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Looking for changepoints in spatio-temporal earthquake data

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Abstract. *This work presents an application of a new method for changepoint detection on spatio-temporal point process data. We summarise the methodology, based on building a Bayesian hierarchical model for the data and priors on the number and positions of the changepoints, and introduce two approaches to taking decisions on the acceptance of potential changepoints. We present the dataset collecting Italian seismic events over 30 years and show results for multiple changepoint detection. Finally, concluding comments and suggestions for further work are provided.*

Keywords. *earthquake data; changepoint analysis; spatio-temporal point processes; log-Gaussian Cox processes*

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1 Introduction

This work provides an application of new methodology for changepoint analysis on spatio-temporal point process data as proposed in [1]. The case study consists of all Italian seismic events exceeding a specific magnitude recorded in the last 30 years.
The collected data are provided by INGV (the National Institute of Geophysics and Vulcanology) and are free to download at `http://terremoti.ingv.it/it/`. They are published in real time and cover all seismic events from January 1985 onwards. For each event, the spatial coordinates, the hypocentre depth and the magnitude are reported. Data come from 390 monitoring stations located over the Italian territory, which operate 24 hours a day, 7 days a week. We analyse a set of 19774 events of magnitude 2.5 and above (earthquakes below this limit are not felt by people). The study period covers from January, 1985 to December, 2014. A map of the hypocentre locations is presented in Figure 1. We split the dataset into yearly patterns and obtain a time series of spatial point processes (where timepoints are years) with a number of seismic events ranging from 304 to 1592, with an average of 659 per year.
A changepoint analysis can answer many questions concerning the evolution of the seismic phenomenon over the Italian territory. Issues that need to be met are listed in many recent articles in the INGV website and highlight concerns about changes occurring in the distribution and magnitude of earthquakes. Since it is sensible to assume spatial correlation and temporal dependence among the events, the development of a methodology able to face such a complex dataset now allows these questions to be answered. Secondly,

as stated in [1], there is a need to provide a proper application of the new method. Indeed, the motivating case study presented in [1], though interesting, is limited as regards the length of the time series ($T = 15$) and the low number of events in some years. We aim here to show a more complex case study and to answer practical questions about changes in earthquake phenomena.

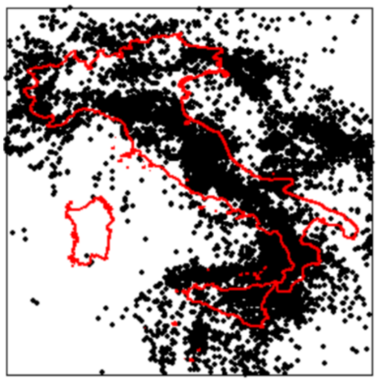

Figure 1: Seismic events of magnitude≥2.5, 1985-2014.

2 Changepoint detection on spatio-temporal point processes

At every time point the datum consists of a realisation of a spatial point process, therefore different types of change over time may occur: a change in scale (expected number of points), in spatial distribution or in both. Moreover, in real situations issues of spatial dependence among points and temporal dependence within time segments must be considered. Recent work [1] has developed a new Bayesian method for the detection of an unknown number of temporal changes over a spatio-temporal inhomogeneous point process where spatial and temporal dependence within time segments are allowed. The validity of the method has been assessed in a thorough simulation study, and it has been shown to be able to detect different types of change. The use of INLA [4] to compute the segment marginal likelihoods makes the approach computationally tractable.

In a nutshell, the method consists in choosing a model and fitting it multiple times to the dataset assuming different changepoint positions. Every time a changepoint is assumed at a timepoint $\theta = 1, \dots, T$, the data vector is split into two segments based on the changepoint location and the model is fitted separately to the two segments (independence across segments is assumed here). Two segment log-likelihoods values are obtained and summed to give the marginal log-likelihood conditional on θ. For different changepoint locations, a vector of log-likelihoods is computed. The posterior distribution of the changepoint location is obtained via the Bayes Rule by multiplying the log-likelihood vector for a vector of prior probabilities over the changepoint positions θ. Once a posterior probability is obtained for every time point, decisions must be made as to which changepoints are to be accepted. For a multiple changepoint search, we implement a binary segmentation algorithm as in [2], i.e. an iterative procedure which looks for a single changepoint for the whole dataset and, if found, iteratively splits the data at the changepoint dealing with the resulting segments separately until no more changes are detected in any segment. This procedure can be matched with either method for a single changepoint detection proposed in [1].

1. The **Bayes Factor method** (BF): a changepoint is found in location θ^* iff $\gamma = \pi(\theta^*) + l_1^* - l_0 > 0$, where θ^* is the location returning the highest marginal log-likelihood, $\pi(\theta^*)$ is the prior probability assigned to that value, l_1^* is the corresponding log-likelihood and l_0 is the log-likelihood under the null hypothesis of no changepoint.

2. The **Posterior Threshold method** (PT): a threshold is chosen and if there are posterior probability values above the threshold, the highest peak marks the detected changepoint location. For discussion about the choice of the threshold, we refer to [1].

