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Supply Information Sharing and Updating in Supply Chain Management

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Dedication

To all those who supported me each step of the way

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Chapter 2

h_i	Unit holding cost for product i, i= 1, 2,
p_i	Unit penalty cost for product i, i= 1, 2,
C_i	Unit ordering cost for product i, i= 1, 2,
$\mathrm{K_i}$	Fixed cost for product i, i= 1, 2,
q_n	The probability of machine being down in period n,
p_n	The probability of machine being up in period n
θ	The probability of machine/ supplier availability,
(α, β)	Machine reliability status at the beginning of a period
Λ	The number of supply available periods
r	Discount factor,
x_i	Initial inventory level for product i, i=1,2,
X_i	Inventory level after production for product i, i=1,2,
ξ_i	The one-period demand for product i, i=1,2,
$\varphi_i(\xi)$	Demand probability density function for product i, i= 1, 2,
ϕ_i (ξ)	Demand cumulative distribution function product i, i= 1, 2,
$L_n(.)$	The one-period expected holding and shortage cost function,
$f_n(\ x_1, x_2, \alpha, \beta)$	Minimum expected discounted cost function,
$D(x_1, \alpha, \beta)$	Switching curve,
$E(x_1, \alpha, \beta)$	Switching curve,
$s_i(\alpha, \beta)$	Ordering threshold level for product i, i=1,2,
$S_i(\alpha, \beta)$	Up to order level (cost minimizer) for product i, i=1,2,
σ_j	Ordering region, j=0,1,2,

Chapter 3

C	Distributor's unit ordering cost from supplier,
C^s	Spot market price,
w	Distributor's unit selling price to retailer,
P	Retailer's unit selling price,

Q	Distributor's order quantity from supplier,
q	Retailer's order quantity to distributor,
h	Retailer's unit holding cost for unsold inventory,
g	Retailer's unit shortage cost for demand that exceeds inventory level,
θ	Random yield variable,
d	Retailer's Flexibility level,
ξ	Retailer's demand,
$(u_0, \mathcal{U}_0)$	Random yield interval
$\phi(\xi)$	Demand probability density function,
$\Phi(\xi)$	Demand cumulative distribution function,
π_D^{NBSM}	Distributor's expected profit for the Non-Bayesian spot market model,
Q_{NBSM}^*	Distributor's optimal ordering quantity for the Non-Bayesian spot market model,
Q_{BLS}^*	Distributor's optimal ordering quantity for the Bayesian lost sale model,
Q_{QFC}^*	Distributor's optimal ordering quantity for the quantity flexibility model,
π_D^{BLS}	Distributor's expected profit for the Bayesian lost sale model,
π_D^{QFC}	Distributor's expected profit for quantity flexibility model,
π_R^{NBSM}	Retailer's expected profit for the Non-Bayesian spot market model,
q_{NBSM}^*	Retailer's optimal ordering quantity for the Non-Bayesian spot market model,
q_{BLS}^*	Retailer's optimal ordering quantity for the Bayesian lost sale model,
q_{QFC}^*	Retailer's optimal ordering quantity for quantity flexibility model,
π_R^{BLS}	Retailer's expected profit for the Bayesian lost sale model,
π_R^{QFC}	Retailer's expected profit for quantity flexibility model.

Chapter 4

θ	Random yield,
$f(\theta)$	Random yield probability density function,
$F(\theta)$	Random yield cumulative distribution function,
$[\underline{\gamma}, \bar{\gamma}]$	The interval of manufacturer's belief on shared forecast,
$g(\theta)$	Probability density function of manufacturer's belief on shared forecast,

$G(\theta)$	Cumulative distribution function of manufacturer's belief on shared forecast,
g_M	Manufacturer's shortage cost,
g_S	Supplier's shortage cost,
c	Supplier's production cost per unit,
p	Manufacturer's purchasing cost per unit,
r	Manufacturer's revenue,
h	Manufacturer's holding cost per unit,
$\underline{\theta}$	Random yield lower bound,
$\bar{\theta}$	Random yield upper bound,
$\hat{\theta}$	Supplier observation on random variable forecast,
m	Supplier's random yield signal,
q_o	Manufacturer's optimal order quantity for non-cooperative case,
$\hat{q}^*$	Manufacturer's optimal order quantity for cooperative case,
$\pi_M(\theta, q)$	Manufacturer's expected profit,
$\pi_S(\theta, q)$	Supplier's expected profit,
$\Phi(\xi)$	Demand cumulative distribution function,
$\varphi(\xi)$	Demand probability density function,
π_M^*	Manufacturer's optimal profit for untruthful case,
$\hat{\pi}_M^*$	Manufacturer's optimal profit for truthful case,
$\hat{\pi}_S^*$	Suppliers' optimal profit for truthful case,

1. INTRODUCTION

1.1 Introduction

The performance of manufacturing/ production planning part of a company is effected by different types of uncertainty. Graves (2011) categorized uncertainties that affect production planning section of a firm into three major types: “Uncertainty in Demand Forecast”, “Uncertainty in External Supply Process”, and “Uncertainty in Internal Supply Process”. Many supply chain uncertainties in most industries arise from the demand side. Supply uncertainty, although infrequent, is another cause of uncertainty. It has considerable effect on the performance of a supply chain and supply decisions. Many researchers in the field of production planning, inventory, and supply chain management have studied demand uncertainty. Recently, supply uncertainty has been receiving considerable attention in the literature. There are several factors which causes major varieties in supply like natural disasters (weather condition, fire), equipment breakdowns, supplier’s capacity constraint, unplanned maintenance, market volatility, terrorist attacks, war, transportation, and so on (Tajbakhsh et al. 2007).
Due to all these supply disruption causes, supply uncertainty can be categorized in three different aspects: Lead time, Quantity/ Quality of supply, and Purchase price (Snyder et.al (2012), Tajbakhsh et al. 2007).

In this research, supply disruption from quantity perspective is in the center of our attention. Therefore, we focus on the concept and types of supply disruption from quantity perspective.

Quantity supply disruptions are classified in three major categories: Randomness in supplier capacity, Random Supplier availability (disruption), and Random yield. (Tajbakhsh et al. 2007).

Yield uncertainty is mostly applicable in the area of electronic devices and chemical process. Form inventory management point of view, yield uncertainty occurs in situation either order quantity to supplier is not match with received quantity or imperfect units are delivered form supplier.

Supply disruption can be viewed as a random supplier availability in which a supplier's availability refers to the form of all-or-nothing (Bernoulli yield). This can be referred as a second type of supply quantity availability (Snyder et.al (2012).

Variation in supplier capacity creates the other type of supply quantity uncertainty. In such environment, the risk of supply increases. To encounter the risk, some operational strategies/tools such as Stockpile Inventory, Multiple Suppliers, Backup Supply Source, Manage Demand, and Strengthen Supply Chain can be used. (Tomlin and Wang 2011).

We illustrate uncertainties in supply chain in Figure 1.1. The focus of this thesis is on special type of supply uncertainties, uncertainty in supplier availability and uncertainty in random yield, which are shown with dashed lines square in Figure 1.1.

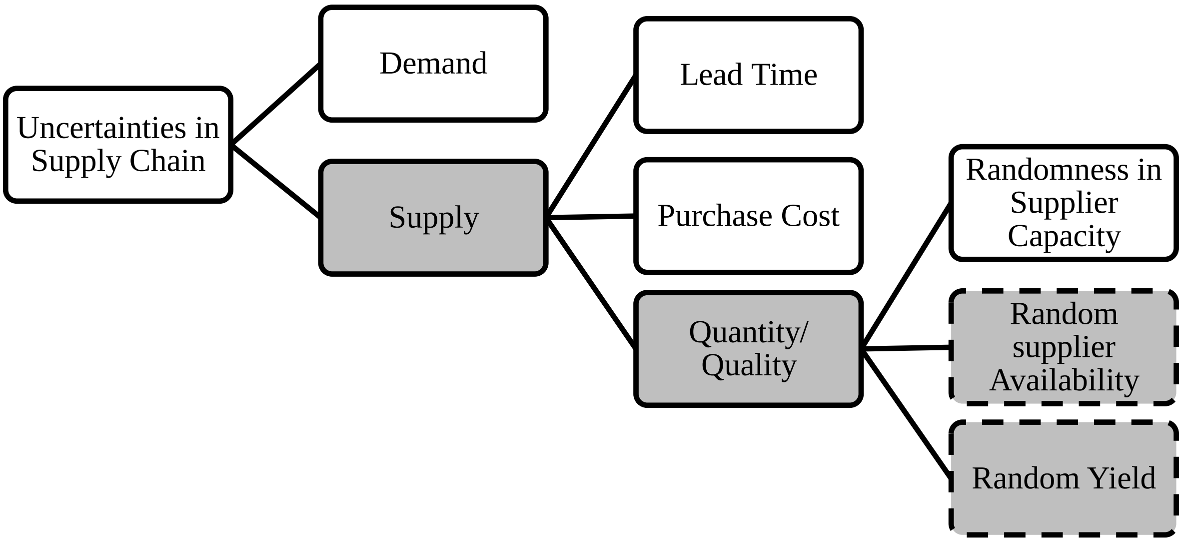

Figure 1.1 Uncertainties in Supply Chain

The other stream of this thesis is on the role of information sharing and updating on supply chain performance. Supply disruption affects the performance of supply chain members. To improve the performance of a supply chain by coordination, the role of information sharing cannot be neglected. Many studies have adopted coordination contract design and information sharing to deal with supply chain uncertainty (Arshinder et.al 2011). On the other hand, many papers have studied different forecasting tools to mitigate supply chain uncertainty that leads to accurate decision making. When supplier's reliability is uncertain, firm uses past information to forecast future supply availability. Since Bayesian approach can be applied for forecasting using past history, it seems to be an appropriate tool. As additional observation/ information reveals, Bayesian approach updates the probability of an event using new information.

Empirical Bayesian approach introduced by Scarf (1960) is a tool to manage optimal base stock level and update or anticipate demand distribution at the same time through demand observation over time.

Although the majority of attention in literature is focused on the value/flow of information from downstream to upstream in the form of realized demand, updated demand, and advanced demand information, less attention is paid to information sharing from upstream to downstream (supply-side information).

In the literature, many inventory models assume supply is available continuously for future (Parlar & Perry 1995). In reality, a true distribution of either demand or supply in not known with certainty.

1.2 Thesis Objectives and Research Questions

This thesis deals with the effect of supply information sharing and updating on different areas of supply chain management: Production Planning/inventory Management, Marketing, and Behavioral Management. The objectives of this thesis are three-fold.
First, from production planning/inventory management perspective, the objectives are

- To characterize the special pattern of optimal production/ordering policy for a multi-period inventory model in which a machine produces two different products with a fixed (setup) cost for each.
- To investigate the benefits of supply uncertainty forecasting on the behavior of the system.

Second, we investigate procurement management under the assumption of uncertain random yield in a three-level supply chain consisting of a supplier, a distributor, and a retailer. The aims are

- To investigate the effect of random yield forecasting by learning from previous supply observations on the supply chain parties.
- To induce the retailer for yield risk forecast sharing.
- To discover under which condition and what contract the distributor shares the knowledge of supply risk with the retailer.
- To explore the role of information sharing on the ordering decision and profit maximization.

In chapters 2 and 3, we highlight the role of information sharing and updating from supply side on inventory and procurement management. While in chapter 4, we investigate the role of trust on shared forecast. The goals are:

- To investigate whether supplier shared supply forecasting information truthfully.
- To explore under which condition manufacture should trust supplier's report.
- To find an optimal ordering policy for two situation of information sharing, truthfully and untruthfully.

1.3 Research Methodologies

To answer the research questions and reach the objectives stated in the previous section, mathematical modeling is proposed and analytical results are obtained. For the first work, a mathematical model is developed for multi-period inventory management where an unreliable machine produces two different products with two different fixed costs. The machine availability is updated using Bayesian updating learning from previous periods' information. Mathematical proofs are presented to support analytical results, theorem and lemmas. In the second work, to share the risk of supply unavailability between supply chain members, a distributor and a retailer, we propose four various contracts in a three-echelon supply chain. For two contracts, Bayesian updating is used to forecast uncertain random yield and an algorithm is proposed to obtain the optimal ordering quantity. While for the two other contracts, analytical results are obtained. In all four contracts, game theory approach is used to make an appropriate procurement decision. Finally, in the last work of the thesis, the problem of truthful forecast information sharing is modeled mathematically and results are obtained intuitively and numerically.

In this thesis, analytical results are supported by numerical examples as well.

1.4 Thesis Outline

The organization of the thesis is as follows: Chapter 2 determines the optimal ordering policy of a two-product, periodic-review inventory problem in which the probability of supply availability is unknown. In Chapter 3, different coordination contracts are proposed to share yield risk uncertainty in a three-echelon supply chain. Chapter 4 investigates the role of trust in supply forecast information sharing and supply chain member's fundamental decisions. Thesis conclusion, findings, limitations, and future recommendations are presented in Chapter 5.

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Snyder, Lawrence V. and Atan, Zumbul and Peng, Peng and Rong, Ying and Schmitt, Amanda J. and Sinsoysal, Burcu, OR/MS Models for Supply Chain Disruptions: A Review (November 8, 2012). Available at SSRN: http://ssrn.com/abstract=1689882 or http://dx.doi.org/10.2139/ssrn.1689882

2. TWO-PRODUCT INVENTORY MANAGEMENT WITH FIXED COSTS AND SUPPLY UNCERTAINTY

This Chapter determines the optimal ordering policy of a two-product, periodic-review inventory problem in which the probability of supply availability is unknown. Moreover, there are two different fixed costs assigned to each product. Demand rates are random variables with known probability density functions, and the supply availability of each product is updated at the beginning of each time period. We prove the optimality of (s,S) policy with a monotone switching curve that indicates the priority of production, where the order-up-to levels and the reorder points are functions of supply availability information. A simple computation is proposed to calculate the two thresholds levels. Bayesian updating helps to manage the optimal ordering policy by updating supply disruption information. Numerical results show that improving the accuracy of the forecast leads to making a better ordering decision and eliminating the negative effect of supply disruption on the total cost.

2.1 Introduction

There are predictable and unpredictable factors that cause supply disruption, which negatively impact the performance of a supply chain and its members, such as uncertainty of demand and supply (e.g., Tajbakhsh et.al , 2007, Atasoy et.al, 2012). Demand uncertainty has been studied by many researchers in the field of production planning, inventory and supply chain management. Recently, supply uncertainty has been receiving considerable attention in the literature. First, due to its importance and, second, because not many studies are available in the literature. Traditional production and inventory policies set by firms to encounter the risks that arise from supply disruptions of products may be ineffective (Tomlin, 2009). A firm can learn from its experience with its supplier, for instance, the history of supply availability, to determine how reliable that supplier is. Through observations, a firm can acquire additional information that leads to more accurate ordering decisions.

This Chapter addresses the optimal ordering/production scheduling policy for a system that produces two products with finite planning horizon, where inventory is reviewed periodically due to variation in demand and uncertainty in supply availability. Two switching costs are incurred, for example, when the facility switches production between two products. The developed model adopts a structure of supply availability of all-or-nothing; i.e., in a given period the supply is either fully available or unavailable. The model is used to determine the optimal ordering/production policy for each of the two products. The developed model is also applicable for periodic production planning problems with unreliable manufacturing facilities.

There is a plethora of published works in the literature on inventory with probabilistic and uncertain demand, however it would not be effective and feasible to review them here. Thus, the following review focuses on the ordering/production planning literature with fixed costs, uncertain supply, and (s, S) inventory policy. The review of the literature classifies the works on inventory/production planning problem into two groups. These include studies that considered fixed cost(s) and those that considered uncertainty of supply. The earliest works of the first category are those of Scarf(1960) and Veinott(1966) who dealt with the concept of K-convexity in stochastic inventory management, for a single product case where a K-convex function is defined as for any $0 < \alpha < 1$, $K \geq 0$ (fixed cost), and $x \leq y$, $f(\alpha x+(1-\alpha)y) \leq \alpha f(x) +(1-\alpha)(f(y)+K)$. They showed that a periodic review (s,S) inventory policy for a single product is optimal under certain assumptions. Johnson (1967) investigated a multi-product periodic review inventory problem with a single fixed cost for all products, with an order cost for each. For the infinite planning horizon case, Johnson (1967) showed that the (σ,S) policy is optimal. This policy operates as follows, if the inventory level of a product at the beginning of a period is in the reordering region, σ, then order up to S otherwise do not order. Kalin (1980) redefined σ as a do-not-order set, an increasing set, regarding a specific partial ordering. He, who considered a single product with a fixed cost, recommended an optimal order policy that when the initial inventory level is in the set σ do-not-order, else order up to the vector level S. Elhafsi and Bai (1996) studied a production and setup scheduling problem in which a machine produces two products with fixed setup times and costs for both products. Under the assumption of constant demand rate and constant processing time for each product, the optimal production rate and setup epochs are derived as a function of the system's state for the cases of finite and infinite time horizons.

To compute the optimal ordering policy, Ohno & Ishigaki (2000) presented a time-computing-efficient algorithm, the Policy Iteration Method (PIM), to minimize the expected undiscounted cost function for multi-item continuous time inventory model with fixed replenishment time. Shaoxiang (2004) showed that the optimality of the hedging point policy is based on the concept of μ differential monotone for finite and infinite planning horizons, for a two-product periodic review stochastic inventory system with no setup (fixed) costs, where demands are random and the production facility is unreliable. A function, $G_n(x, y)$, is said to be μ-differential monotone if: (1) $G'_{nx^+}(x, y)$ when $\uparrow x$ and $\uparrow y$, (2) $G'_{ny^+}(x, y)$ when $\uparrow x$ and $\uparrow y$, and (3) $\mu_1 G'_{nx^+}(x, y) - \mu_2 G'_{ny^+}(x, y)$ when $\uparrow x$ and $\downarrow y$, where, e.g., $\uparrow x$ is non-decreasing in x and $\downarrow y$ is decreasing in y (Shaoxiang, 2004; p.314)). A generalization of the K-convexity in R^n was proposed by Gallego & Sethi (2005) for the case of joint and individual setup costs to determine the optimal policy for a multi-product inventory problem. In addition to the definition of K-convexity in R^n, some properties of the function were developed. Numerically, they showed that the (σ, S) policy is optimal for a two-product, two-period inventory problem with deterministic demand and a joint setup cost. Chen and Simchi-Levi [5] Investigated periodic review of stochastic cash balance problem when there are fixed costs for both ordering and return. The objective was to make ordering or return decisions to minimize the total expected cost for N-period planning horizon. To characterize the optimal policy, they developed the concept of symmetric K-convexity and (K, Q)-convexity.

Demirag et al. (2012 a) developed two heuristic policies for a firm in which an ordering (fixed) cost is incurred when the order quantity of the previous period does not cross a specified threshold level. They partially characterized an optimal policy that is simpler and easier for practical implementation. In a follow-up paper, Demirag et al. (2012 b) determined the optimal policy for a single-product stochastic periodic review inventory problem operating under three different forms of the fixed cost: (1) If the order size exceeds a threshold C, then a fixed cost K_2 is incurred, otherwise $K_1 < K_2$, (2) an incremental value of K is incurred per batch for any additional order quantity higher than batch capacity C, and (3) an additional fixed cost is incurred for any order as well as a fixed cost charged for batches. They introduced an optimal policy for a new concept of $(C,\ K_1 + K_2)$-convexity for (1); an X-Y band type optimal policy applies for (2) and (3).

For continuous review of the capacitated inventory/production problem, Chao et. al (2012) developed an (r,S) policy and presented an algorithm to compute parameters r and S. They assumed that demand arrivals follow a Poisson distribution and setup costs are incurred with every production run. Their optimal production policy suggests turning the machine on to produce, if the inventory level of a certain product is below r, and off if it is above S.

The second group of studies of interest focuses on reviewing stochastic inventory/production planning where product supply availability is uncertain. The review first considers those studies that deal with where the supply distribution is known. Parlar et al. (1995) showed that the (s,S) policy is optimal and a reorder point, s, is dependent on the previous supply state, and that, s, increases if the current period of supply is fully filled. To compute the optimality of order-up-to level policy, Gullu et al. (1997) presented a newsboy-like formula for a single-product, periodic review inventory order-up-to level policy with supply uncertainty for the cases of deterministic and dynamic demand. They assumed that supply availability follows a Bernoulli process and supply to be either available in full or unavailable. Özekicia and Parlar (1999) considered a periodic review inventory-production planning control problem with and without fixed costs in an environment in which the cost parameters and demand and supply availability are randomly changing. A base-stock policy and an (s,S) policy were shown to be optimal when a supplier is unreliable.

The aforementioned papers consider supply uncertainty concept with perfect knowledge on supply distribution. Scarf (1959), Azoury (1985), Lariviere and Porteus (1999) are examples of studies that deal with demand Bayesian learning and its effect on production decisions. On the contrary, scarce are those works on reducing supply uncertainty by learning. To the best of the authors' knowledge, there are two papers that address dual-sourcing and inventory management when supply distribution is unknown and supply uncertainty is forecasted by the Bayesian learning approach. Tomlin (2009) was the first to investigate the effect of supply learning on inventory and sourcing decisions. Using Bayesian updating, he assumed that a supplier's yield distribution is updated for the Bernoulli case of all-or-nothing yields. His results showed that a supplier is more enticed to order when its supply reliability uncertainty increases. In turn, an improvement in estimating the accuracy of successful arrivals from a supplier decreases the risk of a future supply disruption, and consequently reduces investing in inventory. In another study, Chen et al. (2010) investigated a problem where a manufacturer is faced by a dual sourcing problem. Unlike Tomlin (2009), one of the suppliers is unreliable with lower ordering cost, while the other is reliable with a higher order costing. Using the Bayesian learning process, they updated information about the unreliable supplier to find the optimal sourcing decision for a multi-period dynamic problem. They assumed that supplier reliability state and distribution of arrival proportion levels are unknown for the manufacturer at order placement time. Recently, Atasoy et al. (2012) provided an improvement to earlier works by investigating a single-product inventory problem, where a supplier provides its customer (a manufacturer) with information on its future supply availability. They considered a production-inventory situation for non-stationary deterministic demand with fixed and no fixed ordering costs. They proposed a heuristics algorithm to prove the optimality of state-dependent (s,S) policy and showed that supply information sharing is not beneficial when the ordering cost is high and supply availability is low.

The models available in the literature that are of relevance to this study, are either to solve inventory management problems for a single-product or multiple products where supply availability follows a Markovian process, or optimal-sourcing strategies by updating the knowledge of supply disruption. To the best of the authors' knowledge, no study in the literature has considered the effect of supply availability with Bayesian learning updating on the ordering policy for a two-item inventory problem, with a setup/order (fixed) cost for each. The aim of this study is to characterize the special pattern of the optimal production/ordering policy for such a problem. Moreover, the research investigates the benefits of supply uncertainty forecasting on the behavior of the system.

The rest of the Chapter is organized as follows. The mathematical model and the supply availability structure are presented in section 2.2. Section 2.3 presents the characterization of the optimal policy by a simple algorithm and some numerical examples. Discussion of results and managerial insights are provided in Section 2.4. Finally, Section 5 includes a summary, conclusions and possible future extensions.

2.2 Mathematical Model

Suppose a single supplier (e.g., machine) fills orders for two products, one at a time, under an unlimited capacity constraint. It is assumed that the demands for the two products are identically independent, and distributed (i.i.d) random variables in each order period with density function $\varphi(\xi)$. To meet the uncertain demand, it is also assumed that the supplier places an order at the beginning of each period. Then, we investigate a periodic review of N-period ordering/production problem in which the probability of supply availability, say supplier reliability, is unknown. Using Bayesian learning, a manufacturer can update its initial knowledge of supply availability over time. At the end of the period, either holding or shortage costs are incurred, if either an excess or shortage of on hand inventory occurs. The unit production/ordering cost, C_i , and the fixed cost, K_i, are incurred for item $i = 1,2$. The future expected cost is discounted by rate r.

We propose to find the optimal production quantity in each period for each product where the supply availability will be forecasted at the start of each period. We assume supply availability is a random variable that follows the Bernoulli distribution with an unknown parameter, probability of supply availability, θ, with prior Beta density function. Bayesian updating is used as a tool to predict and update the expected supply availability using information received from previous periods.

2.2.1 Supply Availability Structure

Supply availability is of all-or-nothing type. That is, at the beginning of each period an order is placed to a machine/supplier. If the supplier is in an up status, then the order is successfully filled, otherwise it is not. Bayesian updating is used to learn about the reliability of a supplier (supply availability). Suppose that the supply availability probability follows the Bernoulli distribution with an unknown parameter, θ, which is the probability of the supplier's reliability. The prior distribution of the unknown parameter is Beta with parameters α and β. The posterior distribution is also from the Beta distribution family in which the parameter α updates to α+1 as one observes the supplier satisfies a placed order in a period and β increases by the number of unavailability periods. This assumption was used in Tomlin (2009). For modelling reliability uncertainty, Tomlin (2009) applied Beta family distribution. He stated that the distribution is attractive to be chosen because a large range of belief can be reflected by Beta family. Moreover, the Beta family properties are preserved for posterior distribution after a Bernoulli trail.

The initial belief of the probability of the supplier reliability, θ, can be estimated given the two parameters α and β. As the system learns more about the supply availability, it would be possible to forecast the expected supplier reliability using the new observation to find a more accurate one. The initial estimate of θ is $\frac{\alpha}{\alpha+\beta}$ and the supplier is reliable for the next periods with a forecasted mean of $\frac{\alpha+\Lambda}{\alpha+\beta+n}$ and it is unreliable with a probability of $\frac{\beta+n-\Lambda}{\alpha+\beta+n}$, where $\Lambda = \sum_{i=1}^{n} u_i$ is the number of supply available periods where $u_i = 1$ if the supplier is up and 0 if it is down.

2.2.2 Dynamic Programming Notation and Formulation

Consider an N-period, two-product dynamic programming in which a firm incurs two distinct switching/fixed costs any time a positive quantity is ordered from a supplier. Note that, we generalize the meaning of a supplier to either be an outsource supplier or a production facility. In each period, the firm is allowed to order one product only from the supplier. Demands from the two products, $i = 1, 2$, are i.i.d random variables with pdfs and Cdfs $\varphi_i(\xi)$ and $\phi_i\ (\xi)$, respectively. A unit holding cost, h_i , and a penalty cost, p_i, are charged for any leftover inventory and shortage quantity at the end of each period for item i=1,2. Let c_1 and c_2 denote the unit ordering/production cost for items 1 and 2, respectively, and r is a one-period discount factor where $r \in (0,1)$. Define x_i and X_i as the initial and after-order inventory levels of item i=1,2, respectively.

