MME
2017

35th International Conference
Mathematical Methods in Economics
September 13th - 15th, 2017, Hradec Králové, Czech Republic

Conference Proceedings

35th International Conference

Mathematical Methods in Economics

MME 2017

Conference Proceedings

Hradec Králové, Czech Republic
September 13th – 15th, 2017

ISBN 978-80-7435-678-0

Faculty of Informatics and Management, University of Hradec Králové

Programme Committee

doc. RNDr. Helena Brožová, CSc.
doc. Mgr. Ing. Martin Dlouhý, Dr. MSc.
Mgr. Jan Draesslar, Ph.D.
doc. Ing. Jan Fábry, Ph.D.
prof. RNDr. Ing. Petr Fiala, CSc., MBA
doc. Ing. Jana Hančlová, CSc.
prof. Ing. Josef Jablonský, CSc.
doc. RNDr. Ing. Miloš Kopa, Ph.D.
doc. RNDr. Ing. Ladislav Lukáš, CSc.
prof. RNDr. Jan Pelikán, CSc.
doc. Dr. Ing. Miroslav Plevný
doc. RNDr. Petra Poulová, Ph.D.
doc. RNDr. Pavel Pražák, Ph.D.
prof. RNDr. Jaroslav Ramík, CSc.
Ing. Karel Sladký, CSc.
prof. RNDr. Hana Skalská, CSc.
doc. Ing. Tomáš Šubrt, Ph.D.
doc. RNDr. Jana Talašová, CSc.
prof. RNDr. Milan Vlach, DrSc.
prof. RNDr. Karel Zimmermann, DrSc.
prof. Ing. Miroslav Žižka, Ph.D.

Editor: Pavel Pražák

Contents

Systemic Risk and Community Structure in the European Banking System

Gabriele Torri[1]

Abstract. Financial contagion and systemic risk have become increasingly relevant after the financial crisis in 2008. Network theory is a powerful framework for the analysis of these phenomena and is becoming a standard tool in the literature. This paper investigates the properties of the European banking system, focusing on the community structure of the network to identify the potential channels for the propagation of financial distress. The network structure is estimated from the sparse partial correlation of CDS spreads using *tlasso*, a robust technique that induces sparsity in the network. The optimal community structure is then estimated by a procedure that maximises modularity. The analysis shows that, despite the high level of internationalization of the financial system, it exist a clear community structure that mirrors the geographical location of the banks. Finally, a decomposition of *strength centrality* based on the estimated community structure is provided. Such decomposition represents a useful and easy-to-implement tool to monitor the exposure to financial contagion, integrating the traditional risk management tools.

Keywords: Community detection, systemic risk, network theory

JEL classification: C44
AMS classification: 90C15

1 Introduction

Systemic risk is an increasingly relevant issue, especially after the 2008 financial crisis. Given the complexity of the phenomenon, it is difficult to provide a unique definition: concerning the banking sector, some authors put particular emphasis on the effect of macroeconomic shocks on economic fundamentals (see for instance [9]), while other authors focus on the interaction between the public and the financial sectors, stressing the spillover effects of the crisis from the financial system to governments' balances [2]. Most of the works however, stress the idea of shocks affecting financial institutions and/or markets, that can propagate to the entire system [15]. In this context, network theory has become in recent years a fundamental tool for the analysis of systemic risk, capable of describing the structure of the network and to model the diffusion of distress.

Here we focus on the issue of estimating and analysing the structural properties of the banking system network. Such network is often identified as the bilateral exposures on the interbank market [8], that however are often non disclosed and they have to be inferred in absence of bilateral data. A strand of literature reconstructs the network using the total exposures of each bank towards the entire banking system through statistical techniques such as maximum entropy [12]. An alternative approach is to consider the co-movement of time series as proxies for the banks' interdependencies, and to use them to infer the network structure. (e.g. [4] and [14]). We follow this strand of literature, estimating the network from the partial correlations of banks' Credit Default Swap (CDS) spreads. In particular we use the *tlasso* algorithm, an efficient procedure to estimate the sparse partial correlation structure under the assumption of a multivariate t-Student distribution [6]. *Tlasso* can be considered an extension of *glasso*[10], an algorithm that relies on the normality assumption. Compared to the latter, *tlasso* is more appropriate for data with fatter tails and is more robust to misspecification and outliers.

