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Network conditional tail risk estimation in the European banking system

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Abstract

CoVaR is one of the most popular measures of systemic risk. It is the VaR (Value at Risk) of the system (represented as a broad market index) conditional to the fact that a certain institution is in distress (i.e. at its VaR). One of the limits of CoVaR is that it does not consider the relations among institutions in the system, failing to represent interconnectedness, that is a relevant component of systemic risk. Instead, it reflects more the *systematic* component of risk, that is, the one related to a common component. A popular approach to analyze interconnectedness is to consider an economic system as a network. In this work we deal with network-ΔCoVaR, a multivariate extension of ΔCoVaR, that measures the marginal tail dependence among institutions while controlling for the effects of the others. We discuss the properties of the model and we propose an estimation methodology based on quantile regression with Smoothly Clipped Absolute Deviation (SCAD) penalty. Finally, we use these tail risk networks to develop systemic risk indicators and to study the characteristic of the European banking system and its evolution over time.

Key words: ΔCoVaR; Systemic risk; Quantile regression; Financial networks

JEL Classification: C58, G01, G15

1. Introduction

Since the global financial crisis in 2008, systemic risk became a relevant topic for scholars, investors and regulators. Many approaches have been defined to measure and handle it. One of the main challenges is the lack of a shared definition. This leads to very different modelization approaches, each one focused on different aspects of systemic risk.

Some works take systemic risk in terms of potential for the spreading of financial distress, measuring this increase in tail comovement. These works focus on the analysis of tail risk under stress scenarios, measuring either the effect of a systemic shock to the value of an institution, or the effect of the distress of an institution to the entire system. The most known approaches in this group are probably the CoVaR and the closely related ΔCoVaR [1]. In particular, CoVaR measures the Value at Risk (VaR) of the system conditional to a particular asset being distressed, and ΔCoVaR compares the CoVaR to its VaR in a non distressed situation.

Other works use a network approach to study the interdependence of assets. They embrace a vision of systemic risk more focused on the presence of risk spillover and contagion (see e.g. the definition of systemic risk given by [2]. These approaches model institutions as nodes in a network, and their relations as edges. They can be used to uncover structural features of the system that may not emerge from aggregated data. A main challenge is the estimation of the network: it can be modelled in several ways, either considering physical measures of interconnectedness or with statistical measures based on time series.

In this work we join the network approach to the ΔCoVaR approach, by studying systemic risk from a network perspective, extending some of the results developed in the CoVaR

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framework to a multivariate setting, studying the theoretical properties of these networks and comparing them in an empirical application focused on the European market.

The extension of ΔCoVaR is based on a concept recently introduced in the statistical literature. That is, quantile graphical models [3,4]. This approach models the quantiles of a variable in a system conditional to the value of the other variables, and provides a rich and flexible modelization of a multivariate system. The estimation is typically done using quantile regression. We define network-ΔCoVaR (also denoted in matrix form as **ΔCoVaR**) and we show how such model can also be considered an extension of partial correlation networks, in which we consider conditional quantiles instead of the conditional means.

After discussing some of the properties of the model under parametric specifications, we propose an estimation procedure based on SCAD penalized quantile regression and an empirical application focused on the European banking system.

2. Market-based systemic risk measures

2.1 CoVaR and -ΔCoVaR

Value at Risk (VaR) is a popular risk measure that indicates the potential loss of a position in a given time period with a certain level of confidence $1 - \tau$. We can define it implicitly as the τ - quantile:

$$\Pr\{X_i \leq VaR_\tau^{X_i}\} = \tau.$$

Such measure is based on the univariate distribution of the variable X_i, and it does not allow to measure the risk related to interaction among assets. CoVaR is an alternative measure proposed by [1] that allows to consider conditional tail risk. In particular, CoVaR measures what happens to the system's VaR when one institution is under stress.
we denote CoVaR as a value such that:

$$\Pr\left\{X_i \leq CoVaR_\tau^{X_{sys}|X_i} | X_i = VaR_\tau^{X_i}\right\} = \tau,$$

where X_{sys} is the return of the entire system, and X_i the returns of the *i*th institution.
A high CoVaR could be caused by an overall high VaR of the system. Therefore, we compare CoVaR to a reference value, typically the VaR of the system when institution i is in a normal state (i.e. its median value). We consider therefore ΔCoVaR:

$$\Delta CoVaR_\tau^{X_{sys}|X_i} = CoVaR_\tau^{X_{sys}|X_i} - CoVaR_{50\%}^{X_{sys}|X_i}.$$