Let us define $f_n(\,x_1,\ x_2, \alpha, \beta)$ be the minimum expected discounted cost function given the initial inventory levels x_1 and x_2 of items 1 and 2, and any state (α, β) at the beginning of period n. In the model, (α, β) is not subscribed by a time index because its time index is clear from the formula. The dynamic programming problem can be formulated as follows:

$$f_n(x_1, x_2; \alpha, \beta\ ) = q_n\big(G_n(x_1, x_2, \alpha, \beta)\big) + p_n\big(min\{G_n(x_1, x_2, \alpha, \beta), \min_{X_1 \geq x_1}[K_1 + G_n(X_1, x_2, \alpha, \beta)]\,, \min_{X_2 \geq x_2}[K_2 + G_n(x_1, X_2, \alpha, \beta)]\}\big) - c_1 x_1 - c_2 x_2 \qquad (1)$$

where, $q_n = \frac{\beta}{\alpha+\beta}$, $p_n = \frac{\alpha}{\alpha+\beta}$. The other terms of Eq. (1) are given as:

$$G_n(x_1, x_2, \alpha, \beta) = g_1(x_1) + g_2(x_2) + rE\big(f_{n+1}(x_1 - \xi_1, x_2 - \xi_2, \alpha, \beta)\big), \qquad (2)$$

$$g_1(x_1) = c_1 x_1 + L_n(x_1)\ , \quad g_2(x_2) = c_2 x_2 + L_n(x_2)\ ,$$

$L_n(.)$: The one-period expected holding and shortage cost function,

ξ_i: The one-period demand for product i, i=1,2,

Where $L_n(.) = h_1 \int_0^{.}(. - \xi_1)\varphi(\xi_1)\, d\xi_1 + p_1 \int_{.}^{\infty}(\xi_1 - .)\varphi(\xi_1)\, d\xi_1$ for $x_1, x_2, X_1, and\ X_2$. Letting

$$G_n^*(x_1, x_2, \alpha, \beta) = min\{q_n\big(G_n(x_1, x_2, \alpha, \beta)\big) + p_n\big(min\{G_n(x_1, x_2, \alpha, \beta), \min_{X_1 \geq x_1}[K_1 + G_n(X_1, x_2, \alpha, \beta)]\,, \min_{X_2 \geq x_2}[K_2 + G_n(x_1, X_2, \alpha, \beta)]\}\big)\} \qquad (5)$$

, the expected discounted cost function in Eq. (1) can then be expressed as follows:

$$f_n(x_1, x_2, \alpha, \beta) = -c_1 x_1 - c_2 x_2 + G_n^*(x_1, x_2, \alpha, \beta) \qquad (6)$$

Given an unavailable supply period, the state of the system does not change and the parameter β will be updated to $\beta + 1$. On the other hand, when the supplier fills an order, depending on the product type, it costs the firm the corresponding ordering and fixed switching costs and the updated parameter α is $\alpha + 1$.

At the beginning of each period, starting at state (x_1, x_2), there are three choices to consider depending on the supplier's status and the system's state. If the supplier is down, then no action is taken. On the other hand, we may order either product 1 or product 2 or nothing when the supply is up. Given supply availability information using Bayesian updating, the optimal ordering policy that minimizes the total expected discounted cost function is determined.

2.3 Optimal Policy Condition

The mathematical model discussed in the previous section states that if the supplier is unavailable, ordering/production quantity is zero, otherwise the optimal ordering policy is either produce item 1 or item 2 or nothing. To characterize the optimal policy, first we introduce some notations that serve the sufficient conditions of optimality. Let us define V^* as a set of functions defined on R^2 which are (K_1, K_2)-convex**,** supermodular and diagonal dominance. The definition and properties of (K_1, K_2)-convexity introduced by Gallego & Sethi (2005) are presented in Appendix E.2. We assume that $G_n(x_1, x_2, \alpha, \beta) \in V^*$ and $\lim_{\|X\| \to \infty} G_n(X) = \infty$ where $X = (x_1, x_2)$ states the vector form of initial inventory level therefore, S_1 and S_2 are the two global minimizers of $G_n(x_1, x_2, \alpha, \beta)$ with respect to x_1 and x_2 respectively.

To characterize the optimal policy, some critical points and operators are defined as follows.

We define $S_1(\alpha, \beta)$ and $S_2(\alpha, \beta)$ as the minimisers of $G_n(x_1, x_2, \alpha, \beta)$ with respect to x_1 and x_2 respectively. Let us define the two useful operators $\Delta_1$and Δ_2 as follows:

$$\Delta_1 G_n(x_1, x_2, \alpha, \beta) = K_1 + G_n(S_1(\alpha, \beta), x_2, \alpha, \beta) - G_n(x_1, x_2, \alpha, \beta)$$

$$\Delta_2 G_n(x_1, x_2, \alpha, \beta) = K_2 + G_n(x_1, S_2(\alpha, \beta), \alpha, \beta) - G_n(x_1, x_2, \alpha, \beta)$$

These two operators calculate the additional cost of producing product 1 and 2 up to their service level, $S_1(\alpha, \beta)$ and $S_2(\alpha, \beta)$.

Here, we note that in this model, the fixed cost is not a function of a product's inventory level. It is also assumed that both products are identical in terms of manufacturing propose.

To support the characterization of optimal policy, it is assumed that the operators have the following properties:

Assumptions

(I) $\Delta_1 G_n(x_1, x_2, \alpha, \beta) \quad \uparrow x_1, \uparrow x_2,$

(II) $\Delta_2 G_n(x_1, x_2, \alpha, \beta) \quad \uparrow x_1, \uparrow x_2,$

(III) $\Delta_2 G_n(x_1, x_2, \alpha, \beta) - \Delta_1 G_n(x_1, x_2, \alpha, \beta) \quad \downarrow x_1, \uparrow x_2.$

Since we assume that the machine can either produce one product or produce nothing at each time given the initial inventory levels, we need to define two switching curves to separate the ordering regions. This assumption is also addressed by Ha (1997). In such a system, it is important to schedule production of each product according to priority. The two switching curves are as follows:

$D(x_1, \alpha, \beta) = min\{x_2: \Delta_1 G_n(x_1, x_2, \alpha, \beta) \geq 0 \,, \Delta_2 G_n(x_1, x_2, \alpha, \beta) \geq 0\,\}$

$E(x_1, \alpha, \beta) = min\{x_2: \Delta_2 G_n(x_1, x_2, \alpha, \beta) - \Delta_1 G_n(x_1, x_2, \alpha, \beta) \geq 0\}$

Function $E(x_1, \alpha, \beta)$ separates the production region of item 1 from that of item 2 and $D(x_1, \alpha, \beta)$ forms the production and non-production regions. We symbolize these three regions by σ_0, σ_1, and σ_2. The mathematical representation of the ordering regions is defined as follows:

$$\sigma_0 = \{(x_1, x_2) \leq (S_1(\alpha, \beta), S_2(\alpha, \beta))\, | \Delta_1 \,\&\, \Delta_2 > 0\}$$

$$\sigma_1 = \{(x_1, x_2) \leq (S_1(\alpha, \beta), S_2(\alpha, \beta))\, | \Delta_1 \leq 0 \,\&\, \Delta_1 < \Delta_2\}$$

$$\sigma_2 = \{(x_1, x_2) \notin \sigma_0 \,\&\, \sigma_1\}$$

Non-production region σ_0 is located above $D(x_1, \alpha, \beta)$ and σ_1 is above $E(x_1, \alpha, \beta)$ and below $D(x_1, \alpha, \beta)$, while σ_2 is below $D(x_1, \alpha, \beta)$ and $E(x_1, \alpha, \beta)$. The following lemma is useful when we analyze optimal policy.

Lemma 1: If $G_n(x_1, x_2, \alpha, \beta)$ is $(K_1,\, K_2)$-convex, then

a)

$$G^1{}_n(x_1, x_2, \alpha, \beta) = min\{K_1 + G_n(X_1, x_2, \alpha, \beta)\} = \begin{cases} K_1 + G_n(S_1, x_2, \alpha, \beta) & (x_1, x_2) \in \sigma_1 \\ G_n(x_1, x_2, \alpha, \beta) & (x_1, x_2) \in \sigma_0 \end{cases}$$

b)

$$G^2{}_n(x_1, x_2, \alpha, \beta) = min\{K_2 + G_n(x_1, X_2, \alpha, \beta)\} = \begin{cases} K_2 + G_n(x_1, S_2, \alpha, \beta) & (x_1, x_2) \in \sigma_2 \\ G_n(x_1, x_2, \alpha, \beta) & (x_1, x_2) \in \sigma_0 \end{cases}$$

are $(K_1\,,\, K_2)$-convex where $S_1(\alpha, \beta)$ and $S_2(\alpha, \beta)$are the global minimizers.

Lemma 2 below shows that the $(K_1,\, K_2)$-convexity, supermodularity and diagonal dominance properties of $G_n(x_1, x_2, \alpha, \beta)$ are preserved under minimization. Its usefulness will be shown later in the Chapter.

Lemma 2:

$$G^*{}_n(x_1, x_2, \alpha, \beta) = min\{G^0{}_n(x_1, x_2, \alpha, \beta),\ G^1{}_n(X_1, x_2, \alpha, \beta),\ G^2{}_n(x_1, X_2, \alpha, \beta)\} = \begin{cases} K_1 + G_n(S_1, x_2, \alpha, \beta) & (x_1, x_2) \in \sigma_1 \\ K_2 + G_n(x_1, S_2, \alpha, \beta) & (x_1, x_2) \in \sigma_2 \\ G_n(x_1, x_2, \alpha, \beta) & (x_1, x_2) \in \sigma_0 \end{cases}$$

is $(K_1,\, K_2)$-convex, supermodular and diagonal dominance.

Proposition 1: $f_n(x_1, x_2, \alpha, \beta)$ is $(K_1,\, K_2)$-convex, supermodular and diagonal dominant in x_1 and x_2 for α, β.

The proofs of Lemma 1, Lemma 2 and Proposition 1 are presented in Appendices A and B, respectively.

$(K_1,\, K_2)$-convexity of f_n implies that it is customary to show the optimality of (s,S) policy, but is it not sufficient since we need to switch ordering with respect to ordering priority.

2.3.1 The Structure of Optimal Policy

In this section, we show the optimality of (s,S) policy together with a monotone switching curve.

Proposition 2:

a) Given x_1 and x_2, if the inventory level of item 2 is above or equal to $D(x_1, \alpha, \beta)$, then do not order. Otherwise, order product 2 if x_2 is below $E(x_1, \alpha, \beta)$and order item 1 if x_2 is above $E(x_1, \alpha, \beta)$,

b) $D(x_1, \alpha, \beta)$ is decreasing in x_1 for α, β,

c) $E(x_1, \alpha, \beta)$ is increasing in x_1 for α, β.

See the Appendix C for the proof of Proposition 2.

We define the two threshold levels $s_1(\alpha, \beta)$ and $s_2(\alpha, \beta)$ of item 1 and item 2, respectively, by the following expressions

$$s_1(\alpha, \beta) = min\{x_1 : E(x_1, \alpha, \beta) \geq \ D(x_1, \alpha, \beta)\},$$

$$s_2(\alpha, \beta) = E(s_1(\alpha, \beta))$$

where $E(x_1, \alpha, \beta)$ and $D(x_1, \alpha, \beta)$ are two switching curves. The value of x_1 at the first intersection point of $E(x_1, \alpha, \beta)$ and $D(x_1, \alpha, \beta)$ is $s_1(\alpha, \beta)$. Figure 1 portrays the optimal scheduling policy.

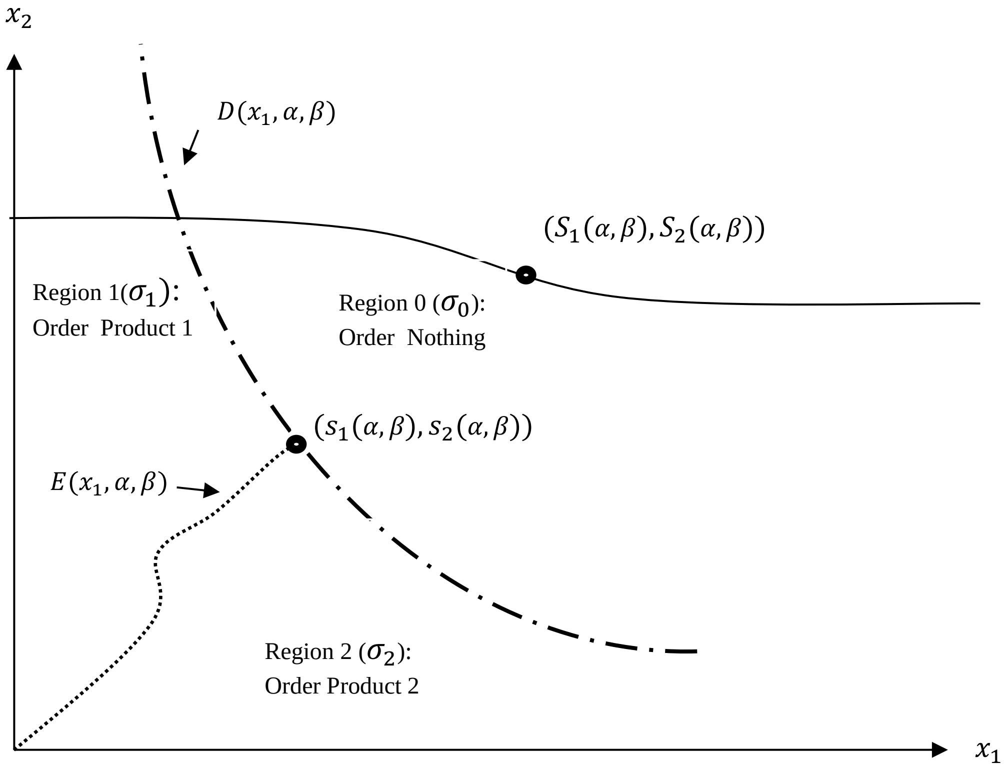

Figure 2.1 Optimal policy and Ordering Regions

(s,S) policy for the problem is defined as follows:

If the initial inventory level of each product is at or above its threshold level, then stop ordering; otherwise, order it up to its base stock level. Next, we define ordering priority for products when both products inventory level are below their threshold level. For the following region, we characterize the optimal ordering policy in Theorem 1.

$$x = (s_1(\alpha,\beta),\ s_2(\alpha,\beta)) = \{(x_1, x_2) | x_1 \leq\ s_1(\alpha,\beta),\ \ x_2 \leq\ s_2(\alpha,\beta)\}$$

Theorem 1:

Given the initial inventory levels, the optimal ordering policy is $(s_i^n(\alpha,\beta), S_i^n(\alpha,\beta))$, together with a monotone switching curve $E(x_1, \alpha, \beta)$ for item $i = 1,2$ in period n when the initial inventory levels are at or below their threshold levels. The proof of Theorem 1 is provided in Appendix D.

Lemma 3: If $K_1' > K_1$ then $s_1'(\alpha,\beta) < s_1(\alpha,\beta)$ for any x_2.

Proof: We define $G_n(x_1,x_2,\alpha,\beta) = K_1 + G_n(S_1,x_2,\alpha,\beta)$ and $G_n'(x_1,x_2,\alpha,\beta) = K'_1 + G_n(S_1,x_2,\alpha,\beta)$. For $K_1' > K_1$, and $K_1 + G_n(S_1,x_2,\alpha,\beta) < K'_1 + G_n(S_1,x_2,\alpha,\beta)$. Therefore, $G_n(x_1,x_2,\alpha,\beta) < G_n'(x_1,x_2,\alpha,\beta)$. Since $s_1(\alpha,\beta)$ and $s'_1(\alpha,\beta)$ minimize $G_n(x_1,x_2,\alpha,\beta)$ and $G_n'(x_1,x_2,\alpha,\beta)$, respectively. $s'_1(\alpha,\beta)$ must be less than $s_1(\alpha,\beta)$.

2.4 Numerical Results

In this section, we present numerical examples to illustrate how the optimal ordering policies for the model developed in the previous section behave for different situations, and for the cases of high and low volume demand. We are interested to see how the initial Bayes estimation of the probability of supply availability, and its updating affect a firm's ordering strategy. To do so, we analyze the results for two models; Bayesian and non-Bayesian. We also investigate the effectiveness of prior information of the probability of supply availability on the optimal order quantity for the Bayesian and non-Bayesian models with and without fixed costs. In the developed model, it is assumed that the prior distribution of the probability of supply availability follows Beta distribution with parameters α and β. For the non-Bayesian model, the probability of supply availability is the expected value of $\theta = \frac{\alpha}{\alpha+\beta}$ with respect to the prior mean in every period, while it is updated for Bayesian model by observing the supplier's status in the previous period.

To compute the total expected discounted cost and the optimal order levels, we assume that the initial inventory levels of both products are zero. The numerical results are obtained by the following input cost parameters: h =1, p =10, $r = 0.9$, $c_1 = 3$, and $c_2 = 1$. For the high demand case, we assume that the demands are uniformly distributed over the intervals [0,5] and [3,6] for products 1 and 2, respectively. On the other hand, low demand follows the same distribution for the intervals [1,3] and [0,2], for products 1 and 2, respectively. Furthermore, in this example, unmet demands will not be filled in the next period, and therefore it is lost at no cost. Six scenarios are considered in this section, which are summarized in Table 2.1. The numerical results are obtained for different scenarios that are presented in Table 2.1. Three production/order periods are considered, T= 1, 2, 3, where the performance measure used is the total expected discounted cost for the optimal order quantity for each of the T periods.

Table 2.1 Demand and Fixed Costs Scenarios

Scenario	Demand	Setup costs	Scenario	Demand	Setup costs
1	High	$K_1 = 0 \quad K_2 = 0$	4	Low	$K_1 = 0 \quad K_2 = 0$
2	High	$K_1 = 18 \quad K_2 = 13$	5	Low	$K_1 = 18 \quad K_2 = 13$
3	High	$K_1 = 11 \quad K_2 = 13$	6	Low	$K_1 = 11 \quad K_2 = 13$

The numerical examples will be classified into four subsections: (1) high demand with and without fixed (setup) costs, (2) low demand with and without fixed (setup) costs, (3) fixed costs effect, and (4) analysis of absolute value of information (VOI). There is a common finding among the results of the six scenarios where the optimal ordering level for 1- period problem is always less than those of 2 and 3-period. This is because we assume that there is no terminal salvage value. The optimal ordering policy consists of the optimal ordering levels shown in the various tables; that will follow together with a switching curve similar to the one depicted in Figure 1, and as stated in Theorem 1. It will be clear after comparing the results summarized in the tables (Appendix) that the optimal ordering policy of a product is more sensitive to the values of the probability of supply availability when the demand is high. For the case when the demand of product 1 is the same as that of product 2, the ordering levels for both products are sensitive to changes in the value of the probability of supply availability. In all scenarios, for the Bayesian model and same prior mean, the slope of cost reduction decreases faster when the probability of supply availability is low especially for high demand case. For example, in Table 2.A4, when (α, β)= (1,1) and (3,3) the total cost decreases by 5.9% (from 181.21 to 170.46) while for (α, β)= (4,1) and (12,3) the cost reduction is 4% (from 144.51 to 138.68).

2.4.1 High Demand with and without setup costs

Tables 2.2 and 2.A1 to 2.A5 summarize the results, which include the optimal order quantity and the total expected discounted for scenarios 1 to 3, with and without Bayesian updating for the three periods, and for different α and β values, where α (β)

the number of periods the machine is up (down). It can be seen that as the sample size increases, the total expected discounted cost decreases. This means that the more knowledge from the past, the greater the value of Bayesian updating especially for the case of low initial prior estimation. For example, in Table 2.2, for the case of determining the order policy for three future periods (or 3-Period) where (α, β)= (1,1) and (3,3) the total cost decreases from 167.08 to 154.10, respectively, as more information is available from the second, 3+3=6 periods, than from the first, 1+1=2 periods.

For the same prior mean, when the initial prior mean on the probability of supply availability, θ, is low, the optimal order quantities for the Bayesian model are less than those produced from the non-Bayesian model for both the 2-period and 3-period, respectively (i.e. when $\theta = 0.5$ where $(\alpha, \beta) = (1,1), (2,2)$, and (3,3) and when $\theta = 0.667$ where $(\alpha, \beta) = (2,1), (4,2)$, and (6,3)). On the other hand, for high prior mean on the probability of supplier availability, the optimal order quantity of the Bayesian model converges to those of a non-Bayesian model (compare the results in Tables 2.2 and 2.A4). For instance, for 3-Period, the optimal order quantity for the Bayesian model with (α, β) = (4,1) is (6.4, 6.3) from Table 2.2 which is the same for the non-Bayesian model when $\theta = 0.8$. The results also show that the optimal order quantities are sensitive to changes in the value of the probability of supply availability, when there is supply scarcity. Moreover, for the same prior mean, the optimal order level of a product with higher expected demand increases in sample size, especially when the probability of supply availability is low (e.g. in Table 2.2, for (α, β) = (1,1) and (3,3) the optimal ordering level of product 2, which has the higher expected demand increases from 6.7 to 6.9) to be more secure about future supply disruption.

One more observation is that with low prior mean on the probability of supply availability, e.g., $\theta = 0.5$ corresponding to (α, β) = (3,3) for 3-Period, the optimal order quantities are strictly higher than those with high prior mean, (e.g., $\theta = 0.8$ corresponding to (α, β) = (8,2) for 3-Period, for both the Bayesian model, $(S_1, S_2) = (6.4, 6.9)$, and non-Bayesian model, $(S_1, S_2) = (6.4, 6.3)$). This can be explained by the fact that under a low initial prior mean, the supplier is less reliable. Hence, to avoid incurring shortages and additional costs, the manufacturer would increase the order levels because holding inventory would be less costly than experiencing a stock-out situation.

Table 2.2 Optimal Ordering Levels for Bayesian model, Scenario 1

(α, β)	1-Period		2-Period		3-Period	
	(S_1, S_2)	Cost	(S_1, S_2)	Cost	(S_1, S_2)	Cost
(1, 1)	(3.2, 5.5)	59.229	(5, 6.7)	107.696	(6.4, 6.7)	167.083
(2, 2)	(3.2, 5.5)	59.229	(5, 6.8)	106.624	(6.4, 6.8)	157.723
(3, 3)	(3.2, 5.5)	59.229	(5, 6.9)	106.148	(6.4, 6.9)	154.104
(2, 1)	(3.2, 5.5)	50.139	(5, 6.5)	90.497	(6.4, 6.5)	140.686
(4, 2)	(3.2, 5.5)	50.139	(5, 6.6)	89.791	(6.4, 6.6)	132.596
(6, 3)	(3.2, 5.5)	50.139	(5, 6.6)	89.500	(6.4, 6.6)	129.694
(3, 1)	(3.2, 5.5)	45.594	(5, 6.3)	82.335	(6.4, 6.3)	126.930
(6, 2)	(3.2, 5.5)	45.594	(5, 6.4)	81.865	(6.4, 6.4)	120.333
(9, 3)	(3.2, 5.5)	45.594	(5, 6.4)	81.684	(6.4, 6.4)	118.045
(4, 1)	(3.2, 5.5)	42.867	(5, 6.3)	77.589	(6.4, 6.3)	118.554
(8, 2)	(3.2, 5.5)	42.867	(5, 6.3)	77.261	(6.4, 6.3)	113.088
(12, 3)	(3.2, 5.5)	42.867	(5, 6.3)	77.138	(6.4, 6.3)	111.223

2.4.2 Low Demand with and without setup

In this section, we repeated the analysis of Section 4.1, for scenarios 4, 5, and 6. The results are summarized in Tables 2.A6-2.A11 for the Bayesian and non-Bayesian models. For the case of a no setup cost, and by comparing the results in Tables 2.2 and 2.A1 (Scenario 1) with those in 2.A6 and 2.A7 (Scenario 4), respectively, one can notice that the expected costs for 1-Period, 2-Period, and 3-Period and all values of (α, β) are much lower than those for the high demand case. In addition, the S_1 and S_2 values are also lower than those for the high demand case, but less sensitive to past information. Similar interpretation of the results can be made when comparing the Tables for Scenarios 2 and 5, and 3 and 6, respectively, for the Bayesian and non-Bayesian models with setup costs and for high and low demand cases. This suggests that obtaining supply availability information for the high demand case is more beneficial than for the low demand case. Moreover, for the same initial prior mean, the total expected discounted cost decreases in sample size smoothly especially for high prior estimation.

2.4.3 The effect of fixed costs

Where $K_1 > K_2 > 0$, we compare the results summarized in Tables 2.2 and 2.A2 and 2.A1 and 2.A3 for Bayesian and Non-Bayesian models respectively. The results comparison indicates that the cost is more sensitive for values of θ for the non-Bayesian model than the Bayesian model. For example, from Tables 2.2 and 2.A2, for 3-Period, when (α, β) = (1, 1), (2, 2) and (3, 3) the costs increase by 8.5% (from 167.08 to 181.21), 10% (from 157.72 to 173.50) and 10.6% (from 154.10 to 170.47) when comparing the corresponding rows in Tables 2.2 and 2.A2. Whereas when (α, β) = (4, 1), (8, 2) and (12, 3) the costs increase by 21.89% (from 118.55 to 144.51), 23.96% (from 113.09 to 140.18) and 24.69% (from 111.22 to 138.69). When comparing Tables 2.A1 and 2.A3, one notices as θ increases in value from 0.5 to 0.8 to 0.98, for 3-Period, the costs increase by 11.93% (from 146.05 to 163.47), 26.23% (from 107.47 to 135.65) and 39.68% (from 87.017 to 121.54). We also notice that S_2–s_2 are sensitive to (α, β) while S_1–s_1 values are not. This is based on the assumption that the expected demand for product 2 is higher than that of product.

When the fixed cost is high, e.g. $K_1 = 18$, for the same initial prior estimation, the availability of more information becomes less important, especially when the initial prior mean of the probability of supply availability, θ, is low. This happens because the manufacturer orders in high volume to avoid high setup/ fixed cost as the optimal order points are independent of the values of α and β. Therefore, it may not be beneficial to invest for collecting the information and learning from previous observations when the fixed cost is high. This is true only when the demands for products 1 and 2 are low. However, when the demand is high the system behaves similar to the high demand with no fixed cost. This can be observed by comparing the results of Tables 2.A2, 2.A4, 2.A6 and A8 (in the Appendix) for the Bayesian model with high demand and low demand cases, respectively, when the fixed ordering costs of a product increases. Considering Scenario 3, where $K_1 < K_2 > 0$, the results are summarized in Table 2.A4 in the Appendix. The behavior of the model as for the case when $K_1 > K_2 > 0$, when compared to the results of Scenario 1 (Table 2.2). The behavior of the order policy remained the same, $s_1 < s_2$ and $S_1 < S_2$ for 1-Period, 2-Period, and 3-Period, with a switch in behavior for 3-Period when $\theta \geq 0.8$. It is most likely that the consistency in the behavior of the inventory policy is due to the assumption that the expected demand for product 1 is lower than that for product 2.

2.4.4 Analysis of the absolute value of information (VOI)

The absolute value of information (VOI) is analyzed for three levels of past information, Low (L), Medium (M), and High (H). Here, we make an example for two cases of initial prior mean, 0.5 and 0.8. The following formulas are used to calculate the absolute VOI for a 1-period, 2-period and 3-period problem, respectively, where VOI is calculated as

$$\begin{aligned}\mathrm{VOI}_{\mathrm{L-M}} = {} & \text{The total expected cost with Low information} \\ & - \text{The total expected cost with Medium infromation}\end{aligned}$$

$$\begin{aligned}\mathrm{VOI}_{\mathrm{M-H}} = {} & \text{The total expected cost with Medium information} \\ & - \text{The total expected cost with High information}\end{aligned}$$

For example, for an initial prior mean of $\theta = 0.5$ with low demand and $\mathrm{K}_1 = 18, K_2 = 13$, the absolute VOI is $\mathrm{VOI}_{\mathrm{L-M}}$ = 84.459-83.917= 0.542. Table 2.3 presents the absolute value of information for the probability of supply availability (θ= 0.5 and θ= 0.8) when $K_1 = 18$, $K_1 = 11$, and $K_1 = 0$.

