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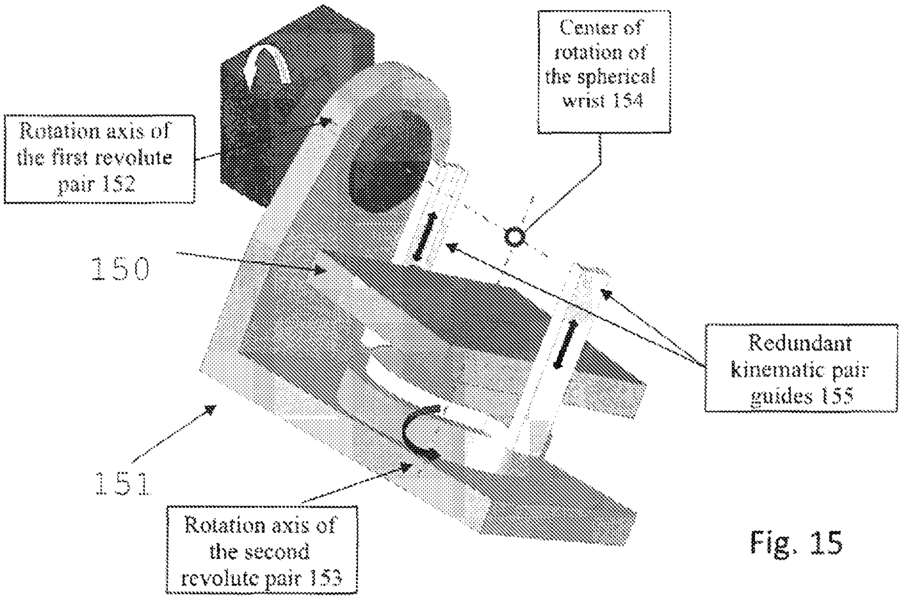

Fig. 15

(57) **Abstract:** A machining apparatus (100) to machine an object (101) is described, comprising: a machining machine (103) provided with a tool (104) such as to operate on the object (101); a handling device (102; 105) designed to orient and position the object (101) relative to the tool (104), causing at least one relative rotation movement, of the object (101) with respect to the tool (104), an instantaneous center of rotation (C) being associated to the latter; a control module (110) designed to control the handling device according to a machining target of said object (104). The control module (110) is designed to manage the positioning and orientation of the object (101) with respect to the tool (104) as well as to achieve, during the implementation of the machining target, a target of minimizing a distance (L) between the center of rotation (C) and the tool (104).

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"Apparatus for machining an object"

TEXT OF THE DESCRIPTION

Field of the invention

The present invention relates to machine tools operating by removing material or those operating by adding material.

Known Art

In the machining operations by adding material, or those by removing chip, systems are known having on the whole five degrees of freedom for positioning the workpiece. For example, three degrees of freedom are managed by prismatic pairs in order to position the workpiece in the space, whereas two revolute pairs are used in order to orient the workpiece.

Document US9266288 describes the structure of a 3D printer wherein the worktable is moved by means of a spherical wrist and a prismatic guide while the shaping head is moved by a slide.

Document WO2015168799 describes a parallel mechanism, a frame and a platform; the frame supports a plurality of single arms and a plurality of double arms causing the movement of the platform. The parallel mechanism described in this prior art document has a redundant number of degrees of freedom.

The Applicant noticed that in the machining apparatuses made according to the known art, when subsequent positioning of the workpiece are carried out in the machining, a progressive degradation of the machining quality can occur.

Dimitris Mourtzis's document: "computer numerical control of machine tools-chapter 16: 5-axis Machining, October 31, 2013, pages 1-61, XP055567496 does not consider the possibility of varying the configuration of the mechanism in order to limit the errors, but only the control strategy, which means control on the velocity module, whereas all the remaining is constrained to follow the path.

S.L.Chiu's document: "Task Compatibility of Manipulator Postures", International Journal of Robotics Research, vol.7, no.5, October 1, 1988, pages 13-21, XP055362753, generally describes a redundant mechanism being able to be arranged such that torques as low as possible are required, given a determined force on the end effector.

Summary of the Invention

It is a problem addressed by the present invention to provide a machining apparatus to machine an object wherein the degradation of the machining quality upon subsequent positioning of the workpiece, which is found in the known art apparatuses, is reduced or removed.

An additional issue is that in the known devices there are limited workspaces that thus do not allow large workpieces to be machined.

By considering for example a 300 mm travel in a given direction, de facto, a piece having a bulk of 300 mm in that direction can not be machined. The reason for this is that the workspace depends not only on the kinematic pairs of the platform, but also on the piece bulk and the machine configuration. More in detail, the workspace is not only the ability of positioning the tool (or nozzle) tip in a given position relative to the blank in terms of coordinates, but doing this according to a given arrangement (thus with a given angle between tool and piece). This results in a further "shrinkage" of the workspace.

The above mentioned issues are sorted out by a machining apparatus as defined by the independent claim 1 and by its preferred embodiments described by the dependent claims.

According to another object, the present invention relates to a machining method as defined by the independent claim 10.

Further characteristics and improvements of the method are object of the dependent claims.

Due to the addition of one or more degrees of kinematic redundancy to the worktable, the afore mentioned problems are overcome.

Indeed, in addition to reducing the error of relative positioning between piece and tool, the available work space for machining the pieces increases, the kinematic pairs travels remaining the same, thus increasing the range of pieces that can be machined by the same machine.

As it will also outlined in the following, adopting the solution according to the present invention for the workpiece platform "optimizes" the shear strains. The forces involved are thus lower and the platform and the machine tool can be designed to be made with lighter structures, since the reduction of the component stiffness is compensated by the inherent increase of the mechanism stiffness given by its optimized "configuration".

Furthermore the lighter structures also have a synergic effect on the workspace, allowing the travels of the kinematic pairs to be increased and an additional increase of the workspace is obtained, the machine bulk remaining the same.

An additional advantage of the invention is the reduction of velocity, accelerations and jerk. It results indeed that the "displacement" of the instantaneous rotation center of the table causes the reduction of accelerations and jerk of the various parts of the machine. This results in huge benefits from the stress containment point of view.

Figures 17 and 18 schematically depict the phenomenon referred and show a machine tool at two machining times.

In reference to figure 17, the object is to carry out the machining, first of the surface A denoted by 161, and then of surface B denoted by 162, by the conventional

5-axes machines, at the corner between the two surfaces, the worktable 160 has to be rotated around the center 163.

In this operation, the tool 164 needs to "trace" the piece 165, thus, as shown in figure 18, the tool needs to travel a circular path centered on the axis of rotation of the table.

Considering the arc of a circle swept from point P between times t0 and t1, figures 19 a), b), c), d) and e) show the path of the center P of the tool 164, the x component of the position of said center, the x component of the displacement velocity of the center P, the x component of the acceleration of the center P and the x component of the jerk of the center P, respectively.

The kinematic components are calculated based on the assumption that the curvilinear path spans along an arc of a circle corresponding to an angle of 90° and is traveled at constant velocity w whereas the angular velocity is defined as the first derivative of angle θ with respect to time:

$$\dot{\theta} = \frac{d\theta}{dt} = \frac{w}{r} \Rightarrow \theta = \frac{d\theta}{dt} \cdot t = \frac{w}{r} \cdot t \qquad (1)$$

The kinematic quantities of interest are velocity, acceleration and jerk, that are derived from the motion law and while limiting for the sake of simplicity only to the X component, one has:

$$x = -rcos\theta = -rcos\left(\frac{w}{r}t\right) \qquad (2)$$

in order to obtain the velocity the derivative with respect to time is executed:

$$v_x = \frac{dx}{dt} = wsen\left(\frac{w}{r}t\right) \qquad (3)$$

which first derivative is the acceleration:

$$a_x = \frac{dv_x}{dt} = \frac{d^2x}{dt^2} = \frac{w^2}{r}cos\left(\frac{w}{r}t\right) \qquad (4)$$

further differentiating with respect to time the jerk is obtained:

$$j_x = \frac{da_x}{dt} = \frac{d^3x}{dt^3} = -\frac{w^3}{r^2}sen\left(\frac{w}{r}t\right) \qquad (5)$$

As known to any person skilled in the art, non-null accelerations and jerk lead to dynamic phenomena that stress the mechanical components of the machine tool, as the figures in the present example show, compelling the designers to properly size such parts. The measures according to the present invention mitigate these effects and are an unquestionable benefit in terms of increase of machining quality, bulks, and sizes of the pieces that can be machined. Indeed, the "displacement" of the instantaneous center of rotation of the worktable obtained by the present invention results in the reduction of accelerations and jerk of the various parts of the machine. By using a conventional 5-axes machine, the point of instantaneous rotation is fixed on the worktable. In machining, due to the distance between two points along a path of the tool, as a result of the table rotation, the tool has to "trace" the piece, and thus an acceleration has to be imposed, not being constant over time (jerk different from zero). This effect is more pronounced as longer is the distance between the afore said points and thus the tool path along the piece. In the case according to the present invention, instead, in which the platform has at least one redundant degree of freedom, and is controlled such as to minimize (o cancel) the distance between instantaneous center of rotation of the piece with respect to the tool and revolute pair of the platform, the situation is different and a minimization of the accelerations and their variation are obtained, i.e. jerk, resulting in benefits on the machining quality.

Brief description of the figures

The present invention is described in detail in the following, for illustration purposes and without limitation, in reference to the accompanying drawings, wherein:

- figure 1 schematically shows an embodiment of an apparatus for machining objects according to the invention;
- figures 2-4 show a particular example of the machining apparatus, in different operative steps.
- Figure 5 shows a first general application example related to a serial mechanism (open kinematic chain).
- Figures 6 and 7 show how the Lagrangian variables related to the example of figure 5 are defined.
- Figure 8 schematically shows the solution of the optimization procedure according to the present invention, applied to a general serial kinematic chain as that of figure 5.

- Figure 9 shows an example of parallel, or closed kinematic chain, mechanism.

- Figures 10 and 11 graphically show the steps of defining the Lagrangian variables of the system and active forces, similarly to figures 6 and 7, but in this case for the closed kinematic chain of figure 9.

- Figure 12 schematically shows the solution of the optimization procedure according to the present invention, applied to a general closed kinematic chain as that of figure 9.

- Figure 13 shows a flowchart of an embodiment of the method according to the present invention.

- Figure 14 shows an embodiment variation of the method according to the present invention.

- Figures 15 and 16 show an example embodiment related to the three-dimensional extension of the simplified bi-dimensional example of figures 2 to 4.

- Figures 17 and 18 show an example of machining a piece with a conventional type 5-axes machine according to the state of the art.

Figure 1 schematically shows an embodiment of an apparatus 100 for handling and machining an object 101. The handling and machining apparatus 100 (in the following, also named "apparatus 100", for the sake of brevity) can be structured in order to carry out a machining by removing material (subtractive manufacturing) or a machining by adding material (additive manufacturing).

The apparatus 100 comprises a tool 104 able to operate on the object 101 according to the required machining and a relative handling device 200 to handle the device 101 with respect to the tool 104.

The tool 104 is a tool suitable for machining by removing material, or it can be a nozzle suitable for machining by adding material.

The relative handling device 200 is designed to orient and position the object 101 relative to the tool 104 and cause at least one rotation movement of the object 101 that allows an instantaneous center of rotation (symbolically denoted by character C in figure 1) to be defined.

Since the positioning and orientation of the object 101 with respect to the tool 104 are to be meant in a relative sense it is possible that, with respect to a fixed reference frame, the object 101 only is handled, the tool 104 only or both are moved.

According to a particular example, the relatively handling device 200 comprises a first handling device 102 to handle the object 101 and a second handling device 105 to handle to tool 104.

The first handling device 102 is designed to support, orient and position the object 101 with respect to the tool 104 during the machining steps and, according to an example, cause the rotation movement of the object 101 around the instantaneous center of rotation C.

The second handling device 105 is designed to support, orient and position the tool 104 with respect to the object (101) during the machining steps. The tool 104 and the second handling device 105 are part of a machining machine 103.

The apparatus 100 also comprises a control module 110 to control the handling device 200. The control module 110 is designed to manage the positioning and the orientation of the object 101 with respect to the tool 104 such as to implement a machining target.

The machining target (working task) is that corresponding to the machining the apparatus 100 carries out as its primary function. For example, the machining target can be: milling, turning, grinding if the apparatus 100 carries out a machining by removing material. Or, the machining target can be producing a predetermined object, if the apparatus 100 carries out a machining by adding material (for example, 3D printing).

Furthermore, the control module 110 is designed to manage the positioning and orientation of the object 101 with respect to the tool 104 to achieve, during the implementation of the machining target, also a target of minimizing a distance L between the instantaneous center of rotation C and the tool 104.

In other words, the relative positioning and orientation of the object 1 relative to the tool 4 is managed for the purposes of the machining target also causing the minimization of the distance L between the instantaneous center of rotation C and the tool 104. As it will also be illustrated hereinafter, in reference to the apparatus 100, minimizing the distance L (vector distance) allows reducing the sensitivity, with respect to errors of angular positioning the device 102, of positioning the object 101 with respect to the tool 104.

In particular, in the control of the apparatus 100 carried out by the control module 100, the machining target has the role of primary target, whereas the target of minimizing the distance L has the role of secondary target.

Returning now to the structure of the apparatus 100, the first handling device 102 comprises, in particular, first kinematic pairs 106 (KPs) for the movement of the object 101 and related first actuators 107 (ACT_S).

For example, the first kinematic pairs 106 comprise at least one pair allowing the rotation of the object 101 to be machined according to one, two or three degrees of freedom. In particular, in order to obtain the rotation of the piece 101, the kinematic pairs of the following group can be employed (alternatively or in combination): revolute pair, cylindrical pair, screw pair, spherical pair.

