

Analytical modelling of heating and evaporation of drop clouds with temperature dependent gas properties

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Abstract

An analytical approach to evaluate the effects of droplet neighbouring on heating and evaporation is proposed, solving the conservation equations through the point source method, accounting for the dependence on temperature and composition of gas thermo-physical properties. The procedure to select the more appropriate boundary conditions on the drop surface is deepened and a method to mitigate the effect is proposed. The analytical solution for different arrays of droplets is compared for validation with an exact analytical solution for pairs of drops and with previous numerical solutions for drop arrays.

Keywords

Evaporation; analytical solution; drop array; thermo-physical properties.

Introduction

Drop heating and evaporation in CFD spray simulations are usually taken into account through the implementation of analytical models (like [1]). Those models were derived and validated for single drops and they may become questionable when used to model dense sprays. However, the effect of interaction between the Stefan flow from neighbouring droplets cannot be ignored when the mutual distance becomes comparable with their size. As an example, in [2] it was shown experimentally that the effect of interaction between droplets on heating and evaporation cannot be ignored when the distance between droplets is less than about 10 diameters, and it is known that increasing the number of droplets per unit volume reduces the evaporation rate [3]. Analytical models for a couple of interacting drops, under the assumption of constant thermo-physical properties, are available in the open literature [4]. The effect of more complex droplet distribution and variable gas properties was numerically investigated [5], and recently an exact analytical solution of energy and species conservation equations for the drop pair case, considering the explicit dependence of gas density on temperature and composition, was reported [6]. More complex arrays of interacting droplets were studied (see [7]), and a further complexity is added when considering that in a real spray the mutual position of the evaporating droplets is randomly distributed and the number of interacting droplets may be large. Exact analytical solutions of the energy and species equations are not available for complex drop dispositions, but a method based on the use of point mass sources to model the evaporating droplets was proposed [8], and this method allows the study of array of any complexity, under the assumption of constant gas properties, although it was shown [9] that the solution for multi-drop structures does not respect the exact boundary conditions on the drop surface.

The present work reports an analytical approach to evaluate the effects of droplet neighbouring on heating and evaporation, solving the species conservation equations through the point source method, taking into account the dependence on temperature and composition of gas thermo-physical properties (conductivity, heat capacity, density, diffusion coefficients) and composition. The analytical solutions for different arrays of droplets of different size were compared with exact analytical solution in bispherical coordinates, for the drop pair case, and with numerical solution of the conservation equations for drop arrays, to assess the accuracy.

Mathematical model

The heat and mass transfer from single component evaporating drops in a quiescent gaseous environment, under quasi-steady conditions, can be modelled through the solution of the steady-state, momentum, species and energy conservation equations for general shapes and number of the evaporating drops. Regarding the momentum conservation equation, it can be written in non-dimensional form as (see [7]):

$$\tilde{\nabla}_k \tilde{P}_T = \frac{1}{\Lambda} \left[Sc^M \tilde{\nabla}^2 \tilde{U}_k - \tilde{P}_T \tilde{T}^{-1} \tilde{\nabla}_j H \tilde{\nabla}_j \tilde{U}_k \right] \quad (1)$$

where $\tilde{\nabla} = R_d \nabla$ is the non-dimensional nabla operator and the following non-dimensional quantities have been introduced:

$$\tilde{P}_T = \frac{P_T}{c_{ref}RT_\infty}; \tilde{T} = \frac{T}{T_\infty}; \Lambda = \frac{RT_\infty R_d^2}{Mm^{(1)}D_{10,ref}^2}; \tilde{U} = U \frac{R_d}{D_{10,ref}}; S_C^M = \frac{\mu_{ref}}{D_{10,ref}c_{ref}M_m^{(1)}}; H = \log(1 - y^{(1)}) \quad (2)$$

and P_T is the ambient pressure, c is the molar gas density evaluated as a function of temperature by assuming ideal gas behaviour $c_T = \frac{P_T}{RT}$, R is the universal gas constant, T_∞ is the gas temperature far from the drop

surface, R_d is the drop equivalent radius, $Mm^{(1)}$ is the molar mass of the vaporising species, D_{10} is the species diffusion coefficient, U the drop velocity, $y^{(1)}$ is the molar fraction of the vaporising species; to be noticed that the gas mixture properties are assumed at a reference condition, as explained below. The assumption of an asymptotic value for the parameter Λ ($\Lambda \rightarrow \infty$) in equation (1), which was justified (see [10] and [11]) by showing that it becomes large for a variety of conditions of interest for applications, allows the reduction of the momentum equation to $\nabla \tilde{P}_T = 0$, i.e. $P_T = \text{const}$. The constancy of P_T allows disregarding the dependence of the thermo-physical properties on the pressure, although, when calculations are performed, the correct values of the properties at the given pressure can be used.

