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Naked singularity in a modified gravity theory

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Abstract. The cosmological constant induced by quantum fluctuation of the graviton on a given background is considered as a tool for building a spectrum of different geometries. In particular, we apply the method to the Schwarzschild background with positive and negative mass parameter. In this way, we put on the same level of comparison the related naked singularity ($-M$) and the positive mass wormhole. We discuss how to extract information in the context of a $f(R)$ theory. We use the Wheeler-De Witt equation as a basic equation to perform such an analysis regarded as a Sturm-Liouville problem. The application of the same procedure used for the ordinary theory, namely $f(R) = R$, reveals that to this approximation level, it is not possible to classify the Schwarzschild and its naked partner into a geometry spectrum.

1. Introduction

Almost ten years ago, observation about type I supernovae data revealed that the Universe is in an acceleration phase [1]. Since then, no satisfactory explanation has been given. Indeed if the Friedmann-Robertson-Walker model of the universe, based on the Einstein's field equations is correct, the explanation of such an expansion should be due to approximately a 76% of what is known as *Dark Energy*. Dark Energy is based on the following equation of state $P = -\rho$ (where P and ρ are the pressure of the fluid and the energy density, respectively). Dark Energy changes into *Phantom Energy* when $P < -\rho$. Nevertheless, neither Dark Energy nor Phantom Energy models appear to be satisfactory. In this scenario, the idea that General Relativity could be modified in something more general has been considered in recent years. In particular, the Einstein-Hilbert action ($\kappa = 8\pi G$)

$$S = \frac{1}{2\kappa} \int d^4x \sqrt{-g} R + S^{matter} \quad (1)$$

is replaced by[2]

$$S = \frac{1}{2\kappa} \int d^4x \sqrt{-g} f(R) + S^{matter}. \quad (2)$$

It is clear that other more complicated choices could be done in place of $f(R)$ [4]. In particular, one could consider $f(R, R_{\mu\nu}R^{\mu\nu}, R_{\alpha\beta\gamma\delta}R^{\alpha\beta\gamma\delta}, \dots)$ or $f(R, G)$ where G is the Gauss-Bonnet invariant or any combination of these quantities¹. Note that in principle one could consider

¹ For a recent review on $f(R)$, see Refs[3, 4], while a recent review on the problem of $f(G)$ and $f(R, G)$ can be found in[5].

the replacement $R - 2\Lambda_c \rightarrow f(R)$ in such a way to avoid the use of the cosmological constant Λ_c . It is well known indeed, that there exists a factor of 120 orders of magnitude of discrepancy between the observed and the computed value. This huge disagreement is known as *cosmological constant problem*. It is important to remark that the cosmological constant plays an alternative rôle with respect to Dark Energy and modified gravity theories in explaining the acceleration of the Universe. Nevertheless, nothing forbids to consider them together in the context of the Wheeler-DeWitt equation (WDW)[6]

$$\mathcal{H}\Psi=0, \tag{3}$$

where ²

$$\mathcal{H} = (2\kappa) G_{ijkl}\pi^{ij}\pi^{kl} - \frac{\sqrt{g}}{2\kappa}({}^3R - 2\Lambda_c), \tag{4}$$

³ R is the scalar curvature in three dimensions and G_{ijkl} is called the super-metric. \mathcal{H} represents the time-time component of the Einstein's field equations without matter fields. It represents a constraint at the classical level and the invariance under *time* reparametrization. One can formally re-write the WDW equation as an eigenvalue problem

$$\frac{1}{V} \frac{\int \mathcal{D}[g_{ij}] \Psi^*[g_{ij}] \int_{\Sigma} d^3x \hat{\Lambda}_{\Sigma} \Psi[g_{ij}]}{\int \mathcal{D}[g_{ij}] \Psi^*[g_{ij}] \Psi[g_{ij}]} = \frac{1}{V} \frac{\langle \Psi | \int_{\Sigma} d^3x \hat{\Lambda}_{\Sigma} | \Psi \rangle}{\langle \Psi | \Psi \rangle} = -\frac{\Lambda_c}{\kappa}, \tag{5}$$

where

$$\hat{\Lambda}_{\Sigma} = (2\kappa) G_{ijkl}\pi^{ij}\pi^{kl} - \frac{\sqrt{g}}{2\kappa} {}^3R \tag{6}$$