3 A Log-Gaussian Cox Process for earthquake data with changepoints

A Bayesian changepoint model needs prior settings on number and positions of the changes, plus a hierarchical model for the data segments. We look for an unknown number of changes at unknow timepoints. We take a uniform prior for the number $m = 1,\ldots,M$ of changepoints and we assume a minimum segment length of d time points in order to avoid unrealistic adjacent changes. Considering that changepoints are looked for sequentially, our prior setting can be written as

$$\begin{aligned} \pi(m) &= (M+1)^{-1} \text{ for } m = 0,\ldots,M \\ \pi(\theta_1,\ldots,\theta_m|m) &= \pi(\theta_m|\theta_{m-1},m)\pi(\theta_{m-1}|\theta_{m-2},m)\ldots\pi(\theta_1|m) \text{ where } \pi(\theta_1|m) = (T-2\times d)^{-1} \end{aligned} \tag{1}$$

The conditional priors for $\theta_2,\ldots,\theta_m$ can be computed sequentially as the binary segmentation algorithm proceeds. As for the data segment likelihood, we build a model as follows:

$$Y_{ts} \sim Poi(\lambda_{ts}|C|) \text{ with } \log(\lambda_{ts}) = \beta_0 + \phi_t + \psi_s \tag{2}$$

Here Y is a response vector of cell counts for each cell C in a regular grid admitted to the observation window. To model the parameter λ_{ts} (where t indexes time and s space) we use a spatio-temporal Log-Gaussian Cox Process (LGCP) [3], i.e. the logarithm of the intensity function at every location s is assumed to be a Gaussian field and depend on an intercept $\beta_0 \sim N(0,\sigma_\beta^{-2})$ and on two random effects modelled as Intrinsic Gaussian Markov Random Fields. In particular, $\phi \sim IGMRF(0,\tau_\phi K_\phi)$ is a RW(1) over time, and $\psi \sim IGMRF(0,\tau_\psi K_\psi)$ is a RW in two dimensions on a regular grid [1]. The same hyperprior is taken on the precision parameter $\tau_\phi,\tau_\psi \sim Gamma(1,.00005)$ because the IGMRFs are scaled in order to have the same variance, following [5]. LGCPs constitute a broad and flexible class of point process models whose estimation issues have been recently overcome by gridding data and using GMRF processes. They also allow spatial and temporal dependence to be included in the model. Goodness-of-fit tests for point processes based on interevent distances that are routinely used in point process analysis (see for example [3]), indicate that the LGCP model fits the data well; we can thus proceed to the changepoint analysis.

4 Results and discussion

At this first stage, considering the length of the series we assume there are no more than $M = 4$ changepoints, and we assume $d = 2$. Following the prior setting in (1), we write $\pi(m) = 5^{-1}$ and $\pi(\theta_1|m) = 26^{-1}$. As for the data likelihood, we estimate model (2) which we label as 'spatio-temporal', as well as a 'fixed' model including β_0 only, a 'temporal' model including ϕ and a 'spatial' model including ψ.
As regards the detected changepoint locations (Table 1), some findings should be dealt with carefully since 2012 is very close to the end of the series and other changes are only detected in one model scenario. Overall, we can appreciate the detection of a changepoint in 2008; indeed, after 2008 two major seismic events (in L'Aquila and in the Emilia-Romagna region) shocked Italy. We can see in Figure 2 that the average intensity of the process, i.e. the expected number of events per cell, increased due to the mentioned shocks (left panel). Moreover, the spatial distribution changed: until 2008 earthquakes

were evenly distributed all along the Appennini (central panel); afterwards, we see a clusterisation of the process around the central-east part of Italy (where Emilia-Romagna and L'Aquila are) and the volcanic islands close to Sicily (right panel), while a decrease occurs in the Adriatic sea and south-eastern area. As in several applications, it would be of interest to include extra knowledge (such as covariates or informative priors) in order to improve the reliability of the results. Useful information regards number and sensitivity of the monitoring stations and their evolution over time. The detection of earthquakes is related to the distance from the hypocentre and to the magnitude of the event; it might be of interest to investigate whether a higher density of the process is partially due to an increased ability to record seismic events. Moreover, the depth of the hypocentre may be exploited in order to check if it is negatively correlated to the earthquake magnitude; besides, a changepoint analysis of the depth itself may bring useful knowledge to the interpretation of the phenomenon.

Model	Bayes Factor	Posterior Threshold
fixed	1987-2008-2012	1987-2008-2012
temporal	2012	1991-2001-2012
spatial	—	2008
spatio-temporal	—	2008

Table 1: Detected changepoints. Changepoints coloured in red are the ones detected first.

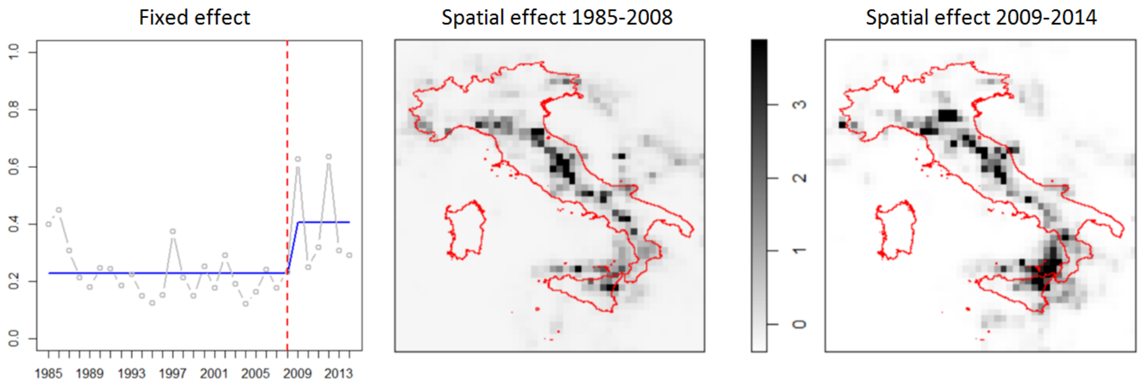

Figure 2: Estimate for the mean intensity function (blue line) and number of events per cell (grey line); spatial effect before and after the main changepoint.

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