Optimal portfolio performance with exchange-traded funds

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Abstract

In this paper, the portfolio selection problem in exchange-traded fund (hereafter ETF) markets is considered. Since the ETFs track some market indexes with lower costs than the indexes, their development and popularity is grown enormously in the last decade. Moreover, ETF characteristics also present several advantages for the investors that we briefly examine for the U.S. and European markets of ETFs. In particular, we first introduce a new performance measure consistent with the optimal choices of non-satiable risk-averse investors and then we discuss the optimization of a few performance measures on the U.S. and European ETF markets. Finally, we propose an empirical comparison among the ex-post wealth obtained by optimizing the new performance measure, the Sharpe ratio and the Rachev ratio.

Keywords

Conditional expected value, ETF, performance measure, portfolio selection.

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1. Introduction

Exchange-traded funds are among the most successful financial innovations of the last few decades. These new financial instruments were first launched as a proxy for equity indices in North America in the late 1980s. They were introduced in Europe at the end of the 1990s, and the European ETF market has continued to grow rapidly over the last decade.

It is worth noting that the European and the U.S. market are substantially different: the former is almost exclusively composed of institutional investors, while the latter consists in equal measure of institutional and retail investors. The development of ETFs in both continents is due to the many advantages of these instruments, which match a wide set of needs of different classes of investors. Specifically, an ETF is an investment fund traded on stock exchanges: it holds assets such as stocks, commodities or bonds, and it trades close to its net asset value over the course of the trading day. Most ETFs track an index, such as a stock index or a bond index.

These new financial instruments are also utilized by a growing number of fund managers as a liquid cash substitute and their attractiveness is underpinned by their relatively lower costs, speed of execution and transparency compared with traditional mutual funds. However, ETFs allow for wide, diversified portfolios, not excess returns, and, for this reason, they are particularly suitable for the retail segment.

ETFs can be easily combined according to a portfolio strategy. They may not be suitable for all investors, but can be used to construct a well-diversified and tax-efficient portfolio as well as to offer hedging opportunities.

The aims of this paper are twofold. First, we discuss the portfolio selection problem using different performance measures and propose a new performance measure (PCEV) that is consistent with the choices of non-satiable risk-averse investors. Then, we examine and compare the characteristics of ETF returns in the U.S. and European markets from 2006 to 2012. The statistical analysis of ETF returns suggests that we cannot reject the idea that ETF returns are Gaussian distributed. Therefore, the empirical evidence from the ETF market is completely different from that of stock and derivative markets, in which several anomalies (heavy tails, skewness, etc.) are generally tested and verified (see among others Fama, 1963, 1965a, b; Mandelbrot, 1963a, b; 1967a, b; Rachev and Mittnik, 2000).

Clearly, these simple observations have a strong impact on the optimal portfolio selection, which we implement practically using the U.S. and European ETFs and optimizing three different performance measures: the Sharpe ratio, Rachev ratio and PCEV. Since ETF returns have not so far been found to be Gaussian distributed, we observe a very good performance of the strategy based on the maximization of the Sharpe ratio, although all the other performance measures present satisfactory results in terms of final wealth.

The paper is organized as follows: in Section 2, we introduce a new performance measure consistent with the choices of non-satiable risk-averse investors. In Section 3, we analyse the daily log returns for ETFs in both the U.S. and the European market. Then, we compare the ex-post wealth obtained by maximizing different performance measures. The conclusion is outlined in section 4.

2. Portfolio selection with performance measures

In this section, we discuss the portfolio selection problem using performance measures and we introduce a new performance measure consistent with the choices of non-satiable risk-averse investors. Suppose that in the market there is a benchmark with return Y and n assets with vector of returns $= [R_1, \ldots, R_n]'$. Then, if we indicate with $x = [x_1, \ldots, x_n]'$ the vector of the percentage of wealth invested in each asset, a portfolio of returns is given by $x'R = \sum_i x_i R_i$.

In a reward–risk framework, the investors either maximize the reward for a fixed risk or minimize the risk for a fixed reward. Moreover, the investors optimize their performance several times, maximizing the reward for unity of risk and this strategy still gives an efficient portfolio in terms of reward and risk that is generally called a *market portfolio* (see Stoyanov et al., 2007).

