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FURTHER OPEN PROBLEMS IN COOPERATIVE GAMES

In 2013, the International Game Theory Review published two special issues on open problems in cooperative games: the first regarding theory and the second applications. In this paper, our aim is to present some problems currently "on our table" that were not included in the two issues mentioned above, either because the topics were too specific or they arose after publication. The problems are divided into theoretical problems, general models that may be applied to different fields and applicative problems.

Keywords: *cooperative games, values, applications*

1. Introduction

In 2013, the International Game Theory Review published two special issues on open problems in cooperative games: the first regarding theory [17] and the second applications [18]. The aim was to provide a unified survey for those scholars wishing to deal with recent developments in game theory, without having to search for them in various journals devoted not only to game theory or operations research but also to more specific topics such as economics, politics, finance, social sciences, medicine, and so on. In this paper, our intention is to present some problems that are now "on our table" and were not included in the two issues mentioned above, either because the topics were too specific or they arose after publication.

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We want to specify that the wide use of self-quotation does not derive from the belief that the open problems mentioned in our papers have particular importance compared with others but simply from the better knowledge that we have of these problems and our decision to present what we are working on.

We would also like to point out that our decision to present these problems and models in a simple and informal way is to enable scholars from fields other than game theory to understand them in a format closer to their own.

The paper is organized as follows: Section 2 is devoted to some theoretical topics, i.e. links between two general forms of games (Subsection 2.1), values of games as barycentres (Subsection 2.2), extensions of bankruptcy problems and solution rules (Subsection 2.3). In Section 3, we present some general models, each of which has different fields of application: collusion (Subsection 3.1), interfering elements (Subsection 3.2), indirect control of assemblies (Subsection 3.3) and power indices with incompatible agents (Subsection 3.4). Finally, in Section 4 we deal with some applications: international economics (Subsection 4.1), marketing cooperatives (Subsection 4.2), blackmailing behaviour (Subsection 4.3) and cost allocation for infrastructure problems (Subsection 4.4).

2. Theory

This section contains those topics that may be viewed as theoretical ones: links between two general forms of games, values of games as barycentres and extensions of bankruptcy problems and solution rules.

2.1. Links between two general forms of games

There are three main forms used to represent games: normal form, extensive form and characteristic function form (in short cff). In 1944, John von Neumann and Oskar Morgenstern created a connection between the extensive form and the normal form of a game by means of a transformation from the first to the second. In this operation, some information is lost but, in general, the developments of the theory regarding these two forms of games have, to a certain extent, developed along parallel lines. On the other hand, as far as games presented in cff are concerned, the two authors formulated a transformation to the normal form, but only for constant sum games. Due to the lack of a general transformation, the development of the theory of games in cff proceeded separately, with its logic leaning more towards the satisfaction of the requirements of the various coalitions, thus losing sight of the strategic aspect of the players' choices. More recently, the latter transformation quoted above has been generalized to variable sum games in [24]. This has enabled:

• Strategic choices to be returned into the hands of the players.

• To make use of a lot of results in the literature (because, as previously mentioned, the normal form and the cff games had previously been studied using completely different methods).

• To define particularly stable solutions for cff games, i.e. a Nash equilibrium leading to Pareto optimal payoffs. The existence of such a solution has been proved for all inessential games, all subadditive games, all two-person games and all n-person games having the interior of the core empty. A game lacking such a solution has yet to be found.

In the paper introducing this transformation, various problems were left open concerning, for example, cases in which coalitions are not equiprobable. The main ones, however, concern existence and uniqueness of the solution. Regarding existence, a general theorem must be found or the class of games, for which the solution is empty, must be identified. Regarding uniqueness, some criteria for restricting the set of solutions could be studied. The achievement of existence and uniqueness would lead to a game value which would be particularly stable, due to the above-mentioned features of this solution.

Along the same research line, Fragnelli [21] considers a cooperative linear programming game [35] in which the players control a bundle of resources and the worth of a coalition is the best output that the players can obtain using just their resources. Then, starting from this cooperative situation, a particular non-cooperative game is defined, where the players have to choose the price for selling their bundle of resources to an external agent who will decide for each player, whether to buy the whole bundle or nothing, on the condition that the total cost paid by the external agent is not larger than the worth of the outcome. The strategy of each player is to set the price that will give him the highest worth. The efficient Nash equilibria of this non-cooperative game are related to the Owen set [28], i.e. the set of core-allocations corresponding to optimal solutions of the associated dual linear programming problem.

