Multi-Population Longevity Risk

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Outline

- Chapter 1: Literature review on mutipopulation longevity risk
- Chapter 2: Multipopulation Longevity risk across Canadian provinces
- Chapter 3: Multipopulation longevity risk life expectancy across Canadian provinces
- Chapter 4: Modeling multi-population life expectancy by races

Outline

- Introduction of the context on longevity risk
- Literature review on single and multi-population
- Financial applications
- Measuring multi-population longevity risk across mortality indices in Canada

Literature

- Lee Carter Model(1992), Lee Miller(2001), Booth Maindonal Smith Variant(2002), Hyndman and Ullah(2005), De Jong and Tickle(2006), Renshaw Haberman(2006) with cohort effect, Currie(2004) with P-Splines, and Currie(2006) with Age period Cohort, Cairns-Blake-Dowd(2009).
- Darkiewicz(2004): Lee Carter validity as a cointegration approach; Lazar and Denuit(2009): common trends between 5 age groups mortality; Njenga and sherris(2011): cointegration among Heligman Pollard parameters; D'Amato(2013): Multi-Population longevity risk among countries; Sharon S. Yang et al. (2009) pricing of longevity bonds derivatives among 4 countries
- Salhi and Loisel(2010) and Zhou et al(2012) on the basis risk; Jarner and Kryger(2011).

Motivations

Here, we contribute to the modeling of multi-populations mortality indices with applications of annuities by cohorts. We model multi-population life expectancy with applications on the engineering of new type longevity bond. This work is based on multi-population rather than 1 as in the existing literature. Why multi-provinces longevity risk in general?

- Pricing of life insurance annuities accross countries or regions within a country
- Engineering of longevity bonds derivatives: EIB & BNP Paris and Swiss Re longevity bond based on mortality indices
- Survivor bond proposed by Burrow(2001) based on the age of the last survivor in the portfolio
- Hedging variations of life expectancy pattern

Methodology

- We retrieve the mortality indices produced by the Lee Carter model for the 9 mortality rates
- The determination of order of integration for each of the 9 mortality indices using the Augmented Dickey Fuller, Philips-Perron as well as KPSS Test
- The computation of the optimal value of lag of the vector of autoregressive model
- the Johansen cointegration test which test the cointegration rank and specify which variable will enter in the cointegrated equations and in the Vector of Error correction model
- The estimation of VECM and the VAR models and the forecasting of derived model.

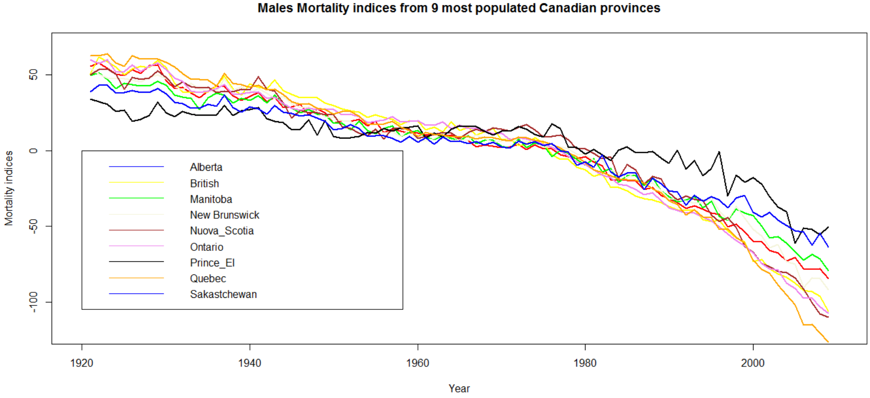
Lee Carter Model for each of 9 mortality rates groups

We retrieved the singular mortality indice from the 9 provinces through Lee Carter model.
The Lee carter Model is described as followed:

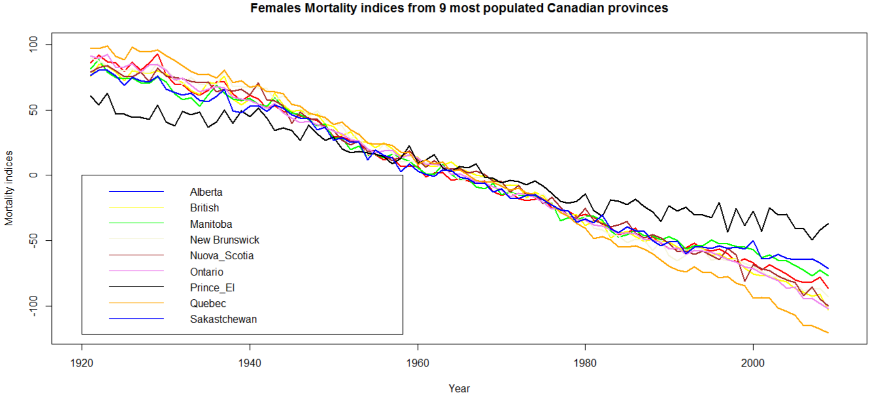
$$ln(m_1(t,1)) = a_{1,x} + b_x k_{1,t} + e_{1,t} \tag{1}$$

where:
a_x describes the shape of age profile of mortality;
b_x coefficient describes the variation of death rates to variation in the level of mortality;
k_t is the mortality index;
$e_{x,t}$ is the error term with $e_{x,t} \sim N(0, \sigma_u^2)$ is white noise which is the age feature mortality not captured by the model.

