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On the processing of earthquake-induced structural response signals by suitable Operational Modal Analysis identification techniques

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Abstract

In the present study, two different Operational Modal Analysis (OMA) techniques, namely a *refined Frequency Domain Decomposition (rFDD)* and an *improved Data-Driven Stochastic Subspace Identification (SSI-DATA)*, are specifically conceived towards assessing current modal dynamic properties of buildings under seismic excitation and simultaneous heavy damping (in terms of identification challenge). Synthetic earthquake-induced response signals, generated from a set of different frame structures and earthquake base-excitations, have been investigated, in order to make a serious step forward in assessing the OMA effectiveness of the two techniques at seismic response input. According to the present investigation, best up-to-date, re-interpreted, output-only algorithms may be effectively used to characterize the current dynamic behaviour of structures and to identify potential structural modifications along the experienced seismic histories, thus allowing for possible Structural Health Monitoring approaches in the Earthquake Engineering range.

1 Introduction

Achieving an appropriate knowledge of structural modal properties constitutes a fundamental target in the dynamical characterization of structural and civil engineering systems. This may be connected not only to ambient or experimentally-controlled excitations, but also to earthquake-induced vibrations. In Earthquake Engineering, structural modifications may be detected along the experienced seismic histories, through specific identification procedures, in order to carefully achieve strong ground motion modal parameter estimates, either in the linear or in the nonlinear range.

In the Earthquake Engineering range of application, most of known identification procedures pertain directly to Experimental Modal Analysis (EMA), where both input and output recordings need to be available for achieving an appropriate operation of the relevant estimation procedures [1]. During the last decades, Operational Modal Analysis (OMA) has been also increasingly widespreading in this field [2]. With OMA procedures, only structural response recordings (input signals for the identification technique) need to be known, which makes it particularly suitable for treating ambient vibrations, but in principle also for earthquake excitations. Generally, the unknown excitation input acting on the structure shall be considered to be similar, in main characteristics, to that of a white noise signal, as it may be recorded under typical ambient or operational excitations.

In the field of OMA algorithms, the adoption of input channels coming from earthquake-induced structural response signals has been considered quite a few times in the dedicated literature, either in the Time or in the Frequency Domain, e.g. in [3–5]. By making reference to OMA techniques, the present work adopts earthquake-induced linear structural responses as input channels for a *refined Frequency Domain Decomposition (rFDD)* technique [6] and an *improved Data-Driven Stochastic Subspace Identification (SSI-DATA)* [7] algorithm. Both were implemented autonomously within MATLAB, for rFDD in [8–11] and for SSI-DATA starting from [12] and as presented here. The traditional versions of these algorithms rely on the assumption of white noise input, which no longer holds with general seismic response input. On the contrary, the present methods have been specifically developed to deal with earthquake-induced structural response signals and simultaneous heavy damping (in terms of identification challenge, i.e. with modal damping ratios larger than a few percents and up to 10% or higher).

In the present paper, the implemented rFDD and SSI-DATA techniques are separately adopted to identify the modal properties of a single heavy-damped three-storey shear-type frame, under ten different selected strong ground motions. The work shall constitute a first basis for an innovative comparison between the two OMA identification methods within the Earthquake Engineering range, in order to inspect their positive and negative aspects, concerning their reliability in correctly identifying strong ground motion modal parameters, in the linear range of seismic response. From the performed analyses, the achieved estimates are compared among them and also with the known target values computed before stage, in order to extract general and specific considerations on the efficiency and consistency of both OMA algorithms and to investigate and compare their effectiveness in identifying all current strong ground motion modal parameters.

The presentation of this work is structured as follows. In Section 2 and in Section 3, main theoretical backgrounds (Sections 2.1 and 3.1) and novelties (Sections 2.2 and 3.2) of the developed rFDD and SSI-DATA algorithms are outlined, respectively. The selected earthquake dataset and the adopted numerical models are briefly presented in Section 4 (Section 4.1), jointly with the outcomes from the performed analyses and the results separately achieved by the two OMA identification methods (Section 4.2). Finally, salient conclusions on the whole study are gathered in closing Section 5.

2 Fundamentals of the present refined Frequency Domain Decomposition algorithm

2.1 Theoretical background

Classical FDD theory is based on the general input/output expression for a MDoF system [6]:

$$\mathbf{G}_{yy}(\omega) = \overline{\mathbf{H}}(\omega)\mathbf{G}_{xx}(\omega)\mathbf{H}^{\mathrm{T}}(\omega) \tag{1}$$

where $\mathbf{G}_{xx}(\omega)$ is the input Power Spectral Density (PSD) matrix (excitations), $\mathbf{G}_{yy}(\omega)$ is the output PSD matrix (responses) and $\mathbf{H}(\omega)$ is the Frequency Response Function (FRF) matrix; overbar denotes complex conjugate and apex symbol T transpose. Therefore, the first step of classical FDD methods is the estimation of the PSD matrix of system responses $\mathbf{G}_{yy}(\omega)$, from time correlation and Fourier Transform.