ΔCoVaR is a popular measure of systemic risk. The estimation is commonly performed using quantile regression. When estimated using quantile regression, ΔCoVaR can be expressed as:

$$\Delta CoVaR_\tau^{X_{sys}|X_i} = \beta_{tau}^{X_{sys}|X_i}\left(VaR_\tau^{X_i} - VaR_{50\%}^{X_i}\right),$$

where $\beta_{tau}^{X_{sys}|X_i}$ is the τ-quantile regression parameter of the quantile regression of X_{sys} over X_i.

2.1.1 Limits of ΔCoVaR

Although very popular in the literature, CoVaR and ΔCoVaR have some issues as systemic risk indicators. First, as highlighted by [5], ΔCoVaR can be interpreted as a measure of *systematic* risk, rather than *systemic*. Indeed, [5] show that this measure under certain assumptions is simply a proxy for systemic risk beta, that is easier to estimate.

Two more arguments regard the shape of the quantile functions: first, the slope of the quantile function can be influenced by the characteristics of the bivariate distribution, and it is not always related to the level of tail dependence. Then, the estimation performed using quantile regression impose the linearity of the conditional quantile function, leading to potential underestimation of the conditional tail risk.
Finally, the ability of conditional tail measures to predict actual distress in financial institions has been questioned by empirical analyses, see e.g. [6].

2.2 Network models for systemic risk – partial correlation networks

Network models allow to consider the component of contagion across companies or financial institutions. In particular, several works use network centrality measures to assess the relevance of a bank in a system. One of the most relevant challeges is the estimation of the network structure, and the literature describes several approaches. A relevant strand of literature focuses on measures of co-movement of time series, e.g. [7,8]. We follow this strand, and we focus in particular on partial correlation networks.
Partial correlation networks allow to model the correlation between any couple of variables while controlling for all the others in the system. The edge *ij* of the network is defined as the partial correlation between nodes *i* and *j*, conditional to all the other variables in the system. An important form of partial correlation networks are Gaussian graphical models that, under the assumption that data have a jointly normal distribution, imply that two nodes that are not connected by an edge are conditionally independent. The reader can refer to [8,9] for more details, where partial correlation models are presented, together with estimation procedures under Gaussian and t-Student distribution.

3. Network-ΔCoVaR

Here we extend ΔCoVaR to the network case, and we highlight some of the properties. The concept is strictly related to the quantile graphical models [3]. In general, we can express the quantiles of the distribution of an item conditional to the others as

$$Q_{X_{i,\tau}} = f\left(X_{\setminus i}, \tau\right),$$

where f is a generic function. The estimation of f is particularly challenging in high dimension, and several simplifying assumptions can help the estimation. A common choice is to use an additive form: $f\left(X_{\setminus i}, \tau\right) = \sum_{j \neq i} f_j\left(X_j, \tau\right)$, where $f_j\left(X_j, \tau\right)$ are smooth functions [4]. A further simplification is to assume that $f_i\left(X_j, \tau\right)$ are linear. In this case we have $f_i\left(X_j, \tau\right) = a_i + \beta^{i|\cdot} X_{\setminus i}$.
The coefficients $\beta^{i|\cdot}$ can be conveniently represented by the matrix B, consisting in the squared matrix consisting in the stacked vectors $\beta^{i|\cdot}$, with zero on the main diagonal.
Given the quantile graphical model, we can extend the concept of CoVaR to the multivariate setting: we measure the quantile function of asset i conditional to the fact that asset j is in distress, and the other assets in the system are in their normal (i.e. median) state. More formally:

$$\Pr\left\{X_i \leq CoVaR_\tau^{X_i|X_j} | X_j = VaR_\tau^{X_j}, X_{\setminus\{ij\}} = VaR_{50\%}^{\setminus\{ij\}}\right\} = \tau.$$

We explicitly we have:

$$CoVaR_\tau^{X_i|X_j} = VaR_\tau\left(X_i \middle| X_j = VaR_\tau^{X_j}, X_{\setminus\{ij\}} = VaR_{50\%}^{\setminus\{ij\}}\right).$$

Then, we can compute ΔCoVaR as:

$$\Delta CoVaR_\tau^{X_i|X_j} = CoVaR_\tau^{X_i|X_i} - CoVaR_{50\%}^{X_i|X_j}.$$

We can construct a weighted and directed network using the set of all bilateral $\Delta CoVaR_{\tau}^{X_i|X_j}$. For convenience we define the weighted adjacency matrix Δ**CoVaR**, where the element ij is equal to $\Delta CoVaR_{\tau}^{X_i|X_j}$.

