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Gravity's Rainbow: a bridge towards Hořava–Lifshitz gravity

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Abstract We investigate the connection between Gravity's Rainbow and Hořava–Lifshitz gravity, since both theories incorporate a modification in the ultraviolet regime which improves their quantum behavior at the cost of the Lorentz invariance loss. In particular, extracting the Wheeler–De Witt equations of the two theories in the case of Friedmann–Lemaître–Robertson–Walker and spherically symmetric geometries, we establish a correspondence that bridges them.

Contents

1 Introduction

The idea that general relativity (GR) is not the fundamental gravitational theory and that it needs to be modified or extended is quite old. On the one hand, the idea of a small-scale, ultraviolet (UV) modification of GR arises from the non-renormalizability of the theory and the difficulties towards its quantization [1]. In particular, since the usual loop-expansion procedure gives rise to UV-divergent Feynman diagrams, the requirement for a UV-complete gravitational theory, which has GR as a low-energy limit, becomes necessary. On the other hand, we know that the large-scale, infrared (IR) modifications of GR might be the explanation of the observed late-time universe acceleration (see [2] and references therein) and/or of the inflationary stage [3]. Due to their significance, both directions led to a huge amount of research.

Concerning the modification of the UV behavior, it was realized that the insertion of higher-order derivative terms in the Lagrangian establishes renormalizability, since these terms modify the graviton propagator at high energies [1]. However, this leads to an obvious problem, namely that the equations of motion involve higher-order time derivatives and thus the application of the theory leads to ghosts. Nevertheless, based on the observation that it is the higher spatial derivatives that improve renormalizability, while it is the higher time derivatives that lead to ghosts, some years ago Hořava had the idea to construct a theory that allows for the inclusion of higher spatial derivatives only. In order to achieve this, and motivated by the Lifshitz theory of solid state physics [4], he broke the "democratic treating" of space and time in the UV regime, introducing an anisotropic, Lifshitz scaling between them [5–8]. Hence, higher spatial derivatives are not accompanied by higher time ones (definitely this corresponds to Lorentz violation), and thus in the UV the theory exhibits power-counting renormalizability but still without ghosts. Finally, the theory presents GR as an IR fixed point, as required, where Lorentz invariance is restored and space and time are handled on equal footing.

On the other hand, in [9] the authors followed a different approach. In particular, instead of modifying the action, they

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constructed an UV modification of the metric itself, in a construction named Gravity's Rainbow (GRw) [9]. Hence, the deformed metric in principle exhibits a different treatment between space and time in the UV, namely on scales near the Planck scale, depending on the energy of the particle probing the space-time, while at low energies one recovers the standard metric, and General Relativity is restored. Physically, one can think of it as a deformation of the metric by the Planck-scale graviton. This deformation has been shown to cure divergences (at least to one loop) avoiding any regularization/renormalization scheme [10,11]. Hence, due to this advantage, a large amount of research has been devoted to GRw [12–33].

In the present work we are interested in examining whether there is a correspondence between Hořava–Lifshitz gravity and GRw, since both directions result in a modification of the equations in the UV regime, while they both present GR as their low-energy limit. In particular, since GR provides a natural scheme for quantization of the gravitational field, namely the Wheeler–De Witt (WDW) equation [34], which is a quantum version of the Hamiltonian constraint obtained from the Arnowitt–Deser–Misner decomposition of space-time, we will impose the requirement that the WDW equation must be satisfied by GRw and Hořava–Lifshitz gravity, respectively. We will examine this correspondence on the Friedmann–Lemaître–Robertson–Walker (FLRW) metric at the mini-superspace level, where the problem with the scalar graviton is absent, as well as in spherically symmetric geometries.

The manuscript is organized as follows: in Sect. 2 we review the basic elements of Hořava–Lifshitz theory, while in Sect. 3 we extract the corresponding WDW equation in the case of FLRW space-time. In Sect. 4 we extract the WDW equation for GRw in the case of FLRW space-time. In Sect. 5 we establish the correspondence between the two theories, while in Sect. 6 we obtain this relation for spherically symmetric space-times. Finally, we summarize our results in Sect. 7.

Throughout this manuscript we use units in which $\hbar = c = k = 1$.

2 Hořava–Lifshitz gravity

We start with a brief review of Hořava–Lifshitz gravity [5–8]. As we stated in the Introduction, the central idea of the theory is the different treatment of space and time, which allows us to introduce higher spatial derivatives without inserting also the annoying higher time derivatives. Thus, a convenient framework to perform the construction in is the Arnowitt–Deser–Misner ($\mathcal{ADM}$) metric decomposition, namely

$$\mathrm{d}s^2 = -N^2\mathrm{d}t^2 + g_{ij}(\mathrm{d}x^i + N^i\mathrm{d}t)(\mathrm{d}x^j + N^j\mathrm{d}t). \tag{2.1}$$

The dynamical variables are the lapse N and shift N_i functions, and the spatial metric g_{ij} (Latin indices denote spatial coordinates). The coordinate scaling transformations are written as

$$t \to \ell^3 t \quad \text{and} \quad x^i \to \ell x^i, \tag{2.2}$$

i.e. it is a Lifshitz scale invariance with a dynamical critical exponent $z = 3$.

