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RISK AVERSE TWO-STAGE STOCHASTIC OPTIMIZATION MODEL FOR THE ELECTRIC POWER GENERATION CAPACITY EXPANSION PROBLEM

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Abstract. We consider the optimal electric power generation capacity expansion problem, over a multi-year time horizon, of a price-taker power producer who has to choose among thermal power plants and power plants using renewable energy sources (RES), while taking into account regulatory constraints on CO_2 emissions, incentives to generation from RES and risk due to fuel price volatility which affects the generation variable costs. A two-stage stochastic mixed integer model is developed that determines the number of new power plants for each chosen technology, as well as the years in which the construction of the new power plants is to begin. The solution allows determining the evolution of the power producer's generation system along the time horizon, so that the expected total profit is maximized, with revenues from sale of electricity and of Green Certificates and costs for the annual debt repayment of new power plants, purchase costs of CO_2 emission permits and of Green Certificates, fixed and variable production costs of new power plants and of power plants owned by the producer at the beginning of the planning period. Alternative risk measures are considered and tested.

Keywords: power generation capacity expansion, stochastic mixed-integer model, risk measures.

1. Introduction

The incremental selection of power generation capacity is of great importance for energy planners. In this paper we deal with the case of a price taker power producer, who has to determine the optimal mix of different technologies for power generation, ranging from coal, nuclear and combined cycle gas turbine to hydroelectric, wind and photovoltaic, taking into account the existing plants, the cost of investment in new plants, the maintenance cost, the purchase and sales of CO_2 emission trading certificates and green certificates to satisfy regulatory requirements over a long term planning horizon (generally, 30 years or more). Uncertainty of prices (fuels, electricity, CO_2 emission permits and Green Certificates) should be taken into account, see [1, 2, 4, 9, 15]. We propose a two-stage stochastic model for finding an optimal trade-off between expected profit and the risk of getting a negative impact on the profit due to the occurrence of a not-wanted scenario. The model can be seen as a generalization of the Levelized Cost of Electricity (LCoE), the standard business tool that finds the technology which provides the lowest electricity selling price to break even: indeed the model finds the technology mix that provides the highest expected profit taking into account risk.

2. The risk-neutral two-stage stochastic model

Given a planning horizon consisting of a set I of years, the two-stage stochastic model determines the optimal power generation expansion plan, i.e. the number of new power plants of candidate technology j, belonging either to the set J^T of thermal technologies or to the set J^R of Renewable Energy Sources (RES) technologies, whose construction is to start in year $i \in I$, taking into account the set K^T of thermal plants and the set K^R of RES plants already owned by the producer at the beginning of the planning horizon. The uncertainty of prices and of power producer's market share along the planning horizon is represented by a set Ω of scenarios on the following stochastic parameters:

$\pi_{i,\omega}^{GC}$: electricity price in year i in scenario ω;

$\pi_{i,\omega}^{GC}$: price of Green Certificates in year i in scenario ω;

$\pi_{i,\omega}^{CO_2}$: price of CO_2 emission permit in year i in scenario ω;

$v_{j,\omega}^{J}$: fuel costs of candidate thermal power plant $j\in J^T \cup J^R$ in scenario ω; $v_{k,\omega}^{K}$: fuel costs of existing thermal power plant $k\in K^T \cup K^R$ in scenario ω;

$\overline{M}_{i,\omega}$: power producer's market share in year i in scenario ω.

The optimal power generation expansion plan is defined by the nonnegative integer variables $w_{j,i}$, that represent the number of new power plants of candidate technology j whose construction is to start in year i. Variables $W_{j,i}$ represent the number of power plants of candidate technology $j\in J^T \cup J^R$ available for production in year $i\in I$. The optimal values of the following decision variables are also determined by the model:

$E_{j,i,\omega}^{J}$: electricity produced by a plant of technology $j\in J^T \cup J^R$ in year $i\in I$ under scenario $\omega\in\Omega$;

$E_{k,i,\omega}^{K}$: electricity produced by power plant $k\in K^T \cup K^R$ in year $i\in I$ under scenario $\omega\in\Omega$;

$G_{i,\omega}$: Green Certificates sold $(G_{i,\omega} \geq 0)$ or bought $(G_{i,\omega} \leq 0)$ in year $i\in I$ under scenario $\omega\in\Omega$;

$Q_{i,\omega}$: CO_2 produced in year $i\in I$ under scenario $\omega\in\Omega$.

