbol{f_3^+}$= 0.03425, $\boldsymbol{f_1^-}$= 82.25, $\boldsymbol{f_2^-}$= 0.05325 and $\boldsymbol{f_3^-}$= 0.05525. Furthermore, it can be seen that the first and second objectives are in conflict. That is the suppliers with a better price have poor quality and vice versa. The third objective, while not directly in conflict with the other two objectives, shows the best delivery in mid range of price and quality.

Here, we assume there are three conditions that the DMs are going to incorporate in their decisions for suppliers' evaluation and order allocation:

Condition 1: $f_{k,min}$=f_k^+ and $f_{k,max} < f_k^-$, $\forall k$, so that $f_{1,max}$=68, $f_{2,max}$=0.0461 and $f_{3,max}$=0.04475. That is, each objective has a critical point, $f_{k,max}$ $\forall k$, and two ranges, MDR and LDR.

Condition 2: The second objective is more important than the first one: its achievement, $f_2(X)$, should be more preferably in the MDR and as close as possible to $f_{2,min}$.

Condition 3: The first objective, while being less important than the second one, should not significantly exceed $f_{1,max}$: its achievement, $f_1(X)$, may be in the LDR but preferably as close as possible to $f_{1,max}$.

Here, we apply the *New-MCGP* model to the above example.

The New-MCGP model:

We first set $\boldsymbol{f_{k,min}}$ and $\boldsymbol{f_{k,max}}$ $\forall \boldsymbol{k}$ as defined in *Condition* 1. In order to consider *Condition* 2 (i.e., the second objective is the most important one), we should have $\boldsymbol{w_2^{\alpha}} \gg \boldsymbol{w_1^{\alpha}}$ which in turn will cause $\boldsymbol{\alpha_2}$ to increase and $\boldsymbol{f_2(X)}$ to approach $\boldsymbol{f_{2,min}}$. Since the first and second objectives are in conflict, $\boldsymbol{f_1(X)}$ approaches $\boldsymbol{f_1^-}$. For taking into account *Condition* 3 (i.e., the first objective should not significantly exceed $\boldsymbol{f_{1,max}}$), we should have $\boldsymbol{w_2^{\beta}} \ll \boldsymbol{w_1^{\beta}}$ that causes $\boldsymbol{\beta_1}$ to decrease and $\boldsymbol{f_1(X)}$ to get far from $\boldsymbol{f_1^-}$ and towards $\boldsymbol{f_{1,max}}$. As a result, the weights may be allocated as $\boldsymbol{w_1^{\alpha}}$=0.1, $\boldsymbol{w_2^{\alpha}}$=0.8, $\boldsymbol{w_3^{\alpha}}$=0.1,

$\boldsymbol{w_1^{\beta}}$=0.8, $\boldsymbol{w_2^{\beta}}$=0.1 and $\boldsymbol{w_3^{\beta}}$=0.1. We employ Solver in Excel to solve this problem by the *New-MCGP* model. The results are as follows:

$f_1(X)$ = 68 = $f_{1,max}$ that holds the second condition,

$f_2(X)$ = 0.044 (within MDR) that holds the first condition,

$f_3(X)$ = 0.039,

X_1=2.75, X_2=0, X_3=3.5, X_4=6, X_5=3.75, X_6=0.

Here, we see that the first objective did not exceed $f_{1,max}$. As demonstrated here, the *New-MCGP* model allows the DMs that by adjusting w_k^{α} and w_k^{β} to move one objective closer to its $f_{k,min}$, and at the same time, to keep another objective not far from its $f_{k,max}$.

Analysis of weights:

The weights (w_k^{α}, w_k^{β}) are user-selected parameters by which the DMs can incorporate their strategies in SSP. For instance, if the strategy is to produce a high quality product, the weight of quality (rejects in this numerical example) should be higher than others. In this model, the more important objective is assigned a higher w_k^{α} that results in the objective to stay in MDR and to approach $f_{k,min}$. If the less important objective is in conflict with the first one, it may fall in LDR and far from $f_{k,max}$. In this case, we can assign a higher w_k^{β} to this objective such that it stays closer to $f_{k,max}$. When an equal value is assigned to all w_k^{β}, it means the DMs are not concerned with the objectives getting far from $f_{k,max}$. In order to investigate the effect of different weight values on the results, we gradually increase w_1^{β}. As we increase w_1^{β} from 0.33 to 0.80, we distribute the remaining values equally among the other two weights. We solve the model for different combinations of weights as presented in Table 3.

Table 2.
The weight analysis for w_1^β.

$\boldsymbol{w_1^\beta}$	**0.33**	**0.60**	**0.80**
w_2^β	0.33	0.20	0.10
w_3^β	0.33	0.20	0.10
$f_1(X)$	82.00	76.45	68.00
$f_2(X)$	0.032	0.036	0.044
$f_3(X)$	0.050	0.045	0.039

Table 3 shows that as w_1^β increases from 0.33 to 0.80, $f_1(X)$ improves from f_1^- to $f_{1,max}$. The gain for the first objective comes at a slow loss for the second objective; i.e., $f_2(X)$ moves farther from $f_{1,min}$ but it does not surpass $f_{2,max}$. Figure 2 depicts the above analysis.

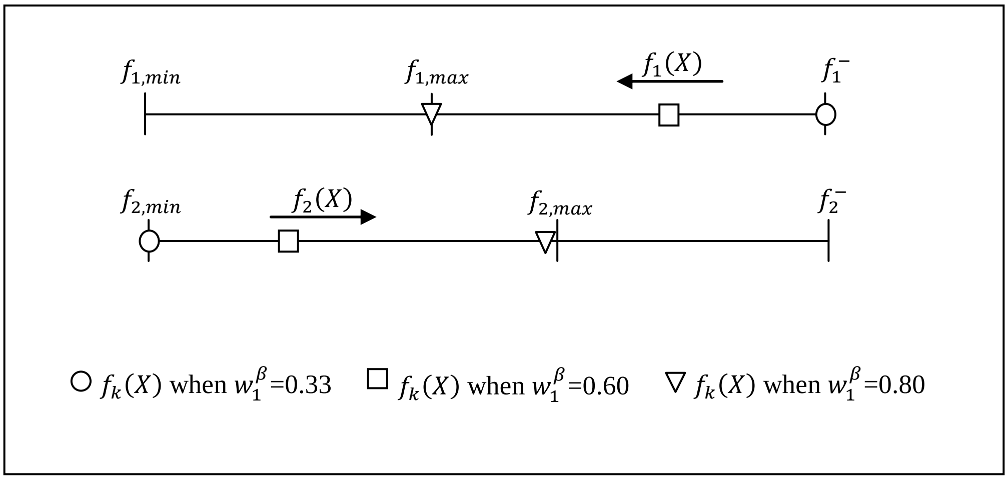


Fig. 2. The sensitivity of $f_1(X)$ and $f_2(X)$ to increased w_1^β and decreased w_2^β in the *New-MCGP*

This analysis shows that the *New-MCGP* enables DMs to better incorporate their preferences in the model for making a more desirable decision on supplier selection and order allocation. DMs are able to consider interval goals or aspiration levels, and at the same time, take more accurately into account their relative importance.

5. Conclusions

In this study, a single product supplier selection problem (SSP) was formulated as a multi-objective optimization model. It was assumed that: (1) the objectives can have different relative importance levels, and (2) it may be easier for decision makers to determine an interval goal or aspiration level for every objective. In order for these two assumptions to be incorporated into the solution methodology, the *New-MCGP* approach was then proposed. In comparison with the previous studies, we set the lower bound of the interval goal at the positive ideal solution (PIS) that drives the objective towards itself if it falls within the interval goal, and at the same time, set the upper bound as a magnetic point if the objective exceeds it. This type of model can be used in reality when managers try to determine the maximum purchasing price of components and raw materials before selecting their suppliers. Then, this maximum cost can be considered as the upper bound from which its achieved objective should not significantly exceed. In addition, the numerical example illustrated that variation in priority of criteria will change the order quantities assigned to the suppliers. This means that our proposed model effectively incorporates DMs' preferences and conditions for SSP and order allocation by providing the DMs with control on both of the more desirable range (MDR) and the less desirable range (LDR).

In this chapter, we assumed that the demand and suppliers' capacities are known. However, these two parameters may be uncertain in reality. The study of SSP where the demand and suppliers' capacities are uncertain can be considered as a direction for future research.

In addition, the supplier selection problem can be integrated with supply chain coordination. The supply chain coordination, which generally concentrates on inventory management, tries to improve the whole supply chain profitability by aligning the partners' strategies and goals. However, to the best of our knowledge, supplier selection studies have mainly considered the buyers' strategies and preferences rather than those of the entire supply chain. Taking supply chain coordination into account for a SSP is also open for further study.

References

Aissaouia, N., M. Haouaria, E. Hassini, Supplier selection and order lot sizing modeling: A review, Comput. & Oper. Res. 34 (2007) 3516 – 3540.

Amid, A., S.H. Ghodsypour, C. O'Brien, A fuzzy multiobjective linear model for supplier selection in a supply chain, Int. J. of Product. Econ. 104 (2006) 394–407.

Amid, A., S.H. Ghodsypour, C. O'Brien, A weighted additive fuzzy multiobjective model for supplier selection in a supply chain under price breaks, Int. J. of Product. Econ. 121 (2009) 323–332.

Amid, A., S.H. Ghodsypour, C. O'Brien, A weighted max–min model for fuzzy multi-objective supplier selection in a supply chain, Int. J. of Product. Econ. 131 (2011) 139–145.

Amin, S.H., and G. Zhang, An integrated model for closed-loop supply chain configuration and supplier selection: Multi-objective approach, Expert Syst. with Appl. 39 (2012) 6782–6791.

Chang, C.-T., Multi-choice goal programming with utility functions, Eur. J. of Oper. Res. 215 (2011) 439–445.

Chang, C.T., Multi-choice goal programming, Omega. 35 (2007) 389 – 396.

Chang, C.T., Revised multi-choice goal programming, Appl. Math. Model. 32 (2008) 2587–2595.

Charnes, A., W.W. Cooper, Management model and industrial application of linear programming, Wiley, New York, 1961.

Chen, H.K., H.W. Chou, 1996. Solving multi-objective linear programming problems-a generic approach, Fuzzy Sets and Syst. 82 (1), 35–38.

Demirtas, E.A., O. Ustun, Analytic network process and multi-period goal programming integration in purchasing decisions, Comput. & Ind. Eng. 56 (2009) 677–690.

Dickson, G.W., An analysis of vendor selection: systems and decisions, J. of Purch. 1 (1966) 5–17.

Ghodsypour, S.H., C. O'Brien, The total cost of logistics in supplier selection under conditions of multiple sourcing, multiple criteria and capacity constraint, Int. J. of Product. Econ. 73 (2001) 15–27.

Jadidi, O., S. Zolfaghari, S. Cavalieri, A new normalized goal programming model for multi-objective problems: A case of supplier selection and order allocation. Int. J. of Product. Econ., 148 (2014) 158–165.

Jolai, F., S.A. Yazdian, K. Shahanaghi, M.A. Khojasteh, Integrating fuzzy TOPSIS and multi-period goal programming for purchasing multiple products from multiple suppliers. J. of Purch. and Supp. Manag. 17 (2011) 42-53.

Kumar, M. P., Vrat, R. Shankar, A fuzzy goal programming approach for vendor selection problem in a supply chain, Comput. & Ind. Eng. 46 (2004) 69–85.

Liao, C.N., H.P. Kao, An integrated fuzzy TOPSIS and MCGP approach to supplier selection in supply chain management, Expert Syst. with Appl. 38 (2011) 10803–10811.

Liao, C.N., H.P. Kao, Supplier selection model using Taguchi loss function, analytical hierarchy process and multi-choice goal programming, Comput. & Ind. Eng. 58 (2010) 571–577.

Liker, J.K., T.Y. Choi, Building deep supplier relationships, Harvard Bus Rev 82 (2004) 104-113.

Lin, C.C., A weighted max–min model for fuzzy goal programming, Fuzzy Sets and Syst. 142 (2004) 407–420.

Lin, R.-H., An integrated model for supplier selection under a fuzzy situation, Int. J. of Product. Econ. 138 (2012) 55–61.

Nazari-Shirkouhi, S. H., Shakouri, B. Javadi, A. Keramati, Supplier selection and order allocation problem using a two-phase fuzzy multi-objective linear programming, Appl. Math. Model. 37 (2013) 9308–9323.

Shaw, K., R. Shankar, S.S. Yadav, L.S. Thakur, Supplier selection using fuzzy AHP and fuzzy multi-objective linear programming for developing low carbon supply chain, Expert Expert Syst. with Appl. 39 (2012) 8182–8192.

Tamiz, M., D. Jones, C. Romero, Goal programming for decision making: An overview of the current state-of-the-art, Eur. J. of Oper. Res. 111 (1998) 569-581.

Tiwari, R.N., S. Dharmahr, J.R. Rao, Fuzzy goal programming - an additive model, Fuzzy Sets and Syst. 24 (1987) 27–34.

Ustun, O., E.A. Demirtas, An integrated multi-objective decision-making process for multi-period lot-sizing with supplier selection. Omega. 36 (2008) 509-521.

Ustun, O., E.A. Demirtas, Multi-period lot-sizing with supplier selection using achievement scalarizing functions. Comput. & Ind. Eng. 54 (2008) 918-931.

Wadhwa, V., A.R. Ravindran, Vendor selection in outsourcing, Comput. & Oper. Res. 34 (2007) 3725 – 3737.

Wang, G., S.H. Hang, J.P. Dismukes, Product-driven supply chain selection using integrated multi-criteria decision making methodology, Int. J. of Product. Econ. 91 (2004) 1–15.

Weber, C.A., J.R. Current, W.C. Benton, Vendor selection criteria and methods, Eur. J. of Oper. Res. 50(1991) 2–18.

Zimmermann, H.J., Fuzzy programming and linear programming with several objective functions, Fuzzy Sets and Syst. 1 (1978) 45–55.

CHAPTER 4 - A MULTI-PERIOD SUPPLIER SELECTION PROBLEM UNDER PRICE BREAKS: A DYNAMIC NEWSVENDOR MODEL

The main aim of this chapter is to develop an algorithm for finding optimal inventory level and suppliers' order allocation for a stochastic multi-period supplier selection model where the suppliers may offer discount quantity to the buyer. In this chapter, we consider a multi-period supplier selection problem where a buyer purchases a single product from a set of capacitated suppliers to meet its stochastic demand. The suppliers may offer quantity discount as a competitive factor to induce the buyer to purchase more. The problem under consideration is non-stationary in terms of selling price, purchasing price, holding cost, and demand. The objective is to obtain the suppliers' optimum order quantity in order to maximize the buyer's expected profit. We first model the problem by mixed integer nonlinear programming, and then propose an algorithm for solving the model. A sensitivity analysis is also conducted to examine the effect of changing the value of model parameters (selling price, purchasing price, holding cost, and demand) during the time horizon on the buyer's performance. The numerical results show that selling price and holding cost have, respectively, the highest and lowest impact on the buyer's profit.

1. Introduction

The competitive environment puts continuously pressure on manufacturers to decline cost. Purchasing departments' role in the supplier selection process is determinant to decrease the purchase cost of the necessary components. For example, up to 70% of the product cost in manufacturing industries can be associated to the cost of component parts and raw materials (Ghodsypour and O'Brien, 1998). Boer et al. (2001) states that "in industrial companies, purchasing share in the total turnover typically ranges between 50-90%". Supplier selection, and the related order allocation among them, has been expansively studied in years (Zhang and Ma, 2009). Although in reality the product demand is mostly uncertain, which increases the complexity of problem, the vast majority of the studies assume it is known precisely (Zhang and Ma, 2009).

The supplier selection problem (SSP) with uncertain demand can be modeled using probability distributions. One of the most famous problems in this area is probably the newsvendor problem that can be extensively employed in reality because of the reduction of product life cycle (Zhang and Zhang, 2011). Dada et al. (2007) and Yang et al. (2007) studied SSP under random yield supply and uncertain demand. They did not consider a limitation on minimum and maximum order sizes, while some economic issues, such as transportation and production setups, cause suppliers to set restriction on minimum order size (Awasthi et al, 2009). Therefore, Burke et al. (2007), Awasthi et al. (2009) and Zhang and Zhang (2011) studied supplier selection and purchase problems where the suppliers have restriction on minimum and maximum order sizes under uncertain demand.

