Order of factors in multiplying decimal numbers and gender differences: a comparison of tasks

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Differential Item Functioning (DIF) analysis carried out within the framework of the Rasch model has been used to compare scholastic performance in sub-groups of students matched on ability and clustered by some features (such as, for example, gender). In this paper, we use DIF to understand if misconceptions about multiplication with decimal numbers act on boys and girls differently, i.e. if this can lead boys and girls to approach a mathematics item differently and, in some cases, to encounter it not successfully. We present empirical evidences revealed on a sample of 1647 students attending grade 8 of lower intermediate school. Results highlighted that misconception about multiplication of decimal numbers strongly disadvantages girls relative to boys and pose new research questions about mechanisms causing these results.

Keywords: Mathematics achievement test, Misconceptions, Gender gap, Rasch model, Differential item functioning.

Introduction.

Italian National standardized assessments, called INVALSI test, are administered every year to all Italian students in different grades from primary to secondary school. In recent years the importance given to these tests has increased and researchers in mathematics education are starting to use the INVALSI results.

Results of national and international surveys have highlighted that in many countries there is a strong gender gap in mathematics in favour of males and Italy is one of the countries in which this gap is more remarkable (Mullis et al., 2016; OECD, 2016; INVALSI, 2017). This phenomenon has been deeply studied in mathematics education research. Recent works overcome the idea that the main causes of gender gap are related to biological differences (Hill, Corbett & St Rose, 2010): the main reasons of this gap must be sought considering factors related to the social and cultural context in which the students live, such as the level of gender inequality which is strictly related to the gap in mathematics (Guiso, Monte, Sapienza & Zingales, 2008). Furthermore, the beliefs of teachers and parents on students' ability and differences in metacognitive aspects such as self perception, self concept and anxiety related to mathematics are fundamental to explain the reason of the observed gender gap (Fredericks & Eccles, 2002; Cargnelutti, Tomasetto, & Passolunghi, 2016; OECD, 2016; Pajares, 2005). Finally, recent research has revealed that factors related to classroom practices, learning strategies and curriculum variables have a strong influence on gender differences in mathematics (Leder & Forgasz, 2008; Giberti, Zivelonghi, & Bolondi, 2016; OECD, 2016). Moreover, in previous studies we have shown the influence on gender differences of micro-social factors, strictly related to the *milieu* habits (Brousseau, 1988) and to the interrelation between the student and the teacher. From this perspective we highlighted that misconceptions, as a product of classroom practices, have a stronger impact on females rather than males (Bolondi, Cascella & Giberti, 2017) and our aim is to investigate more in depth this issue.

In this paper we propose the analysis of a specific item, which is part of a wider research program called "Variazioni 2" and financed by INVALSI, aimed at exploring the relationship between variations in item formulation and in its psychometric functionality. Much research in mathematics education has studied the impact of a variation in item formulation with a particular attention to 'word problems'. An updated literature review has been proposed by Daroczy and colleagues (2015). Literature considers the influence of linguistic variations as well as other kinds of minor changes on students' responses and solving strategies (e.g. Nesher, 1982; De Corte, Verschaffel & Van Coillie, 1988; D'Amore, 2000). Duval in 1991 defines all these modifications as "redactional variables", and Laborde in 1995 uses this term to include also non-verbal changes, e.g. introduction or modification of pictures. Using a specific statistical strategy, in this paper we analyze data collected in the project "Variazioni 2" to understand if and how a specific variation might affect differently male and female solving strategies to solve a mathematics item. In particular, we analyze an item included into a mathematics achievement tests to explore a specific didactical issue concerning misconceptions in multiplying decimal numbers (Fischbein, Deri, Nello, & Marino, 1985; Sbaragli, 2012).

Research questions.