Males Mortality indices for each province in Canada



Females Mortality indices for each province in Canada



VAR and VECM models

The VAR model is derived as described below: The vector autoregression for p lags is written in Lutkepohl(2005) as:

$$k_t = \nu + \eta_1 k_{t-1} + \eta_2 k_{t-2} + ...\eta_p k_{t-p} + e_t \tag{2}$$

where $k_t = (k_{1,t}, k_{2,t}, ...k_{N,t})'$ is a N-dimentional time series, η_i are matrices with the coefficient parameters $(S * S)$, $\nu = (\nu_1, \nu_2, ...\nu_p)'$ is the intercept term, e_t is the residuals part with white noise and $t = 0, 1, ...T$ and p the last lag order..

According to Pfaff(2008), the VAR (p) can be converted into VECM as follows:

$$\Delta k_t = \Gamma_1 \Delta k_{t-1} + \Gamma_2 \Delta k_{t-2} + ... + \Gamma_{p-1} \Delta k_{t-p+1} + \Pi k_{t-p} + \nu + \varepsilon_t \tag{3}$$

where $\Gamma_i = -(I - \eta_1 + ... - \eta_i)$, for $i = 1, ...p - 1$ and $\Pi = -(I - \eta_1 - ...... - \eta_p)$.

Evidence of the cointegrated equations for Canadian provincial mortality level with critical values at 5%, 10% and 1%

Information Crtieria: HQ, SC and FPE indicate 1 optimal lags while AIC is 6. According to Lutkepohl(2005), the preference will be given to SC which is 1.

r	test value	5%	10%	1%
$r <= 8$	3.34	9.24	7.52	12.97
$r <= 7$	11.38	19.96	17.85	24.6
$r <= 6$	25.50	34.91	32	41.07
$r <= 5$	46.40	53.12	49.65	60.16
$r <= 4$	84.23	76.07	71.86	84.45
$r <= 3$	127.73	102.14	97.18	111.01
$r <= 2$	175.99	131.7	126.58	143.09
$r <= 1$	229.25	165.58	159.48	117.2
r = 0	300.68	202.92	196.37	215.74

Backtesting of the two models VAR and VECM

Out-of-samples	VAR(M)	VAR(F)	VECM(M)	VECM(F)
Portmanteau test	0.81	0.68	0.97	0.75
JB Multivariate	0.18	0.31	0.04	0.16
Skewness	0.88	0.17	0.17	0.062
Kurtosis	0.02	0.56	0.0507	0.59

Table 2: Diagnostics of residuals for VAR and VECM models in both genders cases

Backtesting of the two models VAR and VECM

Sex group	Females	Males
Out-of-samples	VAR | VECM	VAR | VECM
h=2005-2009	5.63% | 5.13%	6.85%| 5.73%
h=2002-2009	6.66% | 6.52%	9.47%|10.96%
h=2000-2009	12.89%|7.43%	8.42%|22.91%
h=1995-2009	16.38%|9.79%	10.66%|2.45%
h=1990-2009	19.36%|15.14%	29.67%|24.51%
h=1984-2009	21.77%|16.80%	39.80%|30.01%

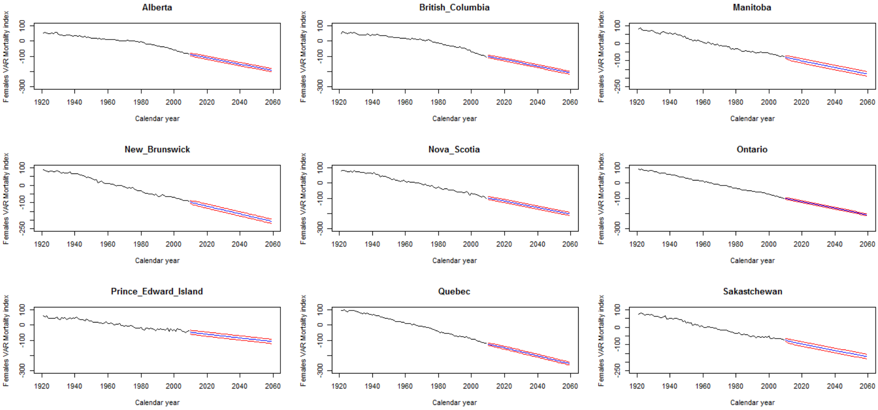
Table 3: The average MAPE for models VAR and VECM for the 9 provinces