The species and energy conservation equations, under quasi-steady assumption are:

$$\nabla_j N_j^{(p)} = 0 \quad p = (0,1) \quad (3)$$

$$c_{p,v}\rho U_j \nabla_j T = \nabla_j (k \nabla_j T) \quad (4)$$

where $N_j^{(p)} = N_j^{(T)} y^{(p)} - c D_{10} \nabla_j y^{(p)}$ are the molar fluxes and the simplified form of the energy equation was obtained taking into account inter-diffusional terms but neglecting dissipation by viscous stresses and further minor terms (refer to [12] p. 465, or [13] p. 589, for a more complete form of the equation, see also [14]). The index $p=(0,1)$ stands for the gas and the evaporating species, respectively, and $N_j^{(T)} = N_j^{(1)} + N_j^{(0)}$ is the mixture molar flux. At infinite distance from the drop the mixture temperature and molar fractions are assumed uniform and equal to the constant values T_∞ and $y_\infty^{(p)}$, respectively. On the drop surface the temperature is assumed uniform and equal to T_s and the molar fraction is defined by the surface temperature assuming equilibrium conditions. The usual approach to this problem is to assume constant gas properties, then a simple solution is obtained for spherical drops [15], spheroidal and triaxial ellipsoidal [16] and for drop pairs [4]. In these cases, the properties are evaluated at reference temperature (T_{ref}) and composition ($\chi_{ref}^{(p)}$) conditions generally defined as:

$$T_{ref} = \alpha_{ref} T_\infty + (1 - \alpha_{ref}) T_s; \quad \chi_{ref}^{(p)} = \alpha_{ref} \chi_\infty^{(p)} + (1 - \alpha_{ref}) \chi_s^{(p)} \quad (5)$$

and α_{ref} is the averaging parameter. It has often been noticed that the prediction of constant properties models shows an important dependence on the reference conditions [17], [18], and the ‘‘correct’’ choice of α_{ref} is still matter of debate (it is usually assumed to be equal to 1/2 or to 1/3 [19]). A way to take explicitly into account the dependence of the thermo-physical properties on temperature and still obtaining analytical solutions was proposed in [17], where it was shown that the prediction of this new analytical model is not influenced by the chosen value of the reference temperature while it is only weakly influenced by the value of the reference composition (see [17] for a complete analysis). Following [17], the diffusion coefficient (D_{10}), the gas mixture molar density (c), thermal conductivities ($k^{(p)}$) and vapour specific heat ($c_{p,v}$) where calculated as function of the temperature through the following laws:

$$D_{10} = D_{10,ref} \hat{T}_{ref}^{-m} \hat{T}^m; \quad c = c_{ref} \hat{T}_{ref} \hat{T}^{-1}; \quad k^{(p)} = k_{ref}^{(p)} \hat{T}_{ref}^{-q_p} \hat{T}^{q_p}; \quad c_{p,v} = c_{p,v,ref} \hat{T}_{ref}^{-b} \hat{T}^b \quad (6)$$

In [17] the chosen dependence on temperature was justified against available experimental data, for a variety of substances, with satisfactory agreement. To notice that $m=7/4$ is the value assumed in the widely used FSG correlation [20] for binary diffusivity coefficients while the coefficients q_p and b can be found by interpolation of available data on conductivity and specific heat for pure substances and the reference conductivity of the gas mixture (k_{mix}) is evaluated from the conductivities of the pure substances by Wassilieva [21] relation:

$$k_{mix} = \frac{k^{(0)} y_{ref}^{(0)}}{y_{ref}^{(0)} + A_{01} y_{ref}^{(1)}} + \frac{k^{(1)} y_{ref}^{(1)}}{y_{ref}^{(1)} + A_{10} y_{ref}^{(0)}} \quad (7)$$

where the coefficients A_{jk} are obtained from the Lindsay and Bromley relationship [22], neglecting their temperature dependence.

The analytical solution, reported in [17], was found introducing an auxiliary harmonic function Φ (i.e. $\nabla^2 \Phi = 0$) equal to 1 over the drop surface and nil at infinity. This approach was first introduced by Laboswky [23] and later used by many authors to solve similar problems [4], [7], but it is worth to mention that in the present case the problem is fully non-linear and that the model [17] yields an analytical solution. The details of the solution procedure can be found in [17], where it was also shown that, with a good approximation, the same simple relations for evaluating the evaporating mass flux under constant properties assumption:

$$n_n^{(1)} = -\rho_{ref} D_{10,ref} \ln(1 + B_M) \nabla_n \Phi \quad (8)$$

may be used by selecting a reference parameter α_{ref} related to the mass transfer number B_M by the rule:

$$\alpha_{ref} = \frac{A_{\alpha,ref}}{\log(1 + B_M)} + \frac{1}{1 - (1 + B_M)^{\frac{1}{A_{\alpha,ref}}}} \quad (9)$$

where the constants A_{ref} depends on the species and the gas temperature [17].