After the estimation of the network, we analyse its structural properties. The literature is mostly focused on national banking systems, that typically present a highly sparse and tiered structure [12]. International banking systems are less studied; the available literature describes a more complex structure compared to national systems (see [1] and [5]). In this paper we focus on a particular structural properties of a network: the presence of a community structure. We identify the communities using the algorithm in [13], that maximises modularity to get the optimal partition, selecting also the optimal number of communities. Thanks to a rewiring procedure, we can also test the significance of the community structure that we identify against an appropriate null model. We find that the European banking system is characterized by a strong and stable community structure, and that this structure is largely overlapping to geographical divisions. In the final part we discuss the policy implication of our results and we provide a simple application that highlights the usefulness of community detection in the regulatory framework.

---

[1] University of Bergamo, Via dei Caniana 2, 24127 Bergamo, Italy, gabriele.torri@unibg.it

2 Methods

2.1 Network Estimation

We briefly present the topic of partial correlation networks in the context of Gaussian graphical models, for more details on the mathematical derivation the reader is referred to [11].

Let $X \sim \mathcal{N}_m(\boldsymbol{\mu}, \boldsymbol{\Sigma})$ be a multivariate Gaussian distribution. This distribution can be associated to an undirected graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ where the nodes in $\mathcal{V}$ correspond to each element of X, the edges $\mathcal{E}$ consist of the pairs of random variables with non-zero partial correlations: $\mathcal{E} = \{(i,j) \in \mathcal{V} \times \mathcal{V} | \rho_{ij} \neq 0\}$ and the edge weights consist in the corresponding partial correlations $\rho_{i,j}$. Partial correlations ρ_{ij} are the linear dependence between two variables conditional on all other variables, and they can be related to the variance-covariance matrix of X. If we define the matrix $\boldsymbol{\Omega} = (Cov[X])^{-1}$ (hereafter defined the *precision matrix*), we can express the following relation:

$$\mathbf{P} = [\rho_{ij}] = -\boldsymbol{\Delta}\boldsymbol{\Omega}\boldsymbol{\Delta} \tag{1}$$

where $\mathbf{P}$ denotes the partial correlation matrix and $\boldsymbol{\Delta} = diag(\frac{1}{\sqrt{\omega_{ii}}})$ [16]. Under the Gaussian assumption we can estimate efficiently the partial correlation matrix using the *glasso* model introduced in [10], that has the advantage of providing a sparse estimate of $\boldsymbol{\Omega}$ by penalizing the 1-norm of the precision matrix. The main disadvantage of this procedure is the normality assumption, therefore we use the alternative *tlasso* algorithm, that is based on *glasso* but relies on the assumption of multivariate t-Student distribution. *Tlasso* estimates are computed efficiently using an Expectation Maximization (EM) algorithm and in simulation studies proved to be more robust to misspecification and outliers in the data. We refer the reader to [10] and [6] for the technical details of *glasso* and *tlasso* respectively. Finally we underline that in this work we use the convenient representation of a network in terms of an *adjacency matrix* $\boldsymbol{A}$, i.e. a square matrix in which each entrance $[\boldsymbol{A}]_{ij} \quad \forall i \neq j$ represents the weight of the edges i, j and the elements on the main diagonal are equal to 0. In this work we use *tlasso* to estimate the network structure of the European banking system, assuming that partial correlations between financial time series (in particular CDS spreads) can represent a proxy of the the interconnections between banks and therefore the channel of propagation of financial distress as in [4].