Then, its transpose shall be decomposed by performing a Singular Value Decomposition (SVD) at each discrete frequency $\omega = \omega_i$:

$$\mathbf{G}_{yy}^{\mathrm{T}}(\omega = \omega_i) \simeq \mathbf{\Phi}_i \left\{ \mathrm{diag} \left[\mathrm{Re} \left(\frac{2\mathrm{d}_j}{\mathrm{i}\,\omega_i - \lambda_j} \right) \right] \right\} \mathbf{\Phi}_i^{\mathrm{H}} = \mathbf{U}_i \mathbf{S}_i \mathbf{U}_i^{\mathrm{H}} \tag{2}$$

where $\mathbf{\Phi}_i$ is the i^{th} eigenvector matrix, gathering all n eigenvectors $\boldsymbol{\phi}_{ij}$ as columns, d_j is a real scalar, λ_j is the j^{th} pole of the system, $j = 1, \ldots, n$, being n the number of input channels. In parallel, $\mathbf{U}_i$ is a unitary complex matrix holding singular vectors $\mathbf{u}_{ij}$ and $\mathbf{S}_i$ is a real diagonal matrix holding Singular Values (SV) s_{ij}.

Starting from the SVD in Eq. (2), the identification of mode k can be made around a modal peak in the frequency domain, which can be located by an appropriate peak-picking procedure on the SV representations. Then, the response PSD matrix of the k^{th} mode, in correspondence of identified damped modal frequency ω_k, can be approximated as [6]:

$$\mathbf{G}_{yy}^{\mathrm{T}}(\omega_i = \omega_k) \simeq \boldsymbol{\phi}_k \left\{ \mathrm{diag} \left[\mathrm{Re} \left(\frac{2\mathrm{d}_k}{\mathrm{i}\,\omega_k - \lambda_k} \right) \right] \right\} \boldsymbol{\phi}_k^{\mathrm{H}} = \mathrm{s}_1 \mathbf{u}_{k1} \mathbf{u}_{k1}^{\mathrm{H}} \tag{3}$$

where first singular vector $\mathbf{u}_{k1}$ at k^{th} resonance frequency ω_k leads to an estimate of related mode shape vector $\hat{\phi}_k = \mathbf{u}_{k1}$. Associated Singular Value (SV) s_1 is the Auto-PSD function of the corresponding SDoF system, which may be detected by comparing the identified mode shape with the surrounding singular vectors around the modal peak. For this purpose, the Modal Assurance Criterion (MAC) index may be classically used [6].

Then, the inverse Fourier Transform (time domain) of the located k^{th} Auto-PSD function (frequency domain) allows to obtain an estimate of the SDoF Auto-Correlation Function (ACF) related to the resonance peak. In this process, remaining parts of the Auto-PSD function are simply reset to zero [6]. All ACF extrema (i.e. peaks and valleys), which represent the free amplitude decay of a damped SDoF system, may be detected within an appropriate time window. The logarithmic decrement, which is classically defined as $\delta_k = (2/m)\ln(r_0/|r_m|)$, can be estimated by a linear regression on $\delta\, m$ and $2\ln(|r_m|)$, where $m = 1, 2, \ldots$ is an integer index counter of the m^{th} ACF extreme and r_0, r_m are the initial and the m^{th} extreme value of the ACF, respectively. Then, modal damping ratio ζ_k can be tipically estimated as [6]:

$$\zeta_k = \frac{\delta_k}{\sqrt{4\pi^2 + \delta_k^2}} \tag{4}$$

Finally, knowing the estimated modal damping ratio, the undamped natural frequency can be obtained from the estimated damped modal frequency by dividing it by factor $\sqrt{1-\zeta_k^2}$ [6].

2.2 Main novelties of the present rFDD algorithm

Main assumptions of classical FDD methods consist of stationary white noise input, light damping (modal damping ratios in the order of 1%) and geometrically-orthogonal mode shapes of close modes [6].

The present rFDD method, whose original theoretical background has been reported in [9, 10], conceptually derives from classical FDD methods [6], but has been specifically developed to deal with earthquake-induced structural response signals and concurrent heavy damping (in terms of FDD identification challenge, i.e. for realistic modal damping ratios up to 10%).

Pioldi et al. [8–10] have discussed the theoretical validity and efficacy of the present rFDD technique, through the use of synthetic seismic response signals in the linear range. Trials with real earthquake responses and damage scenarios in the non-linear range have been effectively performed as well in [13]. In [11, 14], further rFDD computational strategies have been introduced, by adopting excitation data from the complete FEMA P695 earthquake database, towards achieving an extensive validation in the Earthquake Eigineering range. In [15, 16], the rFDD technique has been also applied to frames under Soil-Structure Interaction (SSI) effects, towards obtaining the identification of flexible- and fixed-base modal parameters from earthquake-induced structural response signals.