The accuracy of this simplified approach compared to the exact one was shown to yield discrepancies of the order of few percent. The corresponding evaporation rate is obtained by integrating the flux over the surface S :

$$m_{ev} = -\rho_{ref} D_{10,ref} \ln(1 + B_M) \int_S \nabla_n \Phi dS \quad (10)$$

It can be observed that the effect of drop geometry on mass fluxes is given by the gradient $\nabla_n \Phi$, which is independent of the thermo-physical properties, while the effect of the properties and operating conditions is contained into the multipliers of $\nabla_n \Phi$ in equations (8) and (10), and they do not depend on the geometry of the problem. This was already observed in [7], where the solution to the equation $\nabla^2 \Phi = 0$ was obtained by numerical means, also for non-uniformly spaced regular arrays. The present approach is aimed to extend the results of [7] to non-regular drop arrays and generally to drop clouds made by drops unequally distributed in space but, differently from that, fully analytical methods will be used to find the function Φ , relying of the model reported in [17], and above sketched, to take into account the effect of temperature dependence of the gas mixture thermo-physical properties.

Application to arrays of drops

The method used below comes from electrostatics applications and it was proposed for the solution of drop evaporation problems by [8]. The method is based on the observation that, for a single spherical drop, the solution of the problem:

$$\nabla^2 \Phi = 0; \quad \Phi_s = 1; \quad \Phi_\infty = 0 \quad (11)$$

is equivalent to the solution of the problem:

$$\nabla^2 \Phi = C \delta^3(r); \quad \Phi_\infty = 0 \quad (12)$$

where $\delta^3(x)$ is the 3D Dirac function that has the following property:

$$\int_V \delta^3(\mathbf{x} - \mathbf{x}_0) dV = \begin{cases} 1 & \text{if } \mathbf{x}_0 \in V \\ 0 & \text{if } \mathbf{x}_0 \notin V \end{cases} \quad (13)$$

and $C = 4\pi R_d$, i.e.: $\Phi = R_d / r = C / 4\pi r$. The extension to the case of a drop array is obtained substituting to (11) the problem:

$$\nabla^2 \Phi = \sum_{j=1}^N C_j \delta^3(\mathbf{x} - \mathbf{x}_j); \quad \Phi_\infty = 0 \quad (14)$$

where \mathbf{x}_j are the coordinates of the drop centres and $C_j = 4\pi R_{d,j}$, and the solution is in this case is:

$$\Phi = -\sum_{j=1}^N \frac{C_j}{4\pi |\mathbf{x} - \mathbf{x}_j|} \quad (15)$$

However, as pointed out by [9], this solution cannot satisfy the B.C. over the whole drop surfaces, and it must be considered an approximation of the exact solution.

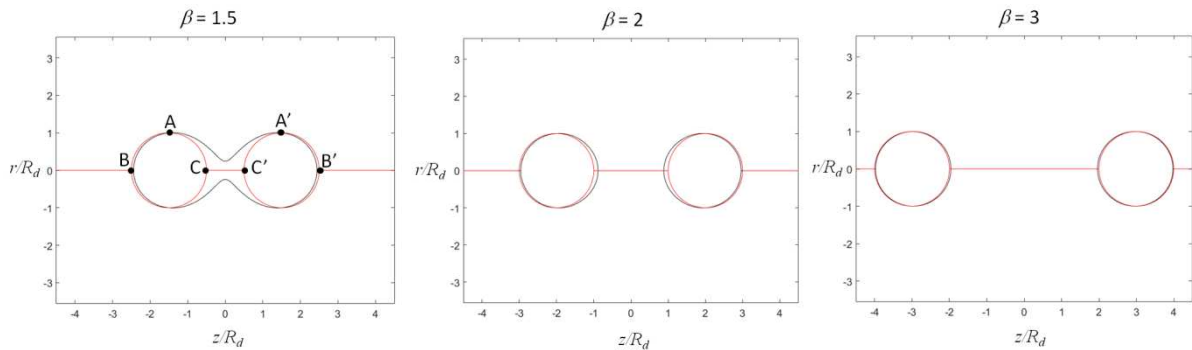


Figure 1. Locus of points where $\Phi=1$; red line: exact solution; black line: approximate solution, equation (16), for three drop distance cases.

Taking for example the case of a pair of identical drops, the exact solution of the problem (11) is given in [4], equation (12) and in [24] equation (33), while that of (12) is, in cylindrical coordinates (see figure 1):

$$\Phi = -\frac{C_1}{4\pi\sqrt{r^2 + (z - z_1)^2}} - \frac{C_2}{4\pi\sqrt{r^2 + (z - z_2)^2}} \quad (16)$$

The first one assumes the value $\Phi = 1$ on the surface of the two drops (red line in figure 1) while the second one assumes this value on the surfaces given by the black lines in figure 1.

It can be appreciated that the approximate solution gets closer to the exact one as the drop distance increases. It should be noticed that for the approximate solution, the B.C. can be satisfied on one point of the drop surface, and solution (16), reported in figure 1, is found by setting the condition on the drops surfaces at $z = z_j$ (points AA' in figure 1). Choosing different points, for example points BB' or CC' in figure 1, the solution is different. A way to compare quantitatively the solutions is by evaluating the so called screening coefficient:

$$\phi = \frac{m_{ev}}{m_{ev, is}} \quad (17)$$

defined as the ratio between the evaporation rate of a drop and that of the same drop infinitely far from the others. The exact solution for the two identical drops is (see [24]):

$$\phi = \sqrt{\beta^2 - 1} \sum_{p=0}^{\infty} \frac{2}{(\beta + \sqrt{\beta^2 - 1})^{2p+1} + 1} \quad (18)$$

while from the approximate solution (16) is:

$$\phi = 1 - \frac{\sqrt{1 + 4\beta^2} - 1}{4\beta^2} \quad (19)$$

and figure 2 shows the comparison; the differences among the two evaluations is better than 0.34%.