The breaking of the four-dimensional diffeomorphism invariance allows for a different treatment of the kinetic and potential terms for the metric in the action, namely the kinetic term can be quadratic in time derivatives while the potential term can have higher-order space derivatives. Thus, in general, the action of Hořava–Lifshitz gravity is written as

$$S = \frac{1}{2\kappa}\int_{\Sigma\times I}\mathrm{d}t\mathrm{d}^3x\,(\mathcal{L}_K - \mathcal{L}_P), \tag{2.3}$$

with $\kappa = M_{\mathrm{pl}}^{-2}$ the Planck mass, where the kinetic term reads

$$\mathcal{L}_K = N\sqrt{g}\left(K^{ij}K_{ij} - \lambda K^2\right), \tag{2.4}$$

with K_{ij} the extrinsic curvature defined as

$$K_{ij} = \frac{1}{2N}\left\{-\dot{g}_{ij} + \nabla_i N_j + \nabla_j N_i\right\}, \tag{2.5}$$

$K = K^{ij}g_{ij}$ its trace, and g is the determinant of the spatial metric g_{ij}. The constant λ is a dimensionless running coupling, which takes the value $\lambda = 1$ in the IR limit. The potential part $\mathcal{L}_P$ can in principle contain many terms. However, one can make additional assumptions in order to reduce the possible terms, thus resulting to various versions of the theory. In the following we review the basic ones.

2.1 Detailed-balance version

The assumption of "detailed balance" [7] allows for the establishment of a quantum inheritance principle [5], that is, the $(D+1)$-dimensional theory exhibits the renormalization properties of the D-dimensional one. Physically, it corresponds to the requirement that the potential term should arise from a superpotential. This condition reduces significantly the potential part of the action, resulting in

$$\mathcal{L}_{Pdb} = N\sqrt{g}\left\{\frac{\kappa^2}{w^4}C_{ij}C^{ij} - \frac{2\kappa^{3/2}\mu}{w^2}\frac{\epsilon^{ijk}}{\sqrt{g}}R_{il}\nabla_j R^l_k + \frac{\mu^2}{\kappa}R_{ij}R^{ij} - \frac{\mu^2}{1-3\lambda}\left[\frac{1-4\lambda}{4}R^2 + \Lambda R - \frac{3\Lambda^2}{\kappa}\right]\right\}, \tag{2.6}$$

where $C^{ij} = \epsilon^{ikl}\nabla_k\left(R^j_l - \delta^j_l R/4\right)/\sqrt{g}$ is the Cotton tensor (it is concomitant with the metric and in three dimensions it is the analog of the Weyl tensor), the covariant derivatives are defined with respect to the spatial metric g_{ij}, and ϵ^{ijk} is the totally antisymmetric unit tensor. Finally, apart from the running coupling λ, we have three more constants, namely w, μ and Λ. We mention that the detailed-balance condition, apart from reducing the possible terms in the potential part of the action, additionally correlates their coefficients, and thus the total number of coefficients is smaller than the total number of terms.

2.2 Projectable version

Independently of the detailed-balance condition one can impose the "projectability" condition, which is a weak version of the invariance with respect to time reparametrizations, namely that the lapse function is just a function of time, i.e. $N = N(t)$ [7]. Such a condition allows also for a significant reduction of terms in the potential, since it eliminates the spatial derivatives of N. In this case, and neglecting parity-violating terms, the potential part of the action becomes [35,36]

$$\mathcal{L}_P = N\sqrt{g}\left\{g_0\kappa^{-1} + g_1 R + \kappa\left(g_2 R^2 + g_3 R^{ij}R_{ij}\right) + \kappa^2\left(g_4 R^3 + g_5 R R^{ij}R_{ij} + g_6 R^i_j R^j_k R^k_i + g_7 R\nabla^2 R + g_8 \nabla_i R_{jk}\nabla^i R^{jk}\right)\right\}, \tag{2.7}$$

where the couplings g_a $(a = 0\ldots 8)$ are all dimensionless and running; moreover, we can set $g_1 = -1$. Finally, note that if, apart from the projectability condition, one additionally imposes the detailed-balance condition, then it will again result in the potential term (2.6) but with $N = N(t)$.

2.3 Non-projectable version

In the general case where neither the detailed-balance nor the projectability conditions are imposed, one can have in the potential part of the action many possible curvature invariants of g_{ij} and, moreover, invariants including also the vector $a_i = \partial_i \ln N$, which is now non-zero. In this case the potential part of the action becomes [37]

$$\mathcal{L}_{Pnp} = N\sqrt{g}\left\{-\xi R - \eta a_i a^i - \frac{1}{M_A^2}\mathcal{L}_4 - \frac{1}{M_B^2}\mathcal{L}_6\right\}, \tag{2.8}$$

where $a_i a^i$ is the lowest-order new term, of the same order as R, and $\mathcal{L}_4$ and $\mathcal{L}_6$, respectively, contain all possible fourth and sixth order invariants that can be constructed by a_i and g_{ij} and their combinations and contractions. Clearly, the above potential term contains much more terms than the projectable or the detailed-balance versions. Lastly, in order to recover GR in the IR limit, apart from the running of λ to 1, η should run to zero too, while ξ can be set to 1.