and

$$V = \int_{\Sigma} d^3x \sqrt{g}. \tag{7}$$

It is clear that, what we interpret as an eigenvalue is an induced cosmological constant and, as pointed out in Ref.[8], we can use such an eigenvalue evaluated to one loop in different backgrounds as a tool to compute a geometrical “*spectrum*” based on Zero Point Energy (ZPE). In particular, in Ref.[8], we have computed the induced cosmological constant generated by a naked singularity associated to the Schwarzschild metric. The reason for such a choice is simple: if the Schwarzschild solution is

$$ds^2 = - \left(1 - \frac{2MG}{r}\right) dt^2 + \left(1 - \frac{2MG}{r}\right)^{-1} dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\phi^2), \tag{8}$$

replacing M with $-M = \bar{M}$, we obtain a naked singularity with the following line element

$$ds^2 = - \left(1 + \frac{2\bar{M}G}{r}\right) dt^2 + \left(1 + \frac{2\bar{M}G}{r}\right)^{-1} dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\phi^2). \tag{9}$$

It is immediate to see there is no horizon protecting the singularity. An immediate consequence of a negative Schwarzschild mass is that if one were to place two bodies initially at rest, one with a negative mass and the other with a positive mass, both will accelerate in the same direction going from the negative mass to the positive one. Furthermore, if the two masses are of the same magnitude, they will uniformly accelerate forever ³. Another reason to consider a naked singularity described by the line element 9 is that it naturally represents a form of “*Dark Energy*” and therefore it deserves attention. In this paper we would like to extend the analysis

² See Ref.[7] for more details.

³ See Ref.[9], for negative mass analysis in General Relativity. See also the problem of the *Cosmic Censorship Conjecture* postulated by R. Penrose[10]

of Ref.[8] to $f(R)$ theories. Although the subject of this investigation is quite delicate, because as far as we know the subject of $f(R)$ theories combined with naked singularities has not been considered even at the classical level, it seems quite reasonable to apply the scheme of Eq.(5) to the negative Schwarzschild mass. Even in this case, we exclude the contribution of matter fields and the final result will be due only to quantum fluctuations. In practice, we desire to compute

$$\Delta\Lambda_c = \Lambda_c^S - \Lambda_c^N \geq (\leq) 0, \quad (10)$$

where $\Lambda_c^{S,N}$ are the induced cosmological constant computed in the different backgrounds. Moreover, the Schwarzschild solution for both masses, namely $\pm M$ is asymptotically flat. Therefore we are comparing backgrounds with the same asymptotically behavior. Nevertheless, in Eq.(3), surface terms never come into play because \mathcal{H} as well as Λ_c/κ are energy densities and surface terms are related to the energy (e.g. ADM mass) and not to the energy density. We want to point up that even in the case of $f(R)$ theories, we are neither discussing the problem of forming the naked singularity nor a transition during a gravitational collapse. Rather the singularity is considered already existing. The semi-classical procedure followed in this work relies heavily on the formalism outlined in Refs.[8, 15].

2. Positive and negative Schwarzschild mass in a $f(R) = R$ theory

The Schwarzschild background is simply described by Eq.(8). In terms of the induced cosmological constant of Eq.(5), we get

$$\frac{\Lambda_{0,c}(\mu_0, r)}{8\pi G} = \frac{1}{64\pi^2} \sum_{i=1}^2 \left(\frac{3MG}{r^3} \right)^2 \ln \left(\left| \frac{4r^3\mu_0^2}{3MG\sqrt{e}} \right| \right), \quad (11)$$

where we have removed the ultraviolet divergence with the help of a zeta function regularization and applied a Renormalization Group equation in order to avoid a dependence on the mass scale μ .

We know that an extremum appears, maximizing the induced cosmological constant for

$$\frac{3MG\sqrt{e}}{4r^3\mu_0^2} = \frac{1}{\sqrt{e}} \quad (12)$$

and leading to

$$\frac{\Lambda_{0,c}(\mu_0, r)}{8\pi G} = \frac{\mu_0^4}{4e^2\pi^2} \quad (13)$$

or ⁴

$$\frac{\Lambda_{0,c}(\mu_0, r)}{8\pi G} = \left(\frac{3MG}{r^3} \right)^2 \frac{1}{64\pi^2} \quad r \in \left[r_t, \frac{5}{4}r_t \right]. \quad (14)$$

Therefore, it appears that there exists a bound for $\Lambda_{0,c}$

$$\frac{9}{256\pi^2 r_t^4} \leq \frac{\Lambda_{0,c}(\mu_0, r)}{8\pi G} \leq \frac{225}{4096\pi^2 r_t^4}. \quad (15)$$