Table 2.3 Absolute VOI for Two probability of Supply Availability

	$K_1 = 0$			$K_1 = 11$			$K_1 = 18$		
θ= 0.5	1-Period	2-Period	3-Period	1-Period	2-Period	3-Period	1-Period	2-Period	3-Period
VOI_{L-M}	0	0.37	2.753	0	0.096	1.297	0	0	0.542
VOI_{M-H}	0	0.088	0.927	0	0.041	0.516	0	0	0.298
θ= 0.8	1-Period	2-Period	3-Period	1-Period	2-Period	3-Period	1-Period	2-Period	3-Period
VOI_{L-M}	0	0.075	1.685	0	0.035	0.673	0	0	0.345
VOI_{M-H}	0	0.028	0.528	0	0.013	0.227	0	0	0.114

The results in Table 2.3 indicate that the absolute VOI decreases as the fixed cost increases. Moreover, when the supplier is more likely to be available, the absolute VOI is less than when the supplier is less likely to be available. For instance, compare $\mathrm{VOI}_{\text{L-M}}$ (K_1=0, θ= 0.5) = 2.753 > $\mathrm{VOI}_{\text{L-M}}$ (K_1=0, θ= 0.8) = 1.685. The results in Table 2.3 also show that the absolute VOI increases by the number of periods, especially for the case of a zero fixed cost. The value of obtaining additional information is less important when the arrival information level arises to High level from Medium level.

2.5 Conclusion

In this Chapter, we investigated the optimal ordering/production policy for a two-item, finite-horizon dynamic problem, with two different fixed costs where the probability of supply availability is forecasted by Bayesian learning. Although in literature more attention is paid to demand updating, our model contributes to the learning effect on supply availability literature. The existence of fixed costs leads to show the optimality of $(s_i^n(\alpha,\beta), S_i^n(\alpha,\beta))$ policy, but the ordering priority is yet to be determined. By a simple computation, we show that a monotone switching curve separates the ordering region for each product. Numerically, we analyzed the results for low and high demand patterns. The results showed that the system was more sensitive to supply availability information when demand is high. We also compared the numerical results for two models, Bayesian and non-Bayesian. Both order and reorder points for Bayesian models were found to be less than those of non-Bayesian, especially when a product demand is high. The results indicated that information obtained from learning could be more profitable and cost-effective for a low value of the fixed cost. Furthermore, the best results were for the case when there is no fixed cost and demand is low. An application of the model could be used in companies dealing with products like the oil and cement industry, to see how mathematical modeling can help managers deal with issues relating to supply uncertainty. Political and economical issues and price violations cause crude oil supply disruption, which consequently affect the production of product made from the crude oil, such as gasoline and petrol. To avoid future unmet demands, supply history would help companies better manage ordering/ production their policies. For example, we present two real applications of Bayesian updating for the cases of demand and failure process updating. Sakauchi [19] and [20] implemented Bayesian updating to accurately estimate the accurate number of heating oil demand for residential and commercial customers, to minimize the supplier's operational cost. Rathnayaka et. al [21] presented SHIPP methodology which helps a system to increase its safety, and provides accurate information to predict the disruption (accident) a system may exprience. They conducted a case study on a liquefied natural gas (LNG) facility to test the model. New observation helps to update the failure probability using Bayesian updating, and the forecasting model predicts the number of next time abnormal events.

Extending the work developed in this work could be in different directions. It would be interesting to investigate the optimal ordering policy for a multi-supplier case with capacity constraints. It may also be worthy to investigate the behavior of the system described in this chapter when there is lead time for delivery, and/or when the manufacturer has advance lead time information. Moreover, the incorporation of demand uncertainty into this model would also be interesting to investigate.

Appendix

A: Proof of Lemma 1:

Proof:

For $x = (x_1, x_2)$, $y = (y_1, y_2)$, and $x \leq y$, we proof the $(K_1\ K_2)$-convexit of for the following cases.

We prove for (a). The proof for (b) is similar to (a). We examine the three following cases:

i. $x \in \sigma_1, y \in \sigma_1$
ii. $x \in \sigma_0, y \in \sigma_0$
iii. $x \in \sigma_1, y \in \sigma_0$

In (i) $x \in \sigma_1, y \in \sigma_1$,

$G^1{}_n(x_1, x_2, \alpha, \beta) = K_1 + G_n(x_1, x_2, \alpha, \beta)$ and $G^1{}_n(y_1, y_2, \alpha, \beta) = K_1 + G_n(y_1, y_2, \alpha, \beta)$. Since $G_n(.,.)$ is (K_1, K_2)-convex, then $G^1{}_n(x_1, x_2, \alpha, \beta)$ is (K_1, K_2)-convex.

In (ii) $x \in \sigma_0, y \in \sigma_0$,

$G^1{}_n(x_1, x_2, \alpha, \beta) = G_n(x_1, x_2, \alpha, \beta)$ and $G^1{}_n(y_1, y_2, \alpha, \beta) = G_n(y_1, y_2, \alpha, \beta)$. Since $G_n(.,.)$is (K_1, K_2)-convex, then $G^1{}_n(x_1, x_2, \alpha, \beta)$ is (K_1, K_2)-convex.

In (iii) $x \in \sigma_1, y \in \sigma_0$,

For $Z \in [x, y]$, let define $Z = (1 - \lambda)x + \lambda y$ for $\lambda \in [0,1]$

$G^1{}_n(z_1, z_2, \alpha, \beta) \leq K_1 + G_n(S_1, x_2, \alpha, \beta)$

$$= \lambda G^1{}_n(x_1, x_2, \alpha, \beta) + (1 - \lambda) G^1{}_n(y_1, y_2, \alpha, \beta) + (1 - \lambda)[K_1 + G_n(S_1, x_2, \alpha, \beta)] - (1 - \lambda)[G_n(y_1, y_2, \alpha, \beta)]$$

$$= \lambda G^1{}_n(x_1, x_2, \alpha, \beta) + (1 - \lambda) G^1{}_n(y_1, y_2, \alpha, \beta) + (1 - \lambda)[K_1 + G_n(S_1, x_2, \alpha, \beta) - G_n(y_1, y_2, \alpha, \beta)]$$

$$\leq \lambda G^1{}_n(x_1, x_2, \alpha, \beta) + (1-\lambda)G^1{}_n(y_1, y_2, \alpha, \beta) + (1-\lambda)[K_1 + G_n(S_1, x_2, \alpha, \beta) - K_2 - G_n(y_1, S_2, \alpha, \beta)]$$

$$\leq \lambda G^1{}_n(x_1, x_2, \alpha, \beta) + (1-\lambda)G^1{}_n(y_1, y_2, \alpha, \beta) + (1-\lambda)[K_1 + G_n(S_1, x_2, \alpha, \beta) - K_2 - G_n(x_1, S_2, \alpha, \beta)]$$

$$\leq \lambda G^1{}_n(x_1, x_2, \alpha, \beta) + (1-\lambda)G^1{}_n(y_1, y_2, \alpha, \beta) + (1-\lambda)[K_1 + K_2]$$

From the definition of $G^1{}_n(x_1, x_2, \alpha, \beta)$, the first inequality holds. The first equality follows for the assumption that $x \in \sigma_1$ and $y \in \sigma_0$. The second equality follows from Assumption (III) and the definitions of σ_0. The third equality holds from definitions of σ_1. The proof is same for $G^2{}_n(x_1, x_2, \alpha, \beta)$.

Proof of Lemma 2:

By Lemma 1, we already proved that $G^1{}_n(x_1, x_2, \alpha, \beta)$ and $G^2{}_n(x_1, x_2, \alpha, \beta)$ are (K_1, K_2)-convex. The only case that remains is to show the preservation of (K_1, K_2)-convexity for the following cases.

For $y \geq x$, we examine the following cases where $x = (x_1, x_2)$ and $y = (y_1, y_2)$.

i. $x \in \sigma_1$, $y \in \sigma_2$ and $Z \in [x, y]$, let define $Z = (1-\lambda)x + \lambda y$ for $\lambda \in [0,1]$.

$$G^*{}_n(z_1, z_2, \alpha, \beta) \leq K_1 + G_n(S_1, x_2, \alpha, \beta)$$

$$= \lambda G^*{}_n(x_1, x_2, \alpha, \beta) + (1-\lambda)G^*{}_n(y_1, y_2, \alpha, \beta) + (1-\lambda)[K_1 + G_n(S_1, x_2, \alpha, \beta)] - (1-\lambda)[K_2 + G_n(y_1, y_2, \alpha, \beta)]$$

$$\leq \lambda G^*{}_n(x_1, x_2, \alpha, \beta) + (1-\lambda)G^*{}_n(y_1, y_2, \alpha, \beta) - (1-\lambda)[K_1 + G_n(S_1, x_2, \alpha, \beta) - K_2 - G_n(y_1, S_2, \alpha, \beta)]$$

$$\leq \lambda G^*{}_n(x_1, x_2, \alpha, \beta) + (1-\lambda)G^*{}_n(y_1, y_2, \alpha, \beta) - (1-\lambda)[K_1 + G_n(S_1, x_2, \alpha, \beta) - K_2 - G_n(x_1, S_2, \alpha, \beta)]$$

$$\leq \lambda G^*{}_n(x_1, x_2, \alpha, \beta) + (1-\lambda)G^*{}_n(y_1, y_2, \alpha, \beta) - (1-\lambda)[K_2 - K_1 + K_1 - K_2]$$

$$\leq \lambda G^*{}_n(x_1, x_2, \alpha, \beta) + (1-\lambda)G^*{}_n(y_1, y_2, \alpha, \beta) + (1-\lambda)[K_1 + K_2]$$

The first inequality is from the definition of $G^*{}_n(x_1, x_2, \alpha, \beta)$. The first equality holds since $x \in \sigma_1$, $y \in \sigma_2$. Definition of σ_1 and Assumption (III) result in the rest of inequalities.

ii) $x \in \sigma_2, y \in \sigma_1$

The proof is similar to (i).

We also want to prove preservation of the supermodularity and diagonal dominance under the minimization. We define

$$
\begin{aligned}
G_n{}^a(x_1, x_2, \alpha, \beta) &= G^*{}_n(x_1, x_2, \alpha, \beta) \\
&= min\Big\{G_n(x_1, x_2, \alpha, \beta),\ \min_{X_1 \geq x_1}[K_1 + G_n(X_1, x_2, \alpha, \beta)],\ \min_{X_2 \geq x_2}[K_2 \\
&\quad + G_n(x_1, X_2, \alpha, \beta)]\Big\}
\end{aligned}
$$

Let υ be a function on the set of admissible action, $a \in \{0,1,2\}$, which is defined on a lattice.

$$
\begin{aligned}
&\upsilon(a, x_1, x_2, \alpha, \beta) \\
&\quad = \frac{1}{2}(1-a)(2-a)G_n(x_1, x_2, \alpha, \beta) + a(2-a)\big(K_1 + G_n(X_1, x_2, \alpha, \beta)\big) \\
&\quad + \frac{1}{2}(a-1)a(K_2 + G_n(x_1, X_2, \alpha, \beta))
\end{aligned}
$$

Therefore, we have

$$G_n{}^a(x_1, x_2, \alpha, \beta) = min_{a \in \{0,1,2\}}\{\upsilon(a, x_1, x_2, \alpha, \beta)\}.$$

Since a is the minimizer of $\upsilon(a, x_1, x_2, \alpha, \beta)$. We assume that a_1 and a_2 are the minimizers of $G_n{}^a(x_1, x_2, \alpha, \beta)$ at state $(x_1 + 1, x_2 + 1)$ and (x_1, x_2), respectively for α, β. Therefore by definition of $G_n{}^a(x_1, x_2, \alpha, \beta)$

$$G_n{}^a(x_1 + 1, x_2 + 1, \alpha, \beta) = \upsilon(a_1, x_1 + 1, x_2 + 1, \alpha, \beta),$$

$$G_n{}^a(x_1, x_2, \alpha, \beta) = \upsilon(a_2, x_1, x_2, \alpha, \beta).$$

Similar to Ha (1997) we show the supermodularity preserves for two cases:

1) $a_1 \geq a_2$.

Here we seek to show that

$$G_n{}^a(x_1+1, x_2, \alpha, \beta) + G_n{}^a(x_1, x_2+1, \alpha, \beta) \\ \leq G_n{}^a(x_1+1, x_2+1, \alpha, \beta) + G_n{}^a(x_1, x_2, \alpha, \beta)$$

By Theorem (8.2) in Porteus (2002), we can write

$G_n{}^a(x_1+1, x_2, \alpha, \beta) + G_n{}^a(x_1, x_2+1, \alpha, \beta) \leq\ v(a_1, x_1+1, x_2, \alpha, \beta) + v(a_2, x_1, x_2+1, \alpha, \beta)$ [definition of $G_n{}^a(x_1, x_2, \alpha, \beta)$]

$\leq v((a_1, x_1+1, x_2, \alpha, \beta) \vee (a_2, x_1, x_2+1, \alpha, \beta)) + v((a_1, x_1+1, x_2, \alpha, \beta) \wedge (a_2, x_1, x_2+1, \alpha, \beta))$

[v is supermodular]

$= \quad v((a_1 \vee a_2), (x_1+1 \vee x_1), (x_2 \vee x_2+1)) + v((a_1 \wedge a_2), (x_1+1 \wedge x_1), (x_2 \wedge x_2+1))$

[definition]

$=\ v(a_1, x_1+1, x_2+1, \alpha, \beta) + v(a_2, x_1, x_2, \alpha, \beta)$

$$= G_n{}^a(x_1+1, x_2+1, \alpha, \beta) + G_n{}^a(x_1, x_2, \alpha, \beta)$$

2) $a_1 < a_2$

$$G_n{}^a(x_1+1, x_2, \alpha, \beta) + G_n{}^a(x_1, x_2+1, \alpha, \beta) \\ \leq\ v(a_1, x_1+1, x_2, \alpha, \beta) + v(a_2, x_1, x_2+1, \alpha, \beta)$$

$\leq\ v((a_1, x_1+1, x_2, \alpha, \beta) \vee (a_1, x_1, x_2+1, \alpha, \beta)) + v((a_1, x_1+1, x_2, \alpha, \beta) \wedge (a_1, x_1, x_2+1, \alpha, \beta)) - v(a_2, x_1, x_2+1, \alpha, \beta) + v(a_2, x_1, x_2+1, \alpha, \beta)$

[Supermoduarity of v in (x_1, x_2)]

$$= \upsilon(a_1, x_1 + 1, x_2 + 1, \alpha, \beta) + \upsilon(a_1, x_1, x_2, \alpha, \beta) - \upsilon(a_1, x_1, x_2 + 1, \alpha, \beta) + \upsilon(a_2, x_1, x_2 + 1, \alpha, \beta)$$

$\leq \upsilon(a_1, x_1 + 1, x_2 + 1, \alpha, \beta) + \upsilon(a_1, x_1, x_2, \alpha, \beta) + \upsilon(a_2, x_1, x_2 + 1, \alpha, \beta) -$
$\upsilon((a_1, x_1, x_2 + 1, \alpha, \beta) \vee (a_2, x_1, x_2, \alpha, \beta) + \upsilon((a_1, x_1, x_2 + 1, \alpha, \beta) \wedge$
$(a_2, x_1, x_2, \alpha, \beta)) + \upsilon(a_2, x_1, x_2, \alpha, \beta)$

[Supermoduarity of υ in (a, x_2)]

$$= \upsilon(a_1, x_1 + 1, x_2 + 1, \alpha, \beta) + \upsilon(a_1, x_1, x_2, \alpha, \beta) - \upsilon(a_1, x_1, x_2 + 1, \alpha, \beta) + \upsilon(a_2, x_1, x_2 + 1, \alpha, \beta)$$

$$= \upsilon(a_1, x_1 + 1, x_2 + 1, \alpha, \beta) + \upsilon(a_2, x_1, x_2, \alpha, \beta)$$

For Diagonal dominance, since the proof is similar to Ha (1997) we refer the readers to Lemma 2, Ha (1997).

B: Proof of Proposition 1

Proof: K- Convexity

By induction, we show K-Convexity property of $f_n(x_1, x_2, \alpha, \beta)$ where $K = (K_1, K_2)$. Knowing the fact that $f_{N+1}(x_1, x_2, \alpha, \beta) = 0$ for the last period, we first prove K-Convexity of $G_n(x_1, x_2, \alpha, \beta)$ in x_1 and x_2 for $n = N$. Suppose $g(.)$ is twice differentiable.

We already proved in Lemma 2 that $min\{G_N(x_1, x_2, \alpha, \beta), min_{X_1 \geq x_1}[K_1 + G_N(X_1, x_2, \alpha, \beta)], min_{X_2 \geq x_2}[K_2 + G_N(x_1, X_2, \alpha, \beta)]\}$ is (K_1, K_2)-convex. By properties (a) and (b) stated in Lemma E.1, we can say $f_N(x_1, x_2, \alpha, \beta)$ is (K_1, K_2)-convex.

Now we assume that $f_n(x_1, x_2, \alpha, \beta)$ is (K_1, K_2)-convex for period $n = t + 1$. We want to show that it is also (K_1, K_2)-convex for period $n = t$.

$$
\begin{aligned}
f_t(x_1, x_2; \alpha, \beta\ ) \\
&= q_t\big(G_t(x_1, x_2, \alpha, \beta)\big) \\
&+ p_t\left(min\left\{G_t(x_1, x_2, \alpha, \beta),\ \min_{X_1 \geq x_1}[K_1 + G_t(X_1, x_2, \alpha, \beta)],\ \min_{X_2 \geq x_2}[K_2\right.\right. \\
&\left.\left.+ G_t(x_1, X_2, \alpha, \beta)]\right\}\right) - c_1x_1 - c_2x_2
\end{aligned}
$$

Where $G_t(x_1, x_2, \alpha, \beta) = g_1(x_1) + g_2(x_2) + rE\big(f_{t+1}(x_1 - \xi_1, x_2 - \xi_2, \alpha, \beta)\big)$

As we assume $f_{t+1}(x_1 - \xi_1, x_2 - \xi_2, \alpha, \beta)$ is $(K_1,\ K_2)$-convex, then by property (C) stated in Lemma E.1, $E\ (f_{t+1})$ is $(K_1,\ K_2)$-convex. By property (b) from Lemma E.1, $rE\big(f_{t+1}(x_1 - \xi_1, x_2 - \xi_2, \alpha, \beta)\big)$ is $r(K_1,\ K_2)$-convex. Finally property (a) in Lemma E.1 proves that $rE\big(f_{t+1}(x_1 - \xi_1, x_2 - \xi_2, \alpha, \beta)\big)$ is $(K_1,\ K_2)$-convex. Thus by Lemma E.2 , $G_t(x_1, x_2, \alpha, \beta)$ is $(K_1,\ K_2)$-convex. By properties (a) and (b) stated in Lemma E.1 and Lemma 1, we complete the proof of the $(K_1,\ K_2)$-convexity of f_t.

Proof: Supermodularity and Diagonal dominance

By induction, we conduct the proof. Assume that $f_{N+1}(x_1, x_2, \alpha, \beta) = 0$. It is trivial to show that $f_N(x_1, x_2, \alpha, \beta)$ is supermodular and has diagonal dominance for $n = N$. $G_N(x_1, x_2, \alpha, \beta)$ is the sum of two separable functions and it is also belongs to V^*. Therefore, knowing the fact that these properties preserve under adding and multiplying by positive scalars, here, $p\ and\ q$, and by Lemma 2, $f_N(x_1, x_2, \alpha, \beta)$ is supermodular and has diagonal dominance. Now, assume $f_{t+1}(x_1, x_2, \alpha, \beta) \in V^*$. Hence, $f_t(x_1, x_2, \alpha, \beta)$ is by lemma 2 and Lemma (8.3) Porteus (2002) and thus, we complete the proof.

C: Proof of Proposition 2:

a) Select a point x_2 above $D(x_1, \alpha, \beta)$. By the definition of $D(x_1, \alpha, \beta)$, $\Delta_1 \geq 0$ and $\Delta_2 \geq 0$ and by (III) in Assumption, for all $x_2' > x_2$,
$\Delta_2 G_n(x_1, x_2', \alpha, \beta) - \Delta_1 G_n(x_1, x_2', \alpha, \beta) \geq \Delta_2 G_n(x_1, x_2, \alpha, \beta) - \Delta_1 G_n(x_1, x_2, \alpha, \beta) \geq 0.$

Thus for all points above x_2, the optimal decision is order nothing. If we pick x_2 below $E(x_1, \alpha, \beta)$, it follows from the definition of $E(x_1, \alpha, \beta)$ that $\Delta_2 G_n(x_1, x_2, \alpha, \beta) -$

$\Delta_1 G_n(x_1, x_2 \alpha, \beta) \leq 0$. So, it is optimal to order product 2. As x_2 increases it moves above $E(x_1, \alpha, \beta)$, the priority of ordering also moves to product 1. The rational follows by Assumption (III), $\Delta_2 G_n(x_1, x_2, \alpha, \beta) - \Delta_1 G_n(x_1, x_2, \alpha, \beta)$ increasing in x_2.

b) By Assumptions (I) and (II), for $x_1 < x_1' < S_1(\alpha, \beta)$ we have the following

$$\Delta_2 G_n(x_1', D(x_1, \alpha, \beta), \alpha, \beta) > \Delta_2 G_n(x_1, D(x_1, \alpha, \beta), \alpha, \beta) \geq 0$$

$$\Delta_1 G_n(x_1', D(x_1, \alpha, \beta), \alpha, \beta) > \Delta_1 G_n(x_1, D(x_1, \alpha, \beta), \alpha, \beta) \geq 0$$

By the definition of $D(x_1, \alpha, \beta)$, we must have $D(x_1', \alpha, \beta) < D(x_1, \alpha, \beta)$.

c) It follows from the definition of $E(x_1, \alpha, \beta)$ that $\Delta_2 G_n(x_1, E(x_1, \alpha, \beta), \alpha, \beta) - \Delta_1 G_n(x_1, E(x_1, \alpha, \beta), \alpha, \beta) \geq 0$. For $x_1 < x_1' < S_1(\alpha, \beta)$, we have by Assumption (III),

$$\Delta_2 G_n(x_1', E(x_1, \alpha, \beta), \alpha, \beta) - \Delta_1 G_n(x_1', E(x_1, \alpha, \beta), \alpha, \beta) \leq 0,$$

and since $\Delta_2 G_n(x_1, x_2, \alpha, \beta) - \Delta_1 G_n(x_1, x_2, \alpha, \beta)$ is increasing in x_2, we must have $E(x_1', \alpha, \beta) > E(x_1, \alpha, \beta)$.

D: Proof of Theorem 1:

In region 0, it follows from the definition of K-convexity that for any $x_1 \geq s_1(\alpha, \beta)$ and $x_2 \geq s_2(\alpha, \beta)$, $\Delta_2 G_n(x_1, x_2, \alpha, \beta) \geq 0$ and $\Delta_1 G_n(x_1, x_2, \alpha, \beta) \geq 0$. Thus, it is not beneficial to order any product. For $x_1 < s_1(\alpha, \beta)$ and $x_2 < s_2(\alpha, \beta)$, switching curve $E(x_1, \alpha, \beta)$ determines the ordering priority. If $\Delta_2 G_n(x_1, x_2, \alpha, \beta) - \Delta_1 G_n(x_1, x_2, \alpha, \beta) \geq 0$, then it is optimal to order product 1. As the inventory level of product 1 increases, by Assumption (III) $\Delta_2 G_n(x_1, x_2, \alpha, \beta) - \Delta_1 G_n(x_1, x_2, \alpha, \beta)$ decreases thus ordering product 2 in more advantageous.

E: Complimentary Definitions, Notations, and Results Table:

E.1. Model Notations

h_i: Unit holding cost for product i, i= 1, 2,

p_i: Unit penalty cost for product i, i= 1, 2,

C_i= Unit ordering cost for product i, i= 1, 2,

K_i: Fixed cost for product i, i= 1, 2,

q_n: The probability of machine being down in period n,

p_n: The probability of machine being up in period n

θ: The probability of machine/ supplier availability,

(α, β): Machine reliability status at the beginning of a period

Λ: The number of supply available periods

r: Discount factor,

x_i: Initial inventory level for product i, i=1,2,

X_i: Inventory level after production for product i, i=1,2,

ξ_i: The one-period demand for product i, i=1,2,

$\varphi_i(\xi)$: Demand probability density function for product i, i= 1, 2,

ϕ_i (ξ): Demand cumulative distribution function product i, i= 1, 2,

$L_n(.)$: The one-period expected holding and shortage cost function,

$f_n(x_1, x_2, \alpha, \beta)$: Minimum expected discounted cost function.

E.2. $(\boldsymbol{K_1}, \boldsymbol{K_2})$-convexity

To characterize the optimal policy, we define $\boldsymbol{K}-$ convex function in R^n where $\boldsymbol{K} = (K_1, K_2, \ldots K_n)$-. The definition follows the definition from Gallego & Sethi (2005).

Definition 1: A function $f: R^n \to R^1$ is $\boldsymbol{K}$-convex function in R^n if

$$f(\lambda x + (1-\lambda)y) \leq \lambda f(x) + (1-\lambda)\left[f(y) + K(y-x)\right]$$

for all $\lambda \in [0,1]$ and all $x \leq y$ where $x_i \leq y_i$ for all $i = 1,2,\ldots n$.

Lemma E.1 presents some properties of K-convex function in $\mathrm{R^n}$.

Lemma E.1: (Gallego and Sethi (2005) [Gallego & Sethi (2005)])The following properties are defined for K-convex function in $\mathrm{R^n}$:

a) If f is K-convex function in $\mathrm{R^n}$, then it is L-convex function in $\mathrm{R^n}$ for any $\mathrm{L} \geq \mathrm{K}$. Particularly, a convex function in $\mathrm{R^n}$ is K -convex function in $\mathrm{R^n}$ for any $\mathrm{K} \geq 0$.

b) If f^1 is K-convex function in $\mathrm{R^n}$ and f^2 is L-convex function in $\mathrm{R^n}$, then for $a \geq 0$, $b \geq 0$, $f = af^1 + bf^2$ is $(\mathrm{aK} + \mathrm{bL})$–convex.

c) If f is K-convex function in $\mathrm{R^n}$, for a random variable $\xi = (\xi_1, \xi_2, \ldots, \xi_n)$, $\mathrm{Ef}(x - \xi) < \infty$ is also K-convex in $\mathrm{R^n}$ for all x.

Proof: See Gallego and Sethi (2005, page: 4 and 5).

The following lemma was defined by Gallego & Sethi (2005)for individual setup costs case to show that the sum of independent k-convex functions is K-convex in $\mathrm{R^n}$.

Lemma E.2: (Theorem 3.1, Gallego and Sethi, 2005).

