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A novel framework to include P-Δ effects in displacement based seismic assessment

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Abstract

During an earthquake, P-Δ effects could significantly increase the horizontal displacements in a structure; hence, they should be duly accounted for in seismic assessment procedures. Considering the displacement based assessment (DBA) procedure, P-Δ effects influence the single degree of freedom substitute structure adopted in the procedure both in terms of shear-displacement relationship and of equivalent viscous damping. The present technical note considers a novel framework to include P-Δ effects in the DBA procedure. The application of the proposed procedure is illustrated considering the Takeda hysteresis model commonly adopted for reinforced concrete structures.

Keywords:
P-Δ effects; displacement based assessment; equivalent viscous damping; substitute structure; post-yield stiffness ratio;

Introduction

In the last two decades, several performance-based design approaches have been developed and applied to the seismic assessment of existing structures. Among these procedures, increasing attention has been placed on the Displacement Based Assessment (DBA) methodology, which considers structural displacements, in terms of inter-story and roof drift, and material strain limits as the main seismic vulnerability indicators [Priestley et al., 2007; Sullivan and Calvi, 2013]. This approach is based on the substitute structure theory and the structural response is evaluated by means of an equivalent single degree of freedom (SDOF) system. The equivalent SDOF system accounts for the inelastic behavior of the building by introducing an effective stiffness and an equivalent viscous damping (EVD). In this context, the appropriate definition and response evaluation of the equivalent SDOF system is fundamental, as it significantly affects

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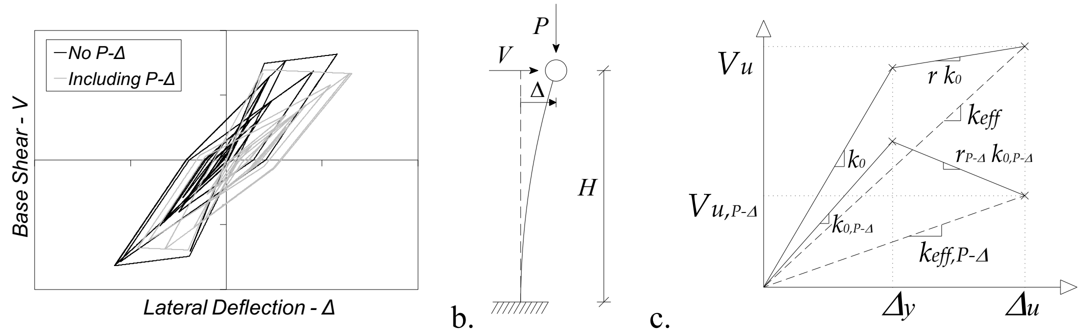
the results. P-Δ effects have been widely acknowledged to be responsible for increased displacement demand (**Figure 1a**) during an earthquake and to represent a possible source of dynamic instability [Rosenblueth, 1965; Bernal, 1987; Priestley et al., 2007; Adam and Jaeger, 2012], thus requiring to be accounted for in the seismic assessment procedure.



a. b. c.

Figure 1 - Influence of P-Δ effects on the structural response

P-Δ effect is typically included in seismic codes [AASHTO, 2009; CEN, 2005; BSSC, 2003] through an amplification factor 1/(1-θ), where θ is the stability index, defined as the ratio between second order moment ($P\cdot\Delta$) and first order moment ($F\cdot H$), **Figure 1b**. Considering a SDOF system (**Figure 1b**) with nonlinear behavior associated to the development of a flexural plastic hinge at the base, it is observed that the system rotation/displacement demand associated to a selected limit state is the same including or not including P-Δ effects. In addition, the moment-rotation relationship is not affected by P-Δ; only the force-displacement loops change shape due to P-Δ while maintaining the same hysteretic energy. For a given lateral deflection (Δ_u), a lower base shear is required to ensure the equilibrium of the system (**Figure 1c**).