When we consider the naked Schwarzschild metric, we obtain an induced cosmological constant of the form

$$\frac{\Lambda_{0,c}^{naked}(\mu_0, r)}{8\pi G}$$

⁴ See Refs.[7, 8] for technical details concerning the reasons of why $r \in [r_t, \frac{5}{4}r_t]$.

$$= \frac{1}{64\pi^2} \left[\left(\frac{15\bar{M}G}{r^3} \right)^2 \ln \left(\left| \frac{4r^3\mu_0^2}{15\bar{M}G\sqrt{e}} \right| \right) + \left(\frac{9\bar{M}G}{r^3} \right)^2 \ln \left(\left| \frac{4r^3\mu_0^2}{9\bar{M}G\sqrt{e}} \right| \right) \right]. \quad (16)$$

In order to find an extremum, it is convenient to define the following dimensionless quantity

$$\frac{9\bar{M}G\sqrt{e}}{4r^3\mu_0^2} = x, \quad (17)$$

then Eq.(16) becomes

$$\frac{\Lambda_{0,c}^{naked}(\mu_0, r)}{8\pi G} = -\frac{\mu_0^4}{4e\pi^2} \left[x^2 \ln x + \frac{25}{9} x^2 \ln \left(\frac{5x}{3} \right) \right]. \quad (18)$$

We find a solution when

$$\bar{x} = \frac{1}{\sqrt{e}} \left(\frac{3}{5} \right)^{\frac{25}{34}} \simeq 0.417 \quad (19)$$

corresponding to a value of

$$\frac{\Lambda_{0,c}^{naked}(\mu_0, r)}{8\pi G} = \frac{\mu_0^4}{4e^2\pi^2} \frac{17}{75} 5^{\left(\frac{9}{17}\right)} 3^{\left(\frac{8}{17}\right)} \simeq 0.328 \frac{\mu_0^4}{4e^2\pi^2} = 0.328 \frac{\Lambda_{0,c}(\mu_0, r)}{8\pi G}. \quad (20)$$

This means that

$$\frac{\Lambda_{0,c}^{naked}(\mu_0, r)}{\Lambda_{0,c}(\mu_0, r)} = 0.328 < 1. \quad (21)$$

A comment to this inequality is in order. Eq.(21) shows that the Schwarzschild naked singularity has a lower value of ZPE compared to the positive Schwarzschild mass. This means that, even if the order of magnitude is practically the same, the naked singularity is less favored with respect to the Schwarzschild wormhole. We now try to apply the same method to a modified gravity theory of the form $f(R)$.

3. Positive and negative Schwarzschild mass one loop energy for a generic $f(R)$ theory

In this section, we report the main steps discussed in Ref.[15] for a $f(R)$ theory in connection with the Sturm-Liouville problem of Eq.(5). Although a $f(R)$ theory does not need a cosmological constant, rather it should explain it, we shall consider the following Lagrangian density describing a generic $f(R)$ theory of gravity

$$\mathcal{L} = \sqrt{-g} (f(R) - 2\Lambda), \quad \text{with } f'' \neq 0, \quad (22)$$

where $f(R)$ is an arbitrary smooth function of the scalar curvature and primes denote differentiation with respect to the scalar curvature. A cosmological term is added also in this case for the sake of generality, because in any case, Eq.(22) represents the most general lagrangian to examine. Obviously $f'' = 0$ corresponds to GR. The generalized Hamiltonian density for the $f(R)$ theory assumes the form ⁵

$$\begin{aligned} \mathcal{H} = f'(R) & \left[(2\kappa) G_{ijkl} \pi^{ij} \pi^{kl} - \frac{\sqrt{g}}{2\kappa} \left({}^{(3)}R - 2\Lambda_c \right) \right] \\ + 2(2\kappa) & \left[G_{ijkl} \pi^{ij} \pi^{kl} + \frac{\pi^2}{4} \right] (f'(R) - 1) + \frac{1}{2\kappa} \left[V(\mathcal{P}) + 2g^{ij} (\sqrt{g} f'(R))_{|ij} \right]. \end{aligned} \quad (23)$$

⁵ See Refs.[16, 15] for technical details.