In particular, the following empirical analysis compares the ex-post wealth obtained either with a new performance measure or with the well-known Sharpe ratio and Rachev ratio (see among others Sharpe, 1994; Biglova et al., 2004).

Sharpe ratio. The Sharpe ratio characterizes how well the return of an asset compensates the investor for the risk taken. The Sharpe ratio is calculated by subtracting the risk-free rate from the rate of return for a portfolio and dividing the result by the standard deviation of the portfolio returns. Formally:

$$SR = \frac{E(x'R) - r_f}{\sigma_{x'R}} \qquad (1)$$

where r_f is the risk-free return and $\sigma_{x'R}$ is the portfolio standard deviation. When comparing two assets with a common benchmark, the one with the higher Sharpe ratio provides a better return for the same risk (or, equivalently, the same return for lower risk). This measurement is very useful because, although one portfolio or fund can reap higher returns than its peers, it is only a good investment if those higher returns are not accompanied by too much additional risk. The greater a portfolio's Sharpe ratio, the better its risk-adjusted performance. A negative Sharpe ratio indicates that a riskless asset would perform better than the security being analysed.

Rachev ratio. The Rachev ratio is built using the CVaR as the risk indicator. The CVaR is the potential loss in a given period of time, with a given confidence interval (usually using a range of 95% or 99%), and can be written formally as

$$CVaR_\alpha(X) = \frac{-1}{\alpha} \int_0^\alpha F_X^{-1}(u) du. \qquad (2)$$

The Rachev ratio proposed by Biglova et al. (2004) is a typical gain–loss ratio determined using the returns CVaR in the numerator and the opposite returns CVaR in the denominator for two different confidence intervals. The mathematical formulation is defined as follows:

$$RR_{i_{\alpha,\beta}}(x'R) = \frac{CVaR_\alpha(r_f - x'R)}{CVaR_\beta(x'R - r_f)}, \qquad (3)$$

where α, β represents a measure of the aversion/tolerance to risk of the investors. This performance measure is often used when the distribution is leptokurtic because it also captures the information in the tails of the distribution. In addition, this measure is suitable for non-satiable individuals who are neither risk-averse nor risk-takers.

2.1 A new performance measure

Recall that if any non-satiable risk-averse investor prefers a random variable X to Y, then, as a consequence of the Strassen theorem (see Strassen, 1965), every non-satiable investor prefers X to $(Y|X)$. Therefore, for any given benchmark Y, the non-satiable risk-averse investors who want to outperform the benchmark Y will choose those portfolios $x'R$ that maximize the part of the portfolio above $E(Y|X)$ and minimize the part of the portfolio below $E(Y|X)$. This observation suggests a simple way to optimize the performance of a given portfolio, since non-satiable risk-averse investors maximize the random variable $(x'R - E(Y|x'R))_+$ and minimize the random variable $(x'R - E(Y|x'R))_-$, which we indicate with

$$Z_+ = \begin{cases} Z & if\ Z > 0 \\ 0 & otherwise \end{cases} \text{ and } Z_- = \begin{cases} -Z & if\ Z < 0 \\ 0 & otherwise \end{cases}. \quad (4)$$

For this reason, in this section, we suggest the use of a new portfolio performance measure called *performance based on conditional expected value* (*PCEV*), given by:

$$PCEV(x'R, Y) = \frac{E[(x'R - E(Y|x'R))_+]}{E[(x'R - E(Y|x'R))_-]}. \qquad (5)$$

Moreover, we observe that $E(Y|x'R)$ is the unique part of the benchmark Y (in the additional decomposition $= E(Y|x'R) + Y - E(Y|x'R)$) that is correlated with the portfolio $x'R$ since

$$cov(Y, x'R) = cov(x'R, E(Y|x'R)) + cov(x'R, Y - E(Y|x'R)) = cov(x'R, E(Y|x'R)). \qquad (6)$$

Thus, when we optimize the performance $PCEV(x'R, Y)$, we just consider the part of the benchmark Y that is strongly related to the portfolio and has a distance that it makes sense to optimize. In this context, we check the solutions to the following optimization problem:

$$max_x \frac{E[(x'R - E(Y|x'R))_+]}{E[(x'R - E(Y|x'R))_-]},$$
s.t. $\sum_i x_i = 1$, where $x_i \geq 0$ for $i = 1, ..., n$. (7)