If $\mathrm{f_i} : \mathrm{R^1} \to \mathrm{R^1}$ is $\mathrm{K_i}$-convex for $\mathrm{i} = 1, \ldots, \mathrm{n}$, then $\mathrm{g}(\mathrm{x_1}, \ldots, \mathrm{x_n}) = \sum_{\mathrm{i}=1}^{\mathrm{n}} \mathrm{f_i}(\mathrm{x_i}) : \mathrm{R^n} \to \mathrm{R^1}$ is K -convex where $\mathrm{K} = (0, \mathrm{K_1}, \ldots, \mathrm{K_n})$.

Proof: See Gallego and Sethi (2005, page: 7)

E.3. Results Tables

Table 2.A1. Optimal Ordering Levels for non-Bayesian model, Scenario 1

Non-Bayesian Model, Scenario 1						
θ	1-Period		2-Period		3-Period	
	(S_1, S_2)	Cost	(S_1, S_2)	Cost	(S_1, S_2)	Cost
0.5	(3.2, 5.5)	59.229	(5, 7)	104.926	(6.4, 7)	146.053
0.6667	(3.2, 5.5)	50.137	(5, 6.7)	88.809	(6.4, 6.7)	123.613
0.75	(3.2, 5.5)	45.594	(5, 6.5)	81.266	(6.4, 6.5)	113.370
0.8	(3.2, 5.5)	42.867	(5, 6.3)	76.867	(6.4, 6.3)	107.468
0.9	(3.2, 5.5)	37.413	(5, 6.1)	68.260	(6.4, 6.1)	98.623
0.98	(3.2, 5.5)	33.049	(5, 5.8)	61.454	(6.4, 5.8)	87.017

Table 2.A2. Optimal Ordering Levels for Bayesian model, Scenario 2

Bayesian Model, Scenario 2									
(α, β)	1-Period			2-Period			3-Period		
	(S_1, S_2)	(s_1, s_2)	Cost	(S_1, S_2)	(s_1, s_2)	Cost	(S_1, S_2)	(s_1, s_2)	Cost
(1, 1)	(3.2, 5.5)	(0, 2.8)	65.729	(5, 6.7)	(1, 4.2)	120.046	(6.4, 6.7)	(2.6, 4.2)	181.210
(2, 2)	(3.2, 5.5)	(0, 2.8)	65.729	(5. 7.7)	(1, 4.6)	118.930	(6.4, 6.8)	(2.6, 4.6)	173.501
(3, 3)	(3.2, 5.5)	(0, 2.8)	65.729	(5, 7.7)	(1, 4.7)	118.396	(6.4, 6.9)	(2.6, 4.7)	170.468
(2, 1)	(3.2, 5.5)	(0, 2.8)	58.806	(5, 6.5)	(1, 3.7)	106.964	(6.4, 6.5)	(2.6, 3.7)	161.060
(4, 2)	(3.2, 5.5)	(0, 2.8)	58.806	(5, 6.6)	(1, 3.9)	106.258	(6.4, 6.6)	(2.6, 3.9)	154.542
(6, 3)	(3.2, 5.5)	(0, 2.8)	58.806	(5, 6.6)	(1, 4)	105.967	(6.4, 6.6)	(2.6, 4)	152.166
(3, 1)	(3.2, 5.5)	(0, 2.8)	55.344	(5, 6.3)	(1, 3.5)	100.860	(6.4, 6.3)	(2.6, 3.5)	150.736
(6, 2)	(3.2, 5.5)	(0, 2.8)	55.344	(5, 6.4)	(1, 3.6)	100.390	(6.4, 6.4)	(2.6, 3.6)	145.467
(9, 3)	(3.2, 5.5)	(0, 2.8)	55.344	(5, 6.4)	(1,3.6)	100.209	(6.4, 6.4)	(2.6, 3.6)	143.619
(4, 1)	(3.2, 5.5)	(0, 2.8)	53.267	(5, 6.3)	(1, 3.4)	97.349	(6.4, 6.3)	(2.6, 3.4)	144.511
(8, 2)	(3.2, 5.5)	(0, 2.8)	53.267	(5, 6.3)	(1, 3.4)	97.021	(6.4, 6.3)	(2.6, 3.4)	140.179
(12, 3)	(3.2, 5.5)	(0, 2.8)	53.267	(5, 6.3)	(1, 3.5)	96.898	(6.4, 6.3)	(2.6, 3.5)	138.688

Table 2.A3. Optimal Ordering Levels for non-Bayesian model, Scenario 2

Non-Bayesian Model, Scenario 2									
θ	1-Period			2-Period			3-Period		
	(S_1, S_2)	(s_1, s_2)	Cost	(S_1, S_2)	(s_1, s_2)	Cost	(S_1, S_2)	(s_1, s_2)	Cost
0.5	(3.2, 5.5)	(0, 2.8)	65.729	(5, 7.7)	(1, 4.9)	117.060	(6.4, 7)	(2.6, 4.9)	163.474
0.666	(3.2, 5.5)	(0, 2.8)	58.804	(5, 6.7)	(1, 4.2)	105.276	(6.4, 6.7)	(2.6, 4.2)	147.101
0.75	(3.2, 5.5)	(0, 2.8)	55.344	(5, 6.5)	(1, 3.7)	99.791	(6.4, 6.5)	(2.6, 3.7)	139.793
0.8	(3.2, 5.5)	(0, 2.8)	53.267	(5, 6.3)	(1, 3.5)	96.627	(6.4, 6.3)	(2.6, 3.5)	135.652
0.9	(3.2, 5.5)	(0, 2.8)	49.113	(5, 6.1)	(1, 3.2)	90.490	(6.4, 6.1)	(2.6, 3.2)	127.730
0.98	(3.2, 5.5)	(0, 2.8)	45.789	(5, 5.8)	(1, 3)	85.660	(6.4, 5.8)	(2.6, 3)	121.543

Table 2.A4. Optimal Ordering Levels for Bayesian model, Scenario 3

Bayesian Model, Scenario 3									
(α, β)	1-Period			2-Period			3-Period		
	(S_1, S_2)	(s_1, s_2)	Cost	(S_1, S_2)	(s_1, s_2)	Cost	(S_1, S_2)	(s_1, s_2)	Cost
(1, 1)	(3.2, 5.5)	(0.1, 2.8)	65.729	(5, 6.7)	(2.8, 4.2)	120.046	(6.4, 6.7)	(3.4, 4.2)	181.198
(2, 2)	(3.2, 5.5)	(0.1, 2.8)	65.729	(5, 7.7)	(2.8, 4.6)	118.893	(6.4, 6.8)	(3.4, 4.6)	173.487
(3, 3)	(3.2, 5.5)	(0.1, 2.8)	65.729	(5, 7.7)	(2.8, 4.7)	118.361	(6.4, 6.9)	(3.4, 4.7)	170.455
(2, 1)	(3.2, 5.5)	(0.1, 2.8)	58.806	(5, 6.5)	(2.8, 3.7)	106.964	(6.4, 6.5)	(3.4, 3.7)	161.043
(4, 2)	(3.2, 5.5)	(0.1, 2.8)	58.806	(5, 6.6)	(2.8, 3.9)	106.258	(6.4, 6.6)	(3.4, 3.9)	154.542
(6, 3)	(3.2, 5.5)	(0.1, 2.8)	58.806	(5, 6.6)	(2.8, 4)	105.967	(6.4, 6.6)	(3.4, 4)	152.166
(3, 1)	(3.2, 5.5)	(0.1, 2.8)	55.344	(5, 6.3)	(2.8, 3.5)	100.867	(6.4, 6.3)	(3.4, 3.5)	150.736
(6, 2)	(3.2, 5.5)	(0.1, 2.8)	55.344	(5, 6.4)	(2.8, 3.6)	100.390	(6.4, 6.4)	(3.4, 3.6)	145.467
(9, 3)	(3.2, 5.5)	(0.1, 2.8)	55.344	(5, 6.4)	(2.8, 3.6)	100.209	(6.4, 6.4)	(3.4, 3.6)	143.619
(4, 1)	(3.2, 5.5)	(0.1, 2.8)	53.267	(5, 6.3)	(2.8, 3.4)	97.349	(6.4, 6.3)	(3.4, 3.4)	144.511
(8, 2)	(3.2, 5.5)	(0.1, 2.8)	53.267	(5, 6.3)	(2.8, 3.4)	97.021	(6.4, 6.3)	(3.4, 3.4)	140.179
(12, 3)	(3.2, 5.5)	(0.1, 2.8)	53.267	(5, 6.3)	(2.8, 3.5)	96.898	(6.4, 6.3)	(3.4, 3.5)	138.688

Table 2.A5. Optimal Ordering Levels for non-Bayesian model, Scenario 3

Non-Bayesian Model, Scenario 3									
θ	1-Period			2-Period			3-Period		
	(S_1, S_2)	(s_1, s_2)	Cost	(S_1, S_2)	(s_1, s_2)	Cost	(S_1, S_2)	(s_1, s_2)	Cost
0.5	(3.2, 5.5)	(0.1, 2.8)	65.729	(5, 7.7)	(2.8, 4.9)	117.030	(6.4, 7)	(3.4, 4.9)	163.447
0.6667	(3.2, 5.5)	(0.1, 2.8)	58.804	(5, 6.7)	(2.8, 4.2)	105.276	(6.4, 6.7)	(3.4, 4.2)	147.101
0.75	(3.2, 5.5)	(0.1, 2.8)	55.344	(5, 6.5)	(2.8, 3.7)	99.791	(6.4, 6.5)	(3.4, 3.7)	139.793
0.8	(3.2, 5.5)	(0.1, 2.8)	53.627	(5, 6.3)	(2.8, 3.5)	96.627	(6.4, 6.3)	(3.4, 3.5)	135.652
0.9	(3.2, 5.5)	(0.1, 2.8)	49.113	(5, 6.1)	(2.8, 3.2)	90.490	(6.4, 6.1)	(3.4, 3.2)	127.730
0.98	(3.2, 5.5)	(0.1, 2.8)	45.789	(5, 5.8)	(2.8, 3)	85.660	(6.4, 5.8)	(3.4, 3)	121.543

Table 2.A6. Optimal Ordering Levels for Bayesian model, Scenario 4

Bayesian Model, Scenario 4						
(α, β)	1-Period		2-Period		3-Period	
	(S_1, S_2)	Cost	(S_1, S_2)	Cost	(S_1, S_2)	Cost
(1, 1)	(2.3, 1.6)	25.649	(3.1, 2.2)	45.550	(3.1, 2.7)	68.110
(2, 2)	(2.3, 1.6)	25.649	(3.2, 2.2)	45.173	(3.2, 2.7)	65.357
(3, 3)	(2.3, 1.6)	25.649	(3.2, 2.2)	45.085	(3.2, 2.7)	64.430
(2, 1)	(2.3, 1.6)	23.282	(3, 2.2)	40.633	(3, 2.7)	59.900
(4, 2)	(2.3, 1.6)	23.282	(3.1, 2.2)	40.187	(3.1, 2.7)	57.425
(6, 3)	(2.3, 1.6)	23.282	(3.1, 2.2)	40.129	(3.1, 2.7)	56.555
(3, 1)	(2.3, 1.6)	22.098	(3, 2.2)	38.165	(3, 2.7)	55.421
(6, 2)	(2.3, 1.6)	22.098	(3, 2.2)	38.062	(3, 2.7)	53.560
(9, 3)	(2.3, 1.6)	22.098	(3, 2.2)	38.022	(3, 2.7)	52.934
(4, 1)	(2.3, 1.6)	21.388	(3, 2.2)	36.725	(3, 2.7)	52.748
(8, 2)	(2.3, 1.6)	21.388	(3, 2.2)	36.650	(3, 2.7)	51.058
(12, 3)	(2.3, 1.6)	21.388	(3, 2.2)	36.622	(3, 2.7)	50.530

Table 2.A7. Optimal Ordering Levels for non-Bayesian model, Scenario 4

Non-Bayesian Model, Scenario 4						
θ	1-Period		2-Period		3-Period	
	(S_1, S_2)	cost	(S_1, S_2)	cost	(S_1, S_2)	cost
0.5	(2.3, 1.6)	25.649	(3.2, 2.2)	44.865	(3.2, 2.7)	62.468
0.6667	(2.3, 1.6)	23.281	(3.1, 2.2)	39.991	(3.1, 2.7)	55.029
0.75	(2.3, 1.6)	22.098	(3, 2.2)	37.989	(3, 2.7)	51.698
0.8	(2.3, 1.6)	21.388	(3, 2.2)	36.560	(3, 2.7)	49.484
0.9	(2.3, 1.6)	19.968	(2.9, 2.2)	34.404	(2.9, 2.7)	45.619
0.98	(2.3, 1.6)	18.832	(2.8, 2.2)	33.057	(2.8, 2.7)	42.759

Table 2.A8. Optimal Ordering Levels for Bayesian model, Scenario 5

Bayesian Model, Scenario 5									
(α, β)	1-Period			2-Period			3-Period		
	(S_1, S_2)	(s_1, s_2)	Cost	(S_1, S_2)	(s_1, s_2)	Cost	(S_1, S_2)	(s_1, s_2)	Cost
(1, 1)	(2.3, 1.6)	(0, 0)	32.75	(3.5, 2.2)	(0.8, 0.1)	58.516	(4.5, 2.7)	(0.4, 0.6)	84.459
(2, 2)	(2.3, 1.6)	(0, 0)	32.75	(3.5, 2.2)	(0.8, 0.1)	58.516	(4.5, 2.7)	(0.4, 0.6)	83.917
(3, 3)	(2.3, 1.6)	(0, 0)	32.75	(3.5, 2.2)	(0.8, 0.1)	58.516	(4.5, 2.7)	(0.4, 0.6)	83.619
(2, 1)	(2.3, 1.6)	(0, 0)	32.75	(3.5, 2.2)	(0.8, 0.1)	57.279	(4.5, 2.7)	(0.3, 0.6)	82.590
(4, 2)	(2.3, 1.6)	(0, 0)	32.75	(3.5, 2.2)	(0.8, 0.1)	57.279	(4.5, 2.7)	(0.3, 0.6)	82.119
(6, 3)	(2.3, 1.6)	(0, 0)	32.75	(3.5, 2.2)	(0.8, 0.1)	57.279	(4.5, 2.7)	(0.3, 0.6)	81.959
(3, 1)	(2.3, 1.6)	(0, 0)	32.75	(3.5, 2.2)	(0.8, 0.1)	56.661	(4.5, 2.7)	(0.3, 0.6)	81.558
(6, 2)	(2.3, 1.6)	(0, 0)	32.75	(3.5, 2.2)	(0.8, 0.1)	56.661	(4.5, 2.7)	(0.3, 0.6)	81.154
(9, 3)	(2.3, 1.6)	(0, 0)	32.75	(3.5, 2.2)	(0.8, 0.1)	56.661	(4.5, 2.7)	(0.3, 0.6)	81.020
(4,1)	(2.3, 1.6)	(0, 0)	32.75	(3.5, 2.2)	(0.8, 0.1)	56.290	(4.5, 2.7)	(0.3, 0.6)	80.907
(8,2)	(2.3, 1.6)	(0, 0)	32.75	(3.5, 2.2)	(0.8, 0.1)	56.290	(4.5, 2.7)	(0.3, 0.6)	80.562
(12,3)	(2.3, 1.6)	(0, 0)	32.75	(3.5, 2.2)	(0.8, 0.1)	56.290	(4.5, 2.7)	(0.3, 0.6)	80.448

Table 2.A9. Optimal Ordering Levels for non-Bayesian model, Scenario 5

Non-Bayesian Model, Scenario 5									
θ	1-Period			2-Period			3-Period		
	(S_1, S_2)	(s_1, s_2)	Cost	(S_1, S_2)	(s_1, s_2)	Cost	(S_1, S_2)	(s_1, s_2)	Cost
0.5	(2.3, 1.6)	(0, 0)	32.75	(3.5, 2.2)	(0.8, 0.1)	58.515	(4.5, 2.7)	(0.5, 0.6)	83.075
0.6667	(2.3, 1.6)	(0, 0)	32.75	(3.5, 2.2)	(0.8, 0.1)	57.278	(4.5, 2.7)	(0.4, 0.6)	81.649
0.75	(2.3, 1.6)	(0, 0)	32.75	(3.5, 2.2)	(0.8, 0.1)	56.660	(4.5, 2.7)	(0.3, 0.6)	80.761
0.8	(2.3, 1.6)	(0, 0)	32.75	(3.5, 2.2)	(0.8, 0.1)	56.29	(4.5, 2.7)	(0.3, 0.6)	80.228
0.9	(2.3, 1.6)	(0, 0)	32.75	(3.5, 2.2)	(0.8, 0.1)	55.548	(4.5, 2.7)	(0.2, 0.6)	79.163
0.98	(2.3, 1.6)	(0, 0)	32.75	(3.5, 2.2)	(0.8, 0.1)	54.954	(4.5, 2.7)	(0.2, 0.6)	78.310

Table 2.A10. Optimal Ordering Levels for Bayesian model, Scenario 6

Bayesian Model, Scenario 6									
(α, β)	1-Period			2-Period			3-Period		
	(S_1, S_2)	(s_1, s_2)	Cost	(S_1, S_2)	(s_1, s_2)	Cost	(S_1, S_2)	(s_1, s_2)	Cost
(1, 1)	(2.3, 1.6)	(0.3, 0)	31.149	(3.5, 2.2)	(1.5, 0.1)	54.535	(3.1, 2.7)	(0.8, 0.6)	80.080
(2, 2)	(2.3, 1.6)	(0.3, 0)	31.149	(3.5, 2.2)	(1.6, 0.1)	54.439	(3.2, 2.7)	(0.8, 0.6)	78.783
(3, 3)	(2.3, 1.6)	(0.3, 0)	31.149	(3.5, 2.2)	(1.6, 0.1)	54.398	(3.2, 2.7)	(0.9, 0.6)	78.267
(2, 1)	(2.3, 1.6)	(0.3, 0)	30.615	(3.5, 2.2)	(1.5, 0.1)	52.132	(3, 2.7)	(0.7, 0.6)	76.866
(4, 2)	(2.3, 1.6)	(0.3, 0)	30.615	(3.5, 2.2)	(1.5, 0.1)	52.064	(3.1, 2.7)	(0.7, 0.6)	75.791
(6, 3)	(2.3, 1.6)	(0.3, 0)	30.615	(3.5, 2.2)	(1.5, 0.1)	52.036	(3.1, 2.7)	(0.7, 0.6)	75.400
(3, 1)	(2.3, 1.6)	(0.3, 0)	30.348	(3.5, 2.2)	(1.5, 0.1)	50.979	(3, 2.7)	(0.6, 0.6)	75.271
(6, 2)	(2.3, 1.6)	(0.3, 0)	30.348	(3.5, 2.2)	(1.5, 0.1)	50.931	(3, 2.7)	(0.6, 0.6)	74.431
(9, 3)	(2.3, 1.6)	(0.3, 0)	30.348	(3.5, 2.2)	(1.5, 0.1)	50.912	(3, 2.7)	(0.6, 0.6)	74.141
(4, 1)	(2.3, 1.6)	(0.3, 0)	30.188	(3.5, 2.2)	(1.4, 0.1)	50.306	(3, 2.7)	(0.6, 0.6)	74.347
(8, 2)	(2.3, 1.6)	(0.3, 0)	30.188	(3.5, 2.2)	(1.4, 0.1)	50.271	(3, 2.7)	(0.6, 0.6)	73.674
(12,	(2.3, 1.6)	(0.3, 0)	30.188	(3.5, 2.2)	(1.4, 0.1)	50.258	(3, 2.7)	(0.6, 0.6)	73.447

Table 2.A11. Optimal Ordering Levels for Bayesian model, Scenario 6

Non-Bayesian Model, Scenario 6									
θ	1-Period			2-Period			3-Period		
	(S_1, S_2)	(s_1, s_2)	Cost	(S_1, S_2)	(s_1, s_2)	Cost	(S_1, S_2)	(s_1, s_2)	Cost
0.5	(2.3, 1.6)	(0.3, 0)	31.149	(3.5, 2.2)	(1.7, 0.1)	53.295	(3.2, 2.7)	(1, 0.6)	77.092
0.6667	(2.3, 1.6)	(0.3, 0)	30.615	(3.5, 2.2)	(1.5, 0.1)	51.972	(3.1, 2.7)	(0.8, 0.6)	74.578
0.75	(2.3, 1.6)	(0.3, 0)	30.348	(3.5, 2.2)	(1.5, 0.1)	50.871	(3, 2.7)	(0.7, 0.6)	73.551
0.8	(2.3, 1.6)	(0.3, 0)	30.188	(3.5, 2.2)	(1.5, 0.1)	50.229	(3, 2.7)	(0.6, 0.6)	72.992
0.9	(2.3, 1.6)	(0.3, 0)	29.868	(3.5, 2.2)	(1.4, 0.1)	48.989	(2.9, 2.7)	(0.5, 0.6)	72.012
0.98	(2.3, 1.6)	(0.3, 0)	29.612	(3.5, 2.2)	(1.3, 0.1)	48.038	(2.8, 2.7)	(0.5, 0.6)	71.331

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3. THE EFFECT OF RANDOM YIELD UPDATING IN THREE-LEVEL SUPPLY CHAIN

This Chapter forms a procurement problem for a three-echelon supply chain (a producer/supplier, a distributor and a retailer) where yield and demand are random variables. The distributor orders from the supplier in which the supplier's shipment quantity is added by random yield. It is assumed that the random yield and demand are uniformly distributed. Using Bayesian updating, the distributor updates its random yield distribution that follows a uniform distribution with two unknown parameters. We model the problem for the distributor and the retailer. For two types of contracts, the effect of information updating and risk sharing is investigated. It is shown how Bayesian updating helps managers to make an appropriate procurement decision. The results also show how supply chain members benefit under different contracts. For models with information updating, an algorithm is proposed to obtain the optimal ordering quantity for each chain member. Analytical results for the Non-Bayesian model are also provided. Sensitivity analysis and the learning effect on the performance of the supply chain members are investigated numerically.

3.1 Introduction

Many supply chain uncertainties in most industries arise from the demand side. Supply uncertainty, although infrequent, is another cause of uncertainty. It has considerable effect on the performance of a supply chain and supply decisions. Supply uncertainty and randomness in yield are caused by many factors that are usually out of humans control. For example, farmers are always worried about the weather conditions that influence planting and quantity and quality of the harvest, where the output might not be the same as the input. On the other hand, since agricultural goods are perishable, some factors, such as transportation, affect the yield for the members of a supply chain, especially the member who deals with the end customers [Keren (2009), He and Zhang (2008)]. Moreover, the quality of inputs or raw materials also has an effect on the production output. For example, the quality of seeds in the agricultural industry and the quality of chips in the semiconductor industry significantly affect random yield that, consequently, has an influence on production planning and ordering, and pricing decisions of the parties in a supply chain. Many studies have adopted coordination contract design and information sharing to deal with supply chain uncertainty (Arshinder et.al 2011). On the other hand, many papers have studied different forecasting tools to mitigate supply chain uncertainty that leads to accurate decision making. In this Chapter, supply contracts in combination with Bayesian updating are used in a three-echelon decentralized supply chain to investigate the effect of information and information sharing on the profits and ordering decisions of the members in the chain.

In this Chapter, we investigate ordering decisions in a three-echelon supply chain consisting of a supplier, a distributor and a retailer. The distributor, who is the middle player of the chain, faces random yield from an upstream supplier. We classify contracts for two scenarios: Bayesian and Non-Bayesian. In each scenario, the distributor shares the random yield risk with the retailer under different contracts. We first study the decentralized case of Non-Bayesian with lost sales and spot market contract. Later, in addition to the lost sales contact, a quantity flexibility contract is proposed for the Bayesian model.

For additive supply yield risk, the distributor's optimal ordering quantity from the supplier and the retailer's optimal ordering quantity from the distributor are derived. We suppose that the yield risk is unknown and the distributor forecasts the yield risk by relying on the observation obtained from the previous period. Under the quantity flexibility contract, the retailer is flexible to receive less than the ordered quantity conditional on the distributor's yield risk sharing and updating and shipment commitment up to a level.

We assume that the retailer is the dominant player. To overcome a stock-out (shortage) situation in the lost sale model, the retailer requests that the distributor shares and updates its random yield information on a regular basis. Moreover, a quantity flexibility contract is offered for the purpose of information sharing where the distributor fills a shortage from a spot market.

Despite the large body of literature that has investigated supply chain management and random yield; we restrict our attention in this Chapter to reviewing recent papers on supply chain interfacing of uncertain random yield and demand with coordination contracts, especially quantity flexibility contract. First, we review the literature on random yield in two-level supply chains, then uncertainty in three-level supply chains.

Gerchak and Grosfeld-Nir (1998), Granot and Yin (2007), Hsu and bassok (1999), Mukhopadhyay and Ma (2009) derived optimal production quantities for a single period under random demand and yield. To overcome double marginalization of a supply chain consisting of a buyer and a supplier, Li et.al (2013) developed two coordination contracts: a shortage penalty contract for deterministic demand and an accept-all contracts for random demand. They assumed that the supplier decides on the production level before supply uncertainty is realized.

To encounter the risks from having uncertain demand, wholesale price and production yield, some researchers (Xu, 2010) suggested applying an option contract to synchronize the production plan of a manufacturer with the procurement policy of a supplier in a decentralized supply chain. Respectively, the optimal production and option order quantities for a manufacturer and a supplier were derived under this contract. Xu's results expressed the benefits of adopting an option contract for the supply chain members.

In a supply chain with one supplier and one retailer, He and Zhang (2008) studied no random yield risk sharing and different risk sharing contracts (underproduction, overproduction, and both underproduction and overproduction) for the decentralized and centralized cases where demand is uncertain. For each decentralized scenario, the retailer's optimal ordering quantity and the supplier's optimal production quantity are calculated. In a later study, He and Zhang (2010) investigated the same supply chain described in their earlier paper where the commitment contract is used to share the random yield risk between the channel members. They also analysed the supply chain performance when demand is fulfilled from a secondary market where the unit price is either dependent or independent of the random yield. For a two- echelon supply chain consisting of a producer and a customer, Keren (2009) considered a single-period inventory problem in which demand is deterministic and supply yield is random. For two types of supply yield, additive and multiplicative, the optimal production quantity was derived. Additionally, he extended the problem in his paper by including one more supply chain member, a distributor, who has the customer's demand information as private. The systems of the distributor and the producer were modelled mathematically and then solved to obtain the respective optimal ordering and production quantities. For the uniform distribution production yield, numerical examples reveal that the customer and the distributor may order more than what they need to overcome supply uncertainty, which is also beneficial for the producer. However, still, some questions with respect to the results obtained by Keren (2009) remain unanswered. Later, Li et. al (2012) extended the work of Keren (2009) to cover issues that were not addressed in his paper. They explored under which condition it is optimal for the distributor to order more than the demand. Moreover, they calculated the effect of yield uncertainty on the performance of the entire supply chain and its members. However, Keren's (2009) results hold for uniformly distributed supply risk, Li et. al (2012) achieved results for the generalized random yield distribution. They showed that above a certain threshold level of marginal profit, the distributor orders more than the demand.

