A robust gradient-based MPC for integrating Real Time Optimizer (RTO) with control

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Abstract— **Recently, a gradient-based model predictive control (MPC) strategy was proposed to reduce the computational burden of integrating real time optimization (RTO) and control: the main idea is to obtain the on-line controller solution by means of the convex combination of a feasible solution and a solution of an approximated (linearized) problem. This formulation, however, is developed only for the nominal case, which significantly reduce its applicability. In this work, an extension of the gradient-based MPC to include bounded additive disturbance is made. Based on the concept of robust set-interval, the uncertainty is explicitly accounted for, while economic performance and stability is maintained. Several scenarios are simulated to show the benefits of the proposal in contrast to the nominal controller.**

Keywords— **Model Predictive Control, Economic Optimization, Robust Control**

1. Introduction

Model predictive control (MPC) is one of the most successful advanced control techniques in the process industries. MPC theoretical background has been widely investigated in the last two decades, showing how MPC is a control technique capable to provide stability, robustness, constraint satisfaction and tractable computation for linear and for nonlinear systems [1].

Recently, researchers are focused on improving the economic performance of MPC. In this context, the study of the hierarchical control structure, typical in process industries, has a certain relevance [2]: at the top of this structure, an economic schedule and planner determines the whole plant production (level, quality, etc.). The outputs of this layer are sent to a Real Time Optimizer (RTO), which is devoted to compute the stationary setpoints according to economic criteria. This optimizer is usually based on a complex nonlinear stationary model of the plant and so has a sampling time different from other layers. Then, the setpoints computed by the RTO are sent to the MPC control level which calculates the control actions necessary for the plant to reach those setpoints, taking into account a simplified dynamic model of the plant and the variable constraints.

One well-known drawback of this hierarchical control structure is, however, that the communication between the economic/stationary and the dynamic layers may be inconsistent, producing in this way problems that go from unreachability of the setpoints to poor economic performances. As a result, a proper strategy to unify this (probably competing) objectives becomes highly desired from an operating point of view.

Two ways to reduced inconsistency are the so-called two-layer and one-layer structures. In the first case, an extra optimization level - the Steady State Target Optimizer, SSTO - is added in between the RTO and the MPC to decide the best admissible target for the MPC, according to a local approximation of the RTO cost function, and using the same simplified model used in the MPC layer. Some examples of this strategy (within different frameworks) can be seen in [3, 4].

In the one-layer strategy the idea is to merge the RTO layer with the MPC layer, by designing controllers that integrates the RTO economic cost function as part of the MPC cost as in [5], or controllers based on a general (economic) cost function, as in [6, 7]. The main problem of this strategy is that the economic objectives are usually represented by a complex nonlinear function that turn the one-layer optimization cost also nonlinear and difficult to solve.

In order to reduce the computational burden, an approximation of the RTO function can be considered, as economic cost function: for instance, in [8], the gradient of the economic objective function is included in the controller cost function, in order to obtain a computational low-cost strategy. This solution allows one to solve the resulting control/optimization problem as a single QP problem, and the results are promising from both, theoretic and practical points of view. This idea has been then extended in [9], in order to obtain a stable formulation.

The novelty of this strategy is that instead of applying to the system the optimal solution of an approximated problem, the applied control action is the convex combination of an arbitrary feasible solution and an approximated solution. In this way, a sub-optimal MPC strategy is obtained, which ensures recursive feasibility and convergence to the optimal steady state in the economic sense, with a reduced computational cost. The aforementioned formulation, however, was developed for the nominal case only.

The aim of this work is to extend the formulation to robust case, considering bounded additive disturbances. Based on the concept of robust set-interval ([10]) the extension is made with the objective to preserve the nominal economic performance and stability. The so obtained control formulation is tested by simulating several economic scenarios on a simple system.

2. Problem Statement

Consider a system described by a linear time-invariant discrete time model

$$\bar{x}^+ \quad = \quad A\bar{x} + Bu + w \tag{1}$$

where $\bar{x} \in \mathbb{R}^n$ is the system state, $u \in \mathbb{R}^m$ is the current control vector, $\bar{x}^+$ is the successor state and $w \in \mathbb{R}^n$ is an unknown but bounded state disturbance. In what follows, $\bar{x}(k)$, $u(k)$ and $w(k)$ denote the state, the manipulable variable and the disturbance respectively, at sampling time k.

