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Income shares, wealth and growth☆

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Abstract

The paper analyzes the relationship between income shares, wealth and growth in an environment where positional goods are taken into account and rent is generated. This hypothesis, which is a macro engine for inequality, creates a gap between profit share and property share and implies a clear-cut distinction between capital and wealth.

The interactions between these aspects are studied in a medium-run growth model led by aggregate demand, where monetary aspects also matter. The results of the dynamic analysis, obtained by means of simulations, are in keeping with some recent stylized facts. Furthermore, the model generates bounded dynamics, where the co-movements between variables are more complex than those obtained in the recent literature. At the same time the disequilibrium processes can create a link between medium-run considerations and a more long-run perspective.

JEL classification: D31; E; E2; 04

Keywords: Property and profit share; Positional goods; Private wealth; Inequality; Demand-led growth; Limit cycles

Resumo

Esse artigo analisa a relação entre participação na renda, riqueza e crescimento em um ambiente em que bens de ‘status’ são levados em consideração e renda é gerada. Essa hipótese, que é uma macro motor de desigualdade, cria um hiato entre participação dos lucro e participação de propriedades e implica em uma distinção clara entre capital e riqueza. As interações entre esses aspectos são estudados em um modelo de crescimento de médio prazo impulsionado pela demanda agregada, onde aspectos monetários são importantes também. Os resultados dessa análise dinâmica, obtidos por meio de simulações, estão de acordo com alguns fatos estilizados. Além disso, o modelo gera uma dinâmica restrita, onde o co-movimento entre as variáveis são mais complexos do que

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aqueles encontrados na literatura recente. Ao mesmo tempo, o processo de desequilíbrio pode criar uma ligação entre considerações de médio-prazo e uma perspectiva de mais longo prazo.

Palavras chave: Participação de propriedade e do lucro; Bens de status; Riqueza privada; Desigualdade; Ciclos limites

1. Introduction

If one considers the performance of advanced economies in recent times, there are three stylized facts worth mentioning: (i) a falling labour share; (ii) an increasing wealth output ratio and (iii) stagnating growth that produces negative repercussions on the labour market.

These patterns are in strong contrast with the Kaldorian "stylized facts" that any growth theory was supposed to mimic. While the first one violates the hypothesis of constant share, which, given a constant capital/output ratio, implies also a stationary rate of profit, the second one was not even mentioned as a macro theme.

In fact, wealth has mainly been a subject of microeconomics, welfare theory or fiscal policy, where the personal distribution of income matters. One of Piketty's (2014) main contributions has been to cast the two themes, i.e. income share and wealth distribution, into a macro dimension, where inequality provides the profound links between the two aspects. These relationships are studied in a very long time horizon and within a steady state environment.

With respect to this analysis, the paper introduces three substantial changes. First, positional goods and rents are introduced into the analysis. Even though from an empirical point of view they are not the major vehicle of capital gains, they seem to represent the "*zeitgeist*" of the present economic situation, dominated by bubbles and their aftermath. Furthermore, from an analytical point if view, they oblige to draw a clear-cut distinction between capital and wealth and to deal with profits and rents, as far as income shares are concerned. Second, the time horizon of the present paper is shorter because it refers to a medium-run perspective (see also Ferri, 2011) which is better suited to capture the events of the "Great Recession" (see also Cynamon et al., 2013). Third, the model put forward is characterized by the following features: its dynamics are driven by aggregate demand, where investment plays a fundamental role. Furthermore, the model operates in a monetary economy, as defined by Keynes. Finally, steady state considerations are supplemented by a dynamic process of interdependence, where the feedbacks between variables make the co-movements between them more complex. In this context, one must possibly consider how instability can affect inequality and how the latter can feedback on the former.

The results of the dynamic analysis are obtained by means of simulations. They not only can mimic the stylized facts before mentioned, but they generate bounded dynamics where accelerating expansions are endogenously followed by opposite movements. At the same time they indicate that the relationship between income share, wealth and growth can assume different features depending on the nature of the model, the type of hypotheses put forward and the quantitative value of the parameters.

The structure of the paper is the following. Section 2 reviews the recent literature. Section 3 introduces rents and positional goods, and identifies the proper income share. Section 4 deals with capital and wealth ratios and defines technology. Section 5 presents a demand-led growth model, discusses steady state relationships and uses simulations in order to carry out a disequilibrium analysis. Section 6 discusses the robustness of a linearized version of the model, it also considers an extension of the model and discusses the implications of different growth scenarios. Section 7 concludes. Appendices A and B includes mathematical details.