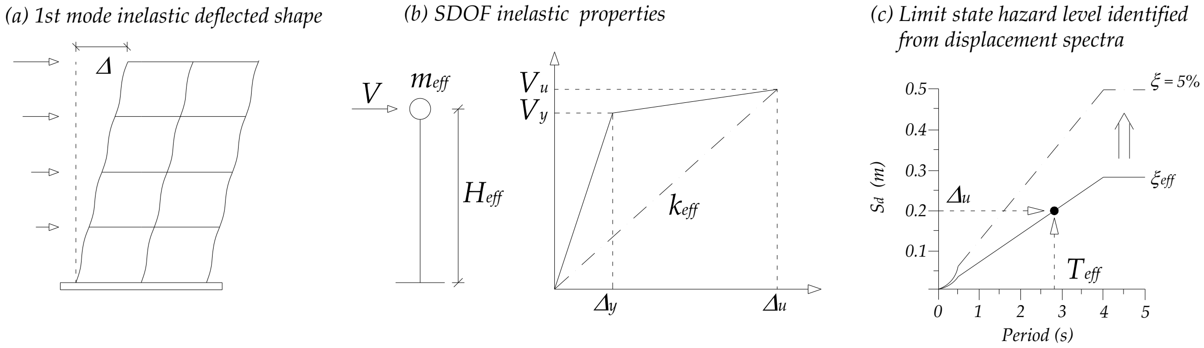
P-Δ affected SDOF systems exhibit the same stability index in the elastic and inelastic range [MacRae, 1994], however in the case of multi degrees of freedom (MDOF) systems, the bilinear idealization of the pushover curve exhibits a different stability index in the elastic and inelastic range, being the latter typically larger than the former [Medina and Krawinkler, 2003; Ibarra and Krawinkler, 2005].

The inclusion of P-Δ effects in the displacement based design procedure has been previously investigated [Priestley et al., 2007; Asimakopoulos et al., 2007; Wei et al., 2012; Lopez et al., 2015]. In the present paper a novel way to include P-Δ effects in the assessment of existing structures is proposed, acting directly on the equivalent viscous damping formulation. The procedure, which overcomes the need of defining the stability index in the elastic and inelastic range, is illustrated considering the Takeda hysteresis model, suitable for reinforced concrete structures.

Accounting for P-Δ effects in the Displacement Based Assessment

The displacement based assessment (DBA) procedure followed herein is based on the direct displacement based design (DDBD) procedure [Priestley et al., 2007]. The fundamental steps of DBA are graphically reported in **Figure 2** and briefly summarized herein. The first step is the definition of the structural deflected shape resembling the fundamental inelastic vibration mode. The deflected shape allows the definition of the parameters of an elastic SDOF substitute structure with stiffness equal to the secant stiffness of the original structure at a selected target displacement (Δ_u). The pushover analysis represents the most efficient way to take into account structural nonlinearities in the definition of the inelastic deflected shape. The effective mass m_{eff}, stiffness k_{eff} and period T_{eff} of the SDOF substitute structure are obtained. The point corresponding to T_{eff} and Δ_u lies on the damped displacement spectrum ($S_{D,in}$). The EVD of the SDOF substitute structure allows to retrieve the elastic displacement spectrum and therefore the hazard level associated to the selected target displacement.

To consider P-Δ effects in the DBA procedure, the substitute structure capacity curve needs to account for the second order moments (**Figure 1**). This is accomplished directly by performing a pushover analysis of the Multi Degree of Freedom (MDOF) system. The chosen method to bi-linearize the capacity curve needs to allow for negative post-yield stiffness. The target displacement of the substitute SDOF system including P-Δ (**Figure 1c**) is characterized by a lower effective stiffness ($k_{eff,P\text{-}\Delta}$) compared to the case without P-Δ (k_{eff}), which leads to an increase of the effective period T_{eff}.