He and Zhao (2012) investigated a three-echelon supply chain consisting of a raw material supplier, a manufacturer and a retailer where both demand and supply are uncertain. They analytically characterized the retailer's optimal inventory decision and the supplier's optimal production planning. Moreover, to obtain the supply chain coordination, they examined two types of coordination contracts, which are the wholesale price contract and the return policy combined with wholesale price contract. The results revealed that the supply chain achieves a win-win situation under the combined return policy and wholesale contracts offered by the manufacturer and the retailer, and the supplier and the manufacturer, respectively, while, solely, the wholesale contract does not coordinate with the immediate member of the supply chain. They also explored the effect of supply uncertainty on the supplier's production decision.

Ding and Chen (2008) studied a single-period coordination model of a three level supply chain for short-lived products. The contract is first negotiated by the manufacturer with the retailer, and then with the supplier. To achieve coordination, a flexible return policy was established allowing for the postponement of decision on final contract prices.

Random yield in assembly systems was studied by Guler and Bilgic (2009) and Shi-hua and Zhe (2012). For the random demand and supplier's yield, Guler and Bilgic (2009) proposed a combination of buyback and revenue sharing contracts to coordinate the channel under forced compliance. Shi-hua and Zhe (2012) proposed an option contract to counter the risk of production uncertainty of an assembly component where the product demand is deterministic and price-dependent. Another stream of related research to our work is the one on channel coordination under quantity flexibility contract. Although the positive affect of flexibility on the performance of the supply chain is clear, it still seems interesting to determine the optimal level of flexibility. In this regard, Tang and Tomlin (2008) conducted a thorough investigation of the role of flexibility to mitigate supply chain risks, which are: supply risks, process risks, intellectual property risks, behavioural risks, demand risks, and political/social risks. For various flexibility strategies, they analytically showed that for some types of supply chain risks a low level of flexibility reduces supply chain risk.

Güray and Keskin (2013) analysed supply chain coordination for wholesale price, buy back, quantity flexibility, quantity discount and revenue sharing contracts in a decentralized setting where the random yield is assumed to be multiplicative. They showed that, apart from the wholesale price contract, the randomness characteristics of the yield have no effect on supply chain coordination for all contracts. For a finite horizon, Lian and Deshmukh (2009) developed two heuristic models to derive the optimal ordering policy that minimises the buyer's total expected cost. They considered a supply contract with quantity flexibility where the supplier offers a discount to the buyer on an advance purchase commitment. Moreover, in their model, demand forecasting was shown to increase the buyer's order quantities for future periods with respect to its inventory condition. For a single period model and under uncertain production yield and demand, Hu et.al (2013) developed a flexibility ordering policy for a supply chain with a manufacturer and a supplier. They investigated a supply chain with centralized and decentralized decisions to find the supplier's optimal procurement quantity when the manufacturer's optimal flexible ordering quantity lies between a minimum value and a maximum value. The analysis of the results showed that for the decentralized model, flexibility is not beneficial (profits) for the manufacturer while it is when the policy is centralized. Therefore, to achieve coordination, they proposed a new contract that combines "revenue sharing" and "order penalty and rebate (OPR)" contracts.

In this Chapter, we model a single period problem with random yield risk sharing and information updating, where an additive random yield that follows the uniform distribution with two unknown parameters is considered. Forecasting by learning from observations has been used wildly to make appropriate production and procurement management decisions. There are many papers published on the effectiveness of demand forecasting on supply chain's profit or cost resulting in appropriate decisions for single period and multi-period dynamic problems.

Bayesian updating was used by Scarf (1959), Azoury (1985), Lariviere and Porteus (1999) as a forecasting tool to update uncertain demand for dynamic production planning. Under a quantity flexibility contract, Tasy (1999) and Wu (2005) investigated a decentralized model where uncertain demand is forecasted by a Bayesian method. We refrain from reviewing the broad literature related to demand information updating since it falls outside the scope of this Chapter. We only focus on reviewing the literature relating to information sharing from the supply side, especially where Bayesian learning has been applied. In a supply chain with one manufacturer and one supplier, a design theory was used by Yang et.al (2009) to derive the optimal contract by the manufacturer to obtain the supplier's private information. The supplier is categorized as either high-reliable or low-reliable. When disruption happens, the supplier either pays a shortage penalty or fills the shortage quantity with backup production. They investigated the effect of asymmetric information on using backup production for low- and high-reliable supplier.

Tomlin (2009) and Chen et al. (2010) investigated the effect of supply learning on inventory and sourcing decisions. Using the Bayesian learning process, they updated information about the unreliable supplier in their model to find the optimal sourcing decision for a multi-period dynamic problem.

Every year at Christmas time or other high demand seasons/occasions, managing inventory level is challenging for many producers and sellers not only from the demand side but also from the supply side. In such a situation, the distributor may not fill the retailer's entire order quantity because the supplier faces a shortage situation. There are some factors that affect supply availability such as emergency orders from other buyers, selling-priority, trust on the buyer's ordering quantity, and so on. To encounter the negative effect of supply unavailability and unmet demand, learning from previous supply observations might contain useful information to the buyer.

In this Chapter, a three-echelon supply chain consisting of a supplier, a distributor and a retailer is considered. We investigate the role of information sharing on the ordering decision and profit maximization. For an unknown additive random yield, an observation from the previous supply period is used to update the distribution of the random variable, random yield variable, with two unknown parameters that indicate the range of the random variable. Moreover, we propose a quantity flexibility contract to induce the distributor for sharing the random yield information to the retailer. Keren's (2009) and He and Zhang (2008, 2010) studied channel coordination and yield risk sharing for additive and multiplicative random yield. To the best of our knowledge, there is no study in the literature that investigates whether random yield forecasting is beneficial for the supply chain parties or not. If it is, we investigate under which condition and what contract the distributor shares the knowledge of supply risk with the retailer. We also explore the role of quantity flexibility, spot market, and lost sale contracts on the performance of the supply chain and its associated parties.

The rest of the Chapter is organized as follows. Section 3.2 presents model analysis including supply risk structure and mathematical formulation of the model. Optimal solutions are derived in section 3.3. Managerial insights and discussion of results are presented in section 3.4. In the last section, there are concluding remarks and some future extensions of the work presented in this Chapter.

3.2 Model Analysis

3.2.1 Supply Risk Structure

The supply chain model of this Chapter assumes that the distributor orders Q units from the supplier whose shipment quantity is affected by an additive random variable θ, i.e. $Q + \theta$. To be specific, the supply risk model arises from the fact that the supplier may ship less or more than the distributor's order quantity. Furthermore, it is assumed that the distributor's yield risk is an unknown parameter. To model the supply risk distribution, a generalization of the uniform distribution is used, which allows uncertainty outside its boundaries. The overall supply risk distribution consist of three parts. From left and right, it captures the Pareto distribution when the yield risk is out of its normal range and behaves uniformly in the middle.

We assume that the supplier delivers $\theta + Q$ units where θ, the yield, is a random variable with $a \leq \theta \leq b$, for $-Q \leq a \leq b < \infty$ and $\psi(\theta|a,b)$ is the random yield distribution function on the range $[a,b]$ where a and b are unknown. The total random yield distribution follows a Doubly-Pareto Uniform distribution and is given as

$$\psi(\theta|a,b) = \begin{cases} \dfrac{1}{b-a}, & a \leq \theta \leq b \\ 0, & Otherwise \end{cases}$$

By Degoort (1970), the conjugate prior on $[a,b]$ is a Bilateral Bivariate Pareto distribution with parameters $\mathcal{u}_0$ and $\mathcal{U}_0$ and α where $\mathcal{u}_0 < \mathcal{U}_0$ and $\alpha > 0$. The prior probability density function is given as

$$g(a,b|\mathcal{u}_0,\mathcal{U}_0) = \begin{cases} \dfrac{\alpha(\alpha+1)(\mathcal{U}_0 - \mathcal{u}_0)^{\alpha}}{(b-a)^{\alpha+2}}, & a < \mathcal{u}_0, \quad b > \mathcal{U}_0 \\ 0, & Otherwise. \end{cases}$$

Therefore, the distribution of θ defined as

$$\psi(\theta) = \frac{\alpha(\mathcal{U}_0 - \mathcal{u}_0)^{\alpha}}{(\alpha+2)[\max(\theta,\mathcal{U}_0) - \min(\theta,\mathcal{u}_0)]^{\alpha+1}}$$

$$
= \begin{cases} \dfrac{\alpha(\mathcal{U}_0 - u_0)^\alpha}{(\alpha+2)[\mathcal{U}_0 - \theta]^{\alpha+1}}, & \theta < u_0 \\ \dfrac{\alpha}{(\alpha+2)(\mathcal{U}_0 - u_0)}, & u_0 \le \theta \le \mathcal{U}_0 \\ \dfrac{\alpha(\mathcal{U}_0 - u_0)^\alpha}{(\alpha+2)[\theta - u_0]^{\alpha+1}}, & \theta > \mathcal{U}_0 \end{cases}
$$

Observe that the distribution of θ is a weighted uniform distribution. On the interval $(-\infty, u_0)$ and $(\mathcal{U}_0, \infty)$, it is a Pareto distribution scaled by 1/(α +2) which indicates random yield surges while on the interval $(u_0, \mathcal{U}_0)$, the scaling factor is α /(α +2) implying the normal range of the random yield. The supply risk probability density function is increasing on the interval $(-\infty, u_0)$ and is decreasing on the interval $(\mathcal{U}_0, \infty)$.

The following lemma states the scaled supply risk probability density function of the Uniform distribution with two unknown parameters.

Lemma 1:

$\psi(\theta) = \frac{1}{(\mathcal{U}_0 - u_0)} f_0(\frac{\theta - u_0}{\mathcal{U}_0 - u_0})$ is the scaled total supply risk p.d.f. where $f_0\left(\frac{\theta - u_0}{\mathcal{U}_0 - u_0}\right) = \frac{\alpha}{(\alpha+2)[max(1, 1-t, t)]^{\alpha+1}}$ for $t = \frac{\theta - u_0}{\mathcal{U}_0 - u_0}$.

Proof:

$$
\frac{1}{(\mathcal{U}_0 - u_0)} f_0\left(\frac{\theta - u_0}{\mathcal{U}_0 - u_0}\right) = \frac{\alpha}{(\alpha+2)\left[max(1,\ 1 - \frac{\theta - u_0}{\mathcal{U}_0 - u_0},\ \frac{\theta - u_0}{\mathcal{U}_0 - u_0})\right]^{\alpha+1}}
$$

$$
= \begin{cases} \dfrac{\alpha(\mathcal{U}_0 - u_0)^\alpha}{(\alpha+2)\left[1 - \dfrac{\theta - u_0}{\mathcal{U}_0 - u_0}\right]^{\alpha+1}}, & \theta < u_0 \\ \dfrac{\alpha}{(\alpha+2)}, & u_0 \le \theta \le \mathcal{U}_0 \\ \dfrac{\alpha(\mathcal{U}_0 - u_0)^\alpha}{(\alpha+2)\left[\dfrac{\theta - u_0}{\mathcal{U}_0 - u_0}\right]^{\alpha+1}}, & \theta > \mathcal{U}_0 \end{cases}
$$

The following lemma states a property that is used to update the random yield distribution.

Lemma 2: (Theorem 2, Degoort [28], Page 173)

Suppose that $\theta_1 \ldots \theta_n$ is a random sample from a uniform distribution on the range $[a, b]$ where the values of a and b are unknown. Further suppose that the prior distribution of the unknown properties a and b is a bilateral bivariate Pareto distribution with parameters $\mathcal{u}_0$, $\mathcal{U}_0$ and α. Then, the posterior distribution of a and b is a bilateral bivariate Pareto with parameters $min(\mathcal{u}_0, \theta_1 \ldots \theta_n)$, $max(\mathcal{U}_0, \theta_1 \ldots \theta_n)$, and $\alpha + n$.

From Lemma 2, the posterior distribution of a and b after an observation of the initial supply risk θ_0 is a bilateral bivariate Pareto distribution with parameters $\mathcal{u}_1 = min(\theta_0, \mathcal{u}_0)$ and $\mathcal{U}_1 = max(\theta_0, \mathcal{U}_0)$ and $\alpha + 1$. Thus, the probability density function supply risk θ given an observation θ_0 is:

$$\psi(\theta|\theta_0) = \frac{(\alpha+1)(max(\theta_0,\mathcal{U}_0)-min(\theta_0,\mathcal{u}_0))^{(\alpha+1)}}{(\alpha+3)[max(\theta,\theta_0,\mathcal{U}_0)-min(\theta,\theta_0,\mathcal{u}_0)]^{\alpha+2}}$$

The conditional probability density function of supply risk is:

$$\psi(\theta|\theta_0) = \frac{1}{(\mathcal{U}_1-\mathcal{u}_1)} f_1\left(\frac{\theta-\mathcal{u}_1}{\mathcal{U}_1-\mathcal{u}_1}\right) \qquad (1)$$

Where $f_1\left(\frac{\theta-\mathcal{u}_1}{\mathcal{U}_1-\mathcal{u}_1}\right) = \frac{\alpha+1}{(\alpha+3)[max(1,1-t,t)]^{\alpha+2}}$ for $t = \frac{\theta-\mathcal{u}_1}{\mathcal{U}_1-\mathcal{u}_1}$.

Therefore, in our model, given an initial supply risk information, the supply risk distribution would be updated with different parameters and preserving its original shape. Moreover, we normalized the distribution by $f_1(.)$ and it is scalable by the parameters $\mathcal{u}_1 = min(\theta_0, \mathcal{u}_0)$ and $\mathcal{U}_1 = max(\theta_0, \mathcal{U}_0)$.

3.2.2 Model Notation and Formulation

3.2.2.1 Notation

C Distributor's unit ordering cost from supplier,

C^s Spot market price,

w Distributor's unit selling price to retailer,

P Retailer's unit selling price,

Q Distributor's order quantity from supplier,

q Retailer's order quantity to distributor,

h Retailer's unit holding cost for unsold inventory,

g Retailer's unit shortage cost for demand that exceeds inventory level,

θ Random yield variable,

d Retailer's Flexibility level,

ξ Retailer's demand,

$\phi(\xi)$ Demand probability density function,

$\Phi(\xi)$ Demand cumulative distribution function,

π_D^{NBSM} Distributor's expected profit for the Non-Bayesian spot market model,

Q^*_{NBSM} Distributor's optimal ordering quantity for the Non-Bayesian spot market model,

Q^*_{BLS} Distributor's optimal ordering quantity for the Bayesian lost sale model,

Q^*_{QFC} Distributor's optimal ordering quantity for the quantity flexibility model,

π_D^{BLS}	Distributor's expected profit for the Bayesian lost sale model,
π_D^{QFC}	Distributor's expected profit for quantity flexibility model,
π_R^{NBSM}	Retailer's expected profit for the Non-Bayesian spot market model,
q^*_{NBSM}	Retailer's optimal ordering quantity for the Non-Bayesian spot market model,
q^*_{BLS}	Retailer's optimal ordering quantity for the Bayesian lost sale model,
q^*_{QFC}	Retailer's optimal ordering quantity for quantity flexibility model,
π_R^{BLS}	Retailer's expected profit for the Bayesian lost sale model,
π_R^{QFC}	Retailer's expected profit for quantity flexibility model.

3.2.2.2 Non- Yield Risk Updating

3.2.2.2.1 Spot market Model

The members of a decentralized supply chain maximize their profits independently. In this section, we model the distributor and the retailer's profit functions where the distributor tolerates a random yield risk from its supplier. This Non-Bayesian model is similar to the model studied by He and Zhang (2008) where the supplier's random yield is multiplicative and the spot market cost is split between the supply chain members.

In this model, the distributor decides on the size of the ordering quantity Q from the supplier by considering random yield to meet the demand from the retailer. The supplier ships $\theta + Q$ unis to the distributor. If $\theta + Q$ is less than the retailer's order quantity q, then the distributor incurs a secondary market ordering cost. It is common to assume that the secondary market ordering cost is higher than the distributor's ordinary ordering cost He and Zhang (2008). On the other hand, if $\theta + Q$ is greater than q, the distributor then carries the over ordering cost, and its profit function is expressed as follows:

$$\pi_D^{NBSM} = wq - C\int_{-\infty}^{\infty}(\theta + Q)\,\psi(\theta)d\theta - C^s\int_{-\infty}^{q-Q}(q-(\theta+Q))\,\psi(\theta)d\theta \qquad (2)$$

The first term of Eq. (2) is the profit from selling q units to the retailer, the second term is the ordinary ordering cost and the third term is the secondary market ordering cost to fulfil the shortage quantity. The distributor's profit function is shown to be concave in Q in lemma 3.

Lemma 3:

The distributor's profit function is concave in Q and the optimal ordering quantity is:

$$Q^*_{NBSM} = q - \Psi^{-1}\left(\frac{C}{C^s}\right) \tag{3}$$

Proof:

$$\frac{\partial \pi_D^{NBSM}}{\partial Q} = -C\int_0^{\infty} \psi(\theta)d\theta \; + C^s \int_0^{q-Q} \psi(\theta)d\theta$$

$$-C \; + C^s\,\Psi(q-Q) = 0$$

$$Q^*_{NBSM} = q - \Psi^{-1}\left(\frac{C}{C^s}\right)$$

$$\frac{\partial^2 \pi_D^{NBSM}}{\partial Q} = -C^s\psi(q-Q) < 0$$

For uniform random yield distribution the distributor's optimal ordering quantity is $Q^*_{NBSM} = q - a - \frac{C.(b-a)}{C^s}$. The retailer's optimal ordering quantity from the distributeor is the solution of a standard newsvendor model, i.e. $q^* = \frac{p+g-w}{p-h-g}$.

3.2.2.2.2 Lost Sale Model

In this model, the distributor decides on the size of the ordering quantity Q from the supplier by considering the random yield parameter to meet the demand from the retailer. The supplier ships $\theta + Q$ units to the distributor. If $\theta + Q$ is less than the retailer's order quantity q, then a lost sales situation occurs. On the other hand, if $\theta + Q$ is greater than q, then the distributor carries over the ordering cost. The Non-Bayesian lost sale model is similar to the lost sales model studied by He and Zhang (2008) where the supplier's random yield was multiplicative. The distributor's profit function is expressed as

$$\pi_D^{NBLS} = w\int_{-\infty}^{q-Q}(\ \theta + Q)\ \psi(\theta)d\theta + w\int_{q-Q}^{\infty} q\ \ \psi(\theta)d\theta - C\int_{-\infty}^{\infty}(\ \theta + Q)\ \psi(\theta)d\theta \qquad (4)$$

The first term of Eq. (4) is the profit from selling q units to the retailer, the second term is the profit obtained by selling $\theta + Q$ units, and the third term is the ordinary ordering cost. The distributor's profit function is shown to be concave in Q in lemma 4.

Lemma 4:

The distributor's profit function is concave in Q and the optimal ordering quantity is a linear function of the retailer's order quantity q:

$$Q_{NBLS}^{*} = q - \Psi^{-1}\left(\frac{C}{w}\right) \qquad (5)$$

Proof:

$$\frac{\partial \pi_D^{NBLS}}{\partial Q} = -C\int_{-\infty}^{\infty}\psi(\theta)d\theta\ + w\int_{-\infty}^{q-Q}\psi(\theta)d\theta$$

$$-C\ (1) + w.\left[\ \Psi(q - Q)\right] = 0$$

$$Q_{NBLS}^{*} = q - \Psi^{-1}\left(\frac{C}{w}\right)$$

$$\frac{\partial^2 \pi_D^{NBLS}}{\partial Q^2} = -w.\psi(q - Q) < 0$$

For a uniform random yield distribution the distributor's optimal ordering quantity is $Q_{NBLS}^{*} = q - a - \frac{C.(b-a)}{w}$.

Lemma 4 shows that Q_{NBLS}^{*} is a linear function of the retailer's order quantity. We rewrite$Q_{NBLS}^{*} = q - A$, where$A =\ \Psi^{-1}\left(\frac{C}{w}\right)$.

The retailer's optimal ordering quantity given the distributor's optimal ordering from lemma 4 is expressed as follows:

$$\pi_R^{NBLS} = pE_{\xi,\theta}[\min(\xi, \theta + Q_{NBLS}^*, q)] - wE_\theta\left[q - \left(q - (\theta + Q_{NBLS}^*)\right)^+\right] - hE_{\xi,\theta}[\min(q, \theta + Q_{NBLS}^*) - \xi] - gE_{\xi,\theta}[\xi - \min(q, \theta + Q_{NBLS}^*)].$$

$$\begin{aligned}
\pi_R^{NBLS} = PE[\xi] - P\int_{q-Q_{NBLS}^*}^{\infty}\int_{q}^{\infty}(\xi - q)\,\phi(\xi)\,\psi(\theta)\,d\xi\,d\theta \\
- P\int_{-\infty}^{q-Q_{NBLS}^*}\int_{\theta+Q_{NBLS}^*}^{\infty}\left(\xi - (\theta + Q_{NBLS}^*)\right)\phi(\xi)\,\psi(\theta)\,d\xi\,d\theta \\
- h\int_{-\infty}^{q-Q_{NBLS}^*}\int_{0}^{\theta+Q_{NBLS}^*}\left((\theta + Q_{NBLS}^*) - \xi\right)\phi(\xi)\,\psi(\theta)\,d\xi\,d\theta \\
- h\int_{q-Q_{NBLS}^*}^{\infty}\int_{0}^{q}(q - \xi)\,\phi(\xi)\,\psi(\theta)\,d\xi\,d\theta \\
- g\int_{q-Q_{NBLS}^*}^{\infty}\int_{q}^{\infty}(\xi - q)\,\phi(\xi)\,\psi(\theta)\,d\xi\,d\theta \\
- g\int_{-\infty}^{q-Q_{NBLS}^*}\int_{\theta+Q_{NBLS}^*}^{\infty}(\xi - (\theta + Q_{NBLS}^*))\,\phi(\xi)\,\psi(\theta)\,d\xi\,d\theta \\
- w\int_{-\infty}^{q-Q_{NBLS}^*}(\theta + Q_{NBLS}^*)\,\psi(\theta)d\theta - w\int_{q-Q_{NBLS}^*}^{\infty} q\ \psi(\theta)d\theta
\end{aligned}$$

The following lemma shows the concavity and the calculation of the optimal ordering quantity.

Lemma 5:

Given the distributor's optimal ordering level, the retailer's optimal ordering quantity is the solution of the following equation where the retailer's profit function is concave and it is given as:

$$[\Phi(q)\overline{\Psi}(A)] + \int_{-\infty}^{A}\Phi(\theta + q - A)\,\psi(\theta)d\theta = \frac{\mathrm{p+g-w}}{\mathrm{p+g+h}} \tag{7}$$

Where $\overline{\Phi}(q) = 1 - \Phi(q)$ and $\overline{\Psi}(A) = 1 - \Psi(A)$.

The proof of lemma 5 is presented in Appendix.

3.2.2.3 Random Yield Updating

Facing lost sales is costly for both the distributor and the retailer. If the risk of random yield is high, then ordering from the distributor is not beneficial and the retailer may decide to order from another supplier. Here, information updating may help the distributor to deal with a future shortage situation that might occur at the supplier's side. On the other hand, if the distributer covers the shortage in inventory by ordering from the spot market, then this will be costly, especially when the product ordering time occurs in the high demand season. To overcome these two problems, learning from a previous observation is proposed to reduce these negative effects. For the lost sales model, updating the information is beneficial for the distributor since it can reduce the risk of losing customers. While for the spot market model, on one hand, random yield forecasting may not be beneficial for the distributor when considering that it pays for shortage quantity at the spot market price. On the other hand, forecasting may help the distributor to avoid ordering from the spot market or, if needed, order less. Consequently, the retailer benefits from not having shortages, especially at a high demand season that shows the retailer's loyalty to customers. Therefore, the retailer offers a quantity flexibility contract to the distributor in the case of ordering from the spot market so that both benefit from information updating and sharing.

3.2.2.3.1 Bayesian Updating-Lost Sale (BLS)

The model presented here is similar to the NBLS model but with the following difference. In BLS model, it is assumed that the distributor updates the random yield before ordering from the supplier. After ordering, the information would be shared with the retailer for making an accurate ordering decision considering the shared information. In section 3, the model and its solution are explained.

3.2.2.3.2 Quantity Flexibility Contract (QFC)

In this model, the distributor forecasts the supplier's random yield risk and decides on the ordering quantity from the supplier by the help of a yield risk observation to fill the retailer's order. Since information gathering along with payments to the spot market is costly for the distributor, the retailer offers a quantity flexibility contract to the distributor as an incentive of sharing updated supply risk information. The framework of the contract gives the retailer the flexibility to receive $q(1-d)$ and the distributor to commit product availability not to drop below $q(1-d)$ units and to supply up to q. If the distributor's shipment quantity exceeds the retailer's ordering quantity, then the distributor has unused items of a product. On the other hand, if the shipment quantity to the retailer falls below its shipment commitment, then the distributor has to supply the shortage quantity from the spot market with a price higher than the ordinary purchasing price. The performance of the supply chain parties are commonly affected by the parameters $w, C, C^s, d, and\ \theta$ where w is the transfer price between the retailer and the distributer, C and C^s are the distributer-supplier transfer price and the spot market cost, d is the flexibility level and θ is the unknown supply risk.

The model events are described as follows. First, the retailer orders q units from the distributor with a level of flexibility d. Then, the distributor receives an update θ_0 quantity on the yield risk distribution and the random yield distribution, which is updated from an observation. Given the updated information and the retailer's flexibility level, the distributor submits an order quantity of Q units to the supplier. After random risk realization, the supplier delivers a production quantity, $\theta + Q$, to the distributor. Then, the distributor shares supply risk information with the retailer. The retailer makes its actual purchase after demand realization and given supply random yield risk information.

The problem is solved for the general demand distribution and the random yield distribution with unknown parameter following the uniform distribution

3.3 Optimal Solutions:

3.3.1 Bayesian Lost sale model

The standard Stackelberg game is employed where the retailer is the dominant player. First, the expected cost of the distributor for the Bayesian lost sales model with random yield information is given as

$$\pi_D^{BLS} = w \int_{-Q}^{q-Q} (\theta + Q)\, \psi(\theta|\theta_0) d\theta - w \int_{q-Q}^{\infty} q\ \psi(\theta|\theta_0) d\theta - C \int_{-Q}^{\infty} (\theta + Q)\, \psi(\theta|\theta_0) d\theta \tag{8}$$

where $\psi(\theta|\theta_0)$ is given by Equation (1).