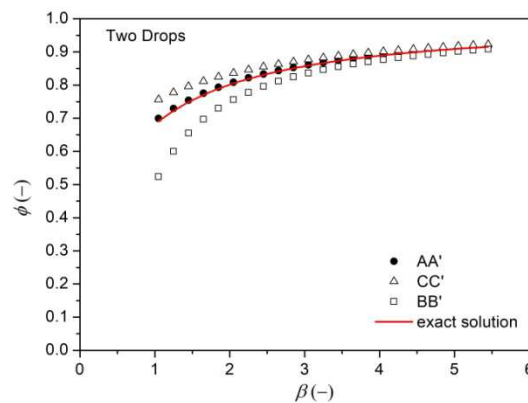


Figure 2. Screening coefficient as function of non-dimensional drop distance, calculated from the approximate solution using three B.C.s located at points AA', BB' and CC' (see figure 1) and from the exact solution, equation (18).

Setting the B.C. on points BB' or CC' (see figure 1) would yield the values also reported in figure 2, showing how in that case the approximation is much worse. Since the position where to set the B.C. for problem (12) influences the solution, a choice must be done, in particular when drop arrays are analysed. The rationale of the choice proposed below relies on the observation, sketched in figure 3, that the best position to set the B.C. on the drop surfaces, for the drop pair case, is that obtained intersecting the plane passing through the drop centre and orthogonal to the segment joining the two drop centres, with the drop surface.

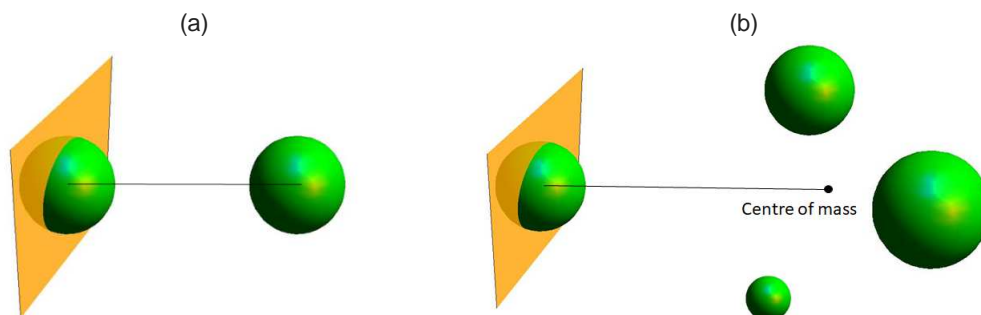


Figure 3. Schematic of the procedure used to calculate the B.C. for (a) two drops and (b) random drop arrays.

Consider the drop array shown in figure 3b, for any given drop, a line joining the drop centre with the centre of mass of the remaining drop cloud can be uniquely defined. Then the plane normal to that line and passing for the centre of the drop is also uniquely defined, and its intersection with the drop surface define a set of points where to impose the B.C. For two drops each of those points is equivalent to the others, while this is not in general true for drop arrays and its influence was analysed by some tests, finding that in practical cases one may disregard this effect choosing randomly the point on the circumference.

The developed model is applied to predict the evaporation rates for clouds of interacting drops taking into account the effect of dependence on temperature and composition of gas mixture properties.

Results and discussion

This section discusses some results obtained implementing the approximate solution proposed in this work, where the gas phase thermo-physical properties are estimated at a reference temperature, which is calculated using the averaging parameter from equation (9). Table 1 reports the values of the constant $A_{\alpha,ref}$ for six species (water, acetone, ethanol, n-hexane, n-octane and n-dodecane) and two gas temperatures at free stream conditions (500 K and 1000 K), while figure 4 shows the corresponding values of the averaging parameter α_{ref} as function of the Spalding mass transfer number B_M , calculated in the range of drop temperature from 270 K up to the species boiling conditions. The results evidence that, for all the selected operating conditions, at low drop temperatures (i.e. low evaporation) the ‘best’ value of the averaging parameter is close to $\frac{1}{3}$ and its value monotonically reduces increasing the drop temperature (i.e. high evaporation). The maximum variation of the averaging parameter is reported for n-dodecane, with the minimum value equal to 0.13 when the drop temperature approaches the boiling conditions, far below the ‘ $\frac{1}{3}$ ’ value commonly used in spray modelling. The averaging constant $A_{\alpha,ref}$ for water and n-dodecane is almost independent on the gas temperature at free stream conditions (refer to Table 1) and this reflects on the corresponding values of the averaging parameter α_{ref} . For the other species the increase of the gas temperature results in an increase of the averaging parameter, except for ethanol where the averaging parameter is lower at higher free stream gas temperature.