For the sake of completeness, the following computational steps (and references quoted therein) summarize the main workflow of the present rFDD algorithm:

- Suitably-developed filtering applied to the structural response input signals (earthquake-induced structural responses) before starting the modal identification process [9].
- Coupling of rFDD to a time-frequency Gabor Wavelet Transform (GWT), towards achieving a correct evaluation of the time-frequency features of the signals and a best setup for rFDD identification [11].
- Processing of the auto- and cross-correlation matrix entries, by aiming at obtaining clearer and well-defined SVs out of seismic response signals [10].
- Integrated PSD matrix computation, implementing simultaneously both Wiener-Khinchin and Welch's modified periodogram methods [9, 10]. The Wiener-Khinchin's approach works especially well with short signals, allowing for a clearer peak detection, not only on the first SV curve, but also on the subsequent ones. Welch's method, instead, implements an averaging and windowing before the frequency-domain convolution, allowing to achieve slightly better mode shapes, despite for the not so good

separation of the signals in the modal space. Then, the integrated PSD matrix computation aims at extracting better modal estimates, by taking simultaneous advantage of both PSD evaluation methods.

- Iterative loop and optimization algorithm towards achieving effective modal damping ratio estimates, especially under heavy-damping identification conditions [9].
- Coupled Chebyshev Type II bandpass filters computational procedure, aiming at enhancing the SDoF spectral bells towards estimation improvement, when challenging seismic input and heavy-damping conditions apply [11].
- Estimation of modal parameters by operating on different SVs and on their composition, to detect each SV contribution and to reconstruct the original SDoF spectral bells [8, 9].
- Inner procedure for frequency resolution enhancement, without the need of higher frequency sampling, as first outlined in [9].
- Combined use of different MAC indexes towards modal validation purposes [9, 13]. After a preliminary "peak-picking" [9], the use of Modal Assurance Criterion (MAC) and Modal Phase Collinearity (MPC) indexes [10] becomes necessary to discern "spurious peaks" from true (physical) modal ones. In particular, these indexes have been used to discard spurious peaks that exhibit a complex-number character (i.e. displaying modal deflection phases that significantly deviate from 0 or π).

Consistently, rFDD results reported later in Section 4 demonstrate the robustness of the developed rFDD algorithm in returning reliable modal parameter estimates at seismic response input and concurrent heavy damping. These are going to be compared to those independently achieved by a separate SSI-DATA implementation, as introduced in the next section.

3 Fundamentals of the present Data-Driven Stochastic Subspace Identification algorithm

3.1 Theoretical background

Classical SSI-DATA theory takes as a typical starting point the following OMA Stochastic State-Space model, in discrete-time notation [17]:

$$\begin{cases} \mathbf{x}_{k+1} = \mathbf{A}\mathbf{x}_k + \mathbf{w}_k \\ \mathbf{y}_k = \mathbf{C}\mathbf{x}_k + \mathbf{v}_k \end{cases} \tag{5}$$

being the first the *state equation*, in terms of *state vector* of responses $\mathbf{x}_k = \{\mathbf{u}_k \ \mathbf{u}_{k+1}\}^{\mathrm{T}}$ and of its derivative $\mathbf{x}_{k+1} = \{\mathbf{u}_{k+1} \ \mathbf{u}_{k+2}\}^{\mathrm{T}}$, and the second the *observer equation*, in terms of *observer vector* of responses $\mathbf{y}_k$ (either displacements, velocities and/or, typically, accelerations); notation $\mathbf{u}_k$, $\mathbf{u}_{k+1}$ and $\mathbf{u}_{k+2}$ refers to discrete-time counterparts of continuous-time vectors of displacement $\mathbf{u}(t)$, velocity $\dot{\mathbf{u}}(t)$ and acceleration $\ddot{\mathbf{u}}(t)$ responses, respectively. Further, $\mathbf{A}$ is the *state matrix* and $\mathbf{C}$ is the *output matrix* of the identification problem, and vectors $\mathbf{w}_k$ and $\mathbf{v}_k$ are zero mean, stationary white noise stochastic processes, representing process noise and measurement noise, respectively [17]. Here, index $k = 1, \ldots, N$ spans discrete time instants t_k, being N the total number of sampling points of the response signal.

The first step of classical SSI-DATA identification algorithms is the computation of so-called *block Hankel matrix* $\boldsymbol{H}_{0|2i-1} = [\boldsymbol{Y}_p \, \boldsymbol{Y}_f]^{\mathrm{T}}$, which is directly computed from the measurement data [17], i.e. system structural responses $\mathbf{y}_k$ (in the present case, accelerations). The two partition sub-matrices of the Hankel matrix refer to past $\boldsymbol{Y}_p = \boldsymbol{H}_{0|i-1}$ and future $\boldsymbol{Y}_f = \boldsymbol{H}_{i|2i-1}$ output channels matrices, where subscripts denote the first and the last element of the first column of block Hankel matrix $\boldsymbol{H}_{0|2i-1}$. So, matrices $\boldsymbol{Y}_p$ and $\boldsymbol{Y}_f$ are defined by splitting matrix $\boldsymbol{H}_{0|2i-1}$ into two equal parts of i block rows, where number of block rows i shall be determined in agreement with condition $m \cdot i \geq n$ [18], being m the number of output (acquisition) channels and n the so-called *system order* (i.e. the dimension of square matrix $\mathbf{A}$ in the identification process, as defined below). Number of columns of block Hankel matrix $\boldsymbol{H}_{0|2i-1}$ is usually taken as $j = N - 2i + 1$, which implies that all recorded data samples are used [17].