However, the abovementioned studies did not consider a SSP under price breaks. Suppliers may propose quantity discount as an incentive for buyers to purchase higher quantities (Amid, 2009), although it increases the complexity of the problem (Xia and Wu, 2007). There are two types of quantity discounts in literature: incremental quantity discounts and all-units discounts. In all-units discounts, all items in an order are discounted if the order exceeds a certain level, while in incremental discounts, units over a certain level are discounted and items before the certain level are not. Xia and Wu (2007), Amid et al., (2009), and Mansini et al., (2012) have recently developed supplier selection models with quantity discount where it was assumed that the demand is deterministic. Despite the benefit of higher selling, this strategy causes the over-order that may increase inventory risk, mainly due to the uncertainty of the demand (Zhang and Ma, 2009). Kim et al. (2002) considered a problem in which a manufacturer buys multiple raw materials from suppliers to produce different type of products with uncertain demand. They proposed an iterative algorithm to solve the problem that was mathematically modeled. Subsequently, Zhang and Ma (2009) extended the model of Kim et al. (2002) to consider the all-units quantity discounts as well. Zhang and Ma (2009) employed mixed integer nonlinear programming (MINLP) formulation for modeling the problem in order to maximize the expected profit of the buyer. To solve the problem, they first used an external module to define the integration function of the expected profit arises due to the stochastic demand, and then integrated the module with GAMS/SBB (Simple Brach & Bound that is one of the GAMS solvers for MINLP models). In another study, Yin and Nishi (2013) developed a solution procedure for the model of Zhang and Ma (2009) with both of incremental and all-units discounts, in which the uncertain demand is assumed to follow the standard normal distribution. They first applied a novel outer-approximation method to solve the mixed-integer programming problem. Since the problem formulation included integral terms due to demand uncertainty, a normalization technique was then used to reformulate the model that replace the integral terms.

The afformentioned studies, dealing with uncertain demand, have only considered single period SSP. However, companies have to ensure that the product is available on an ongoing basis during a year. Due to some restrictions such as warehouse capacity, holding cost, etc, placing an order once a year is not economic, and thus, they usually split a single order to multiple ones (multi-period inventory control). There are studies that have considered SSP over a planning horizon (consisting of multiple periods) such as Sen et al. (2013), Mak et al. (2011), Woarawichai et al. (2011), Demirtas and Ustun (2009), Demirtas and Ustun (2008), Ustun and Demirtas (2008), Basnet and Leung (2005) and Rungreunganaun and Woarawichai (2013). However, these studies only took into account supplier selection models with deterministic demand, while demand in reality is mostly uncertain.

Multi-period models are dynamic and any surplus inventory at period t can be used at period $t + 1$ (Porteus, 2002). Therefore, for calculating the buyer's expected profit in period t, we have to count the value of each leftover inventory going to period $t + 1$ (i.e., C_{t+1}, the purchasing price of one unit in period $t + 1$). When an order quantity should be bought from multiple suppliers with different prices, it becomes difficult for the buyer to calculate C_{t+1}. Moreover, it becomes more difficult if the suppliers offer different price levels due to quantity discounts.

In this study, we used MINLP formulation to model a single product multi-period SSP to find the suppliers' optimum order quantity that maximizes the buyer's expected profit. It is assumed that the demand is stochastic and the suppliers have capacity restriction. In addition, the suppliers may utilize the all-unit quantity discount as a competitive factor to stimulate the buyer for a greater order. The problem is a non-stationary multi-period problem as selling price, purchasing price, holding cost, and demand may change from one period to another. We first develop an algorithm to solve the MINLP model over a single period. In order to verify that our algorithm efficiently finds the optimum order quantities, we also develop a GAMS-based solution program to compare the solutions. Subsequently, we extend the algorithm for the multi-period problem by the concept of dynamic programming. In other words, the multi-period problem is solved by backward induction: the optimum order allocations of the last period is computed first, the optimum order allocations of the second last period is obtained next, and so on (Porteus, 2002). We also investigate a sensitivity analysis to study the effect of changing the value of selling price, purchasing price, holding cost, and demand during the time horizon on the buyer's performance.

The rest of the chapter is organized as follows: Section 2 models the single period problem by using MINLP and develops the algorithm. The algorithm is also extended to the multi-period problem in this section. A numerical example and sensitivity analysis are discussed in section 3. Finally, section 4 draws conclusion and future studies.

2. Supplier selection model with discount

2.1. Single Period Model

Consider a situation in which suppliers, who may offer all-unit quantity discounts, are going to meet a buyer's uncertain demand for a single product. All the suppliers in the identified set are evaluated based only on price (or discounted price) since it is assumed they already satisfied other qualitative and quantitative criteria such as quality, financial strength, delivery, etc. The buyer needs to allocate the optimum order quantity to each supplier in order to maximize his expected profit.

We first develop a single period SSP. The notations for formulating the problem are presented as follows:

Indices

$i = 1,2, \dots, I$	index of suppliers, (I indicates number of suppliers)
$j = 1,2, \dots, m_i$	index of price level (m_i indicates number of price levels offered by supplier *i*).

Parameters

p	selling price per unit at the market, determined exogenously
h	holding cost per unit
Sh	shortage cost per unit
C_{ij}	price per unit offered by supplier *i* at price level *j* ($C_{i,j+1} < C_{ij}$)
V_{ij}^{U}	upper bound of the order that can be allocated to supplier *i* at price level *j*
V_{ij}^{L}	lower bound of the order that can be allocated to supplier *i* at price level *j*
ξ	market demand (random variable)
$f(\xi)$	known probability density function of the demand
$F(\xi)$	known cumulative distribution function of the demand

The relationship between the discount segments are as follows: $V_{i,j-1}^{L} < V_{i,j-1}^{U} < V_{ij}^{L} < V_{ij}^{U}$.

Decision variables

X_{ij}	order allocated to supplier *i* at price level *j*

Y_{ij} is binary variable: Y_{ij} =1 if order is placed to supplier *i* at price level *j*; otherwise Y_{ij} =0.

For single period problem, we assume the inventory level before ordering is zero. Let $X = \sum_{i,j} X_{ij}$ denote the inventory level after ordering. Then, for $X > \xi$, we have positive inventory at the end the period, and for $X < \xi$, we have negative inventory (shortage). As a result, the expected holding and shortage cost function related to X can be formulated as follows:

$$L(X) = h \int_0^X (X - \xi) f(\xi) d\xi + Sh \int_X^{+\infty} (\xi - X) f(\xi) d\xi$$

The purchasing cost is presented as follows:

$$Pur(X) = \sum_{i=1}^{I} \sum_{j}^{m_i} C_{ij} X_{ij}$$

The expected revenue function is also formulated as follows:

$$R(X) = p \int_0^X \xi f(\xi) d\xi + p \int_X^{+\infty} X f(\xi) d\xi$$

The MINLP model of a single period newsvendor problem for maximizing buyer's expected profit is presented as follows:

<u>*Model 1*</u>:

$$Max\ T(X) = R(X) - L(X) - Pur(X) \quad (1.1)$$

Subject to:

$$X = \sum_{i=1}^{I} \sum_{j=1}^{m_i} X_{ij} \quad (1.2)$$

$V_{ij}^{L}Y_{ij} \leq X_{ij}$ $\qquad$ i=1,2,…,n; j=1,2,…,m_i, $\qquad$ (1.3)

$V_{ij}^{U}Y_{ij} \geq X_{ij}$ $\qquad$ i=1,2,…,n; j=1,2,…,m_i, $\qquad$ (1.4)

$\sum_{j=1}^{m_i} Y_{ij} \leq 1$ $\qquad$ i=1,2,…,n, $\qquad$ (1.5)

$X_{ij} \geq 0$ $\qquad$ i=1,2,…,n; j=1,2,…,m_i, $\qquad$ (1.6)

Eq. (1.1) maximizes the buyer's profit. Constraint (1.2) guarantees that the inventory level after ordering equals the summation of all orders assigned to the suppliers. Constraints (1.3) and (1.4) ensure that the order allocated to supplier *i* at price level *j* places in the right interval. By constraint (1.5), at most one discount segment is selected for every supplier.

For taking into account quantity discounts, the model was formulated as an MINLP, which is considered as NP-hard (Awasthi et al., 2009; Yin and Nishi, 2013). In addition, $T(X)$ is concave and the corresponding optimal quantity assigned to supplier *i* at price level *j* without bound constraints is determined using the classical fractal formula $X_{ij}^{*} = F^{-1}(\frac{P+S-C_{ij}}{P+h+S})$ (Burke et al., 2007).

Proposition 1. *Let X_{ij}^{+} and X_{ij}^{*} be the optimum order quantity to supplier i at price level j for Model 1 with and without bound constraints, respectively. Also, let Q be the summation of order quantities assigned to other suppliers except (or before) supplier i* (*note that at most one order can be assigned to each supplier*). *Then we have* (*see Figure. 1*):

(i) *If* $0 \leq V_{ij}^{L} \leq X_{ij}^{*} - Q \leq V_{ij}^{U}$, *then* $X_{ij}^{+} = X_{ij}^{*} - Q$.

(ii) *If* $0 < V_{ij}^{U} < X_{ij}^{*} - Q$, *then* $X_{ij}^{+} = V_{ij}^{U}$.

(iii) *If* $0 < X_{ij}^{*} - Q < V_{ij}^{L}$, *then* $X_{ij}^{+} = V_{ij}^{L}$.

(iv) *If* $\max\left(X_{ij}^{*} - Q, 0\right) = 0$, *then* $X_{ij}^{+} = 0$.

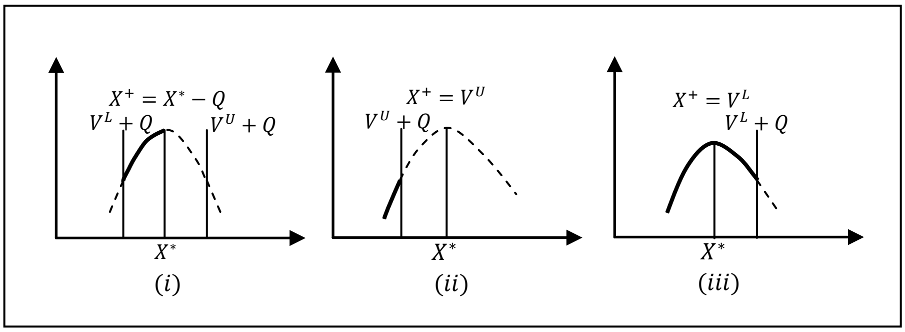


Fig. 1. The optimum order quantity with and without bound constraints

See appendix for proof. Proposition 1 was also proved by Zhang (2010) for a single supplier model.

To better show how the proposed algorithm performs, the algorithm is developed into two stages. We first use Proposition 1 to construct Algorithm A1 to solve Model 1 for finding an initial solution or an initial orders allocation (stage 1). Subsequently, we will discuss that the initial solution may not be optimal, and two more actions should be added to Algorithm A1 for finding the optimal solution. Therefore, Algorithm A1 is then extended to perform those two actions for assessing other neglected solutions (stage 2). The extended algorithm is called Algorithm A2.

Algorithm A1

First, each price interval is named by S_{ij}^{K} in which the index of K is used to show their rank in term of price (i.e., $S_{..}^{K=1}$ has the lowest price and $S_{..}^{K=\sum_{i\in N} m_i}$ has the highest price). From $K = 1$, by using Proposition 1, we assess each interval if the optimal order quantity without bound, $X_{..}^{+}$, can be positive. If so, the order quantity is denoted by $X_{..}^{+K}$. We go for the next interval, $S_{..}^{K+1}$, if $X_{..}^{+K} = V_{..}^{UK}$, we stop otherwise. In addition, when a supplier is assigned an order, his other intervals no longer assessed for further allocation (one order to each supplier). The vector of positive allocations is represented by $X^A = \{\dots, X_{..}^{+K}, \dots\}$, and the corresponding revenue is denoted by T^A. X^* and T^* represent the best allocations vector and its revenue up to a new order is placed (i.e., when an order is placed to a supplier, new T^A is computed and compared with T^*. If $T^A > T^*$, then T^* and X^* are replaced by T^A and X^A, respectively). Furthermore, the vector of selected intervals is denoted by $A = \{\dots, S_{..}^{K}, \dots\}$.

For the ease in the presentation of Algorithm A1, we introduce some task as follows. For starting point, *Initial Task* is used:

Initial Task: Arrange all S_{ij} in increasing order of C (i.e. $C_{..}^{K} < C_{..}^{K+1}$). If $C_{..}^{K} = C_{..}^{K+1}$ then $V_{..}^{KL} \leq V_{..}^{K+1,L}$. Also set $A = \emptyset$, $X^A = \emptyset$, $X^* = \emptyset$, K=0, Q=0, T^*=0, $Kr = M$.

where M is a number greater than $\sum_{i \in N} m_i$; Kr will be only used in Algorithm A2 and is explained later.

Also, every time that a positive order is allocated to an interval, $S_{..}^{.}$, we perform *Task 1* as follows:

Task1: $A = A \cup \{S_{..}^{.}\}$, $X^A = X^A \cup \{X_{ij}^{+k}\}$, $Q = Q + X_{ij}^{+k}$ and compute T^A. If $T^A > T^*$, then replace T^* and X^* by T^A and X^A, respectively.

Algorithm A1 is developed, presented by Figure. A1 in appendix, to find an initial solution. In Algorithm A1, when $X_{..}^{+K} = V_{..}^{UK}$, we have to assess the next interval, whose supplier has not been selected yet, for order allocation. Assume $X_{..}^{+K} = V_{..}^{UK}$ that results in an order allocation to Kth interval, and thus $Q = Q + V_{..}^{UK}$. Then, we have to evaluate the next interval: whether $Q \leq X_{..}^{*(K+1)}$? If so, then we have $(X_{..}^{*(K+1)} - Q) \geq 0$ that according to Proposition 1 in (i), (ii) or (iii), $X_{..}^{+K}$ will be positive. In Addition, allocating order to the intervals stops in following four conditions (*stop-conditions*):

- First condition is when $K > \sum_i m_i$: in this case, no interval left for allocation.
- Second is $X_{..}^{+K}$=0: in this case, $X_{..}^{+(K+1)}$ is also zero because $C_{..}^{K} < C_{..}^{K+1}$.
- Third situation is $X_{..}^{+K} = X_{..}^{*K} - Q$: since in this condition we have $\partial T(X)/\partial X_{..}^{K} = 0$.
- Fourth condition is $X_{..}^{+K} = V_{..}^{LK}$: we know that $X_{..}^{*(K+1)} \leq X_{..}^{*K}$ since $C_{..}^{K} \leq C_{..}^{K+1}$. Therefore, when $X_{..}^{+K} = V_{..}^{LK}$, then $Q = Q + V_{..}^{LK}$ and $X_{..}^{+(K+1)} = X_{..}^{*(K+1)} - Q < 0$.

In next subsection, we investigate on finding a global optimal solution based on initial solution obtained by Algorithm A1.

Algorithm A2

Any time that Algorithm A1 faces one of the four stop-conditions, other intervals, if any, does not evaluated for order allocation. When $X_{..}^{+K} = V_{..}^{LK}$ (fourth condition), the final solution of Algorithm A1 may not be globally optimal: Assume there exist three suppliers with limitation on minimum V_i^L and maximum V_i^U, $i = 1,2,3$, order quantity so that $C_i < C_{i+1}$. If Algorithm A1 arrives to the solution of $X_1^+ = V_1^U$, $X_2^+ = V_2^L$ and $X_3^+ = 0$, according to the forth condition Algorithm A1 terminates. However, there are other possible allocations (*Poss-Allocations*) that may increase the buyer's profit:

Poss-Allocation1: the solution obtained by Algorithm A1: $X_1^+ = V_1^U$, $X_2^+ = V_2^L$ and $X_3^+ = 0$.

