A two-stage stochastic model for distribution logistics with transshipment and backordering: stochastic vs deterministic solutions

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Abstract We present a two-stage stochastic program for a distribution logistic system with transshipment and backordering under stochastic demand and we first argue that it is NP-hard. Then, we perform a computational analysis based on a distribution network. In the case with two retailers, we show that modeling uncertainty with a stochastic program leads to better solutions with respect to the ones provided by the deterministic program, especially if limited recourse actions are admitted. Although there are special cases in which the deterministic and the stochastic solutions select the same retailers towards which sending items, in general, the deterministic solution cannot be upgraded in order to find the optimal solution of the stochastic program. Finally, in the case with four retailers, transshipment can provide more flexibility and better results.

Keywords: Optimization under Uncertainty, Transshipment, Backordering, Stochastic solution analysis

1 Introduction

In recent years, competition pressure has increased and logistics has become more and more crucial for the success of companies due to its impact on costs and service levels. An efficient distribution system is fundamental to satisfy customers' requests with reduced lead times and with a good service level. Traditionally, the distribution network is organized as a hierarchical process in which the flow of goods is shipped

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from the uppermost level of the distribution chain to the lowest. One of the purposes of this paper is to study a more flexible distribution network, where the shipment of products between locations at the same level of the distribution system is admitted. This strategy is called *transshipment* and it allows companies to reduce stock out risks, to share surplus stocks and to improve warehouses management, coping with demand uncertainty.

Based on the inventory system, ordering and transshipment characteristics, [12] present a complete review of the transshipment literature. Examples of stochastic transshipment problems are [5], where fixed replenishment costs are taken into account, while [11] considers the unidirectional transshipment problem, where locations have different backordering and stockout costs. Backordering is not considered in [15], while [16] studies the multi-location transshipment problem including lead times. Finally, [14] proposes a stochastic transshipment model for humanitarian emergencies.

Our contribution is to provide insights about the importance of considering uncertainty in a distribution system with transshipment and backordering.

The remainder of the paper is organized as follows. Sect. 2 presents the problem description and formulation. Sect. 3 shows our computational results and, finally, in Sect. 4, conclusions and research perspectives are outlined.

2 Problem Description and Formulation

The analyzed problem deals with a *single echelon* distribution system composed of a single supplier and a set $\mathscr{I}$ of M retailers with a *centralized decision making*. Transshipment is admitted and, in order to keep track of the origin and destination of product flows, we represent retailers performing transshipment by index i and retailers receiving transshipped quantities by index j ($i \in \mathscr{I}, j \in \mathscr{I}$). In this problem transshipment is *intra-level* (since it involves only retailers), *bi-directional* (each retailer can both transship products to other retailers and receive products from them) and *reactive* (it is performed in emergency situations, after demand realization). We deal with a *single product complete pooling* transshipment (retailer i can not keep any inventory quantity if retailer j has a shortage of product), where the *priority principle* is respected (each retailer satisfies its demand at first and then transshipment is performed if necessary), backordering to supplier is allowed and, consequently, the demand can potentially be covered with supplied quantities, with transshipment quantities and with backordered quantities. The unsatisfied demand represents a lost sale. Since retailers are supposed to be close to each other, lead times are considered negligible. Our problem is described on two time intervals: t_0, which represents the time at which we have to take the decision about the quantities to ship from the supplier to retailers and t_1, in which, after demand realization, we decide the quantities to transship and the quantities to backorder.

Moreover, the problem is characterized by risk presence: the demand is a phenomenon which can not be exactly forecast, but it is stochastic. We denote by d all possible values for the demand, that is a random variable having discrete (mutually indipendent) probability distributions $\mathscr{D}_i$, defined over the support $\mathscr{U}_1 = \{\underline{d}, \ldots, \overline{d}\}$, where $0 < \underline{d} \leq \overline{d}$. Furthermore, we represent by $\mathscr{S}$ the set of scenarios s, $s = 1, \ldots, S$ and by pr^s the probability of each scenario $s \in \mathscr{S}$, so that d_i^s denotes the demand realization for retailer i in scenario s. The measure adopted to evaluate the system performance is the total expected cost.