The term misconception is widely used in research in education and takes different meanings; in this research, following D'Amore & Sbaragli (2005), we define a misconception as a concept momentarily incorrect, standing by for a more critical and developed cognitive arrangement. A typical misconception (D'Amore & Sbaragli, 2005; Hart et al., 1981) is due to the premature formation of a conceptual model of multiplication when students operate exclusively with natural numbers. Students learn multiplication with natural numbers and, observing that the product of two numbers is always greater than its factors, they are led to believe that "multiplication always increases". This is true for natural numbers (excluding 0 and 1), but this is not always true for decimal numbers. This misconception, thus, leads students to make mistakes when they operate with decimal numbers, and, for instance, D'Amore and Sbaragli (2005) explain that the answer of many students of different grades to the question "What is the result of 4 × 0.5?" is 8. In order to explore this phenomenon, we compared two versions of the same item (Table 1).

Booklet F1 and F2	Booklet F3 and F4
D9. Which is the result of 4 x 0.5? Choose one of the following options. A. □ 8 B. □ 4 C. □ 2 D. □ 20	D9. Which is the result of 0.5 x 4? Choose one of the following options. A. □ 8 B. □ 4 C. □ 2 D. □ 20

Table 1: Different formulations of the item D9

The two formulations differ only in factor order. This allows us to understand if the idea that "multiplication always increases" is related to both factors of the multiplication or mostly to one of the two factors. In particular, our hypothesis is that the intuitive model (Fischbein, Deri, Nello & Marino, 1985; Mulligan & Mitchelmore, 1997) of multiplication as a repeated sum is easier to apply in the second version (0.5 × 4) and this strategy led students to overcome easier the misconception. This is of course also related to the language used: in Italian we read the multiplication 4 × 0.5 as "4 repeated 0.5 times" while in English and in many other languages it is "4 times 0.5".

This only one of several misconception related to the transition between natural numbers and decimals and, in previous studies focused on misconception in comparing decimal numbers (Bolondi, Cascella & Giberti, 2017; Giberti, 2018) we observed that females have more difficulties than males. For this reason, we decided to investigate if this misconception and the variation carried out, affects differently boys and girls. Therefore, our research questions are: 1) Does the misconception here analyzed act on boys and girls differently? In other words, does this misconception cause different item functioning in male relative to female subgroup? 2) What is the impact of the inversion of factors into a multiplication (e.g., 4×0.5 in place of 0.5 ×4) on students' answers? What is its impact on item functionality? Does the inversion of factors have different effects on boys and girls?

Participants and measures.

Starting from a mathematics achievement test developed by INVALSI for students attending grade 8 (lower intermediate school), in the project 'Variazioni2', we developed three further mathematics achievement tests, complete in terms of mathematical content and level of difficulty. These four forms (F1, F2, F3 and F4) were administered to a probabilistic sample of 1647 students attending grade 8.

Analytic strategy.

Our four mathematics achievement tests were administered by means of a spiraling process (according to which different forms are administered to different students within each classroom) in order to randomly assign forms, in this way each form was administered to approximately 400 students. When using this design, differences between group-level performance on administered forms are taken as direct indications of the differences in difficulty between forms (Kolen & Brennan, 2004) and it is particularly adequate to make comparable answers given by different subgroups of students. In addition, all achievement tests were equated to make estimations directly comparable. Both item and person parameters estimated via the Rasch model (Rasch, 1960) were scaled in an empirical range equal to [-4; +4] logits, where 0.00 logit does not mean absence of ability but it is the ability-difficulty level in relation to which students have a probability equal to 0.5 of encountering an item successfully.

In the result section, we provided a summary of psychometric characteristics of the item D9 in F1, F2, F3, and F4. In order to compare, male and female performances, we provided a visual display of the set of observed means for each person factors level (i.e., for boys and girls) across each of the class interval present in the item-trait test-of-fit specifications. Each level is plotted in relation to the Item Characteristic Curve, i.e. the theoretical curve estimated by the Rasch model according to which no factor other than students' intrinsic ability can explain the probability of a correct answer. At the bottom of the table, we reported the distractor plots drawn for males and female, separately, in order to explore their answer behavior in relation to each answer option.

Results.

