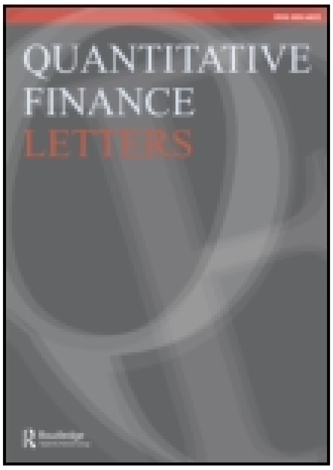

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Applying stochastic programming to insurance portfolios stress-testing

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Applying stochastic programming to insurance portfolios stress-testing

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The introduction of the Solvency II regulatory framework in 2011 and unprecendented property and casualty (P/C) claims experienced in recent years by large insurance firms have motivated the adoption of risk-based capital allocation policies in the insurance sector. In this article, we present the key features of a dynamic stochastic program leading to an optimal asset-liability management and capital allocation strategy by a large P/C insurance company and describe how from such formulation a specific, industry-relevant, stress-testing analysis can be derived. Throughout the article the investment manager of the insurance portfolio is regarded as the relevant decision-maker: he faces exogenous constraints determined by the core insurance division and is subject to the capital allocation policy decided by the management, consistently with the company's risk exposure. A novel approach to stress-testing analysis by the insurance management, based on a recursive solution of a large-scale dynamic stochastic program, is presented.

Keywords: Property and casualty liabilities; Risk-adjusted returns; Stochastic quadratic programming; Dynamic risk–reward trade-off; Stress-testing

JEL Classification: C61, D81, G22, G31

1. Introduction

Record *property and casualty* (P/C) insurance claims reported by global players in recent years (CEA 2010, Europe Economics 2009) and the introduction of the Solvency II regulatory framework (European Parliament 2009) have induced a majority of global P/C firms to increase their technical reserves and at the same time revise their capital allocation policies under growing market competition. An integrated approach to capital allocation and asset-liability management (ALM) by insurance companies has emerged as a managerial requirement and resulted in several internal modelling efforts (Mulvey *et al.* 2007, Consigli *et al.* 2011).

In this article, we present the key features of a *dynamic stochastic program* (DSP) leading to an optimal ALM by a P/C insurance company (Bertocchi *et al.* 2011, Birge and Louveaux 2011) and describe how from such formulation a specific, industry-relevant, stress-testing analysis can be derived. No reinsurance option is considered and the portfolio manager is assumed to pursue a 10-year strategy driven by a set of time-dependent performance *targets*. Throughout the article the investment manager of the P/C portfolio is regarded as the relevant decision-maker: he faces exogenous constraints determined by the core insurance (so-called *technical*) division and is subject to the capital allocation policy determined by the management, consistently with the risk exposure estimated by an independent risk management division.

The literature dedicated to dynamic asset-liability models for P/C firms is limited. After the seminal paper by Cariño *et al.* (1994) focusing, as here and in Consigli *et al.* (2011), on a real-world application of DSP techniques, a discussion on the role of reinsurance decisions by P/C managers can be found in de Lange *et al.* (2003). A rich set of contributions originates from the extended cooperation with the insurance sector by Mulvey, who concentrates in Mulvey and Erkan (2005), Mulvey *et al.* (2007) on the general structure of the decision process for multinational insurers operating in global markets and the implications on the optimal capital allocation decisions. The general relationship between risk measures and capital allocation is considered in Dhaene *et al.* (2003). In Consigli *et al.* (2011), we presented a first version of the long-term P/C portfolio problem under a set of simplifying assumptions, including the lack of any risk capital constraint, the absence of risk-adjusted reward measures and associated targets and a simplistic liability model. We follow up from those contributions and focus here on the relationship between risk capital constraints, insurance claim scenarios and risk-adjusted performance of a P/C portfolio. The introduction across the financial and insurance sectors of performance measures based on *risk-adjusted* returns (Modigliani and Modigliani 1997) is one of the relevant evidences of recent market history. Our contribution lies primarily on (i) the explicit introduction in the optimization problem of control equations on the allocated risk capital (as a function of the portfolio strategy and leading to risk capital bounds for different risk factors correlation matrices), (ii) the extension of the DSP approach to accommodate a post-optimality stress-test analysis (with stress generated by the insurance business) and the associated iterative solution method for worsening input technical conditions. From (i) the evaluation of the difference between allocated and absorbed investment risk capital along specific scenarios can also be retrieved. The following open questions are addressed: to which extent a condition of increasing trouble in the core insurance business needs to be compensated by a relaxation of the risk cap-

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ital constraints? Furthermore, in the presence of a trade-off between short-, medium- and long-term targets in the decision model, how would the targets attainability be affected by deteriorating technical ratios? We tackle these issues relying on a stochastic optimization framework. From a mathematical viewpoint the decision problem is formulated as a quadratically constrained DSP (Bertocchi *et al.* 2011).