Lemma 6:

a) For the Bayesian lost sales model, the distributor's expected profit is concave in Q and has an optimal ordering quantity Q^*_{BLS} , which satisfies the following condition:

$$\frac{\int_{-Q}^{q-Q} \psi(\theta|\theta_0)d\theta}{\int_{-Q}^{\infty} \psi(\theta|\theta_0)d\theta} = \frac{C}{w} \tag{9}$$

Using Lemma 1, Equation (9) can be rewritten as:

$$\frac{F_1\left(\frac{q-Q-u_1}{U_1-u_1}\right) - F_1\left(\frac{-Q-u_1}{U_1-u_1}\right)}{F_1\left(\frac{-Q-u_1}{U_1-u_1}\right)} = \frac{C}{w} \tag{10}$$

b) $Q^*_{BLS} = q - B$, where B can be determined by C, w, and $F_1(.)$.

c) If we assume that $F_1\left(\frac{-Q-u_1}{U_1-u_1}\right) = 0$, then Q^*_{BLS} can be expressed as:

$$Q^*_{BLS} = q - u_1 - F_1^{-1}\left(\frac{C}{w}\right)(U_1 - u_1) \tag{11}$$

$Q^*_{BLS} = q - B$ is a linear function of q, where $B = u_1 + F_1^{-1}\left(\frac{C}{w}\right)(U_1 - u_1)$ and $f_1(t) = \frac{\alpha+1}{(\alpha+3)[max(1,1-t,t)]^{\alpha+2}}$. The distribution $F_1^{-1}\left(\frac{C}{w}\right)$depends on the value of $\frac{C}{w}$. It may be Uniform or Pareto from right or left.

Given the distributor's optimal ordering quantity for Bayesian lost sale model, the retailer's expected profit is:

$\pi_R^{BLS} = PE[\xi] - P\int_{q-Q^*_{BLS}}^{\infty}\int_q^{\infty}(\xi - q)\,\phi(\xi)\,\psi(\theta|\theta_0)\,d\xi\,d\theta - P\int_{-Q^*_{BLS}}^{q-Q^*_{BLS}}\int_{\theta+Q^*_{BLS}}^{\infty}\big(\xi -$
$(\theta + Q^*_{BLS})\big)\,\phi(\xi)\,\psi(\theta|\theta_0)\,d\xi\,d\theta -$
$h\int_{-Q^*_{BLS}}^{q-Q^*_{BLS}}\int_0^{\theta+Q^*_{BLS}}\big((\theta + Q^*_{BLS}) - \xi\big)\,\phi(\xi)\,\psi(\theta|\theta_0)\,d\xi\,d\theta - h\int_{q-Q^*_{BLS}}^{\infty}\int_0^q(q -$
$\xi)\,\phi(\xi)\,\psi(\theta|\theta_0)\,d\xi\,d\theta - g\int_{q-Q^*_{BLS}}^{\infty}\int_q^{\infty}(\xi - q)\,\phi(\xi)\,\psi(\theta|\theta_0)\,d\xi\,d\theta -$

$$g \int_{-Q_{BLS}^*}^{q-Q_{BLS}^*} \int_{\theta+Q_{BLS}^*}^{\infty} (\xi - (\theta + Q_{BLS}^*))\, \phi(\xi)\, \psi(\theta|\theta_0)\, d\xi\, d\theta - w \int_{-Q_{BLS}^*}^{q-Q_{BLS}^*} (\theta + Q_{BLS}^*)\, \psi(\theta|\theta_0) d\theta - w \int_{q-Q_{BLS}^*}^{\infty} q\ \psi(\theta|\theta_0) d\theta \quad (12)$$

Lemma 7:

The retailer's expected profit for the Bayesian lost sales model is concave in q given the updated information, whose solution is given as:

$$(p+g-w)\left[\int_{B-q}^{\infty} \Phi(q)\, \psi(\theta|\theta_0) d\theta\right] = (p+g+h)[\int_{B}^{\infty} \Phi(q)\, \psi(\theta|\theta_0) d\theta + \int_{B-q}^{B} \Phi(\theta+q-B)\, \psi(\theta|\theta_0) d\theta] + (p+g)[\int_{0}^{\infty} \xi\, \phi(\xi)\, \psi(B-q|\theta_0)\, d\xi\,] \quad (13)$$

The proof is presented in the Appendix.

3.3.2 Quantity Flexibility Contract (QFC)

First, we explore the distributor's problem. The distributor's expected profit function for a given retailer's order quantity is expressed as:

$$\pi_D^{QFC} = w \int_{-Q}^{q(1-d)-Q} q(1-d)\, \psi(\theta|\theta_0) d\theta + w \int_{q(1-d)-Q}^{q-Q} (\ \theta + Q) \psi(\theta|\theta_0) d\theta + w \int_{q-Q}^{\infty} q\ \psi(\theta|\theta_0) d\theta - C \int_{-Q}^{\infty} (\ \theta + Q) \psi(\theta|\theta_0) d\theta - C^s \int_{-Q}^{q(1-d)-Q} (q(1-d) - (\ \theta + Q))\, \psi(\theta|\theta_0) d\theta \quad (14)$$

Lemma 8:

Given an order quantity q with flexibility level d and initial random yield θ_0, the distributor's expected profit function is concave in Q and the optimal order quantity Q^* must satisfy the following condition:

$$(w - C^s) \int_{q(1-d)-Q}^{\infty} \psi(\theta|\theta_0) d\theta\ - w \int_{q-Q}^{\infty} \psi(\theta|\theta_0) d\theta\ + (w - C^s)[q(1-d)]\, \psi(-Q|\theta_0) = +(C - C^s) \int_{-Q}^{\infty} \psi(\theta|\theta_0) d\theta$$

(15)

Using Lemma (1), Equation (15) can also be expressed as:

$$(C^s - w)\left[F_1\left(\frac{q(1-d)-Q-u_1}{\mathcal{U}_1-u_1}\right)\right] + w\ \left[F_1\left(\frac{q-Q-u_1}{\mathcal{U}_1-u_1}\right)\right] - (w + C^s)\left[\left(\frac{q(1-d)}{\mathcal{U}_1-u_1}\right)f_1\left(\frac{-Q-u_1}{\mathcal{U}_1-u_1}\right)\right] =$$

$$(C^s - C)\left[\ F_1\left(\frac{-Q-u_1}{\mathcal{U}_1-u_1}\right)\right] + C \qquad (16)$$

The proof for lemma 8 is provided in the Appendix.

It is rather difficult to obtain an explicit expression for Q^* from Equation (13) for $d > 0$. Moreover, the random yield distribution structure is a mix of uniform and Pareto distribution, therefore, depending on the random yield probability density function the profit function form differs at each part of the random yield p.d.f. Thus, this would be difficult to get the explicit solution of Q^* so the model behaviour will be studied numerically.

Given the distributor's shipment quantity $\theta + Q^*_{QFC}$, the retailer's objective is to find the optimal ordering quantity q^* that maximizes the following expected profit function:

$$
\begin{aligned}
\pi_R^{QFC} = PE[\xi] &- P\int_{q-Q_{QFC}^*}^{\infty}\int_{q}^{\infty}(\xi - q)\,\phi(\xi)\,\psi(\theta|\theta_0)\,d\xi\,d\theta \\
&- P\int_{q(1-d)-Q_{QFC}^*}^{q-Q_{QFC}^*}\int_{\theta+Q_{QFC}^*}^{\infty}\left(\xi-(\theta+Q_{QFC}^*)\right)\,\phi(\xi)\,\psi(\theta|\theta_0)\,d\xi\,d\theta \\
&- P\int_{-Q_{QFC}^*}^{q(1-d)-Q_{QFC}^*}\int_{q(1-d)}^{\infty}(\xi - q(1-d))\,\phi(\xi)\,\psi(\theta|\theta_0)\,d\xi\,d\theta \\
&- h\int_{q(1-d)-Q_{QFC}^*}^{q-Q_{QFC}^*}\int_{0}^{\theta+Q_{QFC}^*}\left(\left(\theta+Q_{QFC}^*\right)-\xi\right)\,\phi(\xi)\,\psi(\theta|\theta_0)\,d\xi\,d\theta \\
&- h\int_{-Q_{QFC}^*}^{q(1-d)-Q_{QFC}^*}\int_{0}^{q(1-d)}(q(1-d)-\xi)\,\phi(\xi)\,\psi(\theta|\theta_0)\,d\xi\,d\theta \\
&- g\int_{q(1-d)-Q_{QFC}^*}^{q-Q_{QFC}^*}\int_{\theta+Q_{QFC}^*}^{\infty}\left(\xi-(\theta+Q_{QFC}^*)\right)\phi(\xi)\,\psi(\theta|\theta_0)\,d\xi\,d\theta \\
&- g\int_{q-Q_{QFC}^*}^{\infty}\int_{q}^{\infty}(\xi - q)\,\phi(\xi)\,\psi(\theta|\theta_0)\,d\xi\,d\theta \\
&- w\int_{-Q_{QFC}^*}^{q(1-d)-Q_{QFC}^*}q(1-d)\,\psi(\theta|\theta_0)d\theta - w\int_{q(1-d)-Q_{QFC}^*}^{q-Q_{QFC}^*}(\ \theta \\
&+ Q_{QFC}^*)\psi(\theta|\theta_0)d\theta - w\int_{q-Q_{QFC}^*}^{\infty}q\ \psi(\theta|\theta_0)d\theta
\end{aligned}
\tag{17}
$$

3.3.3 Optimal Solution Calculation

The following proposed algorithm calculates Bayesian optimal ordering quantities and profits for the supply chain parties. To solve the problem numerically, MATLAB 2010 software was used to find the optimal solution by following these steps:

Step 1: Set the input parameters at their values,

Step 2: Determine the appropriate ranges of Q, q and d,

Step 3: Update the random yield information,

Step 4: Find the optimal Q that maximizes Equation (14), the distributor's profit function, given the retailer's order quantity, q, and the flexibility level, d, Step 5: Given Q^*, solve Equation (17) to get the optimal q,

Step 6: Calculate the distributor's and the retailer's profit functions given Q^* and q^*.

Here, we need to mention that the program procedure is similar to the one in Wu (2005).

Proposition 1:

For fixed values of the model parameters, the distributor's optimal ordering quantity Q^* decreases in w, while that of the retailer increases.

When the transfer price increases, the distributor orders less from the supplier since the retailer may order less to avoid higher transfer cost that diminishes its profit. Although the retailer orders less, but the distributor's expected profit improves because of the higher selling price.

Proposition 2:

For fixed values of the model parameters, the distributer's optimal ordering quantity Q^* increases in the spot market price C^s while its profit decreases.

As the spot market price increases, the distributor's willingness to order from the spot market decreases. On the other hand, the distributor commits to fulfill the retailer's order by at least $q(1-d)$, which increases the distributor's order quantity from the supplier. The risk of random yield affects the distributor's profit as the larger the quantity ordered from the supplier the higher will be the yield risk. The retailer's profit is also affected by increasing the spot market price as a result of the distributor's risk sharing.

The proofs for the above propositions are provided in the Appendix.

3.4 Numerical Results

To examine the behavior of the model presented in this Chapter and to obtain some managerial implications and insights several numerical examples are solved with results discussed. These examples study the effect of information sharing between the distributor and the retailer and the impact of random yield updating on the profits and ordering decisions in a three-echelon supply chain is investigated. We assume that demand follows a uniform distribution in these numerical examples. The other input parameters of the model are presented in the following table:

Table 3.1. The values of input parameters.

α	C	C^S	W	h	g	P	ξ
3	1.6	2.4	3.3	1.2	2	8	$U{\sim}[10, 30]$

The next two sections provide comparisons of the numerical results for the cases of: (1) Non-Bayesian, and (2) Bayesian updating. These sections are followed with one that discusses the effect of information updating on the performance of the supply chain and a sensitivity analysis section.

3.4.1 Non- Bayesian Models Comparison

Tables 3.2 to 3.5 summarize the optimal ordering quantities and profits of the supply chain members for uniform demand and supply when there is no random yield updating for different h and g values, where h = 1.2 < g = 2 and h = g = 2 (Tables 3.2 and 3.3), and h = 1.2 < g = 2 when C^S = 2.4 and C^S = 1.6 (Tables 3.4 and 3.5). For both high and low holding cost, the distributor benefits more in the lost sales case since the retailer orders more to deal with shortage effect and consequently the distributor's profit increases. The retailer's profit is less than that of the spot market model since for the lost sale case the retailer orders more to overcome possible supply and demand mismatch. A comparison of the two models shows that the optimal order quantities of the supply chain members are higher for the lost sale model. This is because in the spot market model the parties avert risk yield by ordering from the secondary market. The results in Tables 3.4 and 3.5 show that when the spot market price, C^S, is lower, 1.6 <

2.4, the distributor orders less from the unreliable supplier since a shortage quantity can be filled by ordering from the spot market (for example when $(u, \mathcal{U}) = (6,10)$, Q^*_{NBSM} =13.2976 for $C^S = 2.4$ and $Q^*_{NBSM} = 11.9643$ for $C^S = 1.6$) .

Table 3.2 Non-Bayesian Lost Sale Model (NBLS) for $g = 2$ and $h = 1.2$

$(u, \mathcal{U})$	Q^*_{NBLS}	q^*_{NBLS}	π_R^{NBLS}	π_D^{NBLS}
(6, 10)	16.8524	24.5218	66.0093	40.5103
(2.5, 3.2)	21.1599	23.9993	65.7521	40.5103
(-9.5, -2.7)	31.1681	24.9650	65.4029	39.6382
(-5, -4.3)	28.6599	23.999	66.0093	40.5103

Table 3.3 Non-Bayesian Lost Sale Model (NBLS) for $g = 2$ and $h = 2$

$(u, \mathcal{U})$	Q^*_{NBLS}	q^*_{NBLS}	π_R^{NBLS}	π_D^{NBLS}
(6, 10)	15.6565	23.5959	63.1409	38.4645
(2.5, 3.2)	20.2340	23.0734	63.40618	38.9363
(-9.5, -2.7)	30.2422	24.0391	62.7834	38.0641
(-5, -4.3)	27.7340	23.0734	63.40618	38.9363

Table 3.4 Non-Bayesian Spot Market Model (NBSM) for $g = 2$ and $C^S = 2.4$

$(u, \mathcal{U})$	Q^*_{NBSM}	q^*_{NBSM}	π_R^{NBSM}	π_D^{NBSM}
(6, 10)	13.2976	21.9643	67.0804	36.2976
(2.5, 3.2)	18.9976	21.9643	67.0804	37.1526
(-9.5, -2.7)	26.9310	21.9643	67.0804	35.5260
(-5, -4.3)	26.4976	21.9643	67.0804	37.1526

Table 3.5 Non-Bayesian Spot Market Model (NBSM) for $g = 2$ and $C^S = 1.6$

$(u, \mathcal{U})$	Q^*_{NBSM}	q^*_{NBSM}	π_R^{NBSM}	π_D^{NBSM}
(6, 10)	11.9643	21.9643	67.0804	37.3393
(2.5, 3.2)	18.7643	21.9643	67.0804	37.3393
(-9.5, -2.7)	24.6643	21.9643	67.0804	37.3393
(-5, -4.3)	26.2643	21.9643	67.0804	37.3393

3.4.2 Bayesian Updating Model Comparison

In this section, we compare the results of Bayesian updating of the quantity flexibility contract model and that of Bayesian lost sales model. For both contracts, the risk of random yield and updating information is shared between the supply chain members. Table 3.6 presents the optimal ordering policies that maximize the profits of the retailer and the distributors for the Bayesian lost sales model. The optimal order policies under quantity flexibility contract are shown in Tables 3.7 to 3.12 for different observations and ranges of yield risk. It can be seen that under the quantity flexibility contract the retailer and the distributor benefit (have higher profits) in comparison to the Bayesian lost sales model. When there is no quantity flexibility ($d = 0$), the retailer registers its highest profit for any yield risk range, $\theta_0 =$ 8, 3, -11, and -5 in Tables 3.7-3.10 respectively, the distributor registers its lowest profit even lower than its profit form the lost sale model. The results show that the quantity flexibility contract is more beneficial to the distributor than lost sale when the flexibility level, d, is not very low. The results in Tables 3.7 to 3.12 show that the distributor's profit increases when d increases. This may suggest that the quantity flexibility contract is a profitable one for both members.

Under the Bayesian lost sales model, both the distributor and the retailer register their highest profits when observation keeps its uniform behavior, meaning that observed yield risk is $u_0 \leq \theta_0 \leq U_0$. On the other hand, the retailer's profit was shown to improve slowly when there is a Pareto movement from the left side. It can also be seen that as the yield risk increases, the optimal ordering quantity also increases.

Table 3.6 Bayesian Lost Sale Model (BLS) for $g = 2$.

$(u_0, \mathcal{U}_0)$	θ_0	Q^*_{BLS}	q^*_{BLS}	π_R^{BLS}	π_D^{BLS}
(6,10)	3	17.1	23.4	65.6689	34.6618
	8	14.9	22.8	66.5604	35.8305
	14	13.7	23.5	65.3014	34.1060
(2.5, 3.2)	1	20.4	22.5	66.9067	36.6368
	3	19.3	22.1	67.0645	37.0552
	5	18.8	22.5	66.8662	36.4183
(-9.5,-2.7)	-11	30.6	23.6	65.1637	34.0585
	-5	29.6	23.3	65.74	34.6372
	0	28.8	23.8	64.7407	33.5308
(-5,-4.3)	-7	28.2	22.5	66.8308	36.2718
	-5	26.8	22.1	67.0645	37.0552
	-2	25.4	21.8	68.7680	36.2219

Table 3.7 Quantity Flexibility Contract (QFC) for $g = 2$ when $\theta_0 = 8$ and $(u_0, \mathcal{U}_0) =$ (6,10).

d	Q^*_{QFC}	q^*_{QFC}	π_R^{QFC}	π_D^{QFC}
0	12.3	21.3	70.7721	34.2744
0.1	15	23.1	69.8809	36.4991
0.2	15.5	23.4	69.5080	36.8890
0.3	15.6	23.5	69.3765	37.0348
0.4	15.6	23.5	69.3184	37.0268
0.5	15.6	23.5	69.2877	37.0234
0.6	15.6	23.5	69.2699	37.0218
0.7	15.6	23.5	69.2590	37.0210
0.8	15.5	23.5	69.2522	37.0206

Table 3.8 Quantity Flexibility Contract (QFC) for $g = 2$ when θ_0= 3 and ($u_0, \mathcal{U}_0$) = (6,10).

d	Q^*_{QFC}	q^*_{QFC}	π_R^{QFC}	π_D^{QFC}
0	13.1	21.3	70.8824	32.8081
0.1	15.5	22.9	70.5338	34.4706
0.2	17.1	23.7	69.5954	35.3654
0.3	17.6	24	69.1873	35.7600
0.4	17.7	24.1	68.9294	35.8880
0.5	17.7	24.1	68.7759	35.8691
0.6	17.7	24.1	68.6786	35.8596
0.7	17.8	24.1	68.6918	35.8548

Table 3.9 Quantity Flexibility Contract (QFC) for $g = 2$ when θ_0= -11 and ($u_0, \mathcal{U}_0$) = (-9.5, -2.7).

d	Q^*_{QFC}	q^*_{QFC}	π_R^{QFC}	π_D^{QFC}
0	26.1	21.3	70.9923	32.1642
0.1	28.6	22.9	70.7380	33.7841
0.2	30.4	23.8	69.7698	34.7048
0.3	31	24.1	69.1352	35.0350
0.4	31.2	24.2	69.8076	35.1381
0.5	31.3	24.3	68.5720	35.2772
0.6	31.4	24.4	68.4183	35.4315
0.7	31.4	24.4	68.3168	35.4234

Table 3.10 Quantity Flexibility Contract (QFC) for $g = 2$ when θ_0= = -5 and ($u_0, \mathcal{U}_0$) = (-9.5, -2.7).

d	Q^*_{QFC}	q^*_{QFC}	π_R^{QFC}	π_D^{QFC}
0	25.7	21.3	70.8704	32.9067
0.1	28.2	22.9	70.5050	34.5773
0.2	29.7	23.7	69.5698	35.4958
0.3	30.1	23.9	69.1950	35.7298
0.4	30.2	24	68.9546	35.8608
0.5	30.3	24.1	68.8126	36.0130
0.6	30.3	24.1	68.7235	36.0044
0.7	30.3	24.1	68.6657	35.9999

Table 3.11 Quantity Flexibility Contract (QFC) for $g = 2$ when θ_0= 0 and ($u_0, \mathcal{U}_0$) = (-9.5, -2.7).

d	Q^*_{QFC}	q^*_{QFC}	π_R^{QFC}	π_D^{QFC}
0	23.6	21.2	71.1427	31.3923
0.1	26.1	22.8	70.9238	329920
0.2	28	23.8	70.0128	33.9747
0.3	28.9	24.2	69.1104	34.3939
0.4	29.3	24.4	68.6874	34.6376
0.5	29.4	24.5	68.3583	34.7632
0.6	29.5	24.6	68.1386	34.9100
0.7	29.5	24.5	67.9909	34.8981

Table 3.12 Quantity Flexibility Contract (QFC) for $g = 2$ when θ_0= -5 and $(\mathcal{u}_0, \mathcal{U}_0)$ = (-5, -4.3).

d	Q^*_{QFC}	q^*_{QFC}	π_R^{QFC}	π_D^{QFC}
0	25.9	21.4	70.7609	36.0409
0.1	27.5	22.8	69.6219	38.2459
0.2	27.5	22.8	69.6182	38.2453
0.3	27.5	22.8	69.6177	38.2453
0.4	27.5	22.8	69.6176	38.2453
0.5	27.5	22.8	69.6175	38.2452
0.6	27.5	22.8	69.6175	38.2452
0.7	27.5	22.8	69.6174	38.2452

3.4.3 The effect of Updating

In this section, the results from the Bayesian and Non-Bayesian models are compared to investigate the effect of updating on the performance of the supply chain members.

First, we compare the Non-Bayesian and Bayesian lost sales models. By updating, it was found that the retailer was the only member who benefits from updating when the variance of random yield is low. Moreover, without observation, both members (the distributor and the retailer) order larger quantities to deal with the negative effect of lost sales on their profit, while updating helps them to order more accurately.

By comparing the results obtained from the spot market and the quantity flexibility contract, it was shown that although the retailer benefits under all conditions, the distributer's profit was shown to increase when the observation lies within the range of random variable where a uniform movement happens.

Consider the following numerical example to illustrate how updating affect ordering decision. For $(\mathcal{u}_0, \mathcal{U}_0)$ = (6, 10), the optimal ordering quantity of the retailer is 24.52 for the Non-Bayesian lost sales model, from Table 3.2. So, the distributor in its turn orders

$Q^*_{NBLS} = 16.58$ given the expected random yield $E(\theta) = \frac{6+10}{2} = 8$. On the other hand, for the Bayesian lost sales model, the optimal order quantities varies with respect to the observation. For example, when ($\mathcal{u}_0, \mathcal{U}_0$) = (6, 10), the order quantity differs for different observations, the observation may be on the uniform area or Pareto one. For instance, for $\theta_0 = 8$, given the retailer's order quantity, the distributor's optimal order quantity is $Q^*_{BLS} = 14.9$ and given Q^*_{BLS}, the retailer's optimal ordering quantity is $q^*_{BLS} = Q^*_{BLS} + E(\theta|\theta_0) = 14.9 + (\frac{6+10}{2}) = 22$ .8, while the retailer's and the distributor's order quantities for $\theta_0 = 3$ are 17.1 and 23.4, respectively. For $\theta_0 = 3$ and $\theta_0 = 14$, the calculations are as follows:

Calculated $q^*_{BLS} = Q^*_{BLS} + E(\theta|\theta_0) = 17.1 + \left(\frac{3+10}{2}\right) = 23.6 \approx q^*_{BLS} = 23.5$

Calculated $q^*_{BLS} = Q^*_{BLS} + E(\theta|\theta_0) = 13.7 + \left(\frac{6+14}{2}\right) = 23.7 \approx q^*_{BLS} = 23.5$

The examples presented above indicate how the calculated q^*_{BLS} is determined by the distributor to fill the retailer's order quantity, q^*_{BLS}.

3.4.4 Sensitivity analysis

To investigate the effects of the model parameters on the optimal ordering quantities and the profits of the supply chain member, we vary the values of specific parameters while keeping the values of the other parameters fixed at their values. The results are shown in Tables 3.8, while Figures 3.1, 3.2 and 3.3 relate to Table 3.7. These tables illustrate the effect of increasing the flexibility level on the profits of both parties and ordering decisions. For fixed parameters, the distributor's expected profit increases in d while the retailer's profit decreases. The effect of increasing the spot market price is worse off for both retailer and distributor. As the retailer's flexibility level increases, the distributor's ordering quantity and profit increase. This can be explained as follows. For the retailer, a larger flexibility causes an increase in the retailer's and distributor's order quantities, which also increases profits. Moreover, when the flexibility level is high, the distributor shares more supply risk with the retailer. On the other hand, more flexibility allows the distributor to ship less quantity to the retailer, so the retailer's profit drops because it incurs a shortage cost. The numerical results in a later section show that for small values of the flexibility parameter d is profitable for both the retailer and the distributor, while larger values diminishes the random yield risk by sharing the risk with the retailer.

Increasing in *w* is also beneficial for the distributor while it is worse off for the retailer. This result is consistent with the result obtained in proposition 1.

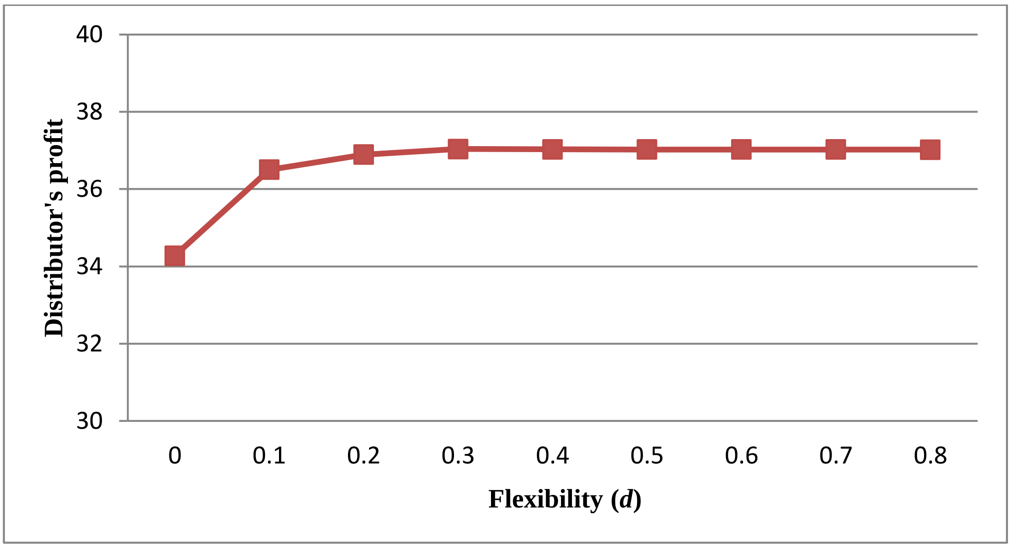

Figure 3.1 The effect of flexibility on the distributor's profit.