3.1 Parametric examples

We consider first a parametric specification characterized by a p-variate Gaussian distribution, and consider the following partition:

$$X = \begin{bmatrix} X_{\setminus i} \\ X_i \end{bmatrix} \sim \mathcal{N}_p(\mu, \boldsymbol{\Sigma}) = \mathcal{N}_p\left(\begin{bmatrix} \mu_{\setminus i} \\ \mu_i \end{bmatrix}, \begin{bmatrix} \boldsymbol{\Sigma}_{\setminus i \setminus i} & \boldsymbol{\Sigma}_{\setminus ii} \\ \boldsymbol{\Sigma}_{i \setminus i} & \boldsymbol{\Sigma}_{ii} \end{bmatrix}\right).$$

Focusing for simplicity on the case $\mu = \mathbf{0}$, we represent the conditional quantile of $X_i|X_{\setminus i}$ as:

$$Q_\tau\left(X_i \middle| X_{\setminus i}, \tau\right) = \phi^{-1}(\tau)\boldsymbol{\Sigma}_{ii|\setminus i}^{0.5} + \mu_{i|\setminus i}$$

where $\mu_{i|\setminus i} = \boldsymbol{\Sigma}_{i\setminus i}\boldsymbol{\Sigma}_{\setminus i\setminus i}^{-1}X_{\setminus i}$ and $\boldsymbol{\Sigma}_{ii|\setminus i} = \boldsymbol{\Sigma}_{ii} - \boldsymbol{\Sigma}_{i\setminus i}\boldsymbol{\Sigma}_{\setminus i\setminus i}^{-1}\boldsymbol{\Sigma}_{\setminus ii}$.
It follows that all the conditional quantile functions are affine, and parallel to the OLS regression coefficients, as we notice that $\beta_{OLS}^{i|\setminus i} = \boldsymbol{\Sigma}_{i\setminus i}\boldsymbol{\Sigma}_{\setminus i\setminus i}^{-1}$.
We can then express network-CoVaR and network-ΔCoVaR in terms of $\beta_{OLS}^{i|\setminus i}$, as:

$$CoVaR_{\tau}^{X_i|X_j} = \phi^{-1}(\tau)\boldsymbol{\Sigma}_{ii|\setminus i}^{0.5} + \beta_{OLS}^{i|\setminus i}X_{\setminus i},$$

$$\Delta CoVaR_{\tau}^{X_i|X_j} = \beta_{OLS}^{i|\setminus i}VaR_{\tau}^{X_j}.$$

In matrix form we can express network-$\Delta CoVaR$ as $\boldsymbol{\Delta CoVaR_{\tau}}$:

$$\boldsymbol{\Delta CoVaR_{\tau}} = \phi^{-1}(\tau)\boldsymbol{B}\,\boldsymbol{D}_{\boldsymbol{\Sigma}}^{0.5},$$

where $\boldsymbol{B}$ is the matrix of OLS coefficients, and $\boldsymbol{D}_{\boldsymbol{\Sigma}}$ is a diagonal matrix with the diagonal elements of the covariance matrix $\boldsymbol{\Sigma}$. Following [8,9,10], we know that the matrix $\boldsymbol{B}$ is a rescaling of the partial correlation matrix. It follows that in the Gaussian case network-ΔCoVaR carry the same information of partial correlation networks.

We consider then a t-Student specification. Let $X \sim t_p(\mu, \boldsymbol{\Sigma}, \nu)$, where ν is the number of degrees of freedom of the distribution, and μ and $\boldsymbol{\Sigma}$ are the mean and scatter parameters, respectively. Similarly to the Gaussian case, we consider the partition $X = \begin{bmatrix} X_{\setminus i} \\ X_i \end{bmatrix}$. Focusing again on the case with $\mu = \mathbf{0}$, according to [11] we can compute the conditional quantile of X_i given $X_{\setminus i}$:

$$Q_\tau\left(X_i \middle| X_{\setminus i}, \tau\right) = Q_{t,\nu+p-1}(\tau)\left(\frac{\nu + d_1\left(X_{\setminus i}\right)}{\nu + p - 1}\boldsymbol{\Sigma}_{ii|\setminus i}\right)^{0.5} + \mu_{i|\setminus i},$$

where $\mu_{i|\setminus i}$ and $\boldsymbol{\Sigma}_{ii|\setminus i}$ are defined as in the Gaussian case, and $d_1\left(X_{\setminus i}\right) = X_{\setminus i}'\boldsymbol{\Sigma}_{\setminus i\setminus i}^{-1}X_{\setminus i}$ is the squared Mahalanobis distance of $X_{\setminus i}$ from the origin with scale matrix $\boldsymbol{\Sigma}_{\setminus i\setminus i}$.

The function is not linear in $X_{\setminus i}$ due to the term $d_1\left(X_{\setminus i}\right)$. It follows that, differently from the Gaussian case, the conditional quantile cannot be computed simply as a translation of the $\beta_{OLS}^{i|\setminus i}$.

We can then compute network-CoVaR and network-ΔCoVaR as:

$$CoVaR_{\tau}^{X_i|X_j} = Q_{t,\nu+p-1}(\tau)\left(\frac{\nu + d_1(X_{\setminus i})}{\nu + p - 1}\boldsymbol{\Sigma}_{ii|\setminus i}\right)^{0.5} + \beta_{OLS}^{i|\setminus i}X_{\setminus i}^{d},$$

$$\Delta CoVaR_{\tau}^{X_i|X_j} = Q_{t,\nu+p-1}(\tau)\boldsymbol{\Sigma}_{ii|\setminus i}^{0.5}\left(\left(\frac{\nu + d_1(X_{\setminus i})}{\nu + p - 1}\right)^{0.5} - \left(\frac{\nu}{\nu + p - 1}\right)^{0.5}\right) + \beta_{OLS}^{i|\setminus i}X_{\setminus i}^{d},$$

Where $X_{\setminus i}^{d}$ denotes a state of the system in which $X_j = VaR_{\tau}^{X_j}, X_{\setminus\{ij\}} = VaR_{0.5}^{X_{\setminus\{ij\}}}$.

In a t-Student setting, a shock to a variable *j* will therefore have a non-linear impact on the conditional value at risk of an asset *i*, and the impact will be higher compared to the Gaussian setting.
Moreover, the estimation of a linear quantile function (as the one obtained using a quantile regression procedure) will underestimate the network- ΔCoVaR, as for $\tau < 0.5$ the quantile function will underestimate the value of the network- ΔCoVaR due to the shape of the quantile function, that is concave for $\tau < 0.5$.
Overall, this t-Student setting show us that interconnectedness may have a particularly strong effect in the tails of the distributions, that does not happen in a Gaussian setting.

4. Estimation of network-ΔCoVaR

Quantile regression is a powerful framework for the estimation of non-parametric quantile graphical model. We consider a linear specification of penalized quantile regression, that allows to perform model selection and estimation at the same time by setting some of the parameters to zero, increasing the efficiency of the estimator in setting with a high number of variables in relation to the observations [3]. Contrarily to previous literature, we use SCAD penalized quantile regression [12].
Quantile regression allows to summarize the relationship between a set of regressors and an outcome variable. Differently from the more common mean regression framework, we model the value of the conditional quantile. It is possible to obtain the quantile regression estimator by minimizing an asymmetric loss function. The optimization problem is linear and can be solved efficiently using the simplex method, or interior point methods [13]. We can obtain a sparse estimate of the model (i.e. a model where some of the parameters are exactly equal to zero) by introducing a penalization in the optimization problem [13, 3]:

$$\min_{\beta_\tau} \mathbb{E}\left[\rho\left(X_i - \beta_\tau X_{\setminus i}\right)\right] + n\sqrt{\tau(1-\tau)}p_\lambda(\beta_\tau)$$

Where $\rho(u) = \left(\tau - \mathbb{I}_{\{u\le 0\}}\right)u$ is an asymmetric loss function, *n* is the number of observations, β_τ the set of parameters and $p_\lambda(\cdot)$ a penalty function. The most common penalty function is the *lasso*, that penalized the sum of absolute values of the vector of parameters inducing sparsity [3]. We consider instead the SCAD penalty that is shaped as follows:

$$p_{\lambda}^{SCAD}(\beta) = \begin{cases} \lambda|\beta| & if\ |\beta| < \lambda \\ -\dfrac{|\beta|^2 - 2a\lambda|\beta| + \lambda^2}{2(a-1)} & if\ \lambda \le |\beta| < a\lambda. \\ \dfrac{(a+1)\lambda^2}{2} & if\ |\beta| \ge a\lambda \end{cases}$$