We close this section by mentioning that in all versions of Hořava–Lifshitz gravity, Lorentz invariance is violated due to both the kinetic term (since λ is in general not equal to 1) and the terms in the potential. It is approximately and asymptotically restored in the IR, where λ runs to 1 and the potential terms will be significantly suppressed. Thus, one can apply Hořava–Lifshitz gravity in order to investigate its implications, which indeed are found to be rich and interesting at both cosmological [38–84] and black hole applications [85–91].

3 The WDW equation in Hořava–Lifshitz gravity

In this section we examine the Wheeler–De Witt (WDW) equation in the framework of Hořava–Lifshitz gravity. For convenience, and in order to simplify the calculations, we focus on the projectable version of the theory, without the detailed-balanced condition, although an extension to the full, non-projectable theory is straightforward.

The WDW equation is a quantum version of the Hamiltonian constraint obtained from the Arnowitt–Deser–Misner decomposition of space-time. Hence, let us consider a simple mini-super-space model described by the FLRW line element,

$$\mathrm{d}s^2 = -N^2\mathrm{d}t^2 + a^2(t)\,\mathrm{d}\Omega_3^2, \tag{3.1}$$

describing a homogeneous, isotropic, and closed universe. $\mathrm{d}\Omega_3^2(k)$ is the metric on the spatial sections, which have constant curvature $k = 0, \pm 1$, defined by

$$\mathrm{d}\Omega_3^2 = \gamma_{ij}\mathrm{d}x^i\mathrm{d}x^j. \tag{3.2}$$

Additionally, $N = N(t)$ is the lapse function and $a(t)$ denotes the scale factor. In this background, the three-dimensional Ricci curvature tensor and the scalar curvature read

$$R_{ij} = \frac{2}{a^2(t)}\gamma_{ij} \quad \text{and} \quad R = \frac{6}{a^2(t)}, \tag{3.3}$$

respectively. With the help of Eq. (2.7), the resulting Hamiltonian is computed by means of the usual Legendre transformation, leading to

$$H = \int_\Sigma \mathrm{d}^3x\mathcal{H} = \int_\Sigma \mathrm{d}^3x\left[\pi_a\dot{a} - \mathcal{L}_P\right], \tag{3.4}$$

where π_a is the canonical momentum. By inserting the FLRW background into $\mathcal{L}_P$ one obtains

$$\mathcal{L}_P = N\sqrt{g}\left[g_0\kappa^{-1} + g_1\frac{6}{a^2(t)} + \frac{12\kappa}{a^4(t)}(3g_2 + g_3) + \frac{24\kappa^2}{a^6(t)}(9g_4 + 3g_5 + g_6)\right]. \tag{3.5}$$

The term $g_0\kappa^{-1}$ plays the role of a cosmological constant. In order to make contact with the ordinary Einstein–Hilbert action in $3+1$ dimensions, we set without loss of generality

$$g_0\kappa^{-1} \equiv 2\Lambda \\ g_1 \equiv -1. \tag{3.6}$$

Note that in the case where one desires to study the negative cosmological constant, the identification will (trivially) be $g_0\kappa^{-1} \equiv -2\Lambda$.

Having set $N=1$, the Legendre transformation leads to

$$\mathcal{H} = \pi_a \dot{a} - \mathcal{L}_K + \mathcal{L}_P, \tag{3.7}$$

and the Hamiltonian constraint becomes [54]

$$\begin{aligned} H &= \int_\Sigma \mathrm{d}^3x\mathcal{H} = -\frac{\kappa\pi_a^2}{12\pi^2 a\left(3\lambda-1\right)} \\ &\quad +2\pi^2a^3\left(t\right)\left[2\Lambda\kappa^{-1} - \frac{6\kappa^{-1}}{a^2\left(t\right)} + \frac{12b}{a^4\left(t\right)} + \frac{24\kappa c}{a^6\left(t\right)}\right] \\ &= \pi_a^2 + \frac{\left(3\lambda-1\right)}{\kappa^2}24\pi^4a^4\left(t\right)\left[\frac{6}{a^2\left(t\right)} - \frac{12\kappa b}{a^4\left(t\right)}\right. \\ &\quad \left. -\frac{24\kappa^2 c}{a^6\left(t\right)} - 2\Lambda\right] = 0, \end{aligned} \tag{3.8}$$

where

$$3g_2 + g_3 = b \\ 9g_4 + 3g_5 + g_6 = c. \tag{3.9}$$

General relativity is recovered when $b = c = 0$, which does not necessarily mean that all the couplings are vanishing. Moreover, all the higher-curvature terms are automatically suppressed, since the curvature becomes small [35]. Let us mention here that the scenario described by the distorted potential Lagrangian (2.7), in the specific case of FLRW geometry, which we are interested in, could be considered to arise equivalently in the framework of $f(R)$ gravity, with R the three-dimensional scalar curvature [11]. Indeed, if one starts from the Lagrangian

$$\mathcal{L}_{fR} = N\sqrt{g}f\left(R\right) \tag{3.10}$$

with

$$\begin{aligned} f\left(R\right) &= g_0\kappa^{-1} + g_1R - \frac{\kappa b}{3}R^2 - \frac{\kappa^2 c}{9}R^3, \\ &= 2\Lambda + R\left(1 - 2\pi b\frac{R}{R_0} - 4\pi^2 c\frac{R^2}{R_0^2}\right), \end{aligned} \tag{3.11}$$

and b and c given by (3.9), and extracts the corresponding field equations in the case of FLRW geometry, one will obtain the same equations as those extracted from $\mathcal{L}_P$ in (2.7). Lastly, note that we have used the definitions (3.6), while we have furthermore set $R_0 \equiv 6/G = 6/l_p^2$.