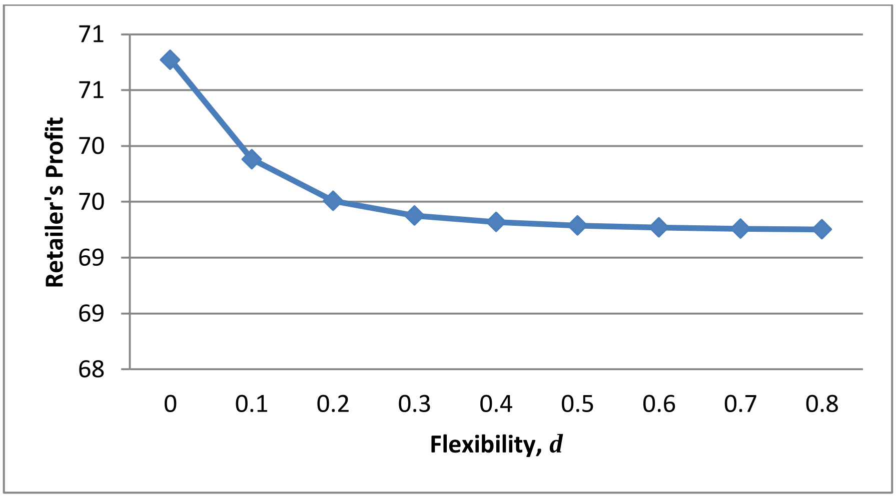

Figure 3.2 The effect of flexibility on the retailer's profit.

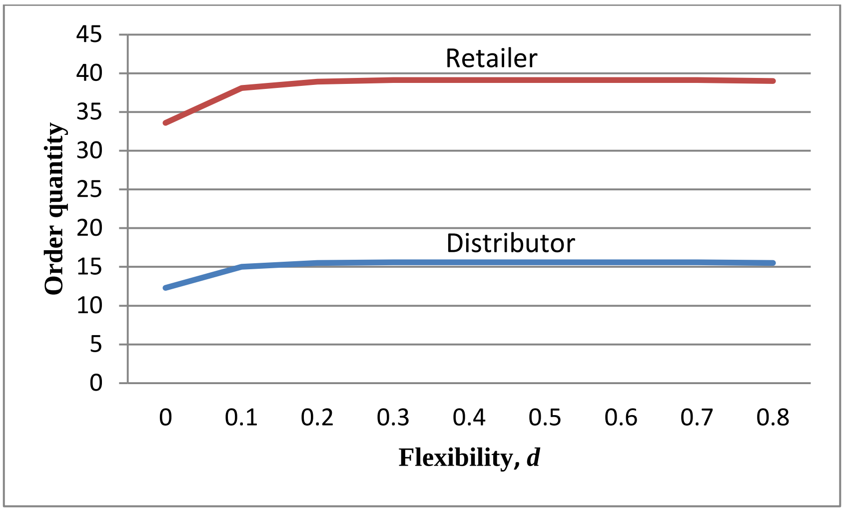

Figure 3.3 The effect of flexibility on the retailer and the distributor's order quantities

3.5 Concluding Remarks

In this Chapter, we investigated the ordering policies for a three-echelon supply chain consisting of a supplier, a distributor and a retailer. The distributor, the middle member of the supply chain, was assumed to face a random yield from the supplier for two scenarios Bayesian and Non-Bayesian. Under additive supply yield risk, the distributor's optimal ordering quantity from the supplier and the retailer's optimal ordering quantity from the distributor are derived. We supposed that the yield risk is unknown and the distributor forecasts the yield risk by using the information from a previous observation. The lost sales model and the spot market model were proposed for the Non-Bayesian case. Moreover, we extended these two contracts for the Bayesian case. We also considered the case of the retailer offering a quantity flexibility contract to the distributor as an incentive of sharing its supply risk information where the distributor incurs spot market cost. The framework of the contract is such that the retailer is flexible to receive quantities less than what it orders given a flexibility level and the distributor commits product availability at least equal to the retailer's flexibility threshold quantity and up to its order quantity. If the distributor's shipment quantity exceeds the retailer's ordering quantity, then the distributor would have unused quantity. On the other hand, if the shipment quantity to the retailer falls below its shipment commitment, then the distributor has to supply the shortage quantity from the spot market at a price higher than the ordinary purchasing price. In addition, to decrease the negative effect of lost sales, updating the distributor supply availability was proposed for this model as well. For some models, analytical results were presented whereas, for Bayesian updating models, the shape of the unknown parameters led to proposing an algorithm to find the optimal solution.

The results obtained would help managers when deciding which type of contract would best work for them. We investigated the effects of random yield updating on the performance of the supply chain members. It was also seen from the results that under the lost sales model only the retailer benefits from information updating when the variance of yield risk is low. Additionally, the most powerful contract was shown to be the quantity flexibility contract for the retailer for any flexibility level. This result is true for the distributor when the flexibility level is not very low.

Apart from the discussed results, sensitivity analysis was performed to observe the effect of changes in the spot market price, transfer price and flexibility level on the performance of the supply chain members for quantity flexibility contract. It was shown, however, that the more the flexibility is, the more the profit for the distributor would be, but the less it would be for the retailer. A future extension would be to investigate demand updating along with the supply risk updating. Additionally, for multi-supplier problems, it would be interesting to investigate the effect of monitoring one supplier's shipment on the ordering from the other supplier.

Appendix:

Proof of Lemma 5:

$$\frac{\partial \Pi_R^{NBLS}}{\partial q} = P\int_A^{\infty}\int_q^{\infty} \phi(\xi)\,\psi(\theta)\,d\xi\,d\theta + P\int_{-\infty}^{A}\int_{\theta+q-A}^{\infty} \phi(\xi)\,\psi(\theta)\,d\xi\,d\theta$$
$$- h\int_{-\infty}^{A}\int_{0}^{\theta+q-A} \phi(\xi)\,\psi(\theta)d\xi\,d\theta - h\int_A^{\infty}\int_0^{q} \phi(\xi)\,\psi(\theta)\,d\xi\,d\theta$$
$$+ g\int_A^{\infty}\int_q^{\infty} \phi(\xi)\,\psi(\theta)\,d\xi\,d\theta + g\int_{-\infty}^{A}\int_{\theta+q-A}^{\infty} \phi(\xi)\,\psi(\theta)\,d\xi\,d\theta$$
$$- w\int_{-\infty}^{A}\psi(\theta)\,d\theta - w\int_A^{\infty}\psi(\theta)\,d\theta$$

$$\frac{\partial \Pi_R^{NBLS}}{\partial q} = (p+g-w) - (p+g+h)\left[\Phi(q)\overline{\Psi}(A) + \int_{-\infty}^{A}\Phi(\theta+q-A)\,\psi(\theta)d\theta\right] = 0$$

$$[\Phi(q)\overline{\Psi}(A)] + \int_{-\infty}^{A}\Phi(\theta+q-A)\,\psi(\theta)d\theta = \frac{p+g-w}{p+g+h}$$

$$\frac{\partial^2 \Pi_R^{NBLS}}{\partial q^2} = -(P+g+h)\left[\phi(q)\overline{\Psi}(A) + \int_{-\infty}^{A}\phi(\theta+q-A)\,\psi(\theta)d\theta\right] < 0$$

Proof of Lemma 6:

a) π_D^{BLS} can be rewritten as:

$$\frac{\partial \pi_D^{BLS}}{\partial Q} = -C\int_{-Q}^{\infty}\psi(\theta|\theta_0)d\theta \ + w\int_{-Q}^{q-Q}\psi(\theta|\theta_0)d\theta = 0$$

$$\frac{\int_{-Q}^{q-Q}\psi(\theta|\theta_0)d\theta}{\int_{-Q}^{\infty}\psi(\theta|\theta_0)d\theta} = \frac{C}{w}$$

Using Lemma 1, the optimal ordering quantity can be calculated as follows:

$$\pi_D^{BLS} = w\int_{-Q}^{\infty}(\theta+Q)\,\psi(\theta|\theta_0)d\theta \;-\; w\int_{q-Q}^{\infty}(\theta+Q-q)\;\psi(\theta|\theta_0)d\theta$$

$$-\,C\int_{-Q}^{\infty}(\theta+Q)\;\psi(\theta|\theta_0)d\theta$$

$$=\;-w(\mathcal{U}_1-u_1)\int_{\frac{q-Q-u_1}{\mathcal{U}_1-u_1}}^{\infty}\left[t-\frac{q-Q-u_1}{\mathcal{U}_1-u_1}\right]f_1(t)dt$$

$$+\,w(\mathcal{U}_1-u_1)\int_{\frac{-Q-u_1}{\mathcal{U}_1-u_1}}^{\infty}\left[t-\frac{-Q-u_1}{\mathcal{U}_1-u_1}\right]f_1(t)dt\;-C(\mathcal{U}_1$$

$$-\,u_1)\int_{\frac{-Q-u_1}{\mathcal{U}_1-u_1}}^{\infty}\left[t-\frac{-Q-u_1}{\mathcal{U}_1-u_1}\right]f_1(t)dt$$

$$\pi_D^{BLS} = -w\int_{\frac{q-Q-u_1}{\mathcal{U}_1-u_1}}^{\infty}f_1(t)dt + w\int_{\frac{-Q-u_1}{\mathcal{U}_1-u_1}}^{\infty}f_1(t)dt - C\int_{\frac{-Q-u_1}{\mathcal{U}_1-u_1}}^{\infty}f_1(t)dt$$

$$\frac{\partial\pi_D^{BLS}}{\partial Q}=0$$

$$\frac{\int_{\frac{q-Q-u_1}{\mathcal{U}_1-u_1}}^{\infty}f_1(t)dt-\int_{\frac{-Q-u_1}{\mathcal{U}_1-u_1}}^{\infty}f_1(t)dt}{\int_{\frac{-Q-u_1}{\mathcal{U}_1-u_1}}^{\infty}f_1(t)dt}=\frac{C}{w}$$

$$-w\left[1-F_1\left(\frac{q-Q-u_1}{\mathcal{U}_1-u_1}\right)\right]+(w-C)\left[1-F_1\left(\frac{-Q-u_1}{\mathcal{U}_1-u_1}\right)\right]=0$$

$$w\left[F_1\left(\frac{q-Q-u_1}{\mathcal{U}_1-u_1}\right)-F_1\left(\frac{-Q-u_1}{\mathcal{U}_1-u_1}\right)\right]=C.\left[1-F_1\left(\frac{-Q-u_1}{\mathcal{U}_1-u_1}\right)\right]$$

$$\frac{\partial^2\pi_D^{BLS}}{\partial Q^2}=\;-w.f_1\left(\frac{q-Q-u_1}{\mathcal{U}_1-u_1}\right)+\;(w-C).f_1\left(\frac{-Q-u_1}{\mathcal{U}_1-u_1}\right)<0$$

b) Assume that $k=q-Q$. We need to show that $R(k)=w\left(\int_{k-q}^{k}\psi(\theta|\theta_0)d\theta\right)-C\left(\int_{k-q}^{\infty}\psi(\theta|\theta_0)d\theta\right)$ is increasing function of k.

$$\frac{\partial R(k)}{\partial k}=w\,\psi(k|\theta_0)+(C-w)\psi(k-q|\theta_0)>0$$

c) The proof follows from a).

$$w\left[\ F_1\left(\frac{q-Q-\mathcal{u}_1}{\mathcal{U}_1-\mathcal{u}_1}\right)\right]=C$$

$$Q_{BLS}^{*}=q-\mathcal{u}_1-F_1^{-1}\left(\frac{C}{w}\right)(\mathcal{U}_1-\mathcal{u}_1)$$

Proof of Lemma 7:

$$\frac{\partial\Pi_R^{BLS}}{\partial q}=P\int_B^\infty\int_q^\infty\phi(\xi)\,\psi(\theta|\theta_0)\,d\xi\,d\theta+P\int_{-q+B}^{B}\int_{\theta+q-B}^{\infty}\phi(\xi)\,\psi(\theta|\theta_0)\,d\xi\,d\theta$$
$$-P\int_0^\infty\xi\,\phi(\xi)\,\psi(B-q|\theta_0)\,d\xi$$
$$-h\int_{-q+B}^{B}\int_0^{\theta+q-B}\phi(\xi)\,\psi(\theta|\theta_0)d\xi\,d\theta$$
$$-h\int_B^\infty\int_0^q\phi(\xi)\,\psi(\theta|\theta_0)\,d\xi\,d\theta+g\int_B^\infty\int_q^\infty\phi(\xi)\,\psi(\theta|\theta_0)\,d\xi\,d\theta$$
$$+g\int_{-q+B}^{B}\int_{\theta+q-B}^{\infty}\phi(\xi)\,\psi(\theta|\theta_0)\,d\xi\,d\theta-g\int_0^\infty\xi\,\phi(\xi)\,\psi(B-q|\theta_0)\,d\xi$$
$$-w\int_{-q+B}^{B}\psi(\theta|\theta_0)\,d\theta-w\int_B^\infty\psi(\theta|\theta_0)\,d\theta$$

$$\frac{\partial\Pi_R^{BLS}}{\partial q}=-(P+g+h)[\int_B^\infty\Phi(q)\psi(\theta|\theta_0)d\theta+\int_{B-q}^{B}\Phi(\theta+q-B)\psi(\theta|\theta_0)d\theta\,]$$
$$+(p+g-w)\left[\int_{B-q}^{\infty}\psi(\theta|\theta_0)\,d\theta\right]-(g$$
$$+p)[\int_0^\infty\xi\,\phi(\xi)\,\psi(B-q|\theta_0)\,d\xi\,]$$

$$\frac{\partial^2\Pi_R^{BLS}}{\partial q^2}=-(P+g+h)[\int_B^\infty\phi(q)\,\psi(\theta|\theta_0)\,d\theta$$
$$+\int_{B-q}^{B}\phi(\theta+q-B)\,\psi(\theta|\theta_0)\,d\theta]+(p+g-w)\,\psi(B-q|\theta_0)<0$$

Proof of Lemma 8:

We can rewrite π_D^{QFC} as follows:

$$\pi_D^{QFC} = w\int_{q(1-d)-Q}^{\infty}[\theta + Q - q(1-d)]\,\psi(\theta|\theta_0)d\theta$$
$$- w\int_{q-Q}^{\infty}[\theta + Q - q]\,\psi(\theta|\theta_0)d\theta$$
$$+ (w - C^s)\int_{-Q}^{\infty}[q(1-d)]\,\psi(\theta|\theta_0)d\theta$$
$$+ (C^s - C)\int_{-Q}^{\infty}[\theta + Q]\,\psi(\theta|\theta_0)d\theta$$
$$+ C^s\int_{q(1-d)-Q}^{\infty}[\theta + Q - q(1-d)]\,\psi(\theta|\theta_0)d\theta$$

$$\frac{\partial\pi_D^{QFC}}{\partial Q} = (w - C^s)\int_{q(1-d)-Q}^{\infty}\psi(\theta|\theta_0)d\theta \; - w\int_{q-Q}^{\infty}\psi(\theta|\theta_0)d\theta$$
$$+ (C^s - C)\int_{-Q}^{\infty}\psi(\theta|\theta_0)d\theta \; + (w - C^s)[q(1-d)]\,\psi(-Q|\theta_0)$$

$$\frac{\frac{\partial\pi_D^{QFC}}{\partial Q}}{\partial Q} = \frac{\partial^2\pi_D^{QFC}}{\partial Q^2}$$
$$= -[(w - C^s).(q(1-d))]\psi'(-Q|\theta_0) - w\,\psi(q - Q|\theta_0)$$
$$+ (C^s - C)\,\psi(-Q|\theta_0) + (w - C^s)\,\psi(q(1-d) - Q|\theta_0) < 0$$

Using Lemma 1, the optimal ordering quantity can be calculated as below:

$$\pi_D^{QFC} = w(\mathcal{U}_1 - u_1)\int_{\frac{q(1-d)-Q-u_1}{\mathcal{U}_1-u_1}}^{\infty}\left[t - \frac{q(1-d)-Q-u_1}{\mathcal{U}_1 - u_1}\right] f_1(t)dt$$
$$- w(\mathcal{U}_1 - u_1)\int_{\frac{q-Q-u_1}{\mathcal{U}_1-u_1}}^{\infty}\left[t - \frac{q-Q-u_1}{\mathcal{U}_1 - u_1}\right] f_1(t)dt$$
$$+ (C^s - C)(\mathcal{U}_1 - u_1)\int_{\frac{-Q-u_1}{\mathcal{U}_1-u_1}}^{\infty}\left[t - \frac{-Q-u_1}{\mathcal{U}_1 - u_1}\right] f_1(t)dt$$
$$+ (w - C^s)(\mathcal{U}_1 - u_1)\int_{\frac{-Q-u_1}{\mathcal{U}_1-u_1}}^{\infty}\frac{q(1-d)}{\mathcal{U}_1 - u_1} f_1(t)dt$$
$$- (C^s)(\mathcal{U}_1 - u_1)\int_{\frac{q(1-d)-Q-u_1}{\mathcal{U}_1-u_1}}^{\infty}\left[t - \frac{q(1-d)-Q-u_1}{\mathcal{U}_1 - u_1}\right] f_1(t)dt$$

$$\frac{\partial \pi_D^{QFC}}{\partial Q} = (C^s - w)[F_1\left(\frac{q(1-d)-Q-u_1}{\mathcal{U}_1 - u_1}\right)] + w[F_1\left(\frac{q-Q-u_1}{\mathcal{U}_1 - u_1}\right)] - (C^s$$
$$- C)\left[F_1\left(\frac{-Q-u_1}{\mathcal{U}_1 - u_1}\right)\right] + (w + C^s)[(\frac{q(1-d)}{\mathcal{U}_1 - u_1})f_1\left(\frac{-Q-u_1}{\mathcal{U}_1 - u_1}\right)] - C$$

$$\frac{\partial^2 \pi_D^{QFC}}{\partial Q^2} = w\left[f_1\left(\frac{q(1-d)-Q-u_1}{\mathcal{U}_1 - u_1}\right) - f_1\left(\frac{q-Q-u_1}{\mathcal{U}_1 - u_1}\right)\right] - C\left[[f_1\left(\frac{-Q-u_1}{\mathcal{U}_1 - u_1}\right)\right]$$
$$- C^s\left[f_1\left(\frac{q(1-d)-Q-u_1}{\mathcal{U}_1 - u_1}\right) - f_1\left(\frac{-Q-u_1}{\mathcal{U}_1 - u_1}\right)\right]$$
$$- (w + C^s)\left[(\frac{q(1-d)}{\mathcal{U}_1 - u_1}) f_1'\left(\frac{-Q-u_1}{\mathcal{U}_1 - u_1}\right)\right] < 0$$

Since $\frac{\partial^2 \Pi_D^{QFC}}{\partial Q^2}$ is < 0 then the distributer's profit function is concave in Q.

Proof of Proposition 1:

$$\frac{\partial Q^*_{QFC}}{\partial w} = \frac{\left(\frac{\frac{\partial \pi_D^{QFC}}{\partial Q}}{\partial w}\right)}{\left(\frac{\frac{\partial \pi_D^{QFC}}{\partial Q}}{\partial Q}\right)}$$

$$\frac{\frac{\partial \pi_D^{QFC}}{\partial Q}}{\partial w} = q(1-d)\psi(-Q|\theta_0) + \int_{q(1-d)-Q}^{q-Q} \psi(\theta|\theta_0)d\theta \; > 0$$

$$\frac{\partial Q_{QFC}^*}{\partial w} = \frac{\left(\frac{\frac{\partial \pi_D^{QFC}}{\partial Q}}{\partial w}\right)}{\left(\frac{\frac{\partial \pi_D^{QFC}}{\partial Q}}{\partial Q}\right)} = \frac{>0}{<0} = <0$$

$$\frac{\partial q_{QFC}^*}{\partial w} = \frac{\frac{\partial \pi_R^{QFC}}{\partial w}}{\frac{\partial \pi_R^{QFC}}{\partial q_{QFC}^*}} = \frac{\left(\frac{\partial \pi_R^{QFC}}{\partial Q_{QFC}^*}\right).\left(\frac{\partial Q_{QFC}^*}{\partial w}\right)}{\left(\frac{\partial \pi_R^{QFC}}{\partial Q_{QFC}^*}\right).\left(\frac{\partial Q_{QFC}^*}{\partial q}\right)}$$

$$\begin{aligned}\frac{\partial \pi_R^{QFC}}{\partial Q_{QFC}^*} = -P\int_{q(1-d)}^{\infty}&(\xi - q(1-d))\,\phi(\xi)\,\psi\left(-Q_{QFC}^*\middle|\theta_0\right)d\xi \\ &+\; h\int_0^q (q-\xi)\,\phi(\xi)\,\psi\left(q-Q_{QFC}^*\middle|\theta_0\right)d\xi \\ &- h\int_0^{q(1-d)} (q(1-d)-\xi)\,\phi(\xi)\,\psi\left(-Q_{QFC}^*\middle|\theta_0\right)d\xi \\ &- g\;\int_{q(1-d)}^{\infty} (\xi - q(1-d))\,\phi(\xi)\,\psi\left(q(1-d)-Q_{QFC}^*\middle|\theta_0\right)d\xi - w.q(1 \\ &- d)).\psi\left(-Q_{QFC}^*\middle|\theta_0\right) - w\int_{q(1-d)-Q_{QFC}^*}^{q-Q_{QFC}^*} \psi(\theta|\theta_0)d\theta \;\; < 0\end{aligned}$$

$$\begin{aligned}\frac{\partial Q_{QFC}^*}{\partial q} = w.(1-d)).\psi&\left(-Q_{QFC}^*\middle|\theta_0\right) + w.\psi\left(q-Q_{QFC}^*\middle|\theta_0\right) + w.(1 \\ &- d)).\psi\left(q(1-d)-Q_{QFC}^*\middle|\theta_0\right) + C^s.(1-d)).\psi\left(q(1-d)-Q_{QFC}^*\middle|\theta_0\right) \\ &- C^s.(1-d)\,.\psi\left(-Q_{QFC}^*\middle|\theta_0\right) \\ &= \;(1-d).\psi\left(q(1-d)-Q_{QFC}^*\middle|\theta_0\right).[w+C^s] + (1 \\ &- d)\,.\psi\left(-Q_{QFC}^*\middle|\theta_0\right).[w-C^s] + w.\psi\left(q-Q_{QFC}^*\middle|\theta_0\right) \; > 0\end{aligned}$$

$$\frac{\partial q^*_{QFC}}{\partial w} = \frac{(< 0).(> 0)}{(< 0).(> 0)} = > 0$$

Proof of Proposition 2:

$$\frac{\frac{\partial \pi_D^{QFC}}{\partial Q}}{\partial C^s} = \int_{-Q}^{q(1-d)-Q} \psi(\theta|\theta_0)d\theta - q(1-d).\psi(-Q|\theta_0) < 0$$

$$\frac{\partial Q^*_{QFC}}{\partial C^s} = \frac{\left(\frac{\frac{\partial \pi_D^{QFC}}{\partial Q}}{\partial C^s}\right)}{\left(\frac{\frac{\partial \pi_D^{QFC}}{\partial Q}}{\partial Q}\right)} = \frac{< 0}{< 0} => 0$$

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4. TRUST IN SUPPLY FORECAST INFORMATION SHARING

In this Chapter, we investigate the role of trust in supply forecast signaling in a supply chain with a supplier and a manufacturer in a one-shot game. The supplier faces with random yield uncertainty. The uncertainty is multiplied by the manufacturer's order quantity. The supplier has private forecast on yield risk. It decides whether share it truthfully or not. On the other hand, the manufacturer faces with two strategies in ordering: if it trusts the supplier's report then it updates its belief on the yield risk providing a forecast signal by the supplier. Otherwise, it orders based on its prior belief. Analytically we obtained the optimal order quantity. Intuitive result indicates that the supplier has a tendency to deviate from reporting true forecast information. Numerical results presented in this Chapter also supports intuitive conclusion.

4.1 Introduction

One of the most important areas in supply chain management is forecast information sharing (Ozer et.al (2011)). On the other hand, information asymmetry is one of the sources of inefficiency in supply chain (Ren et.al 2010). Moreover, since forecast communication is costless, it creates an incentive to informed party for information distortion that affects uninformed parties' decision. (Gumus, 2014 and Ozer et.al, 2011). Researchers have proposed different types of coordination contracts to enhance supply chain performance and information sharing (Tsay and Agrawal, 2004).

On the other hand, when there is information asymmetry in the supply chain, coordination can be obtained through either signaling contract or screening contract. The type of contract depends on who offers it.

Information sharing has been widely used from demand side to achieve coordination in supply chain. In most studies related to forecast signaling, demand forecast signal has been in the center of attention while supply forecast signal sharing cannot be neglected as retailer/manufacturer's order quantity is dependent to the supplier's availability in terms of quantity. Forecast information sharing can be a special supply chain activity where trust and social characteristics are important (Ebrahimi nakhjiri et.al (2011)).

In this chapter, we analyze random yield forecast-sharing in one-shot game between a supplier and a manufacturer under information asymmetry. We assume that the manufacturer faces two strategies about forecast signal, trust or not to trust. In this model, we define trust as the manufacturer's willingness to rely on the supplier's forecast report to determine order quantity. When the manufacturer trusts the supplier, it believes the report with certainty and places it order after updating its belief about yield risk providing a signal by the supplier. On the other hand, when the manufacturer does not trust the supplier's report, it orders based on its prior knowledge on random yield value. It is assumed that random yield has multiplicative form.

We would like to investigate whether the manufacturer should trust the supplier's forecast signal. What is the best ordering decision for the manufacturer when the supplier shares its forecast signal, Trust or Not to Trust?

Here, we refrain to review papers related to the effect of information sharing in supply chain as they have been reviewed in the previous Chapter, Chapter 3. We focus on the review of studies that have embedded the concept of trust and investigated the effect of forecast signaling on supply chain coordination. Papers written by Chu (1992), Lariviere and Padmanabhan (1997), Desai and Srinivasan (1995), Van Mieghem (1999) deal with information sharing in decentralized supply chain in which demand signaling in new product introductions had been investigated.

Bakal et. al (2011) considered a two- echelon model with one supplier and multiple retailers where there is a lack of supplier's capacity information in a single-period. Using game theoretic approach, the retailer's reaction under the assumption of full capacity information ad asymmetric capacity information and the value of information were analyzed in which all retailers are served from the same capacity source.

Uncertainty in quality of product affects buyer's profit. To mitigate the negative effect of such this uncertainty, Wu et.al (2011) investigated a supply chain including a buyer and two competing suppliers who experience product quality uncertainty. They showed that quality information sharing is always beneficial for the buyer and it is dependent on product quality level and price level for the supplier. In the basic EOQ model when there is a level of supply uncertainty, the target production value for a manufacturer differs from an ordered placed by retailer. Chian and Feng (2006) investigated the value of information sharing on the manufacturer's optimal production quantity which will be shipped to a number of retailers under the assumption of production yield variability and demand volatility. Moving average method was used to determine the manufacturer's optimal production level in order of forecasting the retailers' orders for two cases of non-information sharing and information sharing.

To induce a supply chain member to reveal true demand information Özer and Wei (2006) and Cachon and Lariviere (2001) identified a set of contracts. For a channel consisting of a manufacturer and a supplier, Özer and Wei (2006) analyzed credible forecast information sharing problem in which the manufacturer has private information about her end product while the supplier must decide on capacity level before the manufacturer places her order. To make possible credible forecast information sharing, two strategic information sharing contracts were developed by the supplier: a nonlinear capacity reservation contract and an advance purchase contract which enable the supplier to discover the manufacturer's private forecast information and to receive a signal on her forecast information respectively.