In order to maximize $CEV(x'R, Y)$, we propose a consistent estimator of the random variable $E(Y|X)$. Let $X: \Omega \to \mathbb{R}$ and $Y: \Omega \to \mathbb{R}$ be integrable random variables in the probability space $(\Omega, \mathfrak{I}, P)$ and define by $\mathfrak{I}_X$ the σ-algebra generated by X (that is, $\mathfrak{I}_X = \sigma(X) = X^{-1}(\mathcal{B}) = \{X^{-1}(B): B \in \mathcal{B}\}$, where $\mathcal{B}$ is the Borel σ-algebra on $\mathbb{R}$). Notice that $E(Y|X)$ is equivalent to $E(Y|\mathfrak{I}_X)$. We can approximate $\mathfrak{I}_X$ with a σ-algebra generated by a suitable partition of Ω. In particular, for any $k \in \mathbb{N}$, we consider the partition $\{A_j\}_{j=1}^{2^k} = \{A_1, ..., A_{2^k}\}$ of Ω in 2^k subsets, where:

- $A_1 = \{\omega: X(\omega) \leq F_X^{-1}\left(\frac{1}{2^k}\right)\}$,

- $A_h = \{\omega: F_X^{-1}\left(\frac{h-1}{2^k}\right) < X(\omega) \leq F_X^{-1}\left(\frac{h}{2^k}\right)\}$, for $h = 2, ..., 2^k - 1$,

- $A_{2^k} = \Omega - \cup_{j=1}^{2^k-1} A_j = \{\omega: X(\omega) > F_X^{-1}\left(\frac{2^k-1}{2^k}\right)\}$.

Thus, starting with the trivial sigma algebra $\mathfrak{I}_0 = \{\emptyset, \Omega\}$, we can generate a sequence of sigma algebras

generated by these partitions obtained by varying k ($k = 1,\ldots, m$,$\ldots$). Thus, $\mathfrak{I}_1 = \{\emptyset, \Omega, A_1, A_2\}$, where $A_1 = \{\omega: X(\omega) \le F_X^{-1}(1/2)\}$ and $A_2 = \overline{A}_1$ and:

$$\mathfrak{I}_k = \sigma\left(\{A_j\}_{J=1}^{2^k}\right), k \in \mathbb{N}. \qquad (8)$$

Proposition. Given the sequence of σ-algebras $\{\mathfrak{I}_k\}_{k \in \mathbb{N}}$ defined above:

$$E(Y|X) = \lim_{k \to \infty} E(Y|\mathfrak{I}_k) \text{ a.s.} \qquad (9)$$

Moreover, $E(Y|\mathfrak{I}_k)(\omega) = \sum_{j=1}^{2^k} E(Y|A_j)1_{A_j}(\omega)$ a.s. *where* $1_{A_j}(\omega) = \begin{cases} 1 & \omega \in A_j \\ 0 & \omega \notin A_j \end{cases}$.

Proof: Observe that the increasing sequence of simple functions (i.e. $s_k \le s_{k+1}$):

$$s_1 = \inf X, \text{ and } s_k(w) = \sum_{j=1}^{2^k} F_X^{-1}\left(\frac{j-1}{2^k}\right) 1_{A_j}(\omega) \quad (10)$$

converges to X almost surely, *i.e.* $X = \lim_{k \to \infty} s_k$ *a.s.* Moreover, the sigma algebra generated by s_k is $\mathfrak{I}_k$ and $\mathfrak{I}_k \subset \mathfrak{I}_{k+1}$ for $k \in \mathbb{N}$. Thus, the sequence of σ-*algebras* $\{\mathfrak{I}_k\}_{k \in \mathbb{N}}$ is a filtration and $\mathfrak{I}_X = \sigma(\cup_{k \in \mathbb{N}} \mathfrak{I}_k)$. Since the family of random variables $E(X|\mathfrak{I}_k)$ is uniformly integrable, then we obtain $E(Y|X) = \lim_{k \to \infty} E(Y|\mathfrak{I}_k)$ (see among others Chung, 1974). Moreover, observe that $E(Y|A_j) = \frac{1}{P(A_j)} \int_{A_j} Y dP$. Then, assume $E(Y|\mathfrak{I}_k)(\omega) = \sum_{j=1}^{2^k} \frac{1_{A_j}(\omega)}{P(A_j)} \int_{A_j} Y dP$, that is, a $\mathfrak{I}_k$-measurable simple function. Notice that any set $A \in \mathfrak{I}_k$ can be seen as a union of disjoint sets, in particular $A = \cup_{A_j \subseteq A} A_j$. Thus, the definition of conditional expectation is verified because

