

On the Instability of Long-run Money Demand and the Welfare Cost of Inflation in the U.S.

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Abstract

We evaluate the policy implications of measuring the welfare cost of inflation accounting for instabilities in the long-run money demand for the U.S. over the period 1900-2013. We extend the analysis and reassess the results reported in [Lucas \(2000\)](#) and [Ireland \(2009\)](#), also considering the recent theoretical contributions of [Lucas and Nicolini \(2015\)](#) and [Berentsen et al. \(2015\)](#). Breaks in the long-run money demand give rise to regime-dependent welfare cost estimates. We find that the welfare cost is about 0.1% of annual income over 1976-2013, as compared to 0.8% over 1945-1975. Overall, these values are substantially lower than those reported in the literature.

Keywords: Money Demand, Structural Changes, Welfare Cost of Inflation

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1. INTRODUCTION

The main aim of this paper is to measure the welfare cost of inflation for the U.S. in the money demand framework developed by [Lucas \(2000\)](#) and in presence of potentially detected instabilities in the underlying money demand function. The evaluation is undertaken by mapping changes in the structural parameters of the money demand function (mainly changes in the interest-elasticity) to the measures of the welfare cost of inflation. According to [Lucas \(2000\)](#), the welfare cost of inflation can be defined as the social gain/utility obtained by reducing the steady-state nominal interest rate from a positive level to the near-zero level, as prescribed by the [Friedman \(1969\)](#)'s optimal monetary policy rule. Using U.S. data for the period 1900-1994, [Lucas \(2000\)](#) shows that the reduction of the annual inflation rate from 10% to 0% would imply a welfare gain of 1% of income. This result supports the view of strong intervention of monetary authorities targeting anti-inflationary policies. [Lucas \(2000\)](#)' contribution has generated an interesting line of theoretical research on this topic ([Simonsen and Cysne, 2001](#); [Cysne and Turchick, 2012](#)). However, empirical contributions (see, for instance, [Chadha et al., 1998](#), [Bali, 2000](#), and [Serletis and Yavari, 2004](#)) have focused on the case of stable money demand to evaluate the welfare cost in the U.S., although evidence of historical instability has been reported in the literature ([Ball, 2001](#); [Ireland, 2009](#); [Wang, 2011](#)). In particular, the instability of money demand detected at the out-turn of the 70s and the 80s has been interpreted either as changes in the economy's transaction technology ([Ball, 2001](#); [Teles and Zhou, 2005](#); [Ireland, 2009](#); [Berentsen et al., 2015](#)) or as the outcome of financial reforms and monetary policies triggered by high inflation rates ([Reynard, 2004](#); [Lucas and Nicolini, 2015](#)). In both cases, the money demand approach advocated by [Lucas \(2000\)](#), which accounts for the money demand distortion brought about by positive nominal interest rates, appears a valid instrument to analyze the welfare cost of inflation ([Ireland, 2009](#)).¹ Hence, this calls for a reconsideration of the welfare cost of inflation when the economy moves from a regime of sustained inflation to another of moderate inflation as at

the end of the 70s, or even in correspondence of a situation close to the “Friedman rule” as in most recent years, and vice-versa.

In this paper, we address these issues in our implementation of a welfare cost analysis for the U.S. Motivated by the existing literature, we estimate money demand equations using a dataset of yearly observations from 1900 to 2013.² Our contribution focuses on the selection of the best empirical model through a cointegration analysis accounting for the presence of regime changes, that we identify via the implementation of the testing procedure proposed by [Kejriwal and Perron \(2008, 2010\)](#). We estimate long-run money demand models in a single-equation framework and we provide welfare cost estimates accounting for changes in the structural parameters of the money demand function. To the best of our knowledge, this is the first contribution that measures the effect of structural instability of money demand on the welfare cost of inflation.

Our main findings are as follows. First, we find evidence of two structural breaks on the parameters of the long-run money demand relationship, located at the mid-40s and at the end of the 70s. According to our estimates, the interest-elasticity of money demand increased during the post-war from -0.1 to -0.4 , but the demand curve shifted downward and became less elastic afterwards. These results are overall consistent with those reported by [Ball \(2001\)](#) and [Ireland \(2009\)](#) on U.S. data, as well as with the prediction implied by the recent theoretical contributions of [Lucas and Nicolini \(2015\)](#) and [Berentsen et al. \(2015\)](#). Second, once regimes are accounted for, welfare cost estimates are substantially lower than those reported in the literature. For instance, [Lucas \(2000\)](#) finds a value of 1% as opposite to a value of 0.5% in this paper, where the value drops to 0.1% in most recent decades. This means that the target of moderate inflation dictated implicitly or explicitly by the Federal Reserve (FED) would have implied very limited welfare costs to the U.S. economy in latest years.

The remainder of the paper is organized as follows. In [Section 2](#), we briefly review the relevant seminal contributions on the issue of measuring the welfare cost of inflation.

We also discuss the implication of the specification of the money demand function and the computation of the welfare cost. In Section 3, we describe the dataset and we report the empirical results on the selection of the specification of the cointegrating relationship where structural breaks are accounted for. Section 4 evaluates the impact of the instability of the money demand model on the welfare cost estimates. Section 5 concludes.