(a) 1st mode inelastic deflected shape

(b) SDOF inelastic properties

(c) Limit state hazard level identified from displacement spectra

Figure 2 - Overview of DBA procedure (Priestley et al. 2007); P-Δ effect not included

The evaluation of the EVD associated to T_{eff} and to $\mu_\Delta=\Delta_u/\Delta_y$ is another aspect affecting the response of the SDOF substitute structure including P-Δ. It is worth mentioning that the available EVD formulations [Grant et al., 2004; Priestley et al., 2007] have been calibrated based on the force-displacement response of inelastic SDOF systems with positive post yield stiffness ratio (r), typically $r = 0.05$. Therefore, given Δ_u, the actual SDOF system's response including P-Δ is represented by Curve A in **Figure 3**, while the curve considered in the EVD formulation is represented by Curve B; this leads to an underestimation of the net hysteretic energy and, consequently, to an underestimation of EVD.

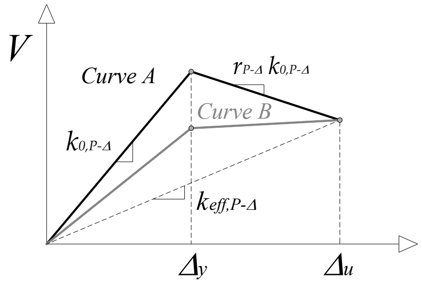


Figure 3 – Curve A: SDOF response including P-Δ effect; Curve B: SDOF response used in EVD formulation

To account for the actual hysteretic energy, a correction factor λ for the available EVD formulations ($\xi_{eq\ r=0.05}$) is proposed herein:

$$\eta_{P-\Delta} = \sqrt{10/\left(5+\lambda\cdot\xi_{eq\ r=0.05}\right)} \tag{1}$$

Where $\eta_{P-\Delta}$, according to EN 1998-1 (CEN 2005), represents the ratio between the equivalent viscous damped displacement spectrum including P-Δ, with EVD equal to $\lambda\cdot\xi_{eq\ r=0.05}$, and the elastic displacement spectrum, with viscous damping equal to 0.05.

The calibration procedure of λ involves the comparison of the dynamic response of two types of SDOF systems: the non-linear SDOF systems including or not including P-Δ effects (Curve A and Curve B, **Figure 3**) and the elastic SDOF system with stiffness equal to $k_{eff,P-\Delta}$ (**Figure 3**). The EVD evaluation procedure starts with the choice of the hysteretic model and from the selection of the displacement ductility (μ_Δ), the effective period (T_{eff}) and post yield stiffness ratio including P-Δ ($r_{P-\Delta}$). The procedure is subdivided in the following steps and graphically represented in **Figure 4**.

