Induced earthquakes and the ETAS model

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Abstract. *The epidemic type aftershock sequence (ETAS) model is widely used in the modelling of earthquake catalogues that include aftershocks. The model has be used successfully in describing tectonic seismicity where the usable catalogue sizes are large. The model is more difficult to apply to induced earthquakes, where catalogue sizes are typically much smaller and the seeding rate of main shocks cannot be assumed to be constant. In both cases, the parameters of the ETAS model are highly correlated under the conventional parameterisation and the resulting log-likelihood function has many flat regions, which can make inference difficult.*
We will introduce issues that arise when modelling induced seismicity caused by gas extraction and put forward an alternative parameterisation for the aftershock component of the ETAS model. The standard ETAS model is nested within our alternative but the correlation of aftershock parameters is greatly reduced. This means that that inference can be made on a broader class of models and more effectively, allowing more model uncertainty to be propagated into earthquake forecasts and simplified parameter interpretation.

Keywords. *Seismic risk; Point processes; Extreme value theory; Spatio-temporal modelling.*

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1 Introduction

The Groningen region of the Netherlands does not experience tectonic seismicity. It does, however, contain the largest field of natural gas in Europe. This gas field supplies homes and industries in the Netherlands, Belgium, Germany and France, where gas-powered appliances are specialised to the gas from this field. Despite the Groningen region not being tectonically active, seismic events have been recorded there since the early 1990s. Gas extraction induces these events but there are still questions on the form of the relationship between the two. Figure 1 shows the relationship between one feature of gas extraction and the density of induced events. Understanding these links is critical to informed decision making about future extraction from the Groningen field, based on the associated seismic hazard.

There has been substantial investment into the investigation of this relationship, including improvements to the network of geophones that cover the gas field. It is important to be able to detect and model events with small magnitudes because the gas field is only 3km below surface level and so small magnitude events are still capable of causing damage. The investment has funded a dense monitoring network across the gas field, which can now reliably detect all events down to 1.1 Mw. This value, known as the

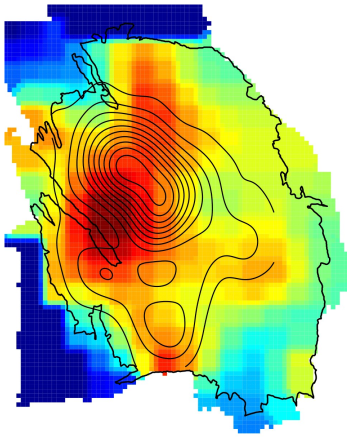

Figure 1: Cumulative compaction caused by gas extraction with event density contours overlaid.

magnitude of completion, has decreased over time but this is not usually accommodated into the model fitting process. The increased ability to detect small magnitude earthquakes is apparent in Figure 2.

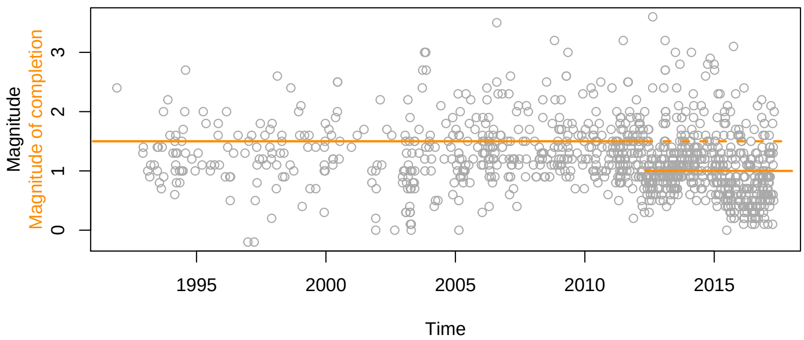

Figure 2: Magnitudes of recorded events and field-wide magnitude of completion.

Modelling seismicity in the Groningen field has additional challenges and opportunities as compared to the usual tectonic setting, these include:

- Covariates such as the cumulative compaction of the gas field in Figure 1 are available but the best way to incorporate these is unclear;
- The variable rate of induced events makes potential aftershock activity difficult to identify;
- The magnitude of completion is decreasing with time but also varies spatially;
- The usable catalogue is small, containing only a few hundred events.

Models that exploit these opportunities and address these challenges could improve our ability to predict induced seismicity, which is the first step in evaluating future seismic hazard and comparing production scenarios.

2 The ETAS model

The epidemic type aftershock sequence (ETAS) model is currently the standard statistical approach to incorporating aftershock activity. This model is a special case of the Hawkes point process, the class of point process models in which the intensity function λ is dependent on the history of the process, $\mathcal{H}_t$. In particular, the ETAS model locally augments a background intensity function, μ, with an increase in intensity after each earthquake. The size of this increase is determined according to the magnitude of each earthquake and reduces with time and distance from the epicentre. Figure 3 shows an example of such an intensity function for a temporal ETAS point process.

