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THE INVESTMENT CERTIFICATES IN THE ITALIAN MARKET: A COMPARISON OF QUOTED AND ESTIMATED PRICES

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Investment certificates are securitized derivatives built as a combination of financial instruments. The financial engineering process aims to create new payoff profiles that allow investors to diversify or to hedge the risk of their portfolios. Such instruments are relatively challenging to price, as highlighted in the recent publications for other European markets. The aim of this paper is to analyze whether in the Italian market there are also differences between the quoted price and the estimated price obtained applying a standard pricing approach well known in the literature. In particular, we analyze three representative certificates, belonging to the classes of target coupon certificates and autocallable certificates, that have been most appreciated by the investors during the last years. To evaluate the price, we propose a Monte Carlo approach that computes directly the payoff of the certificates on a set of scenarios for the evolution of the underlying asset. Moreover, studying the payoff profile of the certificates, we investigate and comment on the recent regulatory debate on "complexity". We show that complexity, the new parameter behind return and risk, should not be necessarily associated with the engineering level of the financial products and that, sometimes, complexity is not associated with risk.

Keywords: Investment certificates; securitized derivatives; Monte Carlo pricing; market complexity.

JEL Classification: 90C15, 91B30

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1. Introduction

Investment certificates are financial instruments that offer a competitive yield and usually include some safety clause regarding the principal amount. These instruments replicate very different investment strategies and, therefore, provide investors with a wide variety of risk–return profiles. More specifically, a certificate is a securitized derivative instrument issued by an investment bank that replicates the trend of the underlying with the possibility of having a leverage effect, a premium in the case of market stability and/or a protection property. Recently, these products have become particularly attractive to investors, both institutional and retail, for several reasons: their asymmetric payoff, their not long-only behavior, and, especially in the Italian system, because they allow a fiscal optimization of the portfolio. Certificates are similar to exchange traded funds (ETFs). Both provide passive investment strategies and are traded on exchanges. ETFs and certificates differ because the latter do not pay dividends but do have a maturity; moreover, the buyer of a certificate assumes the risk of default but does not pay any management fee. Certificates, as well as ETFs, can be written on several types of underlying: corporate bonds, sovereign bonds, indexes, currencies, and commodities.

Certificates can comprise plain vanilla or exotic options. Once the exchange market for certificates grew, many types of certificates arose, replicating structured strategies and realizing a wide variety of payoff profiles. They can be used to provide exposure to positive or negative trends, can provide coupons or not, and can give an early ending premium or a maturity premium. Therefore, implicitly, when investors trade certificates, they are also trading many risk sources, see Guillaume (2015), Chang *et al.* (2013), and Chen (2003). Many certificates also include capital protection, and this can be exploited effectively in portfolio strategies, see Albeverio *et al.* (2017). A comprehensive classification of certificates, proposed by the European Structured Investment Products Association (EUSIPA), divides certificates into four categories: capital-protected, yield enhancement, participation, and leverage products. For the Italian market, an alternative classification has been proposed by the *Associazione Italiana Certificati e Prodotti di Investimento* (ACEPI) and it distinguishes four macro-classes according to the provided protection of capital: Capital Protection, Conditional Capital Protection, No Capital Protection, and Leverage. The first three classes are also called investment certificates. The last class is called leverage certificates. Each class includes several different types and each certificate can be issued with further characteristics. For a complete classification, we refer to the EUSIPA and ACEPI websites.[a]

Structured products can have a quoted price different from the theoretical ones obtained using the standard pricing approaches. Several papers have studied pricing issues related to many structured products, including certificates, see e.g. Chen & Kensinger (1990), Wasserfallen & Schenk (1996), Barth *et al.* (2004), Stoimenov & Wilkens (2005), Baule *et al.* (2008), Wallmeier & Diethelm (2009), Baule (2011),

[a] www.eusipa.org; www.acepi.it.

Bernard *et al.* (2011), Henderson & Pearson (2011), Jessen & Jørgensen (2012), Célérier & Vallée (2015). In particular, as highlighted by Henderson & Pearson (2011), it appears that very often the price offered by the issuing institutions and by the market makers differs from that obtained by using a standard pricing approach. Comparing the price on the primary or secondary market with the "theoretical" price calculated with the Monte Carlo model, they conclude that structured products are "mispriced" by something between 2% and 8%. Moreover, as highlighted in Baule (2011), the possible mismatch between the quoted price and the estimated value of a certificate changes during the life of the certificate according to the expectation of the market maker and the investors. Our paper aims to investigate possible differences between the quoted price and the estimated price of certificates in the Italian market and, eventually, their magnitude. We analyze the case of three representative certificates, computing the estimated price by using a Monte Carlo approach that allows testing different hypotheses about the drift of the underlying asset. In particular, we price two target coupon certificates and one autocallable certificate.

No previous papers have performed such an analysis for the Italian market, which is the second largest in Europe in terms of exchange turnover of structured investment products, see EUSIPA (2017, p. 3). Therefore, the purpose of this paper is threefold. First, we propose a Monte Carlo approach to price directly the whole certificate instead of pricing the single assets that compose the replicating portfolio. Second, we investigate the differences between the price quoted by the market maker and the estimated value computed with the Monte Carlo approach. Third, considering this kind of financial instruments, we discuss whether high complexity implies a high level of risk and, in general, also a particularly exotic payoff structure.