Some questions still remain: what is the relationship between the core and the efficient Nash equilibria? Is it possible to define similar results for the non-efficient Nash equilibria? Is it possible to find a more general setting that enlarges the class of cooperative games behind the linear programming ones?

2.2. Values of games as barycentres

It is easy to check that the Shapley value [36] of each two-person game coincides with the barycentre of the set of the relative imputations. This also occurs for all constant-sum three-person games, but not in general. In [22], a simplex of n-dimensional

Euclidean space has been set up, the vertices of which are located at particular points representing the game. The barycentre of this simplex coincides with the Shapley value in all possible games. An algorithm for the automatic computation of this value, based on such a construction, has been created. This algorithm employs a quick stop theorem.

This result has been extended by Arsen Palestini to the Banzhaf–Coleman –Penrose value but no analogous generalizations are known for other values.

2.3. Extensions of bankruptcy problems and solution rules

A classical bankruptcy problem arises when a firm goes bankrupt, leaving some debts with other firms or agents, i.e. claimants, and the estate available is not sufficient to cover their claims. More generally, bankruptcy problems may be used whenever a set of agents have to share a scarce resource. Given a bankruptcy problem, a solution rule is a function that determines the monetary amount to be obtained by each claimant, while satisfying the conditions that each claimant has to receive a non-negative amount no larger than his claim (rationality) and that the whole estate is assigned (efficiency).

In general, the estate and the claims are assumed to be real numbers, even if in several cases integer numbers seem more appropriate. After the pivotal works by Herrero and Martinez [29, 30], Fragnelli, Gagliardo and Gastaldi [16] considered a situation in which the estate comes in discrete unities, but the claims do not. Possible examples of real-world situations involve the allocations of quotas of hedge funds, minimal stock or bond shares and radio-frequency assignment, in which the claims are coherent with the estate. Nevertheless, the model can also be applied to situations in which claims are not integer, even if the estate is made up of indivisible unities such as emergency intervention units, where the claims may not be integer, depending on the expected number of calls or on the size of the areas to attend to that may correspond to any real number of emergency units.

In [16], a method is presented that, with suitable assumptions, provides a unique integer solution; this rule is in the spirit of the constrained equal losses rule [38]. The authors are looking for assumptions that may enable integer rules to be defined that correspond to the frameworks of the constrained equal awards rule, the Talmud rule and the proportional rule. Other interesting open problems arise from computational aspects related to the number of agents and the values of the claims.

Still within the bankruptcy setting, it is possible to state that the sole element known by all claimants is the claim, which seems too limited to represent all the possible aspects of a claimant. Along this line, some authors have considered the possibility of adding a second element to each claimant, called the weight; consequently, some classical solution rules were modified in order to account for these weights and these rules were characterized by suitable axioms [8, 9].

It is possible to seek new weighted rules, as well as extensions of other existing rules, allowing the weights to play different roles in the definition of the rules, in order to represent the asymmetries of the agents involved more appropriately.

3. General models for multiple applications

This section contains some general models, each with various fields of application: collusion, interfering elements, indirect control of assemblies and power indices with incompatible agents.

3.1. Collusion

Many artistic sports (skating, diving, synchronised swimming and so on) require subjective evaluation by a jury. The relevant federations adopt certain measures to prevent exaggerated scores (excessively high or low) from biasing the overall score of any performance. For this purpose, trimmed means are used, i.e. arithmetical averages of the scores obtained after removing the highest and the lowest scores, or the two highest and the two lowest ones. However, it is easy to see that even such averages do not achieve the objective in the case of asymmetric tails. By means of the coherent majority average introduced in [25], it is possible to pinpoint the exaggerated marks, even if they are all from the same side. This method is based on the assumption that the majority of judges are reliable. However, this assumption may fall short if the jury is divided into sub-juries. In fact, in these cases a certain sub-jury may consist of a number of colluding judges that form a majority in the sub-jury itself. To overcome these problems, the anti collusion average was introduced in [5]. This average is based on an index of collusion assigned to each jury according to the scores given during the whole competition.