Volatility of the two models VAR and VECM

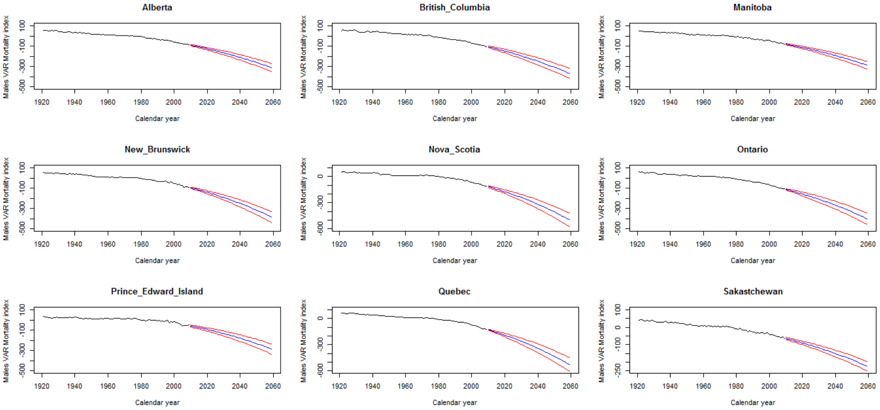
Out-of-samples	Sex	Historic	VAR	VECM
h=1995-2009	Males	166.31	37.23	48.10
	Females	98.16	91.19	78.51
h=1990-2009	Males	172.9	52.17	59.75
	Females	107.77	114.88	107.72
h=1984-2009	Males	213.93	67.46	69.44
	Females	124.45	139.94	136.18

Table 4: Comparison of volatility of historical mortality with out-of-sample forecasts produced by models VAR and VECM with in sample

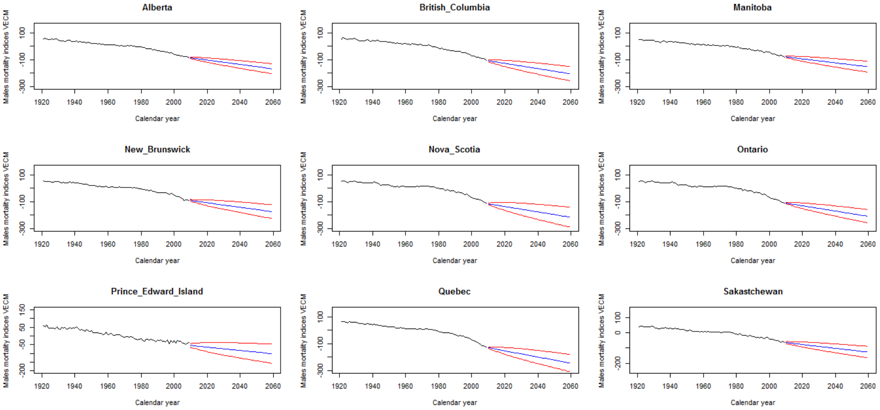
Projecting Males mortality indices for all other provinces with VAR models



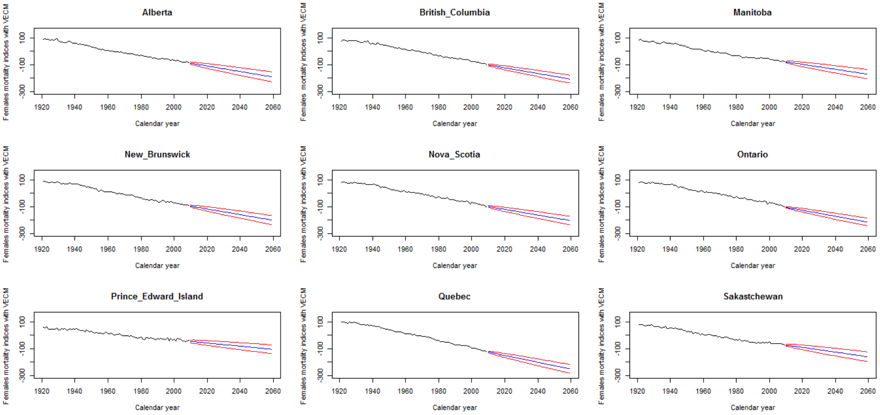
Projecting Females mortality indices for all other provinces with VAR models



Forecasting Canadian Males Mortality indices from the Vector of Error Correction model



Forecasting Canadian females Mortality indices from the Vector of Error Correction model



Pricing annuities of females cohorts 1960, 1970,1980,1990 and 2000

Here we present results from Alberta, but we have found similar conclusions as in Alberta for the other 8 involved in the analysis.

Females	ARIMA	VAR	VECM
Cohorts	Life time | APV	Life time | APV	Life time | APV
1960	16.65 | 7.85	16.73| 7.91	17.81| 8.38
1970	18.25 | 8.16	18.42| 8.23	19.5| 8.79
1980	19.56 | 8.45	19.67| 8.52	20.96| 9.14
1990	20.68 | 8.72	20.86| 8.79	22.29| 9.45
2000	21.54 | 8.97	21.7| 9.03	23.21| 9.71

Table 5: Price of annuity and life time after 65 years old from Alberta provinces cohorts 1960, 1970,1980,1990 and 2000

Pricing annuities of Males cohorts 1960, 1970,1980,1990 and 2000

Here we present results from Alberta, but we have found similar conclusions as in Alberta for the other 8 involved in the analysis.