The Monte Carlo approach generates a set of scenarios for the evolution of the underlying of the certificate and then computes the payoff generated by the certificate on each scenario. Such an approach is particularly effective for these highly structured instruments and, as in the case of some certificates, an early redemption clause is included. The Monte Carlo approach suggests assuming a specific random process for the underlying of the certificate and then generating scenarios for the evolution of the underlying until the maturity of the certificate. On the path of each scenario, the cash flows generated by the certificate are derived considering all specific features of the certificate. Finally, the price of the certificate is computed as the discounted value of the expected payoff. Therefore, the Monte Carlo approach appears to be very useful even if several parameters must be carefully calibrated and the obtained price is very sensitive to these parameters. A similar method has been adopted to price other types of highly structured products, such as CoCos, see Bertocchi *et al.* (2015) and Ramirez (2011).

Despite the fact that the Monte Carlo approach and its extensions have been widely used to price highly structured instruments, see e.g. Moreno & Navas (2003), Stentoft (2004), Areal *et al.* (2008), and Höcht & Zagst (2010), still the price turns out to be relatively different from those proposed by the market maker. Such a mismatch could be motivated by a different pricing model adopted by the market maker, by a conscious misalignment from the fair value to pursue a precise market strategy, or by information asymmetry. To be able to classify the market price as "wrong" it would be necessary to know the quoted prices of the instruments that compose the replicating portfolio and to prove the existence of an arbitrage opportunity. For the certificates that we consider, as for most of the certificates quoted in the Italian market, such a check is not possible because of the lack of replicating assets within the market. Therefore, the market maker is somehow free to fix the price and our aim is to evaluate if this price is generally greater than the one we estimate or not. In other words, we do not want to investigate the possible motivations for the observed differences between the quoted price and the estimated price, but only to measure their magnitude and observe if the differences are only in one direction, i.e. if the market maker proposes constantly a price higher than the estimated one.

Lastly, this paper wants to make a contribution to the discussion about a third element that could help in classifying the financial instruments further than just in terms of risk and reward: namely, in terms of the element of complexity. The European Securities and Market Authority in ESMA (2014, 2015) and the Italian *Commissione Nazionale per le Società e la Borsa* in CONSOB (2014) introduced "complexity" as a further element to take into account when an instrument must be evaluated for retail selling. This debate starts from the hypothesis that if a financial product is complex and produces a particularly exotic payoff, then it is surely risky. In this perspective, complexity should be associated with risk. Our aim is to show that not all instruments with a complex structure are necessarily very risky. Indeed, our empirical analysis shows that the first two considered certificates have a complex structure, but the parameters that define the payoff are set in such a way that the final payoff replicates a plain vanilla bond and the only difference from a fixed coupon bond is the definition of a trigger event for the coupon payment. Therefore, we prove that there are real cases in which complexity does not necessarily imply risk. Moreover, such structured products should be analyzed in terms of surveillance and cross-product manipulation, see Cumming & Johan (2008) and Cumming *et al.* (2011). However, the Italian market authorities have already imposed a very strict set of rules, such as the mandatory presence of a market maker, the imposition of a maximum bid–ask spread, and the definition of a pre-negotiation phase [for more details see Borsa Italiana (2016)]. Hence, since these products are strictly regulated by surveillance, we focus on whether the market maker exploits the complexity of the instrument itself to manipulate the price.

To summarize, in the Italian market there is a strong demand, especially from retail investors, for financial instruments that have nonsymmetric payoffs, that allow optimizing the fiscal profile of the portfolio, and that do not increase the portfolio risk too much. This demand has been satisfied by investment certificates. This market is strongly regulated and the presence of a market maker is mandatory and, thus, guaranteed. However, the complexity of this class of products raises the question whether the price offered by the market maker is manipulated or not. Applying a naive benchmark model very suitable for pricing highly structured products, i.e. the Monte Carlo approach, we show that the prices can diverge in both directions, weakening the hypothesis of price manipulation.

This paper is organized as follows. In Sec. 2, we present the Monte Carlo approach adopted for pricing the certificates. In Sec. 3, we describe and price three representative certificates. Section 4 concludes the paper discussing the issue regarding complexity.

2. The Definition of the Model

In order to apply the Monte Carlo approach, we need to make some hypotheses about the random process followed by the underlying. In particular, we assume that the price S_t at time t of the underlying of the certificate follows a Geometric Brownian Motion (GBM) of the classical form

$$\frac{dS_t}{S_t} = \mu dt + \sigma dZ_t, \tag{2.1}$$

where μ is the drift, σ is the volatility of the underlying return process, and Z_t is the standard Brownian motion, see Wilmott (1998) and Hull (2015). Given a fixed window length, the drift and the volatility are estimated using the historical series of the underlying. For the discount interest rate, many possible models can be used, see James & Webber (2000) and Khramov (2013). We adopt, as the risk-free rate, the AAA yield curve published by the European Central Bank (ECB). Thus, the discount rate is supposed to be deterministic. The ECB yield curve is estimated with the Svensson model, see Svensson (1994), that generates the spot rate for each Time To Maturity (TTM) with the formula that follows:

$$\begin{aligned} z_{TTM} = \beta_0 &+ \beta_1 \left[\frac{1 - e^{\left(\frac{-TTM}{\tau_1}\right)}}{\frac{TTM}{\tau_1}} \right] \\ &+ \beta_2 \left[\frac{1 - e^{\left(\frac{-TTM}{\tau_1}\right)}}{\frac{TTM}{\tau_1}} - e^{\left(\frac{-TTM}{\tau_1}\right)} \right] \\ &+ \beta_3 \left[\frac{1 - e^{\left(\frac{-TTM}{\tau_2}\right)}}{\frac{TTM}{\tau_2}} - e^{\left(\frac{-TTM}{\tau_2}\right)} \right], \end{aligned} \tag{2.2}$$

where β_i, $i = 1, \dots, 3$, and τ_j, $j = 1, 2$, are the parameters estimated by the ECB. Given a series of daily returns ρ_t of the underlying of the certificate, the process to price the certificate at time T employs the following steps:

- **Step 1.** We compute the drift and the volatility of the underlying on the last l days:

$$\mu_T = \mathbb{E}[\rho_{T-k} | 1 \le k \le l], \tag{2.3}$$

$$\sigma_T^2 = \mathbb{E}[(\rho_{T-k} - \mu_T)^2 | 1 \le k \le l]. \tag{2.4}$$

- **Step 2.** Given the initial price p_T and a number S of scenarios, for each scenario s, we generate the daily evolution of the price $p_{t,s}$ of the underlying following the GBM hypothesis (2.1), i.e.

$$p_{t,s} = p_{t-1,s} \cdot e^{\left(\mu_T - \frac{\sigma_T^2}{2}\right) + \sigma_T \xi_t}, \quad \forall s, T < t \le H, \tag{2.5}$$

where ξ_t is a random effect sampled by a standard normal distribution. The scenarios are generated until t reaches the maturity H of the certificate. An alternative procedure will adopt the risk-free rate z_M at time M as the drift in the GBM, i.e.

$$p_{t,s} = p_{t-1,s} \cdot e^{\left(z_M - \frac{\sigma_T^2}{2}\right) + \sigma_T \xi_t}, \quad \forall s, T < t \le H. \tag{2.6}$$

- **Step 3.** At each day t, on all scenarios, we take into account the cash flows $y_{t,s} = f(p_{t,s})$ generated by the certificate. The function $f(\cdot)$ considers all the specific properties of the certificates.
- **Step 4.** We estimate the price of the certificate as the discounted value of the average over the scenarios of the cash flows, i.e.

$$P_T = \sum_{t=T+1}^{H} \frac{\sum_{s=1}^{S} y_{t,s}}{S} \cdot e^{-z_{t-T} \cdot (t-T)}. \tag{2.7}$$

3. Case Study and Numerical Analysis

In order to replicate the yield curve, the parameters β_i and τ_j of the Svensson model (2.2) are taken from the ECB website. On each quotation day of the certificate, we estimate its price according to Eqs. (2.3)–(2.7) and we observe some stability when the number of scenarios reaches the magnitude of 10^4. Therefore, we present the results obtained with 10000 scenarios. The results are very sensitive to the value of the drift used in the GBM. Therefore, for each analyzed certificate, we propose four different settings:

- Equation (2.5) with a long-term estimation of the drift based on the average daily return over the previous five years of the underlying quotations (U5Y).
- Equation (2.5) with a short-term estimation based on the last year (U1Y).
- Equation (2.6) with the one-year risk-free rate given by the yield curve (RF1Y).
- Equation (2.6) with the six-month risk-free rate (RF6M).

When we adopt the risk-free rate as the drift, the volatility is still based on the historical estimate over the last five years.

The proposed method is applied to three representative certificates quoted on the SeDeX market[b]: two issued by Banca Aletti & C. and one by BNP Paribas Arbitrage Issuance. The certificates issued by Banca Aletti & C. belong to the class of the so-called "target coupon certificates", i.e. certificates that pay a coupon if the value of the underlying touches a certain level, called the coupon threshold. The certificate issued by BNP Paribas Arbitrage Issuance has a more complex structure than the previous certificates, since it is an autocallable certificate, i.e. it offers an early redemption if certain conditions are met.

The daily quotations of the certificates have been downloaded from Bloomberg. All computations have been performed using MATLAB R2013b with an Intel(R) Core(TM) i7-4510U CPU @ 2.60 GHz with 8.00 GB RAM running Windows 10.

3.1. *Final payoff computation*

All certificates share common features and components. Therefore, for a homogeneous description of their specific features, we refer to the terminology presented in Table 1.

Banca Aletti & C. adopts the target coupon certificate type and the final payoff computation follows the rule

- if $SF < PL$, then the payoff is $NA \cdot PP$;
- if $PL \leq SF < SI$, then the payoff is $NA \cdot SF/SI$;
- if $SF \geq SI$, then the payoff is NA.

Figure 1 shows an example of the structure of the coupon payments and maturity payment according to the value of the underlying.

Table 1. Explanations of the acronyms.

Nominal Amount (NA):	The stated value of an issued security; stated value means that it is the value assigned to a corporation's stock for accounting purposes.
Initial Valuation Date (IVD):	The date on which the Share Initial is observed.
Final Valuation Date (FVD):	The date on which the Share Final is observed; it can be also the maturity of the instrument.
Share Initial (SI):	The value of the underlying at the Initial Valuation Date.
Share Final (SF):	The value of the underlying at the Final Valuation Date.
Protection Level (PL):	The minimum level of the underlying value below which the Protection Percentage will be recovered, if certain conditions are met.
Protection Percentage (PP):	The percentage of capital invested that will be recovered, if certain conditions are met.

[b] http://www.borsaitaliana.it/cw-e-certificates/notiziedettaglio/brochurerebrandedmercato13_pdf.htm.