where

$$\mathcal{P} = -6\sqrt{g}f'(R) \quad (24)$$

and

$$V(\mathcal{P}) = \sqrt{g} [Rf'(R) - f(R)]. \quad (25)$$

When $f(R) = R$, $V(\mathcal{P}) = 0$ as it should be. By imposing the Hamiltonian constraint and integrating over the hypersurface Σ , we obtain

$$\int_{\Sigma} d^3x \left\{ \left[(2\kappa) G_{ijkl}\pi^{ij}\pi^{kl} - \frac{\sqrt{g}}{2\kappa} {}^3R \right] + (2\kappa) \left[G_{ijkl}\pi^{ij}\pi^{kl} + \frac{\pi^2}{4} \right] \frac{2(f'(R) - 1)}{f'(R)} + \frac{V(\mathcal{P})}{2\kappa f'(R)} \right\} = -\frac{\Lambda_c}{\kappa} \int_{\Sigma} d^3x \sqrt{g}, \quad (26)$$

where we have assumed that $f'(R) \neq 0$ and we have dropped a divergence form term. Eq.(26) can be cast in the form of Eq.(5), by formally repeating the same procedure. Thus one gets

$$\frac{1}{V} \frac{\langle \Psi | \int_{\Sigma} d^3x [\hat{\Lambda}_{\Sigma}^{(2)}] | \Psi \rangle}{\langle \Psi | \Psi \rangle} + \frac{2\kappa 2(f'(R) - 1)}{V f'(R)} \frac{\langle \Psi | \int_{\Sigma} d^3x [G_{ijkl}\pi^{ij}\pi^{kl} + \pi^2/4] | \Psi \rangle}{\langle \Psi | \Psi \rangle} + \frac{1}{V} \frac{\langle \Psi | \int_{\Sigma} d^3x V(\mathcal{P}) / (2\kappa f'(R)) | \Psi \rangle}{\langle \Psi | \Psi \rangle} = -\frac{\Lambda_c}{\kappa}. \quad (27)$$

From Eq.(27), we can define a “modified” $\hat{\Lambda}_{\Sigma}^{(2)}$ operator which includes $f'(R)$. Thus, we obtain

$$\frac{\langle \Psi | \int_{\Sigma} d^3x [\hat{\Lambda}_{\Sigma, f(R)}^{(2)}] | \Psi \rangle}{\langle \Psi | \Psi \rangle} + \frac{\kappa (f'(R) - 1)}{V f'(R)} \frac{\langle \Psi | \int_{\Sigma} d^3x [\pi^2] | \Psi \rangle}{\langle \Psi | \Psi \rangle} + \frac{1}{V} \frac{\langle \Psi | \int_{\Sigma} d^3x \frac{V(\mathcal{P})}{2\kappa f'(R)} | \Psi \rangle}{\langle \Psi | \Psi \rangle} = -\frac{\Lambda_c}{\kappa}, \quad (28)$$

where

$$\hat{\Lambda}_{\Sigma, f(R)}^{(2)} = (2\kappa) h(R) G_{ijkl}\pi^{ij}\pi^{kl} - \frac{\sqrt{g}}{2\kappa} {}^3R^{lin}, \quad (29)$$

with

$$h(R) = 1 + \frac{2[f'(R) - 1]}{f'(R)} \quad (30)$$

and where ${}^3R^{lin}$ is the linearized scalar curvature. Note that when $f(R) = R$, consistently it is $h(R) = 1$. From Eq.(28), we redefine Λ_c

$$\Lambda'_c = \Lambda_c + \frac{1}{2V} \frac{\langle \Psi | \int_{\Sigma} d^3x \frac{V(\mathcal{P})}{f'(R)} | \Psi \rangle}{\langle \Psi | \Psi \rangle} = \Lambda_c + \frac{1}{2V} \int_{\Sigma} d^3x \sqrt{g} \frac{Rf'(R) - f(R)}{f'(R)}, \quad (31)$$

where we have explicitly used the definition of $V(\mathcal{P})$. In the same spirit of the previous section, we find that by replacing $\Lambda_0(\mu_0, r)$ with $\Lambda'_0(\mu_0, r)$, the TT tensors one loop contribution for a $f(R)$ theory is given by Eq.(11) and the extremum is given therefore by ^{6,7}

$$\Lambda'_0(\mu_0, \bar{x}) = \frac{G\mu_0^4}{2\pi e^2}, \quad (32)$$

⁶ For a complete derivation of the effective action for a $f(R)$ theory, see Ref.[17].

