# Stochastic versus Robust Optimization for a Transportation Problem

Francesca Maggioni[1], Marida Bertocchi[1], Florian Potra[2]

[1] Department of Management, Economics and Quantitative Methods, University of Bergamo Via dei Caniana 2, 24127 Bergamo, Italy
{francesca.maggioni, marida.bertocchi}@unibg.it
[2] Department of Mathematics & Statistic, University of Maryland, Baltimore County, Baltimore, MD 21250, U.S.A.
potra@umbc.edu

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## 1 Abstract

The problem of transporting goods or resources from a set of supply points (production plants) to a set of demand points (destination factories or customers) is an important component of the planning activity of a manufacturing firm. Critical parameters such as customer demands, row material prices, and resource capacity are quite uncertain in real problems. An important issue is then represented by the decision on quantities to acquire and store at each destination factory before actual demands reveal themselves. This is involved in the tactical planning of the firm supply chain operations. The significance of uncertainty has prompted a number of works addressing random parameters in tactical level supply chain planning involving distribution of raw material and products (see for example [7], [18], [15], [8], [9] and [19]).

In this paper we analyze the effect of two modelling approaches, stochastic programming and robust optimization, to a real case of a transportation problem under uncertainty. To the best of our knowledge, a direct comparison between SP and RO on such a class of problems has not been addressed yet in literature. Moreover, robust optimization is relatively new concept and there is very little work applying it in a logistic setting. Stochastic Programming (SP) and Robust Optimization (RO) are considered two alternative techniques to deal with uncertain data both in a single period and in a multi-period decision making process. The main difficulty associated with the former is the need to provide the probability distribution functions of the underlying stochastic parameters. This requirement creates a heavy burden on the user because in many real world situations, such information is unavailable or hard to obtain (see for example [6] and [16]). On the other side Robust Optimization addresses the uncertain nature of the problem without making specific assumptions on probability distributions: the uncertain parameters are assumed to belong to a deterministic uncertainty set. The drawback of this approach is the potentially strong dependence of the solution on the rather arbitrarily chosen uncertainty set. RO adopts a min-max approach that addresses uncertainty by guaranteeing the feasibility and optimality of the solution against all instances of the parameters within the uncertainty set. A vast literature about the hypotheses that have to be imposed on the structure of the uncertainty set in order to have computationally tractable problems are available,

see [17] and [4] for polyhedral uncertainty sets and [3] for ellipsoidal uncertainty sets. The original RO model deals with static problems where all the decision variables have to be determined before any of the uncertain parameters are realized. This is not the typical situation in most transportation problems that are multiperiod in nature, and where a decision at any period can and should account for data realizations in previous periods. An extension of robust optimization to a dynamic framework was introduced by [3] via the concept of affinely adjustable robust counterpart (AARC), where part of the decision variables, the so-called adjustable variables, have to be determined after a portion of the uncertain data is realized. The dependence of the adjustable variables on the realized data is represented by an affine function. Other contributions along this line may be found in [2] and in [5]. Notice that the role of adjustable variables in AARC formulation is quite similar to the role of second stage (or recourse) variables in SP formulation. However, SP allows to compute first stage and second stage variables at once, while this is not the case for AARC formulation.

The transportation problem considered, is inspired by a real case of *gypsum* replenishment in Italy, provided by the primary Italian cement producer. The logistic system is organized as follows: a set of suppliers, each of them composed of a set of several plants (origins) located all around Italy have to satisfy the demand of gypsum of a set of cement factories (destinations) belonging to the same cement company producer. The weekly demand of gypsum at cement factories is considered stochastic. We assume a uniform fleet of vehicles with fixed capacity and allow only full-load shipments. Shipments are performed by capacitated vehicles which have to be booked in advance, before the demand is revealed. When the demand becomes known, there is an option to discount vehicles booked but not actually used from different suppliers. The cancellation fee is given as a proportion of the transportation costs. If the quantity shipped from the suppliers using the booked vehicles is not enough to satisfy the demand of the factories, residual product is purchased from an external company at a higher price, which is also uncertain. The problem consists in determining the number of vehicles to book, at the end of each week, from each plant of the set of suppliers, to replenish gypsum at cement factories in order to minimize the total cost, given by the sum of the transportation costs from origin to destinations (including the discount for vehicles booked but not used) and the cost of buying units of product from external sources in case of inventory shortage. The problem described can be classified as a *transportation problem under uncertainty* where a set of retailers is served by a set of suppliers. A particular case is given by the so-called single-sink transportation problem, in which a single retailer is served by a set of suppliers. This problem has been also deeply studied, in particular when the total cost is given by the sum of a variable transportation cost and a fixed charge cost to use the supplier ([11], [10], [1] and [13]).