Males	ARIMA	VAR	VECM
Cohorts	Life time | APV	Life time | APV	Life time | APV
1960	11.39 | 6.65	12.34| 7.29	12.58| 7.43
1970	13.63 |7.08	15.26 | 8.02	15.54 | 8.15
1980	15.62 | 7.49	17.89 | 8.7	18.15 | 8.81
1990	17.91 | 7.87	20.9 | 9.33	21.11| 9.4
2000	19.53 | 8.22	23.08| 9.88	23.22 | 9.91

Table 6: Pricing annuities and life time after 65 years old from Alberta province of male cohorts 1960, 1970,1980,1990 and 2000

Outline

- Introduction and context on life expectancy longevity risk
- Methodology
- Results with applications on Canadian provinces
- Introduction of new longevity bond based on province life expectancy

Literature review

- The Biological techniques such as Oeppen and Vaupel(2002)
- Extrapolative method such as Lee Carter(1992), Whitehouse(2007), Rusolillo(2005), De Beer and Alders (1999), Keilman, Pham and Hetland (2001), Alders Keilman and Cruijsen (2007)
- Torri(2011) focuses analysis on countries

In general ARIMA is described as:

$$L_t = a_0 + a_1 L_{t-1} + \varepsilon_t \tag{4}$$

where a_0 is the drift term , L_{t-1} is the time series, ε_t is the error term distributed with $\varepsilon \sim (0, \sigma^2)$

The 3 steps of the process are:

Identification: This consists of plotting data and identifying the pattern of the time series.

Order Estimation of the model: combinations of p, d, q where p is the number of autoregressive parameters d is the drift, q is the moving average parameters (q) to obtain the best model

Model Validation: checking the diagnostics of residuals from the chosen models by plotting the autocorrelation residuals and experimenting a Portmanteau test of the residuals

Model Validation ARIMA: Evaluation of the models for the forecasting uses

lags	AL	BC	NB	NS	ON	QC
4 ags	0.83	0.57	0.63	0.23	0.19	0.91
10 lags	0.55	0.54	0.39	0.092	0.55	0.91
15 lags	0.67	0.52	0.57	0.11	0.67	0.26
20 lags	0.83	0.67	0.76	0.83	0.83	0.35

Table 7: P-values results of Portmanteau test resulted from ARIMA models over the period 1921-2009

VAR and VECM models

We analyse the optimal lag length of the VAR model. Information criteria shows contradictory results: AIC and FPE indicate 3 optimal lags while HQ indicate a lag order of 2 and finally SC of only 1. Since they differ, Lutkepohl(2005), shows that the preference will be given to SC. Consequently, the lag length is 1.

Cointegrating relationship	critical values	5pct	1pct
5	3.09	9.24	12.97
4	10.29	19.96	24.60
3	32.45	34.91	41.07
2	71.31	53.12	60.16
1	118.49	76.07	84.45
0	193.08	102.14	111.01

Table 8: The cointegration relations under trace test

VAR and VECM models

Type of test	Specific name	p-values
Autocorrelation	Portmanteau(4 lags)	0.0009
Normality	Both	0.23
	Kurtosis	0.195
	Skewness	0.36

Table 9: The diagnostics tests of residuals under VAR model

Type of test	Autocorrelation	p-values
Autocorrelation	Portmanteau(4 lags)	0.0018
Normality	Both	0.0675
	Kurtosis	0.07
	Skewness	0.195

Table 10: The diagnostics tests of residuals of VECM

ARIMA, VAR and VECM models

Out-Of-Sample	VECM	VAR	ARIMA
h=2001-2009	0.29%	0.31%	5.51%
h=2002-2009	0.27%	0.40%	5.53%
h= 2003-2009	0.34%	0.26%	5.60%
h=2004-2009	0.28%	0.44%	5.62%
h=2005-2009	0.30%	0.23%	5.72%
h=2006-2009	0.28%	0.37%	5.86%

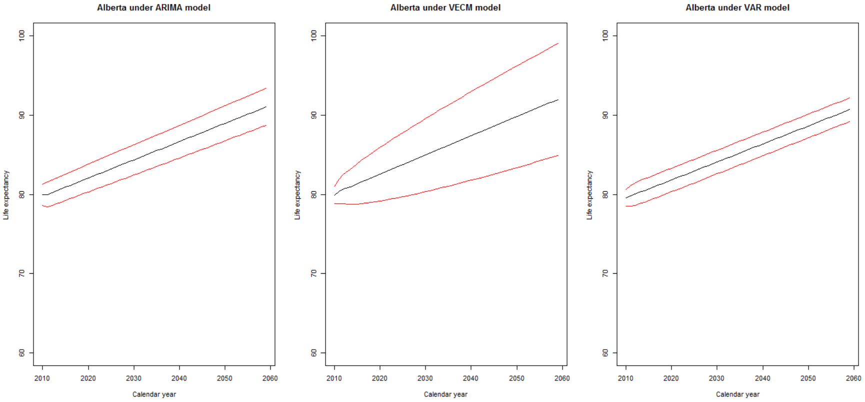
Table 11: The average MAPE for models, ARIMA VAR and VECM for the 6 provinces

VAR and VECM models

Provinces	VECM	VAR	ARIMA
Alberta	(1.04-4.58)	(1.19-1.73)	(1.20-3.44)
British Columbia	(1.07-7.06)	(1.04-1.49)	1.34-2.32
New Brunswick	(1.05-6.52)	(1.18-2.20)	(1.36-5.65)
Nova Scotia	(1.11-6.73)	(1.27-2.09)	(1.32-6.21)
Ontario	(0.65-6.40)	(0.75-1.57)	(0.83-5.88)
Quebec	(1.08-6.33)	(1.24-2.64)	(1.30-6.07)