These methods can be applied not only in the world of sport (as in [26]) but also in various contexts in which the judges are directly involved in the results of their judgements: in economics (e.g. the evaluation of projects), in finance (e.g. company quotations), in insurance (e.g. made-to-measure policies), in the artistic field (e.g. singing and music competitions, in which the judges are involved with record companies) and so on. The problem is to evaluate how much better these new methods are, compared with the traditional ones adopted so far.

To solve the afore-mentioned problem, cooperative games in characteristic function form could be used, according to an idea of Angelo Uristani. For each type of assessment method, a game can be created. To each group of referees, a score is assigned, corresponding to the maximum distortion of the result that the group itself could award by means of strategic evaluations, if that method were adopted. For each game devised in this way, the characteristic function can give important information: for instance, if it is subadditive or inessential, then there is no advantage from the judges colluding. Elsewhere, other properties can help in the evaluation of a rule.

A different form of collusive behaviour is studied in [21] and [6], when the Knaster procedure is adopted for dividing a set of items, whose values are additive, among totally risk-averse agents, with equal rights to them. The division of the bundle among the agents depends on their declared evaluation of the items, so that there are incentives for false declarations by a single agent (manipulation) or by a group of agents (collusion). The total gain and the per capita gain for a group of colluders are not monotonic, so agents may wish not to enlarge a set of colluders. A dynamic mechanism for allocating the gains results in enlargement of the set of colluders always being (slightly) profitable. A cooperative game, namely the collusion game, provides a simple way for allocating the gains.

The set of open problems includes the search for a suitable definition of coalition –strategy–proofness that catches the actual behaviour of agents, a Bayesian game model accounting for the true valuations ascribed by the agents and for their beliefs in the valuations ascribed by other agents and for a mechanism that may allow the optimal coalition formation process to be implemented.

3.2. Interfering elements

Doses of interfering drugs are normally dispensed by the doctor with subsequent adjustments made while keeping the patient monitored. The decision regarding the first dosage is particularly delicate, as the doctor does not always have enough information to hand. Many decisions in other applicative fields must take into consideration the effects that two elements can produce if they are used together. For example, in economics, the demand for a commodity may be influenced by the presence on the market of another commodity with synergic or antagonistic effects. Other cases occur in social choices (for instance in the taxation of various goods), agriculture, zootechnology, and so on. When it is necessary to introduce two factors, often there is a primary interest concerning the effects of one rather than the other. If, for example, the importance of one factor is ten times greater than that of the other, this must also be taken into account when calculating the quantities to be used. Considering the above, a recent model enables the optimum quantities of two interfering factors to be directly calculated (rather than by successive approximations). This computation also accounts for the minimum quantities that must, in any case, be assigned [7]. A method for finding the solution is provided for all cases of continuous effect functions; furthermore, appropriate calculus techniques are given.

Unsolved problems concern methods for non-continuous functions, generalizations for cases of more than two interfering elements and new implementations of the model in terms of cooperative games, for those cases in which the elements are dispensed by various organisms, each of which is interested in optimising its own specific objective.

3.3. Indirect control of assemblies

A problem of some interest concerns the calculation of power indices in the cases of indirect control. For example, this happens when an investor has a shareholding in a certain company, which, in its turn, holds shares in another company and so on, or when a party consists of currents and sub-currents. In situations of this kind, it may be useful to calculate the power of a member in the whole system. The problem has been tackled in [27] by transforming the set of inter-connected games into just one game, using the multi-linear extensions introduced by Owen [34]. The power index that is believed to be the most suitable for describing the situation at hand can then be applied to the unified game.

In certain inter-connected games there may be “loops”: for example, if company A holds shares in company B, which also holds shares in company A. The transformation described above works for all cases without loops and for some cases with loops, but not in general. Moreover, an algorithm for the automatic computation of indirect power indices was introduced by Denti and Prati [12] but this method could be improved to reduce the computation time.

3.4. Power indices with incompatible agents

An interesting problem is how to evaluate the influence of decision makers such as parties in parliaments or members of boards of directors, on final decisions, especially when the agents are not equivalent, for instance depending on different numbers of seats in a parliament or different stock shares. This analysis may be performed, inter alia, by using power indices. The literature includes a large number of power indices, each one designed to emphasize different features of specific situations.