We solve the problem both via a two-stage stochastic programming and robust optimization models with different uncertainty sets. For the former the goal is to compute the minimum expected cost based on the specific probability distribution of the uncertain demand of gypsum at the cement factories and buying cost from external sources based on a set of possible scenarios. Scenarios of demand, for all destinations at the first week of March 2014, are built on historical data directly, using all the weekly values in March, April, May and June of the years 2011, 2012 and 2013. On the other hand, scenarios of buying costs have been generated sampling from a uniform distribution in an interval with a 20% deviation level with respect to the average value. In this way a scenario tree composed of 48 leaves has been built. The resulting linear mixed-integer stochastic programming model is implemented in AMPL and solved using the CPLEX 12.5.1.0. solver. An in-sample stability for an increasing number of scenarios up to 1000 has been verified. The solution is firstly compared with the *Expected Value* (EV) problem under the unique average scenario: the deterministic model will always book the exact number of vehicles needed for the next period, it sorts the suppliers and their plants according to the transportation costs and books a full production capacity from the cheapest one, followed by the next-cheapest. As long as there is enough transportation capacity, the model will never purchase extra gypsum from external sources. The EV model books much fewer vehicles than the stochastic one resulting in a solution costing only 70% of the stochastic counterpart. When using the EV solution, the *expectation expected value* problem EEV is infeasible since a minimum requirement constraint with the biggest supplier is no longer satisfied (see [12] and [14]). The *Expected Value of Perfect Information* EVPI reduces to 20% of total cost showing the advantage of having the information about future uncertainty at the first stage.

However, since the cement demand is highly affected by the economic conditions of the public and private medium and large-scale construction sector, in the last years its variability has been very high. Therefore a reliable forecast and reasonable estimates of demand probability distributions are difficult to obtain. This is the main reason that lead us to consider also robust optimization approaches. First we consider static approaches with uncertainty parameters respectively belonging to boxes, ellipsoidal uncertainty sets or mixture of them, and secondly dynamic approaches, via the concept of affinely adjustable robust counterpart. The main advantage of the RO formulations considered, is that they can be solved in polynomial time and have theoretical guarantees for the quality of the solution which is not the case with the aforementioned SP formulations. A robust solution at the tactical level allows to find a feasible solution for the operational planning problem for each possible realization of demand in the uncertainty set considered. The robust mixed-integer linear optimization model with box uncertainty is modeled in AMPL and solved using the CPLEX 12.5.1.0. solver. Numerical results shows that the robust box-constrained solution is very conservative having a total cost more than twice larger than the expected cost obtained by solving RP. However, the choice of the box uncertainty set is preferable only if the feasibility of all the constraints is highly required. We use also a different uncertainty set in order to get a less conservative outcome: a box-ellipsoid uncertainty set which considers a box for the demand and an ellipsoid for the buying cost is introduced. It requires relaxation of integrality constraints and the solution of a *second-order cone program* SOCP. The box-ellipsoid approach can be solved in polynomial time using the MOSEK solver and it allows to reduce the total cost while guaranteeing that the constraints of the problem are satisfied with high probability. The behavior of the solutions obtained by the two described approaches are different: in the box continuous case, the choice of buying gypsum from external sources, is limited due to the largest buying cost. On the other side, the optimal solution of the box-ellipsoidal model considers more convenient to buy from external sources than using some of the suppliers because the buying cost is modelled with a less conservative ellipsoid uncertainty set. Still the total cost is more than twice larger than the expected cost obtained by solving RP. These are the results obtained using a static robust formulation.