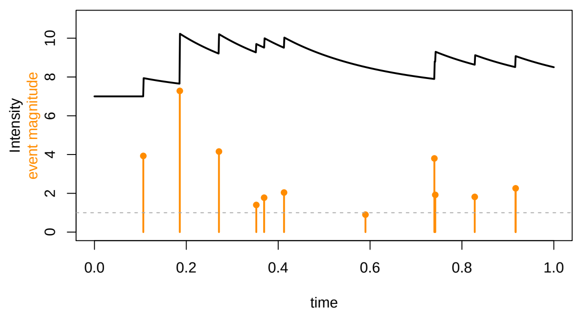

Figure 3: Simulated temporal ETAS catalogue and the associated intensity function.

It is simple to extend the ETAS model to incorporate covariates within the background intensity and to generalise to a spatio-temporal setting, as in Equation (1) . Selecting an appropriate parametric or semi-parametric form for $\mu(x,y,t|X,\theta)$ provides a way of linking gas extraction covariates X and the level of induced seismicity. The functions κ, g and h then describe the aftershock activity by respectively controlling the expected number of aftershocks, their lag and their displacement from the triggering earthquake.

$$\lambda(x,y,t|X,\mathcal{H}_t,\theta) = \mu(x,y,t|X,\theta) + \sum_{i:t_i<t} \kappa(m_i|\theta)g(t-t_i|\theta)h(x-x_i,y-y_i|\theta). \quad (1)$$

3 Reparameterisation

It is well known that the ETAS model is difficult to fit, particularly to small earthquake catalogues like that of Groningen. This is partly because the model was developed in the tectonic setting, where much larger catalogues are available and a temporally constant background rate may be assumed. There are also issues with the conventional choices for the functions κ, g and h, which are motivated by empirical relationships seen in the tectonic setting. The conventional choice is for κ to be an exponentially increasing function above some threshold M_0. The functions g and h are conventionally described by the modified-Omori law, a heavy tailed power-law distribution, in Δt and $r^2 = \Delta x^2 + \Delta y^2$ respectively. This choice of aftershock functions results in a log-likelihood function that is almost flat in many regions of the parameter space and parameters which are strongly correlated [1]. These issues make both frequentist and Bayesian approaches to inference on the ETAS model difficult.

We suggest alternative forms for the aftershock terms in the ETAS model, within which the current standard choices are nested. The reparameterisation centres the effect of magnitude on aftershock productivity and uses a generalised Pareto distribution rather than the modified-Omori law to describe aftershock lags and displacements. The resulting version of the ETAS model is more flexible than the current approach and is able to describe short-tailed delay and displacement distributions. By using the alternative forms the parameter dependence is greatly reduced, and is negligible within the range of models covered by the conventional parameterisation. The reduction in parameter dependence can be seen in Figure 4.

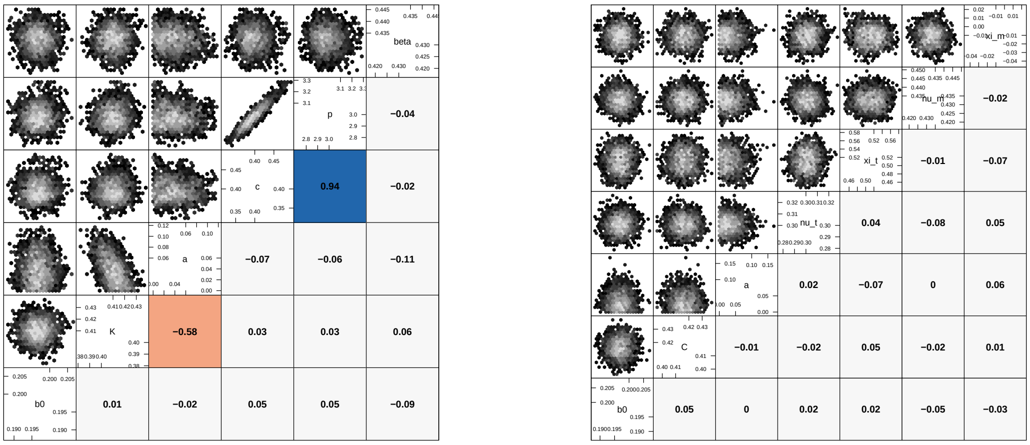

Figure 4: Posterior samples & correlations for a simulated ETAS catalogue using the conventional aftershock parameterisation (left) and the centred generalised Pareto parameterisation (right).

The resulting model allows for more effective inference to be performed, simplifies parameter interpretation and carries uncertainty in the shape of the delay distributions into earthquake forecasts. This is particularly important for application of the model to small catalogues of induced earthquakes such as that of the Groningen gas field.

References

[1] Veen, A. and Schoenberg, F. P. (2008). Estimation of Space-Time Branching Process Models in Seismology Using an EM-Type Algorithm. *Journal of the American Statistical Association* **103** 614–624.