Now, a *projection matrix* $\boldsymbol{P}_i$ can be computed through the orthogonal projection of the row space of future output channels $\boldsymbol{Y}_f$ into the row space of past output channels $\boldsymbol{Y}_p$ as $\boldsymbol{P}_i = \boldsymbol{Y}_f/\boldsymbol{Y}_p = \boldsymbol{Y}_f\boldsymbol{Y}_p^{\mathrm{T}}(\boldsymbol{Y}_p\boldsymbol{Y}_p^{\mathrm{T}})^{\dagger}\boldsymbol{Y}_p = \boldsymbol{O}_i\hat{\boldsymbol{S}}_i$ [2, 17], where symbol † indicates Moore-Penrose pseudo-inverse, whilst $\boldsymbol{O}_i$ and $\hat{\boldsymbol{S}}_i$ are the *observability matrix* and the *Kalman filter state sequence*, respectively [2, 17].

Then, through the application of specific weighting matrices $\boldsymbol{W}_1$ and $\boldsymbol{W}_2$ to projection matrix $\boldsymbol{P}_i$, a SVD may be derived, by holding non-zero singular values only [2, 17], as:

$$\boldsymbol{W}_1\boldsymbol{P}_i\boldsymbol{W}_2 = [\mathbf{U}_1\mathbf{U}_2]\begin{bmatrix}\boldsymbol{\Sigma}_1 & \mathbf{0}\\ \mathbf{0} & \mathbf{0}\end{bmatrix}\begin{bmatrix}\mathbf{V}_1^{\mathrm{T}}\\ \mathbf{V}_2^{\mathrm{T}}\end{bmatrix} = \mathbf{U}_1\boldsymbol{\Sigma}_1\mathbf{V}_1^{\mathrm{T}} \tag{6}$$

where $\mathbf{U}_k$ and $\mathbf{V}_k$, $k = 1, 2$ are the singular vector matrices, and $\boldsymbol{\Sigma}_1$ is the matrix holding the non-zero singular values (which allows for estimating the rank of matrix $\boldsymbol{P}_i$). The selection of the dimensions of matrix $\boldsymbol{\Sigma}_1$ fixes system order n of the State-Space model, which is adopted for the subsequent computational steps.

As concerning weighting matrices $\boldsymbol{W}_1$ and $\boldsymbol{W}_2$, they may be defined according to different weighting proposals [17], namely Principal Component (PC), Unweighted Principal Component (UPC) and Canonical Variate Analysis (CVA). For instance, for the CVA weighting, as used and discussed later in the identification results, weighting matrices $\boldsymbol{W}_1$ and $\boldsymbol{W}_2$ are given as:

$$\boldsymbol{W}_1 = \left(\frac{1}{j}\boldsymbol{Y}_f\boldsymbol{Y}_f^{\mathrm{T}}\right)^{-1/2}, \quad \boldsymbol{W}_2 = \mathbf{I}. \tag{7}$$

At this stage, discrete-time State-Space matrices $\mathbf{A}$ and $\mathbf{C}$ may be computed through an asymptotically-unbiased least squares estimate as:

$$\begin{bmatrix}\mathbf{A}\\ \mathbf{C}\end{bmatrix} = \begin{bmatrix}\hat{\boldsymbol{S}}_{i+1}\\ \mathbf{Y}_{i|i}\end{bmatrix}\hat{\boldsymbol{S}}_i^{\dagger}. \tag{8}$$

where Kalman filter state sequences $\hat{\boldsymbol{S}}_i$, $\hat{\boldsymbol{S}}_{i+1}$ and output sequence $\boldsymbol{Y}_{i|i}$ can be calculated as outlined in [17].

Then, the eigenvalue decomposition of discrete-time state matrix $\mathbf{A} = \boldsymbol{\Psi}\mathcal{M}\boldsymbol{\Psi}^{-1}$ allows for the estimation of the modal parameters, through matrices $\boldsymbol{\Psi}$ and $\mathcal{M}$, holding discrete-time eigenvectors $\boldsymbol{\psi}_r$ and eigenvalues μ_r, respectively. Afterwards, discrete-time eigenvalues μ_r are converted to continuous-time eigenvalues λ_r, as $\lambda_r = \ln(\mu_r)/\Delta t$ [2], so that the so-called *system poles* are obtained. Finally, the r^{th} mode shape, natural frequency, damped modal frequency and modal damping ratio estimates may be computed as [2]:

$$\boldsymbol{\phi}_r = \mathbf{C}\boldsymbol{\psi}_r; \quad f_r = \frac{|\lambda_r|}{2\pi}, \quad f_{r,d} = \frac{\mathrm{Im}(\lambda_r)}{2\pi}, \quad \zeta_r = -\frac{\mathrm{Re}(\lambda_r)}{|\lambda_r|}. \tag{9}$$