The risk-neutral two-stage stochastic model is as follows.

$$\max \quad F = \sum_{\omega\in\Omega} p_\omega F_\omega \tag{1}$$

where

$$\begin{aligned} F_\omega = \sum_{i\in I} \frac{1}{(1+r)^i} \Bigg[& \pi_{i,\omega}^{E} \Bigg(\sum_{j\in J^T \cup J^R} E_{j,i,\omega}^{J} + \sum_{k\in K^T \cup K^R} E_{k,i,\omega}^{K} \Bigg) + \pi_{i,\omega}^{GC} G_{i,\omega} - \pi_{i,\omega}^{CO_2} Q_{i,\omega} - \sum_{j\in J^T} v_{j,\omega}^{J} E_{j,i,\omega}^{J} - \\ & - \sum_{k\in K^T} v_{k,\omega}^{K} E_{k,i,\omega}^{K} - \sum_{j\in J^R} v_j^{J} E_{j,i,\omega}^{J} - \sum_{k\in K^R} v_k^{K} E_{k,i,\omega}^{K} - \sum_{j\in J^T \cup J^R} \left(R_j + f_j\right) W_{j,i} - \sum_{k\in K^T \cup K^R} f_k \Bigg] \end{aligned} \tag{2}$$

subject to

$$w_{j,i} \ \text{non-negative integer} \qquad j\in J^T \cup J^R, \quad i\in I \tag{3}$$

$$\sum_{i\in I} w_{j,i} \leq \overline{Z}_j \qquad j\in J^T \cup J^R \tag{4}$$

$$W_{j,i} = \sum_{i-(S_j+L_j)+1 \,\leq\, l \,\leq\, i-S_j} w_{j,l} \qquad j\in J^T \cup J^R, \quad i\in I \tag{5}$$

$$\sum_{i\in I} \frac{1}{(1+r)^i} \Bigg(\sum_{j\in J^T \cup J^R} R_j W_{j,i} \Bigg) \leq B \tag{6}$$

$$0 \leq E_{j,i,\omega}^{J} \leq \overline{E}_{j,i}^{J} W_{j,i} \qquad j\in J^T \cup J^R, \quad i\in I, \quad \omega\in\Omega \tag{7}$$

$$0 \leq E_{k,i,\omega}^{K} \leq \overline{E}_{k,i}^{K} \qquad k\in K^T \cup K^R, \quad i\in I, \quad \omega\in\Omega \tag{8}$$

$$\sum_{j\in J^T \cup J^R} E_{j,i,\omega}^{J} + \sum_{k\in K^T \cup K^R} E_{k,i,\omega}^{K} \leq \overline{M}_{i,\omega} \qquad i\in I, \quad \omega\in\Omega \tag{9}$$

$$\frac{\sum_{j\in J^R} E_{j,i,\omega}^{J} + \sum_{k\in K^R} E_{k,i,\omega}^{K} - G_{i,\omega}}{\sum_{j\in J^T \cup J^R} E_{j,i,\omega}^{J} + \sum_{k\in K^T \cup K^R} E_{k,i,\omega}^{K}} = \beta_i \qquad i\in I, \quad \omega\in\Omega \tag{10}$$

$$Q_{i,\omega} = \sum_{j \in J^T} \theta_j \cdot E_{j,i,\omega} + \sum_{k \in K^T} \theta_k \cdot E_{k,i,\omega} \qquad i \in I, \quad \omega \in \Omega . \tag{11}$$