2. MONEY DEMAND AND THE WELFARE COST OF INFLATION

2.1. Welfare cost measures

Let us define $m(r)$ the money demand function, r the nominal interest rate, and $\psi(m)$ the inverse demand function. In what follows, we consider the following measures of the welfare cost of inflation, based on the money demand approach developed by Bailey (1956) and Friedman (1969) and further popularized by Lucas (2000): the Bailey (1956) partial-equilibrium welfare cost formula, $B(r) = \int_{m(r)}^{m(0)} \psi(\mu) d\mu = \int_0^r -\rho m'(\rho) d\rho$, corresponding to the area under $\psi(m)$; the Lucas (2000) general-equilibrium formula derived from a simplified Sidrauski (1967) money-in-utility framework, $\bar{w}(r)$, obtained as solution to the differential equation $\bar{w}'(r) = -\psi\left(\frac{m(r)}{1+\bar{w}(r)}\right) m'(r)$, with $\bar{w}(0) = 0$; and the Lucas (2000) general-equilibrium formula derived from a McCallum and Goodfriend (1987) shopping-time framework, $s(r)$, obtained as solution to the differential equation $s'(r) = -\frac{rm'(r)(1-s(r))}{1-s(r)+rm(r)}$, with $s(0) = 0$. A detailed overview of these measures is reported in Mogliani and Urga (2017).

As shown by Simonsen and Cysne (2001) and Cysne and Turchick (2012), the welfare cost measures considered above do not have any obvious closed-form solution, but they can be conveniently arranged in an ascending order and hence approximated by a bounded interval. First, consider the welfare cost measure arising from the Sidrauski framework, $\bar{w}(r)$, which is equivalent to an increase in income necessary to leave the representative household indifferent between the current positive steady-state nominal interest rate and

the optimal policy *à la* [Friedman \(1969\)](#) ($r = 0$). Taking as reference a zero steady-state nominal interest rate, the welfare cost of inflation is equivalent to the decrease in income necessary to leave the household indifferent between the current optimal policy and $r > 0$, and the associated formula takes the form $\underline{w}(r) = 1 - e^{-\int_{m(r)}^{m(0)} \frac{\psi(\mu)}{1+\mu\psi(\mu)} d\mu}$. Second, the measure $s(r)$ can be reasonably approximated by the bounded interval $\underline{A}(r) < s(r) < \bar{A}(r)$, where $\bar{A}(r) = \int_0^r -\frac{\rho m'(\rho)}{1+\rho m(\rho)} d\rho$ and $\underline{A}(r) = 1 - e^{-\int_0^r -\frac{\rho m'(\rho)}{1+\rho m(\rho)} d\rho} = 1 - e^{-\bar{A}(r)}$. It turns out that $s(r)$ can be accurately approximated by $\bar{A}(r)$, and it follows that $\bar{A}(r) < B(r)$. Further, with $\mu \equiv m(\rho)$, we have $\bar{A}(r) = \int_{m(r)}^{m(0)} \frac{\psi(\mu)}{1+\mu\psi(\mu)} d\mu$, leading to $\underline{A}(r) = \underline{w}(r)$, *i.e.* the same measure obtained under two different theoretical frameworks. Noting that $\int_0^r \bar{w}'(\rho) d\rho > \int_0^r -\psi(m(\rho))m'(\rho) d\rho \equiv \int_0^r -\rho m'(\rho) d\rho$, for $\rho \in (0, r]$, it follows that $B(r) < \bar{w}(r)$. An inequality chain can then be formed to order the measures considered:

$$\underline{w}(r) = \underline{A}(r) < s(r) < \bar{A}(r) < B(r) < \bar{w}(r), \quad (1)$$

so that for a given r the width of the region of cost estimates is given by $\mathcal{R}(r) = \bar{w}(r) - \underline{w}(r)$ and the relative percentage difference is given by $\mathcal{D}(r) = \bar{w}(r)/\underline{w}(r) - 1$.

2.2. Money demand specification

The specification of the money demand function $m(r)$ is crucial in determining the accurate size of the welfare cost of inflation. [Lucas \(2000\)](#) and [Ireland \(2009\)](#) consider two standard competing empirical specifications, namely the semi-log and the log-log. The former, derived from the class of inventory-theoretic models ([Baumol, 1952](#); [Tobin, 1956](#); [Miller and Orr, 1966](#); [Bar-Ilan, 1990](#)), relates the natural logarithm of the money-income ratio m to the level of the nominal interest rate r ([Cagan, 1956](#)), and implies an increasing interest-elasticity of real balances, meaning that as r increases real balances converge to zero, and a finite satiation point as $r \rightarrow 0$. The latter, a direct development of the theoretical solution proposed, for instance, in [Sidrauski \(1967\)](#), [Brock \(1974\)](#) and [McCallum and Goodfriend](#)