At time t_0, the decision variables of this model are x_i, which represent the decisions to take at the first stage, i.e. the quantity to ship from the supplier to each retailer i, taking into account the supplier's total inventory availability q and the associated unit inventory cost h_0. We introduce a capacity C_i for each vehicle employed in the shipment of units from the supplier to retailer i and an integer variable v_i, standing for the number of total vehicles used to serve retailer i by direct shipping. The transportation cost between the supplier and each retailer is represented by a variable cost f_i, proportional to the number of shipped units and by a fixed component F_i, paid for each vehicle used.

If retailer j has to face a demand d_j^s greater than the initial inventory level $\overline{I}_{i0}$ plus the quantity x_i received from the supplier, transshipment and/or backordering can be used to avoid stock-out. Thus, at t_1 the decision variables are represented by y_{ij}^s which stand for the quantity to transship from retailer i to retailer j, for each possible scenario s, after the demand realization d_i^s and by b_i^s which represent the quantity to backorder from the supplier for each retailer and for each possible scenario s, after demand realization d_i^s. On one hand, we introduce a capacity C^T for vehicles used to transship units (note that the capacity of vehicles used to ship units from supplier to retailers is typically bigger than the capacity of vehicles used for transshipment) and integer variables V_{ij}^s representing the number of vehicles employed for transshipment from retailer i to retailer j for each scenario s. The total transshipment cost is composed of a unit cost t_{ij} for each transshipped unit and a fixed cost T_{ij} for each vehicle used. On the other hand, backordering is done by using vehicles with the same capacity C_i of vehicles used for the shipment from the supplier to retailer i and we represent the number of vehicles used for backordering with the variables r_i^s. The total backordering cost is composed of a unit backordering cost g_i for each backordered unit and a fixed cost G_i for each vehicle used. Finally, the variables I_i^s represent the balance quantity at each retailer i for each scenario s and they are given by the sum of the initial inventory level $\overline{I}_{i0}$ plus the quantity received from the supplier, the quantity received through transshipment and through backordering minus the sum of the customers' demand and of the transshipped units. If this quantity is positive, it stands for the inventory level and the associated unit cost is represented by h_i. If the quantity is negative, then the balance quantity stands for the stock-out quantity and retailer j has to pay a unit penality cost p_j. In particular, if the product surplus at retailer i is transshipped to retailer j, but it is not sufficient to fully cover the shortage of product of retailer j, and no quantities are backordered, retailer i has neither inventory nor stock-out costs, while retailer j has to face stock-out costs for the unsatisfied demand. We also consider the warehouse capacity Q_i for each retailer i.

Consequently, we formulate the following integer non linear two stage stochastic programming model.

Model $\mathscr{T}$

$$\begin{aligned} \min \; & h_0(q-\sum_{i\in\mathscr{I}} x_i)+\sum_{i\in\mathscr{I}}(f_i x_i+F_i v_i)+ \\ & +\sum_{s\in\mathscr{S}} pr^s[h_0(q-\sum_{i\in\mathscr{I}} x_i-\sum_{i\in\mathscr{I}} b_i^s)+\sum_{i\in\mathscr{I}}(g_i b_i^s+G_i r_i^s)+ \\ & +\sum_{i\in\mathscr{I}} h_i \max\{I_i^s,0\}+\sum_{i\in\mathscr{I}}\sum_{j\in\mathscr{I}:i\neq j}(t_{ij}y_{ij}^s+T_{ij}V_{ij}^s)-\sum_{j\in\mathscr{I}} p_j \min\{I_j^s,0\}] \end{aligned} \quad (1)$$

s.t.