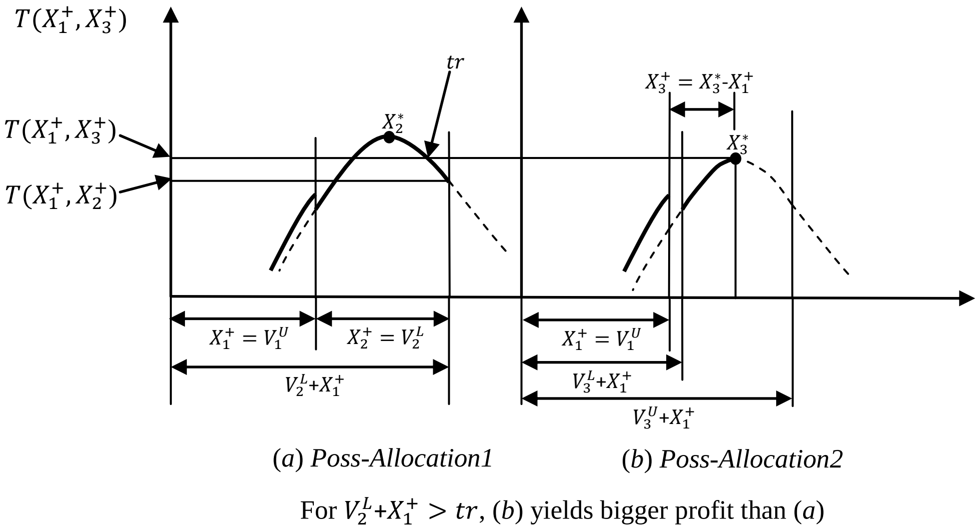
Poss-Allocation2: $X_1^+ = V_1^U$, $X_2^+ = 0$, $\quad X_3^+ = \max\left[\min(X_3^* - Q, V_3^U), V_3^L\right]$. Where $Q = V_1^U$.

Poss-Allocation3: $X_1^+ = \max\,[\min(X_1^* - Q, V_1^U)\,, V_1^L], X_2^+ = V_2^L, \qquad X_3^+ = 0$. Where $Q = V_2^L$.

where it is assumed $X_3^* - V_1^U > 0$, $X_1^* - V_2^L > 0$, $X_i^* = F^{-1}(\frac{p+S-c_i}{p+h+S})$.

Then the global optimal allocation is one of the *Poss-Allocations* that gives a higher profit to the buyer. (In numerical example section, we sometimes refer the solutions to *Poss-Allocation1 & 2 & 3* for better interpretation).

It may happen that *Poss-Allocation2*, $T(X_1^+, X_3^+)$, yields bigger revenue than *Poss-Allocation1*, $T(X_1^+, X_2^+)$, as illustrated by Figure. 2. In *Poss-Allocation3*, if we have to order V_2^L to the second supplier, the optimality condition for the first supplier should be again evaluated, that is $\partial T(\sum_{i=1}^{2} X_i)/\partial X_1 = 0$ should hold. It is straight forward that a solution with higher profit is globally optimal allocation in our example.



(*a*) *Poss-Allocation1* (*b*) *Poss-Allocation2*

For V_2^L+$X_1^+ > tr$, (*b*) yields bigger profit than (*a*)

Fig. 2. Comparison of *Poss-Allocation1*&*2*.

However, it is obvious that if any of the first three conditions (*stop-condition*) is fulfilled, the solution is globally optimal, because there is no any possible allocation that increases the profit of the buyer.

These three *Poss-Allocations* tell us that every time $X_{..}^{+K} = V_{..}^{L}$, some more actions should be added to Algorithm A1 for finding the global optimal solution. The extra actions are presented as follows:

Action 1. Set $X_{..}^{+K} = 0$ and then assess if other intervals after $S_{..}^{+K}$ yield a better solution (as *Poss-Allocations2*).

Action 2. Keep $X_{..}^{+K} = V_{..}^{L}$ and then evaluate again if optimality condition holds for other intervals before $S_{..}^{+K}$ (as *Poss-Allocation3*).

As a result, we incorporate these two actions in Algorithm A1 to develop Algorithm A2 for obtaining global optimal solution (see Figure. 3). In Algorithm A2, we again define some tasks similar to Initial Task and Task 1. Before setting $X_{..}^{+K} = 0$ and going for Action 1, we first need to memorize all allocations and its associated profit, because we later need to use the memorized information for carrying out Action 2.

Task 2 memorizes the information as:

Task 2: $X^{AR} = X^{A}$, $T^{AR} = T^{A}$, $K^{R} = K$, $Q^{R} = Q$, and $A^{R} = A$.

where R is a counter whenever $X_{..}^{+K} = V_{..}^{L}$, R is added by one; $X^{A,R=1} = X^{A}$, as instance, is the vector of positive allocations when $X_{..}^{+K} = V_{..}^{L}$ for the first time, i.e., $R = 1$.

Task 3 set $X_{..}^{+K}$ to zero, because we then need to do Action 1:

Task 3: $X^A = X^A \setminus \{X_{..}^{+K}\}$, $A = A \setminus \{S_{..}^K\}$, $Q = Q - X_{..}^{+K}$, and $X_{..}^{+K} = 0$.

For performing Action 2, Task 4 is also employed that recalls the allocations memorized in Task 2 and then sets all allocations to zero except the one that was equal to $V_{..}^{L.}$. Subsequently, Action 2 can be done.

Task 4: $X^A = X^{AR} \setminus \{X_{..}^{+f}, f = 1, \dots, K^R - 1\}$, $A = A^R \setminus \{S_{..}^f, f = 1, \dots, K - 1\}$, $Q = X_{..}^{+K^R}$, Kr=K^R, $R = R - 1$, $K = 0$.

In Algorithm A2, when $R > 0$, we set Kr=K^R in Task 4. Then by adding the condition of $K < Kr$ in Algorithm A2, we guarantee that only the optimality condition of all intervals before $X_{..}^{+K^R}$ is evaluated.

Algorithm A2

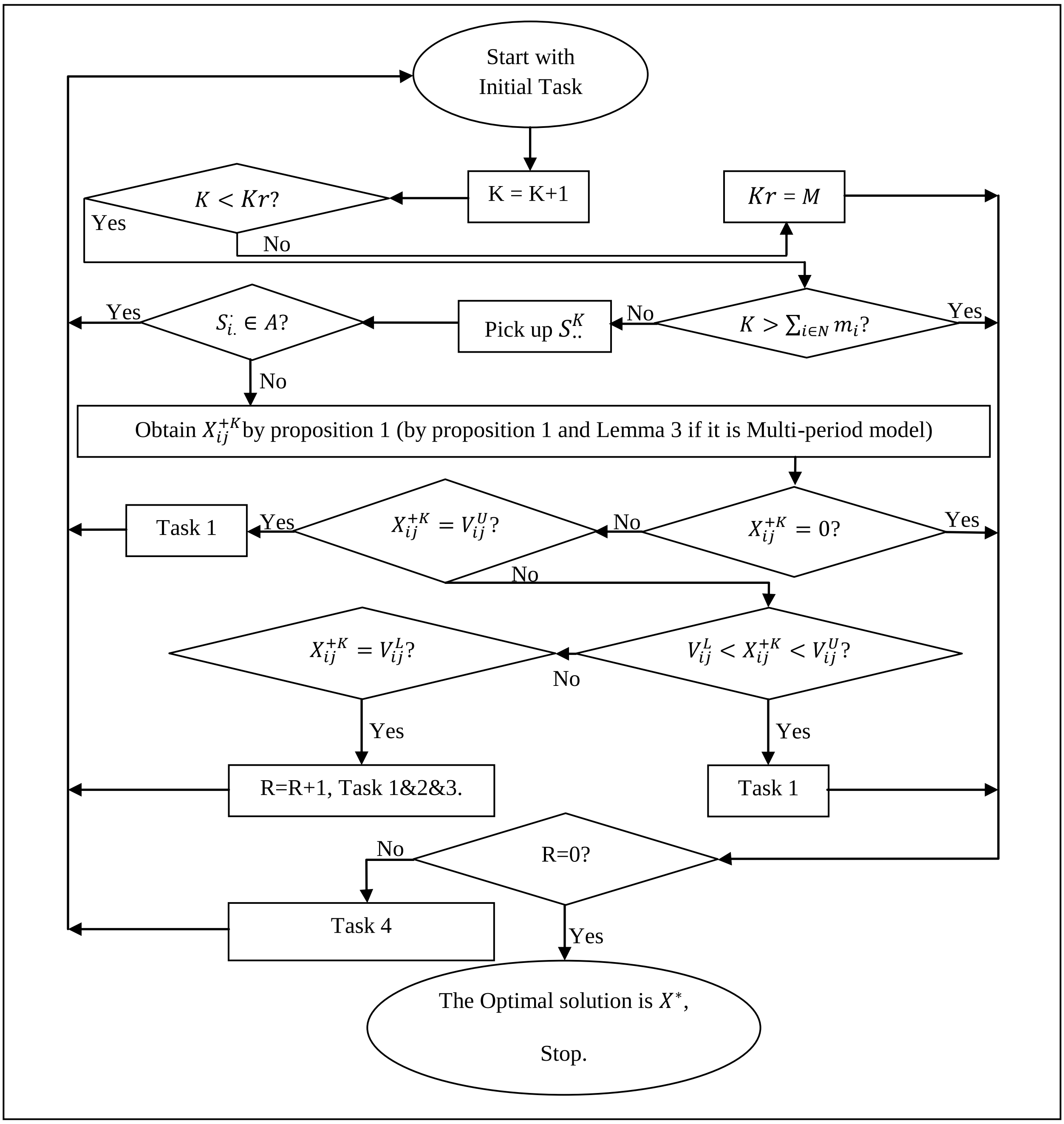


Fig. 3. Flowchart of Algorithm A2

2.2. Multi-Period Model

Despite the single period problem, this model is dynamic and operates over N ($t = 1,2,\ldots,N$) periods so that any surplus inventory in one period will be used at following period. In this study, the multi-period model is solved recursively so that the last period is solved first, the second last period is solved second and so on (Porteus, 2002). We assume that backlogging is not allowed. All parameters and variables in previous section are used here but we only add subscript t to them, because the model is non-stationary (for instance, h_t denotes holding cost per unit in period t). Let x_t and $X_t = x_t + \sum_{i,j} X_{tij}$ indicate respectively the inventory level before and after ordering in period t. Therefore, the expected holding and shortage cost function of period t related to X_t can be formulated as follows:

$$L_t(X_t) = h_t \int_0^{X_t} (X_t - \xi_t) f(\xi_t) d\xi_t + Sh_t \int_{X_t}^{+\infty} (\xi_t - X_t) f(\xi_t) d\xi_t.$$

The purchasing cost of period t is:

As the total order quantity purchased in period t is $OR_t = \sum_{i=1}^{I} \sum_{j}^{m_i} X_{tij} = X_t - x_t$, therefore, $Pur_t(OR_t) = \sum_{i=1}^{I} \sum_{j}^{m_i} C_{tij} X_{tij}$. Since, we need to declare $Pur_t(\cdot)$ in term of X_t, we will have:

$$Pur_t(X_t) = C_t(X_t - x_t), \text{ in which } C_t = \sum_{i=1}^{I} \sum_{j}^{m_i} C_{tij} X_{tij} \Big/ \sum_{i=1}^{I} \sum_{j}^{m_i} X_{tij}.$$

The expected revenue function of period t is also formulated as follows:

$$R_t(X_t) = p_t \int_0^{X_t} \xi_t\, f(\xi_t) d\xi_t + p_t \int_{X_t}^{+\infty} X_t\, f(\xi_t) d\xi_t.$$

Finally, the buyer's expected profit of period t when the state is x_t, is formulated as:

$T_t(x_t) =$

$\max_{X_t \geq x_t}\{R_t(X_t) - L_t(X_t) - C_t(X_t - x_t) + \alpha \int_0^{\infty} T_{t+1}(X_t - \xi_t)\, f(\xi_{t+1}) d\xi_{t+1}\}$.

where $\alpha \in (0, 1]$ denotes the one-period discount factor.

$T_t(x_t)$ can be written as:

$T_t(x_t) = \max_{X_t \geq x_t}\{G_t(X_t) + C_t\, x_t\}$

where $G_t(X_t) := R_t(X_t) - L_t(X_t) - C_t\, X_t + \alpha \int_0^{\infty} T_{t+1}(X_t - \xi_t)\, f(\xi_{t+1}) d\xi_{t+1}$.

Therefore, optimal inventory level after ordering (base stock level: X_t^*) in period t can be obtained by maximizing $G_t(X_t)$ over $\{X_t \mid X_t \geq x_t\}$. In other words, X_t is a maximizer of $G_t(X_t)$.

As a result, the maximizer of buyer's expected profit in each period can be obtained by solving Model 2, formulated using MINLP:

<u>*Model 2*</u>:

$Max\ G_t(X_t)$ (2.1)

Subject to:

$X_t = x_t + \sum_{i=1}^{I} \sum_{j=1}^{m_i} X_{tij}$ $\quad t = 1,2,\ldots,N$ (2.2)

$V_{tij}^{L} Y_{tij} \leq X_{tij}$ $\quad t = 1,2,\ldots,N;\ i = 1,2,\ldots,n;\ j = 1,2,\ldots,m_i$, (2.3)

$V_{tij}^{U} Y_{tij} \geq X_{tij}$ $\quad t = 1,2,\ldots,N;\ i = 1,2,\ldots,n;\ j = 1,2,\ldots,m_i$, (2.4)

$\sum_{j=1}^{m_i} Y_{tij} \leq 1$ $\quad t = 1,2,\ldots,N;\ i = 1,2,\ldots,n$, (2.5)

$X_{tij} \geq 0$ $\quad t = 1,2,\ldots,N;\ i = 1,2,\ldots,n;\ j = 1,2,\ldots,m_i$, (2.6)

In order to employ Algorithm A2 to solve Model 2, we need to find a way to calculate X^*_{tij} (X^*_{tij} is the optimum order quantity to supplier *i* at price level *j* in period *t* for Model 2 without bound constraints). Recall that in a single period problem, we had $X^*_{ij} = F^{-1}(\frac{P+S-C_{ij}}{P+h+S})$. Assume at the end of last period, each leftover unit has a value of C for the buyer (i.e., the slope of the terminal value function is $-C$). Also, assume that there exist only one supplier whose price in period N is C_N. By taking the first derivative of $G_N(X_N)$ over X_N and setting it to zero we obtain $X^*_N = F^{-1}(\frac{P+S-C_N}{P+h+S-\alpha C})$. Furthermore, the second derivative shows that it is concave (i.e., $G''_N(X_N) \geq 0$). By following Porteus (2002) in chapter six, it is demonstrated that $X^*_t = F^{-1}(\frac{P+S-C_t}{P+h+S-\alpha C_{t+1}})$ is optimal in period t for each period, in which C_t and C_{t+1} are the unit purchasing price of the supplier in period t and $t+1$, respectively. If we have multiple suppliers (assume without discount), for computing X^*_{ti} the difficulty is estimating C_{t+1}, because of the different unit purchasing price in period $t+1$ (i.e., $C_{(t+1,i)}\ \forall i$). However, we know that $C^{min}_{t+1} \leq C_{t+1} \leq C^{max}_{t+1}$, where C^{min}_{t+1} and C^{max}_{t+1} are the lowest and highest unit purchasing price in period $t+1$: Assume there are three suppliers in period $t+1$, where their price are $C_{t+1,i=1}$ =5, $C_{t+1,i=2}$ =6, $C_{t+1,i=3}$ =7; Then, C^{min}_{t+1} =5 and C^{max}_{t+1} =7. As a result, $F^{-1}(\frac{P+S-C_{ti}}{P+h+S-\alpha C^{min}_{t+1}}) \leq X^*_{ti} \leq F^{-1}(\frac{P+S-C_{ti}}{P+h+S-\alpha C^{max}_{t+1}})$. Thus, for determining the maximizer of G_t for each interval in each period, X^*_{tij}, Lemma 3 is defined as follows.

Lemma 3. Let $G_t(X_{tij}) \coloneqq R_t(X_{tij}) - L_t(X_{tij}) - C_{tij}\,X_{tij} + \alpha\int_0^\infty T_{t+1}(X_{tij} - \xi_t)\,f(\xi_{t+1})d\xi_{t+1}$ for the interval of S_{tij}. *Then,* $X^*_{tij} = \arg\max_{X_{tij}} G_t(X_{tij})$ *can be determined using following algorithm (see Figure. 4):*

Step 0: Set $X_{tij} = a = F^{-1}(\frac{P+S-C_{tij}}{P+h+S-\alpha C^{min}_{t+1}})$ *and* $b = F^{-1}(\frac{P+S-C_{tij}}{P+h+S-\alpha C^{max}_{t+1}})$.

Step 1. Compute $G_t(X_{tij})$ *and then* $X_{tij} = X_{tij} + \varepsilon$. *If* $X_{tij} \leq b$, *start again Step 1, else go to step 2.*

Step 2. $X^*_{tij} = \arg\max_X G_t(X_{tij})$.

where ε *is a small number so that smaller* ε *yields more exact* X^*_{tij}.