4 The WDW equation in Gravity's Rainbow

In this section we review briefly GRw [9], focusing on the Hamiltonian analysis and the WDW equation. In this formulation, the space-time geometry is described by the deformed metric

$$\mathrm{d}s^2 = -\frac{N^2\left(t\right)}{g_1^2\left(E/E_{\mathrm{Pl}}\right)}\mathrm{d}t^2 + \frac{a^2\left(t\right)}{g_2^2\left(E/E_{\mathrm{Pl}}\right)}\mathrm{d}\Omega_3^2\,, \tag{4.1}$$

where $g_1(E/E_{\mathrm{Pl}})$ and $g_2(E/E_{\mathrm{Pl}})$ are functions of energy, which incorporate the deformation of the metric. Concerning the low-energy limit one is required to consider

$$\lim_{E/E_{\mathrm{Pl}}\to 0} g_1\left(E/E_{\mathrm{Pl}}\right) = 1 \quad \text{and} \quad \lim_{E/E_{\mathrm{Pl}}\to 0} g_2\left(E/E_{\mathrm{Pl}}\right) = 1, \tag{4.2}$$

and thus to recover the usual FLRW geometry. Hence, E quantifies the energy scale at which quantum gravity effects become apparent. For instance, one of these effects would be that the graviton distorts the background metric as we approach the Planck scale.

As has been extensively shown in the literature [10–33], GRw can be used to cure or alleviate the usual GR divergences, at least to one loop, avoiding any regularization and renormalization schemes. If one allows the energy E to evolve depending on t, one finds that the extrinsic curvature of the metric (4.1) reads

$$\begin{aligned} K_{ij} &= -\frac{g_1\left(E\left(a\left(t\right)\right)/E_P\right)}{2N}\frac{\mathrm{d}}{\mathrm{d}t}\left[\frac{g_{ij}}{g_2^2\left(E\left(a\left(t\right)\right)/E_P\right)}\right] \\ &= \frac{g_1\left(E\left(a\left(t\right)\right)/E_P\right)}{g_2^2\left(E\left(a\left(t\right)\right)/E_P\right)}\left[\tilde{K}_{ij} + \tilde{g}_{ij}\frac{A\left(t\right)}{N\left(t\right)}\dot{a}\left(t\right)\right], \end{aligned} \tag{4.3}$$

where

$$A\left(t\right) = \frac{1}{g_2\left(E\left(a\left(t\right)\right)/E_P\right)E_P}\frac{\mathrm{d}}{\mathrm{d}E}\left[g_2\left(E\left(a\left(t\right)\right)/E_P\right)\right]\frac{\mathrm{d}E}{\mathrm{d}a}, \tag{4.4}$$

dots denoting differentiation with respect to time. In the above expressions the tildes indicate the quantities computed in the absence of the rainbow's functions.

The next step is to find the corresponding canonical momentum. After a short calculation, presented in Appendix A, the canonical momentum can be written

$$\pi_a = \frac{\delta S_K}{\delta\dot{a}} = \frac{g_1^2\left(E\left(a\left(t\right)\right)/E_P\right)}{g_2^3\left(E\left(a\left(t\right)\right)/E_P\right)}f\left(A\left(t\right),a\right)\tilde{\pi}_a, \tag{4.5}$$

where

$$f\left(A\left(t\right),a\right) = \left[1 - 2a\left(t\right)A\left(t\right) + A^2\left(t\right)a\left(t\right)^2\right] \tag{4.6}$$

and

$$\tilde{\pi}_a = \frac{6\pi^2}{\kappa}\frac{\left(1-3\lambda\right)}{N\left(t\right)}\dot{a}a. \tag{4.7}$$

Finally, we can now assemble the Hamiltonian density, which is defined as

$$\mathcal{H} = \pi_a \dot{a} - \mathcal{L}_K + \mathcal{L}_P, \tag{4.8}$$

where $\mathcal{L}_P$ is the potential term whose form is

$$\mathcal{L}_P = \frac{N(t)\sqrt{\tilde{g}}}{16\pi G g_2(E(a(t))/E_P)}\left[\tilde{R} - \frac{2\Lambda}{g_2^2(E(a(t))/E_P)}\right]. \tag{4.9}$$

Concerning the kinetic term we have

$$\begin{aligned}\mathcal{H}_K &= \pi_a \dot{a} - \mathcal{L}_K = \frac{\kappa N(t)}{12\pi^2 a}\left[\frac{g_2^3(E(a(t))/E_P)}{g_1^2(E(a(t))/E_P)}\right] \\ &\quad\times \frac{\pi_a^2}{(1-3\lambda)\, f(A(t),a)} \\ &= \left[\frac{\kappa N(t)}{12\pi^2 a}\right]\left[\frac{\tilde{\pi}_a^2}{(1-3\lambda)}\right]\left[\frac{g_1^2(E(a(t))/E_P)}{g_2^3(E(a(t))/E_P)}\right] \\ &\quad\times f(A(t),a),\end{aligned} \tag{4.10}$$

thus the classical Hamiltonian constraint reduces to

$$\begin{aligned}\mathcal{H} &= \frac{\kappa}{12\pi^2 a}\frac{\tilde{\pi}_a^2}{(1-3\lambda)}\frac{g_1^2(E(a(t))/E_P)}{g_2^3(E(a(t))/E_P)} f(A(t),a) \\ &\quad - \frac{\pi^2 a^3(t)}{\kappa g_2(E(a(t))/E_P)}\left[\frac{6}{a^2(t)} - \frac{2\Lambda}{g_2^2(E(a(t))/E_P)}\right] \\ &= 0.\end{aligned} \tag{4.11}$$