$$\sum_{i\in\mathscr{I}}(x_i+b_i^s)\leq q \qquad s\in\mathscr{S} \quad (2)$$

$$I_i^s=\bar{I}_{i0}+x_i+b_i^s-d_i^s+\sum_{j\in\mathscr{I}:i\neq j}(y_{ji}^s-y_{ij}^s) \qquad i\in\mathscr{I},\ s\in\mathscr{S} \quad (3)$$

$$I_i^s\leq Q_i \qquad i\in\mathscr{I},\ s\in\mathscr{S} \quad (4)$$

$$x_i\leq C_i v_i \qquad i\in\mathscr{I} \quad (5)$$

$$b_i^s\leq C_i r_i^s \qquad i\in\mathscr{I},\ s\in\mathscr{S} \quad (6)$$

$$y_{ij}^s\leq C^T V_{ij}^s \qquad i\in\mathscr{I},\ j\in\mathscr{I}:j\neq i,\ s\in\mathscr{S} \quad (7)$$

$$x_i\geq 0\ integer \qquad i\in\mathscr{I} \quad (8)$$

$$y_{ij}^s\geq 0\ integer \qquad i\in\mathscr{I},\ j\in\mathscr{I}:j\neq i,\ s\in\mathscr{S} \quad (9)$$

$$b_i^s\geq 0\ integer \qquad i\in\mathscr{I},\ s\in\mathscr{S} \quad (10)$$

$$v_i\geq 0\ integer \qquad i\in\mathscr{I} \quad (11)$$

$$r_i^s\geq 0\ integer \qquad i\in\mathscr{I},\ s\in\mathscr{S} \quad (12)$$

$$V_{ij}^s\geq 0\ integer \qquad i\in\mathscr{I},\ j\in\mathscr{I}:j\neq i,\ s\in\mathscr{S} \quad (13)$$

$$I_i^s\ free \qquad i\in\mathscr{I},\ s\in\mathscr{S} \quad (14)$$

where the objective function (1) represents the minimization of the total expected cost, obtained through the sum of the supplier's inventory cost, the total shipment costs from supplier to retailers, the expected supplier's inventory costs, the total expected backordering cost, the total expected retailers' inventory cost, the total expected transshipment costs and the expected stock-out costs. Constraints (2) implies that the total quantity shipped from the supplier to all retailers (through usual shipment and backordering) cannot be greater than the supplier's initial inventory. Constraints (3) are the balance constraints. Constraints (4) imply that the balance quantity (computed as in (3)) cannot exceed the warehouse capacity Q_i for each retailer i. Constraints (5), (6) and (7) link together the decision variables x_i, b_i^s and y_{ij}^s with the respective integer variables v_i, r_i^s and V_{ij}^s so that if the first ones are positive, these quantities are splitted in a certain number of vehicles represented by the latter ones, considering the respective vehicles capacities C_i and C^T and, consequently, the associated fixed costs F_i, G_i and T_{ij} are charged in the objective function. Finally, constraints from (8) to (14) are variables definition constraints. Due to the non-linearity of Model $\mathscr{T}$, we linearize it following the approach described in [4] and we call the linearized problem "Model $\mathscr{T}^{\mathscr{L}}$". Finally, we notice that Model $\mathscr{T}^{\mathscr{L}}$ can be reduced to the *Fixed Charge Transportation Problem* (see [6] and [13]) and hence, it is NP-hard.

3 Computational results

Model $\mathscr{T}^{\mathscr{L}}$ was implemented in Python 3.6.1 using the Gurobi 7.5.1 solver, and run on an Intel Core i7-7500U 2.70 GHz and 8GB RAM personal computer. Due to the complexity of Model $\mathscr{T}^{\mathscr{L}}$, the running is stopped when a 1% relative gap to the optimal solution or a time limit of 1 hour is reached.