Constraints (3) impose integrality of nonnegative variables $w_{j,i}$. Constraints (4) state that for every candidate technology j the total number of new power plants constructed along the planning horizon is bounded above by the number $\overline{Z}_j$ of sites ready for construction of new power plants of that technology, i.e. sites for which all necessary administrative permits have been released. Constraints (5) define the number $W_{j,i}$ of new power plants of technology j available for production in year i, i.e. plants for which construction is completed and industrial life is not ended. Constraint (6) states that the sum of actualized annual debt repayments $R_j W_{j,i}$ cannot exceed the available budget B. Constraints (7) require the annual electricity production of all new power plants of technology j to be nonnegative and bounded above by the capacity of new power plants of technology j available for production in year i, which is the product of $W_{j,i}$ times the capacity of one plant of technology j. Analogous restrictions are imposed by constraints (8) on the annual production of existing power plants $k \in K^T \cup K^R$. Constraints (9) require the total electricity generated in every year i not to exceed the power producer's market share in that year. The Green Certificates incentive scheme is taken into account by contraints (10). The ratio β_i is required in year i between the electricity produced from RES and the total electricity produced: if the power producer produces less energy using RES, he must buy Green Certificates ($G_{i,\omega} < 0$); if he produces more electricity from RES than the required amount, he can sell Green Certificates ($G_{i,\omega} > 0$). Constraints (11) define the amount $Q_{i,\omega}$ of CO_2 emissions for which he must buy emission permits in year i under scenario ω, being θ_k and θ_j the CO_2 emission rates of thermal power plant $k \in K^T$ and thermal power plant of candidate technology $j \in J^T$ respectively. The total profit F_ω under scenario $\omega \in \Omega$ is given by equation (2). The variable production costs v_j^J, of a RES power plant of candidate technology $j \in J^R$, and v_k^K, of the RES power plant $k \in K^R$, are assumed to be known with certainty. Parameters f_k and f_j represent the fixed production costs of power plant $k \in K^T$ and of a power plant of technology $j \in J^T$, respectively. In the risk neutral approach the expected profit (1) over scenarios $\omega \in \Omega$ is maximized subject to constraints (3)–(11).

3. Risk aversion strategies

We evaluate the impact of introducing five alternative risk measures in our model.

3.1. Risk aversion strategy 1: Conditional Value at Risk (*CVaR*)

The objective function (1) is substituted by

$$\max \ (1-\rho)\left(\sum_{\omega \in \Omega} p_\omega F_\omega\right) + \rho\left[V - \frac{1}{\alpha}\left(\sum_{\omega \in \Omega} p_\omega d_\omega\right)\right] \tag{12}$$

i.e. by the convex combination of the expected profit and of a term that equals the *CVaR* [12, 13] at the optimal solution. The auxiliary variables d_ω and V are defined by constraints

$$d_\omega \geq V - F_\omega \quad , \qquad d_\omega \geq 0, \qquad \omega \in \Omega, \tag{13}$$

the parameter $\rho \in [0, 1]$ is the risk aversion factor and $\alpha \in [0, 1]$ is the confidence level. The optimal value of V is the Value-at-Risk (*VaR*).

3.2. Risk aversion strategy 2: Shortfall Probability (*SP*)

Given a profit threshold ϕ, the shortfall probability, see [14], is the cumulative probability of the scenarios with a profit smaller than ϕ, i.e.

$$\sum_{\omega \in \Omega} p_\omega \cdot \mu_\omega \tag{14}$$

where the binary variable μ_ω, defined by the constraint

$$\phi - F_\omega \leq M \cdot \mu_\omega \qquad \forall \omega \in \Omega \tag{15}$$

takes value 1 if $F_\omega \le \phi$, i.e. if ω is a non-wanted scenario, with M sufficiently large constant. The profit risk can be hedged by simultaneously pursuing expected profit maximization and shortfall probability minimization: this is done by maximizing the objective function, with $0 \le \rho \le 1$,

$$(1-\rho)\left(\sum_{\omega\in\Omega} p_\omega F_\omega\right) - \rho\left(\sum_{\omega\in\Omega} p_\omega \mu_\omega\right) \tag{16}$$

subject to (2)–(11).