In order to make a fair comparison with the stochastic programming methodology, dynamic approaches via the concept of adjustable/affinely adjustable robust counterpart are also considered. The variables are then partitioned in nonadjustable, i.e. the ones to be determined before the actual data “reveals itself” and in adjustable ones. The adjustable variables, once the uncertain data become known, should be able to adjust themselves by means of some decision rules, eventually specified by affine functions of their arguments. Numerical experiments show that also adjustable robust approach result in around 30% larger objective function values with respect to RP solutions due to the certitude of constraints satisfaction. Conversely, the computational complexity is higher for the stochastic approach.

In conclusion, the SP approach allows the company to reach higher profits, even if the computational effort is expensive expecially due to the scenario generation procedure. On the other hand RO forces the firm to consider an higher cost solution which is strongly dependent on an arbitrarily chosen uncertainty set, but with probability guarantee of costraints satisfaction.

## References


[1] B. Alidaee and G.A. Kochenberger, "A note on a simple dynamic programming approach to the single-sink, fixed-charge transportation problem", *Transportation Science* 39(1), 140-143 (2005).

[2] A. Ben-Tal, D. Bertsimas and D.B. Brown, "A Soft Robust Model for Optimization Under Ambiguity", *Operations Research* 58(4), Part 2, 1220-1234 (2010).

[3] A. Ben-Tal, A. Goryashko, E. Guslitzer and A. Nemirovski, "Adjusting robust solutions of uncertain linear programs", *Mathematical Programming* 99(2), 351-376 (2004).

[4] D. Bertsimas and M. Sim, "The price of robustness", *Operations Research* 52(1), 35-53 (2004).

[5] D. Bertsimas and V. Goyal, "On the power and limitations of affine policies in two-stage adaptive optimization", *Mathematical Programming* Ser. A, 134, 491-531, (2012).

[6] J.R. Birge and F. Louveaux, *Introduction to stochastic programming*, Springer-Verlag, New York, (2011).

[7] R.K. Cheung and W.B. Powell, "Models and Algorithms for Distribution Problems with Uncertain Demands", *Transportation Science* 30, 43-59 (1996).

[8] L. Cooper and L.J. LeBlanc, "Stochastic transportation problems and other network related convex problems", *Naval Research Logistics Quarterly* 24, 327-336 (1977).

[9] T.G. Crainic and G. Laporte, "Planning models for freight transportation", *European Journal of Operational Research* 97 (3), 409-438, (1997).

[10] B.W. Lamar and C.A. Wallace "Revised-modified penalties for fixed charge transportation problems", *Management Science* 43(10), 1431-1436 (1997).

[11] B.W. Lamar, Y. Sheffi and W.B. Powell, "A capacity improvement lower bound for fixed charge network design problems", *Operations Research* 38(4), 704-710 (1990).

[12] F. Maggioni and S.W. Wallace, "Analyzing the quality of the expected value solution in stochastic programming", *Annals of Operations Research* 200, 37-54 (2012).

[13] F. Maggioni, M. Kaut and L. Bertazzi, "Stochastic Optimization models for a single-sink transportation problem", *Computational Management Science* 6, 251-267 (2009).

[14] F. Maggioni, E. Allevi and M. Bertocchi, "Bounds in Multistage Linear stochastic Programming", *Journal of Optimization, Theory and Applications* 163(1), 200-229 (2014).

[15] W.B. Powell and H. Topaloglu, "Stochastic Programming in Transportation and Logistics", in *Handbooks in Operations Research and Management Science* 10, 555-635 (2003).

[16] A. Ruszczinski and A. Shapiro, *Stochastic Programming*, Elsevier, Amsterdam (2003).

[17] A.L. Soyster, "Convex programming with set-inclusive constraints and applications to inexact linear programming", *Operations Research* 21, 1154-1157 (1973).

[18] H. Van Landeghem and H. Vanmaele, "Robust planning: a new paradigm for demand chain planning", *Journal of Operations Management* 20(6), 769-783 (2002).

[19] C. Yu and H. Li, "A robust optimization model for stochastic logistic problems", *International Journal of Production Economics* 64(1-3), 385-397 (2000).