We first consider the case with two retailers (i.e. $|\mathscr{I}|$=2). Our instances are inspired by a real case presented in [1], in which the uncertain demand of pallets should be satisfied by using trucks with limited capacity. The support of the demand probability distribution is in the set of integer numbers in the interval $[30,130]$, while the probability distribution is given by a Beta distribution (α, β), where α=20 and β=16, having average demand $\mathbb{E}(d) = 85.55556$ pallets. The supplier's inventory level q is equal to 200 pallets, the capacity C_i of the vehicles used for shipment and backordering to all retailers is equal to 34 pallets, the capacity C^T of the vehicle used for transshipment is 17 pallets, while the retailers' warehouse capacity Q_i is equal to 170 pallets. Furthermore, we define the value P of a pallet to be equal to 1053 Euros, and since the unit inventory costs approximatively correspond to 5% of the value of a pallet of 100 kilograms, we set the supplier's inventory cost equal to 5% P, and the retailers' inventory costs equal to 6% P. Moreover, since the penalty cost corresponds to a lost sale and to a reputation damage, we let p_j equal to 1.5 P. As in [1], we consider a unit shipment cost of a pallet with 100-200 kilograms weight on a distance up to 500 kilometers equal to 93.60 Euros and a fixed shipment cost equal to $\frac{f_i C_i}{\theta}$, where $\theta = 0.5$. Finally, considering that the fixed transshipment and backordering costs are computed as a function of the unit transshipment and backordering costs, 25 different instances are generated by combining all possible values, as displayed in Table 1. We notice that Model $\mathscr{T}^{\mathscr{L}}$ can be reduced into different special cases, which facilitate a trade-off analysis. In particular, in the "Extremely High case", obtained by assigning to transshipment and backordering costs a very high value (for example, equal to infinity), we get one instance in which both transshipment and backordering are not allowed, four instances in which only backordering is allowed and four instances in which only transshipment is allowed. The same parameters are considered also in the case with four retailers, (i.e. $|\mathscr{I}| = 4$), apart from q which is equal to 350 pallet.

In order to determine the right number of scenarios which have to be considered for the stochastic setting, we perform the in-sample stability analysis identifying as

Table 1 Transshipment and backordering fixed and unit costs

Cost	Extremely Low case (EL)	Low (L)	Medium (M)	High (H)	Extremely High case (EH)
t_{ij}	0	$\frac{0.75f_i}{2} = 35.1$	$\frac{f_i}{2} = 46.8$	$\frac{1.25f_i}{2} = 58.5$	$+\infty$
T_{ij}	0	$\frac{t_{ij}C^T}{0.5} = 1193.4$	$\frac{t_{ij}C^T}{0.5} = 1591.2$	$\frac{t_{ij}C^T}{0.5} = 1989$	$+\infty$
g_i	0	$0.75f_i = 70.2$	$f_i = 93.6$	$1.25f_i = 117$	$+\infty$
G_i	0	$\frac{g_iC}{0.5} = 4773.6$	$F_i = 6364.8$	$\frac{g_iC}{0.5} = 7956$	$+\infty$

benchmark scenario tree, the one with 500 scenarios. The out-of-sample stability analysis in the benchmark tree is obtained with 300 scenarios.