$$\forall A \in \mathfrak{I}_k \quad \int_A E(Y|\mathfrak{I}_k) dP =$$
$$\sum_{j=1}^{2^k} \frac{\int_{A_j} Y dP}{P(A_j)} \int_A 1_{A_j}(\omega) dP(\omega) = \sum_{A_j \subseteq A} \int_{A_j} Y dP =$$
$$\int_A Y(\omega) dP(\omega) \qquad (11)$$

and we obtain $E(Y|\mathfrak{I}_k)(\omega) = \sum_{j=1}^{2^k} E(Y|A_j)1_{A_j}(\omega)$ c.v.d.

Observe that, given N iid observations of Y, we are able to estimate $E(Y|\mathfrak{I}_k)$, since $\frac{1}{n_{A_j}} \sum_{y \in A_j} y$ (where n_{A_j} is the number of elements of A_j) is a consistent estimator of $E(Y|A_j)$. Therefore, we are able to estimate properly the new performance measure $PCEV(x'R, Y)$ as the previous proposition suggests

that $E(Y|\mathfrak{I}_k)$ is a consistent estimator of the conditional expected value $E(Y|X)$.

3. Practical portfolio selection on the U.S. and European ETF markets

In this section, we first analyse the daily log returns for ETFs in both the U.S. and the European market. Second, we optimize the previous portfolio strategies in these markets.

The original data were downloaded from the *Thomson Reuter Datastream*. In particular, we consider 702 ETFs in the U.S. and 587 in Europe, from 1 January 2006 to 1 April 2012. We also use a liquidity filter in order to exclude from our analysis the market indices characterized by unsatisfactory volumes of transactions. Specifically, for each portfolio recalibration, we exclude those ETFs with an average daily volume lower than 10000. We use a window of 125 daily observations to determine the optimal portfolio. Then, considering the liquidity filter in the ETF market, the consecutive optimal portfolios are recalibrated every 5 days, based on the previous 125 observations: the resulting overall number of recalibration processes is 288, for both the U.S and the European market. Finally, in order to simplify the optimization process, we implement a preselection of the 100 most relevant stocks, according to the Sharpe ratio (see Ortobelli et al., 2011 for some justifications for this practice). This procedure obviously concerns both the U.S. and the European market.

3.1 Empirical evidence from ETF markets

In this section, we describe and compare, from a statistical point of view, the returns of the selected data samples for the two different stock markets. The data analysis involves many statistical indices described below, evaluated during each recalibration process. In particular, in Table 1, we summarize the average values (over the 288 recalibration processes considered) of the mean, standard deviation, skewness, kurtosis, max(max), mean(max), min(min), mean(min) and percentage of normality rejection with the Jarque–Bera test (95%).

First of all, observing the daily means of the log returns, we note that the ETFs from the U.S. market exhibit higher returns (on average) than the ETFs from the European market. The returns of the ETFs from the European market are especially lower during the

Table 1 Principal statistics evaluated over the 288 recalibration processes considered

	mean	st.dev.	sk.	kurt.	max(max)	mean(max)	min(min)	mean(min)	J.B.95
U.S.	0.0012	0.0167	−0.2474	4.7101	0.1483	0.0040	−0.1804	−0.0536	0.5697
EU	−0.0001	0.0162	−0.1339	5.5814	0.1560	0.0027	−0.1443	−0.0525	0.5715

years 2008 and 2009. This result may be due to the presence, in the U.S. market (since 2008), of innovative structured financial instruments, such as short ETFs and leveraged ETFs, that allowed ETFs to achieve optimal performances even during the subprime crisis. Since these instruments were recently introduced into the European market, we observe signs of recovery during the years 2009 and 2010.

Analysing the standard deviation of the samples, we notice that the volatility of the ETF market is quite restrained in the considered time interval, except for the worst period of the financial crisis (specifically from the second half of 2008 until the beginning of 2010). In particular, from the data it seems that the U.S. market is characterized by slightly higher volatility than the European market. Even this may be due to the diffusion (in the U.S. market) of many structured financial instruments, which allow higher returns in exchange for higher risk. However, it is well known that restrained volatility is one of the main features of the ETF market and one of the main reasons for its popularity.