3.3. Risk aversion strategy 3: Expected Shortage (*ES*)

Given a profit threshold ϕ, the expected shortfall, see [4], is given by

$$\sum_{\omega\in\Omega} p_\omega d_\omega \tag{17}$$

where d_ω is a nonnegative variable that satisfies constraint

$$\phi - F_\omega \le d_\omega \quad , \quad d_\omega \ge 0, \quad \forall \omega \in \Omega \tag{18}$$

The expected shortage is then defined as

$$\phi - \frac{1}{\sum_{\omega\in\Omega \mid F_\omega \le \phi} p_\omega}\left(\sum_{\omega\in\Omega} p_\omega d_\omega\right) \tag{19}$$

The profit risk is hedged by simultaneously pursuing expected profit maximization and expected shortfall minimization: this is done by maximizing the objective function, with $0 \le \rho \le 1$,

$$(1-\rho)\left(\sum_{\omega\in\Omega} p_\omega F_\omega\right) - \rho\left(\sum_{\omega\in\Omega} p_\omega d_\omega\right) \tag{20}$$

subject to (2)–(11).

3.4. Risk aversion strategy 4: First-Order Stochastic Dominance (*FOSD*)

A benchmark is given by assigning a set P of profiles (ϕ^p, τ^p), $p \in P$, where ϕ^p is the threshold to be satisfied by the profit at each scenario and τ^p is its failure probability. The profit risk is hedged by maximizing the expected value of profit (1), while satisfying the so called first-order stochastic dominance constraints, see [11],

$$\phi^p - F_\omega \le M\mu_\omega^p \quad \forall \omega \in \Omega, \quad \forall p \in P \tag{21}$$

where μ_ω^p are 0–1 variables by which the shortfall probability with respect to the threshold ϕ^p is computed,

$$\sum_{\omega\in\Omega} p_\omega \mu_\omega^p \le \tau^p \quad \forall p \in P \tag{22}$$

and equations (2)–(11). This risk measure is related to Shortfall Probability. The drawback of this approach is the increase of the number of constraints and binary variables.

3.5. Risk aversion strategy 5: Second-Order Stochastic Dominance (*SOSD*)

A benchmark is assigned which is defined by a set of profiles (ϕ^p, e^p), $p \in P$, where ϕ^p is the threshold to be satisfied by the profit at each scenario and e^p is the upper bound to the expected shortfall over the scenarios. The profit risk is hedged by maximizing the expected value of profit (1), while satisfying the so called second-order stochastic dominance constraints, see [10],

$$\phi^p - F_\omega \le d_\omega^p \quad , \quad d_\omega^p \ge 0 \, , \quad \forall \omega \in \Omega, \quad \forall p \in P \tag{23}$$

$$\sum_{\omega\in\Omega} p_\omega d_\omega^p \le e^p \quad \forall p \in P \tag{24}$$

This measure is closely related to the risk measure introduced in [8], where the thresholds $p \in P$ are considered as the benchmark. It is interesting to point out that in the risk aversion strategies 4 and 5 the hedging is represented by the requirement of forcing the scenario profit to be not smaller than a set of thresholds with a failure probability for each of them in strategy 4 and an upper bound on the expected shortage in strategy 5. The price to be paid is the increase of the number of constraints and variables (being 0–1 variables in strategy 4).

4. Numerical results

The stochastic model introduced in Section 2 jointly with the risk aversion strategies presented in Section 3 have been implemented in GAMS and CPLEX 12.1.0 has been used for computing the optimal solution. Only scenarios on fuel prices are considered, with gas price more volatile than coal price and with the nuclear fuel price being the less volatile among the three. All other parameters are deterministic. The obtained results are shown in Tables 1–5. When the power producer is risk neutral, the technology of choice is CCGT; as the risk aversion increases, CCGT plants are gradually substituted by coal plants first and by RES plants eventually. Nuclear plants are never chosen as the budget is not large enough; wind power is the only renewable plant technology in the optimal mix, as other RES technologies either are not economically convenient or there are no sites ready for construction.