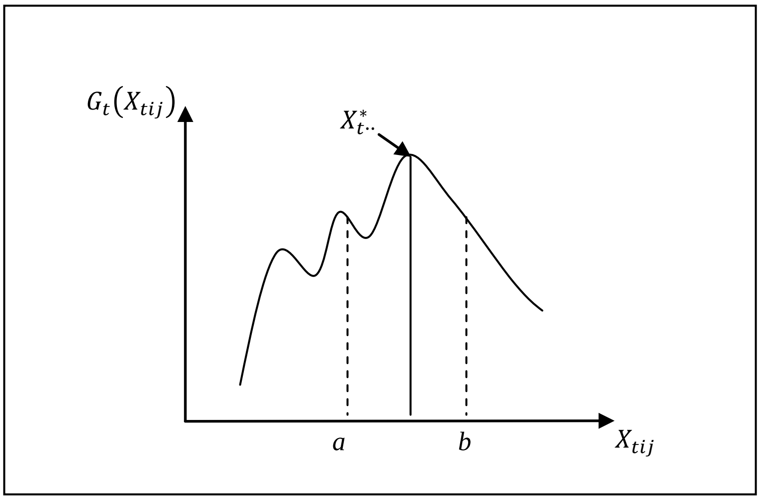


Fig. 4. Optimal inventory level (base stock level) after ordering

(iii) *Base stock policy can be represented as follows: if* $x_t \leq X^*_{tij}$, *then we should increase the stock level up to* X^*_{tij}, *else we do nothing.*

We here employ again Proposition 1 to obtain X^+_{tij}. Note that for multi-period model in proposition 1, Q is the summation of x_t and positive order quantities assigned in period t to other suppliers except (or before) supplier i. Now, Algorithm A2 can be used for multi-period problem where all parameters and variables get subscript t, indicating period t, and the algorithm should be recursively operated over N periods.

3. Numerical experiments

To demonstrate the performance of the proposed algorithm, two subsections are conducted here. In the first subsection, single period problem is considered, and some examples are solved by the proposed algorithm, coded using Visual Basic programming. In addition, different GAMS solvers for MINLP models, such as BARON (Branch-And-Reduce Optimization Navigator), DICOPT (DIscrete and Continuous OPTimizer) and SBB, are used to compare the solutions. In the second subsection, a non-stationary multi-period problem is solved and discussed.

3.1. Single-Period problem

We consider a situation in which demand follows uniform distribution $U(12,18)$, $p = 11$, $h, Sh = 0$. Five different cases are conducted and their parameters are illustrated in Table 1. In Table 1, S_{12}, as an example, represents first supplier in second price level. It is in line with naming each price level by S_{ij}^{K} where the index of K was used to show the rank of each interval among all in term of price. In Case 1, the first supplier has two price levels: If the order assigned to him is between 0 and 17, the purchasing price would be 5.5; and if it is between 17.01 and 20, the price is 5. In Case 3, we only changed V_{32}^{L} from 8.01 to 8.05, in comparison with Case 2.

Table 1.
Data related to the different cases

Scenarios	S_{12}	S_{11}	S_{22}	S_{21}	S_{32}	S_{31}	S_{41}
Case 1							
C_{ij}	5	5.5	5.5	6	6	6.5	6.6
V_{ij}^U	20	17	5.5	2.5	15	8	6
V_{ij}^L	17.01	0	2.51	0	8.01	0	2
Case 2							
C_{ij}	5	5.5	5.5	6	6	6.5	6.6
V_{ij}^U	5	3	5.5	2.5	15	8	6
V_{ij}^L	3.01	0	2.51	0	8.01	0	2
Case 3	In this case, $V_{32}^L = 8.05$, compared to case 2.						
Case 4	In this case, $V_{31}^L = 5.00$, compared to case 3.						
Case 5							
C_{ij}	5	5.5	5.5	-	6	6.5	-
V_{ij}^U	5	3	15	-	15	10	-
V_{ij}^L	3.01	0	12	-	10.01	0	-

By using the proposed algorithm, we solve the problem for the different cases. In order to be able to compare the solutions, the problem is also formulated in GAMS. The solutions are presented using Table 2.

Table 2.
The solutions of GAMS and the proposed algorithm

	Proposed Algorithm		GAMS			
			BARON		DICOPT	
Scenarios	Orders	Profit	Orders	Profit	Orders	Profit
Case 1	$X_{12} = 17.0$	79.08	$X_{12} = 17.0$	79.08	$X_{12} = 17.0$	79.08
Case 2	$X_{12} = 4.78$	72.570	$X_{12} = 4.78$	72.570	$X_{12} = 4.78$	72.570
	$X_{22} = 2.50$		$X_{22} = 2.50$		$X_{22} = 2.50$	
	$X_{32} = 8.00$		$X_{32} = 8.00$		$X_{32} = 8.00$	
Case 3	$X_{12} = 5.00$	72.520	$X_{12} = 4.73$	72.518	$X_{12} = 5.00$	72.520
	$X_{22} = 5.50$		$X_{22} = 5.50$		$X_{22} = 5.50$	
	$X_{31} = 3.96$		$X_{32} = 8.05$		$X_{31} = 3.96$	
Case 4	$X_{12} = 4.73$	72.518	$X_{12} = 4.73$	72.518	$X_{12} = 4.73$	72.518
	$X_{22} = 2.50$		$X_{22} = 2.50$		$X_{22} = 2.50$	
	$X_{32} = 8.05$		$X_{32} = 8.05$		$X_{32} = 8.05$	
Case 5	$X_{12} = 3.27$	75.818	$X_{12} = 3.27$	75.818	$X_{12} = 3.27$	75.818
	$X_2 = 12.0$		$X_2 = 12.0$		$X_2 = 12.0$	

The solutions of the proposed algorithm are interpreted as follows. In Case 1, one can see that the algorithm finds the global optimal solution in line with *Poss-Allocation1* (i.e. $X_{12} = 17.0$). In Case 2, although the price of suppliers 1 and 2 (i.e. C_{12} and C_{22}) are lower in comparison with third supplier's, the buyer does not order up to their full capacity (V_{12}^U and V_{22}^U). It is in line with *Poss-Allocation3*. In Case 3, even though the price of S_{32} is lower than that of S_{31}, the buyer prefers to order from S_{31}. It can be justified by *Poss-Allocation2*. In addition, by looking at the solution of BARON in Case 3, one can see that ordering from S_{32} yields worse profit than S_{31}, and BARON was not able to find the optimal solution. For more discussion assume in Case 3, V_{32}^L was equal to 8.04, then by using our algorithm the optimal solution is obtained as $X_{12} = 4.74$, $X_{22} = 2.50$, $X_{31} = 8.04$ and $T(X) = 72.528$. One can again see that the profit increases by 0.008 compared to original parameters of Case 3. Since $V_{31}^{L} = 5$ in Case 4, it is better to order from S_{32} (i.e. $X_{32} = 8.05$) that is obtained from comparison of *Poss-Allocation*1 and *Poss-Allocation3*. Example 5 can be interpreted the same as Case 2.

These five cases, especially cases 2, 3 and 4, demonstrate the accuracy of the proposed algorithm in finding global optimal solution. SBB in GAMS was also used for solving the cases, but we did not present the solutions in Table 2. We observed that SBB was not able to find global optimal solution for cases 3 and 4. Also, Table 1 illustrates that BARONS could not find global optimal solution in Case 3. However, the proposed algorithm and DICOPT obtained global optimal solution in all cases.

3.2. Multi-Period problem and sensitivity analysis

After demonstrating the accuracy of our algorithm in previous subsection, we here only perform sensitivity analysis in a non-stationary multi-period situation. As we have different parameters, such as C_{tij}, h_t, p_t and market demand, we analyze the effect of changing the value of those parameters on buyer’s profit over a three-period problem. In Case 1, we consider a stationary three-period problem as benchmark. Then, four other cases are introduced to see the effect of %20 improvement in C_{tij}, h_t, p_t and demand on buyer’s profit.

The benchmark case (Case 1) considers a stationary three-period problem in which the demand follows uniform distribution $U_t(12, 18)$, $p_t = 7.2$, $h_t = 4$, $\forall t$, $\alpha = 0.9$. Each leftover unit of last period values $C = 4.5$. In addition, Suppliers' information over the three-period is as follows:

	S_{t12}	S_{t11}	S_{t22}	S_{t21}
C_{tij}	5	5.5	5.5	6
V_{tij}^{U}	5	3	16	6
V_{tij}^{L}	3	0	6	0

The information of non-stationary three-period problems (Case2-5) is presented in Table 3.

Table 3.
The information of different cases

The cases	Scenarios	Model
Case 2	Selling price in second period increases by %20.	$p_2 = 8.64$
Case 3	C_{212} and C_{222} decrease by %20.	$C_{212} = 4.0$, $C_{222} = 4.4$
Case 4	Demand in period 2 increases by %20.	$U_2(14.4, 21.6)$
Case 5	Holding cost per unit in period 2 drops by %20.	$h_2 = 3.2$

The multi-period is solve recursively and the solutions of the different cases are presented in Table 4, only when the initial inventory level in each period is 0, 5 and 10:

Table 4.
The result of different cases

	Period 1		*Period 2*		*Period 3*	
Case 1 as benchmark						
$x_t\ \forall t$	$T_1(x_1)$	Orders	$T_2(x_2)$	Orders	$T_3(x_3)$	Orders
0	65.68	$X_{112} = 5.00$	49.99	$X_{112} = 5.00$	24.11	$X_{112} = 5.00$
		$X_{122} = 8.61$		$X_{122} = 8.61$		$X_{122} = 8.42$
5	92.27	$X_{112} = 3.15$	72.58	$X_{112} = 3.15$	50.42	$X_{112} = 3.00$
		$X_{122} = 6.00$		$X_{122} = 6.00$		$X_{122} = 6.00$
10	120.12	$X_{112} = 4.15$	100.43	$X_{112} = 4.15$	78.43	$X_{112} = 3.85$
Case 2 improves selling price per unit						
$x_t\ \forall t$	$T_1(x_1)$	Orders	$T_2(x_2)$	Orders	$T_3(x_3)$	Orders
0	83.43	$X_{112} = 5.00$	65.73	$X_{112} = 5.00$	24.11	$X_{112} = 5.00$
		$X_{122} = 8.61$		$X_{122} = 9.44$		$X_{122} = 8.42$
5	109.89	$X_{112} = 3.15$	92.55	$X_{112} = 3.84$	50.42	$X_{112} = 3.00$
		$X_{122} = 6.00$		$X_{122} = 6.00$		$X_{122} = 6.00$
10	137.89	$X_{112} = 4.15$	120.55	$X_{112} = 4.85$	78.43	$X_{112} = 3.85$
Case 3 improves purchasing price per unit						
$x_t\ \forall t$	$T_1(x_1)$	Orders	$T_2(x_2)$	Orders	$T_3(x_3)$	Orders
0	79.06	$X_{112} = 5.00$	61.07	$X_{112} = 5.00$	24.11	$X_{112} = 5.00$
		$X_{122} = 8.40$		$X_{122} = 9.65$		$X_{122} = 8.42$
5	105.35	$X_{112} = 3.00$	82.62	$X_{112} = 4.07$	50.42	$X_{112} = 3.00$
		$X_{122} = 6.00$		$X_{122} = 6.00$		$X_{122} = 6.00$
10	133.37	$X_{112} = 3.81$	105.01	$X_{112} = 5.00$	78.43	$X_{112} = 3.85$
Case 4 improves the demand						
$x_t\ \forall t$	$T_1(x_1)$	Orders	$T_2(x_2)$	Orders	$T_3(x_3)$	Orders
0	69.61	$X_{112} = 5.00$	50.35	$X_{112} = 5.00$	24.11	$X_{112} = 5.00$
		$X_{122} = 8.61$		$X_{122} = 11.39$		$X_{122} = 8.42$
5	96.04	$X_{112} = 3.15$	77.85	$X_{112} = 5.00$	50.42	$X_{112} = 3.00$
		$X_{122} = 6.00$		$X_{122} = 6.39$		$X_{122} = 6.00$
10	124.04	$X_{112} = 4.15$	104.81	$X_{112} = 5.00$	78.43	$X_{112} = 3.85$
				$X_{122} = 0.80$		
Case 5 improves the holding cost per unit						
$x_t\ \forall t$	$T_1(x_1)$	Orders	$T_2(x_2)$	Orders	$T_3(x_3)$	Orders
0	65.87	$X_{112} = 5.00$	46.20	$X_{112} = 5.00$	24.11	$X_{112} = 5.00$
		$X_{122} = 8.61$		$X_{122} = 8.85$		$X_{122} = 8.42$
5	92.31	$X_{112} = 3.15$	72.77	$X_{112} = 3.44$	50.42	$X_{112} = 3.00$
		$X_{122} = 6.00$		$X_{122} = 6.00$		$X_{122} = 6.00$
10	120.31	$X_{112} = 4.15$	100.77	$X_{112} = 4.44$	78.43	$X_{112} = 3.85$

Case 1 shows that order allocation in period 3 is slightly less than those of periods 1 and 2. It is due to $C = 4.5$ (each leftover unit value at the end of third period). In periods 1 and 2, the leftover units values C_{tij} for their following period that are greater than $C = 4.5$. For Case 2, one can see that the increase of selling price in period 2 has only influence on order allocations at the same period. In addition, it increases the total profit by %16 $\forall x_t = 5$, compared to Case 1. In Case 3, the decrease of suppliers' prices in second period causes the order allocations to increase in same period and to decrease in period 1. It can be interoperated in two ways: (1) since the purchasing cost in period 2 will decrease by %20, it is better that we buy less in period 1 and more in period 2, and (2) since the left over units in period 1 will have less value in next period, it is better that we have less over stock at the end of period 1. The total profit of Case 3, $\forall x_t = 5$, improved by %12 in comparison with Case 1. For Case 4 also one can see that the increase of demand in period 2 raises the order allocations at the same period only. The total profit of Case 4, $\forall x_t = 5$, was also improved by %4 in comparison with Case 1. In Case 5, improving the holding cost also enhances the total profit in comparison with Case 1, while it is not considerable.

By considering these five cases, we can conclude that (1) selling price and holding cost per unit have respectively highest and lowest impact on total profit (see Figure. 5), and (2) only purchasing cost alteration in one period changes the order allocation of the same period as well as that of in its previous period.

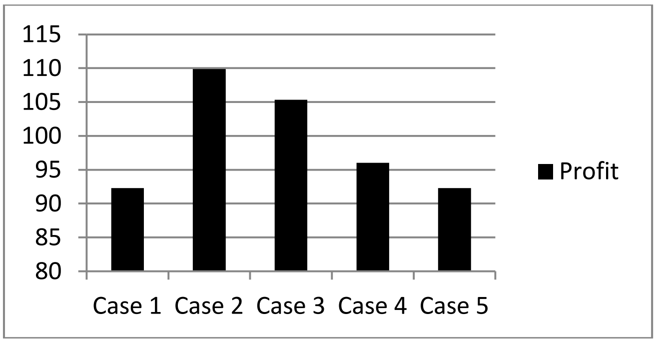


Fig. 5. The profit of the different cases

4. Conclusion

This study considered a single-product multi-period SSP subject to the quantity discount schemes and demand uncertainty. We employed MINLP formulation to model the problem in order to maximize the expected profit of the buyer under multiple sourcing scenario. We developed an algorithm to solve the single period problem, and in order to be able to verify the efficiency and accuracy of the proposed algorithm, we also developed a GAMS-based solution program to compare the solutions. Comparison of the solutions demonstrated the capability of the proposed algorithm for obtaining global optimal solution. Subsequently, we extended the algorithm for solving the non-stationary multi-period problem. In multi-period problem, we examine the effect of changing some model parameters (selling price, purchasing price, holding cost, and demand) on buyer’s expected profit. The result revealed that selling price and holding cost had highest and lowest impact on the buyer profit, respectively.

This study enables DMs to maximize the buyer expected profit by simultaneously considering the demand uncertainty, planning horizon, and suppliers’ quantity discount in supplier selection model that make it very realistic and complicated. This study also helps managers to consider some other restrictions such as buyer’ warehouse capacity, suppliers’ capacity, holding cost, etc in planning the inventory level on an ongoing basis during a year. Moreover, they can ensure the production managers that the raw materials and components parts are available during a time horizon especially when the demand is not known precisely.

In reality, suppliers may allow buyers to return unsold products at the end of each period (buyback). Literature on inventory management shows that buyback can be used as an incentive for the buyer to increase the order. As a result, considering a SSP, where the buyer can return unsold products to the suppliers, is still open for future. Another direction for future work can be considering a situation in which the suppliers have binomial random yields.

Appendix

Proof of Proposition 1.