(1987), relates the natural logarithm of m to the natural logarithm of r (Meltzer, 1963), and implies a constant interest-elasticity. Thus, the shape of the welfare cost function depends on the specification of $m(r)$: the semi-log specification implies a bounded concave (upwards to downwards) function of the interest rate, while the log-log specification implies an unbounded strictly concave function. This means that the money demand specification must be accurately chosen in order to fit the data properly and avoid miscalculation of the cost of inflation. Following Lucas (2000) and Ireland (2009), we choose the log-log specification which fits quite well the historical annual U.S. data used in the present empirical analysis (see also Bae and De Jong, 2007). With the log-log specification, the implied money demand function is $m = \exp(\alpha)r^\beta$, where α is the constant and β the interest-elasticity of money demand (expected to be negative), and the theoretical bounds of the welfare cost region defined by the inequality (1) have the following closed-form solutions:

$$\underline{w}(r) = 1 - (1 + \exp(\alpha)r^{1-|\beta|})^{\frac{|\beta|}{|\beta|-1}} \quad (2a)$$

$$\bar{w}(r) = -1 + (1 - \exp(\alpha)r^{1-|\beta|})^{\frac{|\beta|}{|\beta|-1}} \quad (2b)$$

It is worth noting that real solutions for $\bar{w}(r)$ can be obtained only for $r \in [0, \exp(\alpha)^{\frac{1}{|\beta|-1}}]$, which represents a realistic economic interval for reasonable values of α and β . It follows that the width of the region of cost estimates $\mathcal{R}(r)$ has also a bounded real solution, which is strictly increasing in r with $\mathcal{R}'(r) > 0$ and $\mathcal{R}''(r) > 0$. The relative percentage welfare cost difference $\mathcal{D}(r)$ (which, for the log-log specification, can be well approximated by $\bar{w}(r)/|\beta|$ for reasonable values of r ; see Cysne and Turchick, 2012) has also a bounded real solution and is strictly increasing in r , where $\mathcal{D}'(r) > 0$ but $\mathcal{D}''(r) \geq 0$.

In the next section we estimate a long-run money demand specification for the U.S. and we investigate the presence of long-run instabilities in a cointegrating framework. We will then make use of the relevant estimated parameters to map a correct measure of the welfare cost of inflation.

3. MONEY DEMAND FOR THE U.S.: DATA AND EMPIRICAL ESTIMATES

3.1. *The dataset*

For the empirical analysis, we extended to 2013 the dataset used by [Ireland \(2009\)](#), which in turn is closely comparable to that of [Lucas \(2000\)](#). We have $T = 114$ annual observations spanning from 1900 to 2013 for money, income and interest rates. Money is measured in terms of M1, which includes mainly currency held by the public, non-interest-bearing demand deposits, and, since 1980, interest-bearing Negotiable Order of Withdrawal (NOW) accounts. Further, we follow the recent literature ([Ireland, 2009](#); [Berentsen et al., 2015](#)) and we consider a retail sweep adjusted measure of money from 1994 onward, in order to avoid a downward estimate of M1 consistent with the introduction of retail deposit sweep programs ([Dutkowsky and Cynamon, 2003](#)). Income is measured in terms of nominal GDP, computed as the real GDP multiplied by the series of implicit deflators for GNP (from 1900 to 1928) and GDP (from 1929 onward). Finally, the interest rate series is constructed using data on the six-month commercial paper rate (from 1900 to 1997) and the three-month AA nonfinancial commercial paper rate (from 1998 onward), due to a discontinuity in the statistical publication of the former. The data sources are broadly the same as in [Ireland \(2009\)](#), and we hence refer the reader to that contribution for further details.

3.2. *Searching for cointegration: long-run instability and structural changes*

A cointegrating relationship between $\ln(m)$ and $\ln(r)$ is estimated using the Dynamic OLS estimator ([Saikkonen, 1991](#); [Stock and Watson, 1993](#)) to account for potential endogeneity of the interest rate, with the number of leads and lags ($\ell_T = 4$) set consistently with the upper bound condition implied by the data-dependent rule suggested by [Saikkonen \(1991\)](#), *i.e.* $\ell_T < T^{1/3} \approx 5$ for $T = 114$.³ Heteroskedasticity-autocorrelation robust standard errors are

obtained through the Bartlett kernel and the Newey-West truncated automatic bandwidth selection method (Newey and West, 1994). The estimated equation is (standard errors in parenthesis; leads-and-lags omitted):

$$\ln(m_t) = -2.62 - 0.35 \ln(r_t) + \hat{\epsilon}_t \quad (3)$$

(0.23) (0.06)

where the interest-elasticity of the money-income ratio is close to, although somewhat below, the result of -0.5 consistent with a Baumol-Tobin transaction technology and reported by Meltzer (1963) and Lucas (2000). However, a formal test of cointegration based on $\hat{\epsilon}_t$ (Shin, 1994) strongly rejects the null hypothesis of stationary residuals (at 1% significance level), suggesting that the estimated relationship does not cointegrate.⁴ The first important implication for the subsequent analysis is that welfare cost results, as reported by Lucas (2000) and then discussed by Ireland (2009), might be contaminated by the inconsistencies arising from the long-run relationship between $\ln(m)$ and $\ln(r)$ which does not cointegrate.

In contrast with the literature published in the 80s and the 90s (Lucas, 1988; Hoffman and Rasche, 1991; Stock and Watson, 1993), recent empirical studies have pointed out the presence of structural instability in the money demand parameters for the U.S., especially when the estimation sample includes data from the 90s onward (Ball, 2001; Teles and Zhou, 2005; Wang, 2011). This intuition is confirmed by inspecting Figure 1, which plots long-run money demand curves obtained fitting the data with parameters both used by Lucas (2000) and estimated using (3): most of the observations from the 80s onward (see Ireland, 2009), as well as a large number of points located at the center of the plot, seem consistent with different theoretical curves.