The first actuators 107, cooperating with the first kinematic pairs 106, can comprise, for example, electric motors or linear actuators such as: mechanical actuators, hydraulic actuators, pneumatic actuators or electro-mechanic actuators.

The second handling device 105 comprises, in particular, second kinematic pairs 108 (KP_M) for the movement of the tool 104 and related first actuators 109 (ACT_M). The second kinematic pairs 108 and the second actuators 109 can be of similar type as the one mentioned afore in reference to the first handling device 102. The apparatus 100 is also provided with kinematic pairs and related actuators (not shown) for actuating the tool 104 for machining purposes, such as for example a rotation around its own axis or material extrusion.

The control module 110 comprises, for example, at least one computer executing control software 111 comprising a first algorithm SW1 related to the implementation of the machining target, and a second algorithm SW2 related to the minimizing of the distance L, the algorithms cooperating to one another. The apparatus 100 is, preferably, of the numerical control type and the control module 110 is such to send command signals S_{COM} to the apparatus 100.

The apparatus 100 is furthermore provided with sensors (not shown) needed for measuring quantities to be controlled such as, for example: positions, orientations, velocities, applied forces or other.

In reference to considerations related to the design of the apparatus 100, note that the handling device 200 has the degrees of freedom needed for implementing both the machining target and the target of minimizing the distance L.

In greater detail, the design of the handling device

200 can be related to a constrained optimization problem, that can be expressed as in the following relationship a):

$$\left\{\begin{array}{c} \mathit{Target\ function\ (secondary\ target):} \\ \boldsymbol{minimizing\ the\ distance\ between\ the\ tool\ and\ the\ center\ C} \\ \hline \mathit{Contraint\ function\ (main\ target):} \\ \boldsymbol{positioning\ and\ orienting\ (for\ machining\ purposes)} \\ \boldsymbol{the\ object\ with\ respect\ to\ the\ tool} \end{array}\right. \quad (a)$$

Wherein, the *target Function* is related to minimizing the distance L, while the *constraint Function* is associated to the positioning and orientation needed for the purposes of the machining target of the object 101.

The relationship a) defines a determined problem, that allows achieving a unique solution and thus determining and defining completely the degrees of freedom of the handling device 200.

In particular, the relationship a) can lead to define a handling device 200 provided with at least one degree of freedom of redundancy Nr, i.e. at least one degree of freedom additional to the number of degrees of freedom Np needed for only implementing the machining target. In such a case, the apparatus 100 is "redundant" with respect to the only machining target.

As a consequence of what set forth about the degrees of freedom, the number and type of the kinematic pairs 106 and 107 employed on the whole in the handling device 200 depends on both the machining target and the target of minimizing the distance L. In particular, the handling device 200 will be able to comprise not only those kinematic pairs needed in order to achieve the machining target of the object 101, but also one or more additional kinematic pairs needed in order to achieve the target of minimizing the distance L.

From a point of view of the mechanisms adopted by the handling device 200, through the related kinematic pairs, the apparatus 100 can comprise serial mechanisms, parallel mechanisms or hybrid mechanisms.

As known, the distinction among these three classes of mechanisms rests in the analysis of the kinematic chain they made. If the kinematic chain the mechanism makes is an open kinematic chain, the mechanism is serial. If the kinematic chain is closed, the mechanism is named parallel. Instead, the mechanism is defined hybrid if ideally the mechanism can be split in sub-mechanisms, some of which are serial and some parallel.

EMBODIMENT EXAMPLE

Figures 2-4 relate to a particular example of the apparatus 100 and show some machining steps thereof. The apparatus 100 of figures 2-4 is suitable for manufacturing an object 101 by 3D printing and the tool 104 is, therefore, a 3D printing nozzle.

In connection with this example, the example relates to a bi-dimensional instance (2 dimensions), however as it is apparent to the person skilled in the art, the bi-dimensional example can be applied and extended to the three-dimensional mechanisms as well. An apparatus made according to an embodiment that constitutes a three-dimensional extension of the simplified embodiment of figures 2 to 4 is depicted in figures 15 and 16 and will be described in the following.

Still yet in reference to the case of figures 2 to 4, the second handling device 105 related to the positioning and orientation of the nozzle 104 comprises, in this example, second kinematic pairs 108 of prismatic type, such as guide-slide mechanisms.

In particular, the nozzle 104 is mounted on a first slide 201 adapted to slide on a first guide 202, along a horizontal axis O. The first guide 202 (for example, a profile extending longitudinally) comprises, at i6ts ends, a second slide 203 and a third slide 204 which are designed to slide along side guides 205, thus allowing a movement along the vertical axis v of the nozzle 104. As regards to the second handling device 105 of figures 2-4, note that the nozzle 104, regarded as a material point, has two degrees of freedom.

As regards to the first handling device 102 related to the positioning and orientation of the object 101, note that according to the example considered, it comprises a prismatic kinematic pair and a revolute kinematic pair.

In particular, the first handling device 102 comprises a rotating frame 206 having a portion with spherical cap shape 209, adapted to swing, for example, in the plane of figure 2 (plane O-V), within a concave base 207, thus making a revolute pair.

Furthermore, the rotating frame 206 is mechanically coupled with a supporting base 208 of the object 101 being manufactured. Note that the rotating frame 206 is coupled to the supporting base 208 by means of a prismatic kinematic pair that allows the supporting base 208 to translate with respect to the portion having a spherical cap shape 209 of the rotating frame 206.

The first handling device 102 as afore described has, regarding the positioning and orientation of the object 101, a rotational degree of freedom and a translational degree of freedom.

In figures 2-4 the instantaneous center of rotation C, associated to the rotation of the rotating frame 206, and the distance L between the instantaneous center of rotation C and the delivery opening of the nozzle 104, are depicted as well.

Figure 2 shows an example starting situation of the manufacturing of the object 101 by means of 3D printing wherein the nozzle 104 and the supporting base 208 took a first position and the object 104 is formed of a single layer of material.

Figure 3 relates to a situation following that of figure 2, wherein the object 101 is increased in material and the nozzle 104 is translated (upwardly) with respect to the position of figure 2, in accordance with the machining target and with the first algorithm SW1 of the control software 111.

According to the example, the distance L taken in figure 3 is significantly larger than that taken in figure 2, while the instantaneous center of rotation C of figures 2 and 3 is nearly in the same position.

The second algorithm SW2 of the control software 111, related to the minimization of the distance L, carries the apparatus 100 in the situation of figure 4, wherein the distance L has been reduced with respect to that of figure 3, by means of a downwards translation of the guide 202 with which the nozzle 104 is coupled, and a concurrent downwards translation of the support base 208, such that both the minimization target and the machining target are satisfied.

As already mentioned the apparatus 100 allows reducing or removing the degradation of the manufacturing quality consequent to positioning errors and clearances the kinematic pairs and/or the actuators used can be subjected to. Such degradation is related to the "sensitivity" of the positioning of the object 101 with respect to the distance L between the center of rotation C and the nozzle 104.

In the following a sensitivity analysis is reported, that, for the sake of simplicity, relates to the apparatus 100 in the example embodiment of figures 2-4. For the purposes of such analysis, let θ be the angle that describes the rotation associated to the movement with respect to the nozzle 104.

According to the example of figures 2-4, the angle θ is represented as the rotation angle of the rotating frame 206 with respect to a vertical axis V (figure 4).

It is of interest a sensitivity analysis with respect to a positioning error of the actuator controlling the angle θ, i.e. the error of relative positioning between the nozzle 104 and the object 101 when the rotation of the frame 206 is (θ+ε) rather than θ. The relative displacement x between the nozzle 104 and the object 101 along the horizontal axis O, due to a rotation θ of the rotating frame 206, is:

$x = L\ sin(\theta)$ (1)

When there is the error (θ+ε), (1) becomes:

$x(\varepsilon) = L\ sin(\theta+\varepsilon)$ (2)

For the purposes of sensitivity analysis, the first derivative of the expression (1) with respect to θ is determined and thus:

$$\frac{\partial x}{\partial \theta} = \frac{\partial}{\partial \theta}(L\sin(\theta)) = L\cos(\theta) \qquad (3)$$

When there is the error ε, differentiating the expression (2), one has:

$$\frac{\partial x}{\partial \theta} = \frac{\partial}{\partial \theta}(L\sin(\theta+\varepsilon)) = L\cos(\theta+\varepsilon) \qquad (4)$$

In expression (4), the error ε is assumed not to be depending on θ.

Therefore, the maximum influence on the positioning error (taking ε however small), is at the value θ=0, according to (4), and such error Δx will be:

$$\Delta x = L\sin(\theta+\varepsilon) - L\sin\theta = L(\sin(\theta+\varepsilon) - \sin\theta) \quad (5)$$

The relationship (5) shows that the positioning error of the nozzle 104 with respect to the object 101 depends on the error Δx on the angular position θ at the revolute pair (in the example, the frame 206), but it also results that the error Δx depends on the relative distance of the nozzle 104 from the center C of the revolute pair, i.e. the distance L.

In particular, the "sensitivity" of the error Δx as a function of the errors of angular positioning of the rotating frame 206 linearly increases with the distance L according to the expression (5). Therefore, by implementing a control that also has the purpose of minimizing the distance L, the "sensitivity" of the error Δx is reduced as a function of the errors of angular positioning of the rotating frame 206.

Yet still regarding the embodiment of figures 2-4 note that, for example, the degrees of freedom corresponding to the first slide 201, the second slide 203, the third slide 204 and the rotating frame 206 are associated to the machining target. The degree of freedom corresponding to the translation of the supporting base 208 can, instead, be associated to the target of minimizing the distance L.

Note that the case is not excluded in which the apparatus 100 is configured such as to have degrees of freedom of redundancy also for the purposes of other targets such as, for example: avoiding singularity configurations, limiting the extent of some physical quantities (such as, for example, the kinetic energy of its elements). In this case, such redundant degrees of freedom can be also used for the purposes of the target of minimizing the distance L.

The solution described is particularly advantageous since it allows providing apparatuses for machining objects wherein the degradation of the machining quality, that occurs in conventional apparatuses and due to errors of angular positioning, is reduced or eliminated.

So far, a specific example has been represented, simplified in order to provide greater clarity.

Figures 15 and 16 show an embodiment constituting the three-dimensional extension of the embodiment according to figures 2 to 4 that was bi-dimensional only.

Figure 15 depicts the architecture of the device, making a platform 150 connected to a spherical wrist 151 with two revolute pairs having axes 152 and 153 defining the center of rotation 153 of the spherical wrist 151 and with a prismatic kinematic pair 155 making the degree of redundancy of the device.

Figure 16 shows the relative displacement of the table 150 with respect to the center 153 of the spherical wrist, caused by the redundant kinematic pair 155 according to the optimization process similar to that of the example of figures 2 to 4.

Specifically, in reference to figure 4, the herein reported figures and specifically figure 16 are the three-dimensional extension thereof. By moving the workpiece platform 150 with respect to the center of the spherical wrist 154, i.e. by varying the travel of the prismatic pair 155, the distance of the center of rotation (that in figure 4 is just the center of the "spherical cap" 209), and the application point of the shear strain, or the position of the nozzle tip can be varied.

The problem on which the invention is based can be addressed also more generally by providing, in defining the specific boundary conditions, the solution to the simplified problem discussed in reference to the above example.

By considering a more general analysis, the Lagrangian variables are the parameters adequate to describe the configuration of the (handling or worktable) system; in our case these parameters correspond to the position of the actuators (linear in case of linear actuator, angular in case of motor). To each Lagrangian variable an active force or torque corresponds, that is provided by an actuator.

The difference between a serial mechanism and a parallel one is that, in the first case, there is a single vector closure equation, whereas in the second case, there are as many closure equations as the closed kinematic chains, plus of course the equation expressing the end member (end-effector of handling equipment) laying.

The purpose in writing the scalar closure equations corresponding to the vector closure equations is to express the parameters describing the end member laying as a function of the Lagrangian variables of the system.

Starting from the vector closure equations (reference is made to the examples that will be illustrated in the following):

$$\begin{cases} X = X(x,y,\theta,h) \\ Y = Y(x,y,\theta,h) \\ \Omega = Y(x,y,\theta,h) \end{cases} \quad or \quad \begin{cases} X = X(x,y,z,h) \\ Y = Y(x,y,z,h) \\ \Omega = Y(x,y,z,h) \end{cases}$$

the variation of the end member laying can be described as a function of the variations of the Lagrangian variables:

$$\begin{bmatrix} \dot{X} \\ \dot{Y} \\ \dot{\Omega} \end{bmatrix} = \begin{bmatrix} \underset{\sim}{J}(x,y,\theta,h) \end{bmatrix} \cdot \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \\ \dot{h} \end{bmatrix} \quad oppure \quad \begin{bmatrix} \dot{X} \\ \dot{Y} \\ \dot{\Omega} \end{bmatrix} = \begin{bmatrix} \underset{\sim}{J}(x,y,z,h) \end{bmatrix} \cdot \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ \dot{h} \end{bmatrix}$$

It has to be noted that generally, there are two types of Jacobian: the geometrical Jacobian, and the analytical Jacobian. The geometrical Jacobian refers to the description of the link between linear and angular velocity of the end member and the variation of the Lagrangian variables with respect to time (velocity at the joints). The analytical one refers to the description of the end member lying minimally: by differentiating the expressions that represent such a link, the analytical Jacobian is obtained. In case of plane problem, the two Jacobians coincide.