3.2 Main novelties of the present SSI-DATA algorithm

Main assumptions of classical SSI methods consist of stationary white noise input and adequately long structural response signals, to achieve a suitable stabilization of the estimated poles. Light damping (modal damping ratios in the order of $1 \div 2\%$) leads to better estimates, too, since it may reduce the occurrence of noise poles or of mathematical poles (e.g. false stable poles characterized by a positive real part and a negative damping ratio) [2, 17].

The present SSI-DATA algorithm, whose original theoretical background comes from the general formulation in [17], as briefly outlined in Section 3.1 above, may be intended as a first implementation attempt, for SSI algorithms, to deal with earthquake-induced structural response signals, at concurrent heavy structural damping in terms of modal identification challenge.

Therefore, the following items summarize the main steps and issues related to the present SSI-DATA algorithm:

- A first issue is to arrive at an appropriate definition of weighting matrices $\boldsymbol{W}_1$ and $\boldsymbol{W}_2$. After first extensive simulations performed in [12], under both white noise input or seismic excitation, it was shown that the Canonical Variate Analysis algorithm (CVA) [17], with weighting matrices $\boldsymbol{W}_1$ and $\boldsymbol{W}_2$ as given in Eq. (7), turns out to be the most stable and performing weighting option towards achieving reliable estimates at seismic response input and concurrent heavy damping (as demonstrated later in Section 4). This type of weighting, as opposed to widely-used Principal Component (PC) and Unweighted Principal Component (UPC) weightings, returns even less noise or mathematical poles and looks mostly able to separate true physical modes from possible spurious earthquake harmonics.
- For the correct selection of system order n and for the determination of the stable poles (i.e. the poles where frequency, mode shape and modal damping ratio estimates show to be stable and not deriving from noise or mathematical poles), a *stabilization diagram* may be constructed from the SSI-DATA identification outcomes [2]. It displays the poles that are obtained according to different considered system orders, as a function of frequency. The Singular Value (SV) curves extracted from SVD of SSI-DATA output spectral matrix $\mathbf{G}_{yy}(\omega)$ may be reported too, within the same stabilization diagram. This matrix may be typically calculated from the estimated SSI model as outlined in [18], by adopting the estimated *next state-output covariance matrix* and the *output covariance matrix* [2]. As an original alternative in the present work, a novel integrated arrangement considers instead the use of SV curves, which are computed from the SVD of direct output spectral matrix $\mathbf{G}_{yy}(\omega)$. This is calculated through a routine of the previous rFDD algorithm, by adopting Welch's modified periodogram [13]. Such proposed integration of SSI and FDD information demonstrates to provide a reliable tool to support the individuation of the stable SSI poles within the stabilization diagram, especially when dealing with earthquake-induced structural responses and simultaneous heavy damping.
- Anyway, the most severe issue in the present SSI identification keeps lying in the fact that seismic response signals are characterized by rather short durations (specifically with respect to ambient vibration recordings). This directly affects the achievable estimates, since the poles intrinsically display a harder stabilization. So, the maximum system order employed in the analysis shall be increased, jointly with a careful setting of the adopted number of block rows in the Hankel matrix. This has been specifically pursued in the present implementation (see also preliminary investigation results in [12]). In this way, better natural frequency and mode shape estimates may be achieved. As regarding to the modal damping ratios, they appear to be the most challenging parameters to be detected by the present SSI method, especially in relation to the cited very short durations of the seismic response signals.

4 Numerical simulations and identification outcomes

4.1 Adopted earthquake dataset and numerical models

The input response channels to be fed to the present OMA output-only algorithms are obtained from calculated synthetic structural response signals of linear multi-storey frames. Storey accelerations are considered as input for the identification algorithms. These numerical response recordings are generated prior to modal identification by taking as base acceleration an earthquake from a set of ten selected seismic ground motions (see Table 1). The adopted earthquakes have been chosen as representative ones from a wide variety of seismic events, with rather different characteristics, i.e. time-frequency spectra, duration, sampling, magnitude and PGA. Also, they have been specifically selected as expected challenging instances for the present rFDD and SSI-DATA identification purposes, given their non-stationary nature.

The simulated structural responses are calculated by direct time integration of the equations of motion, via Newmark's (average acceleration) method. The use of simulated signals shall fulfil a first necessary condition for the algorithm's effectiveness, since modal parameters are determined via direct modal analysis before identification, and then adopted as known targets for the subsequent validation purposes.