In Section 2, we present the mathematical instance of the decision problem, while in Section 3 we discuss more in detail the definition of stressed insurance scenarios and the interaction between optimal investment processes and risk capital allocation.

2. ALM model

We consider an insurance company within an ongoing P/C business perspective: every year the company is assumed to renew its liability portfolio, collect the premiums, set the insurance reserves for future claims and compensate for current claims. The income of the technical division is determined by the premiums collected net of insurance claims and administrative and operational costs. The investment manager will benefit from the liquidity generated by the premiums and she/he will seek an optimal portfolio strategy typically under an extended set of regulatory, technical and financial constraints. For given insurance reserves and expected evolution of the technical business, the risk management division will determine the capital to be allocated against unexpected technical losses: this is the so-called *technical* risk capital that will be treated as exogenous and not under the portfolio manager control. Unlike the technical risk capital, the *investment* risk capital, reflecting the investment portfolio tail risk exposure, depends in a dynamic setting on the adopted portfolio strategy and will thus be endogenous to the optimal strategy. The company overall risk exposure is the sum of the technical and investment risk capital.

2.1. The optimization problem

We consider an ALM probem over a 10-year planning horizon, split into non-homogeneous decision stages $t \in \mathcal{T}, \mathcal{T} = \{0, 0.5, 1, 2, 3, 5, 10\}$; T is used to denote the 10-year horizon. We adopt a discrete scenario tree representation of the uncertainty underlying the decision problem, with nodal labels $n \in \mathcal{N}_t$ at stage t. For each node n in $\mathcal{N}_t$, $\mathcal{F}_n$ denotes the σ-algebra, or information set, available to the decision-maker at that node. We denote with $a(n)$ the sequence of all ancestors of node n and with $c(n)$ the nodes originating from node n up to the end of the planning horizon: this is the *subtree* originating from node n. We indicate with t_n the time stage for node n and denote the set of immediately descending, or children, nodes of n by $n+$ and its unique parent node by $n-$: for each leaf node n_T the sequence of nodes $n_T-, (n_T-)-, \ldots, n_0$ defines a scenario path.

Given the investment universe I, we denote with $X_n = \sum_{i \in I} \sum_{m \in a(n)} x_{i,m,n}$ the portfolio value in node n as the sum of all holdings of assets $i \in I$ held for periods $t_n - t_m$, for all $m \in a(n)$. Similarly, we indicate with $X_n^+ = \sum_i x_{i,n}^+$ the buying decisions in node n and with $X_n^- = \sum_{i \in I} \sum_{m \in a(n)} x_{i,m,n}^-$ sellings of assets in node n which were bought in node $m \in a(n)$. The need to keep track of holding periods is related to the inclusion of the portfolio capital gain and losses in the model formulation. Let z_n denote the cash, liquid, position of the portfolio. The management board is assumed to specify a set of investment targets $\tilde{V}^j, j = 1, 2, 3$, at the 1-, 3- and 10-year horizons respectively. Slightly abusing notation, we use $\mathcal{N}_t^j$ to specify the set of nodes associated with the jth target, while $\mathcal{F}_n^j$ is the associated information structure. Accordingly $E[V_n^j | \mathcal{F}_n^j]$ will denote the expectation of V_n^j conditional on the information set associated with that node in the scenario tree (Beraldi *et al.* 2011). In the discrete model, all nodes at a given stage are treated as equally probable in the definition of the expectation. In the optimization problem, the three targets $\tilde{V}_j$ are associated with

$V_n^1 = (\Pi_n^f + \Pi_n^t)$: the sum of financial and technical operating profits before taxes: the *operating profit*.