The purpose is to express "the forces and the torques at the joints" (the forces and the torques provided by the actuators), i.e. (fx, fy, τ, fh) in case of serial mechanism, (fx, fy, fz, fh) in case of parallel mechanism, as a function of the torque and force applied to the end member of the mechanism (Fx, Fy, M). Note that the constraint relationships are not of interest, since, assuming holonomic constraints, these do not carry out mechanical work. The link between the forces and the torques provided for the actuators, and the stresses applied on the end member, can be expressed by a linear system, whose parameters are a function of the mechanism configuration. A coefficient matrix is associated to such a system. Note that the just described matrix is nothing else but the transposition of the Jacobian matrix of the system; the explanation of this is in the kinematic-static duality of the mechanisms constituted of stiff bodies connected to one another by means of prismatic pairs (carriages) or revolute pairs (joints), in both open chain and closed chain. This duality exists by virtue of the virtual work principle.

Since the heretofore considered mechanisms are redundant mechanisms, the inverse kinematic problem is undetermined. In order to render it determined, an additional relationship has to be added or, in other terms, in addition to the primary task, that is to obtain the desired positioning of the end member of the device, a secondary task has to be introduced.

In the example of serial mechanism of figures 5 to 8, it is noted how fx, fy and fh do not depend on the Lagrangian coordinates x, y, h and θ. The force fh corresponding to the Lagrangian variable h only depends on θ, that however has to be identically equal to Ω that expresses the angular positioning of the end member. Indeed, the only physical quantity that can be managed by varying the values of the Lagrangian variables is τ.

In order to render thus the problem determined, the condition can be introduced of minimizing the torque required for the motor managing the angular positioning of the end member of the mechanism, and thus, definitely, studying the problem of constrained minimum.

Instead, in reference to the example of the parallel mechanism depicted in figures 9 to 12, the closure equations and the balance equations are more complicated, in general, in the study of the mechanisms characterized by closed kinematic chains. In any case, in principle, a function of the Lagrangian variables can be defined, to be imposed as target function for the optimization problem.

As done above, a force or a torque can be thought of being minimized, for example, for a specific actuator, remembering that, for the already mentioned kinematic-static duality, this equals to minimize the error of lying the end member due to the uncertainty of the corresponding Lagrangian variable, therefore reducing the sensitivity thereof.

Figure 5 shows a first general application example related to a serial mechanism (open kinematic chain).

The mechanism comprises translation means along the axis X, along the axis Y, along an additional axis h, angular displacement means to angularly displace the axis H with respect to the axis Y and angular displacement means with respect to the axis y.

Figures 6 and 7 show how the Lagrangian variables are defined.

$$\begin{cases} displacement\ x \rightarrow force\ f_x \\ displacement\ y \rightarrow force\ f_y \\ rotation\ \theta \rightarrow torque\ \tau \\ displacement\ h \rightarrow force\ f_h \end{cases}$$

The vector closure equations are as follows:

$$\overline{OE} = \overline{OO'} + \overline{O'A} + \overline{AB} + \overline{BE}$$

The scalar closure equations are:

$$\overline{OE} = \overline{OO'} + \overline{O'A} + \overline{AB} + \overline{BE} \implies \begin{cases} X = x - hsen\theta + acos(\alpha + \theta) \\ Y = y + hcos\theta + asen(\alpha + \theta) \\ \Omega = \theta \end{cases}$$

System Jacobian:

$$\begin{vmatrix} \dot{X} \\ \dot{Y} \\ \dot{\Omega} \end{vmatrix} = \underline{\underline{J}}(x, y, \theta, h) \cdot \begin{vmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \\ \dot{h} \end{vmatrix} = \begin{bmatrix} 1 & 0 & (-hcos\theta - asen(\alpha + \theta) & -sen\theta \\ 0 & 1 & (-hsen\theta - acos(\alpha + \theta)) & cos\theta \\ 0 & 0 & 1 & 0 \end{bmatrix} \cdot \begin{vmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \\ \dot{h} \end{vmatrix}$$

Wherefrom the balance equations of the mechanism

$$\begin{vmatrix} f_x \\ f_y \\ \tau \\ f_h \end{vmatrix} = \begin{vmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ (hcos\theta + asen(\alpha + \theta)) & (hsen\theta - asen(\alpha + \theta)) & 1 \\ sen\theta & -cos\theta & 0 \end{vmatrix} \cdot \begin{vmatrix} F_x \\ F_y \\ M \end{vmatrix}$$

i.e.:

$$\begin{cases} f_x = -F_x \\ f_y = -F_y \\ \tau = F_x(hcos\theta + asen(\alpha + \theta)) + F_y(hsen\theta - asen(\alpha + \theta)) - M \\ f_h = F_x sen\theta - F_y cos\theta \end{cases}$$

Without prejudice to the generality and in order to make simpler and more comprehensible the mathematic formalism, the case can be studied in which the component Fy of the stresses at the joints is null:

$$\begin{cases} f_x = -F_x \\ f_y = 0 \\ \tau = F_x\left(h\cos\theta + a\,sen(\alpha + \theta)\right) \\ f_h = F_x\,sen\theta \end{cases}$$

Such assumption does not invalidate the validity of reasoning, that can be applied by analogy to the case wherein both Fx and Fy are not null.

Therefore, the optimization problem is described by the following equations:

$$\begin{cases} \min \tau = \min\left(F_x\left(h\cos\theta + a\,sen(\alpha + \theta)\right)\right) = 0 \\ X = x - h\,sen\theta + a\cos(\alpha + \theta) \\ Y = y + h\cos\theta + a\,sen(\alpha + \theta) \\ \Omega = \theta \end{cases}$$

whose solution is given by:

$$h = -a(sen\alpha + \cos\alpha \cdot tg\theta)$$

Figure 8 schematically shows the solution.

Figure 9 shows an example of parallel, or closed kinematic chain, mechanism. With respect to the example of figure 5, translation means with respect to the axis x and with respect to the axis y are added.

Figures 10 and 11 graphically show the steps of defining the Lagrangian variables of the system and the active forces, similarly to figures 6 and 7.

$$\begin{cases} displacement\ x \rightarrow force\ f_x \\ displacement\ y \rightarrow force\ f_y \\ displacement\ z \rightarrow force\ f_z \\ displacement\ h \rightarrow force\ f_h \end{cases}$$

In this case the vector closure equations are:

$$\begin{cases} \overline{OE} = \overline{OO'} + \overline{O'A} + \overline{AB} + \overline{BE} \\ \overline{OD} = \overline{OO'} + \overline{O'A} + \overline{AB} + \overline{BC} + \overline{CD} \end{cases}$$

and the scalar closure ones are:

$$\begin{cases} \overline{OE} = \overline{OO'} + \overline{O'A} + \overline{AB} + \overline{BE} \\ \overline{OD} = \overline{OO'} + \overline{O'A} + \overline{AB} + \overline{BC} + \overline{CD} \end{cases} \Rightarrow \begin{cases} X = x - (h + a\,\mathrm{sen}\alpha)\left(\frac{z-Y}{b}\right) + a\cos\alpha \cdot \sqrt{1-\left(\frac{z-Y}{b}\right)^2} \\ Y = y + a\cos\alpha\left(\frac{z-Y}{b}\right) + (h + a\,\mathrm{sen}\alpha) \cdot \sqrt{1-\left(\frac{z-Y}{b}\right)^2} \\ \Omega = \mathrm{sen}^{-1}\left(\frac{z-Y}{b}\right) \end{cases}$$

The system Jacobian is thus:

$$\begin{vmatrix} X \\ Y \\ \Omega \end{vmatrix} = \underset{=}{J}(x,y,z,h) \begin{vmatrix} x \\ y \\ z \\ h \end{vmatrix} =$$

$$= \begin{vmatrix} 1 & \left(\frac{(h + a\,\mathrm{sen}\alpha)}{b} + \frac{a\cos\alpha \cdot \left(\frac{z-Y}{b}\right)}{b\sqrt{1-\left(\frac{z-Y}{b}\right)^2}}\right) & \left(-\frac{(h + a\,\mathrm{sen}\alpha)}{b} - \frac{a\cos\alpha \cdot \left(\frac{z-Y}{b}\right)}{b\sqrt{1-\left(\frac{z-Y}{b}\right)^2}}\right) & -\left(\frac{z-Y}{b}\right) \\ 0 & \left(1 - \frac{a\cos\alpha}{b} + \frac{(h + a\,\mathrm{sen}\alpha) \cdot \left(\frac{z-Y}{b}\right)}{b\sqrt{1-\left(\frac{z-Y}{b}\right)^2}}\right) & \left(+\frac{a\cos\alpha}{b} + \frac{(h + a\,\mathrm{sen}\alpha) \cdot \left(\frac{z-Y}{b}\right)}{b\sqrt{1-\left(\frac{z-Y}{b}\right)^2}}\right) & \sqrt{1-\left(\frac{z-Y}{b}\right)^2} \\ 0 & \left(\frac{-b}{\sqrt{b^2-(z-y)^2}}\right) & \left(\frac{-b}{\sqrt{b^2-(z-y)^2}}\right) & 0 \end{vmatrix} \cdot \begin{vmatrix} x \\ y \\ z \\ h \end{vmatrix}$$

and the balance equations of the mechanism are:

$$\begin{vmatrix} f_x \\ f_y \\ f_z \\ f_h \end{vmatrix} = \begin{vmatrix} 1 & 0 & 0 \\ \left(\frac{(h + a\,\mathrm{sen}\alpha)}{b} + \frac{a\cos\alpha \cdot \left(\frac{z-Y}{b}\right)}{b\cos\beta}\right) & \left(1 - \frac{a\cos\alpha}{b} + \frac{(h + a\,\mathrm{sen}\alpha) \cdot \left(\frac{z-Y}{b}\right)}{b\cos\beta}\right) & \left(\frac{-1}{b\cos\beta}\right) \\ \left(-\frac{(h + a\,\mathrm{sen}\alpha)}{b} - \frac{a\cos\alpha \cdot \left(\frac{z-Y}{b}\right)}{b\cos\beta}\right) & \left(\frac{a\cos\alpha}{b} + \frac{(h + a\,\mathrm{sen}\alpha) \cdot \left(\frac{z-Y}{b}\right)}{b\sqrt{\cos\beta}}\right) & \left(\frac{-1}{b\cos\beta}\right) \\ -\mathrm{sen}\beta & \cos\beta & 0 \end{vmatrix} \cdot \begin{vmatrix} F_x \\ F_y \\ M \end{vmatrix}$$

being

$$\beta = sen^{-1}\left(\frac{Z-Y}{b}\right) \Rightarrow sen\beta = \left(\frac{Z-Y}{b}\right) \Rightarrow cos\beta = \sqrt{1-\left(\frac{z-y}{b}\right)^2}$$

i.e.:

$$\begin{cases} f_x = 1 \cdot F_x \\ f_y = \left(\frac{(h + asen\alpha)}{b} + \frac{acos\alpha \cdot sen\beta}{bcos\beta}\right) \cdot F_x + \left(1 - \frac{acos\alpha}{b} + \frac{(h + asen\alpha) \cdot sen\beta}{bcos\beta}\right) \cdot F_y + \left(\frac{-1}{bcos\beta}\right) \cdot M \\ f_z = \left(-\frac{(h + asen\alpha)}{b} - \frac{acos\alpha \cdot sen\beta}{bcos\beta}\right) \cdot F_x + \left(\frac{acos\alpha}{b} + \frac{(h + asen\alpha) \cdot sen\beta}{bcos\beta}\right) \cdot F_y + \left(\frac{-1}{bcos\beta}\right) \cdot M \\ f_b = -sen\beta \cdot F_x + cos\beta \cdot F_y + 0 \cdot M \end{cases}$$

Similarly to before, considering the component Fy null:

$$\begin{cases} f_x = F_x \\ f_y = \left(\frac{(h + asen\alpha)}{b} + \frac{acos\alpha \cdot sen\beta}{bcos\beta}\right) \cdot F_x \\ f_z = \left(-\frac{(h + asen\alpha)}{b} - \frac{acos\alpha \cdot sen\beta}{bcos\beta}\right) \cdot F_x \\ f_b = -sen\beta \cdot F_x \end{cases}$$

the optimization problem is expressed by the equations:

$$\begin{cases} \min f_y = \min\left(F_x\left(\frac{(h + asen\alpha)}{b} + \frac{acos\alpha \cdot sen\beta}{bcos\beta}\right)\right) = 0 \\ X = x - hsen\theta + acos(\alpha + \theta) \\ Y = y + hcos\theta + asen(\alpha + \theta) \\ \Omega = \theta \end{cases}$$

whose solution is

$$h = -asen\alpha - acos\alpha \cdot tan\beta$$

Graphically the solution is shown in figure 11.

Comparing the two solutions of figure 8 and figure 12, it is noted that in both cases, there is the minimization of the positioning error introduced by the uncertainties on the Lagrangian variables when the force

Fx, or better, its action line, "contains" the revolute pair (actuated or not actuated, it does not matter) corresponding to the point A.

The problem at the base of the present invention can thus be generalized as that of increasing the configuration stiffness, i.e. the ability the mechanism has to arrange its elements such that both the torques required for its motors and the forces required for its linear actuators are minimal, in order to counter the active forces it is subjected to. The proposal of the present patent is to do so by using an optimization algorithm in order to control the mechanism provided with redundant degrees of freedom.