2.2 Community Detection

A *community* in the field of complex network can be defined as a group of nodes that are more densely connected among themselves than with nodes outside the group. The problem of identifying the best community structure is well studied in the network literature (see for instance [7])[2]. We consider an optimization-based approach in which the optimal community structure is the one associated with the highest *modularity* [13], a quantity defined as follows. Given a partition $G = \{G_1, \ldots, G_p\}$ the modularity Q is:

$$Q = \frac{1}{2m}\sum_{i,j}\left(a_{ij} - \frac{s_i s_j}{2m}\right)\mathbb{I}_{[g_i = g_j]} \tag{2}$$

where a_{ij} is an element of the adjacency matrix $\boldsymbol{A}$, s_i is the strength of node i, $m = \frac{1}{2}\sum_{i,j} a_{i,j}$, g_i is the group in the partition in which the element i belongs and $\mathbb{I}_{[g_i = g_j]}$ is 1 if $g_i = g_j$ and 0 otherwise. Modularity can assume values between -1 and 1, with positive and high values denoting a good division of the network into communities. The procedure proposed in [13] identifies the optimal partition using a greedy optimization that, starting with each vertex being the unique member of a community, repeatedly joins together the two communities whose amalgamation produces the largest increase in modularity. This approach can be implemented efficiently on large networks and identifies automatically the optimal number of communities. Note that a positive value of modularity is not a sufficient condition for identifying a network divided in communities, therefore we need to test if it is the modularity is statistically significantly higher than the one of a random network. In particular we generate the random networks using a degree-preserving rewiring procedure [7].

3 Empirical Analysis

3.1 Data Description

Our dataset consists of 31 weekly time series of CDS (5 years maturity) of European financial institutions settled in 12 countries. They refer to CDS spreads quoted in Euro and they span the time period from January 2009 to June 2016. 20 of the banks in the sample belong to countries in the Eurozone, the other 11 are located in the United Kingdom, Sweden and Denmark. We observe that our database includes 85% of the banks with total assets over 500 billions that are under the European Central Bank (ECB) supervision and it is also consistent with the

---

[2] In the case of non-overlapping communities we can refer to the optimal community structure as *optimal partition*.

European Banking Authority (EBA) stress-test exercise 2016, representing 47% of the banks involved. For the analysis we consider the log-differences of CDS spreads, computing the partial correlation matrix from them using *tlasso* algorithm. We first estimate the network using the data of the entire sample period, and then we analyse the evolution over time using a rolling analysis using windows of 100 weekly observations each.

3.2 Empirical Results

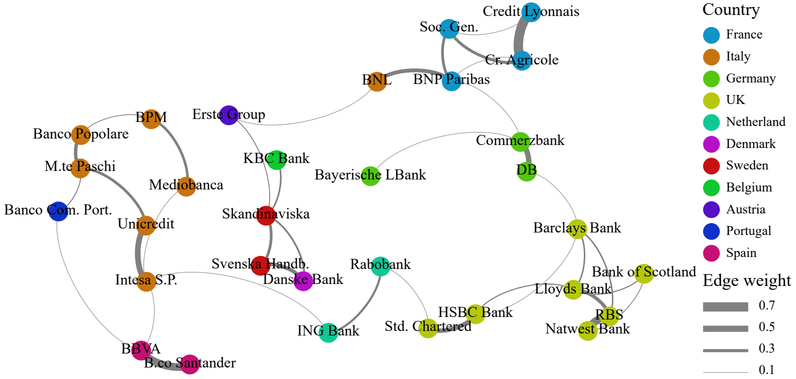
Static Analysis

Figure 1 shows the network represented using a force layout. The visual inspection denotes the clustering of banks in communities aligned with the national groups.

Table 1 shows the composition of the optimal communities identified by Newman's algorithm. The partition consists in five communities and it is possible to notice that it roughly overlaps with geographical divisions, confirming the results of the visual inspection of Figure 1. In particular, community 1 is composed uniquely by banks from Mediterranean Countries, community 2 by British and German banks, communities 3 and 4 include a more diversified group of banks from United Kingdom (UK), central and northern Europe and finally community 5 is composed by French banks (with the exception *BNL*, that is part of the French group *BNP Paribas* but is Italian).

Table 2 reports the value of modularity of the optimal community structure compared to two geographical partitions, one obtained grouping banks by country and one by grouping them in three broad geographical areas: Southern Europe, Central Europe and countries outside Eurozone. For comparison we also consider a partition based on the size of the banks, to check whether banks of similar dimension tend to connect to each other[3]. For each indicator we compute a confidence level based on the empirical distribution of the indicator computed on 1000 random rewirings of the network. We observe that the modularity of the optimal partition is equal to 0.461 and statistically significantly different from the null model with a confidence level higher than 99%, confirming that the network is characterized by a relevant division in communities. We also see that the modularity of geographical partitions (0.352 and 0.335 for the country partition and the area partition respectively), although smaller than the optimal one, are rather high and statistically significant, indicating that the geographical divisions represent a relevant feature of the banking network. Concerning the partition by size, although the modularity is positive and statistically significant, it has a much smaller value compared to the other partitions (0.087), suggesting that is a less relevant factor.