An important aspect is the incompatibility of some agents, for which Myerson [31] proposed to use a communication graph that allows the willingness of players to be in the same coalition to be represented. In [15], the question of coalitions including incompatible agents is discussed and the conclusion is that the idea of the communication graph of Myerson should be integrated with the probability that these coalitions form.

One question that has remained open is how to evaluate, in a computationally simple but coherent way, the probability with which each coalition forms, while also taking into account the possibility that some players may enter or leave it. Another open question is related to the possibility that coalitions with incompatible agents are considered unfeasible, so that a different way of evaluating the marginal contributions of the players is needed, in order to apply indices, like those of Shapley–Shubik [37] and Banzhaf [2], that are based on the comparison of the worth of two coalitions, in the event that one of the two is unfeasible.

4. Applications

This section is devoted to problems related to direct applications: international economics, marketing cooperatives, blackmailing behaviour and cost allocation for infrastructure problems.

4.1. International economics

Developing countries often contract debts with important banking institutions that they are then unable to pay back. In these cases, the real value of the debt decreases compared with the nominal value, to the point that, in some situations, it is worthwhile for the indebted country to buy its debt again at a lower price, to reacquire the possibility of obtaining new credit. Many studies have been made on this subject; some in particular provide analytical models (see, for instance, [23]).

A three-person game could be studied, the players being the indebted country, the credit bank and an international organisation (such as the World Bank or the International Monetary Fund) that is concerned with settling the debt, for instance to avoid detrimental situations with a domino effect on other countries.

4.2. Marketing cooperatives

Some manufacturers pool together to form cooperatives, to improve the marketing of their products, for example by negotiating better prices with large buyers and sharing the risk of lost production amongst the various members. Sometimes the market price may increase after such agreements are made and, as a result, some manufacturers have an incentive to sell part of their production directly, without going through the cooperative. The cooperative may retaliate by applying previously agreed taxes on these manufacturers. It is important to understand which are the best strategies for the cooperative (in terms of regulations to be approved) and for its individual members. A model by Bertini et al. [3] describes this situation and leads to a Nash equilibrium that constitutes an attractive solution.

This result opens the way to more complex models, such as oligopoly markets in which the market price falls as the supply increases.

4.3. Blackmailing behaviour

Power indices are usually designed for evaluating static power, e.g. what happens with reference to a precise proposal under discussion. However, a power index may also be interpreted as a measure of the influence of parties on the formation of a coalition government, bearing in mind the possibility that during the process of forming a majority coalition, a party may choose from amongst various alternatives. This may be viewed as blackmailing power in the sense that a party may ask for more power when it has an actual opportunity to join a more profitable coalition and this naturally leads to the evaluation of dynamic power. This situation may be captured by the concept of a bargaining set formulated by Aumann and Maschler [1], that is based on the idea that no agent may reject a division of the value in favour of a preferable one (objection) if another agent may reject the new division in favour of a third one (counterobjection). Following this remark, Chessa and Fragnelli [10] propose employing the bargaining set to assign to each party a quota of power that cannot be rationally objected to by the other parties.

It is well known that the most negative aspect of the bargaining set is its computational complexity, so the authors propose to reduce it by exploiting the particular structure of such games or by eliminating the possibility of objections and counterobjections involving unfeasible coalitions, or evaluating the situation after a majority has been formed.

Theoretical results and computational instruments may warrant future research, in order to provide further tools for tackling these aspects. Other possible future research questions may arise from the fact that the bargaining set is a set valued solution. The former direction involves applying different definitions of the bargaining set, while the latter involves selecting a unique point in the bargaining set, e.g. the closest to a given power index that does not belong to the bargaining set.

4.4. Cost allocation for infrastructure problems

Infrastructure cost games were introduced by Fragnelli et al. [19]. This class of games permits the fair and efficient computation of fees for transport operators to access the railway system after the European Union established that everyone could operate trains. The resulting fees are strongly rooted in the Shapley value [37]. The proposed model considers players to be a set of users of certain infrastructure, namely the railway network, that can be grouped according to increasing levels of sophisticated requirements regarding the infrastructure. This results in a strong reduction of complexity, as the players in each group are symmetric and have to pay the same fee.