The table below (Table 1) reports on differences in item functionality in F1, F2, F3, and F4. For each of them, in addition to information about fit (i.e., Weighted MNSQ) always close to 1 and thus good, the distribution of correct and wrong answers provided by students is available in order to provide a first glance on the differences between booklets.

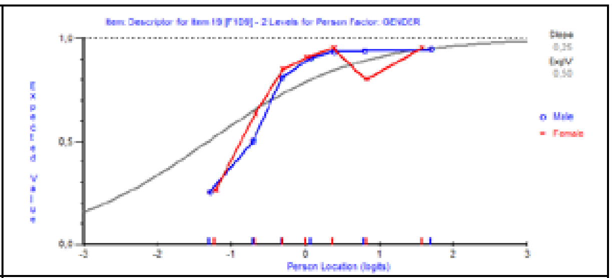
Booklet F1							**Booklet F2**						
item:19(D9)							**item:18(D9)**						
Weighted_MNSQ 0.87							Weighted_MNSQ 1.02						
Label	Score	% of tot	Pt Bis	t (p)	PV1Avg:1	PV1 SD:1	Label	Score	% of tot	Pt Bis	t (p)	PV1Avg:1	PV1 SD:1
A	0.00	3.83	-0.07	-1.40(.161)	-0.12	0.85	A	0.00	4.90	-0.17	-3.47(.001)	-0.67	0.49
B	0.00	4.31	-0.18	-3.68(.000)	-0.77	0.67	B	0.00	4.66	-0.19	-3.83(.000)	-0.81	0.74
C	1.00	75.36	0.49	11.41(.000)	0.27	0.84	C	1.00	69.85	0.51	11.94(.000)	0.19	0.81
D	0.00	15.55	-0.41	-9.23(.000)	-0.68	0.68	D	0.00	19.36	-0.35	-7.61(.000)	-0.65	0.60
Miss	0.00	0.96	-0.12	-2.49(.013)	-0.88	0.41	Miss	0.00	1.23	-0.17	-3.42(.001)	-1.27	0.64
Booklet F3							**Booklet F4**						
item:19(D9)							**item:19(D9)**						
Weighted_MNSQ 0.89							Weighted_MNSQ 0.88						
Label	Score	% of tot	Pt Bis	t (p)	PV1Avg:1	PV1 SD:1	Label	Score	% of tot	Pt Bis	t (p)	PV1Avg:1	PV1 SD:1
A	0.00	1.95	-0.08	-1.54(.124)	-0.60	0.63	A	0.00	4.03	-0.19	-3.98(.000)	-0.75	0.93
B	0.00	2.19	-0.10	-2.13(.034)	-0.64	0.67	B	0.00	2.68	-0.19	-3.99(.000)	-0.94	0.65
C	1.00	81.51	0.45	10.24(.000)	0.18	0.92	C	1.00	80.31	0.47	11.30(.000)	0.23	0.93
D	0.00	13.38	-0.39	-8.66(.000)	-1.01	0.62	D	0.00	12.08	-0.33	-7.46(.000)	-0.91	0.79
Miss	0.00	0.97	-0.16	-3.25(.001)	-1.37	0.95	Miss	0.00	0.89	-0.13	-2.87(.004)	-2.07	1.34

Table 1. Item functionality in F1, F2, F3 and F4

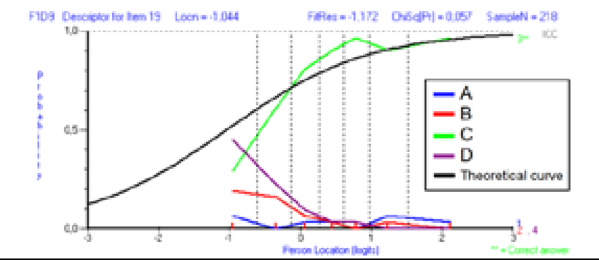
Item formulation shows little variation comparing booklets F1 and F2 and it's the same also comparing booklets F3 and F4. As already explained, in both versions the question intent is the same but the inversion of factors into the multiplication might change the influence of the misconception on decimal numbers and the answering strategy activated by students to solve the item might change. Indeed, we observe a lower percentage of correct answers (Table 1) in the first version, included in booklet F1 and F2, than in the second one, included in F3 and F4.