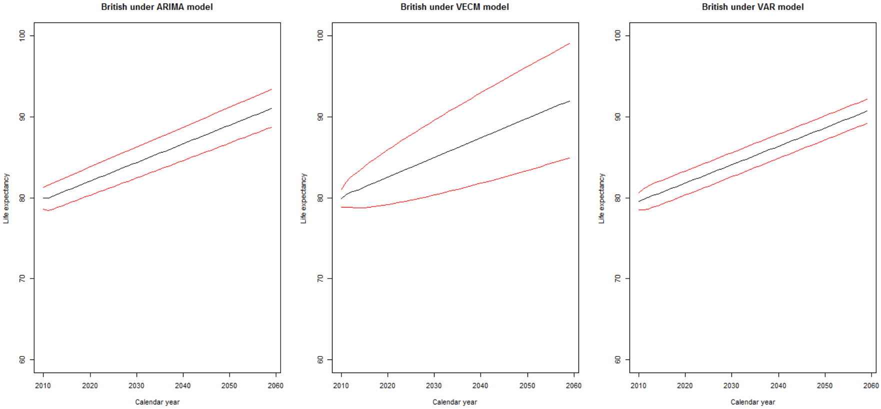
Table 12: The Confidence Interval of models VAR, VECM and ARIMA for the 6 provinces derived from predictions 50 years ahead

Projecting Females mortality indices for all other provinces with VAR models



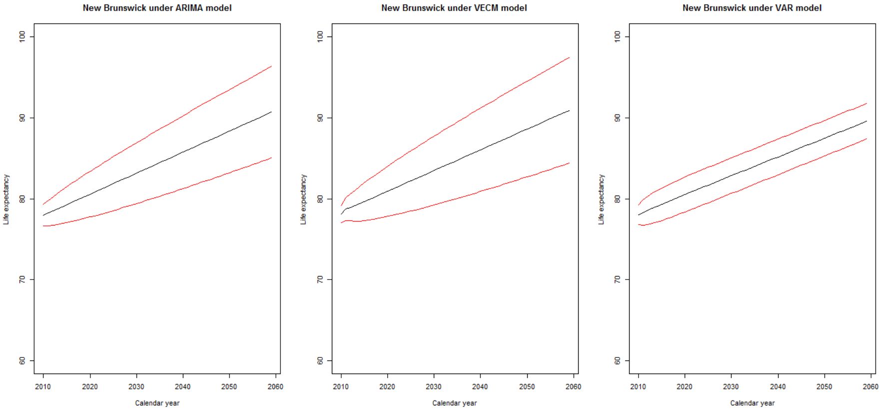
The Alberta's Confidence Interval for VECM is Greater than for VAR and ARIMA

Projecting males life expectancy for all other provinces with each model



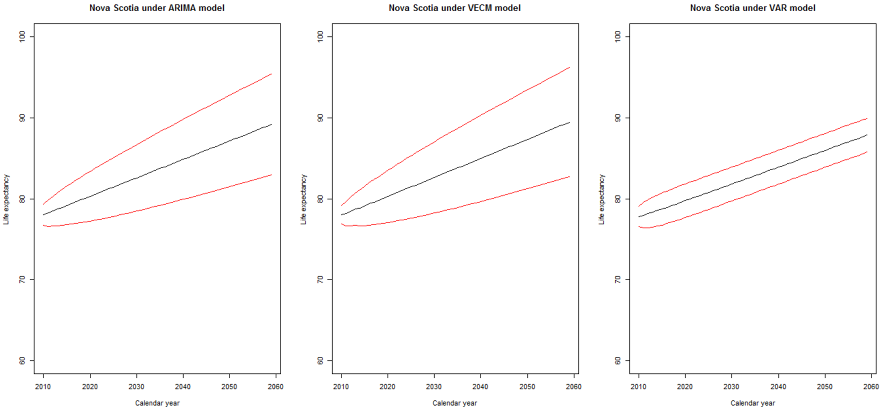
The British Columbia's Confidence Interval for VECM is Greater than for VAR and ARIMA

Projecting males life expectancy for all other provinces with each model



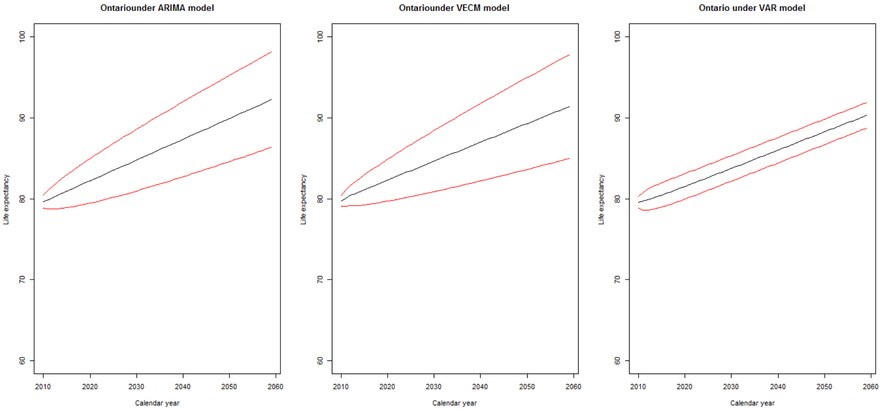
The New Brunswick's Confidence Interval for VECM is Greater than for VAR and ARIMA