Other differences between the U.S. and the European ETF market can be identified by analysing the trend of the maximum values. For this purpose, we use the indicator *mean*(*max*), which evaluates the mean of the maximum values obtained by the selected ETFs, and the indicator *max*(*max*), which evaluates the maximum among those maximum values. The empirical evidence shows that the European market yields better results than the U.S. market in periods of low volatility. On the other hand, in periods of high instability (i.e. crisis periods), the converse holds.

Similarly, we also analyse the trend of the minimum values in the samples. In particular, we use the indicator *mean*(*min*), which evaluates the mean of the minimum values obtained by the selected indexes, and the indicator *min*(*min*), which evaluates the minimum among those minimum values. The data show that, in periods of high volatility, the U.S. market yields higher (in terms of absolute values) minimum values than the European market. The contrary is true in periods of low volatility. Further, jointly observing the minimum and the maximum values, we notice that, in periods of instability, the maximum values become extremely high and, at the same time, the minimum values become extremely low. This may be due to the different kinds of ETFs available in the two markets. For instance, consider that, in a contraction period, short ETFs yield high returns (thus high maximum values), but, at the same time, the traditional ETFs yield extremely low returns (thus low minimum values). Hence, the U.S. market exhibits more accentuated variations, because it contains a greater assortment of different kinds of indices.

Finally, to validate or reject the normality assumption, the Jarque–Bera test is used. It is worth noting that, unlike most distributions used in finance, which are usually leptokurtic, skewed and characterized by heavy tails (see among others Rachev and Mittnik, 2000), the ETF market follows a Gaussian distribution. In particular, it was surprising to observe that in many cases the Gaussian assumption is accepted for more than 40% of the stocks, whereas this hypothesis is usually rejected for more than 90%. This unusual result may be due to the fact that an ETF just represents a collection of stocks, which means that the ETF market consists of an array of random variables, barely observable in the other markets. Thus, in the case of the ETF market, because of the uncommon quantity of random variables involved, the Gaussian approximation (from the Central Limit Theorem) seems to be appropriate.

3.2 An ex-post comparison

In this section, we propose an empirical comparison among the three performance measures introduced in Section 2. Portfolio optimization leads to different results depending on the performance measures adopted. Moreover, we do not use any riskless assets, so $r_f = 1$ in formulas (1) and (2). For the Rachev ratio, we use $\alpha = \beta = 5\%$. Finally, for PCEV, we use as the benchmark Y the optimal portfolio obtained with the Sharpe ratio and we approximate $E(Y|x'R)$ using the estimator $E(Y|\Im_s)$ of Proposition 1 with s = 3. For each strategy, we have to compute the optimal portfolio composition 288 times, and at the *k*-th optimization (b), three steps are performed to compute the ex-post final wealth:

Step 1 Preselect the first 100 assets with the highest Sharpe ratio among all those liquid and active in the last six months. Since this number of assets is still too large, in order to reduce the randomness of the problem, we approximate these returns by regressing them on a few factors obtained by a PCA (see, among others, Papp et al., 2005; Kondor et al., 2007; Ortobelli and Tichy, 2011).

Step 2 Determine the market portfolio $x_M^{(k)}$ that maximizes the performance ratio $\rho(W(x))$ associated with the strategy, i.e. the *ideal* solution to the following optimization problem:

$$max_{x^{(k)}}\rho(W(x))$$
$$s.t. \left(x^{(k)}\right)'e = 1; \; x_i^{(k)} \geq 0, i = 1, \ldots, n \quad (12)$$

Stoyanov et al. (2007) and Angelelli and Ortobelli (2009) observed that, except for the Sharpe problem, the complexity of some portfolio problems is much higher (such as the maximization of the Rachev ratio). In order to overcome this limit, we use Angelelli and Ortobelli's heuristic algorithm, which could be applied

to any complex portfolio selection problem that admits more local optima.

Step 3 Compute the ex-post final wealth given by:

$$W_{t_{k+1}} = \left(W_{t_k} - t.c._{\cdot t_k}\right)\left(x_M^{(k)}\right)' z_{t_{k+1}}^{(ex\ post)} \qquad (13)$$

where $t.c._{\cdot t_k}$ are the proportional transaction costs of 5 basis points that we obtain by changing the portfolio and $z_{t_{k+1}}^{(ex\ post)}$ is the vector of observed gross returns between t_k and t_{k+1}.