Table 1. Values of the averaging constant $A_{\alpha,ref}$ (eq. 7), for six species and two operating conditions.

T_∞	water	acetone	ethanol	n-hexane	n-octane	n-dodecane
500 K	0.752	0.753	0.85	0.66	0.60	0.545
1000 K	0.752	1.240	0.68	0.75	0.65	0.540

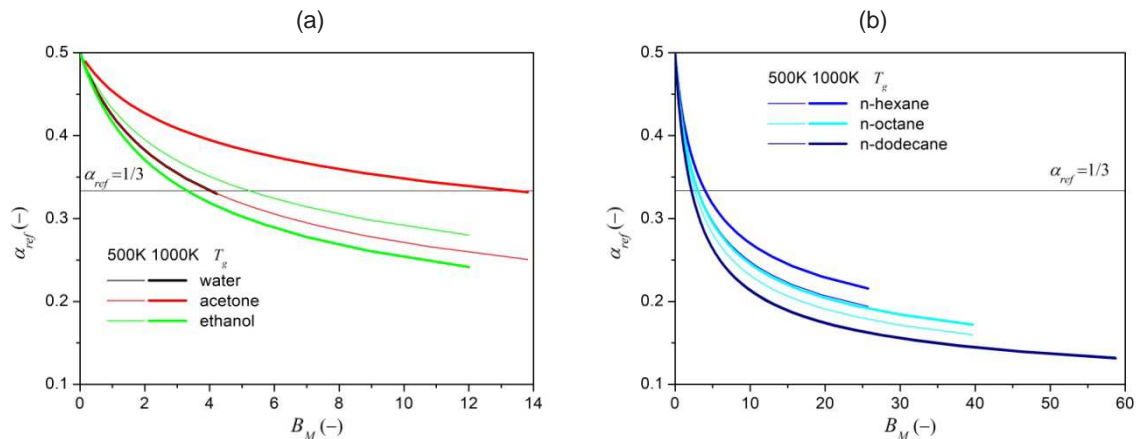


Figure 4. Averaging parameter α_{ref} as function of Spalding mass transfer number for two gas temperatures and different species: (a) water, acetone and ethanol, (b) three hydrocarbons.

The different way of estimating the thermo-physical properties reflects on the calculation of the mass evaporation rate from the proposed approximate solution, equation (10). This effect has been investigated for all the selected species and operating conditions (refer to Table 1). Figure 5 reports the ratio between the mass evaporation rate calculated using the averaging parameter α_{ref} from equation (9) and that calculated using the constant value equal to $\frac{1}{3}$, commonly used in spray applications:

$$\gamma = \frac{m_{ev}(\alpha_{ref} = f(B_M))}{m_{ev}(\alpha_{ref} = 1/3)} \quad (20)$$

for all the operating conditions of figure 4.

The results show that at low drop temperature (i.e. low evaporation) the evaporation rate calculated using the ‘ $\frac{1}{3}$ rule’ is always lower than the one calculated using a more refined averaging parameter. The increase of gas

temperature at free stream conditions results to the reduction on the evaporation rate ratio γ for water, acetone and ethanol, while an opposite behaviour occurs for hydrocarbon species. The maximum variation of the parameter γ within the range of drop temperature selected is around 20% for all the selected operating conditions.

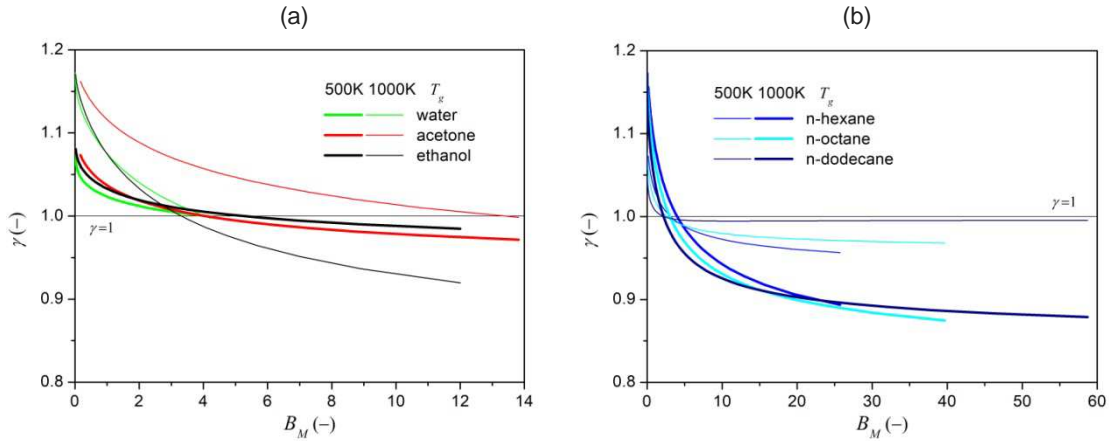


Figure 5. Evaporation rate ratio γ as function of Spalding mass transfer number for two gas temperatures and different species: (a) water, acetone and ethanol, (b) three hydrocarbons.