Projecting males life expectancy for all provinces with each model



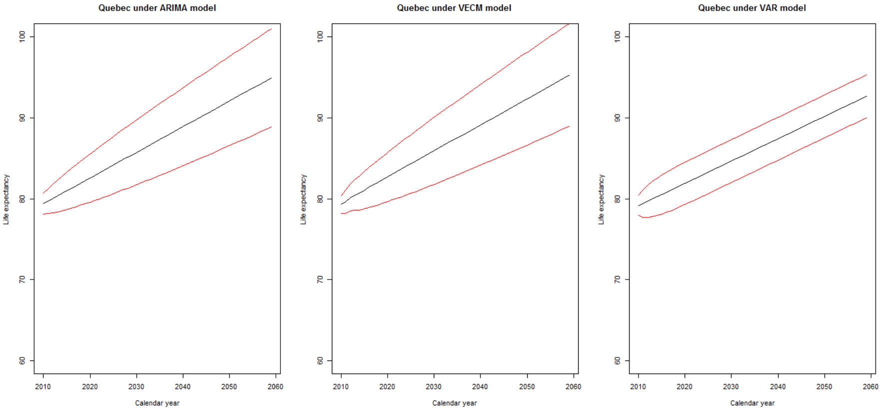
The Nova Scotia's Confidence Interval for VECM is Greater than for VAR and ARIMA

Projecting males life expectancy for all provinces with each model



The Ontario's Confidence Interval for VECM is Greater than for VAR and ARIMA

Projecting males life expectancy for all provinces with each model



The Quebec's Confidence Interval for VECM is Greater than for VAR and ARIMA

VAR and VECM models

Year	A	BC	NB	NS	ON	Q
2010	79.28	78.12	78.02	79.74	79.74	79.30
2020	81.26	82.29	80.67	80.06	82.18	82.36
2030	83.57	84.71	83.23	82.41	84.72	85.59
2040	85.89	87.13	85.79	84.75	87.26	88.82
2050	88.21	89.55	88.35	87.10	89.79	92.05
2060	90.63	91.97	90.92	89.45	92.33	95.27

Table 13: Future forecast of life expectancy with model VECM for the 6 provinces

New product based on the dynamics of life expectancy may be issued.

- No credit risk from both parties involves
- Financial markets are liquid and no basis risk

- It consists of an initial payment of X with successive payment coupon C(depending on the dynamic evolution of life expectancy) with frequency of 10 on a maturity period of 50 years to correspond to potential investors.
- We compute the variation of life expectancy of the considered regions between the period 2000-2009 which is equal to 2.2. Accordingly, pension plan would pay a certain amount C to the investors if future life expectancy is greater than 2.2.
- Coupons are discounted at rate linked to Libor. We build a bond with maturity of 50 years since shorter period do not provide effective hedging(see Dowd et al., 2006b).

Outline

- Introduction of the context on longevity risk by races in the USA
- Review on the existing literature
- Measuring multi-population longevity risk of life expectancy
- Principal component analysis to measure multi-population life expectancy
- Applications of autoregressive and multiregressive models on life expectancy
- Estimation of Vector Autoregressive model(VAR)
- The estimation of VECM the forecasting of derived model(VECM).
- Computations on future life expectancy by races in the USA.

Literature

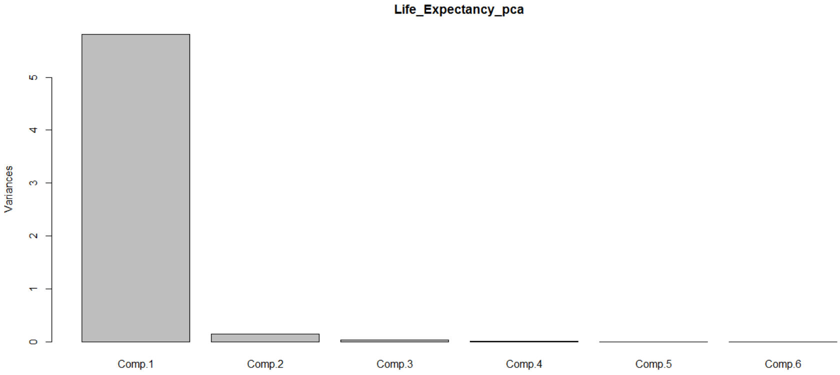
- Life expectancy is getting improved, across developped countries, as several studies such as Tulgapurkar et. al(2007) and Oeppen(2002) have shown. The United States of America are not an exception since they have highlighted also signs of improvements within their national population as well as in different races groups living in the country.
- Most work done on this topic have focused on predicting the pair black-white death rates such as Rives 1977; NCHS (1975), Manton(1980, 1982), Manton et. al(1979) Philipps and Burch(1960), Woodbury et. al(1981), Manton et. al(1979), Carter(2010).
- We will focus not only on two but on more races groups which include life expectancy from asian and latino americans.