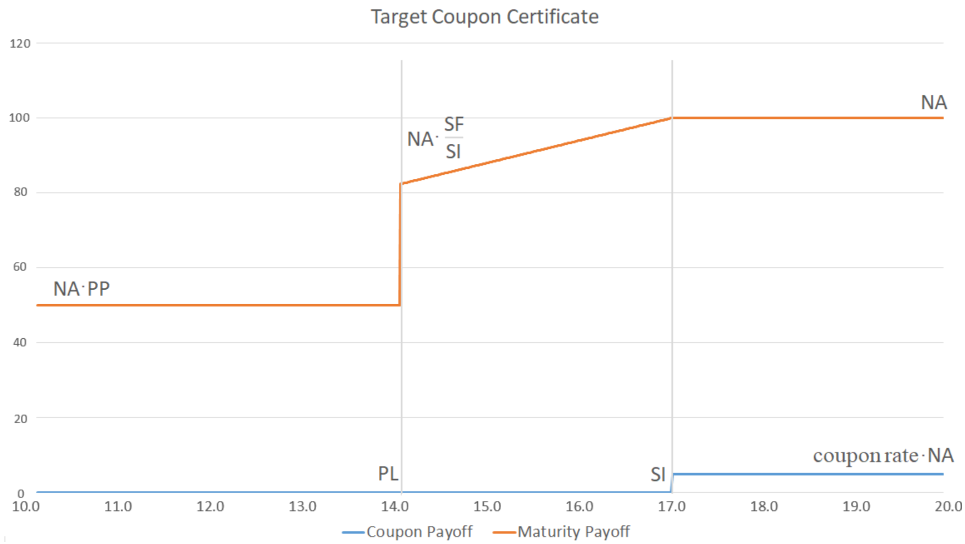

Fig. 1. Representative payoff for the target coupon certificate for $PL = 14$, $SI = 17$, $NA = 100$, $PP = 0.5$, and coupon rate equal to 5%. Orange: maturity payment. Blue: coupon payment.

In contrast, BNP Paribas Arbitrage Issuance adopts the autocallable equity protection certificate, where the final payoff follows the so-called knock-in rule for a given strike K:

- if $SF/SI \geq K$, the payoff is $NA \cdot SF/SI$;
- if $SF/SI < K$, the payoff is $NA \cdot K$.

Figure 2 shows an example of the maturity payment according to the value of the underlying.

After defining the structures for the payoff just mentioned, we generate scenarios and evaluate the payoffs of the certificates following the pricing procedure explained in Sec. 2. Figure 3 presents four representative scenarios for the case of Cert 1 under the RF1Y hypothesis.

3.2. *Certificate Banca Aletti & C. — underlying Generali*

The first certificate (Cert#1, ISIN IT0004956964) is written on Generali's stocks, it has been negotiated since 20/12/2013 and it paid five coupons, on 18/09/2014, 17/09/2015, 15/09/2016, 14/09/2017 and 20/09/2018. The NA is equal to €100. The maturity of the certificate is the last coupon payment date, which is also the FVD. The coupon rate is 5%. Coupons are paid when the underlying is higher than a threshold, which is set to $SI =$ € 14.75. The PL is equal to SI, while PP is 100%. At maturity, the certificate pays a final payoff computed as described in the previous subsection. Clearly, since in this specific case $PL = SI$ and $PP = 100\%$, the final payoff is always equal to NA and, therefore, the payments change from those

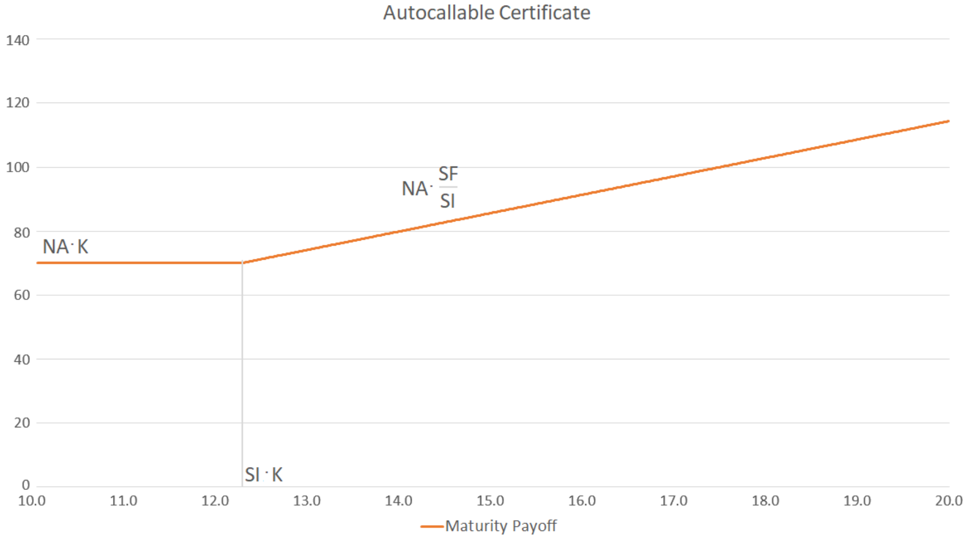

Fig. 2. Representative maturity payoff for the autocallable equity protection certificate for $K = 0.7$, $SI = 17.49$, and $NA = 100$.

presented in Fig. 1 and become simpler, as shown in Fig. 4. As a matter of fact, we obtain a certain and fixed amount at maturity and five cash flows paid under the condition that the underlying is greater than SI.

This kind of payoff limits the uncertainty in the coupon payments, reducing the level of risk of the instrument. This implies that highly structured products do not necessarily coincide with risky instruments. Therefore, referring to the note of CONSOB (2014), we believe that complexity is indeed a third dimension that must be considered for a more comprehensive evaluation of a financial product. Furthermore, it appears clear that assessing the complexity of a product is far from simple, since the general criteria that consider only the product category can lead a product that is indeed very similar to plain vanilla instruments to be classified as complex.

Observing Fig. 3, we notice that the generated cash flows respect the defined rules. For instance, for the first scenario, the price is always above PL and then there is always a coupon equal to €5, at maturity the price is below and then the final cash flow is €100 without coupon; similarly, for the second scenario, the price goes above PL in the last three evaluation dates including the maturity, and the cash flows reflect this behavior. When the price of the underlying is always below PL until maturity, cf. the third scenario, the certificate pays only the final payoff and its coupon. The fourth scenario shows a more volatile movement of the price around PL.