2. A review of the recent literature

The main contribution of Piketty's (2014) analysis is to have studied the macro conditions that favour inequality (see Milanovic, 2014 and the Special issue of Real-World Economics Review, 2014 for a review of the book).

These macro conditions are determined by two laws: the "first fundamental law" is initially derived from an identity, i.e. the definition of profit share (α):

$$\alpha \equiv r\beta \tag{2.1}$$

If r, the average return of capital, is assumed to be given and stationary, then the identity becomes an equation and turns itself in the "first fundamental law", which states that an increase in β, which is both the wealth/output ratio and the capital/output ratio, is accompanied by a surge in the profit share α. Since capital is more unequally distributed than labour, it follows that profit share can be used as a macro proxy of inequality, which ultimately depends on wealth distribution.

The second fundamental law of capitalism is given by:

$$g = \frac{s}{\beta} \tag{2.2}$$

where s is the (net) exogenous propensity to save and g the rate of growth. This formula is in keeping with the long-run steady state relationship à la Harrod-Domar (see also Fazzari et al., 2013; Dutt, 2006). Suppose that g is determined à la Solow, i.e. by the supply side of the economy and therefore is equal to the rate of growth of productivity and labour force. It then follows that, for a given s, a fall of g implies an increase in β. Stagnation induces an increase in wealth. Since a stagnation period is in front of us, a higher wealth/ratio and therefore increasing inequalities are expected. The forecast is based upon the following causal scheme:

$$g \downarrow \rightarrow \beta \uparrow \alpha \uparrow$$

In other words, an exogenous fall in the rate of growth (g) entails an increase in the wealth/output ratio β and an increase in the profit share (α). Since capital is more unequally distributed than labour, the rise in α is accompanied by an increase in inequality. This result is at the root of the macro inference of inequality pursued by Piketty (2014).

The results do not depend only on the neoclassical nature of the model, but they are marked by two aspects that are worth stressing. The first one is that there is no distinction between capital and wealth. It follows that the capital/output ratio v of the growth literature has been replaced by β, the wealth/output ratio in the two formulas above. This fallacy has been stressed by many authors (see also Aspromourgos, 2015; Homburg, 2014). A consequence of the distinction is that a rise of β does not necessarily entails a rise in v and therefore other causes, apart from technology, are to be looked for, as suggested by Stiglitz (2015). The second aspect is that these relationships are characterized by complex dynamic patterns so that it is essential to overcome exercises of comparative statics in order to find meaningful relationships and feedbacks in a dynamic context.

3. Rents and positional goods

3.1. An introduction

Although one is the complement of the other, capital share and labour share do not usually raise the same type of problems. This is the reason why they have been studied with different approaches in time and in different contexts. Usually, in empirical studies, the emphasis is on the labour share defined as:

$$1 - \alpha = \frac{WN}{PY}$$

i.e. the ratio of real wages to output per man, usually measured in terms of value added. This strategy is one way of bypassing the so-called "capital debate" (see Harcourt, 1972). Most of the studies agree on the global decline of the labour share in the recent period (see Karabarbounis and Neiman, 2014), even though different explanations have been put forward, ranging from the impact of globalization to the role of technical change.

At the theoretical level, however, the emphasis has very often been on the profit share (α), since the profit rate, one of its determinants, is a fundamental variable in growth models. To avoid difficult problems, the analysis has been often confined to steady state situations, where past, present and future values are well defined (see Robinson, 1956).

Since one of the aims of the paper is to link income share to wealth, it is rather important to understand that in this case "tertium non datur", in other words along with wages and profit, also rents must be considered, above all in a period when different property bubbles have taken place. In this case, profit share becomes a subset of what may be named the property share.

To better illustrate this aspect, positional goods are introduced into the analysis (see Ferri, 2016 for a generalization). In other words, consider land for building that is exogenously given and that remains constant with respect to GDP.

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Call this ratio θ_1.[1] Furthermore, suppose that it does not interfere with the production of other goods. Finally, suppose that rent (R_t) can increase because rich people invest their savings, especially when monetary policy is permissive.