The function is linear and equivalent to a lasso penalization near the origin, then quadratic for a trait and finally flat [14].
[12] proved that the model estimation performed using SCAD penalty has the oracle property (i.e., asymptotically, it identifies the right subset model and it has an optimal estimation rate). This is an advantage over lasso, that does not. The non-convexity of the penalization makes the optimization problem much harder to solve, however several specific algorithms have been developed. For penalized quantile regression problems, we rely on the procedure outlined by [12]. They propose a Difference Convex Algorithm (DCA) that is based on the representation of SCAD penalty as the difference between a linear and a convex function, and solves a sequence of convex problems to approximate the SCAD problem efficiently. The SCAD penalization requires the calibration of two parameters: the penalization factor λ and the parameter a that regulates the shape of the penalty. We use a value of $\alpha = 2.7$ as proposed by [14], and we calibrate λ using Bayesian Information Criterion (BIC).

Quantile regression typically estimates a linear model, where the conditional quantile functions are straight lines (it is possible to model quantile function using different specifications, but this would increase the number of parameters of the model to estimate and would increase the computational complexity). We have seen in Section 3.1 that, except for the Gaussian case, linearity is not respected. We propose therefore to estimate the penalized quantile regressions in a specific sub-set of the data, that is the data points where the market obtained the lowest returns. In this way, we focus on the slope of the quantile function in the observations of distressed markets. In order to guarantee a good balance between size of the estimation dataset and localization of the quantile, we focus on the lowest 50% observations of an equally weighted index of all the data.

5. Network indicators

Network centrality measures are commonly used to assess the relevance of nodes in a network [7, 8, 10]. We consider here strength centrality as an indicator of relevance of nodes, and we develop a novel indicator that combines network tail risk and a measure of idiosyncratic credit risk (i.e. Non-Performing Loans – NPL). Since network- ΔCoVaR is a directed network, we can compute two set of indicators: in-strength c_{in}^{i}, representing systemic fragility, and out-strength c_{out}^{i}, representing systemic relevance:

$$c_{in}^{i} = \sum_{j=1}^{p} \Delta CoVaR_{\tau}^{i|j} \,, \qquad c_{out}^{i} = \sum_{j=1}^{p} \Delta CoVaR_{\tau}^{j|i}.$$

Another set of indicators includes the NPL in the computation:

$$c_{NPL-in}^{i} = \sum_{j=1}^{p} \Delta CoVaR_{\tau}^{i|j} \; NPL_{j}, \qquad c_{NPL-out}^{i} = \sum_{j=1}^{p} \Delta CoVaR_{\tau}^{j|i} \; NPL_{i},$$

where NPL_i is the NPL ratio of bank i, computed as the ratio between non performing loans over total loans. This measure allows us to provide a richer and nuanced assessment of systemic risk, as interconnection alone may not be enough to cause financial risk. The credit quality of the neighbors may influence the risk of a bank: a strong connection with a solid bank may indeed not represent a threat, while a fragile company may significantly affect the stability of the neighbors.

6. Empirical analysis

We estimate network-ΔCoVaR on the equity returns of a set of 36 large European banks. Due to the presence of heteroskedasticity in the data, we proceed in two steps: first we fit a CCC-GARCH model to the data, and then we estimate the conditional quantile model on the residuals. We report here for brevity the result of the analysis performed in the period Jan2007 - Dec2012. The analysis has been repeated also for the period Jan2013-Jun2018, and a more extensive analysis will be presented in future works.

For comparison, we compute two networks: network-ΔCoVaR and a network based on the quantile graphical model with $\tau = 0.5$, that is, the conditional medians (median QGM). In this way we highlight the different behavior of the tails and the center of the distributions. Note that for the network-ΔCoVaR we consider $\tau = 0.1$; we did not choose a smaller value to avoid estimation instability in finite samples.