Then, using (2.7), we compute the expectation of the cash flows over all scenarios and we discount this value to get the estimated price. This procedure is repeated for all days until maturity to estimate the evolution of the price of Cert 1. The results for the pricing of Cert 1 are presented in Fig. 5 for the four proposed cases of the underlying drift.

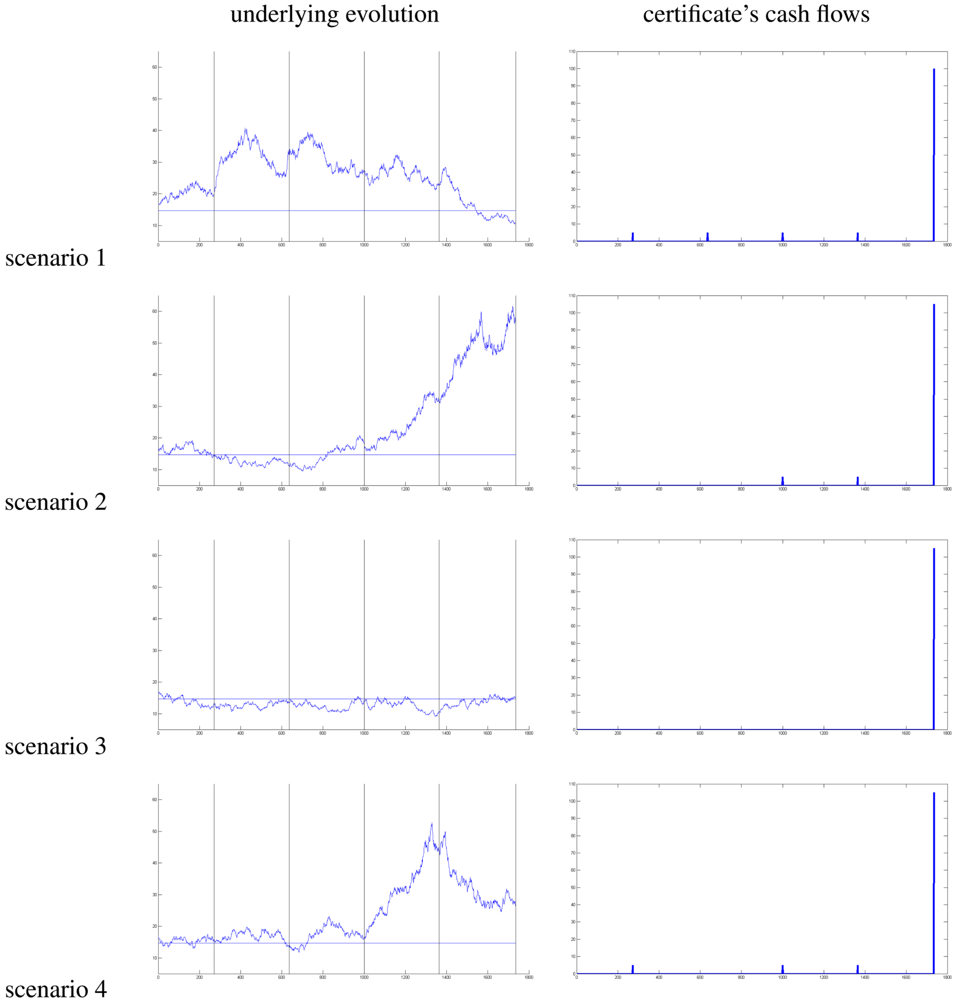

Fig. 3. Representative scenarios for the pricing of Cert 1. On the left-hand side is the evolution of the price of Generali under the assumption RF1Y; on the right-hand side are the corresponding cash flows generated by the certificate. In the left-hand side, the vertical lines represent the coupon payment dates, whereas the horizontal line, the PL.

The estimated prices are almost always higher than the market prices. Only the case U1Y offers too optimistic estimates until the beginning of 2016 while the other cases are more aligned with the trading prices. The estimated prices are far from the quoted prices until the middle of 2015, then all the cases show a convergence approaching the maturity. In particular, observing Table 2, we notice that on average all the estimates are higher than the market price. The U1Y case diverges significantly from the market observations, by an average distance of 5.73%. The U5Y estimate has the lowest average distance, 1.11%, with some periods of high

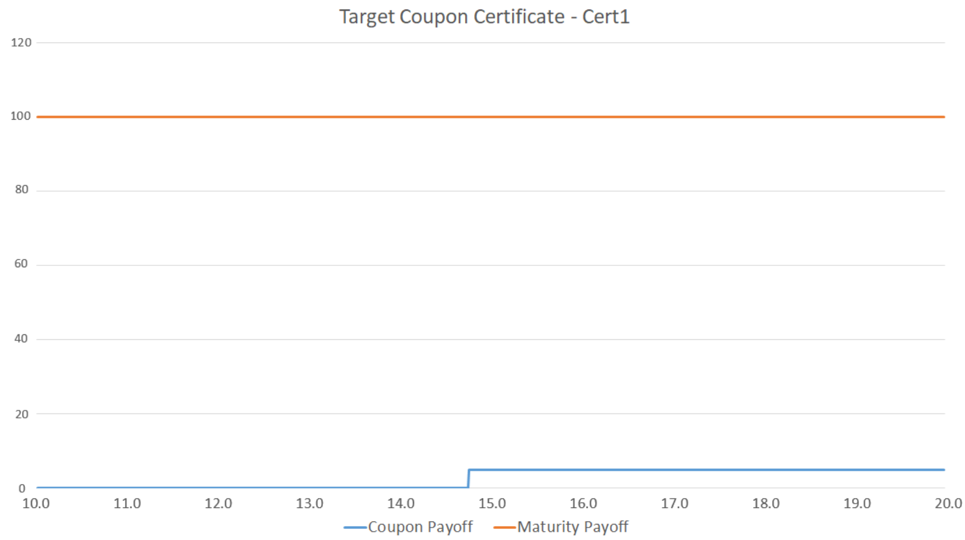

Fig. 4. Representative payoff for the target coupon certificate Cert 1, i.e. for $PL = SI = 14.75$, $NA = 100$, $PP = 1$, and coupon rate equal to 5%. Orange: the maturity payment. Blue: the coupon payment.