Table 1. Results with *CVaR*: $\alpha = 5\%$, budget $B = 3.84$ *G€*

ρ	**0**	**0.072**	**0.1**	**0.15**	**0.4**	**0.6**
CCGT	9	7	4	2	1	0
Coal	1	2	4	5	5	0
Wind	0	0	1	2	4	24
Expected Profit [*G€*]	12.43	12.01	11.34	10.79	10.26	7.24
VaR [*G€*]	–3.40	–0.60	3.48	5.87	6.30	7.24
CVaR [*G€*]	–14.80	–9.33	–1.22	3.43	4.57	7.24

Table 2. Results with *SP*: $\phi = 6$ *G€*, budget $B = 3.84$ *G€*

ρ	**0**	**0.07**	**0.8**	**0.9**
CCGT	9	7	4	2
Coal	1	2	4	5
Wind	0	0	1	2
Expected Profit [*G€*]	12.43	12.01	11.34	10.79
Shortfall Probability	0.12	0.10	0.047	0

Table 3. Results with ES: $\phi = 6$ G€, budget B = 3.84 G€

ρ	**0**	**0. 578**	**0. 866**	**0. 958**	**0.976**
CCGT	9	4	2	1	0
Coal	1	4	5	5	2
Wind	0	1	2	4	17
Expected Profit [*G€*]	12.43	11.34	10.79	10.26	8.22
Expected Shortage [*G€*]	–6.51	1.92	3.86	4.42	5.30

Table 4. Results with *FOSD*: budget B = 3.84 *G€*

	Benchmark 1	Benchmark 2
(ϕ^1, τ^1)	(3, 0.02)	(5.0, 0.02)
(ϕ^2, τ^2)	(6, 0.08)	(6.5, 0.08)
(ϕ^3, τ^3)	(8, 0.28)	(7.5, 0.28)
(ϕ^4, τ^4)	(9, 0.90)	(8.0, 0.90)
CCGT	2	0
Coal	5	2
Wind	2	17
Expected Profit [*G€*]	10.79	8.22
Expected Shortage [*G€*]	3.86	5.30

Table 5. Results with *SOSD*: *Budget* = 3.84 *G€*

	Benchmark 1	Benchmark 2
(ϕ^1, τ^1)	(2, 0.01)	(4.0, 0.01)
(ϕ^2, τ^2)	(5, 0.05)	(6.0, 0.05)
(ϕ^3, τ^3)	(6, 0.10)	(6.5, 0.10)
(ϕ^4, τ^4)	(7, 0.20)	(7.0, 0.20)
CCGT	0	0
Coal	4	2
Wind	10	17
Expected Profit [*G€*]	10.79	8.22
Expected Shortage [*G€*]	3.86	5.30

5. Conclusions

Risk neutral strategy could be "a fiasco", if there is a great variability in the scenario objective function values, so any risk aversion measure presented above is a better choice for risk minimization. When choosing the risk measure three aspects are relevant: ease of implementation, model complexity and information on the risk level. As regard to the ease of implementation, *CVaR* is the easiest to be used, since it only requires assigning the confidence level α (tipical values are 1%, 2% o 5%); *ES* and *SP* require a profit threshold, which needs to be carefully chosen; *FOSD* and *SOSD* require more information, since a set of thresholds (benchmark) needs to be chosen. As regard to model complexity, *CVaR*, *ES* and *SOSD* do not use binary variables, while *SP* and *FOSD* need binary variables. Finally, concerning the information on the risk level, *SOSD* forces the shape of profit distribution which is particularly useful for modeling the left tails; *CVaR* and *ES* give information about the expected value of left tail of distribution; *FOSD* focuses only on where distributions intersect the thresholds and *SP* focuses only on where distributions intersect the profit threshold. When the problem dimension increases, *ad hoc* algorithms are required for computing the optimal solution, see [3, 5, 6, 7].

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