This model has revealed itself to be extremely stable and reliable, so it has been successfully applied to other situations involving infrastructure, e.g. cost allocation for a consortium of municipalities sharing a common collection and disposal plant for solid urban waste [20, 14] and the calculation of rent fees in water management situations [11].

Some problems remain on the table. Above all, an analysis of the theoretical properties of such games and of other game theoretical solutions, following the path traced by Norde et al. [33]; in the case of applications to solid urban waste, it could be interesting to modify the fees in order to provide incentives for waste reduction, waste segregation and recycling; in the case of water management, it is of interest to improve the balance of the quota related to fixed costs and to variable costs, to provide incentives for reducing water usage and to increase the fairness of the fees charged to small users.

5. Conclusions

We avoided quoting other open problems "on our table", related to political and financial applications of power indices, as these topics already appear in [4]. We will be happy to provide further material and suggestions to those deciding to cooperate on any of the previously mentioned topics.

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References

[1] AUMANN R.J., MASCHLER M., *The bargaining set for cooperative games*, [in:] M. Dresher, L.S. Shapley, A.W. Tucker (Eds.), *Advances in game theory*, Princeton University Press, Princeton 1964, 443–476.

[2] BANZHAF J.F., *Weighted voting doesn't work: A mathematical analysis*, Rutgers Law Review, 1965, 19, 317–343.

[3] BERTINI C., GAMBARELLI G., SCARELLI A., VARGA Z., *Equilibrium solution in a game between a cooperative and its members*, AUCO Czech Economic Review, 2011, 5, 162–171.

[4] BERTINI C., GAMBARELLI G., STACH I., *Some open problems in the application of power indices to finance and politics*, Homo Oeconomicus, 2014 (to appear).

[5] BERTINI C., GAMBARELLI G., URISTANI A., *Indices of collusion among judges and an anti-collusion average*, [in:] S. Greco, R. Marques Pereira, M. Squillante, R.R. Yager, J. Kacprzyk (Eds.), *Preferences and decisions. Models and applications in studies in fuzziness and soft computing*, Springer, Berlin 2010, 195–210.

[6] BRIATA F., DALL'AGLIO M., FRAGNELLI V., *Dynamic collusion and collusion games in Knaster's procedure*, AUCO Czech Economic Review, 2012, 6, 199–208.

[7] CARFÌ D., GAMBARELLI G., URISTANI A., *Balancing pairs of interfering elements*, [in:] *Financial management. Measurement of performance and evaluation of investment appraisal*, Zeszyty Naukowe Uniwersytetu Szczecińskiego nr 760, Szczecin University Press, Szczecin 2013, 435–442 (in Polish).

[8] CASAS-MENDEZ B., FRAGNELLI V., GARCIA-JURADO I., *Weighted bankruptcy rules and the museum pass problem*, European Journal of Operational Research, 2011, 215, 161–168.

[9] CASAS-MENDEZ B., FRAGNELLI V., GARCIA-JURADO I., *A survey of allocation rules for the museum pass problem*, Journal of Cultural Economics, 2014, 38, 191–205.

[10] CHESSA M., FRAGNELLI V., *The bargaining set for sharing the power*, Annals of Operations Research, 2014 (to appear).

[11] DE AGOSTINI R., FRAGNELLI V., *A fair tariff system for water management*, [in:] A. Dinar, J. Albiac, J. Sanchez-Soriano (Eds.), *Game theory and policy making in natural resources and the environment*, Routledge, London 2008. 236–247.

[12] DENTI E., PRATI N., *Relevance of winning coalitions in indirect control of corporations*, [in:] G. Gambarelli (Ed.), *Essays on cooperative games – in Honor of Guillermo Owen*, Special Issue of Theory and Decision, 2004, 36, 183–192.

[13] FRAGNELLI V., *A note on the Owen set of linear programming games and Nash equilibria*, Theory and Decision, 2004, 56, 205–213.

[14] FRAGNELLI V., *How to divide the costs of urban solid wastes collection among citizens?*, Economics and Policy of Energy and the Environment, 2012, 3, 51–64.

[15] FRAGNELLI V., *A note on communication structures*, [in:] M.J. Holler, H. Nurmi (Eds.), *Power, voting, and voting power. 30 years after*, Springer-Verlag, Berlin 2013, 467–473.