It is then straightforward to see that the Hamiltonian density reduces to

$$\begin{aligned}\mathcal{H} &= \tilde{\pi}_a^2 + \frac{12(3\lambda-1)\pi^4 a^4(t)}{\kappa^2 g_1^2(E(a(t))/E_P)\; f(A(t),a)} \\ &\quad\times\left[g_2^2(E(a(t))/E_P)\frac{6}{a^2(t)} - 2\Lambda\right] = 0,\end{aligned} \tag{4.12}$$

where we have integrated out all degrees of freedom apart from the scale factor.

5 Correspondence of Gravity's Rainbow with Hořava–Lifshitz gravity

In the previous sections we have extracted the WDW equation in the cases of Hořava–Lifshitz gravity and GRw, for a FLRW background, that is, Eqs. (3.8) and (4.12), respectively. Hence, observing their forms we deduce that it is possible to create a formal correspondence between the two formulations provided that

$$g_1^2(E(a(t))/E_P)\, f(A(t),a) = 1 \tag{5.1}$$

and

$$g_2^2(E(a(t))/E_P)\frac{6}{a^2(t)} = \frac{6}{a^2(t)}\left[1 - \frac{2\kappa b}{a^2(t)} - \frac{4\kappa^2 c}{a^4(t)}\right]. \tag{5.2}$$

Since we preserve the freedom to fix $g_2(E(a(t))/E_P)$, we impose the requirement that

$$\begin{aligned}g_2^2(E(a(t))/E_P) &= 1 - \frac{2b\kappa}{a^2(t)} - \frac{4\kappa^2 c}{a^4(t)} \\ &= 1 - \frac{16bR}{R_0} - \frac{256cR^2}{R_0^2},\end{aligned} \tag{5.3}$$

where R_0 has been defined in (3.11) as $R_0 \equiv 6/G = 6/l_p^2$. Although at first sight identification (5.3) seems to be imposed *ad hoc*, it can be supported by invoking the dispersion relation of a massless graviton, which, as we show in Appendix B, for a FLRW background acquires the form

$$E^2 = \frac{k^2}{a^2(t)}, \tag{5.4}$$

with k the constant dimensionless radial wavenumber, and thus in the present case of GRw it is modified to

$$\frac{E^2}{g_2^2(E(a(t))/E_P)} = \frac{k^2}{a^2(t)}. \tag{5.5}$$

Since the dispersion relation (5.5) is valid at the Planck scale too, we can write

$$\frac{E^2}{g_2^2(E(a(t))/E_P)} \to \frac{E_P^2}{g_2^2(E_P/E_P)} = E_P^2 = \frac{k^2}{a_P^2}. \tag{5.6}$$

Hence, Eq. (5.3) becomes

$$\begin{aligned}g_2^2(E(a(t))/E_P) &= 1 - \frac{16b\pi R}{R_0} - \frac{256c\pi^2 R^2}{R_0^2} \\ &= 1 - c_1\frac{E^2(a(t))}{E_P^2} - c_2\frac{E^4(a(t))}{E_P^4}.\end{aligned} \tag{5.7}$$

Therefore we deduce that

$$E^2 = R/6k^2 \tag{5.8}$$

with

$$E_P^2 = G^{-1}, \quad c_1 = 16b\pi \quad \text{and} \quad c_2 = 256c\pi^2. \tag{5.9}$$

We mention here that the fact that a relation between the energy of a particle and the scalar curvature can come into play directly in the metric, is not a novelty. Indeed in [92] the scalar curvature enters into the metric via the trace of the Einstein's field equations connecting the energy-momentum tensor with the 4D scalar curvature. Moreover, note that the energy-momentum tensor has dimensions of energy density.

Thus, and in order to take the comparison on general grounds, one can assume that $g_2\left(E\left(a\left(t\right)\right)/E_P\right)$ can be represented by a formal expansion in powers of E/E_P, identifying the coefficients order by order. However, since in the present work we are comparing GRw with the Hořava–Lifshitz gravity with $z=3$, the formal Taylor expansion is truncated at the second order.