Figure 1 Graphical representation of the network with a force layout.



Dynamic Analysis

We perform a rolling analysis to monitor the evolution of the community structure over time. In particular, we consider the evolution of modularity as presented in Figure 2. We see that the modularity of the optimal partition is highest in the period corresponding to the Sovereign crisis, decreases from mid-2012 and then grows again in recent years. The pattern is similar for the geographical partitions, while modularity of the partition generated by the size show a moderate increase across the time period. The high level of modularity during the crisis is consistent with the sovereign-driven nature of the European crisis: the increased relevance of country risk leads to a decrease in confidence in the transnational interbank market, and thus to a "flight to safety"and a tightening of national banking systems. The rise in modularity in the last part of the sample may be related to the low level of the interbank interest rates in recent years, that makes less convenient for banks in core Countries to lend to banks in peripheral Countries, exacerbating the division among national banking systems. In an unreported test, we also

---

[3] In particular we defined 5 classes of homogeneous size based on the *total assets* based on 2015 balance sheet.

Community	Number of banks	Constituents
1	9	M.te Paschi(IT), Banco Popolare(IT), BPM(IT), Intesa S.P.(IT), Mediobanca(IT), Unicredit(IT), B.co Santander(ES), BBVA(ES), Banco Com. Port.(PT)
2	7	Commerzbank(DE), DB(DE), Barclays Bank(UK), Bank of Scotland(UK), Lloyds Bank(UK), Natwest Bank(UK), RBS(UK)
3	6	KBC Bank(BE), Bayerische LBank(DE), Erste Group(AUT), Skandinaviska(SE), Svenska Handb.(SE), Danske Bank(DK)
4	4	Rabobank(NL), ING Bank(NL), HSBC Bank(UK), Std. Chartered(UK)
5	5	BNL(IT), BNP Paribas(FR), Cr. Agricole(FR), Credit Lyonnais(FR), Soc. Gen.(FR)

Table 1 Constituents of Optimal Communities

Partition	Modularity
Optimal partition	0.461***
Countries	0.352***
Geographical area	0.335***
Total assets	0.087***

Table 2 Modularity of 4 partitions. ***,**,* refer to confidence level of 99%, 95% and 90% respectively.

measure the stability of the community structure by testing how well the optimal partition in a given estimation window can describe the community structure in a future window. We found that the optimal partition, despite the variations in the modularity over time, is characterized by a great stability. The results are available from the author upon request.

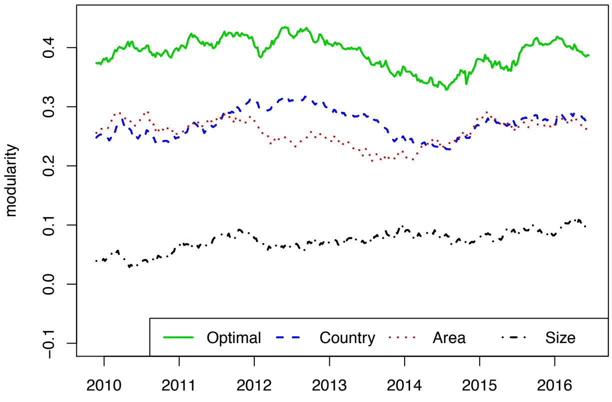


Figure 2 Evolution of Modularity over Time.

4 Economic Implications and Applications to Risk Management

It is well known that the presence of a community structure influences greatly the diffusion of epidemics and, in general, the behaviour of dynamic processes in complex network. Epidemic models find interesting applications in the economic literature, and have been used by several authors to model the diffusion of financial contagion [8]. However, to the best of our knowledge, the role of a community structure has not been explicitly studied in this literature. The results we present here describe the presence of a stable and significant community structure in the data and suggest to further study the topic, opening a new line of research focused in the study of the effect of community structure on the diffusion of financial distress.