Drop arrays

To check the correctness of the approach for the case of many drop a comparison with the results reported by [7] on drop array was performed. [7] solved numerically the problem for many kinds of drop dispositions and their results will be here considered the reference for checking the acceptability of the approximate analytic solution in case of multi-drop structure. Arrays made by 9 drops were selected to reproduce one of the cases reported in [7], precisely 8 drops were positioned on the corners of a parallelepiped (or a cube) and one in its centre (see figure 6). In [7] an average screening coefficient was defined as:

$$\phi_m = \frac{\sum_{j=1}^{N_d} \phi_j}{N_d} \quad (21)$$

where ϕ_j is the screening coefficient of the j -th drop and N_d is the total number of drops in the array ($N=9$ for the present case).

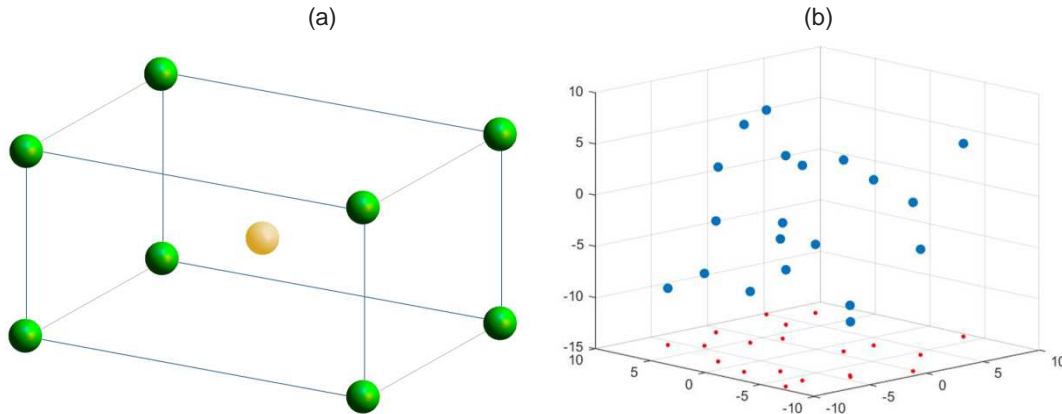


Figure 6. Sketch of drop distribution: (a) a 3D array (parallelepiped) and (b) random distribution.

To correlate the results for many kinds of arrays [7] defined a non-dimensional parameter ξ as:

$$\xi = \frac{d}{R_d N_d^{0.72}}; \quad d = \frac{V^{1/3}}{N_d^{1/3} - 1} \quad (22)$$

where V is the volume of the parallelepipedon, and proposed the following correlation:

$$\phi_m = 1 - \frac{1}{0.7256 \xi^{0.9717}} \quad (23)$$

which was checked against their numerical solutions with very satisfactory results. Following the same procedure, but using the approximate solution (16) instead of the numerical one, the obtained values of ϕ_m are presented in figure 7 against the correlation (23) proposed by [7].

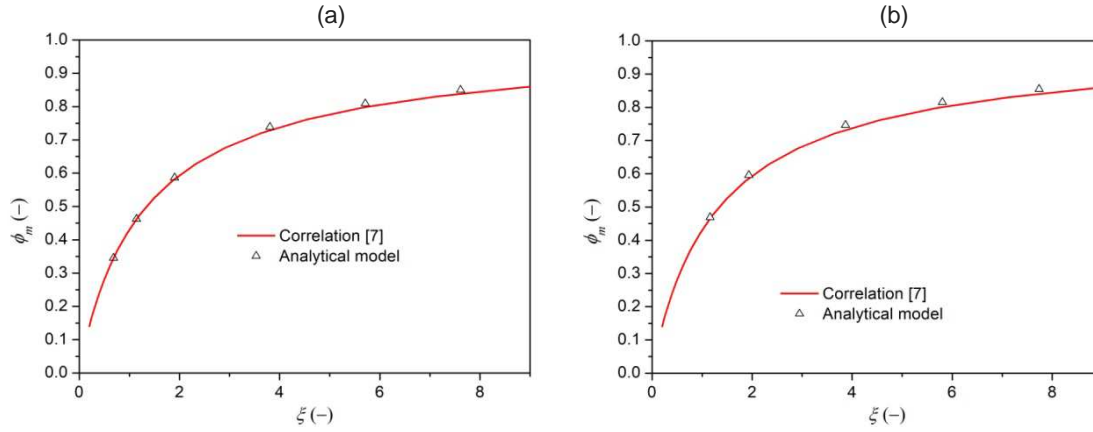


Figure 7. Average screening coefficient ϕ_m as function of the non-dimensional parameter ξ , as predicted by the approximate analytical model (16) and the correlation proposed in [7] for (a) a cubic drop array and (b) parallelepipedon drop array.