The flowchart of figure 13 shows a first detailed embodiment of the method according to the present invention for minimizing the positioning error of a tool, comprising a handling device (200) designed to orient and position the object (101) relative to the tool (104), the device comprising a positioning mechanism provided with redundant degrees of freedom and motorized actuators in order to control the displacements along each provided degree of freedom, the method providing the steps of:

- generating a mathematical model of the mechanism that describes both the geometry of the mechanism and the active forces in the mechanism which are determined by the actuators through the corresponding Lagrangian variables;

- defining the constraints of the mechanism by means of vector and scalar closure equations;

- defining the balance equations of the mechanism, based on the variables and equations defined in the preceding steps;

- defining the functions describing the target conditions (target functions) of the mechanism;

- minimizing the target functions by meeting the constraints defined in the preceding step;

- using the optimized mechanism configuration obtained for the set of Lagrangian variables as a configuration to maximize the configuration stiffness and to minimize the influence of the errors of the variables at the joints corresponding to the Lagrangian variables of the previously defined set and to minimize the torques and forces of the corresponding motors and actuators.

The embodiment illustrated in detail in figure 13 shows the following steps:

- (130) describing the geometry of the mechanism by using vector and scalar equations;

- (131) describing the mechanism by a set of Lagrangian variables of the active forces of the mechanism based on the mechanism geometry;

- (132) generating a set of vector closure equations;

- (133) generating a set of scalar closure equations;

- (134) defining the Jacobian of the mechanism;

- (135) defining the balance equations of the mechanism;

- (136) defining a target function describing an optimization target related to both the values of certain forces or torques and/or the values of these forces or torques with respect to one another;

- (137) defining the optimization algorithm as minimizing algorithm of the target functions in combination with the constraint of satisfying the scalar closure equations;

- (138, 139) determining, by executing the minimizing algorithm, the configuration of the mechanism that increases the configuration stiffness, i.e. the ability the mechanism has to arrange its elements such that both the torques required for its motors and the forces required for its linear actuators are minimal, in order to counter the active forces it is subjected to.

As expressed, the algorithm just described in reference to figure 13 is well suited to a solution at the level of position of the optimization problem.

An embodiment variation of the invention provides, as a process for optimizing the target functions, a so called velocity analysis, wherein the optimization of the target functions is obtained by imposing increments of the variables at the joints (Lagrangian variables) such that the mechanism always takes configurations within the Null-space of the main task.

As it is apparent from figure 14, the steps 137 and 138 have been omitted and are replaced by the following steps:

- starting from step (134) a pseudo-inverse matrix of the Jacobian is generated therefrom, as denoted by (140).

This matrix defines at step 141 the Null-space of the primary task and at step 142 the minimization is determined, i.e. the optimization by resolution of the secondary task projected in the Null-space from which the configuration optimized for the given set of Lagrangian variables.

In the examples seen, the setting and resolution of the optimization problem are simple since:

1. the problem is bi-dimensional, but the proposed method needs to the able to study three-dimensional mechanisms;

2. the forces and the torques required for the actuators can be treated "singularly", being able explicitly link them to the Lagrangian variables. In a general case, however, the quantities can not be expressed explicitly and simply, and thus the method for solving the optimization problem needs to be able to solve more complex systems;

3. the problem addressed relates to the positioning of the end member in a single point (point E), but, in general, a method is needed that allows the control parameters of the mechanism to be optimized when the point E (that is nothing else but the application point of our nozzle/tool) follows a determined path.

In the examples described, the resolution of plane problems has been referred, to simplify matters, i.e. problems in which all the physical quantities such as velocities, displacements and forces belong to all parallel planes (XY), and the quantities such as rotations and torques can all be represented by vectors normal to such a plane. Of course, in practice, the problems are three-dimensional, where all the physical vector quantities can be expressed by three components, the tensor ones of the first order by nine components, etc. It is however possible to extend the inventive concept to the three-dimensional condition.

Therefore, in the three-dimensional case, the (geometrical) Jacobian matrix of the system is in the form:

$$\begin{vmatrix} \dot{X} \\ \dot{Y} \\ \dot{Z} \\ \dot{\phi} \\ \dot{\psi} \\ \dot{\theta} \end{vmatrix} = \begin{vmatrix} \quad \underline{J}(q_1, q_2, \ldots q_n) \quad \end{vmatrix} \cdot \begin{vmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \ldots \\ \dot{q}_n \end{vmatrix}$$

where q1, q2, …, qn are nothing but the variables at the joints, i.e. the Lagrangian variables used to describe the system configuration. Note that these can express a linear parameter in case of carriages moved by linear actuators, angular parameters in case of hinges actuated by respective motors. The degree of redundancy of the mechanism is given by n-6.

In a way absolutely similar to the bi-dimensional case, if Fx, Fy, Fz are the components of the resultant of the forces applied to the end member, $M\phi$, $M\psi$, $M\Omega$, the torques, and $\gamma 1$, $\gamma 2$, …,γn are the actuator actions (forces for the linear actuators, and torques for the motors), corresponding to the variables at the joints q1, q2, …, qn, then the system balance equations can be expressed in matrix form as:

$$\begin{vmatrix} \gamma_1 \\ \gamma_2 \\ \dots \\ \gamma_n \end{vmatrix} = \left| \quad \underline{\underline{J}}^T(q_1, q_2, \dots, q_n) \quad \right| \cdot \begin{vmatrix} F_x \\ F_y \\ F_z \\ M_\phi \\ M_\psi \\ M_\Omega \end{vmatrix}$$

The equations expressing $\gamma 1$, $\gamma 2$, …,γn are, de facto, suitable to be the target function of the optimization algorithm (or being part thereof).

As mentioned above and as evident from the above description, in reference specifically to the embodiment of the method of which at Fig. 14, a fundamental step is the definition of the equation, or the equations that make the reverse kinematic problem determined, i.e. the definition of the secondary task. To do this a function to be minimized is defined and related to the kinematic-static characteristics of the mechanism (in a given configuration), thus defining one (or more) target function for the constrained optimization problem. The constraint equations are the closure equations (i.e. the correct positioning of the end-effector).

There is however an aspect to be considered: in the two examples given above, a closed-form analytical solution has been able to be provided. However, in practice, many times this is not possible, also for what mentioned above as regards to the dependence between the stresses at the joints and the Lagrangian variables, which results in the impossibility of algebraically or also analytically only solving this problem.

There are in literature different methods for solving constrained optimization problems: "gradient based" methods, genetic algorithms, evolution strategies, etc. All of these methods can be in principle used to solve the problem.

The optimization of the secondary task, imposing the satisfying of the primary task, in case of the redundant mechanisms can be solved by using the differential inverse kinematic method, by using the pseudo-inverse matrix. This technique is specifically useful in case of a mechanism tracing the positioning and orientation of the end member along a well defined path.

In this case, the Jacobian matrix, besides being a tool for the kinetic-static analysis of the mechanism, can be used for determining the inverse kinematics at velocity level, or infinitesimal variations of the virtual displacements of the Lagrangian variables. In fact, for example, one has that if

$$=(,,\Omega) \qquad =(,,\theta)$$

then the infinitesimal variations of the end member lying can be expressed as infinitesimal variations:

$$\underline{\delta p} = \underline{\underline{J}} \cdot \underline{\delta q}$$

Therefore, in case the Jacobian matrix is not degenerate in the configuration under study, the problem of inverse kinematics could be thought of being solved at differential level, and i.e. it is wondered, instant by instant, which set of "displacements at the joints" δx, δy, δθ (or, in the general case, δq1, δq2, …, δqn) has to be imposed in order to have a determined set of variations of the main member lying, i.e. δX, δY and δΩ, or in case of three-dimensional problem, δX, δY, δY, δϕ, δψ, δΩ.

To do this, the easiest thing is to invert the Jacobian matrix, in case this is not degenerate:

$$\underline{\delta p} = \underline{\underline{J}} \cdot \underline{\delta q} \implies \underline{\delta q} = \underline{\underline{J}}^{-1} \cdot \underline{\delta p} \; if \; det\left(\underline{\underline{J}}\right) \neq 0$$

In case the mechanism is not redundant, and under the assumption of determinant of the Jacobian matrix different from zero, it is always possible.

Such an approach is not trivial however in case of redundant mechanisms, since in this case the Jacobian is a rectangular matrix. In order to carry out the analysis of the inverse kinematics at velocity level of a redundant mechanism the pseudo-inverse matrix can be used, i.e. a matrix such that:

$$\underline{\delta p} = \underline{\underline{J}} \cdot \underline{\delta q} \implies \underline{\delta q} = \underline{\underline{J}}^{+} \cdot \underline{\delta p} \;\; being \;\; \underline{\underline{J}}^{+} = \underline{\underline{J}}^{T}\left(\underline{\underline{J}} \cdot \underline{\underline{J}}^{T}\right)^{-1}$$

In practice, the use of this formulation allows determining the set of variations of the parameters at the joints (interpretable as the virtual displacements of the Lagrangian variables), compatible with the variation of laying of the end member, and that minimizes a determined function of the variation of the variables at the joints. By modifying the definition of the pseudo-inverse matrix, the target function that is minimized can be ri-defined, by imposing that the variation of configuration of the mechanism is compatible with an increase of stiffness of configuration, which condition, for the kinetic-static duality, equals to minimize the influence of the errors of the variables at the joints, corresponding to the Lagrangian variables considered in the optimization problem, and to minimize the torques and forces of the corresponding motors and actuators.

CLAIMS

1. Machining apparatus (100) to machine an object (101), comprising:

a tool (104) able to operate on the object (101);

a handling device (200) designed to orient and position the object (101) relative to the tool (104), causing at least one relative rotation movement of the object (101) with respect to the tool (104), an instantaneous center of rotation (C) being associated to the latter;

a control module (110) designed to control the handling device according to a machining target of said object (104);

characterized in that:

the control module (110) is designed to manage the positioning and orientation of the object (101) with respect to the tool (104) as well as to achieve, during the implementation of the machining target, a target of minimizing a distance (L) between the center of rotation (C) and the tool (104).

2. Apparatus (100) according to claim 1, which apparatus comprises a worktable, which worktable is mounted swinging around a first axis and possibly further rotating around a second axis perpendicular to said first axis, a prismatic guide of said worktable being further provided to said rotating supports of the worktable, and which worktable is provided in combination with a machining tool, said rotating supports and said prismatic joint being used to hold the tool tip in the center of rotation of the table as a function of the size and/or shape variations of the workpiece.

3. Apparatus (100) according to claim 1 or 2, wherein said apparatus is made according to one of the following types: apparatus operating by removing material, apparatus operating by adding material.

4. Apparatus (100) according to at least one of the preceding claims, in which the handling device (200) is structured to have a sufficient number of degrees of freedom to implement both the machining target and the target of minimizing the distance (L).

5. Apparatus (100) according to claim 4, in which the handling device (200) has at least one degree of redundancy with respect to the machining target only.

6. Apparatus (100) according to at least one of the preceding claims, in which the control module (110) comprises a software control module (111) designed to implement:

an algorithm (SW1,SW2) related to the implementation of both the machining target and the target of minimizing the distance (L) thanks to a constrained optimization based on satisfying the constraint equations and the target function.

7. Apparatus (100) according to at least one of the preceding claims, in which said handling device (200) comprises:

- a first handling device (102) designed to support, orient and position the object (101) with respect to the machining machine (103);

- a second handling device (105) designed to support, position and orient the tool (104) with respect to the object (101).

8. Apparatus according to at least one of the preceding claims, in which the handling device (200) further comprises:

- a plurality of kinematic pairs (106, 108) for the relative movement of the object (101) with respect to the tool (104);

- a plurality of actuators (107, 109) for operating said plurality of kinematic pairs (106, 108).

9. Apparatus according to at least claim 8, in which the plurality of kinematic pairs (106, 108) comprises at least two kinematic pairs belonging to the group: revolute pair, cylindrical pair, screw pair, spherical pair.

10. Apparatus (100) according to at least one of the preceding claims, in which the handling device (200) is such as to achieve one of these mechanisms: serial mechanism, parallel mechanism, hybrid mechanism.

11. Method of minimizing the positioning error of a tool comprising a handling device (200) designed to orient and position the object (101) relative to the tool (104), the device comprising a positioning mechanism provided with redundant degrees of freedom and motorized actuators in order to control the displacements along each provided degree of freedom, the method providing the steps of:

- generating a mathematical model of the mechanism that describes both the geometry of the mechanism and the active forces in the mechanism which are determined by the actuators through the corresponding Lagrangian variables;

- defining the constraints of the mechanism by means of vector and scalar closure equations;

- defining the balance equations of the mechanism, based on the variables and equations defined in the preceding steps;

- defining the functions describing the target conditions (target functions) of the mechanism;

- minimizing the target functions by meeting the constraints defined in the preceding step;

- using the optimized mechanism configuration obtained for the set of Lagrangian variables as a configuration to maximize the configuration stiffness and to minimize the influence of the errors of the variables at the joints corresponding to the Lagrangian variables of the previously defined set and to minimize the torques and forces of the corresponding motors and actuators.

12. Method according to claim 11, comprising the following steps:

- (130) describing the geometry of the mechanism by using vector and scalar equations;

- (131) describing the mechanism by a set of Lagrangian variables of the active forces of the mechanism based on the mechanism geometry;

- (132) generating a set of vector closure equations;

- (133) generating a set of scalar closure equations;

- (134) defining the Jacobian of the mechanism;

- (135) defining the balance equations of the mechanism;

- (136) defining a target function describing an optimization target related to both the values of certain forces or torques and/or the values of these forces or torques with respect to one another;

- (137) defining the optimization algorithm as minimizing algorithm of the target functions in combination with the constraint of satisfying the scalar closure equations;

- (138, 139) determining, by executing the minimizing algorithm, the configuration of the mechanism that increases the configuration stiffness, i.e. the ability the mechanism has to arrange its elements such that both the torques required for its motors and the forces required for its linear actuators are minimal, in order to counter the active forces it is subjected to.