In terms of the national accounts, the following identity holds:

$$Z_t = Y_t + R_t \tag{3.1}$$

where all the items are in real terms, while Z_t is GDP including rents.

By dividing through by GDP net of rents, i.e. Y_t, one obtains:

$$z_t = 1 + \frac{R_t}{Y_t} \tag{3.2}$$

Assume that the ratio in the r.h.s. is constant and equal to θ_2 so that the following relationship is obtained:

$$z_0 = 1 + \theta_2 \tag{3.2'}$$

where the zero subscript refer to steady state values. Two implications follow. The first is that only one rate of growth is relevant (g_t). The second is that the relative price Q_t (i.e. the value of rents deflated by the general level of prices) remains to be determined. For instance, one can suppose the following specification:

$$Q_t = \chi_1 - \chi_2 \left[(R_t - \pi_t) - r_0 \right] + \chi_3 (\beta_{t-1} - \beta_0) \tag{3.3}$$

where R_t and π_t are respectively the nominal rate of interest and the inflation rate and r_0 is the steady state real rate of interest, which in equilibrium also equal the rate of profit. Specification (3.3) then implies that the relative price of positional goods decreases when the monetary policy is tight (i.e. when the actual real rate of interest is bigger than its steady state value) and increases with the wealth ratio.

3.2. The three components of income share

In this context, property share (α) is equal to the sum of profit share ($r_t v_t$), where r_t is the rate of profit multiplied the capital/output ratio v_t, and the rent share.[2] Since the former is expressed in terms of Z_t, while its determinants are valued in terms of Y_t, the corrective term z_0 must be introduced:

$$\alpha_t = \frac{r_t v_t + \theta_2}{z_0} \tag{3.4}$$

This equation can be expressed as determining the rate of profit (r):

$$r_t = \frac{\alpha_t z_0 - \theta_2}{v_t} \tag{3.5}$$

where the negative role of rents on the rate of profit emerges, for a given α (to be determined later on).

The wealth ratio can be written in the following way:

$$\beta_t = v_t + \Omega_t \tag{3.6}$$

where v_t is the capital/output ratio, while Ω_t is the property wealth ratio given by

$$\Omega_t = Q_t \theta_1 \tag{3.7}$$

where θ_1 is an increase in the property ratio can make personal wealth grow without necessarily impacting on capital, which remains a distinct concept.

1 For a justification of this assumption, see Stiglitz (2015).
2 One might consider also r interest on public debt. See Ferri (2016). According to Lawrence (2015) capital share is bigger than profit share.

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4. Capital ratios and technology

In the old growth theory, capital is essentially fixed depreciable capital (see Knibbe, 2014) or reproducible capital, as stressed by Homburg (2014).[3] Given the following accumulation equation (and ignoring depreciation):

$$K_t = K_{t-1} + I_{t-1}$$

where K is capital and I investment, one can divide both sides by Y_t in order to obtain the following capital/output ratio equation:

$$v_t = \frac{K_t}{Y_t} = \frac{v_{t-1}}{(1+g_t)} + \frac{i_{t-1}}{(1+g_t)}$$

In a medium-run perspective, it is not reasonable to assume that the capital/output ratio is constant.[4] This hypothesis implies that the production function is assumed to have fixed coefficients, while technical progress can be assumed to be Harrod-neutral.

Within this perspective, which is rather different from that advanced by Stiglitz (2015), three aspects are to be stressed. The first is that one obtains the following determinants of the rate of growth:

$$g_t = \frac{i_{t-1}}{v^*} \tag{4.1}$$

In the second place, this rate of growth is supposed to be lower than the "natural one" so that spare resources are available and this justifies the absence of an explicit price-wage mechanism. Finally, it must be matched by aggregate demand so that an interdependent process between aggregate demand and supply is set in motion.

5. A nonlinear demand-led growth model

The above nonlinear equations, expressed in intensive form, i.e. divided by Y_t, can be inserted into a growth model that presents three characteristics. First, differently from Piketty (2014) and Stiglitz (2015) investment plays a strategic role. Second, the model also considers inflation and the monetary sector plays a role along with economic policies measures. Third, steady state considerations are supplemented by a dynamic process of interdependence where the feedbacks between variables make the co-movements between them more complex.