Steps 1, 2 and 3 are repeated for all the performance measures until some observations are available.

Portfolio selection in the U.S. ETF market

In our analysis, we first compare the optimal portfolios, obtained with the Sharpe ratio, the PCEV or the Rachev ratio. These results are also compared with the S&P500 index, the main stock benchmark of the U.S. market. Second, we compare the three different strategies in order to identify the most appropriate one for the considered market. The results are summarized and described as follows.

Using the Sharpe strategy, the ETF portfolio outperforms the benchmark S&P500, especially in 2008 and 2009, one of the most difficult periods for the U.S. economy (during this crisis, the index lost about 50%). Conversely, in the pre-crisis period, the S&P500 realized a higher return than the ETF portfolio. The reason can be explained by the nature of the Sharpe ratio: this index, preferred by risk-averse investors, allows lower losses in a bearish market but does not allow high returns in a bullish market.

Comparing the S&P500 with the portfolio selected using the Rachev ratio, we notice that this portfolio never underperforms the market, either before the crisis or during the crisis, although the results improve in the latter period. Therefore (unlike the Sharpe ratio strategy), using the Rachev ratio, the performance is never lower than the benchmark, in any market condition. This can be explained by the fact that Rachev optimization considers investors who are neither risk-averse nor risk-takers.

Finally, using the PCEV strategy, the ETF portfolio performance beats the S&P500 performance during the financial crisis and underperforms it in the previous period, following the trend observed in the Sharpe optimization. This result is quite logical, since in this case the PCEV optimization is subject to the portfolio selected by Sharpe. Furthermore, portfolios composed of ETFs benefit from the increasing presence of leveraged or inverse ETFs: such instruments allow an investor to increase either his gains or his profits from falling asset prices.

The three different strategies can be compared in Figure 1 and Table 2: on the one hand, Table 2 sum-

marizes the results by showing their mean and standard deviation of log returns, evaluated over the whole period considered; on the other hand, from Figure 1 it is apparent that each portfolio prevails in a specific market phase. On average, Sharpe optimization leads to the best portfolio being highly performing, especially in the most difficult periods of the crisis. However, at the beginning of the crisis, optimizing the PCEV performance seems to be the best strategy. In a period of strong contraction and instability, a conservative strategy such as the Sharpe strategy allows the reduction of the impact of high volatility. This feature seems to overcompensate for poorer performance in less volatile markets. On the other hand, Rachev optimization allows better results to be achieved in the period before the crisis but generates lower performance in the worst phases. The higher performance obtained by the Sharpe strategy might be explained by the preference of risk-averse investors for investments in ETFs; we conjecture that a market with products designed for less risk-averse individuals may give different results in terms of performances, and both the Rachev and the PCEV strategy could enable higher gains. However, the main explanation for the results obtained is the normal distribution of the ETF portfolios. This feature, often assumed by financial theory but rarely confirmed in real financial markets, makes these instruments appreciable by less experienced investors as well, since less sophisticated strategies (easier to implement than the Sharpe one) still give optimal results.
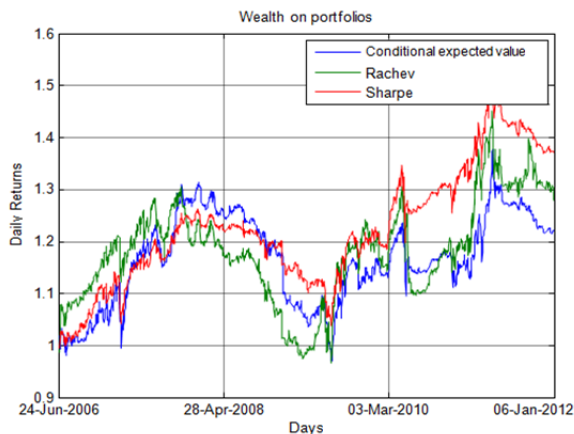
Figure 1 ETFs' PCEV, Rachev and Sharpe portfolio performances in the U.S. market

The results observed in Figure 1 are partially confirmed in Table 2, from which we see that the Sharpe strategy presents the smallest risk (standard deviation) and the highest reward (mean), while the PCEV strategy shows lower risk and reward than the Rachev one.