13. Method according to claim 11 or 12, in which the steps of optimizing the target functions comprise the optimization of the target functions by imposing increments of the variables at the joints (Lagrangian variables) such that the mechanism always takes configurations within the Null-space of the main task.

14. Method according to claim 11 or 12, in which the steps (137 and 138), allowing the mechanism configuration that increases the configuration stiffness to be determined by executing the minimizing algorithm, are omitted, and are replaced by the following steps:

- starting from step (134) a pseudo-inverse matrix of the Jacobian (140) is generated therefrom;

- this matrix defines (141) the Null-space of the primary task (142) and determines the minimization, i.e. the optimization by resolution of the secondary task projected in the Null-space from which the configuration optimized for the given set of Lagrangian variables.

15. Machining method to machine an object (101), comprising:

carrying out a machining operation on the object (101) by a tool (104);

orienting and positioning the object (101) relative to tool (104), causing at least one relative rotation movement of the object (101) with respect to the tool (104), an instantaneous center of rotation (C) being associated to the latter;

controlling the handling device according to a machining target of said object (104);

controlling the positioning and orientation of the object (101) with respect to the tool (104) in order to further achieve, during the implementation of the machining target, a target of minimizing a distance (L) between the center of rotation (C) and the tool (104).

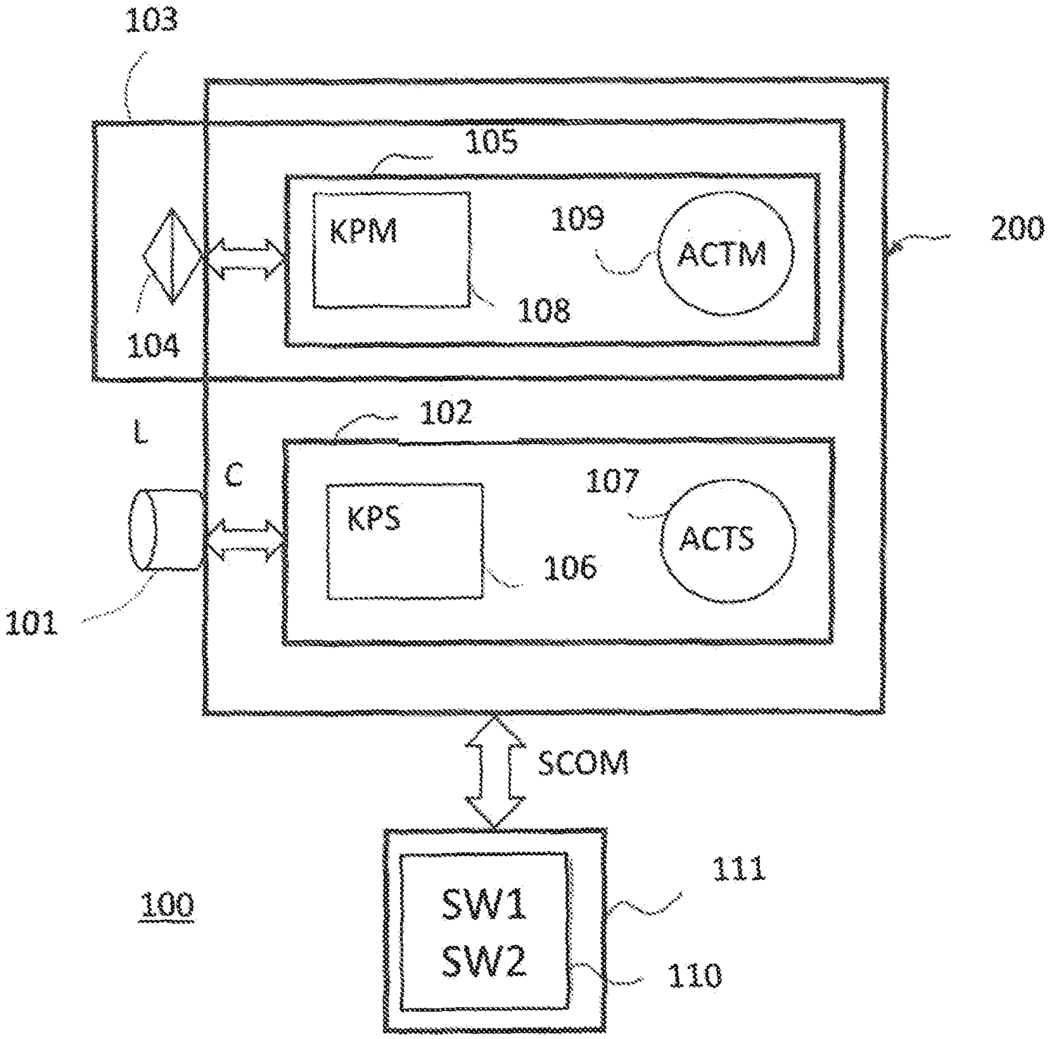
103
105
KPM
109
ACTM
200
104
108
L
102
C
107
KPS
ACTS
106
101
SCOM
111
100
SW1
SW2
110

