Some Results and Open Problems in Applications of Cooperative Games

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Abstract: As it is well known, the theory of cooperative games finds applications in many fields. The goal of this paper is to highlight some applications of cooperative games, resulting from the cooperation between the Universities of Bergamo and Krakow, and some related open problems. The paper is organized in three main sections. The first deals with the description of some specific fields where the theory of cooperative games has been applied (i.e. interfering elements, international economics and marketing cooperatives). The second section discusses general results about power indices. The third section presents several applications to Finance and Politics. We discuss these models and the related open problems in a simple and informal way to facilitate understanding by scholars from fields other than game theory.

Key-Words: cooperative games; power indices; finance; politics; interfering elements; international economics; marketing cooperatives

1 Introduction

As it is well known, the theory of cooperative games finds applications in many different fields.

In this paper, we present some results, obtained in the Universities of Bergamo and Krakow, and some related open problems.

We present these problems and models in a simple and informal way to enable scholars from fields other than game theory to understand them in a format closer to their own.

The paper is organized as follows: Clauses 2, 3 and 4 are devoted to the description of some specific topics, where the theory of cooperative games has been applied, i.e. interfering elements, international economics and marketing cooperatives. In Clause 5, we present general results about the power indices. Then we deal with several applications to Finance (Clause 6) and to Politics (Clause 7).

2 Interfering elements

In therapeutic practice, a doctor often prescribes multiple, interfering drugs. Doses of interfering drugs are normally adjusted in subsequent phases, while keeping the patient monitored. The decision regarding the first dosage is particularly delicate, as the doctor does not always have enough information at hand. Many decisions in other applicative fields must take into consideration the effects that two interacting elements can produce. For example, in economics, the demand for a commodity may be influenced by the presence on the market of another commodity with synergic or antagonistic effects. Other cases occur in social choices, for instance in the taxation of various goods (agriculture, zoo technology, etc.). When two factors interfere with one another, there is often a primary interest concerning the effects of one rather than the other. If, for example, the importance of one factor is ten times greater than that of the other, this must be taken into account when calculating the quantities to be used. Considering the above, a recent model enables the optimum quantities of two interfering factors to be directly calculated (rather than obtained by successive approximations). This computation also accounts for the minimum quantities that are in any case to be assigned [10] and [11]. A method is provided for all cases of continuous effect functions; furthermore, appropriate calculus techniques are given.

Unsolved problems concern methods for non-continuous functions and new implementations of the model in the area of cooperative games. The latter refer, for example, to those cases in which the different factors are introduced and controlled by various bodies, where each body is interested in optimizing its own specific objective.

A generalization for cases of more than two interfering elements has been supplied by Carfì [9].

3 International Economics

Developing countries often contract debts with important banking institutions, but they are then unable to pay back. In these cases, the real value of the debt decreases compared with the nominal value, to the point that, in some situations, it is worthwhile for the indebted country to buy its debt again at a lower price, to reacquire the possibility of obtaining new credit. Many studies have been made on this subject; some in particular provide analytical models, see for instance [20].

A three-person game could be studied, the players being the indebted country, the credit institution and an international organization (such as the World Bank or the International Monetary Fund) that is concerned with settling the debt, for instance to avoid detrimental situations with a domino effect on other countries.

4 Marketing cooperatives

Some manufacturers pool together to form cooperatives, in order to improve the marketing of their products, for example by negotiating better prices with large buyers and sharing the risk of production losses amongst the various members. Sometimes the market price of the products may increase after such agreements are made and, as a result, some manufacturers have an incentive to sell part of their production directly, without going through the cooperative. The cooperative may retaliate by applying previously agreed sanctions on these manufacturers. It is important to understand which are the best strategies for the cooperative (in terms of regulations to be approved) and for its individual members. A model by Bertini et al. [5] describes this situation and leads to a Nash equilibrium that provides an attractive solution.

This result opens the way to more complex models, such as oligopoly markets in which the market price falls as the supply increases.

5 General results regarding power indices

As it is well known, a “simple game” is a cooperative game in which every coalition can be only winning or losing. A power index is a value for simple games, which is a function able to represent a reasonable sharing of power among the players, usually in forecasting or normative context.

Many of the most known power indices have been studied and compared among each other, from the viewpoint of several properties in Bertini et al. [3] and [4], Bertini and Stach [8], and Freixas and Gambarelli [17]. For a few indices, the fulfillment of these properties has not been proved yet, and remains as an open problem.

An interesting problem is how to evaluate the influence of decision-makers, such as parties in parliaments or members of boards of directors, on final decisions, especially when the agents are not equivalent, for instance depending on the different numbers of seats in Parliament or different stock shares. This analysis may be performed, inter alia, by using power indices. The literature includes a large number of power indices, each one designed to emphasize different features of specific situations.

Another problem concerns the calculation of power indices in cases of indirect control. For example, this happens when an investor has a share in a certain company, which, in turn, holds shares in another company and so on. Another case is that of a political party consisting of currents and sub-currents. In situations of this kind, it may be useful to calculate the power of a member in the whole system. The problem has been tackled in [25] by transforming the set of inter-connected games into just one game, using the multi-linear extensions introduced by Owen [33]. The power index that is believed to be the most suitable for describing the situation at hand can then be applied to the unified game.

In certain inter-connected games, there may be "loops": for example, if company A holds shares in company B, which also holds shares in company A. The transformation described above works for all cases without loops and for some cases with loops, but not in general. Moreover, an algorithm for the automatic computation of indirect power indices was introduced by Denti and Prati [14], but this method could be improved to reduce the computation time.

A more promising work on the matter has been provided by Karos and Peters [31].