Thus, a natural follow up of the above results is to test for cointegration in presence of structural changes. To this purpose, we implement the Kejriwal and Perron (2008, 2010) testing procedure ($\sup F$ and $UD \max F$ tests) and we allow both the intercept and the interest-elasticity to change over time/alternative regimes, where the break dates are sequentially estimated via a dynamic programming algorithm (Bai and Perron, 2003). We

find robust evidence of $n = 2$ structural breaks (*i.e.* $n + 1 = 3$ regimes), at 1945 and 1976 (details on both the implemented approach and the results are reported in [Mogliani and Urga, 2017](#)). Interestingly, these findings are broadly consistent with those reported by [Ball \(2001\)](#), who identifies a post-war and a post-82 regimes.⁵

Equation (4) reports the estimated cointegrating Dynamic OLS regression (3) with the selected breaks included (standard errors in parenthesis; leads-and-lags omitted):

$$\ln(m_t) = \underset{(0.07)}{-1.64}\iota_{t_1} - \underset{(0.02)}{0.13}\ln(r_{t_1}) - \underset{(0.06)}{2.83}\iota_{t_2} - \underset{(0.02)}{0.43}\ln(r_{t_2}) - \underset{(0.06)}{2.24}\iota_{t_3} - \underset{(0.02)}{0.11}\ln(r_{t_3}) + \hat{\epsilon}_t, \quad (4)$$

where ι_{t_i} is a regime-dependent intercept, $t_i = T_{i-1} < t \leq T_i$, with $i = 1, \dots, n + 1$, and by convention $T_0 = 0$ and $T_{n+1} = T$. The results suggest that the interest-elasticity of the money-income ratio increased from -0.13 in the pre-war period to -0.43 up to the 80s, and then decreased back again to a low -0.11 in the last part of the sample.⁶ Further, a formal test of cointegration with breaks ([Carrion-i-Silvestre and Sansò, 2006](#); [Arai and Kurozumi, 2007](#)) does not reject the null of stationary residuals at any standard significance level when exact critical values are used (at 10% level with asymptotic critical values), after controlling for initial conditions in the residuals vector. Figure 2 plots the long-run money demand curves obtained by fitting the data with parameters estimated from our structural breaks regression, and it provides strong evidence of the downward shift of the money demand curve in the last regime. As expected, our findings on the interest-elasticity for the pre-war period are in line with those reported, for instance, by [Stock and Watson \(1993\)](#) and [Ball \(2001\)](#). The estimated elasticity for the third regime is also close to that reported by [Ireland \(2009\)](#), although his specification involves quarterly data spanning from 1980 to 2006. The higher elasticity observed in the post-war period up to the mid-70s is a novel result, reflecting the transition from low to high velocity of money driven by changes in the transaction technology such as the creation of near-monies instruments, which is in turn consistent with a rise in the degree of substitution between real balances and alternative assets.⁷ Furthermore, our results

seem also consistent with the predictions implied by the recent theoretical contributions of [Lucas and Nicolini \(2015\)](#) and [Berentsen et al. \(2015\)](#).⁸

4. INSTABILITY IN MONEY DEMAND AND WELFARE COST ESTIMATES FOR THE U.S.

We provided robust econometric evidence of an unstable money demand specification attributed to changes in the structural parameters of the long-run relationship between real balances and interest rates. An interesting but completely unexplored field is represented by the policy implications of measuring the welfare cost of inflation in presence of instabilities in the money demand function. Thus, we now turn to the evaluation of the welfare cost of inflation for the U.S. using the theoretical bounds of the welfare cost region represented by $\underline{w}(r)$ and $\bar{w}(r)$, which can be computed from closed-form solutions (2a) and (2b) and from the estimated calibration parameters ($\hat{\alpha}$ and $\hat{\beta}$). Moreover, it is reasonable to account for the uncertainty affecting the estimated parameters when computing welfare cost estimates. To this purpose, we consider the confidence region of $\underline{w}(r)$ and $\bar{w}(r)$ using the 90% level confidence values for $\hat{\alpha}$ and $\hat{\beta}$. This requires the construction of confidence intervals using the Bonferroni inequality, *i.e.* $\tilde{\alpha} = \hat{\alpha} \pm \mathcal{B}\sigma_{\alpha}$ and $\tilde{\beta} = \hat{\beta} \pm \mathcal{B}\sigma_{\beta}$ with \mathcal{B} the Bonferroni multiple and σ the standard error. Hence, lower and upper bound analogs of $\underline{w}(r)$ and $\bar{w}(r)$, that is $\underline{w}(r)^-$ and $\bar{w}(r)^+$, are computed using these joint confidence bounds.