To analyze gender differences, we provided a visual display of the set of observed means for each person factors level (i.e., for boys and girls) across each of the class interval present in the item-trait test-of-fit specifications. Each level is plotted in relation to the Item Characteristic Curve, i.e. the theoretical curve estimated by the Rasch model according to which no factor other than students' intrinsic ability can explain the probability of a correct answer. Distractor plots drawn for males and female, separately, showed differences in answer behavior between boys and girls in relation to each answer option (Table 2). In particular, option B is more attractive for low-ability male students relative to females with the same ability level. Instead, it is evident that females are more attracted by option D, which is strictly related to the misconception, since they give the result of 4×5 instead of 4×0.5. Little differences can be disclosed also in relation to option A, more attractive for low-ability girls and all these evidences are confirmed also by percentage reported in Table 2. Most of these differences are more remarkable in F2 (Table 3), with a clear advantage of males relative to females. Nevertheless, by exploring Distractor Response Curves drawn by gender, response patterns for the set of distractors associated with D9 presented in F1 and D9 presented in F2 are similar. The main difference between them can be disclosed into the distractor plot drawn for males in Table 3: high-ability boys are not attracted by any distractors, while distractor D is attractive also for high-ability females.

Booklet F1		
	Males	Females
A	3%	5%
B	7%	2%
C (correct)	77%	73%
D	12%	20%
Missing	1%	1%



Males



Females

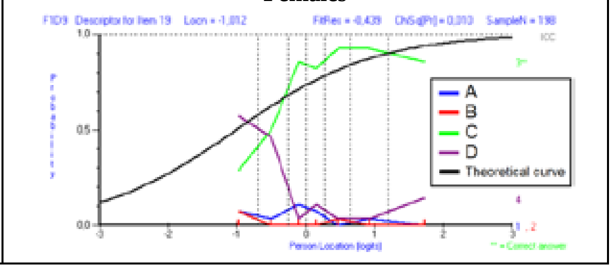
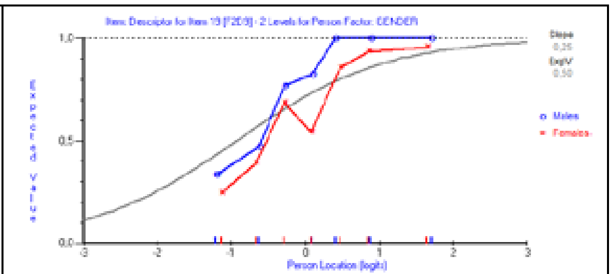
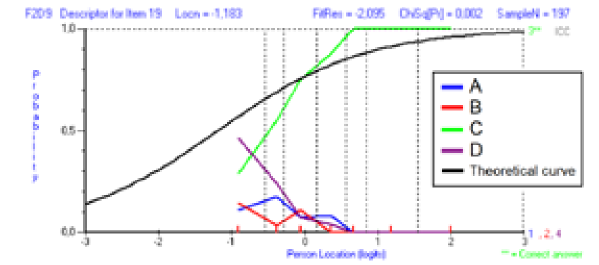


Table 2: Responses by gender and Analysis of Variance by Gender for item D9 administered in booklet F1 (top-left) - DIF-plot (top-right) - Distractor Plot of males (bottom-left) and females (bottom-right)

Booklet F2		
	Males	Females
A	2%	1%
B	3%	3%
C (correct)	74%	60%
D	15%	34%
Missing	6%	1%