In order to close the model the following equations are be considered. The inflation rate is represented by a Phillips curve, where the Okun's law has been used to transform the rate of unemployment into a rate of growth of output:

$$\pi_t = \pi^* + \phi_1(\pi_{t-1} - \pi_0) + \phi_2(g_t - g_0) \tag{5.1}$$

where π^* is the target rate of inflation, and the subscript 0 again refers to the steady state value. The nominal rate of interest (R) is determined according to a (simplified) Taylor equation of the type:

$$R_t = R^* + \psi_1(\pi_t - \pi_0) \tag{5.2}$$

where R^* is the target interest rate. (Later on it will be modified.) The expected (real) rate of profit (r) is formulated in the following simple adaptive way:

$$E_t r_t = (1-\rho_1) r_{t-1} + \rho_1 r_0 \tag{5.3}$$

Given the values of these variables, the investment ratio can be determined in the following way:

$$i_t = i_a + \gamma_1 \{E r_t - [(R_t - \pi_t)]\} \tag{5.4}$$

where i_a is the autonomous component, while the term inside the square parenthesis represents the excess of the expected rate of profit over the real rate of interest. Also for this equation a different specification will be introduced.

[3] As is well known, in the "new growth" theories, human capital is considered. See Aghion and Howitt (1998).
[4] According to Jones (2015), this ratio (referred to reproducible capital) has remained almost constant in the US during the post war period.

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The saving ratio is defined as:

$$s_t = [(s_\pi - s_w)\alpha_t + s_w]z_0 - c_\beta \beta_t$$

where s_π is the propensity to save out of profits, while s_w represents the propensity to save out of wages. The usual inequality holds:

$$s_\pi > s_w$$

On the contrary, c_β stands for the propensity to consume out of wealth (a mix of Pasinetti, 1962; Kaldor, 1956 hypothesis) and represents the channel through which wealth and inequality feedback on the system. Finally, z_0 represents the steady state ratio between GDP with rent.

The macro equilibrium implies that $s_t = i_t$ and this implies that the property share is endogenously given by:

$$\alpha_t = \frac{i_t + c_\beta \beta_t - s_w z_0}{(s_\pi - s) z_{0w}} \tag{5.5}$$

In the above nonlinear system there are 10 unknowns g_t, β_t, Ω_t, Q_t, r_t, π_t, R_t, Er_t, i_t and α_t in 10 equations ((3.3), (3.5)–(3.7), (4.1) and (5.1)–(5.5)).[5]

5.1. Steady state

The steady state values of the model (marked by the subscript 0) are easily obtained in a recursive way. In equilibrium, the rate of profit is equal to the real rate of interest. It then follows that:

$$i_0 = i_a \tag{5.6}$$

and therefore:

$$g_0 = \frac{i_a}{v^*} \tag{5.7}$$

It turns out, from (3.3), that:

$$Q_0 = \chi_1 \tag{5.8}$$

This equation allows one to determine the wealth ratio, so that the steady state property share becomes (from (5.5)):

$$\alpha_0 = \frac{i_0 + c_\beta \beta_0 - s_w z_0}{(s_\pi - s_w) z_0} \tag{5.9}$$

The steady state values of the other variables are also easily obtained. In fact the rate of profit is given by Eq. (3.5):

$$r_0 = \frac{\alpha_0 z_0 - \theta_{2t}}{v^*} \tag{5.10}$$

Given the nominal rate of interest (R^*) and the equality between the rate of profit and the real rate of interest, the inflation rate is given by[6]:

$$\pi_0 = R^* - r_0 \tag{5.11}$$

5.2. A disequilibrium analysis

In order to detect the dynamics of the model and the co-variances between variables in a disequilibrium context (see Hicks, 1965), the nonlinear system of equations has been simulated. The values of the parameters are illustrated in Table 1, while the dynamics are pictured in Fig. 1.

The values of both, parameters and exogenous variables, are chosen by referring to empirical studies. In their absence, values capable of producing steady state close to the empirical values have been adopted.[7]

5 An alternative strategy would let v_t change and assume a given income distribution.
6 The assumption of a constant θ_2 constrains the steady state dynamics of Q_t.
7 The steady state values of the main variables are the following: $g_0 = 0.0235$, $\alpha_0 = 0.2131$, $r_0 = 0.0527$, $\beta_0 = 2.8$ and $\pi_0 = 0.0073$.

Table 1
Parameters and exogenous variables.