Figure 1 shows a graphical representation of the network-ΔCoVaR and the median conditional quantile network. We see that network-ΔCoVaR presents a denser network compared to median QGM, denoting that the interconnectivity is stronger in the tails of the distribution. Moreover, we see that both networks have a community structure (i.e. nodes are grouped in dense clusters), but such structure is stronger in median QGM compared to ΔCoVaR. Together these two facts suggest that conditional tail risk is a relevant channel for the transmission of financial distress, highlighting how shocks to individual institutions can spread to the entire network fast, despite the presence of a community structure.

Figure 2 reports the in- and out-strength centrality for each bank, as well as the NPL-weighted centrality. We see that banks in Northern and Central Europe have particularly low NPL-adjusted in- and out-strength centrality, denoting less systemic relevance. Italian, Irish and Spanish banks on the other hand have high NPL-weighted centrality. The traditional strength centralities instead are more evenly distributed, and banks in central Europe are among the most relevant, suggesting that they are bridges for the transmission of distress.

Figure 1: Representation of ΔCoVaR (τ = 0.1) and median QGM model for the European banking system (2007-12). Colors represent countries.

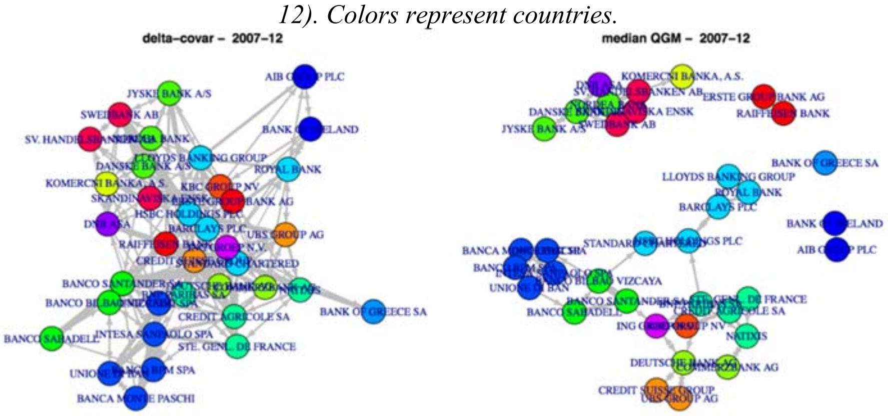
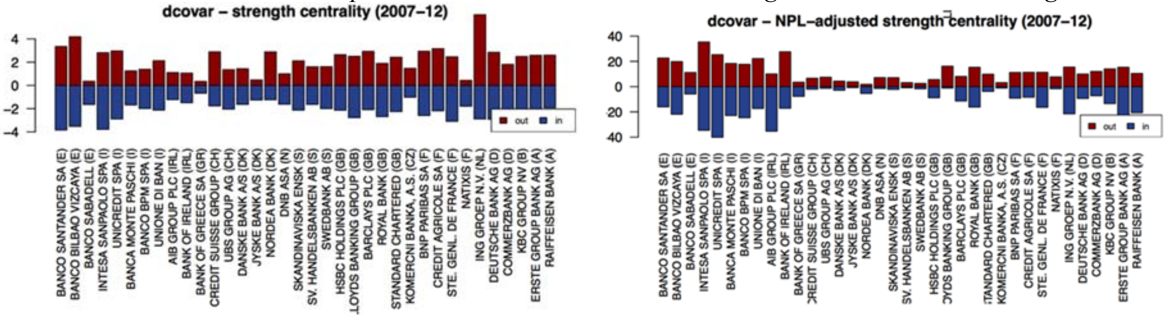


Figure 2: Strength centrality (left) and NPL adjusted strength centrality (right) of the banks in the system for ΔCoVaR networks in the period 2007-12. Blue bars are in-strength and red bars out-strength.



7. Conclusion

We introduced network-ΔCoVaR to characterize the connectivity structure in the tails of banks' equity, extending the concept of ΔCoVaR, and relating it to partial correlation networks.
After showing some properties under Gaussian and t-Student assumptions, we introduce an estimation procedure based on SCAD-penalized quantile regression, and we propose an empirical application to the European banking system, finding that tail risk network is strongly connected and less clustered than median-QGM. This work opens several research lines: first, we may study more in details the theoretical properties of the network-ΔCoVaR, then we can extend the empirical analysis to different time periods and different network indicators, then it would be interesting to assess the ability of the network indicators to forecast realized losses due to systemic risk, and to relate it to regulators' tools. Finally, it would be interesting to use network-ΔCoVaR in a portfolio management framework.

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