In the following two sections we will see further financial and political applications of the power indices.

6 Finance

In this section we will analyze two relevant topics: trading of shares and indirect control.

6.1 Moving shares to gain control

Some models have been devised to determine changes in an investor's power in a company following trading of shares with others [19]. Gambarelli [22] proposed algorithms to compute the variations of Shapley-Shubik [34] index and of Banzhaf-Coleman index [2] and [12], following exchanges of shares. These models may be useful not only to the bidder, but also to the current controller, because they enable him to assess the stability of his position in relation to potential takeovers [21].

Some financial institutions have begun using these techniques, though obviously without divulging related results. Therefore, a comparison between theoretical models and their application remains an open problem. Furthermore, the above mentioned formulae concern the exchange of shares between two shareholders, or among one shareholder and an ocean of small shareholders who cannot control the firm. Some works regarding small shareholders who can control the firm have been developed, starting with Milnor and Shapley [32] onwards. It would be useful to widen such research to include other types of buying and selling.

6.2 Portfolio Theory

Some developments of the above discussed results concern the Theory of Portfolio Selection. It is known that traditional portfolio models imply a diversification of investments to minimize risk. This diversification contrasts with the concentration of shares necessary for takeovers. A method of linking these two theories has been proposed by means of a control propensity index that can be linked to the risk aversion index [18] and [28]. However, possible developments of such theory remain open. Moreover, the authors of these operations usually prefer not to disseminate the details, then it is difficult to do comparisons between models and reality.

Finally, we would like to draw attention to recent work by Crama and Leruth [13], in which they show how techniques such as power indices are more suitable than cut-off methods for describing power-sharing among shareholders.

7 Politics

Regarding Politics, open problems remain in the following areas: Simulation, Forecasting and Apportionment.

7.1 Simulation

With regard to simulations for normative models, the same formulations used to describe variations in shareholder power can be applied to political parties. In this way, each party may obtain preliminary information, when considering changes to electoral regulations: for example, whether to extend the vote to immigrants, emigrants, youngsters, and so on. Nevertheless, in order to be applicable to politics, such models should be generalized to cases of differing affinities or hostilities in the coalition formation.

Other simulations that could be updated involve the enlargement of the European Parliament, with seat apportionment that not only takes population size into consideration, but also the Gross Domestic Products [6] and [7].

For a more detailed analysis of the contents of this section, see [29].

7.2 Forecasting

The models provided for financial applications may be also applied to political forecasting. For example, as we have seen in section 6, models of indirect control may describe a situation in which parties are subdivided into tendencies. Likewise, models of share exchanges in taking control of a firm may also be applied to power changes following a shift of votes between parties. Therefore, many of the open problems mentioned above also remain open in the context of political applications.

A more specifically Politics-related issue concerns bicameral Parliaments. With regard to this, consider a national Parliament divided into two chambers (for example Deputies and Senators). If we apply a power index, a party may have a certain power in one chamber and a different power in the other. How can the party’s overall power be determined? Moreover, if we have to take into consideration the various propensities for alliances amongst the different parties, the situation becomes even more complicated. Previous models have examined these two problems separately. A unifying model was formulated by Gambarelli and Uristani [30]. The resulting algorithm was applied to the political situation, at that time, in Belgium, the Czech Republic, France, Italy, Netherlands, Poland and Rumania, taking into account the positions of the parties along the left-right axis. The model was also applied globally to the European Union, considering each member country as an individual chamber.

The algorithm can be applied to other countries to assess the stability of current government compositions and of the distribution of the centres of power (Ministries, etc.) amongst the governing parties. However, it should be perfected with the use of various power indices, beyond the Banzhaf-Coleman index used in the paper [2] and [12].

7.3 Apportionment

Some classical criteria of fairness should be taken into account when building rules on seat apportionment: equal seats for equal votes, monotonicity (i.e. no fewer seats if more votes), symmetry (i.e., independence from the order by which the parties are taken into consideration), non exceeding rounding up and down, and so on.

Gambarelli and Hołubiec [24] introduced a criterion particularly linked to the principle of democracy: the minimization of power index distortion [26]. These and other criteria seem first and foremost sacrosanct, but there are cases in which it is not possible to satisfy them, singly or jointly. Furthermore, distortions are magnified in systems with several districts, because despite complying with certain criteria within each individual district, infringements may result at a global level. A compromise solution was put forward by Balinski and Young [1]. They suggested using a traditional method (the Highest Divisors), though adapted so as to avoid apportioning seats in a way that infringes the rounding-up criterion. However, the problem of other general infringements remained open. The minimax method of apportionment introduced by Gambarelli [23] has radically overturned techniques adopted so far. In the past, after a method was chosen and applied to a real situation, tears were shed over all the criteria that were violated. The new technique overturns this procedure: first a priority order of criteria is chosen to be complied with, then all seat distributions are produced that comply with the first criterion; subsequently, all those not respecting the second criterion are eliminated from these seat distributions, and so on, until the final criterion is reached. If the application of a particular criterion leaves the set of remaining seats empty, this criterion is removed and the following ones are then applied and dealt with. This method was generalized by Gambarelli and Palestini [27], extending it to the case of several districts. A theorem of solution existence has been found; an algorithm has been created. Nevertheless, an algorithm of this type leads to lengthy elaboration times because of the minimization metrics employed. This leaves the question of adopting alternative metrics, in order to usefully reduce elaboration times.

8 Conclusion

As a conclusion of this work, we believe useful to quote the two Special Issues on the matter edited by Fragnelli and Gambarelli [15] and [16] for the reader interested in other open problems in Cooperative Games.

We hope that the present overview of results and open problems will bring to further studies and improvements.

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