$\varphi_1 = 0.90$; $\varphi_2 = 0.10$;	$R_0 = 0.06$;
$\psi_1 = 1.6$;	$v^* = 2.55$; $i_a = 0.06$;
$\gamma_1 = 2.27$; $\rho_1 = 0.2$;	$\theta_1 = 0.25$; $\theta_2 = 0.10$;
$s_\pi = 0.80$; $s_w = 0.16$; $c_\beta = 0.095$	$\chi_1 = 1$; $\chi_2 = 0.3$; $\chi_3 = 1.50$.

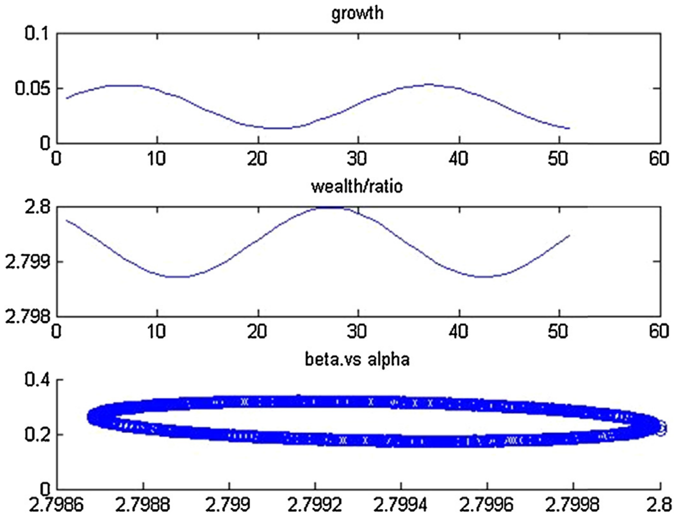

Fig. 1. The dynamics of the nonlinear system.

Starting from the steady state values and by introducing a temporary shock to the investment function, the model is capable of generating persistent fluctuations, which are one of the main stylized facts characterizing these variables. Fig. 1 illustrates the last 50 run of the simulation, where the period considered is 1500. The bounded dynamics are fundamentally the result of two loops generating feedbacks of opposite sign: the classical positive feedback (centrifugal) and the monetary negative (centripetal) feedback.

Investment plays a strategic role in both loops. In fact, it determines saving, and hence income distribution. Expansions feed profits and further growth. However, expansions also feed inflation and make the nominal rate of interest increase. The conflict between these two forces generates, for appropriate values of the parameters, a limit cycle that captures the dynamics of the model.

Fig. 1 also allows us to make a comparison with Piketty's results, even though the context is different. As in Piketty, the dynamics of β are negatively related to g. However, α_t and β_t are not positively related but they rather design a dynamical loop. This weakens the possibility of inferring inequality by simply observing the dynamics of the functional distribution of income. The macro inference hypothesis can be maintained only by considering the origin of wealth changes.

6. The robustness of the model

The model is robust to parameters changes. In order to show some results, the linearized version of the model is considered (see Appendix A).

The system can be formulated in a more compact way in matrix form as:

$$Ax_t = Bx_{t-1}$$

or

$$x_t = A^{-1}Bx_{t-1}$$

where all the variables included in vector x represent deviations from steady state values. Some eigenvalues of this system are complex conjugate and have unitary modulus, $|\lambda| = 1$, while the remaining are less than 1. Thus the fixed point has properties similar to the Neimark bifurcation, with limit cycles being generated (see Kuznetsov, 2004).

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Table 2
Values of the parameters necessary to maintain $|\lambda| = 1$.

γ_1	γ_2
2.6655	0
2.0	0.73
1.27	1.5
1.3824	1.3824

The linearization is important for at least two reasons. On the one hand, it allows us to test the robustness of the model: changes in the values of the parameters maintain the main structural properties of the system. On the other, it identifies the role of the different parameters.

For instance, one might wonder what happens if the propensity to consume out of wealth (c_β) increases. It is worth noting that this is the only channel through which an increase in wealth can affect the dynamics of the system. Furthermore, the nature of this increase is such to allow a macro inference of inequality.

In the above linearized model, an increase in c_β makes the system unstable, because $|\lambda|$ becomes greater than 1. In other words, inequality can feed instability. The same holds true if one increases the values of χ_2, χ_3, θ_1 and θ_2. In other words, whenever the weight of rent increases, not only inequality rises but also the system can become unstable.