(i) Obviously, if we have assigned Q units to other suppliers, $(X_{ij}^{*} - Q)$ should be ordered to supplier *i* at level *j* in order to satisfy $\partial T(X)/\partial X_{ij} = 0$.

(ii) The second derivative of $T(X)$ is $-(p + s + h)f(X_{ij}) \leq 0$. Therefore, it is concave and monotonic increasing for $X_{ij} < X_{ij}^{*}$. Since $\left(V_{ij}^{L} + Q\right) < (V_{ij}^{U} + Q) < X_{ij}^{*}$, we have $T\left(V_{ij}^{L} + Q\right) < T\left(V_{ij}^{U} + Q\right) < T\left(X_{ij}^{*}\right)$. Therefore, $X_{ij}^{+} = V_{ij}^{U}$.

(iii) Similar to (ii), we have $X_{ij}^{+} = V_{ij}^{L}$, since $T(X)$ is monotonic decreasing for $X_{ij} > X_{ij}^{*}$.

(iv) It is straightforward. □

Algorithm A1

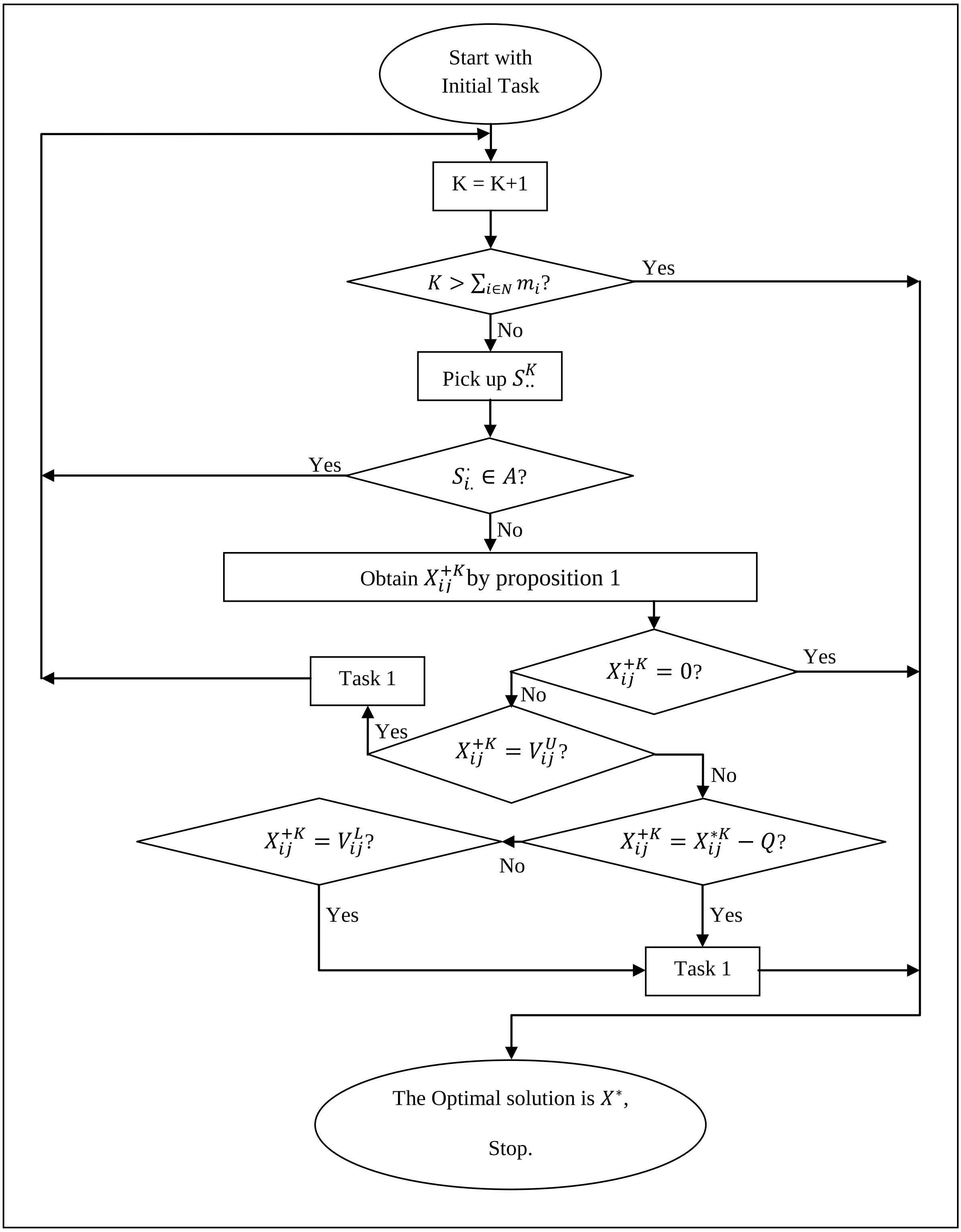


Fig. A1. Flowchart of Algorithm A1

Reference

Amid, A., Ghodsypour, S.H., O'Brien, C., 2009. A weighted additive fuzzy multi-objective model for the supplier selection problem under price breaks in a supply Chain. International Journal of Production Economics 121 (2), 323-332.

Awasthi, S.S., Chauhan, S. K., Goyal, S. K., Proth, Jean-Marie, 2009. Supplier selection problem for a single manufacturing unit under stochastic demand. International Journal of Production Economics 117, 229–233.

Basnet, C., Janny M.Y. Leung, Inventory lot-sizing with supplier selection, Computers & Operations Research 32 (2005) 1–14

Boer, L.D., Labro, E. and Morlacchi, P., 2001. A review of methods supporting supplier selection. European Journal of Purchasing and Supply Management, 7, 75–89.

Burke, G.J.,Carrillo,J.E.,Vakharia,A.J.,2007.Single versus multiple supplier sourcing strategies. European Journal of Operational Research 182, 95–112.

Dada, M., Petruzzi, N. C., Schwarz, L. B., 2007. A newsvendor's procurement problem when suppliers are unreliable. Manufacturing & Service Operations Management 9, 9–32.

Demirtas, E.A., Ustun, O., An integrated multiobjective decision making process for supplier selection and order allocation, Omega 36 (2008) 76 – 90

Demirtas, E.A., Ustun, O., Analytic network process and multi-period goal programming integration in purchasing decisions, Computers & Industrial Engineering 56 (2009) 677–690

Ghodsypour, S. H., O'Brien, C., A decision support system for supplier selection using an integrated analytic hierarchy process and linear programming, 56-57 issue 1 (1998), 199-212

Kim, B., Leung, J.M.Y., Park, K.T., Zhang, G., and Lee, S. (2002), 'Configuring a Manufacturing Firm's Supply Network with Multiple Suppliers', IIE Transactions, 34(8), 663–677.

Mak, K.L., Cui, L.X., and Su, W., Optimal Supplier Selection and Order Allocation for Multi-Period Multi-Product Manufacturing Incorporating Customer Flexibility, Engineering Letters 2011

Mansini, R., Savelsbergh, M.W.P., Tocchella, B., The supplier selection problem with quantity discounts and truckload shipping, Omega 40 (2012) 445–455.

Porteus, E., Foundation of Stochastic Inventory Theory, Standford University Press, Standford, 2002.

Rungreunganaun, V., and Woarawichai, C., Applying Genetic Algorithms for Inventory Lot-Sizing Problem with Supplier Selection under Storage Space, World Academy of Science, Engineering and Technology 74, 2013

Sen, A., Yaman, H., Guler, K., Korpeoglu, E., Multi-period Supplier Selection under Price Uncertainty, Journal of the Operational Research Society (2013), Volume 65 Issue 1

Ustun, O., Demirtas, E.A., An integrated multi-objective decision-making process for multi-period lot-sizing with supplier selection, Omega 36 (2008) 509 ‑ 521

Woarawichai, C., Kullpattaranirun, T., Rungreunganun, V., Inventory Lot-Sizing Problem with Supplier Selection under Storage Space and Budget Constraints, International Journal of Computer Science Issues, Vol. 8, Issue 2, March 2011

Xia, W., Wu, Z., Supplier selection with multiple criteria in volume discount environments, Omega 35 (2007) 494 – 504

Yang, S., Yang, J., Abdel-Malek, L., 2007. Sourcing with random yields and stochastic demand: a newsvendor approach. Computers & Operations Research 34,3682–3690.

Yin, S., Nishi T., 2013. A solution procedure for mixed-integer nonlinear programming formulation of supply chain planning with quantity discounts under demand uncertainty, International Journal of Systems Science, in press

Zhang, G., & Ma L., (2009): Optimal acquisition policy with quantity discounts and uncertain demands, International Journal of Production Research, 47:9, 2409-2425

Zhang, G., The multi-product newsboy problem with supplier quantity discounts and a budget constraint, European Journal of Operational Research 206 (2010) 350–360

Zhang,J., Zhang, M., Supplier selection and purchase problem with fixed cost and constrained order quantities under stochastic demand, Int. J. Production Economics 129 (2011) 1–7

CHAPTER 5 – SUPPLIER SELECTION AND ORDER ALLOCATION PROBLEM UNDER DEMAND AND SUPPLY UNCERTAINTY WITH RETURN POLYCY

The main aim of this chapter is to propose an algorithm for finding optimal inventory level and suppliers' order allocation for a stochastic single-period supplier selection problem where the suppliers may be unreliable in term of quality and/or delivery. In addition, the suppliers may allow the buyer to return unsold products at the end of the period. This study addresses the combination of supplier selection and inventory management under demand and supply uncertainty. For a single-period, a buyer purchases a product from a set of capacitated suppliers whose supply may be unreliable in term of quality and/or delivery. In order to compensate the unreliability, the suppliers might allow the buyer to return unsold products at the end of the period with a buyback price (buyback scheme). As a result, the buyer has to consider three criteria for supplier evaluation: suppliers' wholesale price, unreliability level, and buyback price. Considering multiple criteria along with demand and supply uncertainties contribute to the complexity of the problem. In order to calculate the optimum inventory level and suppliers' order quantity, we develop an algorithm that is the main contribution of our study.

This study numerically reaches to the following managerial results: (1) buyback scheme lets the buyer share the risk of uncertain supply and demand with his suppliers; (2) it is also an effective approach for the suppliers to compensate their unreliability-level; (3) in comparison with lower demand variance, when the demand variance is higher, suppliers can compensate their unreliability-level with fewer buyback price; (4) diversification is not an optimal approach for the retailer, unless the suppliers have limited capacity.

1. Introduction

A supply chain (SC) is composed of a combination of activities associated with planning, acquiring, coordinating, contributing materials, component parts, finished good and service from upstream suppliers to downstream customers (Chopra and Meindl, 2006; Wadhwa, 2008). Managing the SC has been recently considered as a substantial issue by both practitioners and academicians, since the increase of customer's satisfaction and firm's profitability is the important targets of it (Boran et al., 2009; Chou and Chang, 2008; Ha and Krishnan, 2008; Heizer and Render, 2004; Monczka et al., 2001; Simchi-Levi et al., 2003; Stevenson, 2005). A very important element for the success of SC management (SCM) is following an effective purchasing function (Boran et al., 2009; Cakravastia and Takahashi, 2004; Chou and Chang, 2008; Giunipero and Brand, 1996; Porter and Millar, 1985). Prudently selecting of right suppliers brings meaningful savings for the manufacturing firms (Boran et al., 2009; Haq and Kannan, 2006). For most US manufacturers, 40-60% of production costs are associated with raw material cost (Wadhwa, 2008). In addition, Ghodsypour and O'Brien in 2001 stated that, on average, up to 70% of product cost in manufacturing firms is associated with purchased component parts and raw materials. Therefore, selecting the competent group of suppliers and maintaining them are the most important decisions in purchasing function (Wadhwa, 2008).

The literature has addressed two scenarios for supplier selection problem (SSP): (1) Single-sourcing scenario. In this scenario, the buyer needs to select the best supplier if he can meet all his requirements in terms of demand, quality, delivery etc. Although following this scenario causes the buyer to have a close relationship with the supplier, the risk of supply disruptions may increase (Tajbakhsh, 2007). On the other hand, if the buyer needs a high-tech product, obviously a small number of suppliers are available (Tajbakhsh, 2007); (2) Multiple-sourcing scenario. In this situation, the buyer needs to select a competent group of suppliers. This scenario is a good way to decrease the risk of supply disruptions, although it needs more flexibility from the buyer (Aissaoui et al, 2007). In addition, applying multiple-sourcing scenario can decreases the overall costs of procuring and inventory in many cases like just-in-time environment (Hong and Hayya's, 1992; Aissaoui et al, 2007).

For both multiple- and single-sourcing scenarios, the suppliers are evaluated according to some conflicting criteria such as price, quality, delivery etc. By an study in the United States and Canada, Dickson (1966) identified and ranked 23 criteria for SSP. The top six criteria were respectively quality, delivery, performance history, warranty policy, production facilities and capacity, and price. As can be seen from the top six, price, defects and lead-time are the only criteria that are quantitative. That is why, in studies that have used mathematical programming for SSP, price, defects and lead-time are widely used as the top three criteria influencing supplier selection (Roa and Kiser, 1980; Weber and Current, 1993; Ghodsypour and O'Brien, 1998; Kumar et al., 2004, 2006; Wadhwa and Ravindran, 2007; Amid et al., 2006, 2009, 2011).

In addition, at the time of evaluating and selecting the suppliers, it is important for the buyer to simultaneously consider the holding, shortage and salvage costs of the products (inventory management) in order to find the optimal order quantity. This importance is more highlighted when the demand is not precisely known. However, most studies on inventory management focus on single sourcing model and multi-sourcing model has observed less consideration (Minner, 2003; Tajbakhsh, 2007). There are two types of inventory management models: (1) the economic order quantity (EOQ) model that applies when the demand is deterministic. This model determines the optimal order quantity in order to minimize the total cost of ordering, purchasing, holding and storage; (2) the newsvendor model that determines the optimal order quantity when the actual demand is unknown while its stochastic distribution is known. In inventory management, newsvendor model is considered as a very important and basic model for other sophisticated inventory models and is widely used in reality due to the decline in product life cycle (Zhang and Zhang, 2011). Furthermore, in real situation, the buyer's information on market demand as the most significant cause of uncertainty is not always known precisely (Tajbakhsh, 2007). Burke et al. (2007) developed an approach for supplier selection problem with stochastic demand; allocating optimal order quantities to a set of suppliers who must be ordered a positive amount, in which the minimum and maximum of their capacity were limited. Awasthi et al. (2009) proposed a heuristic algorithm for identifying and allocating the suppliers with restriction on suppliers' minimum and maximum capacity. Zhang and Zhang (2011) proposed an algorithm for solving a SSP with stochastic demand and restriction on suppliers' minimum and maximum capacity by considering fixed ordering cost. Zhang and Ma (2009) and Yin and Nishi (2013) developed a mixed integer nonlinear programming model for multi-supplier newsvendor problem where the suppliers offer quantity discounts.

These aforementioned studies only used the suppliers' price (or discounted price) for the evaluation of the suppliers, while the literature shows that *quality* and *delivery* have been wildly used as well. *Quality* can refer to a situation that the delivered quantity by suppliers might include defective items, and *delivery* can refer to a situation that the delivered quantity might be different from the ordered quantity (Tajbakhsh, 2010b). These supply uncertainties in *quality* and/or *quantity* is one of the most important factors that influences the decision of buyers on lot sizing (Tajbakhsh et al., 2010b). Binomial yield, stochastically proportional yield, and random capacity are three approaches used to model yield uncertainty in inventory control (Tajbakhsh et al., 2010b).

However, there are few studies that have dealt with stochastic inventory control (newsvendor) problems with unreliable suppliers (or with suppliers whose supply is uncertain). In this study, the level of supply uncertainty of suppliers is also called the level of suppliers' unreliability. Anupindi and Akella (1993) studied a situation in which a buyer has two uncertain suppliers and face with stochastic demand. The authors investigated on the implication of order allocation on buyer's inventory policies in both single- and multiple periods. Dada et al., 2007 also studied multi-supplier newsvendor problem under uncertain supply. Yang et al, 2007 proposed a solution algorithm for supplier selection newsvendor problem in which a buyer needs to allocate optimal order quantities to a set of suppliers whose supply is uncertain. In addition, there are other studies that have considered multi-supplier inventory problems with deterministic demand. Fadıloglu et al. (2008) considered supplier diversification for an Economic Order Quantity (EOQ) model for a case that suppliers' supply was modeled by binomial yields. They also demonstrated that diversification is not optimal. Subsequently, the study of Fadıloglu et al. (2008) was followed by Tajbakhsh et al. (2010a) and Yan and Wang (2013) as technical not. Yan et al. (2012) considered a supplier diversification problem where the suppliers' supply was uncertain and the price was linearly dependent on the products quantity delivered to the market.