The results are compared to the benchmark values provided by the welfare cost estimates reported by [Lucas \(2000\)](#) and [Ireland \(2009\)](#). Moreover, in this paper we extend their analysis by computing intervals for welfare cost estimates, based on both historical values for nominal and real interest rates and alternative counterfactual scenarios. We compute sample-specific averages of the nominal interest rate (\bar{r}), the inflation rate ($\bar{\pi}$) and the implicit real interest rate ($\bar{\rho}$). Results are reported in Panel A of Table 1. According to the values of the calibration parameters reported by [Lucas \(2000\)](#), average interest and inflation rates computed over the

sample 1900-1994 (4.6% and 3.1%, respectively, leading to $\bar{\rho} = 1.5\%$) imply an average cost of inflation of about 1.0 – 1.1% of income. This value is very close to what we would obtain if we assume that the steady-state real interest rate (ρ_{ss}) ranges between 3% and 5% under a policy of price stability. Under a policy of positive inflation matching the average inflation rate, the cost of positive nominal rates (6.1% and 8.1%, respectively) would range instead between 1.2% and 1.4% of income. Finally, assuming $\rho_{ss} = 3\%$ would imply a cost of 1.1% of income for a policy of 2% inflation, and 1.7 – 1.8% of income for a policy of 10% inflation. All in all, the results based on the calibration reported by Lucas (2000) suggest that the welfare cost of inflation for the U.S. should range between 1% and 2% of GDP.

We now turn to welfare cost estimates based on the cointegrating regression performed over the sample 1900-2013 and the calibration parameters reported in (3). The results, reported in Panel B of Table 1, suggest the welfare cost of inflation for the U.S. should range between 0.3% and 1.4% of GDP. Hence, cost estimates based on regression (3) are fairly low compared to those reported in Panel A of Table 1. These are very important findings that suggest a different quantitative interpretation, with respect to the estimates reported by Lucas (2000), of the welfare gain implied by the Friedman rule for the U.S. We thus recommend at a first stance a downward revision of those benchmark estimates.

4.1. Welfare cost of inflation and regime changes

Let us now consider welfare cost estimates based on the cointegrating regression with regime changes performed over the sample 1900-2013 and the calibration parameters reported in (4). Results are reported in Table 2. Average interest rates do not differ substantially in the first two regimes (3.7% in 1900-1944 and 3.9% in 1945-1975), but the high policy rates observed by the end of the 70s and during the first-half of the 80s leads to a somewhat higher average in the last regime (5.5%). It is worth noting that latest years have been characterized by historically low interest rates, consistently with the easing policy set by the FED in the

aftermath of the Great Recession episode. Inflation is about 1.9% in the first regime, but almost doubled in the second and third regimes (3.8% and 3.3%, respectively). The low average inflation observed in the first regime is mainly due to a few deflationary episodes in the 20s and the 30s. On the other hand, recent years have been characterized by a moderate inflation (around 2%) consistent with the implicit (and explicit since 2012) target of the FED (Goodfriend, 2004). It follows that the average real interest rate is around 2% in both the first and last regime (1.8% and 2.2%), but it is very low in the intermediate regime (0.1%). The results suggest that the implied cost of inflation is about 0.1 – 0.2% in the first and third regimes, and 0.6 – 0.8% in the second regime. Assuming a steady-state real interest rate ranging between 3% and 5%, a policy of price stability would cost the economy about 0.1 – 0.2% of income in the first and third regimes, and about 0.5 – 0.9% of income in the second regime. Under a policy of positive inflation matching $\bar{\pi}$, the cost of positive nominal rates would be about 0.2 – 0.4% of income in the first regime, 0.8 – 1.2% of income in the second regime, and 0.1 – 0.2% in the third regime. Finally, assuming $\rho_{ss} = 3\%$, a policy of 2% inflation would imply a cost of 0.2 – 0.3% of income in the first regime, 0.7 – 0.9% in the second regime, and 0.1% in the third regime. A policy of 10% inflation would cost the economy 0.4 – 0.6% of income in the first regime, 1.2 – 1.5% in the second regime, and 0.1 – 0.3% in the third regime. These findings lead to several interesting conclusions.

First, the size of the cost of inflation for the first and third regimes is broadly comparable across scenarios, which means that the two regimes share some long-term equilibrium features. This is likely related to the fact that the money demand function displays an interest-elasticity which is virtually the same in these two regimes. Of course, striking differences arise from the level of money-income ratio, which is around 30% on average during the pre-war period and only 15% from the late 70s onward. This means that for high interest rates (not considered in Table 2) welfare cost estimates are nevertheless expected to diverge. Second, for moderate interest rates our welfare cost estimates are overall substantially lower than those reported by Lucas (2000). Infrequently exceeding 1%, they rather float mostly

around 0.4 – 0.5%, dropping to 0.1% in most recent decades, suggesting that the priors on the welfare gain implied by the Friedman rule for the U.S. might be substantially revised downward. Third, from an approximate decomposition, we can calculate the contribution of changes in the interest-elasticity of money demand to changes in the welfare cost across regimes. This amounts to about 60% of the (positive) change from the first to the second regime, and about 90% of the (negative) change from the second to the third regime. Fourth, the policy of 2% inflation, dictated implicitly or explicitly in the last two decades by the FED, seems to imply limited welfare costs to the economy: between 0.05% and 0.1%, depending on the assumed steady-state real interest rate. However, after almost three decades of sustained real rates, the monetary policy response to the Great Recession drove nominal interest rates to very low territory, while inflation kept around 1.5%. Thus, the economy has been facing negative real rates since 2009. This policy being not sustainable in the long-run, it is reasonable to expect nominal rates to rise again in the next years. Finally, compared to the literature, our findings are interestingly close to the quantitative results reported by [Ireland \(2009\)](#) and [Calza and Zaghini \(2011\)](#) on the post-80s period, as well to estimates obtained from calibration of theoretical models reported by [Cooley and Hansen \(1991\)](#), [Faig and Jerez \(2007\)](#), and [Berentsen et al. \(2015\)](#), among others. However, they are overall below the estimates reported in [Fischer \(1981\)](#), [Lucas \(1981\)](#), [Craig and Rocheteau \(2008\)](#), and [Gupta and Majumdar \(2014\)](#).