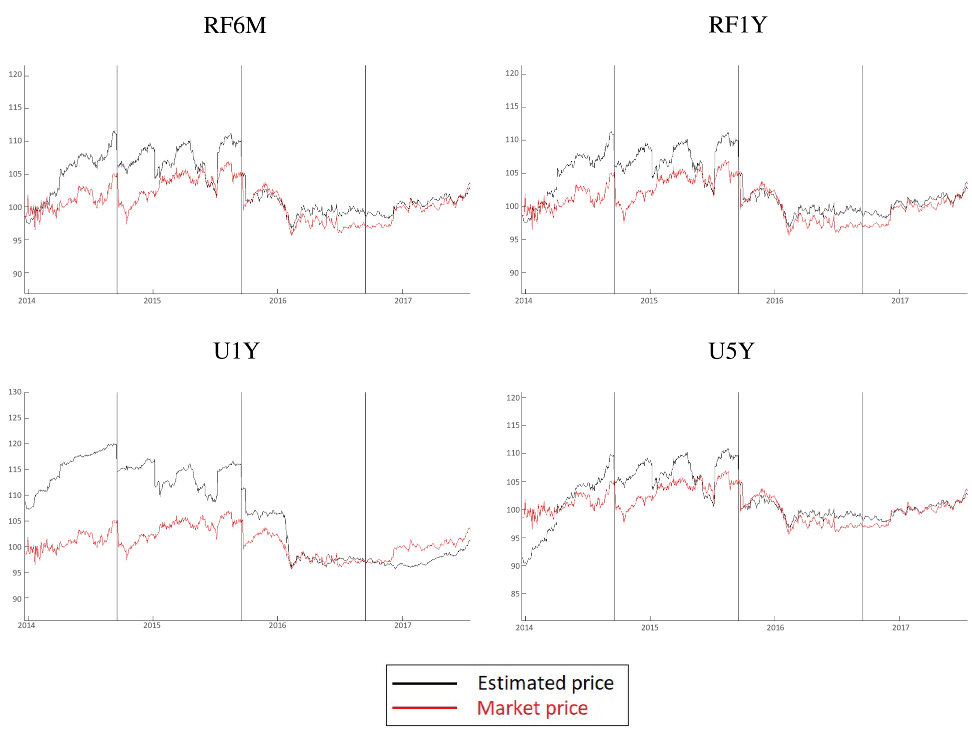

Fig. 5. Results for the Cert 1 quotation under different drift hypotheses.

Table 2. Cert 1 statistics of the relative distances between the estimated and market prices. Values are in %.

	RF6M	RF1Y	U1Y	U5Y
Max	8.0533	7.8064	18.7683	7.4023
Average	2.3392	2.3067	5.7306	1.1168
Min	−4.1333	−4.1057	−5.336	−11.0917
Std. dev.	2.3251	2.2583	6.8085	2.7744

distance touching a negative distance of 11.09%. The RF1Y has the lowest difference in standard deviations, 2.26%, and also a relatively low average distance.

3.3. *Certificate Banca Aletti & C. — underlying Unicredit*

The second certificate (Cert 2, ISIN IT0004963754), i.e. the second target coupon issued by Banca Aletti & C., is written on Unicredit's stocks. It started to be negotiated on 28/02/2014 and paid five coupons, on 16/10/2014, 15/10/2015, 20/10/2016, 19/10/2017, and 18/10/2018. The NA is equal to €100. The FVD of the certificates and its maturity are on 18/10/2018. The coupon rate is 5.25%. The coupon threshold is equal to the initial value of the underlying, i.e. €27.7624. The PL is equal to the initial value of the underlying as well. At maturity, the certificate pays a final payoff computed as described in Sec. 3.1. As with the previous certificate, since $PL = SI$ and $PP = 100\%$, the final payoff is always equal to NA and the payment structure is similar to the one shown in Fig. 4. Therefore, as for the previous certificate, the fact that this instrument is highly structured does not make it a risky instrument. Indeed, the apparent complexity is mitigated by the choice of the values of the parameters and the product becomes a coupon bond where the coupons are paid depending on a specific condition. Then, again, the original complexity of the certificate does not induce a particularly odd payoff. The results for the pricing of Cert 2 are presented in Fig. 6.

The estimate converges to the market prices in all the cases upon getting close to the maturity. The gap is remarkable in the U1Y case. In the other cases, the estimated price and the quoted price often intersect each other even if they rarely show similar movements. From Table 3, we observe that the estimated prices of both the cases RF6M and RF1Y are very close to the market prices with average distances of 0.61% and 0.59%, respectively, and also the differences in their standard deviations are significantly low. The U1Y case, as for Cert 1, shows a largely positive distance with an average of 6.16% and a maximum of 21.21%. The U5Y estimates are on average lower than the market observations. These remarks suggest that the market maker does not offer a price higher than the one estimated under standard assumptions but, sometimes, the quoted price is even cheaper.