Fig. 1

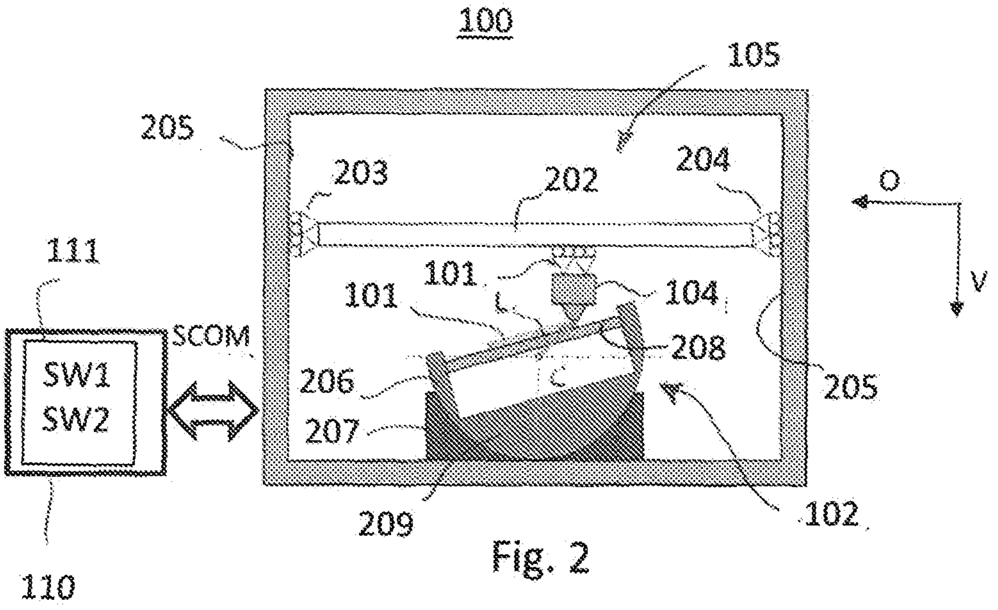

Fig. 2

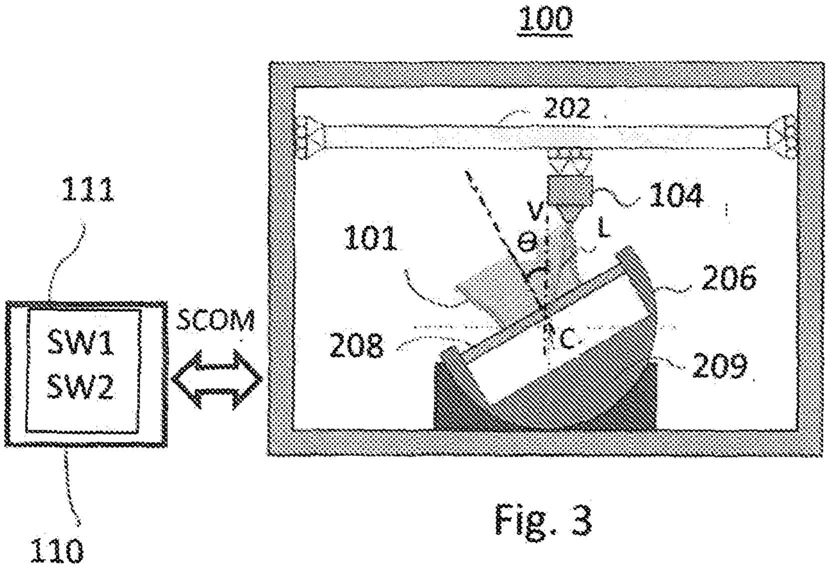

Fig. 3

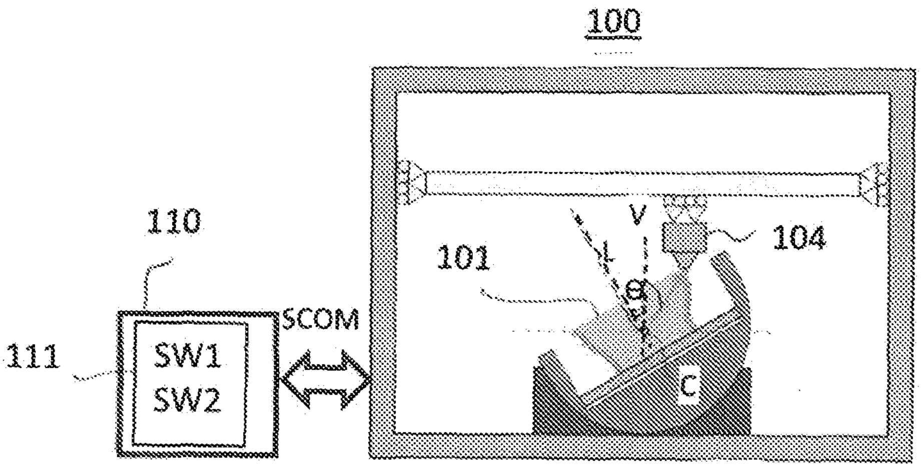

Fig. 4

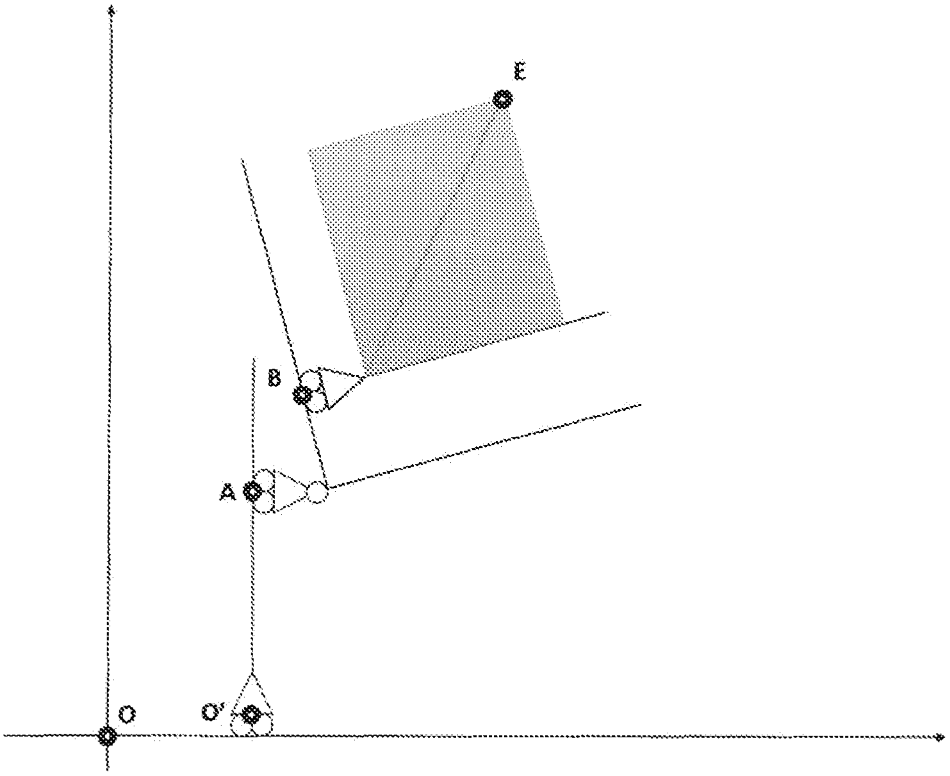

Fig. 5

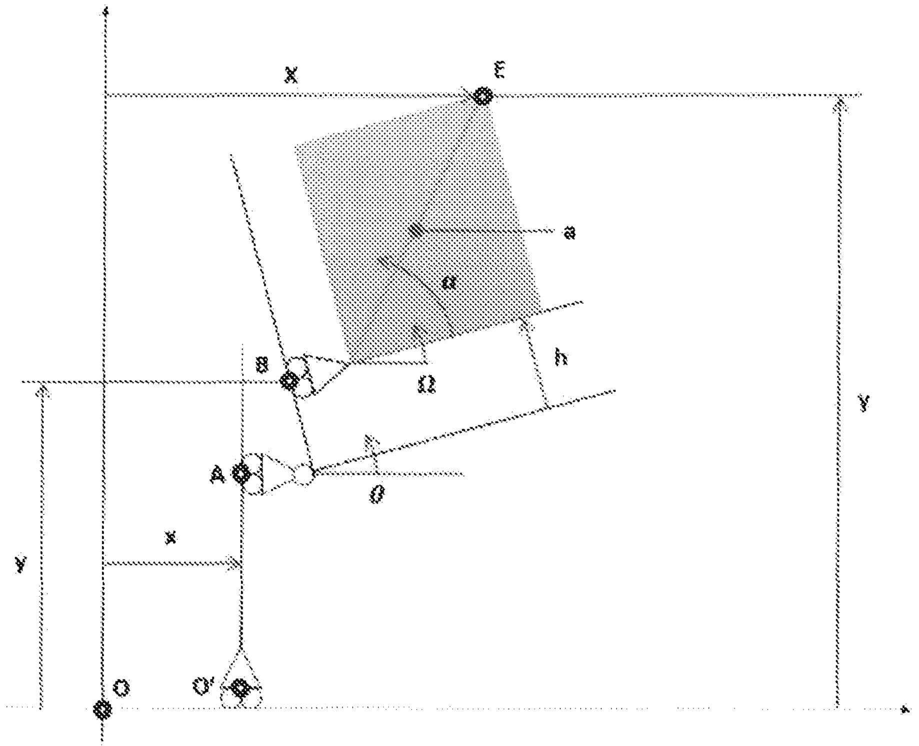

Fig. 6

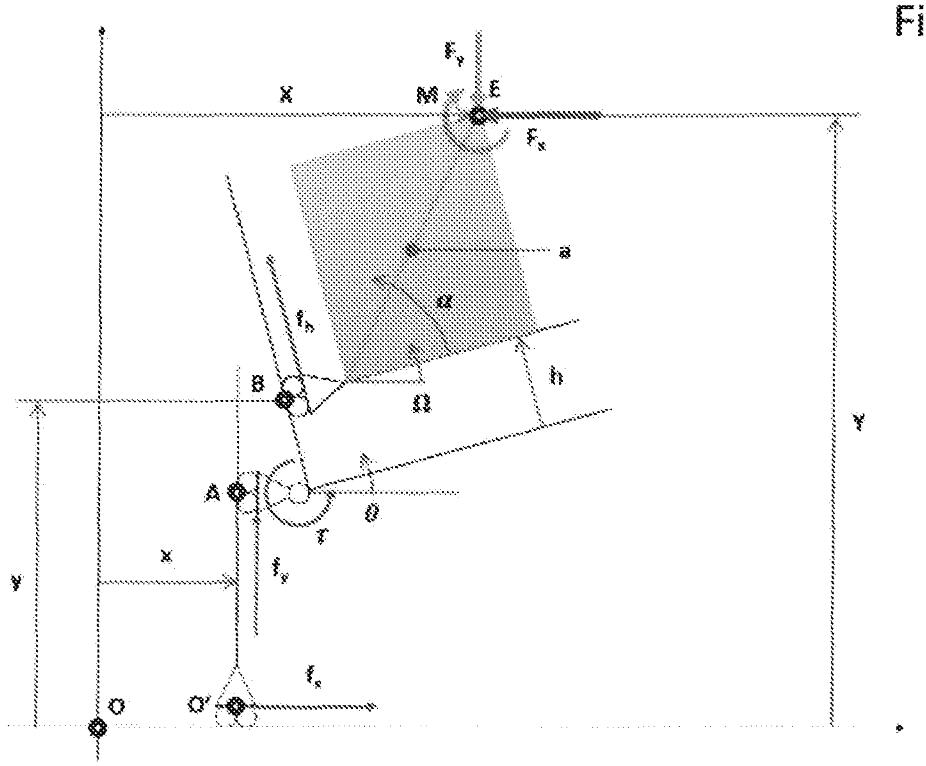

Fig. 7

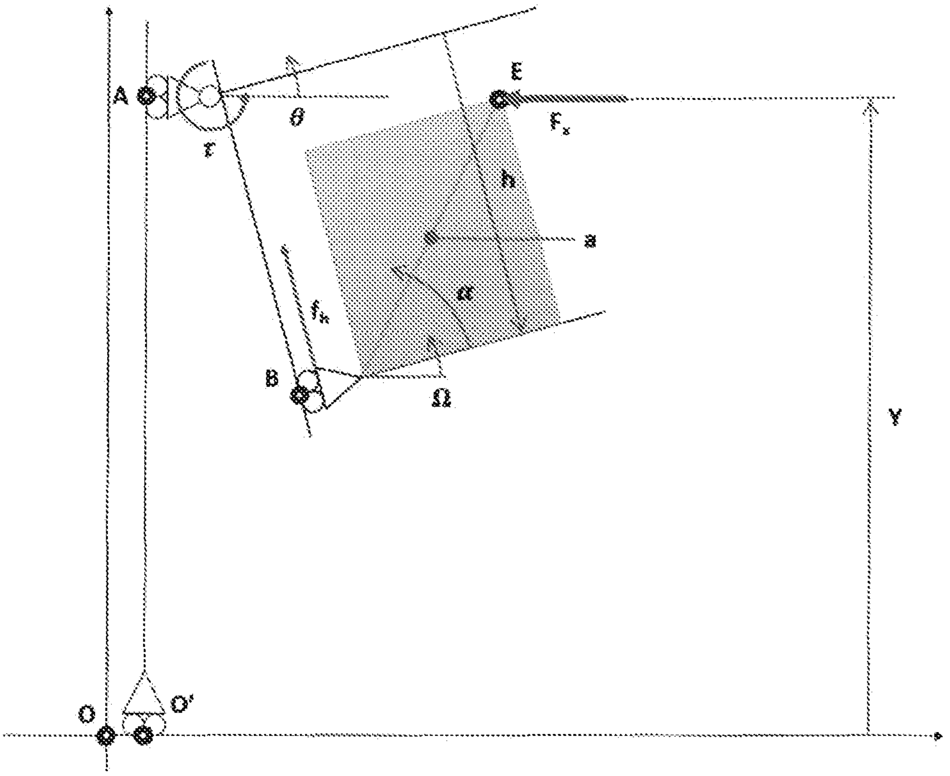

Fig. 8

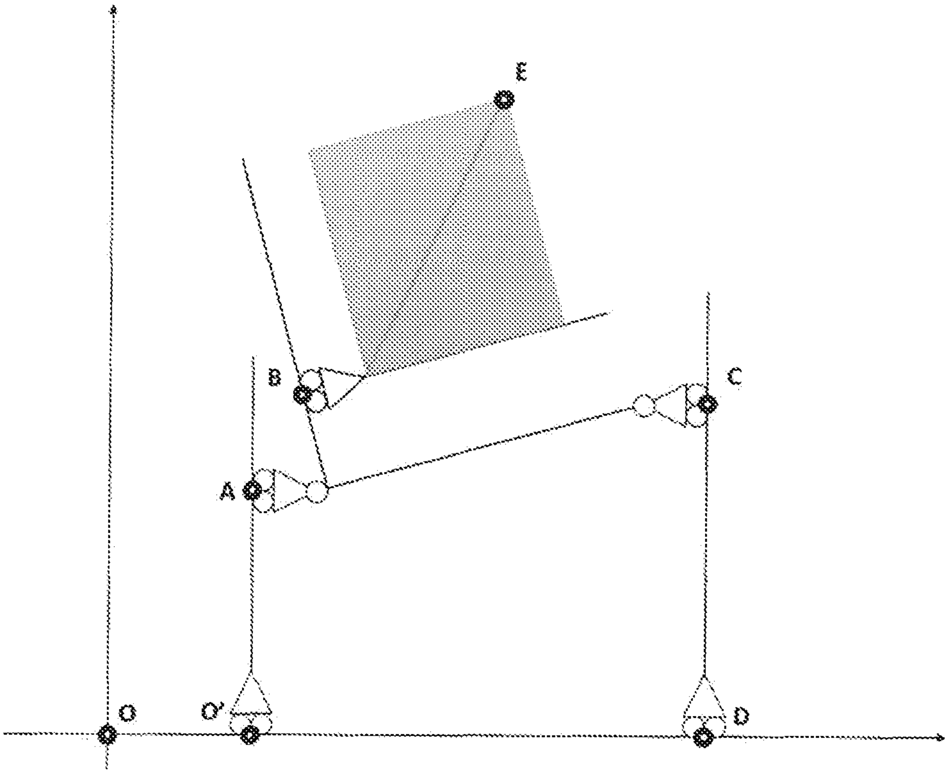

Fig. 9

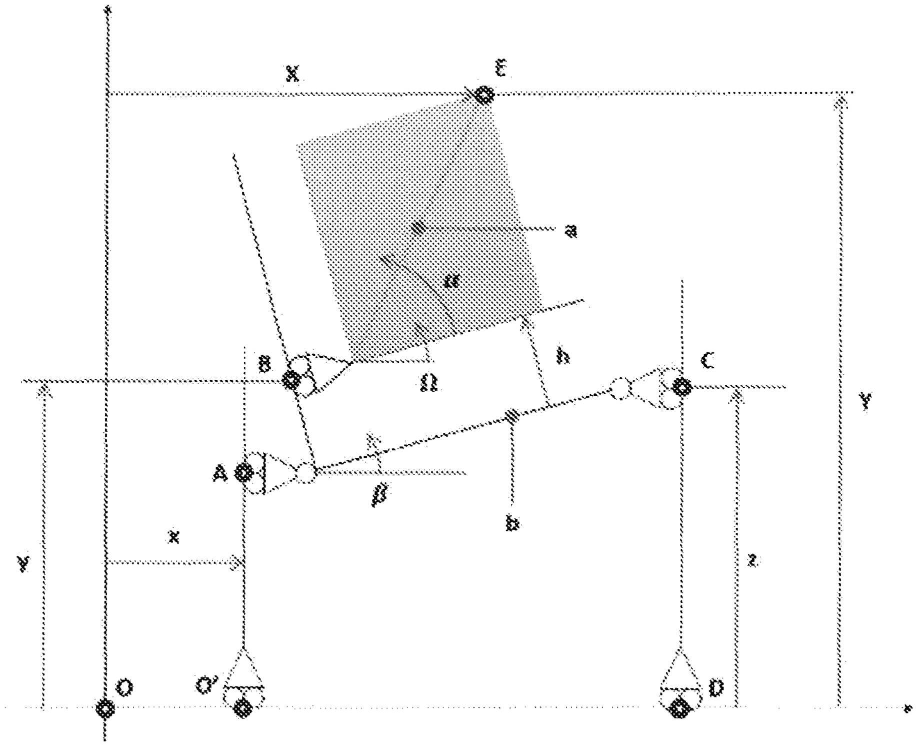

Fig. 10

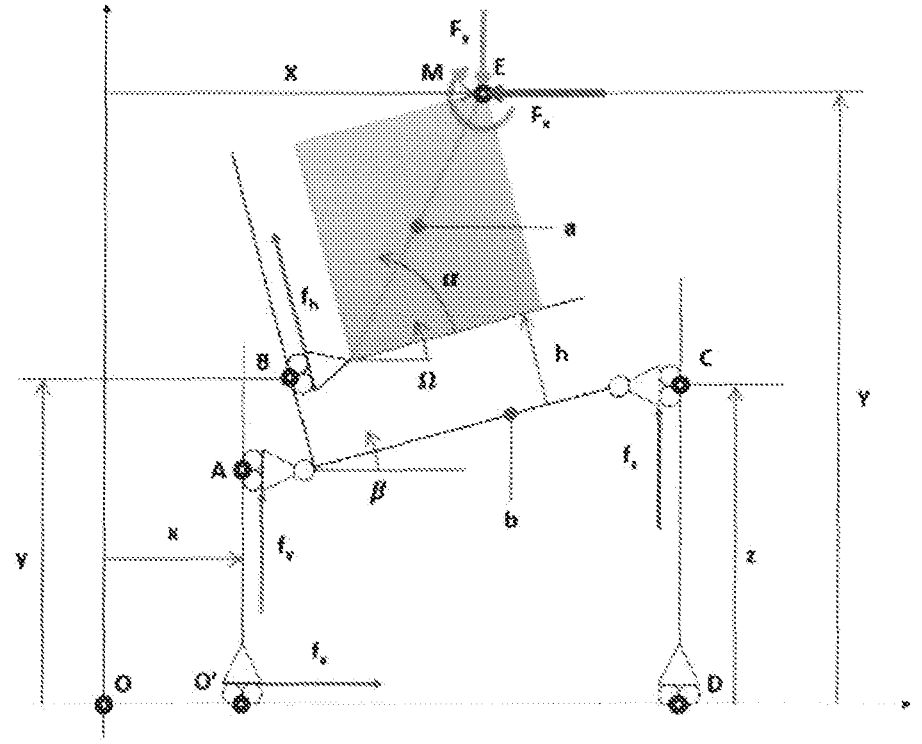

Fig. 11

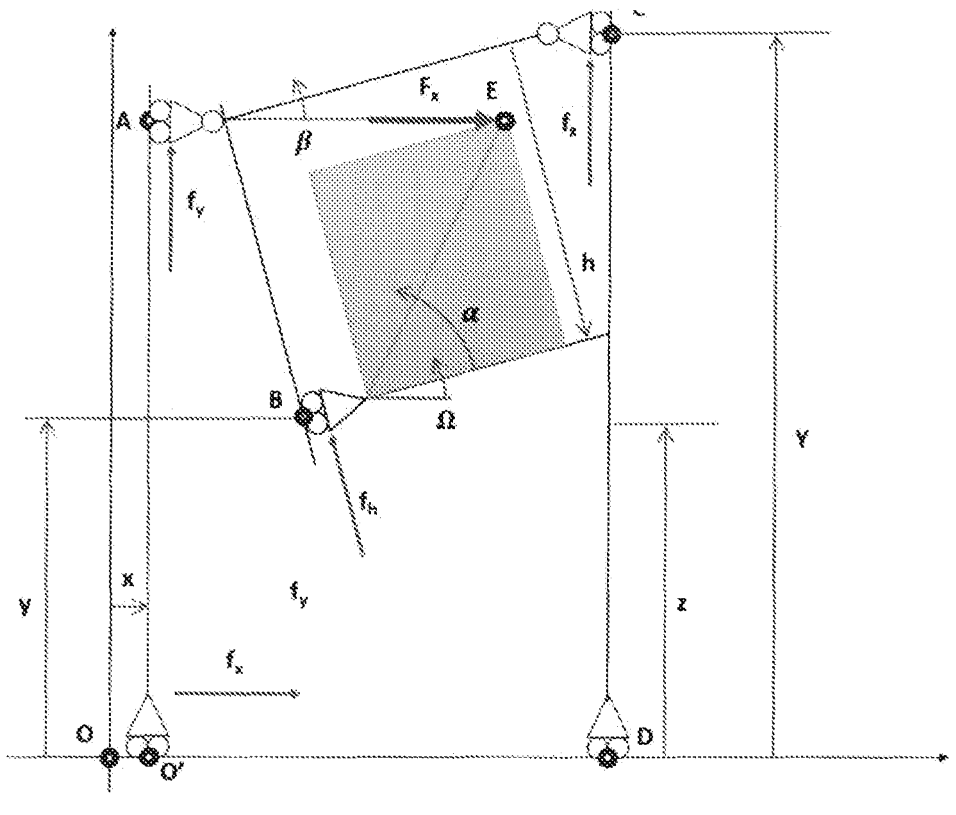

Fig. 12

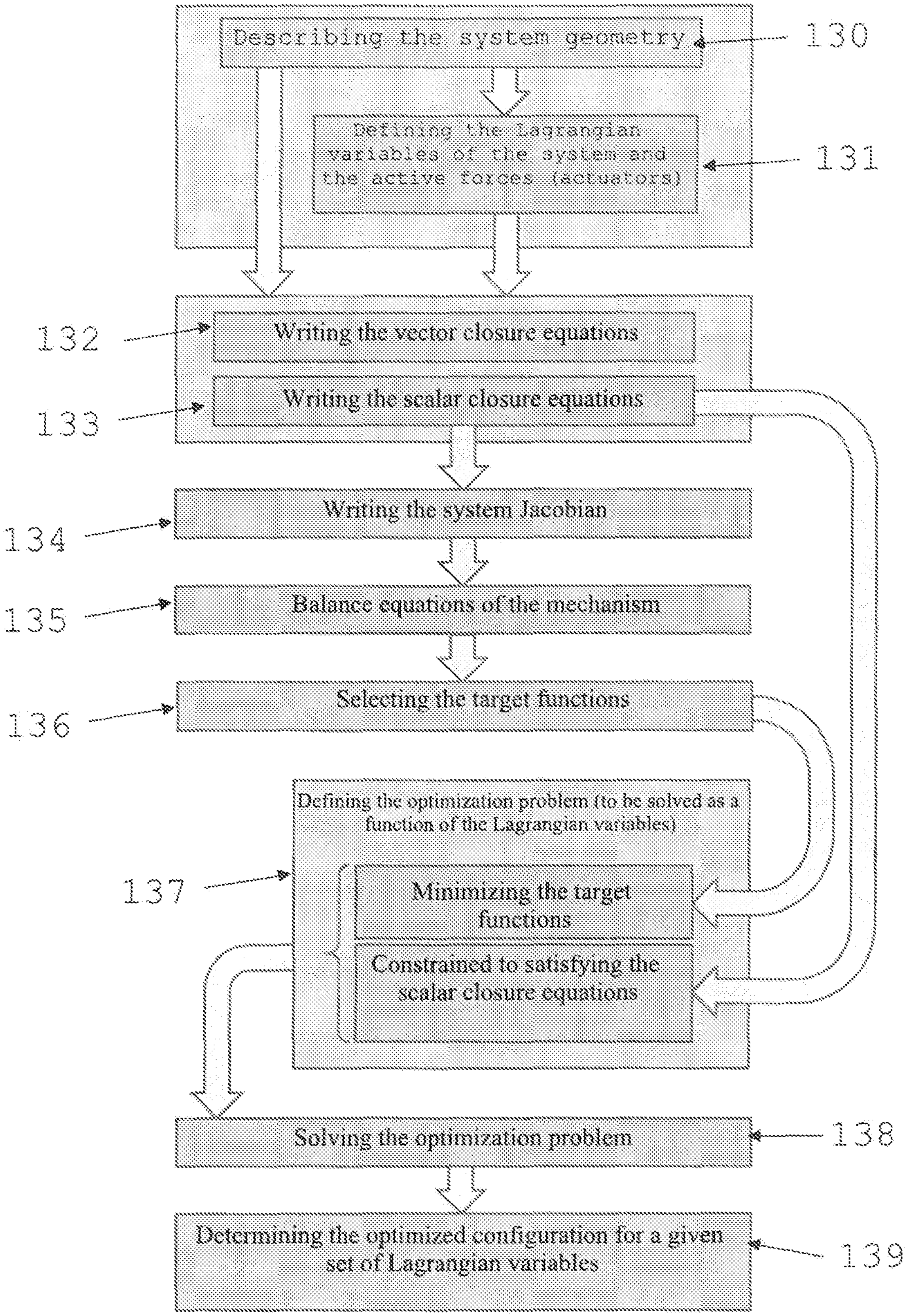
Describing the system geometry
130
Defining the Lagrangian variables of the system and the active forces (actuators)
131
132
Writing the vector closure equations
133
Writing the scalar closure equations
134
Writing the system Jacobian
135
Balance equations of the mechanism
136
Selecting the target functions
Defining the optimization problem (to be solved as a function of the Lagrangian variables)
137
Minimizing the target functions
Constrained to satisfying the scalar closure equations
Solving the optimization problem