Males



Females

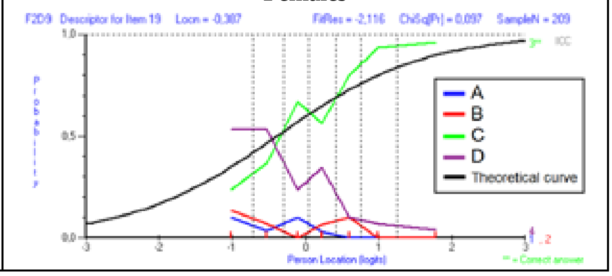
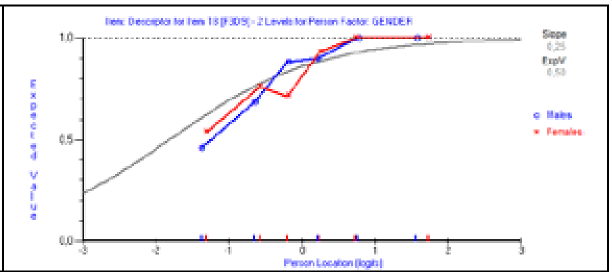


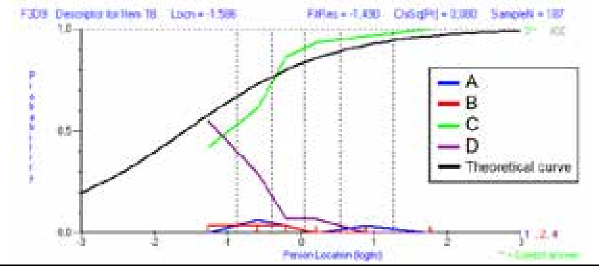
Table 3: Responses by gender and Analysis of Variance by Gender for item D9 administered in booklet F2 (top-left) - DIF-plot (top-right) - Distractor Plot of males (bottom-left) and females (bottom-right)

Both in booklet F3 and F4 (Table 4 and 5), the item D9 proposes the inversion of factors, i.e. 0.5×4 in place of 4×0.5. The analysis of answers to the item D9 in booklet F3 confirms an overall advantage of males. Differently from previous cases, higher differences are observable especially at medium-ability level along the latent trait (around 0.0 logit). Nevertheless, differences observed for low ability students are the most result of this analysis: from -1.5 to -0.5 logit all differences are in favor of females. This evidence is strongly confirmed by analyzing F4 data: the probability of a correct answer is 20% higher in female than in male group, from -1.50 to -0.5 along the latent trait. In both cases, distractor analysis shows interesting dissimilarities. The most interesting differences between boys and girls can be observed in F4. In particular, option B is attractive for males and not attractive for females, while option D is more attractive for girls especially at the lower level of the latent trait, confirming the evidences emerged in F1 and F2.

	Booklet F3	
	Males	Females
A	0%	1%
B	1%	2%
C (correct)	84%	73%
D	12%	22%
Missing	3%	1%



Males



Females

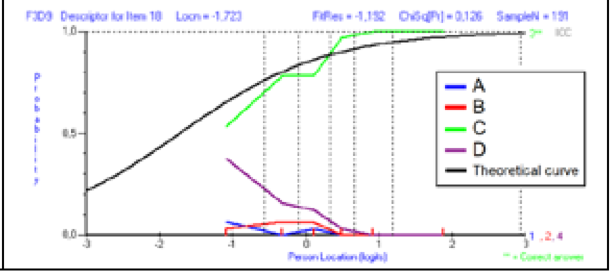
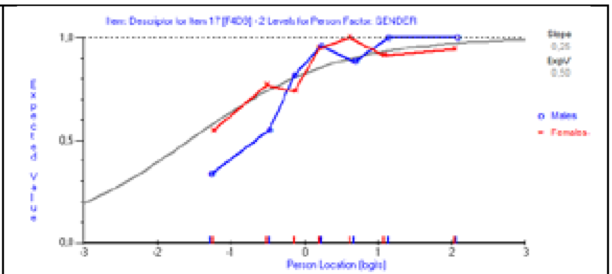
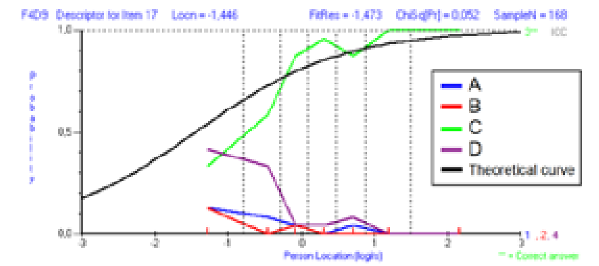