The discrepancy is less than 2.2% for the cubic array for $\xi = 1.14$ and 3.2% for $\xi = 0.68$, which corresponds to distance from the central drop to the edge drops equal to about 2.5 and 1.5 diameters respectively, while for larger distances the discrepancy is better than 1.5%

For the parallelepipedon case, with relative lengths of the edges equal to 0.7x1x1.5, the discrepancy is only slightly larger (2.7% for the minimum distance: $\xi = 1.16$).

All the calculations were performed with mono-sized drops. Two further tests were performed, changing the relative radius (compared to the external drops) of the central drop to 0.5:1 and 2:1. The results are reported in figure 8.

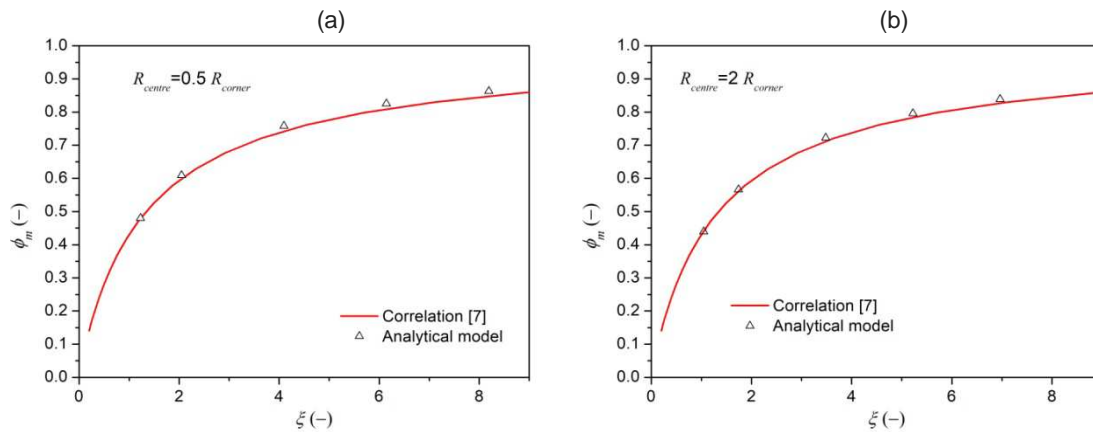


Figure 8. Average screening coefficient ϕ_m as function of the non-dimensional parameter ξ , as predicted by the approximate analytical model (16) and the correlation proposed in [7] for parallelepipedon drop array and relative radius (compared to the external drops) of the central drop equal to (a) 0.5:1 and (b) 2:1.

The discrepancy with respect to [7] results is even better (1.97% for $R_{d,centre}/R_{d,corner} = 0.5$ and $\xi = 1.22$ and 2.1%, for $R_{d,centre}/R_{d,corner} = 2$ and $\xi = 1.04$). These results show the reliability of the method also for more complex structure than the two drops, and allow the next analysis of drop clouds.

Clouds of drops

The simplicity of the proposed method to find an approximate solution to the Laplace equation allows the application to the case of drop clouds, made by large number of drops unequally distributed in space.

First the case of mono-sized drops was analysed. Clouds from 5 to 25 drops are considered here.

The [7] correlation used the array volume to define the parameter ξ , but for a drop cloud, rather than a drop array, the concept of “volume” is unclear, since it is not possible even to define in an unique way the “external surface” for an arbitrary ensemble of points (see figure 6b). However, the “volume” of a cloud can be related to its “compactness” and this hinted the following idea. The parameter d (equation 22) used in [7] is a measure of drop inter-distances; on the other hand, for a general drop distribution the quantity σ :

$$\sigma^2 = \frac{\sum_{j=1}^{N_d} |\mathbf{x}_j - \mathbf{x}_G|^2}{N_d} \quad (24)$$

where \mathbf{x}_j are the coordinates of the each drop centre and \mathbf{x}_G those of the cloud centre of mass, is also a measure for drop inter-distances. Choosing the following definition for d :

$$d = A \frac{\sigma}{N_d^{1/3} - 1} \quad (25)$$

the results from the analytic evaluation of ϕ_m for different drop clouds were used to find the constant A and the value of $A=1.25$ is suggested. Figure 9 shows the comparison between the analytical evaluation for different drop clouds (from $N=5$ to $N=25$, where the coordinates of each drop where chosen randomly) and the correlation (23) proposed in [7]. The agreement for mono-sized clods of drop is rather satisfactory.