$V_n^2 = (\Pi_n^f - \Phi_n)\phi - \kappa K_n^f$: a surplus investment value – the *investment value created* (IVC) – determined by the financial operating profit net of a cost Φ_n and taxes ϕ, minus the portfolio cost of capital κK_n^f. Here κ is a constant cost of capital coefficient and K_n^f, as specified below, is the portfolio investment risk capital in node n. The quantity Φ_n reflects a specific cost of funding assumed to be charged by the actuarial division on the investment manager to compensate, at the current short-term interest rate, for the transfer of resources induced by growing insurance liabilities. From V^1 we denote with

$V_n^3 = V_n^1 \phi$: the company overall profit after taxes. From V_n^3 with associated target $\tilde{V}^3$, the company *return on risk-adjusted capital* (RoRAC) target can be specified as $Z_n = V_n^3 / K_n$: this is the ratio of the total profit to the company overall risk capital K_n (see below), with an associated target $\tilde{Z}$.

Both V_n^2 and Z_n are considered risk-adjusted rewards based on K_n^f and K_n, respectively (Modigliani and Modigliani 1997). The former is a performance indicator specific to the portfolio manager responsible for generating a reward sufficient to compensate the investment risk exposure and the internal cost of funding. The latter instead focuses on the company overall risk-adjusted performance. The financial operating profit Π_n^f represents the key variable for the portfolio manager. It is the sum of the portfolio income returns and the capital gains:

$$\Pi_n^f = X_{n-}\xi_n + \sum_{m \in a(n)} X_{m,n}^- \gamma_{m,n} + \Pi_{n-}^f,$$

where ξ_n denotes the portfolio cash return in node n and $\gamma_{m,n}$ the capital gain and loss coefficients on selling decisions. The other stochastic coefficients of the DSP are clarified in Section 2.2.

We solve the following optimization problem, based on a convex trade-off between a growth criterion and a penalty function defined with respect to the targets $\tilde{V}_j$ for $j = 1, 2, 3$:

$$\max_{x \in X_n} \left\{ (1-\alpha) \sum_j \lambda_j E[V_n^j | \mathcal{F}_n^j] - \alpha \sum_j \lambda_j E[\tilde{V}^j - V_n^j | \mathcal{F}_n^j] \right\} \tag{1}$$

subject, for $t \in \mathcal{T}$, $n \in \mathcal{N}_t$, to the following constraints:

(1) The portfolio inventory balance constraints:

$$X_n = X_{n-}(1+\rho_n) + X_n^+ - X_n^- + z_n. \tag{2}$$

(2) The cash balance constraints (3), which include the cash balance z_n and accumulated interests, the P/C premiums R_n, treated as exogenous and driven by market competition, the insurance claims L_n, the associated operational costs C_n and cash payments generated by the holding portfolio X_n:

$$\begin{aligned} z_n &= X_n^- - X_n^+ + R_n - L_n - C_n \\ &\quad + X_{n-}\xi_n + z_{n-}(1+r_n). \end{aligned} \tag{3}$$

(3) A maximum turnover constraint (4) under which in node n buying and selling decisions cannot exceed a certain proportion ϑ of the current portfolio value:

$$X_n^+ + X_n^- = \vartheta X_{n-}(1+\rho_n). \tag{4}$$

(4) An investment risk capital constraint (5) under which at any point in time the value X_n of the portfolio, net of the reserves Λ_n and the technical risk capital K_n^t must be sufficient to hedge against unexpected investment losses:

$$K_n^f \le X_n - \Lambda_n - K_n^t, \tag{5}$$

Λ_n are statutory inflation-adjusted reserves allocated at time t_n which are consistent with the reserves' so-called *run-off* profile: $\Lambda_n = \sum_{\tau \in [1,\hat{\tau}]} \Lambda_{t_n,\tau}$, where $\hat{\tau}$ denotes the extinction year (or end of the run-off period) of the insurance reserves allocated at time t_n for all future claims from year 1 to $\hat{\tau}$. The constraint in equation (5) provides an upper bound to the capital that can be allocated to the investment division and can be regarded as a default boundary for the overall P/C business. We assume $K_n^t = \Lambda_n \cdot \kappa^t$, where κ^t is a predetermined constant risk multiplier. $K_n = K_n^f + K_n^t$ defines the company global risk exposure in node n. At the root node we have $K_{n_0}^f = K_{n_0}^t = K_{n_0} = 0$.

The DSP solution implies for given α the maximization of equation (1) subject to the constraints (2)–(5).