4.2. Welfare cost of inflation in presence of interest-bearing assets

As mentioned in Section 3, technological innovations and new regulations have increased the liquidity of interest-bearing deposits in the last decades. Thus, in an economy characterized by the presence of these financial technologies, the welfare cost measures presented in Section 2 may be misleading, because they do not account for the existence of a possible trade-off between more liquid non interest-bearing and less liquid interest-bearing monies

(Bali, 2000; Cysne, 2003; Cysne and Turchick, 2010, 2012). Neglecting the existence of interest-bearing monetary assets may hence result in a bias in the evaluation of the welfare costs of inflation.

In this section, we provide an evaluation of this bias by implementing the approach described by Cysne and Turchick (2010). For ease of exposition, we limit our analysis to the simple case of two groups of monetary assets, non-interest and interest bearing. We consider a Cobb-Douglas monetary-aggregator technology, with unit constant elasticity of substitution between the assets. Further, we assume that the interest-elasticity of the demand for non-interest bearing assets and the elasticity of substitution between the monetary assets and the consumption good (ν) in the utility function of the household are both less than 1. These assumptions are quite realistic and not unusual in theoretical monetary models, as they jointly imply a positive interest-elasticity of the demand for interest bearing assets. It is worth noting that these assumptions also imply that the unidimensional measures of the welfare costs of inflation are expected to be biased upward in presence of interest bearing monetary assets. Accordingly, considering the Bailey (1956)'s measure of the welfare cost of inflation, the bias takes the following form:

$$\Omega(r) = \frac{(1 - \nu)(1 - \theta)}{\nu} > 0 \tag{5}$$

where θ is the relative share of the non-interest bearing asset in the Cobb-Douglas monetary-aggregator. We evaluate the bias $\Omega(r)$ by building on the empirical unidimensional results obtained for the last regime estimated in our sample, which is consistent with the presence in the economy of monetary assets used for transaction purposes, beyond currency, paying different interest rates. Further, we consider the benchmark case of a real interest rate at 3% and a policy of 2% inflation, implying a nominal interest rate at 5%, which is fairly close to actual average observations for the last regime, as reported in Table 2.

According to the unidimensional Bailey (1956)'s measure, we evaluate the welfare cost of inflation to 0.10% of output for the last regime, for simplicity neglecting parameters uncer-

tainty. When we consider the bidimensional framework described above, the overestimation bias $\Omega(r)$ is in the range of 140% and 13%, for combinations of reasonable values for ν and θ in the range of 0.3 and 0.7. Accordingly, the “unbiased” welfare cost of inflation would range between 0.04% and 0.09%. These results are very close to those reported in Panel C of Table 2, representing an additional evidence of the low welfare cost of inflation identified for the last regime, as commented in Section 4.1.

5. CONCLUSIONS

In this paper, we evaluated the policy implications of measuring the welfare cost of inflation accounting for instabilities in the long-run money demand for the U.S. over the period 1900-2013. We extended the analysis and reassessed the results reported in Lucas (2000) and Ireland (2009), also in the light of the recent contributions by Lucas and Nicolini (2015) and Berentsen et al. (2015).

We estimated a long-run money demand specification that cointegrates only when breaks are accounted for. We then evaluated the costs to the economy of inflationary policies under the assumption of regime changes and we found out that the existing empirical evaluations, based on likely misspecified money demand models, tend to overestimate the welfare cost of inflation. In particular, we found evidence of two statistically significant structural breaks (in 1945 and 1976) affecting the long-run money demand relationship. According to our estimates, the interest-elasticity of money demand increased during the post-war from -0.1 to -0.4 , but the demand curve shifted downward and became less elastic afterwards. These results are consistent with those reported by Ball (2001) and Ireland (2009) on U.S. data, as well as with the prediction implied by the recent theoretical contributions of Lucas and Nicolini (2015) and Berentsen et al. (2015). Once regimes are accounted for, welfare cost estimates appear substantially lower than those reported, for instance, in Lucas (2000): usually around 0.5%, but only up to 0.1% in most recent decades. This means that the

target of moderate inflation dictated implicitly or explicitly by the FED would have implied very limited welfare costs to the U.S. economy in latest years.

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Notes

¹It is worth noting that the latter interpretation could imply an underestimate of the cost to the post-1980 U.S. economy. According to [Dotsey and Ireland \(1996\)](#), in general equilibrium, the inflation tax distorts a variety of marginal decisions, such as the holding of real cash balances and the allocation of productive resources, which are small taken individually but yield to fairly large welfare cost estimates when combined. Thus, [Ireland \(2009\)](#) argues that if these inefficiencies remain present in the post-1980 U.S. economy, the welfare cost could be underestimated by the measures considered in the present paper. However, according to [Cysne \(2003\)](#) and [Cysne and Turchick \(2010\)](#), the presence in the economy of monetary assets used for transaction purposes, beyond currency, paying different interest rates, may instead lead to an overestimate of the welfare cost of inflation (see [Section 4.2](#)). We thank the referees for raising this point.