138
Determining the optimized configuration for a given set of Lagrangian variables
139

Fig. 13

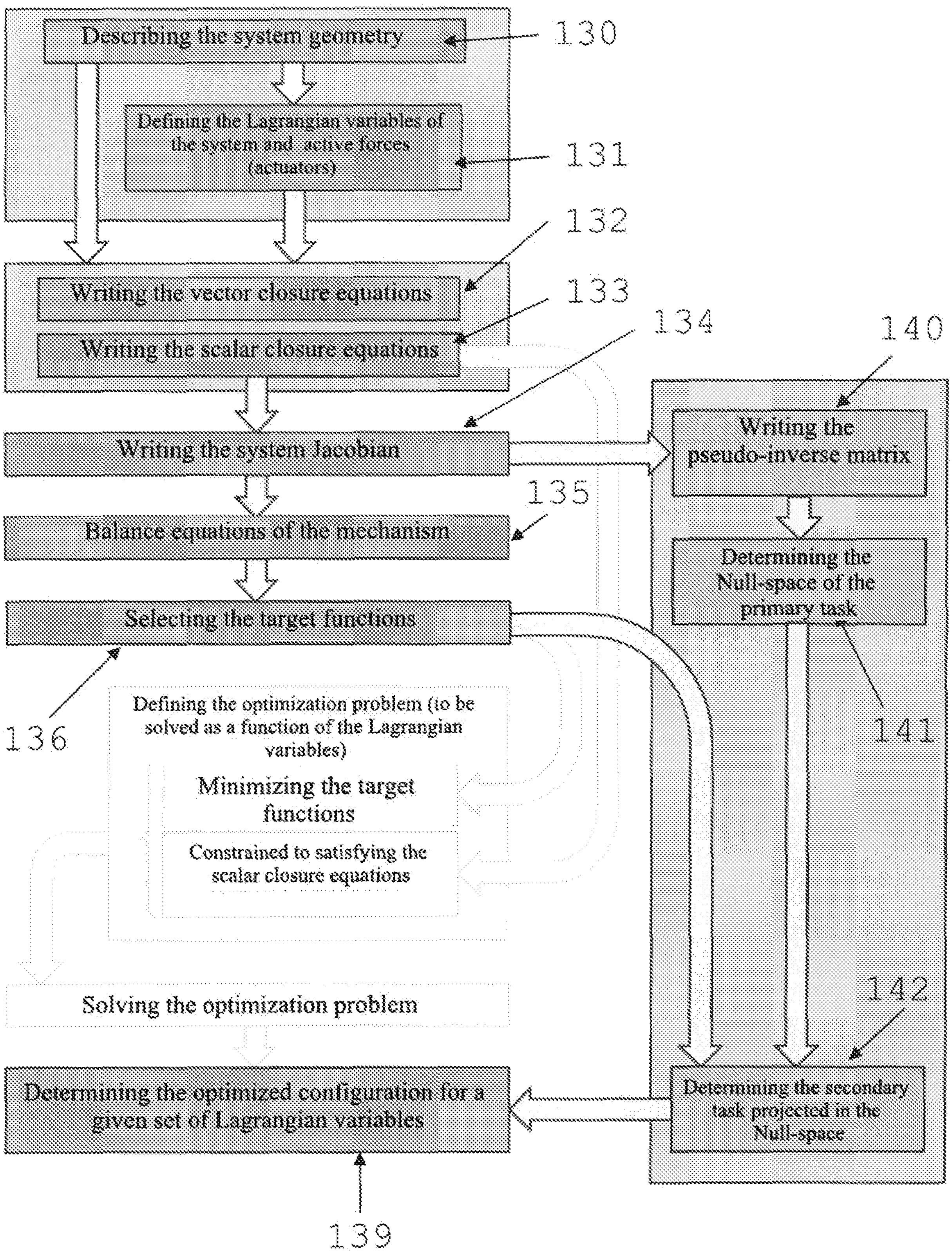
Describing the system geometry
130
Defining the Lagrangian variables of the system and active forces (actuators)
131
132
Writing the vector closure equations
133
Writing the scalar closure equations
134
140
Writing the system Jacobian
Writing the pseudo-inverse matrix
135
Balance equations of the mechanism
Determining the Null-space of the primary task
Selecting the target functions
136
Defining the optimization problem (to be solved as a function of the Lagrangian variables)
Minimizing the target functions
Constrained to satisfying the scalar closure equations
141
142
Solving the optimization problem
Determining the optimized configuration for a given set of Lagrangian variables
Determining the secondary task projected in the Null-space
139

Fig. 14

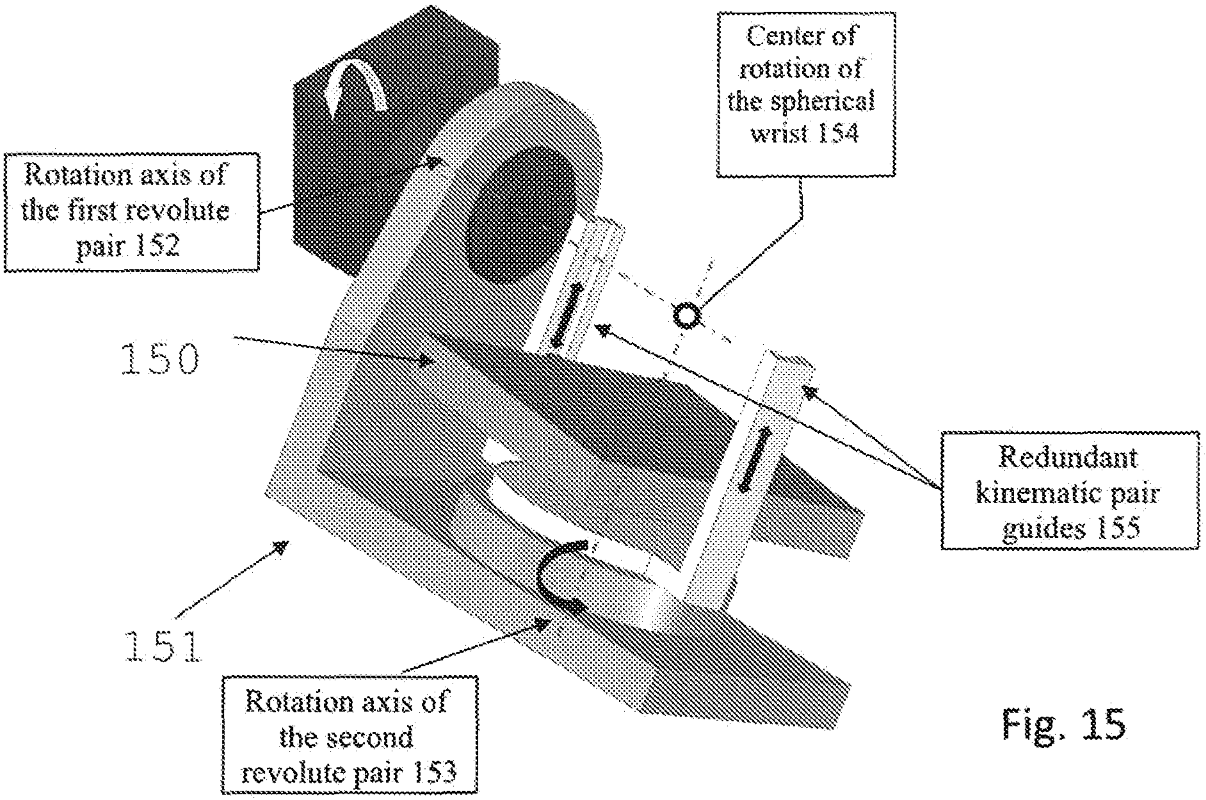
Center of rotation of the spherical wrist 154
Rotation axis of the first revolute pair 152
150
Redundant kinematic pair guides 155
151
Rotation axis of the second revolute pair 153