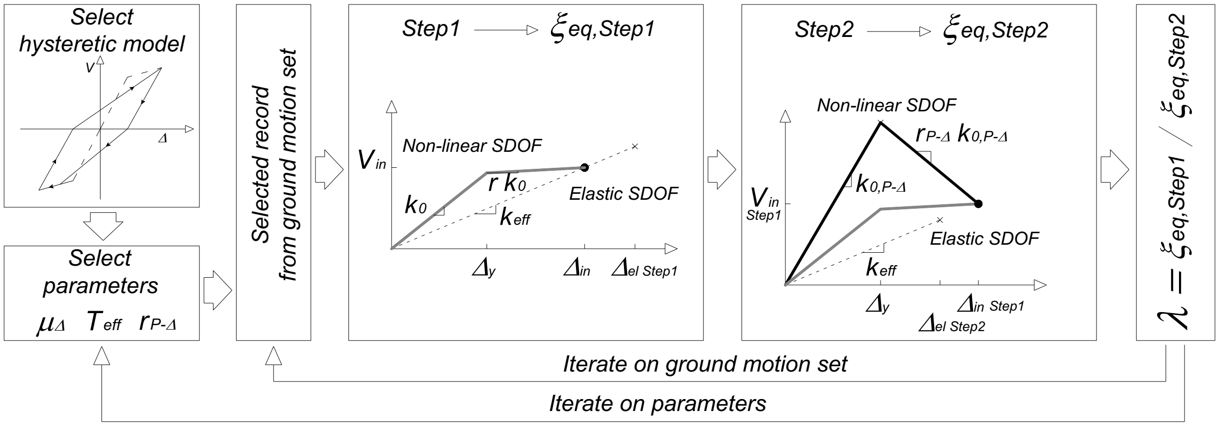


Figure 4 – λ calibration procedure flow chart

In **Step 1**, the EVD of a SDOF system with positive *r*, compatible with EVD available formulations (typically, $r = 0.05$), is evaluated (Curve B, **Figure 3**). The nonlinear SDOF system is subjected to a selected ground motion, which is iteratively scaled in order to obtain the selected μ_Δ and T_{eff}. The maximum shear and displacement are referred to V_{in} and Δ_{in}, respectively. Subsequently, the same scaled ground motion is applied to a linear elastic SDOF system with stiffness equal to V_{in}/Δ_{in} (secant stiffness at maximum displacement) and the maximum displacement $\Delta_{el\ Step\ 1}$ is recorded. The ratio between Δ_{in} and $\Delta_{el\ Step\ 1}$ is equal to [CEN, 2005]:

$$\eta_{Step\ 1} = \Delta_{in}/\Delta_{el\ Step\ 1} = \sqrt{10/\left(5+\xi_{eq\ Step\ 1}\right)} \tag{2}$$

Eq. 2 allows determining $\xi_{eq\ Step\ 1}$.

In **Step 2**, a nonlinear SDOF system with a post yield stiffness equal to $r_{P\text{-}\Delta}$ is considered. The system is subjected to the ground motion of Step 1 iteratively scaled to obtain a maximum displacement equal to Δ_{in} (Step 1). The scaled ground motion is applied to the linear elastic SDOF system of Step 1 and the maximum displacement $\Delta_{el\,Step\,2}$ is recorded. $\xi_{eq\,Step\,2}$ may be determined from the ratio between the inelastic and elastic displacements, Δ_{in} and $\Delta_{el\,Step\,2}$.

$$\eta_{Step\ 2} = \Delta_{in} / \Delta_{el\ Step\ 2} = \sqrt{10/\left(5+\xi_{eq\ Step\ 2}\right)} \tag{3}$$

Finally, a new parameter λ is defined as the ratio between $\xi_{eq\,Step\,2}$ and $\xi_{eq\,Step\,1}$

$$\lambda\left(r_{P-\Delta}, \mu_{\Delta}, T_{eff}\right) = \xi_{eq\ Step\ 2} / \xi_{eq\ Step\ 1} \tag{4}$$

For each displacement ductility (μ_Δ), effective period (T_{eff}) and post-yield stiffness ratio ($r_{P\text{-}\Delta}$), λ is taken as the mean of the values obtained from a set of ground motions. It is worth mentioning that the selected set of ground motions needs to be compatible with the ground motions adopted in the calibration of the available formulations of η and EVD, being the parameters of these expressions interdependent [Pennucci et al., 2014]. Analytical expressions of λ are obtained by means of regression analyses. The proposed procedure allows considering the EVD formulations available in the literature.

For illustration purpose, the EVD evaluation procedure including P-Δ effect is applied to a Takeda "thin" SDOF hysteretic system [Priestley et al., 2007] with the following properties T_{eff}= [1.5; 2.0; 2.5; 3]s, μ_Δ = [2; 3; 4; 5] and r = [0.05; 0; -0.04; -0.08; -0.12; -0.16]. A set of 14 natural ground motions from the European Strong-Motion Database[5] [Ambraseys et al., 2004] is selected.