The drawback of these studies is, however, they assumed the suppliers offer only wholesale price contract, while because of some reason the suppliers may offer buyback contract as well. The importance of buyback contract can be interpreted as follows: since risks are the consequence of uncertain demand, many buyers often request the suppliers to accept the return of unsold products in order to secure the risks (Shi and Su, 2004; Padmanabhan and Png, 1997). The most important reason for this request is risk-sharing, and if the suppliers are able to endure the risk more than the buyer, this request is rational (Shi and Su, 2004; Padmanabhan and Png, 1997). As a result, return policy is implementable for many products facing with uncertain demand, such as newspapers, magazines, books, recorded music, computer hardware and soft-ware, greeting cards, and pharmaceuticals (Shi and Su, 2004; Padmanabhan and Png, 1997; Howard et al. 1995). Despite the popularity of the buyback scheme, to the best of our knowledge, only Yang et al. (2007) considered a multi-supplier newsvendor problem whose suppliers offer buyback contract. They used genetic algorithm for SSP and order allocation.

Despite the importance of buyback scheme and supply uncertainty on production planning and inventory control, studies on SSP have addressed each aspect separately in the literature. In order to fill the gap, we here consider a multi-supplier newsvendor problem under binomial yield supply in which the suppliers may offer buyback contract for alleviating the risks arise due to uncertain demand and supply. The difficulty of this problem is that the suppliers should be evaluated based on three criteria: wholesale price, unreliability level, and buyback price. Consequently, we develop a solution algorithm for SSP and order allocation that is applicable for situations in which the suppliers are evaluated according to single- or multiple criteria.

The rest of the chapter is organized as follows. Section 2 models the problem and then develops the solution algorithm for supplier selection and order allocation. Numerical example is carried out in section 3. Furthermore, the effect of different parameters, such as buyback price, unreliability level, etc., on decision variables and buyer's profit is investigated in the numerical example. Finally, section 4 draws the conclusion.

2. Methodology

2.1. The model

n potential suppliers ($i = 1,2, \dots, n$), with the unit selling price or wholesale price of C_i and the production capacity of V_i, are going to meet the stochastic (single-period) demand, y, of one product for a buyer. In addition to the uncertain demand, the suppliers' supply may be uncertain that can refer to uncertainty in *quality* and/or *delivery*. Therefore, the suppliers may be unreliable with the unreliability level of θ_i: (1-θ_i) is the probability of that a unit, delivered by supplier i, is non-defective or usable (or the probability of that a unit, delivered by supplier i, arrives on time). The Binomial yield is used here to model the unreliability of the suppliers on supply. Q^{θ} is the total order quantity, including defective products, that the buyer allocates to the suppliers, and $0 \leq X_i \leq 1$ is the percent of Q^{θ} assigned to supplier i: $q_i^{\theta} = Q^{\theta} X_i$ is the ith supplier's order. It is assumed that $\sum_{i=1}^{n} V_i > Q^\theta$, therefore, $\sum_{i=1}^{n} X_i = 1$. The number of non-defective (usable) products delivered by supplier i, q_i, is a Binomial random variable with parameters $q_i^\theta (= Q^\theta X_i)$ and $(1 - \theta_i)$, i.e., $\Pr\{q_i = Q_i\} = \binom{q_i^\theta}{Q_i} (1 - \theta_i)^{Q_i} \theta_i^{q_i^\theta - Q_i}$, $Q_i = 0,1, \ldots, q_i^\theta$. As a result, the expected number of non-defective items received from supplier i can be determined by $\mathrm{E}[q_i] = q_i^\theta (1 - \theta_i)$, and similarly, the expected total number of non-defective products is $\mathrm{E}[Q] = R = Q^\theta \sum_{i=1}^{n} X_i (1 - \theta_i)$. In other words, the retailer purchases Q^θ units from all the suppliers, but he can only sell R units to the market (or only R units are useable).

In addition, the suppliers pay the retailer b_i \$/unit for leftover inventory at the end of season (return policy). As the defective items cannot be sold to the market, it is assumed that after receiving the order, the buyer executes 100% inspection and then returns the defective items accompany with leftover inventory to the suppliers by b_i \$/unit. (Note: this way of treating with defective items, is one way out of many other ways).

The known density function of the demand and its cumulative distribution function are presented by $f(y)$ and $F(y)$, respectively. The buyer's selling price, P \$/unit, to the market is determined exogenously. The buyer incurs s \$/unit for each unit of shortage inventory. Throughout the chapter, when we say "supplier i dominates other suppliers" it means that it is more profitable for the buyer to purchase from supplier i as much as possible.

The following assumptions are made:

(a) $0 < C_i < p, \qquad \forall\, i = 1, \ldots, n,$

(b) $0 \leq b_i \leq C_i$, $\forall\ i = 1, \dots, n$,

(c) the remaining products at the end of season are returned to the suppliers corresponding to X_i.

The buyer's expected revenue of selling the product to the market is expressed as follows:

$$ER = P\int_0^R y.f(y)dy + P\int_R^{+\infty} R.f(y)dy$$
$$= P\int_0^{Q^\theta.X} y.f(y)dy + P\int_{Q^\theta.X}^{+\infty} Q^\theta.X.f(y)dy$$

where $X = \sum_{i=1}^n X_i(1-\theta_i)$.

The expected shortage cost for the buyer is presented as follows:

$$ES = s\int_R^{+\infty}(y-R)f(y)dy$$
$$= s\int_{Q^\theta.X}^{+\infty}(y-Q^\theta.X)f(y)dy$$

The buyer's expected revenue of returns of unsold and defective products to the suppliers is stated as follows:

$$EB = \left(\sum_{i=1}^n b_iX_i\right)\int_0^R (R-y)f(y)dy + Q^\theta\sum_{i=1}^n b_iX_i\,\theta_i$$
$$= \left(\sum_{i=1}^n b_iX_i\right)\int_0^{Q^\theta.X}(Q^\theta.X-y)f(y)dy + Q^\theta\sum_{i=1}^n b_iX_i\,\theta_i$$

The purchasing cost is also presented as follows:

$$Pur = Q^\theta\sum_{i=1}^n C_iX_i$$

Therefore, the maximization profit function of the buyer for a multi-supplier newsvendor problem can be presented as follows:

$$\max T(Q^\theta) = ER - ES + EB - Pur \quad (1)$$

Lemma 1. $\max T(Q^\theta)$ *is concave with respect to* Q^θ.

Proof. To show that $\max T$ is concave, we first take the second derivative of it with respect to Q^{θ}, and then show that it is equal or less than zero.

$$\frac{\partial T}{\partial Q} = P\int_{Q^{\theta}.X}^{+\infty} X.f(y)dy + \sum_{i=1}^{n} b_i X_i.\left(\int_{0}^{Q^{\theta}.X} X.f(y)dy\right) + s\int_{Q^{\theta}.X}^{+\infty} X.f(y)dy - \sum_{i=1}^{n} C_i X_i$$

$$\frac{\partial^2 T}{\partial Q^2} = (-P - s + \sum_{i=1}^{n} b_i X_i)X^2.f(Q^{\theta}.X)$$

Since $b_i < P$ and $\sum_{i=1}^{n} X_i = 1$ then $\sum_{i=1}^{n} b_i X_i < P$ and thus $\partial^2 T/\partial Q^2 \leq 0$. Therefore, it is demonstrated that $\max T$ is concave with respect to Q^{θ}. □

If supplier *i* is uncapacitated, the profit function of the buyer if he only purchases from supplier *i* is presented as follows:

$$T^{si}(Q^{\theta}) = P\int_{0}^{R} y.f(y)dy + P\int_{R}^{+\infty} R.f(y)dy - s\int_{0}^{+\infty}(y-R)f(y)dy + b_i\int_{0}^{R}(R-y)fydy+Q\theta bi\theta i+Q\theta Ci \quad (2)$$

By taking the first derivative of $T^{si}(Q^{\theta})$ with respect to Q^{θ} and set it to zero we obtain the optimal order quantity assigned to supplier *i* as

$$Q^{\theta i*} = F^{-1}\left(\frac{(P+s)(1-\theta_i)+b_i-C_i}{(P+s-b_i)(1-\theta_i)}\right)/(1-\theta_i) \quad (3)$$

Before going to the following theorems, without lose of generality, assume that there are only two suppliers for order allocation in our newsvendor problem.

THEOREM 1. *In wholesale-price contract, for any* $Q > 0$, $\theta_1 = \theta_2 = 0$ *and* $C_1 < C_2$, *we have*:

(a) $T^{s1}(Q) > T^{s2}(Q)$ that *means supplier* 1 *dominates supplier* 2.

(b) *It is optimal for the buyer to fill his order, Q, as much as possible by supplier* 1 *first* (*i.e.*,$QX_1 = \min(Q, V_1)$), *and then the remaining, if any, by supplier* 2 (*i.e.*, $QX_2 = \min(Q - QX_1, V_2)$).

Proof.

(a) For wholesale-price contract when $\theta_i = 0$ and $b_i = 0$ $\forall i$, the profit of the buyer if he only purchases from supplier *i* can be presented as Eq(2), $T^{si}(Q) = ER - ES - QC_i$ $\forall i$. Since $C_1 < C_2$, then we have $-QC_1 > -QC_2$. As a result, $T^{s1}(Q) = ER - ES - QC_1 > T^{s2}(Q) = ER - ES - QC_2$. Thus, we can conclude that if the suppliers are uncapacitated, it is more profitable for the buyer to purchase from supplier 1 who has the lowest purchasing price per unit.

(b) If supplier 1 has limited production capacity, by Eq(1) the buyer's profit is presented as $T(Q) = ER - ES - Q\sum_{i=1}^{2} C_i X_i$. If X_1=0 then X_2=1, therefore, we have $T(Q) = T^{s2}(Q)$. If X_1=$\varepsilon < 1$, then X_2=1$-\varepsilon$, therefore, we have $T(Q) = T^{s2}(Q) + Q(C_2 - C_1)\varepsilon$. Since $C_1 < C_2$, it is proved that $T(Q) > T^{s2}(Q)$. That is, when X_1 increases, the buyer's profit increases as well. □

Theorem 1 says that when suppliers are evaluated based on only wholesale price, buying from a supplier whose price is less is always optimal for the buyer. If the suppliers have limited capacity, the lower price supplier should be first ordered as much as possible, and then, the second supplier is ordered, if any.

THEOREM 2. *In buyback contract, for any* $Q > 0$, $\theta_i = 0, \forall i$, *and* $C_1 < C_2$,

(a) *if* $b_2 = b_1 + Q(C_2 - C_1)/ESA$, *then neither supplier dominates another one and* $T^{s1}(Q) = T^{s2}(Q)$,

(b) *if* $b_2 < b_1 + Q(C_2 - C_1)/ESA$, *then supplier* 1 *dominates supplier* 2 *and* $T^{s1}(Q) > T^{s2}(Q)$.

(c) *if* $b_2 > b_1 + Q(C_2 - C_1)/ESA$, *then supplier* 2 *dominates supplier* 1 *and* $T^{s1}(Q) < T^{s2}(Q)$.

where $ESA = \int_{-\infty}^{Q}(Q - y)f(y)dy$.

Proof.

(a) By Eq(1) the buyer's profit function is presented as $T(Q) = ER - ES + ESA\sum_{i=1}^{2} b_i X_i - Q\sum_{i=1}^{2} C_i X_i$. By substituting $b_2 = b_1 + Q(C_2 - C_1)/ESA$ in the profit function, we have $T(Q) = T^{s1}(Q)$. That is, with every portion of $X_i \; \forall \; i = 1{,}2$ the buyer's profit is equal to a case if he purchases from supplier 1 only. On the other hand, by substituting $b_2 = b_1 + Q(C_2 - C_1)/ESA$ in $T^{s2}(Q) = ER - ES + ESA.b_2 - QC_2$ we obtain $T^{s1}(Q) = T^{s2}(Q)$.

(b) Let $b_2 = b_1 + \frac{Q(C_2 - C_1)}{ES} - \varepsilon$ (ε is a positive small number). By substituting b_2 in $T(Q)$, we have $T(Q) = T^{s1}(Q) - ESA.X_2.\varepsilon$. That is, when X_2 increases $T(Q)$ decreases. It is proved that it is more profitable for the buyer to purchases from supplier 1 as much as possible. On the other hand, by substituting b_2 in $T^{s2}(Q) = ER - ES + ESA.b_2 - QC_2$ we have $T^{s2}(Q) = T^{s1}(Q) - ES.\varepsilon$ that means $T^{s1}(Q) > T^{s2}(Q)$.

(c) Similar to (b), (c) can be proved. □

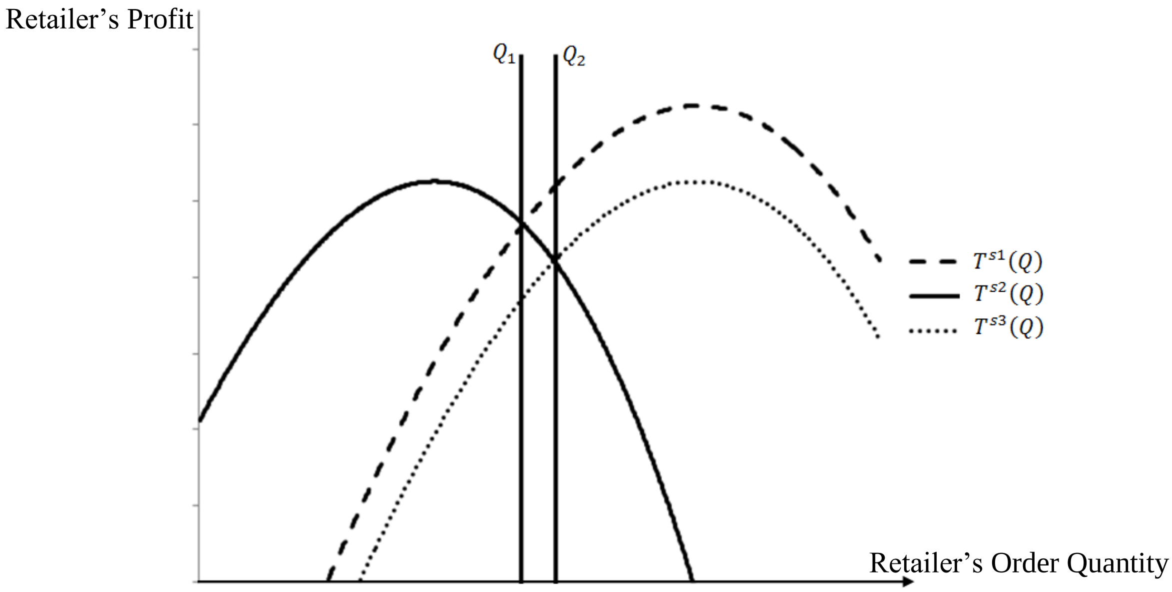


Fig. 1. Buyer's profit for the buyback scheme.

Theorem 2 and Figure. 1 say that for any order quantity, Q, greater than Q_2, supplier 1 is the first one, supplier 3 is the next one, and supplier 2 is the third one, because $T^{s1}(Q) > T^{s3}(Q) > T^{s2}(Q)$. For any order quantity, Q, between Q_1 than Q_2, supplier 1 is again the first one, supplier 2 is the next one, and supplier 3 is the third one, because $T^{s1}(Q) > T^{s2}(Q) > T^{s3}(Q)$. By the same logic, for any order quantity, Q, less than Q_1, supplier 2 is the first one, supplier 1 is the next one, and supplier 3 is the third one.

Now we can generalize these two theorems as follows: when the buyer has *n* (*i*=1,2,…,*n*) suppliers, he has to first arrange the suppliers in decreasing order of $T^{si}(Q)$ $\forall i$, because a supplier with greater $T^{s\cdot}(Q)$ dominates a supplier with smaller $T^{s\cdot}(Q)$. Then he allocates Q to the suppliers as much as possible, respectively. Similar to these theorems, we can conclude that when $0 < \theta_i \leq 1$, supplier i is again dominant for a positive order quantity, Q, if $T^{si}(Q)$ is greater than others'.

2. 2. Solution Methodology

In this section, we develop a solution algorithm for SSP with stochastic demand and uncertain supply. The suppliers may offer buyback contract to compensate their unreliability of supply. In fact, the proposed algorithm should be able to perform supplier evaluation and order allocation based on multiple criteria: wholesale price, unreliability level, and buyback price.