Historical Evolution of race in the USA

Races	1910	1950	1970	2000	2010
White	88.9%	89.5%	87.7%	75.1%	72.4%
Black	10.7%	10%	11.1%	12.3%	12.6%
$American/Indian$	-0.3%	0.2%	0.8%	3.8%	4.9%
Asian	0.2%	0.2%	0.8%	3.8%	4.9%
Hispanic	0.9%	0.8%	0.1%	12.5%	16.3%

Table 14: Statistics census of American population

Projecting males life expectancy for all provinces with each model



Principal Component analysis

with estimations of models ARIMA(p,d,q) All Sex Males(ASM), All Sex Females(ASF), White Females(WF), Black Males(BM), Black Females(BF)

lags	ASM	ASF	WM	WF	BM	BM
4 ags	0.63	0.77	0.09	0.53	0.63	0.57
10 lags	0.66	0.87	0.24	0.91	0.66	0.94
15 lags	0.10	0.66	0.08	0.45	0.10	0.93
20 lags	0.11	0.59	0.13	0.11	0.11	0.75

Table 15: P-values of Portmanteau test resulted from ARIMA models over the period 1921-2009

Cointegrating relationship	critical values	5pct	1pct
5	0.64	8.18	11.65
4	8.02	14.90	19.19
3	13.19	21.07	25.75
2	19.65	27.14	32.14
1	23.58	33.32	38.78
0	57.79	39.43	46.82

Table 16: The cointegration relations under trace test

Type of test	Autocorrelation	p-values
Autocorrelation	Portmanteau(4 lags)	0.91
Normality	Both	0.77
	Kurtosis	0.55
	Skewness	0.78

Table 17: The diagnostics tests of residuals of VAR

Type of test	Autocorrelation	p-values
Autocorrelation	Portmanteau(4 lags)	0.98
Normality	Both	0.5076
	Kurtosis	0.5078
	Skewness	0.42

Table 18: The diagnostics tests of residuals of VECM

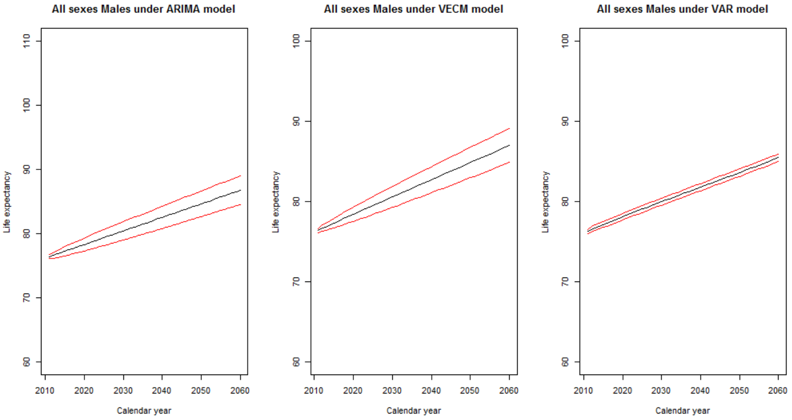
OO-Sample	VECM	VAR	ARIMA
h=2000-2010	0.5%	2.31%	5.1%
h=2001-2010	0.55%	2.3%	5.8%
h=2002-2010	0.4%	0.62%	6.2%
h= 2003-2010	1.02%	0.77%	6.41%
h=2004-2010	1.1%	0.60%	6.69%
h=2005-2010	1.39%	0.48%	7.37%
h=2006-2010	0.280%	0.62%	7.34%
h=2007-2010	0.29%	0.32%	7.9%
h=2008-2010	0.19%	0.42%	8.39%

Table 19: The average MAPE for models, ARIMA VAR and VECM for the 6 provinces

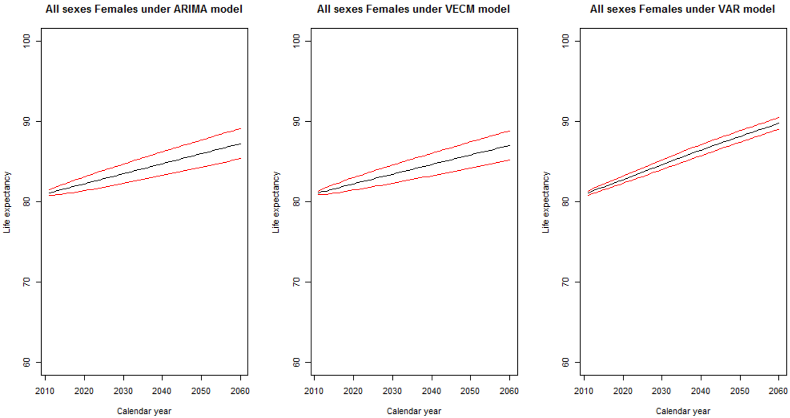
Races	VECM	VAR	ARIMA
All sexes Males	(0.23-2.13)	(0.24-0.46)	(0.31-2.24)
All sex Fem	(0.23-1.82)	(0.26-0.72)	(0.35-1.89)
White fem	(0.21-9.21)	(0.23-0.31)	(0.28-2.04)
White Mal	(0.35-5.21)	(0.23-0.62)	(0.31-3.12)
Black Femal	(0.35-7.66)	(0.80-2.17)	(0.9-6.35)
Black Mal	(1.08-6.33)	(0.40-1.68)	(0.47-4.72)

Table 20: Confidence interval of models VAR, VECM and ARIMA for the 6 provinces derived from predictions 50 years ahead