²The dataset cannot be updated to more recent years, as in 2012 the Board of Governors of the Federal Reserve System discontinued the publication of the retail deposit sweeps data (last observation available: December 2013), which enter the monetary aggregate used in this paper (sweep-adjusted M1).

³Preliminary analysis, performed through a battery of unit-root tests ([Ng and Perron, 2001](#)), confirms that the null hypothesis of unit-root cannot be rejected for both series. See details in [Mogliani and Urga \(2017\)](#).

⁴Similar conclusions are obtained by restricting the sample to the period 1900-1994, the same time span considered by [Lucas \(2000\)](#). The interest-elasticity is estimated to -0.4 , but regression residuals confirm no cointegration. Further, we tested whether the estimated parameters reported in [\(3\)](#) are statistically different from those reported by [Lucas \(2000\)](#). From a Wald test on the joint hypothesis that $\alpha_0 = -3.02$ and $\beta_0 = -0.5$, we can reject the null at less than 1% level. When the hypothesis on the interest-elasticity is tested alone, we can reject the null at 1% level.

⁵[Carlson et al. \(2000\)](#) find a stable long-run money demand, based upon monthly data of the Money-zero-maturity aggregate ([Motley, 1988](#); [Poole, 1991](#)) over 1964-1998, only when the estimation sample is restricted to start in 1976.

⁶For the second regime, the estimated coefficients are close to those reported by [Lucas \(2000\)](#). However, we again reject at less than 1% level both the joint null hypothesis $\alpha_0 = -3.02$ and $\beta_0 = -0.5$ and the simple hypothesis on the interest-elasticity. Thus, we can conclude that in our long-run analysis (with or without breaks) there is no statistical evidence in favor of the Baumol-Tobin transaction technology advocated by [Meltzer \(1963\)](#) and [Lucas \(2000\)](#).

⁷A downward trend in the velocity of money is often explained by a decrease in the income-elasticity of real balances from unity to 0.5. [Lucas \(2000\)](#) suggests that a technical change in the provision of transactions services would produce a downward trend in the money-income ratio.

⁸[Lucas and Nicolini \(2015\)](#) refer explicitly to the regulatory changes on the banking sector implied by Regulation Q, in force from 1933 to 2011, which explicitly banned interest payments on checkable deposits. The authors find that the interest-elasticity of real money balances in the regulated economy is higher than in the free-market economy. [Berentsen et al. \(2015\)](#) suggest instead that the introduction of retail deposit sweep programs in the first half of the 90s reduces the interest-elasticity (agents earn higher rates on their idle balances) and shifts downward the demand curve (the money stock is allocated more efficiently).

Table 1: Welfare cost estimates: no structural changes models

| Panel A. Lucas (2000) money demand parameters | | | | | | |
|---|------------|------------|--------------------|--------------|------------------|------------------|
| ρ_{ss} | r | π | $\underline{w}(r)$ | $\bar{w}(r)$ | $\mathcal{R}(r)$ | $\mathcal{D}(r)$ |
| 3.0 | 3.0 | 0.0 | 0.84 | 0.85 | 0.01 | 1.70 |
| 5.0 | 5.0 | 0.0 | 1.08 | 1.10 | 0.02 | 2.21 |
| 1.5 | <i>4.6</i> | <i>3.1</i> | 1.03 | 1.06 | 0.02 | 2.11 |
| 3.0 | 6.1 | <i>3.1</i> | 1.19 | 1.22 | 0.03 | 2.43 |
| 5.0 | 8.1 | <i>3.1</i> | 1.37 | 1.41 | 0.01 | 2.81 |
| 3.0 | 5.0 | 2.0 | 1.08 | 1.10 | 0.02 | 2.21 |
| 3.0 | 13.0 | 10.0 | 1.73 | 1.79 | 0.06 | 3.58 |

| Panel B. Regression (3) estimated parameters | | | | | | |
|--|------------|------------|----------------------|----------------|------------------|------------------|
| ρ_{ss} | r | π | $\underline{w}(r)^-$ | $\bar{w}(r)^+$ | $\mathcal{R}(r)$ | $\mathcal{D}(r)$ |
| 3.0 | 3.0 | 0.0 | 0.25 | 0.62 | 0.37 | 152.33 |
| 5.0 | 5.0 | 0.0 | 0.36 | 0.82 | 0.45 | 125.02 |
| 1.4 | <i>4.3</i> | <i>2.9</i> | 0.32 | 0.75 | 0.43 | 132.39 |
| 3.0 | 5.9 | <i>2.9</i> | 0.41 | 0.89 | 0.48 | 117.08 |
| 5.0 | 7.9 | <i>2.9</i> | 0.51 | 1.04 | 0.53 | 103.52 |
| 3.0 | 5.0 | 2.0 | 0.36 | 0.82 | 0.45 | 125.02 |
| 3.0 | 13.0 | 10.0 | 0.75 | 1.36 | 0.62 | 82.63 |

Notes: values reported are expressed in percentage points. Values in italic denote the empirical average over the estimation period.