The results of the procedure, in terms of the mean values of the 14 records, are presented in **Figure 5** as a function of λ and $r_{P\text{-}\Delta}$. The results, subdivided in constant T_{eff} and in constant μ_Δ, show a more pronounced dependence from μ_Δ. Based on this observation, a non-linear regression is performed to relate λ to μ_Δ. Owing the definition of λ, the value of λ for $r_{P\text{-}\Delta}$ = 0.05 is 1. The resulting expression is:

$$\lambda\left(r_{P-\Delta}, \mu_{\Delta}\right) = \left(4.57 \cdot \mu_{\Delta} - 5.53\right)\left(r_{P-\Delta}{}^{2} - 0.0025\right) - \left(1.19 \cdot \mu_{\Delta} - 0.80\right)\left(r_{P-\Delta} - 0.05\right) + 1 \tag{5}$$

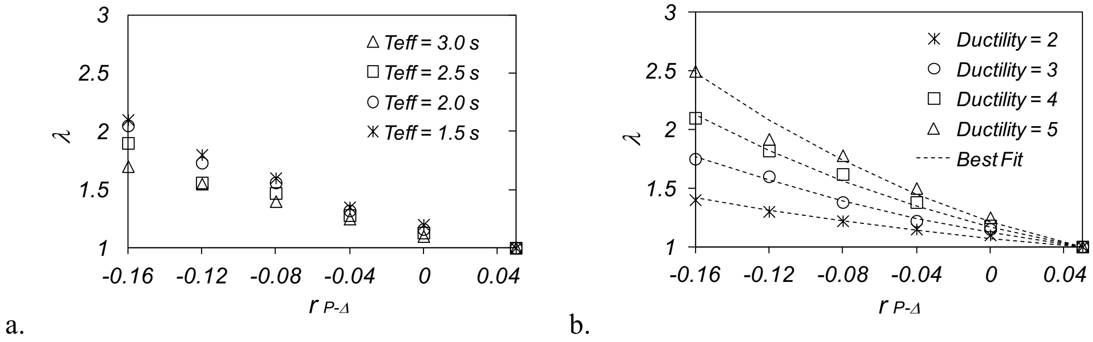


Figure 5 – Results of λ calibration procedure: a. constant T_{eff}; b. constant μ_Δ

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[5] Waveform id (Ambraseys et al. 2004): 000244xa, 000302ya, 000359xa, 000377ya, 005270xa, 005791ya, 005815xa, 000343xa, 000472xa, 000644xa, 000244xa, 000302ya, 000359xa, 000377ya, 005270xa, 005791ya, 005815xa, 000343xa, 000472xa, 000644xa

Conclusions

The influence of P-Δ effects in the displacement based assessment (DBA) procedure has been investigated herein. P-Δ effects reduce the lateral load associated to a selected target displacement in the substitute structure capacity curve, leading to a decrease of the effective stiffness and to an increase of the effective period. In addition, the formulations of equivalent viscous damping (EVD) available in the literature do not account for negative post-yield stiffness which could arise when P-Δ effects are considered.

A procedure to directly account for P-Δ effects in the DBA procedure has been proposed. The procedure is based on the definition of a new parameter λ, which is a function of the displacement ductility and of the post-yield stiffness ratio considering P-Δ. The procedure, suitable for existing EVD formulations, does not involve the evaluation of the stability index in the elastic and inelastic range, because it is based on the bi-linearization of the pushover curve of multi degree of freedom (MDOF) systems including P-Δ.

For illustration purpose, the developed procedure has been applied to single degree of freedom (SDOF) systems with the Takeda "thin" hysteresis; a post yield stiffness ratio in the range -0.16 to 0.05, an effective period in the range 1.5s to 3.0s and a displacement ductility in the range 2 to 5 have been considered. At the current stage, the presented procedure is directly applicable to SDOF like structures such as bridge piers, although future research is necessary to validate its effectiveness in the case of MDOF systems.

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