6.1. An extension

It is worth stressing that the model is robust in a deeper sense. In fact, also changes in the specification of the equations do not necessarily modify the qualitative properties of the model. Two cases are worth considering, the interest rate rule (IRR) and the investment equation.

Eq. (5.2) can be extended in the following way:

$$R_t = R^* + \psi_1(\pi_t - \pi_0) + \psi_2(g_t - g_0) \tag{6.1}$$

The above equation extends Eq. (5.2) because it introduces the gap between actual growth and its steady state value in the equation. Ceteris paribus, the role of the parameter ψ_2 is stabilizing (in other words it diminishes the value of $|\lambda|$ that becomes smaller than 1). This circumstance, however, does not allow one to conclude that the generalized Taylor rule can check instability.

To this purpose, it is important to consider the possibility of deflation and the role of the following inequality:

$$R_t \geq 0 \tag{6.2}$$

In case of both deflation and stagnation, the second and the third terms in the r.h.s. of (6.1) can become negative. It follows that inequality (6.2) creates a trade-off between the values of the parameters ψ_1 and ψ_2. This means that the value of ψ_2 cannot be simply added to the value of ψ_1, as it appears from Table 1. Furthermore, the trade-off is worse the higher the value of γ_1. It follows that the presence of inequality (6.2) is an obstacle to the countercyclical power of monetary policy, as is well known in the literature.[8]

In addition to the extension of the Taylor equation, consider the following specification for the investment function:

$$i_t = i_a + \gamma_1 \{Er_t - [(R_t - \pi_t)] + \gamma_2(g_t - g_0)\} \tag{6.3}$$

In other words, an accelerator has been appended to the classical specification of the investment function. Also in this case, the value of γ_2 is in a dynamic conflict with that of γ_1, even though for different reasons. In fact, as appears from Table 2, the values of the two parameters necessary to maintain $|\lambda| = 1$ stand in a negative relationship (see Appendix B).

Table 2 confirms that the classical hypothesis alone is enough to generate a limit cycle, while the opposite is not true (at least in this model). Furthermore, the value of the accelerator can become >1 if γ_1 is allowed to decrease. Finally, the two parameters can have equal values.

The presence of the accelerator parameter stresses the role that quantity adjustment plays in the model. They reinforce those based upon the changes in the profit margins that allow income distribution to endogenously adjust to the investment dynamics.

[8] One could also include the price of assets. Also in this case, the same considerations would hold. It is important to stress that Eq. (3.3) states that Q_t is a function of monetary policy.

Table 3
The impact of changes in i_0.

	i_0	g_0	$\lvert\lambda\rvert$	max g_t	min g_t
Benchmark	0.06	0.0235	1	0.030	−0.030
Stagnation	0.03	0.0118	1	0.045	−0.045
Sustained	0.09	0.0350	1	0.035	−0.035

6.2. Implications for growth

At this stage of the analysis, suppose that i_0, i.e. the exogenous value of investment, measured by the so-called autonomous component, is changed. Two things happen. First of all, the value of $|\lambda|$ does not change. In other words, the dynamic profile is maintained. However, the amplitude of the oscillations is modified in a considerable way, as appears from Table 3.

It is important to stress that a higher i_0 increases the steady state growth of the system without altering its dynamic profile. Furthermore, it increases the amplitude of the fluctuations only marginally. In fact, the relationship between growth and variability is strongly non linear and therefore asymmetric. A reduction in steady state growth is also accompanied by a greater variability.

Two aspects are worth considering. The first is that episode such as the "Great Contraction" can have a durable negative impact on growth because the higher variability has implied a greater degree of uncertainty that can affect the state of the long-run expectations governing autonomous investment. There is therefore a potential link between fluctuations and growth (as also stressed by Minsky, 1982).

The second aspect is that autonomous investment may also include public investment. In this case, the model should be extended in order to take the fiscal variables into account. However, one can say that it could link medium-run fluctuations to long-run growth, where the role of technology and institutions become strategic (see Ferri and Minsky, 1992).

7. Concluding remarks

The paper has analyzed the relationship between income shares, wealth and growth in an environment where positional goods are taken into account and rent is generated. Even though this is not the most important empirical channel through which capital gains have been generated during the recent bubble experience, its analytical role is relevant. First, the presence of positional goods implies a clear-cut distinction between capital and personal wealth. In the second place, it creates a gap between profit share and property share due to the existence of rent. Thirdly, it is a macro engine of inequality, which is fed, inter alia, by instability. Finally, it allows one to consider the feedback from inequality to inequality.