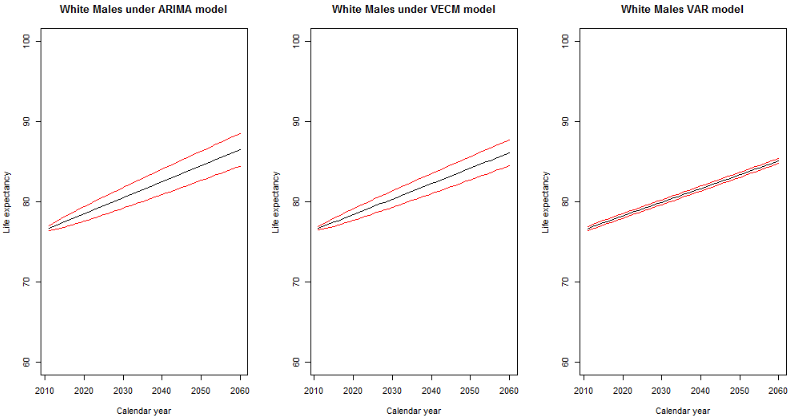
Projecting males life expectancy for each race group



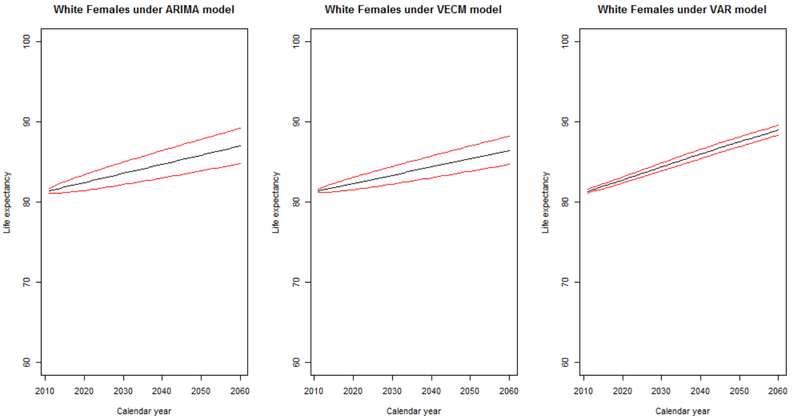
Projecting males life expectancy for each race group



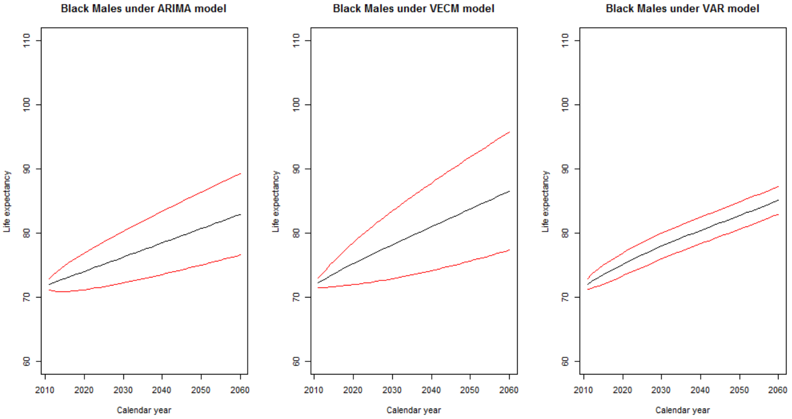
Projecting males life expectancy for all provinces with each model



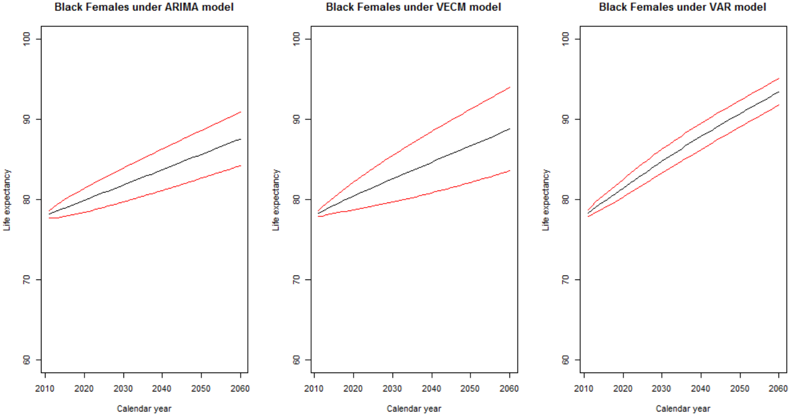
Projecting males life expectancy for all provinces with each model



Projecting males life expectancy for all races with each model



Projecting males life expectancy for all races with each model



Year	ASM	ASF	WM	WF	BM	BF
10	78.43	82.25	78.42	82.32	75.27	80.46
20	80.59	83.46	80.34	83.35	78.18	82.59
30	82.73	84.65	82.27	84.39	80.39	84.67
40	84.87	85.85	84.20	85.43	83.77	86.73
50	87.01	87.05	86.112	86.47	86.56	88.79

Table 21: Future forecast of life expectancy with model VECM for the 6 provinces

Conclusion

- We show dependence in mortality between mortality index through the cointegration analysis. We apply also the pricing of annuity by cohort. This study was published as conference proceedings.
- VECM have shown better performance over ARIMA model in backtesting samples as well as in the evaluation of error components in explaining life expectancy at birth by regions within a country. This study is also available as working paper on ssrn platform.
- We take into account the emergence of new ethnic groups such as hispanic and asians rather than only white and black.
- Measuring Basis risk between Canada national provinces and each province????? future issue we are working on.