When the suppliers' contract is wholesale-price only (θ_i and $b_i = 0\ \forall i$), the buyer arranges the suppliers in increasing order of C_i, because they are evaluated based on only one criterion, C_i. Then the order quantity of *i*th supplier is obtained by

$$Q^{i+} = min(Q^{\theta i*} - Q^{<i}, V_i), \tag{4}$$

where $Q^{\theta i*}$ is calculated by Eq(3) when θ_i and $b_i = 0\ \forall i$, and $Q^{<i}$ is the order quantity assigned to suppliers $j = 1, 2, \dots, i-1$ (see Burke et al. (2007) for more details). In our case that the suppliers are evaluated based on C_i, b_i and θ_i, according to Theorem 1 and 2 we should arrange the suppliers in decreasing order of $T^{si}(Q)$. However, $Q^{\theta i*}$ cannot be obtained by Eq(3) if at least one supplier has been already assigned a positive order quantity (the reason is shown later in numerical example). As a result, we first introduce Algorithm 1 for obtaining $Q^{\theta i*}$, and then develop the main algorithm for finding the optimal set of suppliers and order quantities.

Before introducing Algorithm 1, we discuss how it finds $Q^{\theta i*}$. Assume $Q^{<i}$ is the order quantities assigned to suppliers $j = 1, 2, \dots, i-1$. This algorithm gradually increases the *i*th supplier order quantity while the retailer's profit increases. Since the retailer's

profit is concave on order quantity, once the retailer's profit drops, the algorithm stops. Obviously, the order quantity contributing the maximum profit, is the optimal order, $Q^{\theta i*}$. Then by Eq(4), $Q^{i+} = min(Q^{\theta i*} - Q^{<i}, V_i)$, the *i*th supplier's order quantity is determined. If $Q = Q^{<i} + Q^i$ is the total order quantity assigned to suppliers $j = 1, 2, \dots, i$ (Q^i is the *i*th supplier's order), by using Eq(1), the retailer's profit is calculated as

$$T(Q) =$$
$$P\int_0^Q y.f(y)dy + P\int_Q^{+\infty} Q.f(y)dy - s\int_Q^{+\infty}(y-Q)f(y)dy + \left(\sum_{j=1}^{i} b_j X_j\right)\int_0^Q (Q -$$
y)fydy+Qj=1ibjXjθj−Qj=1iCjXj

Algorithm 1:

Step 1.

1.1. Set Δ as arbitrarily (the smaller Δ is, the more exact Q^{i+} is, so that Δ= 0.1 means Q^{i+} is obtained with only one digit decimal).

1.2. Calculate $T\left(Q^{<i}\right)$, set $T = T\left(Q^{<i}\right)$, $Q^i = \Delta$ and $Q = Q^{<i} + Q^i$. (For the first supplier $Q^{<i}$ =0)

Step 2. Calculate $T(Q)$.

2.1. If $T(Q) \geq T$, then $T = T(Q)$, $Q^i = Q^i + \Delta$, $Q = Q^{<i} + Q^i$and go to Step 2.

2.2. If $T(Q) < T$, then go to Step 3.

Step 3. $Q^{\theta i*} = Q^i - \Delta$, $Q^{i+} = min(Q^{\theta i*} - Q^{<i}, V_i)$, and stop.

From now on, Q^{i+} is calculated by Algorithm 1. The following algorithm is developed to find the optimal set of suppliers and order quantities.

Algorithm 2:

Step 1. Let $Q = \mu$ as an initial order quantity, set *j*=0, and $V_Q = \emptyset$. μ is the mean of the demand, j counts the number of iteration, and V_Q is the vector of positive orders assigned to the suppliers.

Step 2. Calculate $T^{si}(Q)$ ∀ *i*=1,2,…,*n*.

Step 3. Re-index and rearrange the suppliers in decreasing order of $T^{si}(Q)$, so that $T^{si}(Q) \geq T^{si+1}(Q)$. Set *j*=*j*+1, and *k*=1 (*k*=1 indicates the first supplier in the new ranking).

(a) If in iteration *j*>1, the ranking of suppliers is the same as in iteration $j-1$, the solution of iteration $j-1$ is the final solution and stop.

(b) Otherwise, continue.

Step 4. Obtain Q^{k+} by Algorithm 1, that is the order quantity assigned to supplier *k*.

Step 5.

(a) If $k< n$ and $Q^{k+} = V_k$, then $V_Q = V_Q \cup \{Q^{k+}\}$, set *k*=*k*+1, and back to step 4.

(b) If $k\leq n$ and $Q^{k+} = Q^{\theta k*}$, then $V_Q = V_Q \cup \{Q^{k+}\}$, $Q = \sum_{i=1}^{k} Q^{k+}$ and back to step 2.

(c) If $k\leq n$ and $Q^{k+} = 0$, then back to step 2.

(d) Otherwise, $V_Q = V_Q \cup \{Q^{k+}\}$, $Q = \sum_{i=1}^{k} Q^{k+}$ and back to step 2.

In step 3 (a), the stopping condition is sufficient to terminate the algorithm, because it is straightforward when the suppliers' ranking in iteration *j*+1 is the same as iteration *j* the order allocation to the suppliers is the same as previous iteration.

For the wholesale-price scheme, the algorithm finds the optimal order allocation in the first iteration, because according to Theorem 1, suppliers can be arranged according to either C_i or $T^{si}(Q)$ $\forall i$ that results in the same ranking. That is, the second iteration has definitely the same ranking as the first one.

3. Numerical example

Consider a SSP with three unreliable suppliers (S_a, S_b, S_c) whose unreliability levels are $\theta_a = 0.05$, $\theta_b = 0.07$, $\theta_c = 0.06$, prices are $C_a = 501$, $C_b = 510$ and $C_c = 501.8$, buyback prices are $b_a = 35$, $b_b = 285$ and $b_c = 125$, and capacities are $V_a = V_b = V_c = 3000$. Here, a =1, b =2 and c =3, e.g., $S_1 = S_a$, $S_2 = S_b$, $S_3 = S_c$ (in other words, $i = a, b, c$). Product demand follows Normal distribution with $\mu = 7691$ and

$\sigma = 600$. The retailer sells the product to the market with P =550. We also assume that the shortage cost per unit, s, is zero. These data and information that have been introduced here are used throughout the numerical example as benchmark (Benchmark Data).

One very useful way to demonstrate that our algorithm solve the problem properly is that we solve the problem with different initial order quantity: Case 1: $Q = \mu$; Case 2: Q =6000.

Case 1 ($Q = \mu$):

Step 1. $Q = 7691$, *j*=0 and $V_Q = \emptyset$.

First Iteration:

Step 2. Calculate the retailer's profit for each supplier by assuming that the suppliers are uncapacitated: $T^{S_a}(Q) =$ 106,090.59; $T^{S_b}(Q) =$ 118,964.49; and $T^{S_c}(Q)$ =120,864.27.

Step 3. Based on retailer's profit obtained in Step 2, S_c becomes the first, S_b is the second and S_a becomes the last one. We also set *j*=1 (*j*=1 means first iteration), and *k*=1 (*k*=1 means the first supplier that is S_c).

Step 4. By using Algorithm 1, we obtain $Q^{(k=1)+}$ =3000, that is, the order quantity assigned to S_c.

Step 5. (a) Thus, we have $V_Q = \{Q^{(k=1)+} = 3000\}$, set *k*=2 (*k*=2 means the second supplier that is S_b), and back to step 4.

Step 4. By using Algorithm 1, we obtain $Q^{(k=2)+}$ =3000, that is, the order quantity assigned to S_b.

Step 5. (a) As a result, we have $V_Q = \{Q^{(k=1)+} = 3000, Q^{(k=2)+} = 3000\}$, set *k*=3 (*k*=3 means the third supplier that is S_a), and back to step 4.

Step 4. By using Algorithm 1, we obtain $Q^{(k=3)+}$ =1579.71, that is, the order quantity assigned to S_a.

Step 5. (d) We here have $V_Q = \{Q^{(k=1)+} = 3000, Q^{(k=2)+} = 3000, Q^{(k=3)+} = 1579.71\}$, Q =7579.71 as an initial order quantity for the second iteration, and back to step 2.

For the seek of convenience, we show the vector of order quantity as $V_Q = \{Q^{(S_a)+} = 1579.71, Q^{(S_b)+} = 3000, Q^{(S_c)+} = 3000\}$.

Second Iteration:

In the second iteration, we see that the ranking of the suppliers is the same as iteration 1. Therefore, according to *Step* 3, the solution of iteration 1 is the final and optimal solution. Thus, the optimal order quantity vector is $V_Q = \{Q^{(S_a)+} = 1579.71, Q^{(S_b)+} = 3000,\ Q^{(S_c)+} = 3000\}$ that results in the retailer's profit as 111,661.38.

We here employ the proposed algorithm for solving the same problem but with different initial order quantity as *Case* 2.

Case 2 (Q =6000):

In the first iteration, $T^{S_a}(Q)$ =39,909.62; $T^{S_b}(Q)$ =23,139.69; and $T^{S_c}(Q)$ =37,946.92. That is, S_a becomes the first, S_c becomes the second and S_b becomes the last one. The order quantity allocated to each supplier in the first iteration is obtained as $V_Q = \{Q^{(S_a)+} = 3000, Q^{(S_b)+} = 1505.54, Q^{(S_c)+} = 3000\}$. Thus, Q =7505.54 is the initial order quantity for the second iteration.

In the second iteration, according to the initial order quantity, Q =7505.54, S_b becomes the first, S_a becomes the second and S_c becomes the last one. It results in $V_Q = \{Q^{(S_a)+} = 3000, Q^{(S_b)+} = 3000, Q^{(S_c)+} = 1565.21\}$, and Q = 7565.21 as the initial order quantity for the third iteration. Since the Suppliers' order in this iteration is not the same as iteration 1, we go to iteration 3.

In iteration 3, by calculating the retailer's profit based on Q =7565.21 for each supplier, we reach to this that S_c becomes the first, S_b becomes the second and S_a becomes the last one. This ranking is the same as that of *Case* 1-iteration 1. It means that we will reach to the same result of *Case* 1. From *Case* 1 and *Case* 2, we can conclude that different initial order quantity will reach to the same result, but probably, with different number of iterations.

Here, we would like to discuss why Eq(3) cannot be used for gaining $Q^{\theta i*}$. If in this numerical example we used Eq(3), we had $V_Q = \{Q^{(S_a)+} = 1389.77, Q^{(S_b)+} = 3000,$ $Q^{(S_c)+} = 3000\}$ and retailer's profit was 108,953.81. While by Algorithm1, we have $V_Q = \{Q^{(S_a)+} = 1579.71, Q^{(S_b)+} = 3000,\ Q^{(S_c)+} = 3000\}$ that results in retailer's profit as 111,661.38. That is, Eq(3) cannot be used in this problem.

3.1. The impact of return price on supplier selection

In Benchmark Data, we see that S_a is better than S_b in term of reliability rate and wholesale price. That is, if the suppliers are evaluated based on only these two criteria, S_a dominates S_b. However, here that the suppliers are evaluated according to three criteria, the return price offered by S_b ($b_b = 285 > b_a = 35$) helps him to dominates S_a and to be ordered more: $V_Q = \{Q^{(S_a)+} = 1579.71, Q^{(S_b)+} = 3000,\ Q^{(S_c)+} = 3000\}$, and retailer's profit is 111,661.38.

We here decrease the return price of S_b to b_b =200. We again solve the problem by the proposed algorithm with different initial order quantities as *Case* 1 and *Case* 2. Both cases lead us to $V_Q = \{Q^{(S_a)+} = 3000, Q^{(S_b)+} = 1454.87, Q^{(S_c)+} = 3000\}$ that results in the decline of retailer's profit to 103,614.23. It also causes that the retailer switches from S_b to S_a. The decline of return price also decreases the total order quantity from Q =7579.71 in Benchmark Data to Q =7454.87, because the buyer cannot share the risks of uncertain demand and supply with his suppliers as much as before.

We can conclude that implementing return policy is a very effective approach for suppliers to compensate their other disadvantages as well as for retailers to lighten the risks due to the uncertainties of demand and supply.

3.2. The impact of unreliability level on supplier selection

In the solution of Benchmark Data, we see that S_b dominates S_a. In order to see the effect of unreliability level of suppliers on both the supplier selection and retailer's profit, we here improve the unreliability level of S_a from θ_a =0.05 in Benchmark Data to θ_a =0.04. We again solve the problem by the proposed algorithm with different initial order quantities as *Case* 1 and *Case* 2. The both cases release $V_Q = \{Q^{(S_a)+} = 3000, Q^{(S_b)+} = 1468.30, Q^{(S_c)+} = 3000\}$ that results in the growth of retailer's profit to 123,865.29 from 111,661.38 in Benchmark Data. In addition, the improvement of unreliability level decreased the total order quantity from Q =7579.71in Benchmark Data to Q =7468.31. The reason for the retailer's profit growth and order quantity reduction is that the number of useable products received from suppliers increased. Also, S_b no longer dominates S_a.

3.3. The impact of standard deviation, σ, of the demand on supplier selection

In comparison with Benchmark Data, we decrease σ from 600 to 100. We again solve the problem by the proposed algorithm with different initial order quantities as *Case* 1 and *Case* 2. Both cases arrived at $V_Q = \{Q^{(S_a)+} = 3000, Q^{(S_b)+} = 2062.88, Q^{(S_c)+} = 3000\}$ that results in the growth of retailer's profit from 111,661.38 to 167,187.55, and of the total order quantity from Q =7579.71 to Q =8062.88 compared to Benchmark Data. For σ =600, choosing S_b (with $C_b = 510$, $\theta_b = 0.07$, $b_b = 285$) was more profitable than S_a (with $C_a = 501$, $\theta_a = 0.05$, $b_a = 35$), while for σ =100 it is inverse.

The reason can be interpreted as follows. When σ =600, $b_b = 285$ is enough for S_b to compensate his weak-points on price and unreliability level, while for σ =100, causing the retailer's profit to increase, $b_b = 285$ is not enough anymore. That is, S_b needs to increase his buyback price in order to be considered better than S_a as the retailer's profit gets bigger. In other word, if a specific buyback price is effective on a specific retailer's profit, it may not by anymore affective on a bigger retailer's profit, and definitely, it is more affective on a smaller retailer's profit.

In order to have more investigation on the relationship between σ and unreliability level, we consider the following case.

Assume that there are two uncapacitated suppliers (S_a, S_b): $C_a = 501$, $C_b = 501$, $\theta_a = 0.05$, $\theta_b = 0.03$, b_a =?, $b_b = 10.18$, and the market demand is Normal whose $\mu = 7691$ and $\sigma = 100$.

We see that the wholesale prices, offered by the suppliers, are the same for both. The second supplier, S_b, is more reliable than the first one, S_a, because $\theta_a = 0.05 > \theta_b = 0.03$. Obviously, the only item that S_a can utilize to compete with S_b is buyback price. Numerically we observe if $b_a = 207.97$ (while $b_b = 10.18$) neither supplier dominates another one. That is, the retailer's profit if he buys from either supplier is the same and equal to 246,840. It is shown that in order for S_a to compensate his unreliability, he has to offer a greater buyback price compared to S_b.

Now we investigate how much buyback price should be in order for neither supplier to dominate another one if σ increases to 600. We again numerically see that if $b_a = 187.2$, $b_b = 10.18$ neither supplier dominates another one and the retailer's profit becomes equal to 180,516.

This numerical example demonstrates when σ is lower (here σ =100 compared to σ =600) the less reliable supplier (here S_a compared to S_b) needs to offer a greater buyback price (here b_a =207.97 for σ =100 in comparison with b_a =187.2 for σ =600) in order to compensate his unreliability.

3.4. The impact of diversification on Retailer's profit

Consider the previous data: There are two uncapacitated suppliers: $C_a = 501$, $C_b = 501$, $\theta_a = 0.05$, $\theta_b = 0.03$, b_a =187.2, $b_b = 10.18$, and the market demand is Normal whose $\mu = 7691$ and σ =600 (based on this data, retailer's profit is depicted by Figure. 2 for both suppliers).

The Figure. 2 shows that when the retailer only buys from S_b, his profit is 180,516 and order quantity is 7266.46. His profit and order quantity are 180,516 and 7563.83, respectively, if he only buys from S_a. Obviously, any combination between this two suppliers results in changing the retailer's profit between 180,516 and T, and order quantity between 7266.46 and 7563.83. Therefore, diversification is not optimal at all for the retailer. However, when the suppliers have limited capacity, the retailer has no choice except diversification.