2.2. *Scenario generation*

All investment positions $X_{i,n} \in I$ in node n are defined in terms of current market values: the optimal strategy is determined at each stage in terms of portfolio rebalancing decisions expressed in market values along the scenario tree and up to the beginning of the last stage. No rebalancing is allowed at T. A distinction between price and income returns for each investment opportunity is introduced in the model: $\rho_{i,n}$ is a price return in node n for asset i per unit investments, computed as $\rho_{i,n} = (v_{i,n}/v_{i,n-}) - 1$, where $v_{i,n}$ denotes the clean price of benchmark i in node n. Cash returns are computed from income coefficients (coupon rates, dividend yields and rental rates) $\xi_{i,n}$ taking accrual times into account, from holdings in the parent node. The scenario coefficients ρ_n, r_n, ξ_n and $\gamma_{m,n}$ in problem (1)–(5) define, for ease of mathematical representation, the portfolio price return, the interest rate on the money account, the portfolio income return and the capital gain and losses, respectively.

The investment universe includes a set of global market indices for treasuries with different maturity buckets, inflation-linked bond indices, corporate bond indices for speculative and investment grade positions, equity and private equity investments, direct and indirect real estate and, finally, a set of alternative investments including renewable energy, infrastructures and commodities.

As for liability scenarios, these are generated exogenously by the insurance division through the so-called *chain-ladder* methodology. The insurance claims and the reserves run-off profiles over the 10-year horizon are then inflation-adjusted from an underlying evolving inflation process.

From a methodological viewpoint scenarios are generated through a set of dedicated economic and financial models employing a Monte Carlo procedure adapted to the scenario tree conditional structure. For an overview of scenario generation methods, we refer to Bertocchi *et al.* (2011).

We focus next on the dependence of the investment risk capital evolution on the adopted portfolio strategy.

2.3. *Investment risk capital control*

According to regulatory and operational standards (Mulvey *et al.* 2007), the risk exposure K_n^f of the portfolio manager is assumed to carry two components: $K_n^{f_1}$ represents a linear risk exposure induced by an asset-liability duration mismatching and $K_n^{f_2}$ reflects the portfolio market risk exposure:

$$K_n^f = K_n^{f_1} + K_n^{f_2}, \tag{6}$$

$$K_n^{f_1} = (\Delta_{n-}^I - \Delta_{n-}^\Lambda) dr_n (t_n - t_{n-}) + K_{n-}^{f_1}, \tag{7}$$

$$K_n^{f_2} = \sqrt{\sum_{i \in \mathcal{I}} \sum_{j \in \mathcal{I}} X_{i,n-} X_{j,n-} k_{ij} (t_n - t_{n-}) + K_{n-}^{f_2}}. \tag{8}$$

In equation (7), $\Delta_n^I = \sum_i \sum_{m \in a(n)} x_{i,m,n} \Delta_{i,n}$ and $\Delta_n^\Lambda = \sum_{T > t_n} \Lambda_{n,T} \cdot (T - t_n)$ are the asset and liability durations in node n, respectively. In equation (8), k_{ij} are defined as the product between the risk charges k_i and the correlation coefficients ζ_{ij} between assets i and j : $k_{ij} = k_i \cdot k_j \cdot \zeta_{ij}$. The risk charges k_i and the correlation matrix ζ with elements

ζ_{ij} can either be estimated through appropriate statistical procedures or, as we assume in the sequel, set by the regulatory body. We observe that following Solvency II standards, the k_i's are defined between 0 (lack of extreme risk: treasuries, fixed income positions) and 0.40 (maximum tail risk exposure: alternative investments, commodities), while the correlation matrix is positive definite. In the numerical study, we consider three specifications for ζ: as a matrix of all 1 (perfect and positive risk factors correlation), as an identity matrix (uncorrelated risk factors) and as the regulatory matrix (correlation coefficients between 0 and 1). From an economic viewpoint, K_n^f for $n \in \mathcal{N}_t$ defines the risk capital *absorbed* or *consumed* by the investment strategy while the right-hand side in equation (5) represents the maximum risk capital that in a dynamic setting can be allocated to the portfolio manager. The former cannot exceed the latter from a risk management perspective.

In our numerical application, the stochastic program (1)–(5) is solved replacing k_{ij} in equation (8) with $\tilde{k}_{ij}$, a rescaled matrix to preserve feasibility with respect to constraint (5):

$$K_n^f = K_{n-}^f + \left[K_n^{f_1} + \sum_i \sum_j X_{n-}^i X_{n-}^j \tilde{k}_{ij}(t_n - t_{n-}) \right]. \quad (9)$$

The definition leads to a quadratically constrained stochastic program (QCSP). After solution and given the optimal policies along each scenario we recompute the investment risk capital consistently with equation (6) and derive the appropriate values of V^2, V^3 and Z for further analysis.