6 Correspondence in spherically symmetric backgrounds

The discussion on the WDW equations in GRw and Hořava–Lifshitz gravity of the previous section was presented in homogeneous and isotropic backgrounds, namely on the FLRW metric. One could wonder whether these results are an artifact of the space-time symmetries and not of the features of the two theories. Thus, in the present section we repeat the above analysis in the case of spherically symmetric backgrounds. In particular, we consider metrics of the class

$$\mathrm{d}s^2 = -N^2\left(r\right)\,\mathrm{d}t^2 + \frac{\mathrm{d}r^2}{1-b(r)/r} + r^2\left(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\phi^2\right), \tag{6.1}$$

where $N(r)$ and $b(r)$ are arbitrary functions of the radial coordinate r, denoted as the lapse function and the form function respectively. In this case, the energies now depend on the shape function $b\left(r\right)$ and the radial coordinate r, namely

$$\begin{aligned} g_1\left(E/E_P\right) &\equiv g_1\left(E\left(b\left(r\right)\right)/E_P\right) \\ g_2\left(E/E_P\right) &\equiv g_2\left(E\left(b\left(r\right)\right)/E_P\right). \end{aligned} \tag{6.2}$$

Hence, the metric modification appearing for the scalar curvature R is given by

$$R = g^{ij}R_{ij} = \frac{2b'\left(r\right)}{r^2}, \tag{6.3}$$

where the prime denotes derivative with respect to r, and we have used the mixed Ricci tensor R^a_j with components

$$R^a_j = \left\{\frac{b'\left(r\right)}{r^2} - \frac{b\left(r\right)}{r^3}, \frac{b'\left(r\right)}{2r^2} + \frac{b\left(r\right)}{2r^3}, \frac{b'\left(r\right)}{2r^2} + \frac{b\left(r\right)}{2r^3}\right\}. \tag{6.4}$$

When GRw is switched on, the line element (6.1) becomes

$$\begin{aligned} \mathrm{d}s^2 = &-\frac{N^2\left(r\right)}{g_1^2\left(E\left(b\left(r\right)\right)/E_P\right)}\,\mathrm{d}t^2 \\ &+\frac{\mathrm{d}r^2}{g_2^2\left(E\left(b\left(r\right)\right)/E_P\right)\left(1-b(r)/r\right)} \\ &+\frac{r^2}{g_1^2\left(E\left(b\left(r\right)\right)/E_P\right)}\left(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\phi^2\right), \end{aligned} \tag{6.5}$$

and the scalar curvature transforms as

$$\begin{aligned} R \to &\left[1-\frac{b\left(r\right)}{r}\right]\Bigg\{r^4\,g_2\left(E\left(b\left(r\right)\right)\right)\tilde{R}^2 \\ &\times\left\{\frac{\mathrm{d}^2 g_2\left(E\left(b\left(r\right)\right)\right)}{\mathrm{d}E^2}\left[\frac{\mathrm{d}E\left(b\left(r\right)\right)}{\mathrm{d}b}\right]^2 \right. \\ &\left. +\frac{\mathrm{d}g_2\left(E\left(b\left(r\right)\right)\right)}{\mathrm{d}E}\frac{\mathrm{d}^2E\left(b\left(r\right)\right)}{\mathrm{d}b^2}\right\} \\ &-\frac{3}{2}r^4\tilde{R}^2\left[\frac{\mathrm{d}E\left(b\left(r\right)\right)}{\mathrm{d}b}\right]^2\left[\frac{\mathrm{d}g_2\left(E\left(b\left(r\right)\right)\right)}{\mathrm{d}E}\right]^2 \\ &+4\,g_2\left(E\left(b\left(r\right)\right)\right)\frac{\mathrm{d}E\left(b\left(r\right)\right)}{\mathrm{d}b}\frac{\mathrm{d}g_2\left(E\left(b\left(r\right)\right)\right)}{\mathrm{d}E}\frac{\mathrm{d}^2b\left(r\right)}{\mathrm{d}r^2}\Bigg\} \\ &\times g_2\left(E\left(b\left(r\right)\right)\right)\frac{\mathrm{d}g_2\left(E\left(b\left(r\right)\right)\right)}{\mathrm{d}E}\frac{\mathrm{d}E\left(b\left(r\right)\right)}{\mathrm{d}b} \\ &\times\left[-\frac{r^3}{2}\tilde{R}^2 - 3b\left(r\right)\tilde{R} + 4r\tilde{R}\right] + g_2^2(E\left(b\left(r\right)\right))\,\tilde{R}, \end{aligned} \tag{6.6}$$

where the tildes indicate that the quantities are computed in the absence of the rainbow's functions. Although this is not necessary, for simplification we focus on the case where there is no explicit dependence of E on $b\left(r\right)$, that is, we assume $\mathrm{d}E\left(b\left(r\right)\right)/\mathrm{d}b = 0$. In this case the scalar curvature simplifies to

$$R \to g_2^2\left(E\left(b\left(r\right)\right)/E_P\right)\tilde{R}. \tag{6.7}$$

Since the extrinsic curvature K_{ij} becomes

$$K_{ij} = -\frac{\dot{g}_{ij}}{2N} = \frac{g_1\left(E\left(b\left(r\right)\right)/E_P\right)}{g_2^2\left(E\left(b\left(r\right)\right)/E_P\right)}\tilde{K}_{ij}, \tag{6.8}$$

even in this case the kinetic term does not contribute at the classical level and the GRw distortion is completely encoded in the potential term. Hence, if we assume the validity of Eq. (5.7) for the spherically symmetric case too, we find

$$\begin{aligned} g_2^2\left(E\left(a\left(t\right)\right)/E_P\right) &= 1 + g_2\frac{E^2\left(b\left(r\right)\right)}{E_P^2} + g_4\frac{E^4\left(b\left(r\right)\right)}{E_P^4} \\ &= 1 + g_2\frac{R}{R_0} + g_4\frac{R^2}{R_0^2}. \end{aligned} \tag{6.9}$$

Therefore, we conclude that one can establish a correspondence between GRw and Hořava–Lifshitz gravity in the spherically symmetric geometries too. Although we have shown this correspondence in the case of scalar curvature, we expect it to hold in the general case too, although such a feature needs to be proven formally.

