Alessandro Fassò and Francesco Finazzi (*University of Bergamo*)

The proposal of Castruccio and his colleagues for the assessment of spatiotemporal statistical models using a virtual reality environment has interesting potential. In fact, its implementation on cheap devices, easy to adapt to user data and models, is the key to the success of this approach. Moreover, they have introduced the use of the similarity index SSIM (Wang *et al.*, 2004) as a model diagnostic applied to a global climate model output on a three-dimensional regular grid.

This discussion first focuses on the use of the SSIM-index in statistics in general, suggesting a modified asymmetric version. Then, it motivates a virtual reality approach in modelling four-dimensional global climate observational data on a non-regular grid.

Structural similarity index in statistics

The structural similarity index SSIM is a standard for image and movie fidelity assessment, especially in its local, sliding window and multiscale versions (Wang and Bovik, 2009).

Inspired by Castruccio and his colleagues, the adoption of SSIM in statistics as a general model validation diagnostics is considered here. To do this, SSIM in their formula (1) is simplified assuming that $C_1 = C_2 = 0$, non-zero means and positive variances. Moreover, considering the case where $\mathbf{x} = \hat{\mathbf{y}}$ is a statistical model for $\mathbf{y}$, the following asymmetric version of SSIM is suggested:

$$\mathrm{SSIM}(\hat{\mathbf{y}}|\mathbf{y})^2 = \rho_{\hat{\mathbf{y}}\mathbf{y}}^2 \left(1 - \frac{(\mu_{\hat{\mathbf{y}}} - \mu_{\mathbf{y}})^2}{\mu_{\mathbf{y}}^2}\right)_+ \left(1 - \frac{(\sigma_{\hat{\mathbf{y}}} - \sigma_{\mathbf{y}})^2}{\sigma_{\mathbf{y}}^2}\right)_+ \tag{1}$$

where $\rho_{\hat{\mathbf{y}}\mathbf{y}} = \sigma_{\hat{\mathbf{y}}\mathbf{y}}/\sigma_{\hat{\mathbf{y}}}\sigma_{\mathbf{y}}$ and $(x)_+$ is the positive part of x.

Virtual reality and the four-dimensional case

The information content of the radiosonde network depicted in Fig. 2 and known as RAOB is addressed hereby using a four-dimensional statistical model for temperature profiles (Fassò *et al.*, 2018a, b; Finazzi *et al.*, 2018).

Because of the profile structure of data, a spatiotemporal model for functional data is used and estimated by using an expectation–maximization algorithm derived by Finazzi and Fassò (2014). Moreover, the information gaps (Fassò *et al.* 2018a) are assessed by computing the kriging estimate, say $\hat{\mathbf{y}}$, on a fine grid. This gives big four-dimensional objects for $\hat{\mathbf{y}}$, its standard deviation and the local asymmetric SSIM of equation (1). Hence the challenge is to represent in a virtual reality environment such large four-dimensional objects.

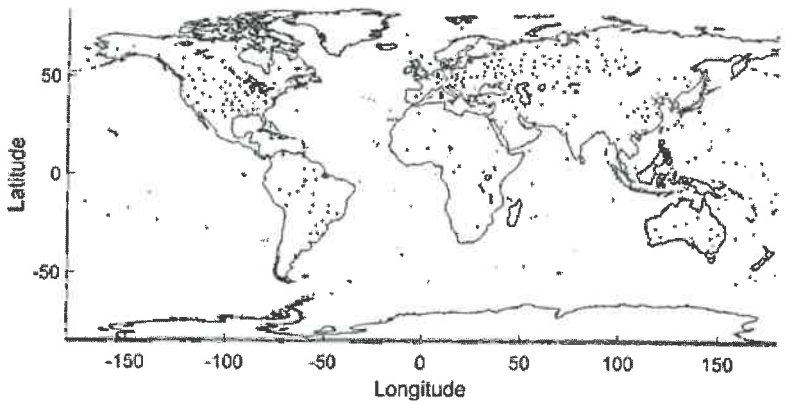

Fig. 2. Universal rawinsonde observation program data set for the year 2015: 12.5 × 10^6 observations come from 684 radiosonde stations, 25 observations per radio sounding and 720 time steps for twice-a-day measurement