3.4. *Certificate BNP Paribas Arbitrage Issuance — underlying Eni*

The third certificate (Cert 3, ISIN NL0010623500) was issued by BNP Paribas Arbitrage Issuance and written on Eni's stocks. The final payoff is subject to a

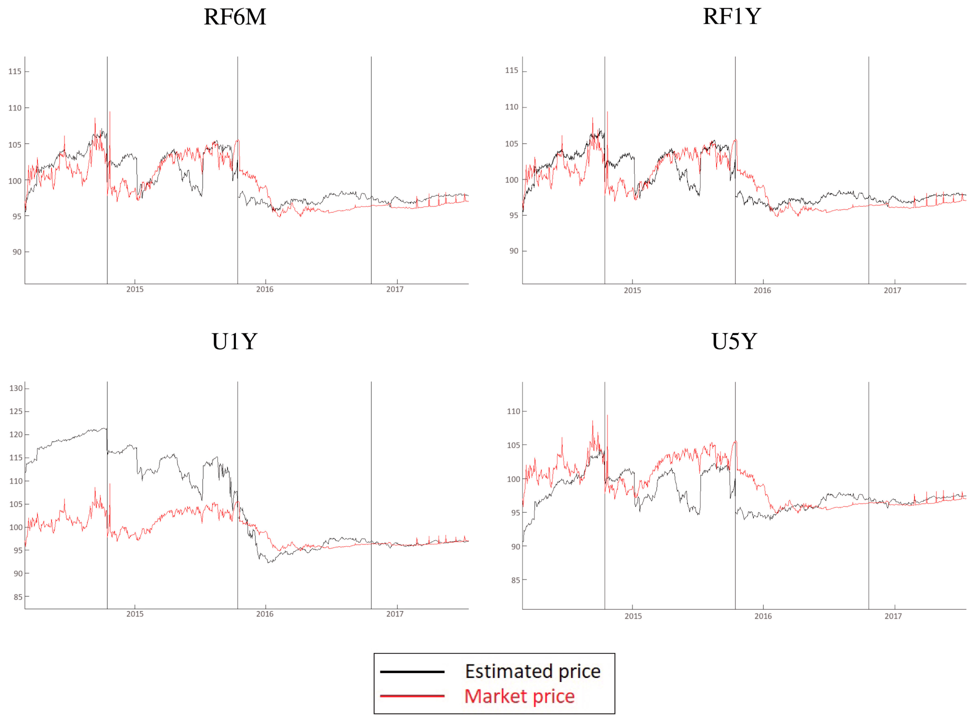

Fig. 6. Results for the Cert 2 quotation under different drift hypotheses.

Table 3. Cert 2 statistics of the relative distances between the estimated and market prices. Values are in %.

	RF6M	RF1Y	U1Y	U5Y
Max	6.1231	6.0298	21.2116	3.8752
Average	0.61	0.5943	6.1606	−1.4169
Min	−7.5652	−7.5498	−5.9648	−10.0903
Std. dev.	2.0422	2.0312	7.5981	2.7435

condition of early expiration of the instrument. The certificate was issued on 30/12/2013 with a nominal amount of €1000 and exercise date 02/01/2018. The autocallable property implies that at the Valuation Dates (cf. Table 4) if the price of the underlying is greater than or equal to a strike price, then we observe an

Table 4. Early redemption condition.

Valuation date	*ExitAERRate*
30/12/2014	8.50%
30/12/2015	17.00%
30/12/2016	25.50%

automatic early redemption (AER) and the certificate expires. The strike price is set to €17.49. In the case of AER, the payoff is computed with the following formula: $NA \cdot (1 + ExitAERRate)$, where the $ExitAERRate$ depends on the valuation date on which the condition is met according to Table 4. In the absence of AER, the final payoff is computed according to the rules described in Sec. 3.1 and in particular Fig. 2 with $K = 70\%$. In this case, unlike the previous ones, the specific properties of the certificates are fully exploited and the financial instrument is and remains particularly complex both in its definition and in the shape of its payoff. Hence, this is a typical case in which the complexity of the instrument turns out to induce a high level of risk for the investor. The results for the pricing of Cert 3 are presented in Fig. 7.

The Monte Carlo approach gives an estimate that follows the movements of the quoted prices with a gap. This gap is negative, i.e. the estimated price is smaller than the quoted price, when we consider the hypotheses RF6M and RF1Y. In the U5Y case, the effect is the opposite, and the estimates are slightly higher than the quoted prices. In the case of U1Y, the estimates reflect too much draw-down of the underlying from the middle of 2015 given by the short historical data window on which we estimate the drift. In particular, in Table 5, we measure such behaviors and we