The relationships between these aspects have been examined within a demand-led model where investment plays a strategic role. Furthermore, the model refers to a monetary economy where rates of profits and nominal rates of interest (corrected by inflation) face each other in determining investment. Finally, the model is studied in a medium-run perspective where actual dynamics vis-à-vis steady state relationships are taken into account in order to study the co-movements between the variables.

The model generates persistent and bounded dynamics, where the stylized facts characterizing the present state of the economy can be considered just a phase of these fluctuations. Within this perspective, changes in the wealth ratio (β) are not accompanied by changes in capital ratio, v, which is assumed to be fixed. Furthermore, α, the property share, and β, the wealth-output ratio, can be either positively or negatively related. Their links do not depend only on the hypotheses underlying the model but also on the values of the parameters chosen and the monetary policy pursued. Yet, the macro inference of inequality from the study of wealth dynamics still holds.

The model can be extended in several ways. A first line would be to allow rent to affect production as happens in the classical authors. In order to accomplish this target, at least a two sector model must be considered. In the second place, the role of monetary and financial aspects could be enriched. The introduction of a cash flow in the investment function, along with the possibility of speculative debt according to a Minskian approach can be pursued (see Ferri, 2016). Thirdly, the impact of these variables on inequality should be deepened, for instance by considering

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heterogeneity between families holding wealth. Finally, the labour market variables should be explicitly taken into consideration.

These enrichments can also allow to pursue more ambitious aims such as to better investigate how the medium-run dynamics can influence the long-run rate of growth. In fact, inequality not only can impact on fluctuations but it may also interfere with growth through the role of institutions and technical change that become essential players (see Cristini et al., 2015).

A. The linearized model

In order to linearize the model, only the following Eqs. (5.5) and (3.5) are to be modified:

$$\alpha_t = \frac{i_t}{(s_\pi - s_w)z_0} + \frac{c_\beta}{(s_\pi - s_w)z_0}\beta_t$$
$$r_t = \frac{z_0}{v^*}\alpha_t$$

The system can be cast into the following matrix form, keeping in mind that all the variables are deviations from their respective steady state and that the following expressions have been used:

$$A_1 = \frac{1}{(s_{pr} - s_w)z_0}; \quad A_2 = \frac{c_\beta}{(s_{pr} - s_w)z_0}; \quad R_1 = \frac{z_0}{v^*}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -\varphi_2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \psi_1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\chi_2 & \chi_2 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\gamma & \gamma_1 & -\gamma_1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\theta_1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -A1 & 0 & -A2 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -R1 & 1 \end{pmatrix} \begin{pmatrix} g_t \\ \pi_t \\ R_t \\ Er_t \\ Q_t \\ i_t \\ \Omega_t \\ \beta_t \\ \alpha_t \\ r_t \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & \frac{1}{v^*} & 0 & 0 & 0 & 0 \\ 0 & \phi_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & (1-\rho_1) \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \chi_3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} g_{t-1} \\ \pi_{t-1} \\ R_{t-1} \\ Er_{t-1} \\ Q_{t-1} \\ i_{t-1} \\ \Omega_{t-1} \\ \beta_{t-1} \\ \alpha_{t-1} \\ r_{t-1} \end{pmatrix}$$

In a more compact way, the system becomes:

$$x_t = Dx_{t-1}$$

where D is the matrix obtained by pre-multiplying the matrix in the r.h.s. by the inverse of the matrix in the l.h.s.

Using the same parameters indicated in Table 1, one obtains that the modulus of the maximum value of the couple of complex roots $(|\lambda|) = 1$.

Furthermore, as one chooses γ_1 as a bifurcation parameter, the derivative of the roots with respect to this parameters are not null and in general the condition set by the Hopf (Neimark-Sacker) theorem are respected so that limit cycles are generated. However, to study the attractiveness of the limit cycles higher order terms must be introduced (see Kuznetsov, 2004).

B. Linearizing the extended model

In this case one has to introduce the two supplementary parameters ψ_2 and γ_2. For instance, if $\psi_2 = 0.2$, then $\gamma_1 = \gamma_2 = 1.3824$ can generate a cycle, given the other values of the parameters shown in Table 1. Other combinations are possible as shown in Table 2.

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