Table 2 Mean and standard deviation of log returns obtained by the Rachev, Sharpe and PCEV portfolio strategies in the U.S. market

	Rachev	Sharpe	PCEV
mean	0.000171	0.000225	0.000144
st.dev	0.008875	0.006215	0.007622

**Portfolio selection in the European ETF market**

Let us assume the Eurostoxx 50 index as the logical stock benchmark of the Eurozone. Regarding the European ETF market, the optimal ETF portfolio gains a better performance than the benchmark in a negative market. In particular, we find evidence of increasing returns on the ETF portfolio within 2008 and 2011 despite the global financial crisis. The three different strategies can be compared in the Eurozone in Figure 2 and Table 3.

In particular, with the Rachev strategy, investment in ETFs generates a higher performance in the crisis period, compared with the European market index. However, unlike the U.S. market, in the pre-crisis period, the returns of the Eurostoxx 50 are significantly higher. Therefore, we can conclude that this strategy is quite conservative and outperforms the market when it is in a down phase and underperforms it when the financial market cycle rises. In the case of portfolios obtained with PCEV and Sharpe, unlike the U.S. market, the returns grow continuously during the observation period and are not affected by the period of uncertainty in the market.

As shown by Figure 2, the worst performance corresponds to the Rachev strategy, while the other two strategies yield very similar returns. The portfolios selected by Sharpe optimization present a more stable performance and higher returns at the end of the considered period. The ETF portfolio obtained with the PCEV, instead, presents much better results in the months before the financial crisis of 2008. Overall, the best results are achieved by the Sharpe strategy. However, note that, unlike what was observed for the U.S. market, the PCEV strategy allows higher gains and the performances of PCEV and Sharpe increase over time.

Even these results can be explained by the ETFs' return distribution in the European market, which is closer to a Gaussian distribution. In this situation, the Sharpe index, as well as the PCEV, yields better results. Besides, the deeper impact of the financial crisis on the European markets had a positive impact on ETFs' performances, as this instrument is more protective and highly required in bearish markets. Finally, extreme (highest and lowest) returns have lower volatility in the European markets, allowing for higher returns in a negative phase. With respect to our period of analysis, we can then conclude that an ETF

investment would be optimal, since it allows better protection, obtaining higher returns that are definitely not reachable in other mature markets.
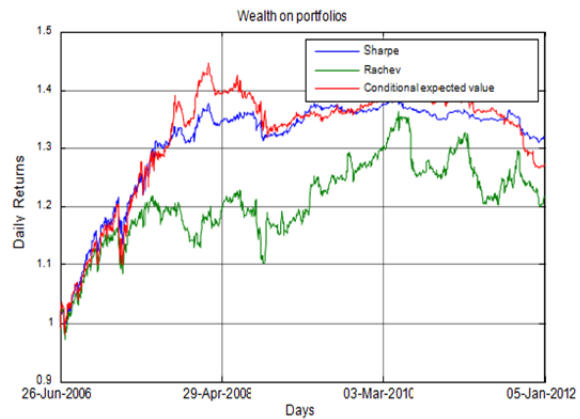
Figure 2 ETFs' PCEV, Rachev and Sharpe portfolio performances in the European market

The results observed in Figure 2 are partially confirmed in Table 3, which shows that the Sharpe strategy presents the smallest risk (standard deviation) and the PCEV strategy achieves the greatest reward (mean).

Table 3 Mean and standard deviation of log returns obtained by the Rachev, Sharpe and PCEV portfolio strategies in the European market

	Rachev	Sharpe	PCEV
mean	0.000140	0.000178	0.000183
st.dev	0.004865	0.003082	0.004189

4. Conclusions

In this paper, an analysis of the main ETF markets (U.S. and European) is proposed and an examination of the portfolio selection problem with the use of performance measures in these markets is undertaken. Moreover, a new performance measure is proposed for which the consistency property of the estimator is discussed and proved. On the one hand, from a preliminary analysis of the markets, we deduce that we cannot reject the Gaussian hypothesis of the portfolio of ETF returns. For this reason, the optimization of the Sharpe performance measure presents very good results in terms of final wealth. On the other hand, the new performance measure is very ductile and in several cases presents greater ex-post final wealth than the Sharpe ratio.

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