⁷ By a canonical decomposition of the gauge part ξ_a into a transverse part ξ_a^T with $\nabla^a \xi_a^T = 0$ and a longitudinal part ξ_a^{\parallel} with $\xi_a^{\parallel} = \nabla_a \psi$, it is possible to show that most of the contribution comes from the longitudinal part (scalar). Evidence against scalar perturbation contribution in a Schwarzschild background has been discussed in Ref.[18].

with \bar{x} expressed by Eq.(12). In terms of $\Lambda_0(\mu_0, \bar{x})$, we find

$$\frac{1}{\sqrt{h(R)}} \left[\Lambda_0(\mu_0, \bar{x}) + \frac{1}{2V} \int_{\Sigma} d^3x \sqrt{g} \frac{Rf'(R) - f(R)}{f'(R)} \right] = \frac{G\mu_0^4}{2\pi e^2} \quad (33)$$

and isolating $\Lambda_0(\mu_0, \bar{x})$, we obtain

$$\Lambda_0(\mu_0, \bar{x}) = \sqrt{h(R)} \frac{G\mu_0^4}{2\pi e^2} - \frac{1}{2V} \int_{\Sigma} d^3x \sqrt{g} \frac{Rf'(R) - f(R)}{f'(R)}. \quad (34)$$

Note that $\Lambda_0(\mu_0, \bar{x})$ can be set to zero when

$$\sqrt{h(R)} \frac{G\mu_0^4}{2\pi e^2} = \frac{1}{2V} \int_{\Sigma} d^3x \sqrt{g} \frac{Rf'(R) - f(R)}{f'(R)}. \quad (35)$$

Let us see what happens when

$$f(R) = \exp(-\alpha R). \quad (36)$$

This choice is simply suggested by the regularity of the function at every scale. In this case, Eq.(35) becomes

$$\sqrt{\frac{3\alpha \exp(-\alpha R) + 2}{\alpha \exp(-\alpha R)}} \frac{G\mu_0^4}{\pi e^2} = \frac{1}{\alpha V} \int_{\Sigma} d^3x \sqrt{g} (1 + \alpha R). \quad (37)$$

For Schwarzschild, it is $R = 0$, and by setting $\alpha = G$, we have the relation

$$\mu_0^4 = \frac{\pi e^2}{G} \sqrt{\frac{1}{(3G + 2)G}}. \quad (38)$$

It is clear that the passage to the naked singularity is straightforward, at least at formal level. The result is identical to Eq.(32) with the replacement

$$\Lambda'_0(\mu_0, \bar{x}) \rightarrow \Lambda_0^{(naked)'}(\mu_0, \bar{x}), \quad (39)$$

which means that also in Eq.(33) we have to replace $\Lambda_0(\mu_0, \bar{x})$ with $\Lambda_0^{naked}(\mu_0, \bar{x})$. However, also in this case we have the freedom to choose the r.h.s of Eq.(33) in such a way to cancel $\Lambda_0^{naked}(\mu_0, \bar{x})$. This means that also a naked Schwarzschild singularity predicts a vanishing cosmological constant. Let us see the consequences on the renormalization point μ_0 . If we further proceed and we fix the form of $f(R)$ like in Eq.(36), we get the relation

$$\mu_0^{(naked)4} = 0.328\mu_0^4 \quad \longrightarrow \quad \mu_0^{naked} = 0.757\mu_0. \quad (40)$$

As we can see, it seems that a generic $f(R)$ theory cannot be more different by the ordinary case, namely when $f(R) = R$. This means that in this approach and at this level of approximation we cannot discriminate the different geometries. The situation is more marked, if we had chosen as a boundary condition for the naked singularity a privileged point $r_0 = 2MG$, namely a “*fictitious throat*”. We use the term fictitious because there is no throat at all. Nevertheless, if one wishes to fix such a boundary condition, there should not be difference at all between positive and negative Schwarzschild mass. Therefore, it appears that we are in a position where we cannot build a spectrum of geometries *including* a naked singularity. On the other hand, the impact of this approach on the cosmological constant problem deserves further investigation.