Earthquake	*Date*	*Station*	*Dur.* [s]	f_s [Hz]	*M*	*Comp.*	*PGA* [g]
(AQ) L'Aquila	06/04/2009	AQV	100	200	5.8	WE	0.659
(CH) Maule	27/02/2010	Angle S/N 760	180	100	8.8	WE	0.697
(EC) El Centro	18/05/1940	0117	40	100	7.1	NS	0.312
(IV) Imperial Valley	15/10/1979	01260	58	100	6.4	NS	0.331
(KO) Kobe	17/01/1995	Nishi Akashi	41	100	6.9	WE	0.510
(LP) Loma Prieta	17/10/1989	47459	40	50	7.0	WE	0.359
(NO) Northridge	17/02/1994	24436	60	50	6.7	WE	1.778
(NZ) Christchurch	03/09/2010	163541	82	50	7.1	NS	0.752
(TA) Tabas	16/09/1978	70 Boshrooyeh	43	50	7.3	WE	0.929
(TO) Tohoku	11/03/2011	MYG004	300	100	9.0	NS	2.699

Table 1: Main characteristics of the adopted set of ten selected earthquake base excitations.

The frame structure that has been adopted for the present analysis is a three-storey shear-type frame, subjected to each single base excitation instance from the set of strong ground motions above. This reference 3-DoF case is characterized by a modal damping ratio $\zeta_k = 7\%$ for all the modes, a rather high value for the present OMA identification purposes, especially in the seismic engineering scenario. Structural and modal dynamic characteristics of the adopted three-storey frame [9] are reported in Table 2.

Floor	1	2	3
Stiffness $\times 10^3$ [kN/m]	202.83	202.83	202.83
Mass $\times 10^3$ [kg]	144	144	144
Mode	1	2	3
Natural frequency [Hz]	2.658	7.448	10.76
Mode shape [1]	$\begin{Bmatrix} 0.328 \\ 0.591 \\ 0.737 \end{Bmatrix}$	$\begin{Bmatrix} -0.737 \\ -0.328 \\ 0.591 \end{Bmatrix}$	$\begin{Bmatrix} 0.591 \\ -0.737 \\ 0.328 \end{Bmatrix}$
Assumed modal damping ratio [%]	7%	7%	7%

Table 2: Properties of the analysed three-storey frame [9].

4.2 Results from output-only modal dynamic identification by rFDD and SSI-DATA

By taking as base excitation the instances from the set of ten selected earthquake recordings presented in Section 4.1, dynamic identification analyses have been performed here with the present rFDD and SSI-DATA algorithms, in order to identify all strong ground motion modal parameters.

A synthesis from all the achieved results is reported in Fig. 1, where the estimates in terms of absolute deviations of rFDD and SSI-DATA identified natural frequencies and modal damping ratios and achieved MAC indexes for the estimated mode shapes are depicted. Estimates are reported in terms of absolute deviations of estimated natural frequencies ($\Delta f = |(f_{est} - f_{targ})/f_{targ}|$) and modal damping ratios ($\Delta\zeta = |(\zeta_{est} - \zeta_{targ})/\zeta_{targ}|$) from the target parameters, and of achieved MAC indexes for the estimated mode shapes on the target ones ($\text{MAC} = |\phi_{est}^{\text{H}}\phi_{targ}|^2/(|\phi_{est}^{\text{H}}\phi_{est}|\,|\phi_{targ}^{\text{H}}\phi_{targ}|)$, where H is the Hermitian symbol, namely complex conjugate and transpose).