The system is subject to constraints on state and input:

$$(\bar{x}(k), u(k)) \in \bar{\mathcal{Z}} \tag{2}$$

for all $k \geq 0$, where $\bar{\mathcal{Z}} = \bar{\mathcal{X}} \times \bar{\mathcal{U}}$ is a compact convex polyhedron containing the origin in its interior.

Define also the plant nominal model, given by 1 neglecting the disturbance input w:

$$x^+ \quad = \quad Ax + Bu \tag{3}$$

The solution of this system for a given sequence of control inputs $\mathbf{u} = \{u(0), \cdots, u(j-1)\}$ and an initial state x is denoted as $x(j) = \phi(j; x, \mathbf{u})$, $j \in \mathbb{I}_{\geq 1}$, where $x = \phi(0; x, \mathbf{u})$.

The plant model is assumed to fulfill the following assumption:

Assumption 1. *(i) The pair (A, B) is controllable and the state is measured at each sampling time. (ii) The uncertainty vector w is bounded and lies in a compact convex polyhedron, $\mathcal{W}$, containing the origin in its interior. (iii) The state of the system is measured, and hence $\bar{x}(k)$ is known at each sample time.*

2.1. The robustness approach

The keystone of the robust MPC presented in [10] is to use predictions based on the nominal system for the MPC cost and to restrict the constraints set $\mathcal{X}$ and $\mathcal{U}$ any step of the prediction horizon. The controller is based on a prestabilization of the plant using a state feedback control gain K, such that $A_K = A + BK$ has all its eigenvalues in the interior of the unit circle. The nominal controlled system is then given by:

$$\begin{aligned} x(k+1) &= A_K x(k) + Bc(k) \\ u(k) &= Kx(k) + c(k) \end{aligned} \tag{4}$$

The notion of robust positively invariant (RPI) set ([11]) plays an important role in the design of robust controllers for constrained systems. This is defined as follows:

Definition 1. *A set Ω is called a robust positively invariant (RPI) set for the uncertain system $\bar{x}(k+1) = A_K\bar{x}(k) + w(k)$ with $w(k) \in \mathcal{W}$ if $A_K\Omega \oplus \mathcal{W} \subseteq \Omega$*

It is also necessary to define the so-called reachable sets, that represents the forced response of the system due to the uncertainty.

Definition 2. *The reachable set in j steps, $\mathcal{R}_j$, is given by*

$$\mathcal{R}_j = \bigoplus_{i=0}^{j-1} A_k^i \mathcal{W} \tag{5}$$

This is the set of states of the nominal closed-loop systems which are reachable in j steps from the origin, under the disturbance input w. This set satisfies the following properties:

(i) It is given by the recursion $\mathcal{R}_j \oplus A_K^j \mathcal{W} = \mathcal{R}_{j+1}$ whit $\mathcal{R}_1 = \mathcal{W}$.

(ii) $A_K^j \mathcal{R}_j \oplus \mathcal{W} = \mathcal{R}_{j+1} = \mathcal{R}_j \oplus A_K^j \mathcal{W}$.

(iii) $\mathcal{R}_j \subseteq \mathcal{R}_{j+1}$

(iv) The sequence of sets $\mathcal{R}_j$ has a limit $\mathcal{R}_\infty$ as $j \to \infty$, and $\mathcal{R}_\infty$ is a robust positive invariant set.

(v) $\mathcal{R}_\infty$ is the minimal RPI set.

Based on this, the sets of **restricted constraints** are defined by:

$$\begin{aligned} \mathcal{X}_j &\triangleq \bar{\mathcal{X}} \ominus \mathcal{R}_j \\ \mathcal{U}_j &\triangleq \bar{\mathcal{U}} \ominus K\mathcal{R}_j \end{aligned} \tag{6}$$

These sets are non-empty if the following assumption holds:

Assumption 2. *The sets $\mathcal{X}_j$ and $\mathcal{U}_j$ exist if and only if $\mathcal{R}_\infty \subset \bar{\mathcal{X}}$ and $K\mathcal{R}_\infty \subset \bar{\mathcal{U}}$*

It is important also to note that the computation of such sets is made off-line, so it has no practical effects on the MPC problem.