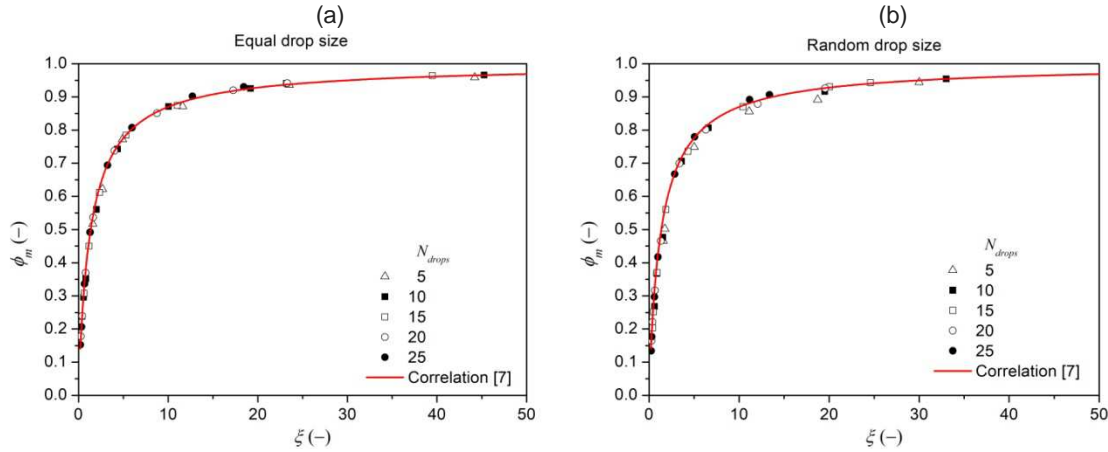


Figure 9. Average screening coefficient ϕ_m as function of the non-dimensional parameter ξ , as predicted by the approximate analytical model (16) and the correlation proposed in [7] for (a) equal and (b) random drop size distribution.

The agreement for the most numerous clouds is quite good, with maximum discrepancies of 4.1% (25 drops) and 3.1% (20 drops). For the 10 and 15 drop case the maximum discrepancy reaches the 6% and 10% for the 5 drop case. The calculations were repeated for clouds of non-uniform drop sizes, drop radii were randomised from 0.91 to 1.45 times the previous ones. The first results show that no significant differences were detected with respect to the previous analysis.

This study shows that the correlation (23), which was derived for uniform and non-uniform drop arrays [7], can be extended with good results to clouds of drops (i.e. randomly distributed drops) by the new definition of the drop inter-distances (equation 25).

Conclusions

An analytical solution to the conservation equations under steady-state conditions, taking into account the effect of drop neighbouring and varying thermo-physical properties, is proposed and analysed.

The model combines an analytical approach to take into account the dependence of all thermo-physical properties on temperature and composition with the point source approach to describe the geometrical disposition of the drops in arrays and clouds.

The application of the model to different evaporating species (water, ethanol, acetone and three hydrocarbons) shows that the effect of varying thermo-physical properties on the prediction of the evaporation rate is of the order of 20%, depending also on the operating conditions.

The model has been compared with existing exact analytical and numerical solutions to assess its validity with satisfactory results. The application to drop clouds of an existing correlation to predict the average screening coefficient for a drop array can be extended with modifications to randomly distributed drop ensembles.

Nomenclature

A_{jk}	Lindsay-Bromley coefficients [-]	$n^{(p)}$	mass flux of species p [$\text{kg m}^2\text{s}^{-1}$]
$A_{\alpha,ref}$	Averaging parameter constant [-]	$N^{(p)}$	Molar flux of species p [$\text{mol m}^2\text{s}^{-1}$]
B_M	Spalding mass transfer number [-]	P	Pressure [Pa]
c	Molar density [mol m^{-3}]	R	Universal gas constant [$\text{J mol}^{-1}\text{K}^{-1}$]
C	Constants, eqs. (10,12,13,14) [m]	R_d	Drop radius [m]
$c_{p,v}$	Heat capacity at constant pressure [$\text{J kg}^{-1}\text{K}^{-1}$]	Sc^M	modified Schmidt number [-]
d	Drop inter-distance [m]	T	Temperature [K]

D_{10}	Mass diffusivity [$\text{m}^2 \text{s}^{-1}$]	U	Velocity [m s^{-1}]
$f(x)$	Function [-]	r, z	Cylindrical coordinates [m]
H	logarithm of gas molar fraction [-]	S	Surface [m^2]
k	Thermal conductivity [$\text{W K}^{-1}\text{m}^{-1}$]	V	Volume [m^3]
m_{ev}	Evaporation rate [kg s^{-1}]	x	Cartesian coordinates [m]
M_m	Molar mass [kg kmol^{-1}]	y	Molar fraction [-]
N_d	Number of drops [-]		

Greek symbols

α_{ref}	Averaging parameter [-]	ρ	Mass density [kg m^{-3}]
β	Non-dimensional drop distance [-]	σ	Cloud compactness parameter [m]
γ	Evaporation rate ratio [-]	ϕ	Drop screening coefficient [-]
δ	Dirac's delta function [-]	ϕ_m	Average screening coefficient [-]
Λ	Non-dimensional parameter [-]	Φ	Harmonic function [-]
ξ	Non-dimensional parameter, eq. (19)	χ	Mass fraction [-]

Subscripts

is	Isolated [-]	s	Surface [-]
mix	Mixture [-]	T	Total [-]
n	Surface normal component [-]	∞	Free stream conditions [-]
ref	Reference conditions [-]		

Superscripts

b	exponent for the specific heat power law [-]	q_j	exponents for conductivity power laws [-]
m	exponent for the diffusivity power law [-]	T	Total [-]
p	Species [-]		

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