3. Stressed technical scenarios and risk capital

The *combined ratio* $\Gamma_n := (L_n + C_n)/R_n = L_n/R_n + C_n/R_n = l_n + c_n$ of claims plus operational costs to premiums is regarded as a key measure of P/C business efficiency and reflects the core activity profitability. Γ_n is expected to remain on average below 1. However, during the last years several large P/C companies have been confronted with combined ratios frequently close to one, resulting into increasing pressure on the investment strategy. L_n in insurance practice is often estimated as a certain constant percentage of expected premiums: $L_n = l \times R_n$. As l (an input to the scenario generator) increases so will the claims and the associated reserves Λ_n. Such increase will impact both the IVC, due to an increasing cost of funding – Φ_n in the variable specification – and the RoRAC, due to an increasing technical risk capital K_n^t and decreasing operating profit.

Consider the following formulation of the financial risk upper bound:

$$K_n^f \leq \hat{\chi} \cdot [X_n - \Lambda_n - K_n^t]. \quad (10)$$

In constraint (5) $\hat{\chi}$ was assumed to be equal to 1. Here we allow $\hat{\chi}$ to vary and in the case study below we allow the coefficient to increase from 0.6 to 1.2. A coefficient below 1 will force the portfolio manager to follow a very conservative and non-risky strategy, while the opposite is true for $\hat{\chi}$ increasing above 1. For different specifications of $\hat{\chi}$, it is of primary importance to the P/C management – and the focus of this analysis – the evaluation of the impact of stressed technical conditions on the IVC and the RoRAC dynamics.

3.1. *Stress-testing methodology*

For given goals $\tilde{V}^2$ and $\tilde{Z}$ on the IVC and RoRAC, respectively, the stress-testing application is based on the evaluation through a sequence of DSP solutions of the impact of an increasing loss ratio l – claims per unit premiums – on the risk-adjusted returns, while relaxing through $\hat{\chi}$ the risk capital constraint.

Let in particular $\Upsilon(l_s, \hat{\chi}_s, 1, \Omega)$ denote the stochastic program solution for given loss ratio l_s and risk capital coefficient $\hat{\chi}_s$, with $\alpha = 1$ in equation (1) and fixed number of financial and insurance scenarios Ω. We wish to evaluate numerically along a representative mean scenario the surfaces $V_s^2 = f(l_s, \hat{\chi}_s)$ and $Z_s = f(l_s, \hat{\chi}_s)$ of the risk-adjusted returns to increasing loss ratio and risk capital tolerance. We analyse $\Upsilon(l_s, \hat{\chi}_s, 1, \Omega)$ for different input values $l_s = \{0.52, 0.56, 0.59, 0.63, 0.66, 0.7\}$ and $\hat{\chi}_s = \{0.6, 0.72, 0.84, 0.96, 1.08, 1.2\}$ on a sequence of problem instances with 768 scenarios and a branching structure of $[6^1 4^2 2^3]$. The weights $\lambda_1 = 0.5$, $\lambda_2 = 0.2$ and $\lambda_3 = 0.3$ are considered as default settings in the objective function, consistently with insurance practice (Consigli *et al.* 2011). The following coefficients are also considered in the case study: a turnover constant $\vartheta = 0.3$, the technical risk capital multiplier $\kappa^t = 0.17$ and the cost of capital $k = 8\%$. Targets (figure 1) are set at the 3-year horizon to $\tilde{V}^2 = 400€$ per year, assumed to be roughly 0.5% of the initial portfolio value, and at the 10 years to $\tilde{Z} = 14\%$, an RoRAC target.

3.2. *Results*

We summarize first the results collected from the solution of one instance of the DSP ((1)–(5)), for $l = 0.6$ and $\hat{\chi} = 1$, and then analyse the evidences emerging from the stress-testing exercise.

All results have been collected on an HP workstation with Intel® Core i3-3220-CPU at 3.30 Ghz processor and 5 Gb of RAM running Windows 8 OS and 453 GB of hard disk. The model has been implemented in the algebraic modelling language GAMS 21.3, with scenarios, graphical user interface and main program all in Matlab R2011b. We have generated the deterministic equivalent (DetEqv) in GAMS and selected the Cplex solver for large-scale quadratically constrained problems.