7 Conclusions

In this work we explored the connection between two Lorentz-violating theories, namely GRw and Hořava–Lifshitz gravity. In GRw, it is the metric that incorporates all the distortion of the space-time when one approaches the Planck scale, while in Hořava–Lifshitz gravity, it is the potential part of the action (or the Hamiltonian) that acquires higher-order curvature terms. Usually GRw is switched on because a Planckian particle distorts the gravitational metric tensor $g_{\mu\nu}$. However, since in the present application we have neglected any matter fields, the only particle appearing is the graviton. Since the graviton is the quantum particle associated with the quantum fluctuations of the space-time, we conclude that it is the gravitational field itself that is responsible for such a distortion. This is also enforced by the dispersion relation relating the graviton energy and the scale factor, namely the scalar curvature, in the case where an FLRW background is imposed, or the graviton energy and the shape function in the case where a spherically symmetric background is imposed.

As we have shown, one can indeed establish a correspondence between the two theories, through the examination of their Wheeler–De Witt equations. However, although we have explicitly shown this in the case of two physically interesting space-times, namely the FLRW and the spherically symmetric ones, and thus we have a strong indication that this correspondence is not an artifact of the space-time symmetries but rather it arises from the features of the two theories, a general proof (or disproof) in the case of arbitrary metrics is still needed. In order to handle this issue, one might use the well-known relation between Hořava–Lifshitz gravity and Einstein-aether theory [93–95].

It is interesting to mention that GRw, in the FLRW background, generates Hořava–Lifshitz gravity under a specific form of $f(R)$ theory, with R the three-dimensional scalar curvature. A similar result was pointed out in [92], where a connection between the rainbow's functions and a specific $f(R)$ form seems to be evident. In our analysis we saw that the obtained correspondence includes information even for the terms of the type $R^{ij}R_{ij}$, $RR^{ij}R_{ij}$ and $R^i_jR^j_kR^k_i$, which were not explicitly included. Hence, we deduce that in order to incorporate higher-curvature terms, it is likely that the rainbow's functions must include terms of the form $R^{ij}R_{ij}$ etc., a possibility that could be encoded in the Kretschmann scalar. These issues reveal that the bridge between GRw and Hořava–Lifshitz gravity could be much richer, and it deserves further investigation.

We close this work by mentioning that in the above analysis we have remained at the background level, as a first step towards bridging the two theories. However, it is required and it is interesting to examine their relation at the perturbation level too, since there are many examples of theories that coincide at the background level, while being distinguishable or different when one incorporates the perturbations. Furthermore, relating the perturbations between GRw and Hořava–Lifshitz gravity becomes necessary having in mind the problems of the extra mode propagation that appears in the simple versions of the latter [96–99]. Since such a detailed analysis lies beyond the scope of the present manuscript it is left for a future investigation.

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Appendix A: Kinetic term in Gravity's Rainbow with a time-dependent energy term

In the case where $E \equiv E(a(t))$, the extrinsic curvature of the metric (4.1) acquires the form of Eq. (4.3), namely

$$K_{ij} = -\frac{g_1(E(a(t))/E_P)}{2N}\frac{\mathrm{d}}{\mathrm{d}t}\left[\frac{g_{ij}}{g_2^2(E(a(t))/E_P)}\right] = \frac{g_1(E(a(t))/E_P)}{g_2^2(E(a(t))/E_P)}\left[\tilde{K}_{ij} + \tilde{g}_{ij}\frac{A(t)}{N(t)}\dot{a}(t)\right], \quad \text{(A.1)}$$

where

$$A(t) = \frac{1}{g_2(E(a(t))/E_P)E_P}\frac{\mathrm{d}}{\mathrm{d}E}\left[g_2(E(a(t))/E_P)\right]\frac{\mathrm{d}E}{\mathrm{d}a}, \quad \text{(A.2)}$$

and with dots denoting differentiation with respect to time. In the above expressions the tildes indicate the quantities computed in the absence of the rainbow's functions. The trace of the extrinsic curvature becomes

$$K = g^{ij}K_{ij} = g_2^2(E(a(t))/E_P)\tilde{g}^{ij}K_{ij} = g_1(E(a(t))/E_P)\left[\tilde{K} + 3\frac{A(t)}{N(t)}\dot{a}(t)\right], \quad \text{(A.3)}$$

while raising the indices in K_{ij} we obtain

$$K^{ij} = g^{il}g^{jm}K_{lm} = g_2^2(E(a(t))/E_P)g_1 \times (E(a(t))/E_P)\left[\tilde{K}^{ij} + \tilde{g}^{ij}\frac{A(t)}{N(t)}\dot{a}(t)\right]. \quad \text{(A.4)}$$

Hence, the kinetic term becomes

$$K^{ij}K_{ij} - \lambda K^2 = g_1^2\left(E\left(t\right)/E_P\right)\left\{\tilde{K}^{ij}\tilde{K}_{ij} - \lambda\tilde{K}^2 + (1-3\lambda)\left\{\frac{2\tilde{K}}{N\left(t\right)}A\left(t\right)\dot{a}\left(t\right) + 3\left[\frac{A\left(t\right)}{N\left(t\right)}\dot{a}\left(t\right)\right]^2\right\}\right\}. \tag{A.5}$$