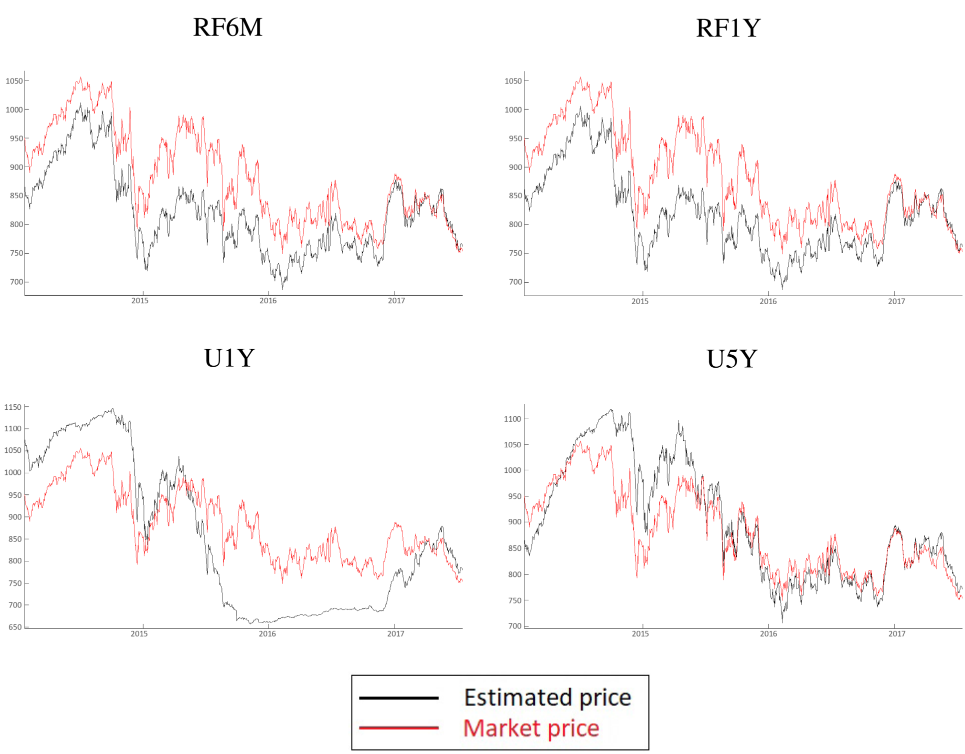

Fig. 7. Results for the Cert 3 quotation under different drift hypotheses.

Table 5. Cert 3 statistics of the relative distances between the estimated and market prices. Values are in %.

	RF6M	RF1Y	U1Y	U5Y
Max	1.5961	1.7527	22.0417	20.4086
Average	−6.8129	−6.9572	−4.2773	1.7681
Min	−14.0447	−13.79	−28.4908	−9.1541
Std. dev.	3.9872	3.9808	12.5575	5.6296

confirm that cases RF6M, RF1Y, and U1Y have a negative average distance and, moreover, the case U1Y presents also a huge standard deviation, 12.56%, highlighting that the two paths, estimated and market prices, do not follow the same pattern. The U5Y case has, in absolute value, the smallest average distance, 1.77%, but shows a higher standard deviation than the RF cases.

To summarize, comparing the distance statistics of the three analyzed certificates, we notice that the U1Y estimate is generally the farthest from the quoted price, having the highest standard deviation and, in two cases of the three proposed, showing the highest average distance. For Cert 2 and Cert 3, the U5Y estimates have a distance of a different sign from that of RF6M and RF1Y, but these three approaches turn out to adapt well to the behavior of the market prices: the U5Y estimates have, in general, the lower distance in absolute value, while the RF6M and the RF1Y have the lowest standard deviation.

4. Conclusion

In this paper, we investigated the price of investment certificates in the Italian market, using a Monte Carlo approach as the pricing model.

We applied the Monte Carlo approach using the historical volatility of the underlying returns and the discount factor given by the AAA yield curve estimated by the ECB. In this paper, we discussed four alternative choices of the drift of the underlying process. In general, when we use the ECB risk-free rate as the drift, the estimated values are closer to the quoted prices.

As far as the RF6M and the RF1Y cases are concerned, for Cert 1 the market price is slightly lower than the estimated price, which is probably due to the fact that the instrument is relatively close to a plain vanilla bond and, therefore, the market maker does not bear a significant uncertainty. For Cert 2, because of the same reason, the two price paths are also very close, crossing each other and finally converging, but the estimated price is, on average, slightly higher than the market price. For Cert 3, the most structured product, the market price is higher than the estimated price and this fact highlights that the market maker wants to hedge against the uncertainty carried by the instrument by overestimating the value of the instrument.

The differences between the estimated price and the market price of the certificates typically decrease as one approaches maturity. As explained in Sec. 1, we cannot state uniquely which price, the estimated or the quoted, is the "correct" one because we would need to evaluate the price of the replicating portfolio, whose components are not traded in the market.

Over the whole of the life of the certificate, the difference between the quoted and the estimated prices could depend on the wish of the market maker to exploit the difficulties of the investor to correctly price such a complex financial instrument, as suggested in Henderson & Pearson (2011). This motivation is particularly well founded when the market price is higher than the estimated value and when the instrument appears artificially over-complicated, since it eventually produces the same payoff as a plain vanilla instrument. In contrast, when the market price is lower than the estimated price, the market maker probably is trying to perform some marketing strategies with respect to the issuer's competitors, as suggested in Baule (2011). However, the aim of this paper is not to investigate the reasons for the differences, which could be the motivation for a further extension of this research, but only to highlight that also the Italian market suffers a significant misalignment between the estimated prices and market prices. Moreover, we want to show that such a misalignment does not have a unique direction, meaning that the Italian market makers do not want to obtain an extra margin penalizing the investors, or, at least, not always.

To summarize, there are several conclusions to be drawn from this analysis. First, we suggest using the Monte Carlo approach not to price the single components of the replicating portfolio, but to compute directly the payoff produced by the certificate. Indeed, the Monte Carlo approach appears to be very useful for pricing highly structured financial products since it allows specifying various types of payoff structures. The input parameters of the Monte Carlo approach must be calibrated by an expert. Second, applying the Monte Carlo approach, we have shown that in the Italian market also, the estimated price of the investment certificates can be remarkably different from the market price. Even if we do not analyze the motivation for this misalignment, we can suggest the investors to carefully evaluate whether the price of a certificate meets their expectations and not to renounce this control because of the apparent complexity of the instrument. Indeed, we remark that in several cases the certificate issuers define the parameters of the certificates in such a way as to make the final payoff much closer to that of a simpler instrument. Such behavior represents an example where the high complexity of a financial instrument can still correspond to a low-risk instrument, but the market maker exploits the complexity to pursue some pricing flexibility without making the investor fully aware of it.

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