The following analyses are relevant for our purposes and can be retrieved from the DSP optimal solution:

(1) for given targets we wish to assess the time evolution of $(\tilde{V}^2 - V_n^2)$ and $(\tilde{Z} - Z_n)$ at the target stages and over the planning horizon along a representative scenario and
(2) estimate the investment risk capital absorption K_n^f relative to the maximum risk exposure in equation (10).

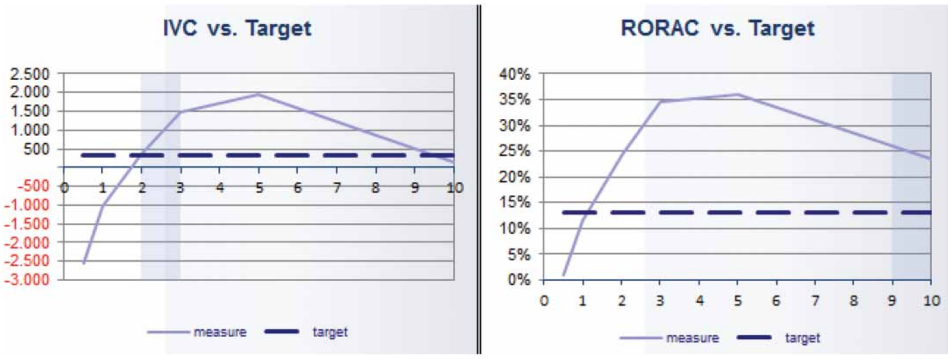

Figure 1. IVC and RoRAC behaviour along a representative mean scenario.

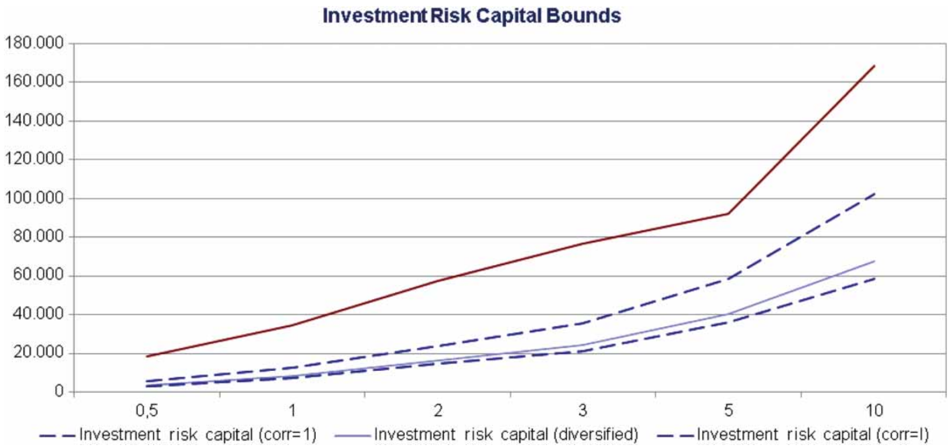

Figure 2. Investment risk capital requirements for different correlation matrices ζ.

From an economic standpoint, the higher the return generated by the portfolio manager, the lower is the required risk capital, the higher is the company long-term profitability. In the presence of increasing insurance costs, due to a worsening combined ratio, increasing investment returns are welcome only if the absorbed risk capital does not compensate such increase. The investment risk capital is evaluated at each stage and cumulates from time 0 up to the 10-year horizon. The focus is on a representative mean scenario, identified as detailed in Consigli *et al.* (2011) by the scenario and portfolio policy leading to a mean operating profit trajectory. A shadow area highlights in figure 1 the target stages. The plots show along the specified scenario the dynamic of the IVC and RoRAC with respect to their targets. Only in the short term both reward measures remain below their targets, when however the portfolio manager is concerned primarily with the end-of-year-1 financial profit target $\tilde{V}^1$.

The same underlying optimal portfolio policy generates under the three different specifications of the correlation matrix ζ (see equation (6)) the investment risk capital requirements plotted in figure 2. The thin solid line is associated with the regulatory correlation matrix consistent with the Solvency II framework. The two dotted lines are instead associated with the case of perfect positive correlation between risk factors (upper line) and uncorrelated risk factors (lower line).

Under any specification of the correlation matrix the investment risk capital *absorbed* by the portfolio policy is lower than the maximum risk exposure specified in equation (5) and displayed in red in figure 2.