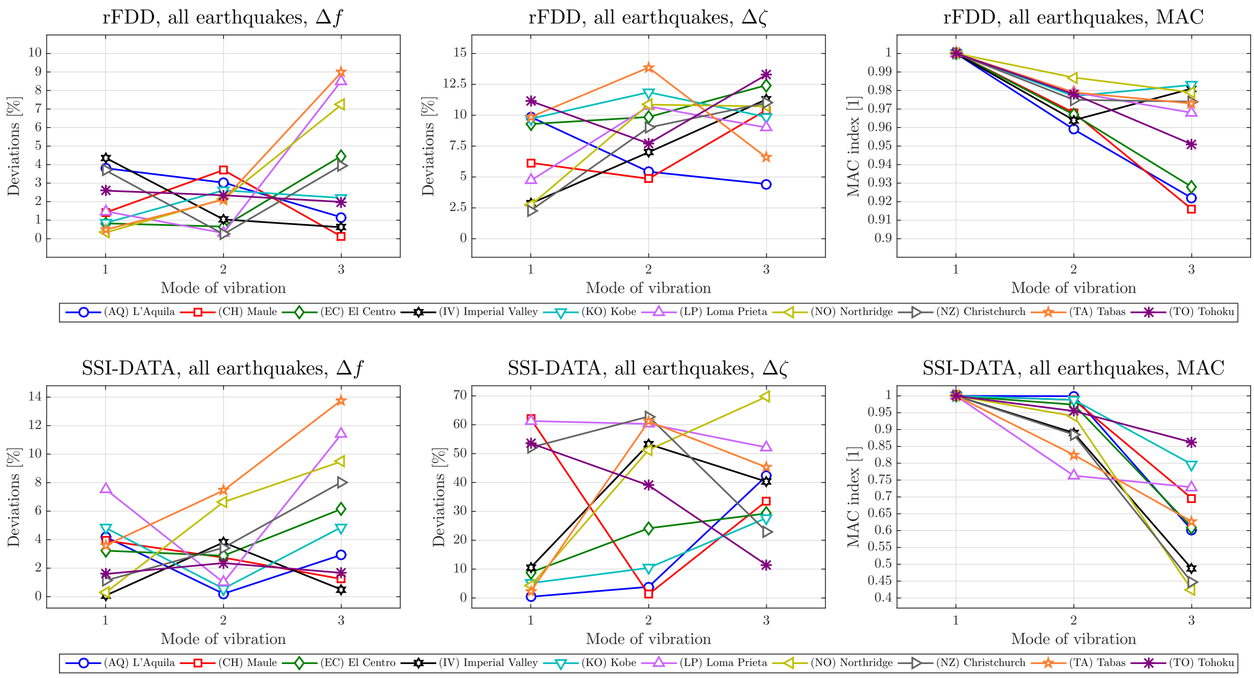

Figure 1: Deviations of estimated natural frequencies (first column), modal damping ratios (second column) and MAC indexes (third column), three-storey frame, rFDD (first row) and SSI-DATA (second row) algorithms, complete considered earthquake dataset.

The rFDD estimated frequencies show deviations that are always below 5%, except for the last modes of NO, LP and TA earthquake cases (Table 1), where deviations increase up to 9%. SSI-DATA estimated frequencies show more scattered deviations, with values up to 14%. The estimated modal damping ratios display very low deviations, at around 10%, for the rFDD algorithm. This is not true for the SSI-DATA algorithm, where modal damping ratios display discrepancies raising up to 70%. By this method, the order of magnitude is more or less caught, but deviations look much higher and often rather unacceptable in engineering terms. However, it should be recalled that really tough heavy-damping identification conditions have been considered here. Generally, the frequency and damping estimates from the rFDD cases show to be much closer to the target values than from the SSI ones. In this sense, Frequency Domain FDD seems superior to Time Domain SSI, in the considered seismic engineering scenario at simultaneous heavy damping and according to the present implementations.

MAC values are always higher than 0.91 for the rFDD instances, for all the modes. With SSI-DATA, MAC indexes perform slightly-less well. For the first two modes, MAC values are always higher than 0.75, with acceptable values in engineering terms. The third modes, instead, show to display some problems, especially with the IV, NO and NZ cases (Table 1), which return quite unreliable mode shapes. Thus, also in terms of mode shape estimates, FDD performs slightly better than SSI, in the present seismic and heavy damping context.

Finally, global results on the achieved modal estimates are further summarized in Fig. 2, where the deviations of estimated natural frequencies and modal damping ratios, and the MAC indexes are represented, in terms of suitably-designed dispersion diagrams [11]. The estimates for the adopted three-storey frame have been condensed all together, by displaying the minimum, the mean and the maximum (absolute) deviations, in blue, black and red coloured lines and markers, respectively. Then, the normalized truncated Gaussian Probability Density Functions (PDF) related to the dispersion of the estimates have been depicted for each mode, jointly with an indication of the standard deviation σ of the estimated values. In the present case, frequency and modal damping ratio deviations must turn out strictly positive (since absolute percentage deviations are adopted), while MAC indexes shall vary between 0 and 1. By taking into account such boundaries, truncated

Gaussians are fitted on the achieved estimates, for each examined case. These truncated Gaussians represent the probability of appearance of a certain deviation as associated to each estimate, between the minimum and the maximum value, and are centred on the mean value. As it is possible to be appreciated, the maximum deviations are always on the Gaussian tails, while the minimum deviations lay in the Gaussian center. This confirms the goodness of the achieved modal identification estimates, especially for the rFDD outcomes.