Table 4: Responses by gender and Analysis of Variance by Gender for item D9 administered in booklet F3 (top-left) - DIF-plot (top-right) - Distractor Plot of males (bottom-left) and females (bottom-right)

	Booklet F4	
	Males	Females
A	1%	1%
B	1%	2%
C (correct)	82%	78%
D	13%	19%
Missing	3%	0%



Males



Females

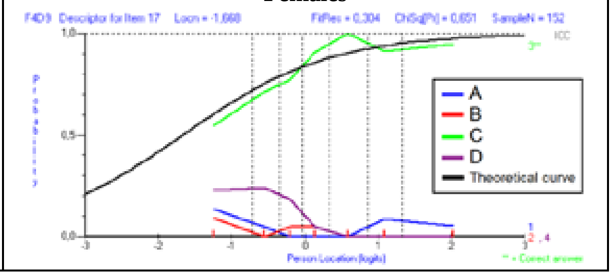


Table 5: Responses by gender and Analysis of Variance by Gender for item D9 administered in booklet F4 (top-left) - DIF-plot (top-right) - Distractor Plot of males (bottom-left) and females (bottom-right)

Conclusions.

In this paper, we compared two versions of the same item with the purpose of analyzing a specific misconception concerning multiplication with decimal numbers. The results showed interesting evidence that will be investigated in further studies from a qualitative point of view. The analysis showed that our new formulation (0.5×4) is simpler (in terms of item difficulty) than the first one (4×0.5) and this is particularly due to the improvement of the performances of girls. This result is coherent to the model of multiplication as a repeated sum proposed by Fischbein, Deri, Nello, & Marino (1985) or by Sbaragli (2012). In particular, our new formulation seemed to affect low-ability girls positively suggesting that they are more prone than boys to the effect of misconception. Moreover, empirical findings showed a sharper difference between boys and girls at the higher ability level, unless boys confirmed their superiority in encountering this task independently on its formulation at the top level of the ability distribution. Furthermore, in both versions, distractor D (i.e., product equal to 20) which is strictly related to the misconception, is always preferred by females. In particular, we observed that the inversion of the two terms of a multiplication has a huge impact on students' answering behavior especially on females: strong gender gap emerges in favor of males in the first version (4×0.5) for students scaling at the upper-tail of the latent trait and in favor of females in the second version (0.5×4), especially at the lower tail of the ability distribution. In the second form the influence of the misconception appears lower and (probably for this reason) the item is easier since students can calculate the product by using the implicit model of multiplication as repeated addition and this particularly helps low-ability girls.

References

Bolondi, G., Cascella, C., & Giberti, C. (2017). Highlights on gender gap from Italian standardized assessment in mathematics. In J. Novotnà & H. Moravà (Eds.), *Diversity in Mathematics Education. Proceedings of the International Symposium Elementary Maths Teaching*. Prague: Università Karlova Press.

Brousseau, G. (1988). Le contrat didactique: le milieu. *Recherches en Didactique des Mathématiques, 9* (3), 309-336.

Cargnelutti, E., Tomasetto, C., & Passolunghi, M. C. (2016). How is anxiety related to math performance in young students? A longitudinal study of Grade 2 to Grade 3 children. *Cognition & Emotion, 31*, 755-764.

D'Amore, B. (2000). Lingua, matematica e didattica. *La matematica e la sua didattica*, 1, 28-47.

D'Amore B., Sbaragli, S. (2005). Analisi semantica e didattica dell'idea di "misconcezione". *La matematica e la sua didattica, 2*, 139-163.