The stochastic program can be solved recursively for different specifications of the loss ratio to perform stress-testing: a new set of liabilities is derived at each run and input to the optimization problem. At the same time we relax the investment risk capital upper bound and analyse along the representative mean scenario the dynamics of the IVC and RoRAC surfaces, displayed in figures 3 and 4.

Consider V^2 first. As the loss ratio increases, so will the endogenous cost of funding: the IVC will deteriorate if, for given income taxes and exogenous cost of capital, the portfolio manager will be unable to push portfolio net returns up without increasing the portfolio cost of capital. From figure 3 as the loss coefficient increases the IVC at the three-year horizon decreases: the derivative is negative and decreasing rapidly as the loss ratio reaches 0.65. On the other hand, V^2 appears relatively independent from the allocated risk capital, suggesting that to achieve the specified target, as the loss ratio deteriorates, the portfolio manager will not find convenient to move into riskier strategies, which would require incremental investment risk capital.

We can also evaluate the dependence of the IVC on the loss ratio only, through the Matlab interactive interface: for $l = \{0.5, 0.6, 0.7\}$, for instance, the IVC achieved

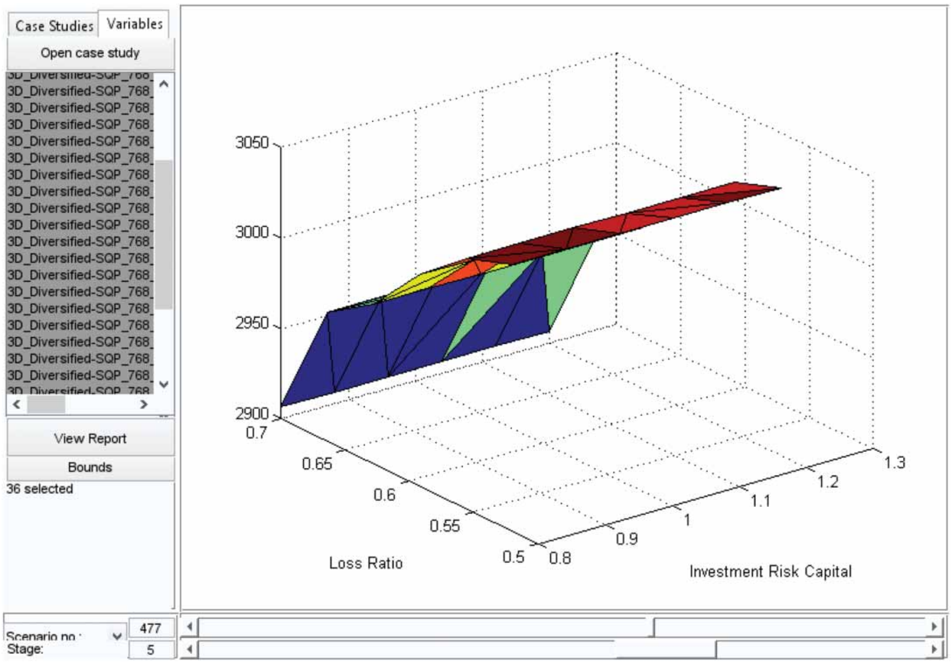

Figure 3. IVC and loss ratio–risk capital trade-off.