For the specific case of a FLRW metric we find that

$$\tilde{K}_{ij} = -\frac{\tilde{g}_{ij}}{N\left(t\right)}\frac{\dot{a}}{a}, \tag{A.6}$$

and thus

$$\tilde{K}^{ij}\tilde{K}_{ij} - \lambda\tilde{K}^2 = 3\frac{(1-3\lambda)}{N^2\left(t\right)}\left(\frac{\dot{a}}{a}\right)^2. \tag{A.7}$$

In this case Eq. (A.5) becomes

$$K^{ij}K_{ij} - \lambda K^2 = 3g_1^2\left(E\left(t\right)/E_P\right)\frac{(1-3\lambda)}{N^2\left(t\right)}\left(\frac{\dot{a}}{a}\right)^2 \times f\left(A\left(t\right), a\right), \tag{A.8}$$

where

$$f\left(A\left(t\right), a\right) = \left[1 - 2a\left(t\right)A\left(t\right) + A^2\left(t\right)a\left(t\right)^2\right]. \tag{A.9}$$

It is now possible to calculate the kinetic part of the action, which is defined as

$$S_K = \int_{\Sigma\times I} \mathrm{d}t\mathrm{d}^3x\mathcal{L}_K, \tag{A.10}$$

where

$$\mathcal{L}_K = \frac{N}{2\kappa}\sqrt{g}\left(K^{ij}K_{ij} - \lambda K^2\right). \tag{A.11}$$

Inserting (A.8) into S_K we obtain

$$S_K = \frac{3}{\kappa}\pi^2\int_I \mathrm{d}t N\left(t\right)a\dot{a}^2\frac{g_1^2\left(E\left(a\left(t\right)\right)/E_P\right)}{g_2^3\left(E\left(a\left(t\right)\right)/E_P\right)}\frac{(1-3\lambda)}{N^2\left(t\right)} \times f\left(A\left(t\right), a\right), \tag{A.12}$$

and thus the canonical momentum reads

$$\pi_a = \frac{\delta S_K}{\delta\dot{a}} = \frac{g_1^2\left(E\left(a\left(t\right)\right)/E_P\right)}{g_2^3\left(E\left(a\left(t\right)\right)/E_P\right)}f\left(A\left(t\right), a\right)\tilde{\pi}_a, \tag{A.13}$$

where

$$\tilde{\pi}_a = \frac{6\pi^2}{\kappa}\frac{(1-3\lambda)}{N\left(t\right)}\dot{a}a. \tag{A.14}$$

To be definite, we restrict ourselves to the case $\lambda \neq \frac{1}{3}$, since in the special case where $\lambda = \frac{1}{3}$ the ultralocal metric (the one-parameter family of supermetrics, which allows one to disentangle gauge modes from physical deformations) [100,101], is not invertible and becomes a projector onto the tracefree subspace.

Appendix B: The Lichnerowicz equation for the graviton

In 3 + 1 dimensions the graviton operator is described by

$$O^{ikjl} = \triangle_L^{ikjl} - 4R^{il}g^{kj} + Rg^{ik}g^{jl} + \frac{\partial^2}{\partial t^2}g^{ik}g^{jl}, \tag{B.15}$$

where we have assumed the absence of mixing of time and space, which naturally follows from the structure of the FLRW metric (3.1). The Riemann tensor in three dimensions becomes

$$R_{ikjl} = g_{ij}R_{kl} - g_{il}R_{kj} - g_{kj}R_{il} + g_{kl}R_{ij} - \frac{R}{2}\left(g_{ij}g_{kl} - g_{il}g_{kj}\right), \tag{B.16}$$

and for a FLRW background the three-dimensional Ricci curvature tensor and the scalar curvature read

$$R_{ij} = \frac{2}{a^2\left(t\right)}\gamma_{ij} \quad \text{and} \quad R = \frac{6}{a^2\left(t\right)}, \tag{B.17}$$

where γ_{ij} is the metric on the spatial sections which have constant curvature $k = 0, \pm 1$, defined by

$$\mathrm{d}\Omega_3^2 = \gamma_{ij}\mathrm{d}x^i\mathrm{d}x^j. \tag{B.18}$$

Hence, the Riemann tensor reduces to

$$R_{ikjl} = -\frac{2}{a^2\left(t\right)}\left(\gamma_{ij}\gamma_{kl} - \gamma_{il}\gamma_{kj}\right). \tag{B.19}$$

Then the operator O^{ikjl} on transverse traceless tensors reduces to

$$O^{ikjl} = a^{-2}\left(t\right)\left(-\nabla^a\nabla_a\gamma^{ik}\gamma^{jl} + 2\gamma^{il}\gamma^{kj}\right) + \frac{1}{N^2}\frac{\partial^2}{\partial t^2}\gamma^{ik}\gamma^{jl}, \tag{B.20}$$

and the dispersion relation becomes

$$\frac{k^2}{a^2\left(t\right)} = E^2, \tag{B.21}$$

where, as usual, in the end of the calculation we have set the lapse function N to 1. Finally, as shown in [10], in the case of GRw the above dispersion relation has to be modified to

$$\frac{k^2}{a^2\left(t\right)} = \frac{E^2}{g_2^2\left(E/E_P\right)}. \tag{B.22}$$

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