Daroczy, G., Wolska, M., Meurers, W. D., Nuerk, H-C. (2015) Word problems: a review of linguistic and numerical factors contributing to their difficulty. *Frontiers in Psychology, 6,* 348. doi: 10.3389/fpsyg.2015.00348

De Corte E., Verschaffel L., & Van Coillie V. (1988). Influence of number size, problem structure, and response mode on children's solutions of multiplication word problems. *Journal of Mathematical Behaviour, 7*, 197–216.

Duval, R. (1991). Interaction des différents niveaux de représentation dans la compréhension de textes. *Annales de Didactique et de sciences cognitives,* 136-193.

Fischbein, E., Deri, M., Nello, M. S., & Marino, M. S. (1985). The role of implicit models in solving verbal problems in multiplication and division. *Journal for Research in Mathematics Education,* 3-17.

Fredricks, J. A., & Eccles, J. S. (2002). Children's competence and value beliefs from childhood through adolescence: Growth trajectories in two male-sextyped domains. *Developmental Psychology, 38(4),* 519.

Guiso, L., Monte, F., Sapienza, P., & Zingales, L. (2008). Culture, gender, and math. *Science,* New York Then Washington, 320(5880), 1164.

Giberti, C. (2018). Differenze di genere e misconcezioni nell'operare con le percentuali: evidenze dalle prove INVALSI. *CADMO*, 2018 (2), 97-114.

Giberti, C., Zivelonghi, A., & Bolondi, G. (2016). Gender differences and didactic contract: analysis of two INVALSI tasks on powers properties. In C. Csikos, A. Rausch & J. Szitanyi (Eds.), *Proceedings of the 40th Conference of the International Group for the Psychology of Mathematics Education* (p. 275). Szeged: IGPME.

Hart, K. M., Brown, M. L., Kuchemann, D. E., Kerslake, D., Ruddock, G., & McCartney, M. (1981). *Children's understanding of mathematics: 11-16.* London: John Murray.

Hill, C., Corbett, C., & Rose, A. (2010). *Why So Few? Women in Science, Technology, Engineering, and Mathematics.* Washington: American Association of University Women.

INVALSI, (2017). Rilevazione nazionale degli apprendimenti 2016-2017. Le rilevazioni degli apprendimenti. Retrieved in January 2019 from http://www.invalsi.it/invalsi/doc_eventi/2017/Rapporto_Prove_INVALSI_2017.pdf

Kolen, M. J., & Brennan, R. L. (2004). *Test equating, scaling, and linking.* Springer: New York.

Laborde, C. (1995). Occorre apprendere a leggere e scrivere in matematica. *La matematica e la sua didattica*, *9(2),* 121-135.

Leder, G., & Forgasz, H. (2008). Mathematics education: new perspectives on gender. *ZDM – The International Journal on Mathematics Education, 40*(4), 513-518.

Mulligan, J. T., & Mitchelmore, M. C. (1997). Young children's intuitive models of multiplication and division. *Journal for research in Mathematics Education,* 309-330.

Mullis, I. V. S., Martin, M. O., Foy, P., & Hooper, M. (2016). *TIMSS 2015 International Results in Mathematics.* Boston: TIMSS & PIRLS International Study Center.

Nesher, P. (1982). Levels of description in the analysis of addition and subtraction word problems. *Addition and subtraction: A cognitive perspective,* 25-38.

OECD (2016). *PISA 2015 – Results (Volume I): Excellence and equity in education.* Paris: OECD Publishing.

Pajares, F. (2005). Gender differences in mathematics self-efficacy beliefs. In A. M. Gallagher & J. C. Kaufman (Eds.), *Gender differences in mathematics: An integrative psychological approach,* pp. 294-315. New York: Cambridge University Press.

Rasch, G. (1960). Probabilistic models for some intelligence and attainment tests. Copenhagen: Denmarks Paedagogiske Institut.

Sbaragli, S. (2012). Il ruolo delle misconcezioni nella didattica della matematica. In Bolondi B., Fandiño Pinilla M.I. (Eds), *I quaderni della didattica. Metodi e strumenti per l'insegnamento e l'apprendimento della matematica,* 121-139.