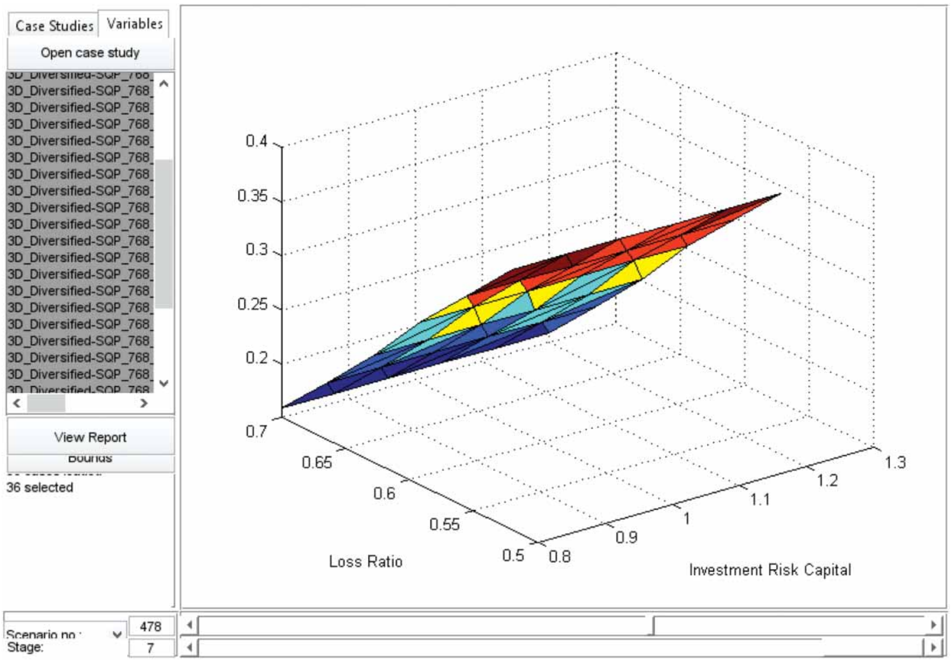

Figure 4. RoRAC and loss ratio–risk capital trade-off.

by the strategy at the three-year horizon changes, respectively, to $V^2 = \{3021, 3003, 2927\}$. Taking all runs into account, we can fit the surface through a nonlinear spline and with sufficient accuracy derive an estimate of the IVC–loss ratio first-order derivative $\partial V^2/\partial l = -5010\,l + 3010$, which depends on the loss ratio l. Through this approximation the insurance management can evaluate the impact of an increasing loss ratio on the IVC achieved by the optimal policy under an average financial market scenario.

A different set of evidences comes from the estimation along the same average scenario of the RoRAC surface against the loss ratio and the investment risk tolerance. The RoRAC is very sensitive to the change of technical coefficients and indeed as the loss ratio deteriorates an increasing limit risk exposure is exploited by the portfolio manager to achieve higher risk-adjusted returns and reduce the target shortfall.

We can again evaluate the dependence of the RoRAC on the loss ratio and the risk tolerance coefficient through the Matlab interactive interface: for $\{l; \hat{\chi}\} = \{0.5; 1.2, 0.6; 1, 0.7; 0.8\}$ the RoRAC achieved by the strategy changes, respectively, to $\{0.38; 0.31; 0.20\}$. Interestingly the 3D surface can be fitted in this case with a hyperplane with equation $Z = 0.864\,K^f - 0.55\,l$, with estimated first-order partial derivatives $\partial Z/\partial l = -0.55$ and $\partial Z/\partial K^f = 0.864$. Contrary to the previous case, the relaxation of the investment risk capital constraint is in this case exploited to compensate the stress generated by increasing losses on the P/C business.

The evidence collected on the sensitivity of the IVC and the RoRAC to worsening technical conditions depends to a certain extent on the financial and insurance scenarios considered in the problem. For given scenarios, however, the different behaviour of the IVC and RoRAC can be related to the different risk exposure considered by the two indicators, relative to the investment risk K_n^f (which is endogenous to the optimal strategy) for V^2 and to the total risk capital K_n for Z. In the first case, in the presence of increasing cost of funding the search for profit by the portfolio manager may be jeopardized by an increasing cost of capital: this is the evidence emerging from our study.

The estimation of the above surfaces through repeated solutions of the DSP problem and the derivation of an analytical relationship between risk-adjusted returns, loss ratio and risk capital tolerance allow the management to evaluate the business forward evolution and adapt medium- and long-term business plans to alternative insurance scenarios. The accurate estimation of an average, representative stress-testing surface is a computationally very challenging application and extensive numerical work is needed to derive a robust surface estimate, a requirement for the subsequent fitting procedure, here briefly outlined, and the stress-testing analysis.

4. Conclusions

Following the historic change of the insurance world towards risk-based capital allocation policies and evidence of stressed conditions in the core business in several advanced economies, we have presented an approach based on dynamic stochastic programming to evaluate how exogenous changes of loss ratios and relaxed risk capital constraints would affect P/C portfolios risk-adjusted performances. A stress-testing analysis has been derived from the iterative solution of a sequence of DSPs leading to the definition of IVC and RoRAC sensitivity surfaces. In the presence of deteriorating insurance coefficients, under the introduced modelling framework and set of assumptions, the following evidences are worth recalling:

- The investment value surplus decreases as the loss ratio increases and an increase of the allocated risk capital would not be beneficial.
- The RoRAC, instead, has shown a negative linear dependence on the loss ratio, compensated in this case by an increasing risk capital absorption.
- The QCSP problem formulation has facilitated the adoption of a decision paradigm taking into account risk factors correlation and intertemporal trade-off between short-, medium- and long-term targets leading to optimal strategies with minimal risk capital requirements over a 10-year horizon.

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