Cachon and Lariviere (2001) considered similar problem to Özer and Wei (2006) in which a manufacturer who has private information about demand offers two contract compliance regimes, forced compliance and voluntary compliance, to a supplier. In turn, the supplier decides on capacity procurement. To share demand forecasts credibility, they considered contracts under each compliance regime. Wang et. al (2009) considered a supply chain consisting of a manufacturer and a retailer who is dominant in the supply chain. The supplier supplies the retailer in which its manufacturing cost is unknown for the retailer and the retailer has only a perception on the prior distribution of the manufacturing cost. For different types of contract: Price-Only, Franchise fee, Two-part tariffs, and Menu of contracts (MC), the retailer investigated conditions in which the manufacturer is interested to share its information about manufacturing cost and how these contracts affect on information sharing conditions.

Researchers have defined trust variously. Trust in interpersonal relationship was defined as an individual's confidence in another person's intensions, motives and sincerity of speech by Mellinger (1995). Mayer et.al (1995) developed definition of trust consisting of the following dimensions: 1) benevolence, 2) integrity, and 3) ability. Donohue and Siemsen (2011) defined trust as a relationship of reliance. We review analytical works on trust to improve the coordination of supply chain. These works have not been commonly studied in literature.

A manager's disclosure credibility of private information was examined by Stocken (2000) in a supply chain including a manager and an investor in a single-game and a repeated game. Although in the one-shot game there was no communication, in the repeated game the manager reveals its information truthfully conditional on sufficient accounting report.

Using a game theoretical approach, Ren et.al (2010) investigated supply chain coordination and forecast information sharing for a channel consisting of a supplier and a customer. The supplier invests on its capacity before the customer realizes demand and relies on the customer forecast sharing. The results showed that for a one-shot game, the supplier behaves rationally on capacity allocation since the customer does not share the forecast information truthfully. On the other hand, for long-term interaction, review strategy was used to update the customer's behavior scoring index to assure whether the customer shares truthful information in each transaction.

Özer et.al (2011) considered a capacity investment decision problem where a supplier who is not aware of a manufacturer's private forecast. A laboratory experiment observation reveals that there is cooperation between the manufacturer and the supplier in the absence of reputation-building mechanism which is the results of trust between the parties. They further developed an analytical model to identify how trust and trustworthiness provide cheap-talk forecast sharing effective for one-time interaction and repeated interaction as well.

In addition to the role of trust in supply chain, Ebrahim-Khanjari et.al (2012) incorporated social characteristics in a multi-period dynamic model with a manufacturer, a salesperson and a retailer. Both the retailer and the salesperson forecast demand independently. For different types of salesperson, they showed that in long relationship, the salesperson shares its information truthfully. They evaluated and examined the impact of trust, referred to forecast sharing, and the salesperson social characteristics on the supply chain parties' decisions.

The most relevant study to our research published very recently by Pun and Heese (2013). They investigated a manufacturer's make-buy decision in which there are potential suppliers who are either high type or low type. The suppliers' type is private information for the suppliers and unknown for the manufacturer. On the other hand, the manufacturer has a prior knowledge about suppliers' type and updates its beliefs about the capabilities of unaudited suppliers by learning from audit supplier.

In this chapter, we investigate the effect of forecast signaling on the manufacturer's optimal order quantity in which the manufacturer has two strategies about forecast signal, trust and not trust. To the best of our knowledge, there is no study in the literature to investigate whether random yield forecast signaling is beneficial for the supply chain parties or not. Moreover, we are interested to discover the best strategy of the manufacturer given shared forecast. We also explore numerically the effect of manufacturer's trust about signal on the supply chain parties' profits and order quantity.

The rest of the chapter is organized as follows. Model analysis and discussion are presented in Section 4.2. In Section 4.3, managerial insights and discussion of results are presented by numerical example. In the last section, there are concluding remarks and some future extensions of the work presented in this chapter.

4.2 The Model

We model a one-shot game for a supply chain consisting of a supplier and a manufacturer in which there is a random yield uncertainty from the supply side. The manufacturer orders a single product from the supplier who faces with uncertain random yield, θ. The manufacturer decides on its ordering decision with regard to the supplier's shared forecast on disruption and uncertain market demand. Our focus is on the role of trust in forecast shared and the manufacturer's optimal ordering policy. It is assumed that market demand is uncertain in which $\Phi(\xi)$ is demand cumulative distribution function which is continuous and differentiable and $\varphi(\xi)$ is its probability density function. The model uncertainty comes from both demand and supply side. The supplier decides in truthful signaling and the manufacturer decides whether trust the received signal.

4.2.1 Random yield Model

To model random yield uncertainty, we assume that the manufacturer orders quantity q to the supplier whose shipment quantity is a scaled random variable $\theta.\mathrm{q}$, where θ is a random variable and represents the supplier's private forecast information.

Random yield uncertainty, θ, belongs to the interval $[\,\underline{\theta}, \bar{\theta}]$ where $\underline{\theta}$ and $\bar{\theta}$ are the lower and upper bounds on random yield. Let $F(\theta)$ be the random yield cumulative distribution function that is continuous and differentiable and $f(\theta)$ is its probability density function. We assume that yield uncertainties are distributed independently.

The forecast is shared with the manufacturer based on two strategies, truthfully and untruthfully. It is assumed that the manufacturer's belief on shared forecast is in the interval $[\,\underline{\gamma}, \bar{\gamma}\,]$ with probability density function $g(\theta)$ and cumulative distribution function $G(\theta)$. If the manufacturer trusts the supplier's signal, then the manufacturer updates the belief interval and places order according to the received signal and uncertain market demand. On the other hand, if it does not believe on the truth of forecast signal, then it orders according to its prior knowledge.

4.2.2 Model Formulation and Discussion

It costs c the supplier per unit production and charges the manufacturer p for each unit purchased from the supplier. The manufacturer earns r revenue per unit sold to market. If the shipment quantity is less than what the manufacturer orders, then both the supplier and the manufacturer incur shortage cost g_M and g_S respectively. The manufacturer is the only member who incurs a unit holding cost of h for unsold units.

The manufacturer's expected sales given an available shipment quantity from the supplier is:

$$E(sales) = E(\xi) - E_{\xi,\theta}\,[\xi - \theta q]^{+} - E_{\xi,\theta}\,[\xi - q]^{+} \tag{1}$$

Given all the cost parameters, the supplier and the manufacturer's expected profits are defined as follow respectively by $\pi_M(\theta, q)$ and $\pi_S(\theta, q)$.

$$\pi_M(\theta, q) = (r-p)E(sale) - h\,.E_{\xi,\theta}\,[\theta q - \xi]^+ - g_M\,.E_{\xi,\theta}\,[\xi - \theta q]^+ - g_M\,.E_{\xi,\theta}\,[\xi - q]^+ \tag{2}$$

$$\pi_S(\theta, q) = p.E_\theta\, min(\,\theta q\,, q) - g_S\,.E_{\xi,\theta}\,[q - \theta q]^+ - cq \tag{3}$$

The following is explained how the game procedure played by the supply chain members. First, the supplier observes the forecast of random variable $\hat{\theta}$ while the manufacturer does not know about $\hat{\theta}$. Then, the supplier sends the random yield signal m to the manufacturer. We assume that the manufacturer faces two strategies about forecast signal, trust or not to trust. If the manufacturer trusts the supplier's signal, it updates its belief by revising interval belief. Then the order is placed according to the received signal and uncertain market demand. On the other hand, if the manufacturer believes that the forecast is shared untruthfully, then, the manufacturer places its order quantity according to its prior knowledge and the belief interval does not change. The manufacturer's optimal order quantity that maximizes its profit is given below:

$$q^* = \arg\max_q E[\pi_R(m, q)]$$

q^* is the solution of the following formula:

$$(r-p)\int_{\underline{\gamma}}^{1} \theta\ g(\theta)\,d\theta + (r-p+g_M)\int_{1}^{\overline{\gamma}} g(\theta)\,d\theta =$$

$$(r-p+h+g_M)\int_{\underline{\gamma}}^{1} \theta\,\Phi(\theta q)\,g(\theta)\,d\theta - (r-p+g_M)\ \int_{1}^{\overline{\gamma}}\ \Phi(q)\,g(\theta)\,d\theta \tag{4}$$

For a special case, where both demand and random yield follow uniform distribution, the manufacturer's order quantity is:

$$q^* = \frac{3\,\bar{\xi}\,(r-p+g_M)\left[\left(1-\underline{\gamma}^2\right)+2\,(\overline{\gamma}-1)\right]+3\,\underline{\xi}\,h\,(1-\underline{\gamma}^2)}{2\,(r-p+h-g_M)\left(\left(1-\underline{\gamma}^3\right)\right)+6\,(r-p+g_M)(\overline{\gamma}-1)} \tag{5}$$

It is assumed that demand varies in the interval $[\,\underline{\xi}, \bar{\xi}]$.

In cooperative game, truthful information-sharing case, the manufacturer trusts the supplier's forecast signal and orders $\hat{q}$ to maximize its profit with the updated belief after signal observation. The manufacturer updates its belief on the random yield availability given the signal, m, from the supplier. The updating process is as follows:

- Updated $\underline{\gamma} = min(\,\underline{\gamma}, m)$,

- Updated $\bar{\gamma} = max(\bar{\gamma}, m)$

The retailer updates the yield risk interval after forecast observation in the same way.

- Update $\underline{\theta} = min(\underline{\theta}, \hat{\theta})$,
- Update $\overline{\theta} = max(\overline{\theta}, \hat{\theta})$.

Alternatively, in the non-cooperative case, the manufacturer does not trust the supplier's shared signal and orders its required quantity q_o regardless of the shared information (received signal).

$$q_o = \arg\max_q E[\pi_R(\theta, q)]$$

Intuitively we may conclude that the supplier has an incentive to recommend another value for random yield when the uncertainty in yield risk is low. It may report the signal over pessimistic to get more orders from the manufacturer to increase its profit and encounter its shortage cost due to mismatch between demand and supply. This signal affects the manufacturer's profit negatively due to increasing holding cost and over purchasing. Therefore, the best strategy for the manufacturer is do not trust the supplier's signal.

4.3 Numerical example

To support the intuitive result presented in this chapter, numerical examples are solved. The analytical answer for the case of Uniform demand and random yield was obtained in the previous section. Here, by numerical example we show that how the information shared by the supplier affects ordering policy of the manufacturer. We also show what the most beneficial strategy for the manufacture and retailer is. Moreover, the impact of trust in random yield forecast signal on both supply chain members profits is investigated. The following table represents the value of input parameters in the model for numerical example.

Table 4.1. The values of input parameters.

C	g_S	p	r	h	g_M	ξ
1.6	1	3.5	8	1.2	2	$U \sim [10, 30]$

The next two sections discuss the effect of truthful and untruthful information sharing on the supply chain members' performance. Moreover, the discussion is followed with the manufacturer's optimal action given forecast signal.

4.3.1 Untruthful Case

In this case, the manufacturer does not trust shared information on random yield availability by the supplier. The manufacturer's order quantity is independent of yield signal. Tables 4.2, 4.3, 4.4, and 4.5 show the results for different scenarios of the manufacturer's belief about yield value based on its prior knowledge on yield risk.

Following, some trivial results are explained. Table 4.2. shows the manufacturer's profit and optimal order quantity regardless to the supplier's signal for different belief about yield risk, $[\underline{\gamma}, \bar{\gamma}]$. Regardless yield risk revealed by the supplier, as the interval range of the manufacture's belief about random yield decreases, the optimal order quantity, q_o, decreases. The reason is, the manufacturer is the only member who pays holding cost. Therefore, if on hand inventory level at the end of period exceeds demand, then the manufacturer incurs holding cost. The smaller length of interval means the more reliable supplier. Consequently, more precise ordering leads to increasing the expected profit of the manufacturer while the supplier's profit decreases. This happens due to reduction in order quantity.

One more result is that for the same yield risk interval, the supplier obtains the most benefit when observation ensures more supply availability. For example, for $[\underline{\theta}, \bar{\theta}] = [0.7, 1.2]$ and $[\underline{\gamma}, \bar{\gamma}] = [0.5, 1.5]$, the supplier's profits are 76.82, 47.64, and 28.28 respectively for observations 1.4, 0.9, and 0.4.

Table 4.2. Manufacturer's profit and optimal order quantity for untruthful case for $[\underline{\theta}, \bar{\theta}] = [0.7, 1.2]$.

$[\underline{\gamma}, \bar{\gamma}]$	q_o	π_M^*
[0.5, 1.5]	31.87	81.76
[0.75, 1.15]	30.31	83.07
[0.9, 1.1]	29.11	84.75

Table 4.3 Supplier's profit for $[\underline{\theta}, \bar{\theta}] = [0.7, 1.2]$ and different observations.

$[\underline{\gamma}, \bar{\gamma}]$	$\hat{\theta} = 0.4$	$\hat{\theta} = 0.9$	$\hat{\theta} = 1.4$
[0.5, 1.5]	28.28	47.64	76.82
[0.75, 1.15]	26.91	45.31	73.07
[0.9, 1.1]	25.83	43.51	70.17

When the supplier's observation is smaller than the lower level of yield risk interval, the supplier's profit decreases as the supplier's uncertainty on random yield increases. To be specific, for the smaller interval range of supplier's yield risk, the supplier obtains the lowest profit when the observed $\hat{\theta}$ is smaller than the lower level of yield risk interval, $\hat{\theta} < \underline{\theta}$. For instance, for $[\underline{\gamma}, \bar{\gamma}] = [0.5, 1.5]$ and $\hat{\theta} = 0.4$, when the range of $[\underline{\theta}, \bar{\theta}]$ moves from $[0.7, 1.2]$ to $[0.9, 1.1]$, the supplier's profit drops from 28.28 to 20.83. This can be explained as follows. The manufacturer does not update its belief and orders based on its prior knowledge, while the supplier's observation is far from the manufacturer's belief. Therefore, the supplier may incur shortage cost since it may ship quantity less than what the manufacturer requested.

Table 4.4 Supplier's profit for $[\underline{\theta}, \bar{\theta}] = [0.8, 1.1]$ and different observations.

$[\underline{\gamma}, \bar{\gamma}]$	$\hat{\theta} = 0.4$	$\hat{\theta} = 0.9$	$\hat{\theta} = 1.4$
[0.5, 1.5]	23.67	50.98	130.12
[0.75, 1.15]	22.52	48.5	123.77
[0.9, 1.1]	21.62	46.57	118.85

Table 4.5 Supplier's profit for $[\underline{\theta}, \bar{\theta}] = [0.9, 1.05]$ and different observations.

$[\underline{\gamma}, \bar{\gamma}]$	$\hat{\theta} = 0.4$	$\hat{\theta} = 0.9$	$\hat{\theta} = 1.4$
[0.5, 1.5]	20.83	55.76	267.29
[0.75, 1.15]	19.82	53.04	254.26
[0.9, 1.1]	19.03	50.94	244.16

4.3.2 Truthful Case

We compare the results for three different scenarios of supplier's observation: I) supplier's observation lies within its yield risk interval, II) it is smaller than the lower bound of the interval, and III) it is greater than the upper bound of the interval. Although the manufacturer trusts the supplier, the supplier may have incentive to reveal another value of random yield. For each scenario, we analyze results and the manufacturer's optimal action for two different cases of information sharing.

Scenario I: The supplier's observation lies within the manufacturer's belief on yield risk interval $[\underline{\gamma}, \bar{\gamma}]$

- Case i) Truthful Supplier

The result is the same as untruthful information sharing case discussed in previous section.

- Case ii) Untruthful Supplier

A comparison of Tables 4.6, 4.7, and 4.8, indicates that as the difference between upper and lower bond of the manufacturer's belief interval decreases, lower uncertainty, the optimal order quantity increases when yield signal is smaller than the lower bound and decreases when it is greater than the upper bound.

For the case where the signal is smaller than $\underline{\gamma}$, the supplier's profit increases as $\bar{\gamma} - \underline{\gamma}$ decreases while there is a significant reduction in the manufacturer's profit. In this situation as it is stated above, the optimal order quantity increases, therefore the supplier sells more that leads to higher profit while the untruthful information hurts the manufacturer by increasing the level of unsold units and incurring holding cost.

Table 4.6 Manufacturer and suppliers' profit and optimal order quantity for untruthful case for $[\underline{\gamma}, \bar{\gamma}] = [0.5, 1.5]$ and $\hat{\theta} = 0.9$.

m	$\hat{q}^*$	$\hat{\pi}_M^*$	$\hat{\pi}_S^*$
0.3	33.35	72.23	49.86
0.9	31.87	81.76	47.64
1.6	31.67	81.71	47.34

Table 4.7 Manufacturer's profit and optimal order quantity for untruthful case for $[\underline{\gamma}, \bar{\gamma}] = [0.75, 1.15]$ and $\hat{\theta} = 0.9$.

m	$\hat{q}^*$	$\hat{\pi}_M^*$	$\hat{\pi}_S^*$
0.3	35.57	48.21	53.14
0.9	30.31	83.07	45.32
1.6	30.14	83.06	45.06

Table 4.8 Manufacturer's profit and optimal order quantity for untruthful case for $[\underline{\gamma}, \bar{\gamma}] = [0.9, 1.1]$ and $\hat{\theta} = 0.9$.

m	$\hat{q}^*$	$\hat{\pi}_M^*$	$\hat{\pi}_S^*$
0.3	36.12	7.78	54
0.9	29.11	84.75	43.51
1.6	29.14	84.65	44.46

On the other hand, when the signal is higher than $\bar{\gamma}$, the supplier's profit decreases as $\bar{\gamma} - \underline{\gamma}$ decreases. This happens due to reduction in the optimal order quantity from the manufacturer as a result of receiving higher forecast signal.

Scenario II) the supplier's signal is smaller than the lower bound of the Manufacturer's belief on yield risk $[\underline{\gamma}, \bar{\gamma}]$

- Case i) Truthful Supplier

As $\bar{\gamma} - \underline{\gamma}$ decreases, the manufacturer orders more thus the supplier's profit increases while the manufacturer's profit goes down.

Case ii) Untruthful Supplier

By comparing Tables 4.9, 4.10, and 4.11, it is quite clear that reduction in $\bar{\gamma} - \underline{\gamma}$ increases the optimal order quantity when the supplier mimic to signal random yield lower than the real observation while it decreases for yield signal higher than the supplier's observation. This leads to increase in supplier's profit and reduction in the manufacturer's profit in comparison with the case of truthful supplier. The same results are observed for Scenario III. Results tables for Scenario III are presented in Appendix.

Table 4.9. Manufacturer's profit and optimal order quantity for untruthful case for $[\underline{\gamma}, \bar{\gamma}] = [0.5, 1.5]$ and $\hat{\theta} = 0.4$.

m	$\hat{q}^*$	$\hat{\pi}_M^*$	$\hat{\pi}_S^*$
0.3	33.35	72.23	29.6
0.4	32.63	77.78	28.96
1.3	31.87	81.8	28.28

Table 4.10 Manufacturer's profit and optimal order quantity for untruthful case for $[\underline{\gamma}, \bar{\gamma}] = [0.75, 1.15]$ and $\hat{\theta} = 0.4$.

m	$\hat{q}^*$	$\hat{\pi}_M^*$	$\hat{\pi}_S^*$
0.3	35.55	48.21	31.55
0.4	34.4	61.11	30.53
1.3	30.22	83.07	26.82

Table 4.11. Manufacturer's profit and optimal order quantity for untruthful case for $[\underline{\gamma}, \bar{\gamma}] = [0.9, 1.1]$ and $\hat{\theta} = 0.4$.

m	$\hat{q}^*$	$\hat{\pi}_M^*$	$\hat{\pi}_S^*$
0.3	36.12	7.8	32.65
0.4	34.87	33.08	30.95
1.3	29.55	84.68	26.22

From above discussion, it can be concluded that it is beneficial for the supplier to mimic high uncertainty and send signal lower than the manufacturer's belief. The manufacturer obtains the most benefit when it does not trust the supplier. This result is consistent with the result obtained intuitively.

4.4 Conclusion

In this chapter, the effect of random yield forecast signaling in a one-shot game for a supply chain consisting of a manufacturer and a supplier was studied. The supplier has superior information on the forecast of yield risk. It was assumed that the manufacturer either trust to supplier's report or do not trust it. We modeled the supply yield risk in a multiplicative form. The manufacturer placed its order after observing the supplier's signal. If the manufacturer absolutely trusted the supplier, it updated its belief about yield risk. On the other hand, in the absence of trust, the manufacturer decided on the optimal order quantity based on its prior knowledge. We were interested in investigating the role of trust in yield risk forecast information sharing. For two types of information sharing situation, truthful and untruthful, we analytically obtained the manufacturer's optimal order quantity. Intuitively, we concluded that the supplier has tendency to deviate from truthful sharing information when the forecast random yield indicates that the supplier is high reliable.

We also obtained the numerical results in which both demand and supplier's yield risk follow uniform distribution. The numerical result is consistent with intuitive result. Numerically we compared the supply chain members' profits for two cases of truthful and untruthful. In truthful case, the results were achieved for three different scenarios of the supplier's observation. Each scenario includes two cases of information sharing.

When the supplier is untruthful, it is not beneficial for the manufacturer trust the supplier.

Although we considered one-shot game in this model, many businesses have long-term relationships. It would be interesting to see how the supply chain members behave in long-term relationship and how the credibility of the supply chain parties are examined. Moreover, it can be investigated under which circumstances information will be shared truthfully when relationships are long term.

Appendix A:

Table 4.A1 Manufacturer's profit and optimal order quantity for untruthful case for $[\underline{\gamma}, \bar{\gamma}] = [0.75, 1.15]$ and $\hat{\theta} = 1.4$.

m	$\hat{q}^*$	$\hat{\pi}_M^*$	$\hat{\pi}_S^*$
0.3	35.55	48.21	57.26
1.4	30.19	83.07	48.62
1.8	30.11	83.06	48.51

Table 4.A2 Manufacturer's profit and optimal order quantity for untruthful case for $[\underline{\gamma}, \bar{\gamma}] = [0.5, 1.5]$ and $\hat{\theta} = 1.4$.

m	$\hat{q}^*$	$\hat{\pi}_M^*$	$\hat{\pi}_S^*$
0.3	33.35	72.23	53.72
1.4	31.87	81.76	51.33
1.8	31.38	81.64	50.54

Table 4.A3 Manufacturer's profit and optimal order quantity for untruthful case for $[\underline{\gamma}, \bar{\gamma}] = [0.9, 1.1]$ and $\hat{\theta} = 1.4$.

m	$\hat{q}^*$	$\hat{\pi}_M^*$	$\hat{\pi}_S^*$
0.3	36.12	7.8	58.18
1.4	29.64	84.67	47.73
1.8	29.8	84.64	47.99

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5. CONCLUSIONS AND FUTURE RESEARCH

5.1 Conclusions

This thesis studies the effect of information sharing and updating on inventory and procurement management in supply chain management for two types of supply uncertainties: uncertainty in randomness and uncertainty in yield risk.
The first part contributes theoretical results for a dynamic inventory model in which the probability of supply availability of a machine is forecasted by Bayesian learning. Supply availability is an all-or-nothing type and follows the Bernoulli distribution with an unknown parameter, θ, which is the probability of the supplier reliability.

We investigate the optimal ordering/production policy for a two-item, finite-horizon dynamic problem, with two different fixed costs. To characterize the optimal policy we prove that cost function is $(K_1\,,\ K_2)$-convex. Then, we prove the optimality of $(s_i^n(\alpha,\beta), S_i^n(\alpha,\beta))$ policy along with a monotone switching curve as a optimal ordering policy for the problem.

We provide numerical example to compare Bayesian and non-Bayesian models with different fixed cost values and demand levels. The results reveal that information obtained from learning could be more profitable and cost-effective for a low value of the fixed cost. Generally, numerical results show that improving the accuracy of the forecast leads to making a better ordering decision and eliminating the negative effect of supply disruption on the total cost.

Second, we investigate a procurement problem for a three-echelon supply chain consisting of a supplier, a distributor and a retailer where yield and demand are random variables. Under additive supply yield risk, the distributor's optimal ordering quantity from the supplier and the retailer's optimal ordering quantity from the distributor are derived. Using Bayesian updating, the distributor updates its random yield distribution that follows a uniform distribution with two unknown parameters. We propose two contracts for Non-Bayesian case. Moreover, we extend these two contracts for the Bayesian case. A quantity flexibility contract is offered by retailer to the distributor as an incentive of sharing its supply risk information where the distributor incurs spot market cost.

We show how supply chain members benefit under different contracts. While analytical results for the Non-Bayesian model are provided, for models with information updating, an algorithm is proposed to obtain the optimal ordering quantity for each chain member. The results obtained would help managers when deciding which type of contract would best work for them.

The effects of random yield updating on the performance of the supply chain members are also investigated. Parts of the results indicate that for the lost sales model, only the retailer benefits from information updating when the variance of yield risk is low. Moreover, the most powerful contract is shown to be the quantity flexibility contract for the retailer for any flexibility level. The learning effect on the performance of the supply chain members and sensitivity analysis are investigated numerically.

Finally, we investigate the effect of trust on random yield forecast signaling in a one-shot game in a supply chain consisting of a manufacturer and a supplier in which the supplier has superior information on the forecast of yield risk. The manufacturer places its order after observing the supplier's signal. If the manufacturer fully trusts the supplier, it updates its belief about yield risk. On the other hand, in untruthful case, the manufacturer orders the optimal quantity based on its prior belief. For two types of information sharing situation, truthful and untruthful, the analytical result is obtained. Intuitive discussion states that the supplier has an incentive to deviate from truthful sharing information when the forecast random yield indicates that the supplier is high reliable.

The numerical results for Uniform demand and supplier's yield risk are obtained. The numerical results are in consistent with intuitive result. The supply chain members' profits for two cases of truthful and untruthful are compared numerically. It is shown that when the supplier is untruthful, it is not beneficial for the manufacturer trust the supplier.

5.2 Limitations and Future Recommendations

Several future researches are presented for the problems discussed in each Chapter.

Future extensions and limitations for the model developed in Chapter 2 could be in different directions. In the model developed in Chapter 2, it is assumed that capacity is unlimited which is not true in reality. Therefore, it would be interesting to investigate the optimal ordering policy with capacity constraints. It may also be worthy to investigate the behavior of the system when there is lead time for delivery, and/or when the manufacturer has advance lead time information. Moreover, the incorporation of multi-supplier and demand uncertainty into this model would also be interesting to investigate.

Second work can be extended by considering the following issues. A future extension would be to investigate demand updating along with the supply risk updating. Additionally, for multi-supplier problems, it would be interesting to investigate the effect of monitoring one supplier’s shipment on the ordering from the other supplier. This work is limited in one-period time horizon so there is an extension for multi-period information sharing.

The limitation of the last work in this study can be considered as an extension of the work as well. Although we considered one-shot game in the model presented in Chapter 4, many businesses have long-term relationships. It would be interesting to see how the supply chain members behave in long-term relationship and how the credibility of the supply chain parties is examined. Moreover, it can be investigated under which circumstances information will be shared truthfully when relationships are long term.

Journal Papers

Firouzi F., Baglieri E., Jaber M.(2014). "Two-Product Inventory Management with Fixed Costs and Supply Uncertainty", *Applied Mathematical Modeling*. In press. http://dx.doi.org/101016/j.apm.2014.03.018.

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Firouzi F. Jaber M., Baglieri E. (2014). "Trust in Supply Forecast Information Sharing", *Working Paper*.