The analysis of contagion dynamics are outside the scope of this work, however we propose here a simple procedure to use the information regarding the community structure for the assessment of the role of each bank in the system. The procedure is based on the decomposition of *strength centrality*, an indicator computed for each node as the sum of the weights of the edges connected to it. We can think to a node characterized by a higher centrality as more interconnected, and therefore more systemically relevant. If the network is characterized by a community structure, a high centrality of a node could be associated to strong bonds to nodes in the same communities or to bonds to nodes in different ones. In a banking system this distinction is particularly relevant for the management of systemic risk: for instance a bank that has a high level of interconnectedness, but whose connections span mostly in a limited neighbourhood, would be less relevant in terms of systemic risk compared to a bank with broader interconnections, that could represent a "bridge" for financial contagion[4]. In particular we decompose *strength centrality* in two components: *strength inside* and *strength outside*.

$$Str_i^I = \sum_j a_{ij} \mathbb{I}_{[g_i=g_j]} \tag{3}$$

$$Str_i^O = \sum_j a_{ij} \left(1 - \mathbb{I}_{[g_i=g_j]}\right) \tag{4}$$

where a_{ij} is an element of the weighted adjacency matrix, G is the optimal partition of the network, g_i is the group in the partition in which the element i belongs and $\mathbb{I}_{[g_i=g_j]}$ is 1 if $g_i = g_j$ and 0 otherwise.

Figure 3 reports the decomposition of *strength centrality*. We can see that for most of the banks the *inside* component (dark) is particularly relevant, representing the largest part of the total *strength*, while the *outside* component (light) is in many cases marginal. Focusing on individual banks, we can use the decomposition to enrich the information coming from centrality measures. For instance, in the first community we notice that *M.te Paschi* and *Mediobanca* have similar value of *strength centrality*, the decomposition however shows that the former is mostly exposed to banks in the same community, while the latter has connection that span more internationally. Comparing to the state-of-art approach to financial regulation, we can make a relation between this indicator and the Global systemically important banks (G-SIB) assessment methodology proposed by Basel Committee. Two of the criteria for the identification of G-SIBs are related to the international exposure of a bank: cross-jurisdictional claims and cross-jurisdictional liabilities. The idea is that the greater a bank's global reach, the more difficult it is to coordinate its resolution and the more widespread the spillover effects from its failure. Our indicator is constructed on different basis, but provides similar information, showing the extent of the interconnection of the node in the network and the potential footprint of a credit event of an institution in the system using a network based model based on easily available data. Finally we underline that the *strength inside* and *strength outside* could find application in the early warning literature for financial distress or for the definition of network based capital requirements for banks (see [3]).

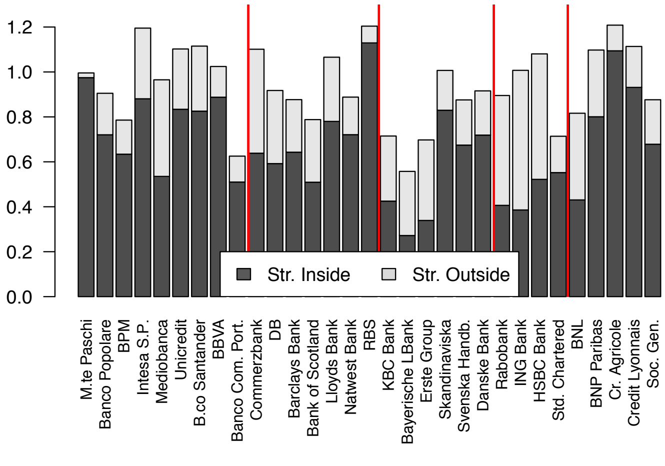


Figure 3 Decomposition of *strength centrality* in *strength inside* and *strength outside*. Vertical red bands divide the optimal communities.

---

[4] Note some centrality measures such as *eigenvector centrality* and *betweenness centrality* already allow to measure the relevance of each country in a more meaningful way, for instance weighting more the connections to more important nodes. Our approach is complementary to these measures, and we claim that it is more flexible, providing more insights on the role of each bank in the network.