References

- [1] Riess A G *et al* 1998 *Astron. J.* **116** 1009 (*Preprint arXiv:astro-ph/9805201*); Perlmutter S *et al* 1998 *Nature* **391** 5 (*Preprint arXiv:astro-ph/9712212*); Riess A G *et al* 1998 1999 *Astron. J.* **118** 2668 (*Preprint arXiv:astro-ph/9907038*); Perlmutter S *et al* 1999 *Astrophys. J.* **517** 565 (*Preprint arXiv:astro-ph/9812133*); Riess A G *et al* 2001 *Astrophys. J.* **560** 49 (*Preprint arXiv:astro-ph/0104455*); Tonry J L *et al* 2003 *Astrophys. J.* **594** 1 (*Preprint arXiv:astro-ph/0305008*); Knop R *et al* 2003 *Astrophys. J.* **598** 102 (*Preprint arXiv:astro-ph/0309368*); Riess A G *et al* 2004 *Astrophys. J.* **607** 665 (*Preprint arXiv:astro-ph/0402512*); Barris B *et al* 2004 *Astrophys. J.* **602** 571 (*Preprint arXiv:astro-ph/0310843*)
- [2] Capozziello S, Carloni S and Troisi A 2003 *Preprint arXiv:astro-ph/0303041*; Carroll S M, Duvvuri V, Trodden M and Turner M S 2004 *Phys. Rev. D* **70**, 043528
- [3] Faraoni V 2008 $f(R)$ gravity: successes and challenges *Preprint arXiv:gr-qc/0810.2602*
- [4] Capozziello S and Francaviglia M 2007 Extended Theories of Gravity and their Cosmological and Astrophysical Applications *Preprint arXiv:astro-ph/0706.1146*
- [5] Nojiri S and Odintsov S D 2007 *Int. J. Geom. Meth. Mod. Phys.* **4** 115 (*Preprint arXiv:hep-th/0601213*)
- [6] DeWitt B S 1967 *Phys. Rev.* **160** 1113
- [7] Garattini R 2006 *J. Phys. A* **39** 6393 (*Preprint arXiv:gr-qc/0510061*); *do.* 2006 *J. Phys.: Conf. Ser.* **33** 215 (*Preprint arXiv:gr-qc/0510062*)
- [8] Garattini R 2008 The induced Cosmological Constant as a tool for exploring geometries *Preprint arXiv:gr-qc/0812.4037*
- [9] Bondi H 1957 *Rev. Mod. Phys.* **29** 423
- [10] Penrose R 1969 *Riv. Nuovo Cim.* **252** 1; *do.* 2002 *Gen. Rel. Grav.* **34** 1141
- [11] Gibbons G W, Hartnoll S A and Ishibashi A 2005 *Prog. Theor. Phys.* **113** 963 (*Preprint arXiv:hep-th/0409307*)
- [12] Gleiser R J and Dotti G 2006 *Class. Quant. Grav.* **23** 5063 (*Preprint arXiv:gr-qc/0604021*)
- [13] Dotti G, Gleiser R J, Ranea-Sandoval I F and Vucetich H 2008 *Class. Quant. Grav.* **25** 245012 (*Preprint arXiv:gr-qc/0805.4306*); Dotti G, Gleiser R J and Pullin J 2007 *Phys. Lett. B* **644** 289 (*Preprint arXiv:gr-qc/0607052*)
- [14] Perez-Mercader J and Odintsov S D 1992 *Int. J. Mod. Phys. D* **1** 401; Cherednikov I O 2002 *Acta Phys. Slov.* **52** 221; *do.* 2004 *Acta Phys. Polon. B* **35** 1607; Bordag M, Mohideen U and Mostepanenko V M 2001 *Phys. Rep.* **353** 1. Inclusion of non-perturbative effects beyond one-loop in de Sitter Quantum Gravity have been discussed in Falkenberg S and Odintsov S D 1998 *Int. J. Mod. Phys. A* **13** 607 (*Preprint arXiv:hep-th/9612019*)
- [15] Capozziello S and Garattini R 2007 *Class. Quant. Grav.* **24** 1627 (*Preprint arXiv:gr-qc/0702075*)
- [16] Querella L 1999 Variational Principles and Cosmological Models in Higher-Order Gravity - Ph.D. Thesis (*Preprint arXiv:gr-qc/9902044*)
- [17] Cognola G, Elizalde E, Nojiri S, Odintsov S D and Zerbini S 2005 *JCAP* **0502** 10 (*Preprint arXiv:hep-th/0501096*)
- [18] Garattini R 2004 *TSPU Vestnik* **44** N7 72 (*Preprint arXiv:gr-qc/0409016*)
- [19] Abdalla M C B, Nojiri S and Odintsov S D 2005 *Class. Quant. Grav.* **22** L35 (*Preprint arXiv:hep-th/0409177*)