3.1 Stochastic solution analysis

In this section, we perform the stochastic solution analysis considering the benchmark scenario tree with 500 scenarios and computing the indicators presented in [10]. Table 2 displays the average results for the two retailers case, where with “Other” we refer to instances not belonging to any special case (i.e. the ones in which both transshipment and backordering are allowed). First, the availability of a perfect information about the future is more important if recourse decisions (i.e. backordering and transshipment) are not allowed or just transshipment is admitted with an $EVPI$ of 12.07% in the first case and approx. 10% in the second. The case in which only backordering is allowed is the most flexible with an $EVPI$ of 1.72%, as new quantities can be introduced in the system through the recourse decision, while when only transshipment is allowed, there can be a flow of goods between retailers, but further quantities are not available. Concerning the Value of Stochastic Solution, *VSS*, results show there are more advantages in including stochasticity in the cases where no recourse actions are admitted or only less flexible recourse actions are allowed (i.e. transshipment). In order to understand why the deterministic solution is worse compared to the stochastic one, we compute the *LUSS* and the *LUDS* indicators. Through the *LUSS*, we see that in the cases where no recourse decisions or just one of them are admitted, the deterministic solution identifies the same retailers selected by the stochastic solution, but with wrong delivered quantities. In the other cases, the retailers receiving zero quantities are different in the stochastic and in the deterministic solution and, as a consequence, the poor performance is due both to the selection of retailers and to the selection of the quantities. Through the *LUDS*, we notice that the solution is perfectly upgreadable only if both backordering and transshipment are not allowed, meaning that these quantities are always lower or equal to the ones suggested by the stochastic program. For all other cases, the *LUDS* is not null, meaning that the deterministic solution is only partially upgreadable (at least in one case, the stochastic solution delivers a lower number of pallets than the one suggested by the deterministic solution).

Finally, we focus on the case with four retailers. Due to the computational complexity of the problem, with the exception of the case “No transshipment, No back-

Table 2 Average values for the stochastic solution analysis indicators for every special case with two retailers

Cases	RP	WS	EVPI	EEV	VSS	ESSV	LUSS	EIV	LUDS
No transshipment No backordering	56941.68	50066.85	12.07%	57688.54	1.31%	56941.68	0.00%	56941.68	0.00 %
Only backordering	40856.24	40153.84	1.72%	40956.26	0.25%	40856.24	0.00%	40876.56	0.05%
Only transshipment	53557.80	48337.35	9.75%	54600.35	1.95%	53557.80	0.00%	53567.35	0.02%
Other	39487.43	38979.73	1.29%	39723.29	0.60%	39512.12	0.06%	39504.06	0.04%

ordering", we analyze only the instances whose costs of the allowed strategy are set at a "Medium" level (i.e. only one instance for each case is considered). Results are displayed in Table 3. We specify that after 549090 seconds, the gap to the optimal solution of the *RP* for the "Other" case was not closed and we calculate only the *EVPI* and the *VSS*, since the other indicators require further constraints which make the model even more difficult to get solved to optimality. Differently from Table 2, now, if only backordering is allowed the cost is higher than the case in which only transshipment is admitted, while for the *EVPI*, the previous results are confirmed. Concerning the VSS, the results are now different, as there are more advantages in including stochasticity in the case where only backordering is allowed. Even if with only backordering, the quantities delivered in the first stage are fewer, transshipment is cheaper if only few quantity adjustments are needed and the presence of more retailers provides more flexibility to the distribution system.

Table 3 Values for the stochastic solution analysis indicators for every special case with four retailers and "Medium" cost level

Cases	RP	WS	EVPI	EEV	VSS
No transshipment No backordering	120144.82	109191.65	10.03%	131098.04	9.12%
Only backordering	112319.31	109191.65	2.86%	122822.55	9.35%
Only transshipment	107613.10	103077.98	4.40%	111960.39	4.04%
Other	105432.62 (2.27%)	103077.98	2.28%	106535.91	1.05%

4 Conclusions

We presented a real problem arising in logistics and after modeling it with an integer stochastic program, we stated that this is NP-hard. Furthermore, we show that with two retailers, a decision-maker has a greater advantage by including uncertainty, especially if no recourse actions or only transshipment is admitted. We also show that in some cases, the selection of retailers to which quantities should be delivered is the same both in the deterministic and in the stochastic solution. Nevertheless, the deterministic solution can be upgrated only in the special case where no recourse actions are allowed. Conversely, with four retailers, transshipment provides more flexibility. Future research could be devoted to analyze the multistage version of this problem by exploiting lower bounds (see [7]-[8]) and, as in [1], to compare the stochastic solution to the one obtained through a rolling-horizon heuristic. Another stream of research could be analyzing robust optimization approaches (see [9]) or adapting approaches presented in [2].

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