5 Concluding Remarks

This work estimates the network structure of the European banking system on the basis of partial correlations using the *tlasso* algorithm. The analysis is focused on the identification of a community structure in the network and its influence on contagion risk. The results support the presence of a strong community structure and indicate that this structure is largely aligned to the geographical distribution of the banks. Furthermore, we propose a simple decomposition of *strength centrality* based on the community structure and we highlight its usefulness for assessing the role and importance of each bank in the network. The work opens new research questions regarding the role of community structure in the diffusion of financial contagion, that haven't been addressed yet in the literature.

Acknowledgements

I would like to thank Sandra Paterlini and Rosella Giacometti for introducing me to the topic and for the valuable help. All errors are mine.

References

[1] Aldasoro, I. and Alves, I., Multiplex interbank networks and systemic importance An application to European data, *Journal of Financial Stability*, forthcoming.

[2] Alter, A. and Schler, Y. S. , Credit spread interdependencies of European states and banks during the financial crisis, *Journal of Banking & Finance*, 36(12), 3444–3468, (2012).

[3] Alter, A., Craig, B. and Raupach, P., Centrality–based capital allocations, *International Journal of Central Banking*, 11(3), 329–376, (2014).

[4] Anufriev, M. and Panchenko, V., Connecting the dots: Econometric methods for uncovering networks with an application to the Australian financial institutions, *Journal of Banking & Finance* 61, S241–S255 (2015).

[5] Craig, B. and Saldías, M., Spatial Dependence and Data-Driven Networks of International Banks, IMF working paper WP/16/184, (2016).

[6] Finegold, M. and Drton, M., Robust graphical modeling of gene networks using classical and alternative t-distributions, *The Annals of Applied Statistics* 5(2A), 1057–1080, (2011).

[7] Fortunato, S., Community detection in graphs, *Physics reports*, 486(3), 75–174, (2010).

[8] Hurd, T. R., Contagion! Systemic Risk in Financial Networks, *Springer*, (2016),

[9] Fabozzi, F. J., Giacometti, R., Tsuchida, N., The ICA-based Factor Decomposition of the Eurozone Sovereign CDS Spreads, *Journal of International Money and Finance*, vol 65, 1–23, (2016).

[10] Friedman, J., Hastie, T. and Tibshirani, R., Sparse inverse covariance estimation with the graphical lasso, *Biostatistics* 9(3), 432–441, (2008).

[11] Lauritzen, S. L., Graphical models, *Clarendon Press* vol 17 (1996).

[12] Mistrulli, P. E., Assessing financial contagion in the interbank market: Maximum entropy versus observed interbank lending patterns, *Journal of Banking & Finance*, 35(5),1114–1127, (2011).

[13] Newman, M., Fast algorithm for detecting community structure in networks, *Physical review E* 69(6), (2004).

[14] Puliga, M., Caldarelli, G., Battiston, S., Credit Default Swaps networks and systemic risk, *Sci. Rep.* 4, 6822; DOI:10.1038/srep06822 (2014).

[15] Schwarcz, S., Systemic risk,*Duke Law School Legal Studies Paper No. 163* , *97(1)*, (2008).

[16] Stevens, G. V. G., On the inverse of the covariance matrix in portfolio analysis, *The Journal of Finance* 53(5), 1821–1827, (1998).

All papers passed a blind review process.

This publication has not been a subject of language check.

Published with the financial support of Czech Software First, Ltd., K Západi 54,
621 00 Brno.

Titlle: 35th International Conference Mathematical Methods In Economics, Conference Proceedings
Author: composite authors participants of the international conference
Intended: for participants of the internationalconference form researchers and professional public
Publisher: Gaudeamus, University of Hradec Králové
Published: in September 2017
Pages: 896
Edition: 1.
Press: Karo Reklama s.r.o.
Reference number of publication: 1615

ISBN 978-80-7435-678-0

University of Hradec Králové
Faculty of Informatics and Management

ČSOV

35th International Conference

Mathematical Methods in Economics

September 13th - 15th, 2017, Hradec Králové, Czech Republic

ISBN 978-80-7435-678-0