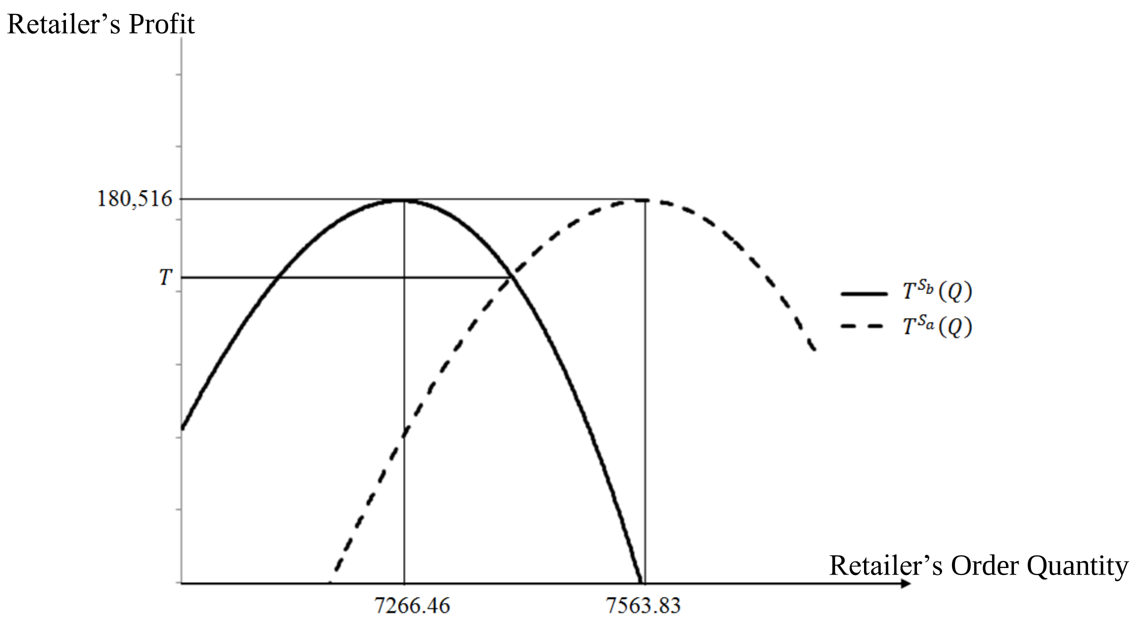


Fig. 2. Buyer's profit on two suppliers with different unreliability level and buyback price.

4. Conclusion

This study simultaneously took into account supplier selection problem (SSP) and stochastic inventory management in which the suppliers' supply was uncertain. In order for the suppliers to compensate their supply unreliability, the suppliers allow the buyer to return unsold products at the end of the period (or season). The buyer needed to consider three criteria for supplier evaluation and order allocation: suppliers' wholesale price, unreliability level, and buyback price. We then proposed an algorithm to evaluate the suppliers for concurrently obtaining the optimum inventory level and suppliers' order allocation. The developed algorithm was the main contribution of our study.

In reality, it may be difficult for purchasing managers to make a decision on optimal ordering quantity in such uncertain situations. Apart from wholesale price, which is common suppliers' competitive factor, suppliers' reliability level and buyback price can be considered as other competitive factors. This study helps purchasing managers to incorporate different criteria, such as (but not limited to) wholesale price, buyback price, and uncertainty in demand and supply, in order to obtain the best set of suppliers and their optimum order allocation. In addition, this study numerically reaches to the following managerial insights: (1) buyback scheme is able to allow the buyer to share the risk of uncertain supply and demand with his suppliers; (2) it is also an effective approach for the suppliers to compensate their unreliability level; (3) in comparison with lower demand variance, when the demand variance is higher, suppliers can compensate their unreliability level with fewer buyback price; (4) diversification is not an optimal approach for the retailer, unless the suppliers have limited capacity.

References

Aissaouia, N. Haouaria, M., and Hassinib, E., Supplier selection and order lot sizing modeling: A review, European Journal of Operational Research 34, page 3537–3540, 2007.

Amid, A., Ghodsypour, SH., OBrien, Ch., 2006. Fuzzy multi-objective linear model for supplier selection in a supply chain. International Journal of Production Economics. 104, 394–407.

Amid, A., Ghodsypour, SH., OBrien, Ch., 2009. A weighted additive fuzzy multi-objective model for the supplier selection problem under price breaks in a supply Chain. International Journal of Production Economics. 121, 323-332.

Amid, A., Ghodsypour, SH., OBrien, Ch., 2011. A weighted max–min model for fuzzy multi-objective supplier selection in a supply chain. International Journal of Production Economics. 131, 193-145.

Anupindi, R., Akella, R., 1993. Diversification under supply uncertainty. Management Science 39, 944–963.

Awasthi, S.S., Chauhan, S.K., Goyal, S.K., Proth, Jean-Marie, 2009. Supplier selection problem for a single manufacturing unit under stochastic demand. International Journal of Production Economics 117, 229–233.

Boran, F.E., Genç, S., Kurt, M., Akay, D., 2009. A multi-criteria intuitionistic fuzzy group decision making for supplier selection with TOPSIS method. Expert Systems with Applications. 36, 11363–11368.

Burke, G.J., Carrillo, J.E., Vakharia, A.J., 2007. Single versus multiple supplier sourcing strategies. European Journal of Operational Research 182, 95–112.

Cakravastia, A., Takahashi, K., 2004. Integrated model for supplier selection and negotiation in a make-to-order environment. International Journal of Production Research. 42, 4457–4474.

Chopra, S. and P. Meindl (2006), Supply Chain Management: Strategy, Planning and Operation, Prentice Hall.

Chou, S. Y., Chang, Y. H., 2008. A decision support system for supplier selection based on a strategy-aligned fuzzy SMART approach. Expert System with Applications. 34, 2241–2253.

Dada, M., Petruzzi, N.C., Schwarz, L.B., 2007. A newsvendor's procurement problem when suppliers are unreliable. Manufacturing & Service Operations Management 9, 9–32.

Dickson, G.W, 1966. An analysis of vendor selection systems and decisions. Journal of Purchasing. 2, 5–17.

Fadıloglu, M. M., Berk, E., Gürbüz, M. Ç., Supplier diversification under binomial yield, Operations Research Letters 36 (2008) 539–542

Ghodsypour SH., Ch. O'Brien, A decision support system for supplier selection using an integrated analytical hierarchy process and linear programming, Int. J. of Prod. Econ. 56–57 (1998) 199–212.

Ghodsypour, S.H., O'Brien, C., 2001. The total cost of logistics in supplier selection, under conditions of multiple sourcing, multiple criteria and capacity constraint. International Journal of Production Economics. 73, 15–27.

Giunipero L., R. Brand, Purchasing's role in supply chain management, International Journal of Logistics Management, 7 (1) (1996), pp. 29–38.

Ha, S.H., Krishnan, R., 2008. A hybrid approach to supplier selection for the maintenance of a competitive supply chain. Expert Systems with Applications. 34, 1303–1311.

Haq, A.N., Kannan, G., 2006. Fuzzy analytical hierarchy process for evaluating and selecting a vendor in a supply chain model. International Journal of Advanced Manufacturing Technology. 29, 826–835.

Heizer, J., Render, B., 2004. Principles of operations management. Prentice Hall, New Jersey.

Hong, J., Hayya, J.C., Just-in-time purchasing: single or multiple sourcing?. International Journal of Production Economics 1992;27:175–81.

Howard P. Marvel and James Peck, Demand Uncertainty and Returns Policies, International Economic Review, Vol. 36, No. 3 (Aug., 1995), pp. 691-714.

Kumar, M., Vart, P., Shankar, R., 2004. A fuzzy goal programming approach for supplier selection problem in a supply chain. Computer and Industrial Engineering 46 (2004), 69–85.

Kumar, M., Vrat, P., Shankar, R., A fuzzy programming approach for vendor selection problem in a supply chain, International Journal of Production Economics, Volume 101, Issue 2, June 2006, Pages 273-285.

Minner, S., (2003) Multiple-supplier inventory models in supply chain management: A review. International Journal of Production Economics 81-82:265–279.

Monczka, R., Trent, R., Handfield, R., 2001. Purchasing and supply chain management (2nd ed.). Cincinnati, OH: South-Western College Publishing.

Padmanabhan, V. and I. P. L. Png. 1997. Manufacturer's returns policy and retail competition. Marketing Science. 16(1). 81-94.

Porter, M.E., Millar, V.E., 1985. How information gives you competitive advantage. Harvard Business Review, 63, 149–160.

Roa, C.P., Kiser, G.E., 1980. Educational buyers' perceptions of vendor attributes. Journal of Purchasing and Materials Management 16, 25–30.

Shi C.-S., and Su, C.-T., A manufacturer's optimal quantity discount strategy and return policy through game-theoretic approach, The Journal of the Operational Research Society, Vol. 53, No. 8 (Aug., 2002), pp. 922-926.

Shi C.-S., and Su, C.-T., Integrated Inventory Model of Returns-Quantity Discounts Contract, The Journal of the Operational Research Society, Vol. 55, No. 3 (Mar., 2004), pp. 240-246.

Simchi-Levi, D., Kaminsky, P., Simchi-Levi, E., 2003. Designing and managing the supply chain: Concepts, strategies, and case studies. McGraw-Hill, New York.

Stevenson, W. J., 2005. Operations management. McGraw-Hill, New York.

Tajbakhsh, M. M., Lee, C.-G., Zolfaghari, S., On the supplier diversification under binomial yield, Operations Research Letters 38 (2010a) 505–509

Tajbakhsh, M. M., Lee, C.-G., Zolfaghari, S., Sole sourcing in EOQ models with Binomial yield, Journal of Purchasing & Supply Management 16 (2010b) 163–170

Tajbakhsh, M. M., Inventory Management and sourcing decision in the presence of upstream uncertainty, 2007, Thesis.

Wadhwa, V., A.R. Ravindran, Vendor selection in outsourcing,Computers & Operations Research 34 (2007) 3725 – 3737.

Wadhwa, V., MULTI-OBJECTIVE DECISION SUPPORT SYSTEM FOR GLOBAL SUPPLIER SELECTION, 2008, Thesis.

Weber, A., and Current, J.R., A multiobjective approach to vender selection, Eur. J. of Oper. Res. 68 (1993) 173–184.

Yan, X., Ji, Y., Wang, Y., Supplier diversification under random yield, Int. J. Production Economics 139 (2012) 302–311

Yan, X., Wang, Y., A note on supplier diversification under random yield, Operations Research Letters 41 (2013) 545–551

Yang, P.C., Wee, H.M., Chung, S.L., and Kang, S.H., A Multi-supplier and Return-Policy Newsboy Model with Limited Budget and Minimum Service Level by Using GA

Yang, S., Yangb, J., Abdel-Malek, L., Sourcing with random yields and stochastic demand:A newsvendor approach, Computers & Operations Research 34 (2007) 3682 – 3690

Yin, S., & Nishi, T., (2013): A solution procedure for mixed-integer nonlinear programming formulation of supply chain planning with quantity discounts under demand uncertainty, International Journal of Systems Science, DOI: 10.1080/00207721. 2013. 769645

Zhang, G., and Ma, L., 2009: Optimal acquisition policy with quantity discounts and uncertain demands, International Journal of Production Research, 47:9, 2409-2425

Zhang, J.-l., Zhang, M.-y., Supplier selection and purchase problem with fixed cost and constrained order quantities under stochastic demand, Int. J. Production Economics 129 (2011) 1–7.

CHAPTER 6 – CONCLUSION AND FURTHER RESEARCH

This study was dedicated to supplier selection problem (SSP) in a manufacturing company: a two echelon supply chain with a buying manufacturer and some capacitated suppliers. The reason for choosing this subject was that SSP is one of the most important tasks in a purchasing department. Selecting the right suppliers can meaningfully decrease the cost of purchasing and improve corporate competitiveness (Willis et al., 1993; Dobler et al., 1990; Xia and Wu, 2007). In addition, SSP is a very complicated problem since Decision Makers (DMs) should consider multiple criteria for evaluating the suppliers. Moreover, information on market demand may be considered as the most important cause of uncertainty in reality (Tajbakhsh, 2007). When uncertainty is an issue in SSP, complexity also increases more because the problem is usually integrated with inventory management so that DMs should simultaneously obtain the optimal inventory level and the suppliers' order quantity.

1. The main Objectives of this study

We considered two different streams in this study: SSP with deterministic demand, and SSP with stochastic demand. In addition, for each stream we had two contributions.

First contribution in deterministic demand: we modeled a multi-objective SSP where the DMs can determine a single goal for every objective. Then we propose a new normalize goal programming approach that can effectively incorporate the DMs' preferences in suppliers' evaluation by making the achieved objectives consistent with their goals.

Second contribution in deterministic demand: We modeled a multi-objective SSP that allows the DMs to determine an interval goal for every objective. We subsequently developed a new approached, namely multi-choice goal programming (MCGP), that can effectively incorporate the DMs' preferences in suppliers' evaluation by providing more control on both the inside and outside of the interval goals.

First contribution in stochastic demand: We developed a multi-period multi-supplier newsvendor problem (dynamic programming) in which the capacitated suppliers may offer quantity discount as a competitive factor. Afterward, we proposed an algorithm to simultaneously obtain the optimal inventory level and suppliers' order quantity for each period.

Second contribution in stochastic demand: At the end, we extended a single-period multi-supplier newsvendor problem where the capacitated suppliers may be unreliable in terms of quality and/or delivery. For compensating the unreliability, the suppliers may allow the buyer to return unsold products at the end of the single period (buyback scheme). As a result, it was a very complicated problem because DMs should concurrently consider three criteria, suppliers' wholesale price, unreliability level, and buyback price, for obtaining the optimal inventory level and suppliers' order quantity. For solving the problem, we proposed an algorithm.

2. The limitations of this study and Future research

For the first two contributions, we employed mathematical programming that only considers tangible criteria, while SSP may be affected by both tangible and intangible factors. In order to obviate this limitation, AHP or ANP can be integrated with our mathematical programming model. In addition, we assumed that the demand and suppliers' capacities are known. However, these two parameters may be uncertain in reality. Therefore, considering multi-objective SSP with uncertain demand can be as future research.

For the second two contributions, we only considered discount and buyback contracts, while other contracts, such as revenue sharing, price protection and rebate, are also very famous in the literature. Considering these contracts for SSP can be considered as future study as well. In addition, the supplier selection problem can be integrated with supply chain coordination. The supply chain coordination, which generally concentrates on inventory management, tries to improve the whole supply chain profitability by aligning the partners' strategies and goals. However, we only considered the buyers' strategies and preferences rather than those of the entire supply chain. Taking supply chain coordination into account for a SSP is also open for further study. Furthermore, we used stochastic distribution to model the uncertain demand. However, if the data related to the demand is not available, stochastic models cannot be employed anymore. Fuzzy set theory is one of the suitable techniques used for lack of data and information. Another direction for future study can be considering a SSP with fuzzy demand.

3. Managerial implications

In the first study (first contribution in deterministic demand), we make the achieved objectives consistent with their goals, which are determined by DMs. This means that purchasing managers (or DMs) can effectively incorporate their preference in decision making process. In addition, by the proposed model, the managers can express their preferences by both fuzzy and deterministic goals.

In the second study (second contribution in deterministic demand), we make the DMs or purchasing managers able to determine an interval goal, instead of a single goal, for every objective. Again, the managers can effectively consider their preferences in evaluating and selecting the suppliers.

In the third study (First contribution in stochastic demand), we model a multi-period supplier selection problem that ensures the managers the availability of raw materials on an ongoing basis for production.

In the fourth study (second contribution in stochastic demand), the managers become able to evaluate unreliable suppliers. In addition, our model helps the suppliers to compensate their unreliability by offering buyback contract (policy) to the buyer.

4. Journal publications

Refereed Journals

Jadidi O., Zolfaghari S., Cavalieri S., "A new normalized goal programming model for multi-objective problems: A case of supplier selection and order allocation", International Journal of Production Economics. Vol. 148 (2014), PP. 158–165.

Jadidi O., Cavalieri S., Zolfaghari S., "An Improved Multi-Choice Goal Programming Approach for Supplier Selection Problems", (Under revision).

In Preparation for submission

Jadidi O., Taghipour Sh., Pinto R., "A Multi-Period Supplier Selection Problem under Price Breaks: A Dynamic Newsvendor Model".

Jadidi O., Cavalieri S., "Supplier selection and order allocation problem under demand and supply uncertainty with return policy".