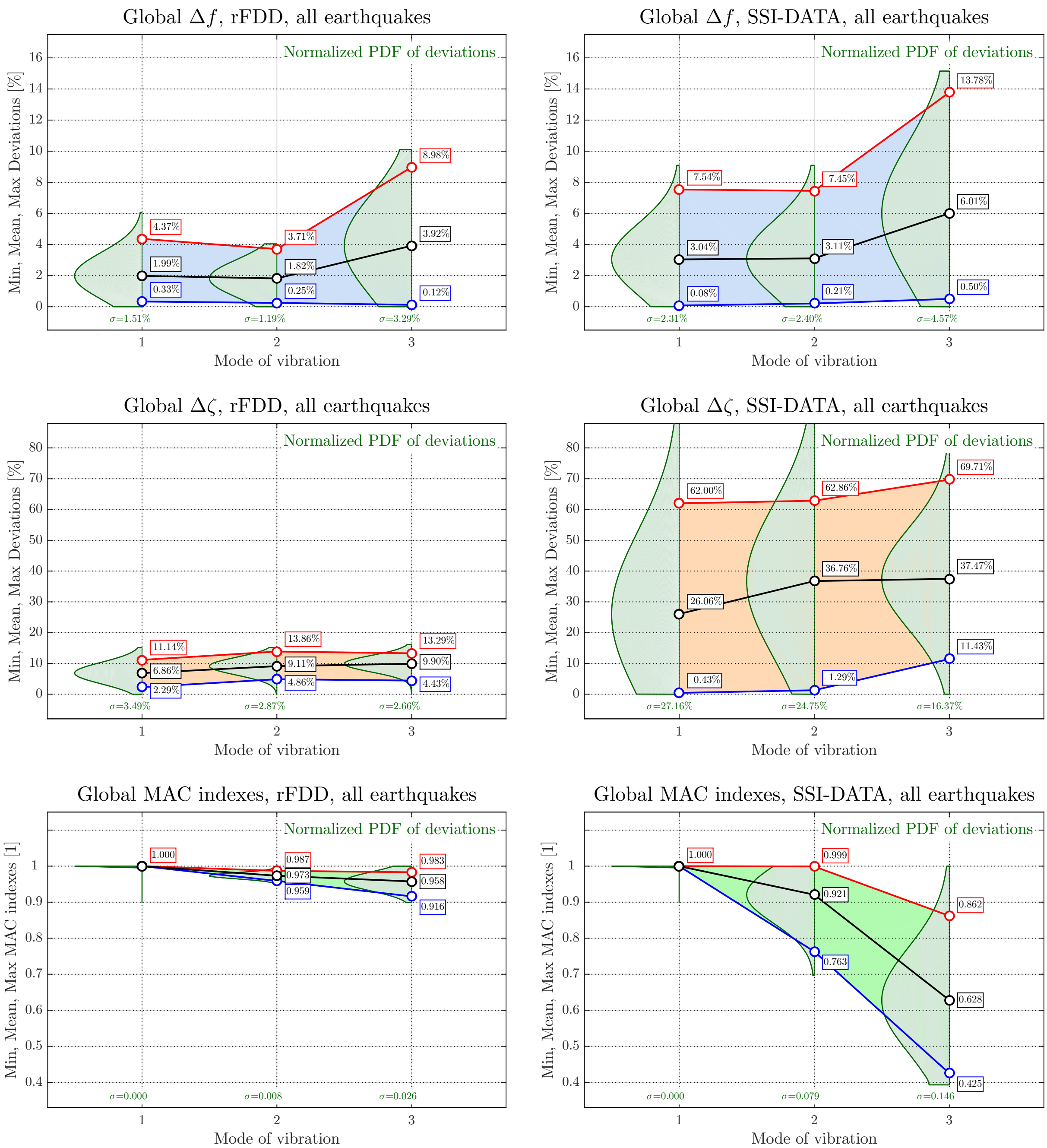

Figure 2: Dispersion diagrams of deviations of estimated natural frequencies (first row), modal damping ratios (second row) and MAC indexes (third row), three-storey frame, rFDD (first column) and SSI-DATA (second column) algorithms, complete considered earthquake dataset. Minimum, mean and maximum values, and standard deviations are indicated.

5 Conclusions

The present study has attempted a first original comparison between the adoption of two different implemented OMA identification algorithms in the Earthquake Engineering range. Concurrently, heavy structural damping in terms of considered identification challenge has been taken into account.

The two adopted OMA methods have been developed by original implementations within MATLAB, either in the Frequency Domain (*refined Frequency Domain Decomposition, rFDD*) or in the Time Domain (*Data-Driven Stochastic Subspace Identification, SSI-DATA*). Such implementations have started from classical algorithmic characteristics, as respectively outlined in Sections 2.1 and 3.1, but have been then developed by specific original features, as respectively described in Sections 2.2 and 3.2. These strategies were specifically devised to handle the present challenging identification scenario in the Earthquake Engineering range and at concurrent heavy damping.

Identification results have been independently derived with the two OMA methods for a sample linear frame structure (three-storey shear-type frame), considering base-excitation instances from a selected earthquake dataset of ten seismic signals. Again, these earthquake recordings have been chosen also among ones that were expected to possibly lead to considerable identification troubles, due to their intrinsic characteristic features and strong non-stationary.

The achieved outcomes have shown both methodologies to turn out rather robust in terms of global modal identification capability in the considered seismic engineering range, by allowing for objective and readable estimates of all the modal parameters. Thus, they may be effectively used, either separately or together, for the stated challenging identification purposes.

As a specific identification outcome, within the considered and analysed cases, rFDD overall looks to yield a superior performance than SSI-DATA, at the present stage of development and implementation, with better achieved modal estimates, especially for the modal damping ratios.

Such encouraging trends shall be confirmed by further, much extensive simulations, which may consider additional refinements of the present SSI-DATA implementation, in order to inspect if the role of the two identification approaches may become much balanced, in terms of the achievable modal estimates and also of conceptual complexity, implementation effort and running effectiveness. All this shall be left for further research work and developments to be pursued within the present field of OMA identification in the Earthquake Engineering range.

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