[16] FRAGNELLI V., GAGLIARDO S., GASTALDI F., *Integer solutions to bankruptcy problems with non-integer claims*, TOP, 2014, 22, 892–933.

[17] FRAGNELLI V., GAMBARELLI G. (Eds.), *Open problems in the theory of cooperative games*, Special Issue of International Game Theory Review, 2013, 15 (2).

[18] FRAGNELLI V., GAMBARELLI G. (Eds.), *Open problems in the applications of cooperative games*, Special Issue of International Game Theory Review, 2013, 15 (3).

[19] FRAGNELLI V., GARCÍA-JURADO I., NORDE H., PATRONE F., TIJS S., *How to share railway infrastructure costs?*, [in:] F. Patrone, I. García-Jurado, S. Tijs (Eds.), *Game practice. Contributions from applied game theory*, Kluwer, Amsterdam 1999, 91–101.

[20] FRAGNELLI V., IANDOLINO A., *A cost allocation problem in urban solid wastes collection and disposal*, Mathematical Methods of Operations Research, 2004, 59, 447–464.

[21] FRAGNELLI V., MARINA M.E., *Strategic manipulations and collusions in Knaster procedure*, AUCO Czech Economic Review, 2009, 3, 143–153.

[22] GAMBARELLI G., *A new approach for evaluating the Shapley value*, Optimization, 1990, 21, 445–452.

[23] GAMBARELLI G., *Strategics for the repayment of external debt*, Proceedings of the International Conference on Information and Systems, Dalian Maritime University Publishing House, Dalian 1992, 566–573.
[24] GAMBARELLI G., *Transforming games from characteristic into normal form*, International Game Theory Review, 2007, 9, 87–104.
[25] GAMBARELLI G., *The coherent majority average for Juries' evaluation processes*, Journal of Sport Sciences, 2008, 26, 1091–1095.
[26] GAMBARELLI G., IAQUINTA G., PIAZZA M., *Anti-collusion indices and averages for the evaluation of performances and judges*, Journal of Sports Sciences, 2012, 30, 411–417.
[27] GAMBARELLI G., OWEN G., *Indirect control of corporations*, International Journal of Game Theory, 1994, 23, 287–302.
[28] VAN GELLEKOM J.R.G., POTTERS J.A.M., REIJNIERSE H., ENGEL M.C., TIJS S.H., *Characterization of the Owen set of linear production processes*, Games and Economic Behavior, 2000, 32, 139–156.
[29] HERRERO C., MARTINEZ R., *Egalitarian rules in claims problems with indivisible goods*, Instituto Valenciano de Investigaciones Económicas (IVIE), WP-AD 2004-20, Valencia, Spain, 2004, 1–24.
[30] HERRERO C., MARTINEZ R., *Balanced allocation methods for claims problems with indivisibilities*, Social Choice and Welfare, 2008, 30, 603–617.
[31] MYERSON R., *Graphs and cooperation in games*, Mathematics of Operations Research, 1977, 2, 225–229.
[32] VON NEUMANN J., MORGENSTERN O., *Theory of games and economic behaviour*, Princeton University Press, Princeton 1944.
[33] NORDE H., FRAGNELLI V., GARCÍA-JURADO I., PATRONE F., TIJS S., *Balancedness of infrastructure cost games*, European Journal of Operational Research, 2002, 136, 635–654.
[34] OWEN G., *Multilinear extensions of games*, Management Science, 1972, 18, 64–79.
[35] SAMET D., ZEMEL E., *On the core and dual set of linear programming games*, Mathematics of Operations Research, 1984, 9, 309–316.
[36] SHAPLEY L.S., *A value for n-person games*, [in:] A.W. Tucker, H.W. Kuhn (Eds.), *Contributions to the theory of games II*, Princeton University Press, Princeton 1953, 307–317.
[37] SHAPLEY L.S., SHUBIK M., *A method for evaluating the distributions of power in a committee system*, American Political Science Review, 1954, 48, 787–792.
[38] THOMSON W., *Axiomatic and game-theoretic analysis of bankruptcy and taxation problems. A survey*, Mathematical Social Sciences, 2003, 45, 249–297.

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