Fig. 15

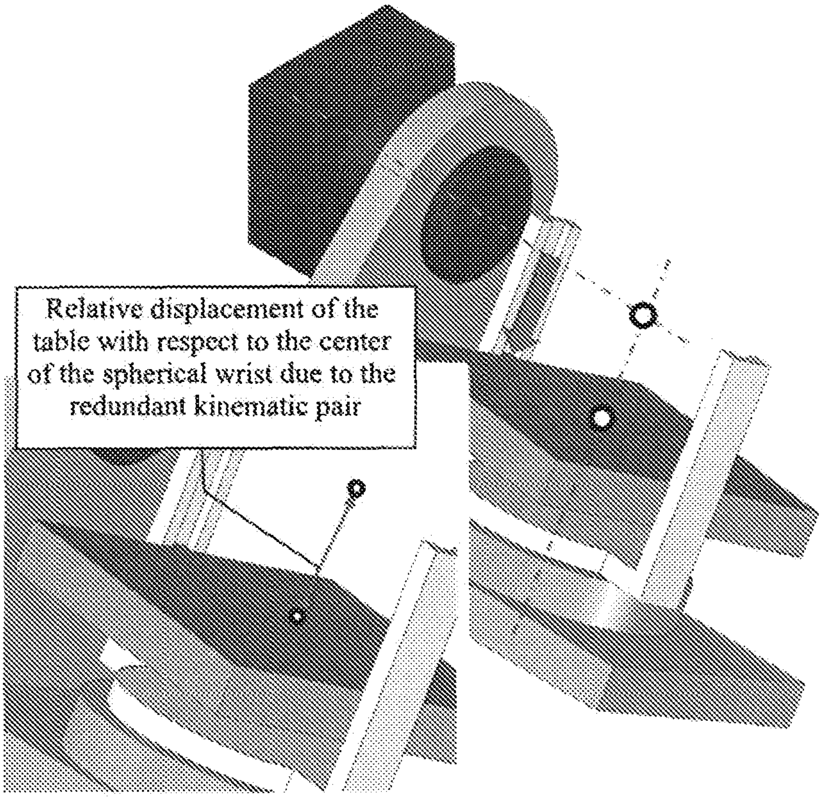
Relative displacement of the table with respect to the center of the spherical wrist due to the redundant kinematic pair

Fig. 16

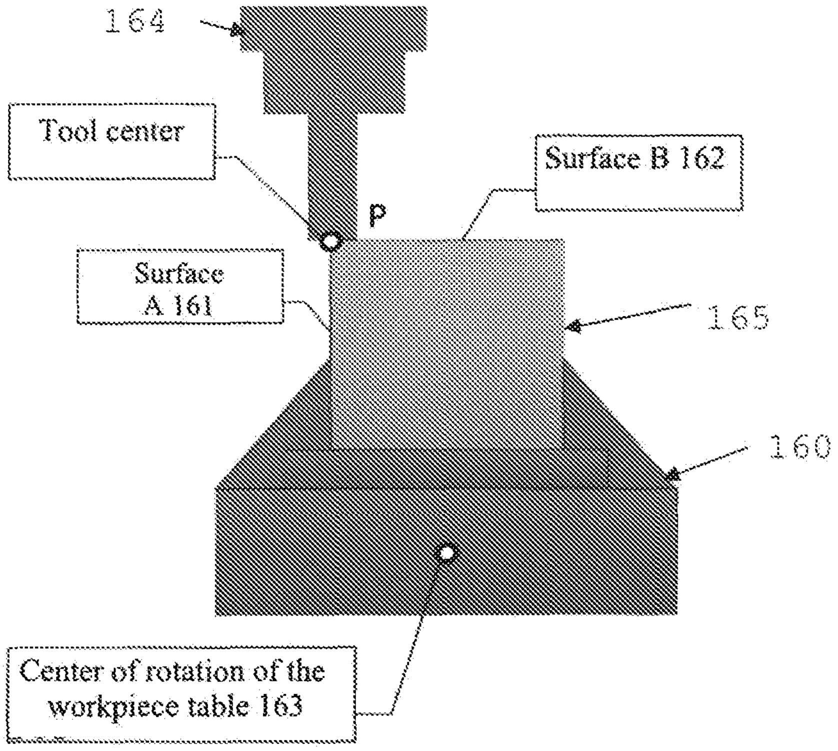
164
Tool center
P
Surface B 162
Surface A 161
165
160
Center of rotation of the workpiece table 163

Fig. 17

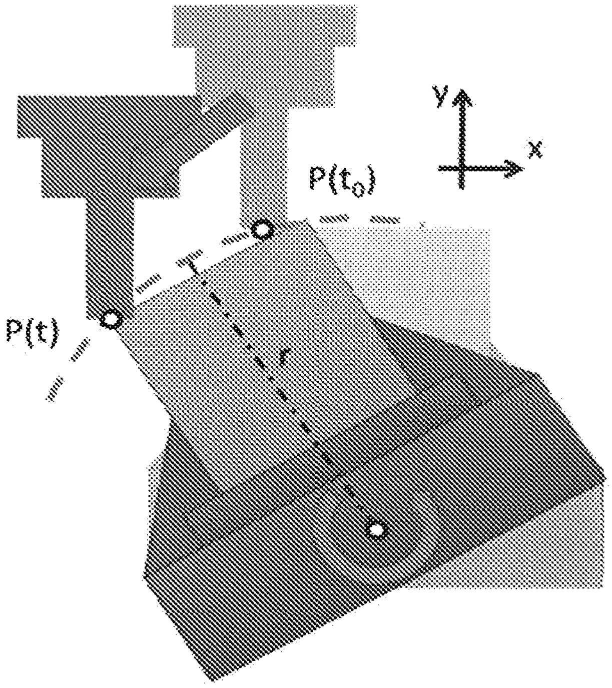
y
x
P(t₀)
P(t)
T
r

Fig. 18

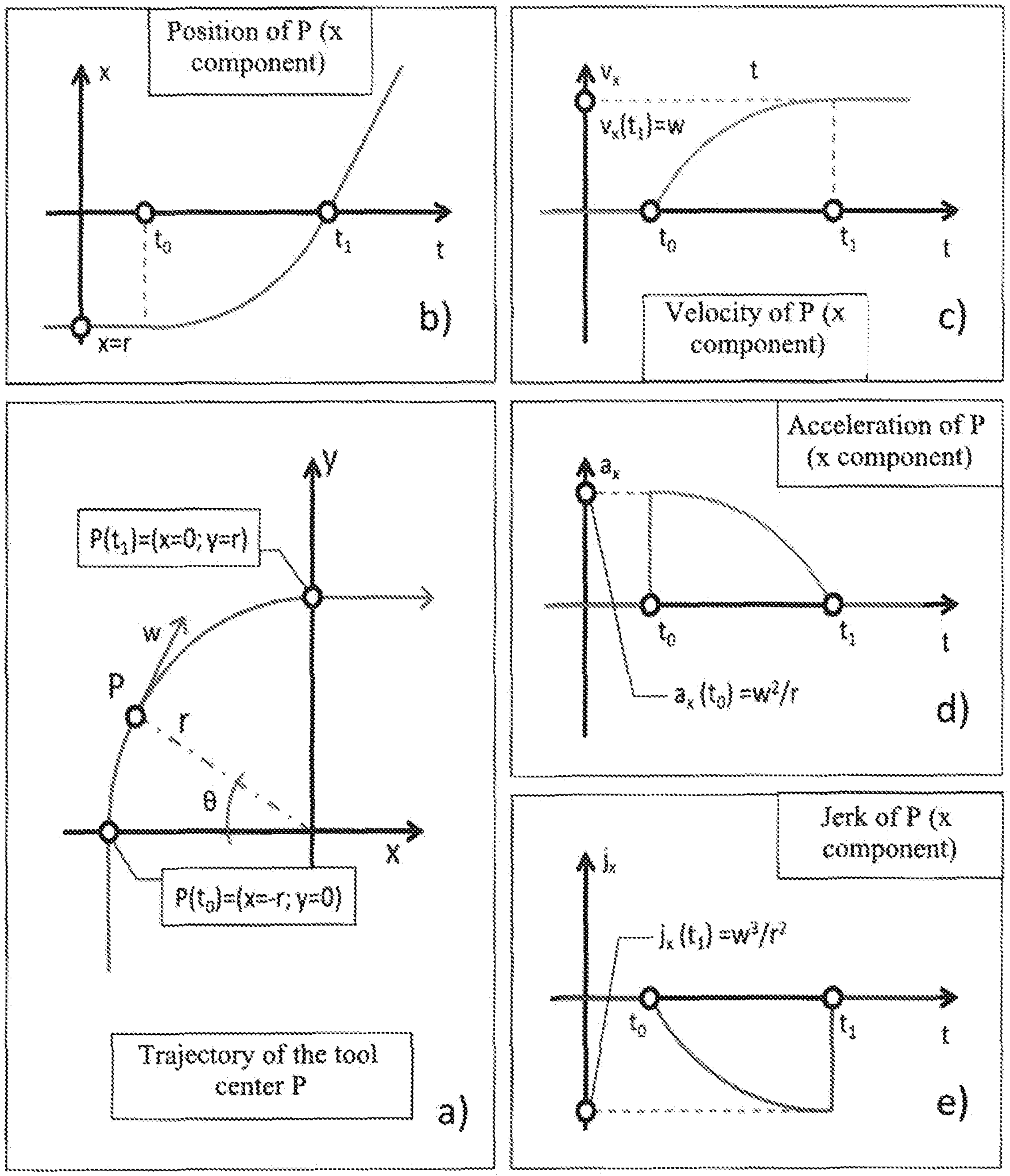
Position of P (x component)
x
t₀
t₁
t
x=r
b)
Velocity of P (x component)
vₓ
t
vₓ(t₁)=w
t₀
t₁
t
c)
y
P(t₁)=(x=0; y=r)
w
P
r
θ
X
P(t₀)=(x=-r; y=0)
Trajectory of the tool center P
a)
Acceleration of P (x component)
aₓ
t₀
t₁
t
aₓ (t₀) =w²/r
d)
Jerk of P (x component)
jₓ
jₓ (t₁) =w³/r²
t₀
t₁
t
e)

Fig. 19

INTERNATIONAL SEARCH REPORT

International application No
PCT/IB2019/055026

A. CLASSIFICATION OF SUBJECT MATTER
INV. G05B19/18 G05B19/4099 G05B19/404
ADD.

According to International Patent Classification (IPC) or to both national classification and IPC

B. FIELDS SEARCHED

Minimum documentation searched (classification system followed by classification symbols)
G05B

Documentation searched other than minimum documentation to the extent that such documents are included in the fields searched

Electronic data base consulted during the international search (name of data base and, where practicable, search terms used)

EPO-Internal

C. DOCUMENTS CONSIDERED TO BE RELEVANT

Category*	Citation of document, with indication, where appropriate, of the relevant passages	Relevant to claim No.
X	Dimitris Mourtzis: "COMPUTER NUMERICAL CONTROL OF MACHINE TOOLS - Chapter 16: 5-Axis Machining", , 31 October 2013 (2013-10-31), pages 1-61, XP055567496, Retrieved from the Internet: URL:http://lms.mech.upatras.gr/LMS/files-1/students-area/arithmetikos-elegkhos-ergaleiomekhanon/subject-files/ekpaideutiko-uliko/CNCChapter16_15012014_LOCKED.pdf/view [retrieved on 2019-03-12]	1,3-10, 15
Y	page 7 - page 16	11-14
A	page 47 - page 53 ----- -/--	2

☒ Further documents are listed in the continuation of Box C.

☐ See patent family annex.

* Special categories of cited documents :

"A" document defining the general state of the art which is not considered to be of particular relevance

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"O" document referring to an oral disclosure, use, exhibition or other means

"P" document published prior to the international filing date but later than the priority date claimed

"T" later document published after the international filing date or priority date and not in conflict with the application but cited to understand the principle or theory underlying the invention

"X" document of particular relevance; the claimed invention cannot be considered novel or cannot be considered to involve an inventive step when the document is taken alone

"Y" document of particular relevance; the claimed invention cannot be considered to involve an inventive step when the document is combined with one or more other such documents, such combination being obvious to a person skilled in the art

"&" document member of the same patent family

Date of the actual completion of the international search

27 September 2019

Date of mailing of the international search report

14/10/2019

Name and mailing address of the ISA/

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NL - 2280 HV Rijswijk
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Authorized officer

De Santis, Agostino

INTERNATIONAL SEARCH REPORT

International application No
PCT/IB2019/055026

C(Continuation). DOCUMENTS CONSIDERED TO BE RELEVANT

Category*	Citation of document, with indication, where appropriate, of the relevant passages	Relevant to claim No.
X	FENG WANG ET AL: "Design and implementation of five-axis transformation function in CNC system", CHINESE JOURNAL OF AERONAUTICS, vol. 27, no. 2, 20 February 2014 (2014-02-20), pages 425-437, XP055567508, AMSTERDAM, NL ISSN: 1000-9361, DOI: 10.1016/j.cja.2014.02.009	1,15
Y	Section 4	11
Y	S. L. CHIU: "Task Compatibility of Manipulator Postures", INTERNATIONAL JOURNAL OF ROBOTICS RESEARCH., vol. 7, no. 5, 1 October 1988 (1988-10-01), pages 13-21, XP055362753, US ISSN: 0278-3649, DOI: 10.1177/027836498800700502 Sections 2,4	11-14
Y	PASQUALE CHIACCHIO ET AL: "GLOBAL TASK SPACE MANIPULABILITY ELLIPSOIDS FOR MULTIPLE-ARM SYSTEMS", IEEE TRANSACTIONS ON ROBOTICS AND AUTOMATION, IEEE INC, NEW YORK, US, vol. 7, no. 5, 1 October 1991 (1991-10-01), pages 678-685, XP000430864, ISSN: 1042-296X, DOI: 10.1109/70.97880 Sections III-IV	11-14