Table 2: Welfare cost estimates: structural changes model

| Panel A. 1900-1944 regime | | | | | | |
|---------------------------|------|-------|----------------------|----------------|------------------|------------------|
| ρ_{ss} | r | π | $\underline{w}(r)^-$ | $\bar{w}(r)^+$ | $\mathcal{R}(r)$ | $\mathcal{D}(r)$ |
| 3.0 | 3.0 | 0.0 | 0.10 | 0.18 | 0.08 | 76.57 |
| 5.0 | 5.0 | 0.0 | 0.16 | 0.28 | 0.12 | 71.61 |
| 1.8 | 3.7 | 1.9 | 0.12 | 0.22 | 0.09 | 74.59 |
| 3.0 | 4.9 | 1.9 | 0.16 | 0.27 | 0.11 | 71.84 |
| 5.0 | 6.9 | 1.9 | 0.22 | 0.37 | 0.15 | 68.85 |
| 3.0 | 5.0 | 2.0 | 0.16 | 0.28 | 0.12 | 71.61 |
| 3.0 | 13.0 | 10.0 | 0.38 | 0.63 | 0.25 | 64.38 |

| Panel B. 1945-1975 regime | | | | | | |
|---------------------------|------|-------|----------------------|----------------|------------------|------------------|
| ρ_{ss} | r | π | $\underline{w}(r)^-$ | $\bar{w}(r)^+$ | $\mathcal{R}(r)$ | $\mathcal{D}(r)$ |
| 3.0 | 3.0 | 0.0 | 0.51 | 0.69 | 0.18 | 34.61 |
| 5.0 | 5.0 | 0.0 | 0.70 | 0.91 | 0.21 | 30.54 |
| 0.1 | 3.9 | 3.8 | 0.60 | 0.79 | 0.19 | 32.57 |
| 3.0 | 6.8 | 3.8 | 0.84 | 1.08 | 0.24 | 28.26 |
| 5.0 | 8.8 | 3.8 | 0.98 | 1.24 | 0.26 | 26.43 |
| 3.0 | 5.0 | 2.0 | 0.70 | 0.91 | 0.21 | 30.54 |
| 3.0 | 13.0 | 10.0 | 1.24 | 1.53 | 0.30 | 23.85 |

| Panel C. 1976-2013 regime | | | | | | |
|---------------------------|------|-------|----------------------|----------------|------------------|------------------|
| ρ_{ss} | r | π | $\underline{w}(r)^-$ | $\bar{w}(r)^+$ | $\mathcal{R}(r)$ | $\mathcal{D}(r)$ |
| 3.0 | 3.0 | 0.0 | 0.04 | 0.09 | 0.05 | 146.41 |
| 5.0 | 5.0 | 0.0 | 0.06 | 0.14 | 0.08 | 137.15 |
| 2.2 | 5.5 | 3.3 | 0.06 | 0.15 | 0.09 | 135.45 |
| 3.0 | 6.3 | 3.3 | 0.07 | 0.17 | 0.10 | 133.21 |
| 5.0 | 8.3 | 3.3 | 0.09 | 0.22 | 0.12 | 128.71 |
| 3.0 | 5.0 | 2.0 | 0.06 | 0.14 | 0.08 | 137.15 |
| 3.0 | 13.0 | 10.0 | 0.14 | 0.32 | 0.17 | 121.98 |

Notes: See Table 1.

Figure 1: U.S. money demand, 1900-2013: Lucas (2000) and regression (3)

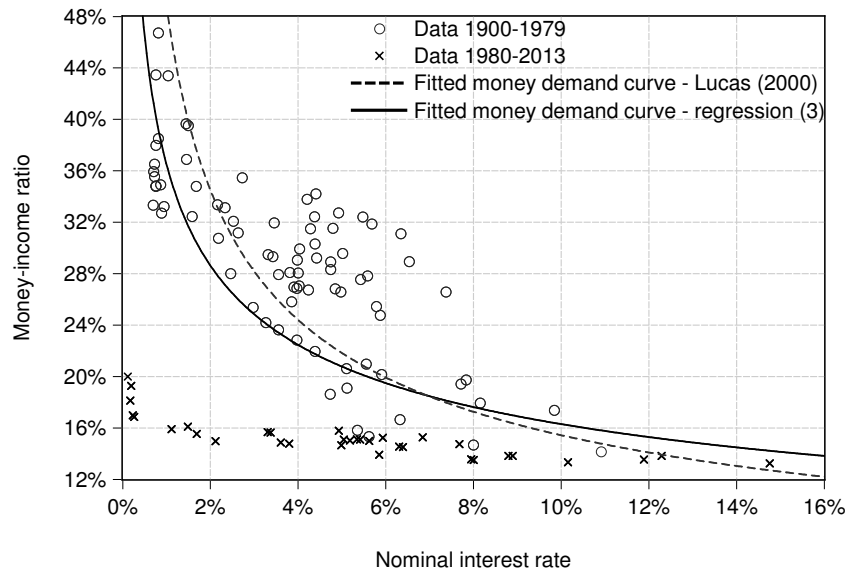


Figure 2: U.S. money demand, 1900-2013: regression (4) with breaks in 1945 and 1976