Remark 1. *The control gain K has an important role in the proposed robust approach, since it determines the dynamic of the closed-loop system in presence of disturbances and hence, it has to ensure that Assumption 2 holds.*

2.2. Equilibrium characterization and optimal point

If we consider the joint variable (x, u), the state and input equilibrium subspace, associated to the nominal model (3), is given by $\mathcal{N}([A - I_n \;\; B])$, where $\mathcal{N}$ is the null space of a matrix. That is

$$[A - I_n \;\; B] \begin{bmatrix} x_s \\ u_s \end{bmatrix} = \mathbf{0}_{n,1}$$

Defining $\mathcal{Z} \triangleq \mathcal{X}_N \times \mathcal{U}_N$, the set of admissible equilibriums - for the nominal system, is given by

$$\mathcal{Z}_s \triangleq \{(x,u) \in \gamma\mathcal{Z} \mid x^+ = Ax + Bu\}$$

where $\gamma \in (0,1)$ is a given parameter added to avoid those steady states and inputs that provide active constraints.

Taking into account the later equilibrium characterization, the optimal operation point that stabilizes the plant is

Definition 3. *The* ***optimal steady state and input****,* (x_s^{eco}, u_s^{eco})*, satisfies*

$$(x_s^{eco}, u_s^{eco}) = \arg \min_{(x,u)\in\mathcal{Z}_s} f_{eco}(x,u,p). \tag{7}$$

where $f_{eco}(x,u,p)$ defines an economic cost function and p is a parameter that takes into account prices, costs or production goals.

Assumption 3. *The economic cost function* $f_{eco}(x,u,p)$ *is strictly convex in* (x,u) *and twice differentiable. This is important for the proof of the theorem of convergence which will be presented later.*

In addition, according to most real cases, it is assumed that $f_{eco}(x,u,p)$ is nonlinear and its evaluation takes a significant computation time, since it is based on complex stationary models of the real plant.

3. The one layer robust economic MPC strategy

In this section, the proposed controller formulation is presented. This controller follows the ideas presented in [9], which considers the offset cost function as the economic objective and provide a suboptimal and easy-to-compute solution, that prevents computational problem originated by the non-linear nature of the economic cost. Furthermore, here we take into account the robustness results presented in section 3, to extend the controller to the robust case. The main challenge is then to maintain the properties of the former formulation (simplicity, feasibility, convergence) when a bounded disturbance is explicitly considered.

The controller cost function is given by:

$$V_N(x,p;\mathbf{u},x_s,u_s) = V_N^{dyn}(x;\mathbf{u},x_s,u_s) + V_{eco}(x_s,u_s,p) \tag{8}$$

where $V_N^{dyn}(x;\mathbf{u},x_s,u_s) = \sum_{j=0}^{N-1} \|x(j) - x_s\|_Q^2 + \|u(j) - u_s\|_R^2$, for appropriate matrices Q and R, and $V_{eco}(x_s,u_s,p) = f_{eco}(x_s,u_s,p)$.

As can be seen in (8), the cost function is formed by two terms, based on nominal predictions. The first term is a pure dynamic term (since the pair (x_s,u_s) defines an artificial target only forced to be in $\mathcal{Z}_s$) while the second one is a pure stationary term (since it only penalizes the artificial target - which is an admissible equilibrium - according to the economic objectives). This is an extension of the so called **MPC for tracking**, which incorporates the artificial target (x_s,u_s) for feasibility reasons ([12]).

Assuming the prestabilization gain K, we can write $V_N(x,p;\mathbf{u},x_s,u_s) = V_N(x,p;\mathbf{c},x_s,u_s)$, where each element of $\mathbf{c}$, $c(j;x)$, fulfill $u(j;x) = K(x(j) - x_s) + u_s + c(j;x)$. This way, for any current state x, the optimization problem $P_N(x,p)$ to be solved is given by:

Problem $P_N(x,p)$

$$\begin{aligned}
\min_{\mathbf{c},x_s,u_s} \quad & V_N(x,p;\mathbf{c},x_s,u_s)\\
\text{s.t.} \quad & x(0) = x,\\
& x(j+1) = A_K x(j) + Bu(j), \;\; j \in \mathbb{I}_{[0:N-1]}\\
& u(j) = K(x(j) - x_s) + u_s + c(j), \;\; j \in \mathbb{I}_{[0:N-1]}\\
& x(j) \in \mathcal{X}_j, \;\; j \in \mathbb{I}_{[0:N-1]}\\
& u(j) \in \mathcal{U}_j, \;\; j \in \mathbb{I}_{[0:N-1]}\\
& x(N) = x_s,\\
& x_s = Ax_s + Bu_s \quad ((x_s,u_s) \in \mathcal{Z}_s)
\end{aligned}$$

In this optimization problem, x and p are the parameters, while the input sequence $\mathbf{c} = \{c(0), \cdots, c(N-1)\}$ and the artificial target variables x_s and u_s are the optimization variables.

Notice that the additional constraints necessary to ensure stability (according to the **MPC for tracking** framework) are the last two ones: first, the artificial variables (x_s,u_s) are only forced to belong to the admissible nominal equilibrium set $\mathcal{Z}_s \subseteq \mathcal{X}_N \times \mathcal{U}_N$, and second, the final predicted state $x(N)$ is forced to be x_s (which means that $x(N) \in \mathcal{X}_s$).

The control law is given by $\kappa_N(x,p) = u^0(0;x) = K(x - x_s^0) + u_s^0 + c^0(0;x)$, where $c^0(0;x)$ is the first element of the solution sequence $\mathbf{c}^0(x)$. In this regards, the following assumption must be done to ensure convergence:

Assumption 4. *For a given system* (A,B)*, the horizon* N *is such that* $\mathcal{R}(Co_{N-1}) \supset \mathcal{X}_s$*, where* $Co_j = [A^{j-1}B \;\; A^{j-2}B \;\; \cdots \;\; B]$ *is the* j*-controllability matrix of system* (A,B) *and* $\mathcal{R}$ *is the range of a matrix.*

4. An easy-to-obtain suboptimal solution

Given that the economic cost is generally based on a complex nonlinear model, the main problem of the later formulation is the high computational burden. In this context, [9] proposed a strategy based on suboptimal solutions to Problem $P_N(x,p)$. This suboptimal solution is constructed by an appropriate convex combination of a feasible solution and an optimal solution of an approximated optimization problem, as described next.

Now, for a given time instant k, define the *feasible solution* to problem $P_N(x,p)$, $\hat{\mathbf{c}} = \{\hat{c}(0), \cdots, \hat{c}(N-2), 0\}, \hat{x}_s, \hat{u}_s$, as the **shifted solution** of the same problem at time $k-1$. Associated to this solution is the feasible state sequence $\hat{\mathbf{x}} = \{\hat{x}(0), \cdots, \hat{x}(N)\}$ where $\hat{x}(N) = \hat{x}_s$ by the terminal constraint.

The cost function corresponding to solution

$(\hat{\mathbf{x}}, \hat{\mathbf{c}}, \hat{x}_s, \hat{u}_s)$ (called *feasible cost*) is given by:

$$V_N(x,p;\hat{\mathbf{c}},\hat{x}_s,\hat{u}_s) = V_N^{dyn}(\hat{x};\hat{\mathbf{c}},\hat{x}_s,\hat{u}_s) + V_{eco}(\hat{x}_s,\hat{u}_s,p) \quad (9)$$

Let us consider now an *approximated optimal solution* to to the original problem $P_N(x,p)$, which is obtained by solving the following approximated problem:

Problem $P_N^{app}(x,p)$

$$\min_{\mathbf{c},x_s,u_s} \quad V_N^{app}(x,p;\mathbf{c},x_s,u_s)$$
$$s.t. \;\; \mathbf{c},x_s,u_s \in \mathcal{C}_N(x)$$

where $\mathcal{C}_N(x)$ define the set of values $\mathbf{c},x_s,u_s$ that fulfill the constrain in Problem $P_N(x,p)$, and the approximated cost is given by

$$V_N^{app}(x,p;\mathbf{c},x_s,u_s) = V_N^{dyn}(x;\mathbf{c},x_s,u_s) + V_{eco}(\hat{x}_s,\hat{u}_s,p) + \nabla V'_{eco}(\hat{x}_s,\hat{u}_s,p)\begin{bmatrix} x_s-\hat{x}_s \\ u_s-\hat{u}_s \end{bmatrix}$$

and $\nabla V_{eco}(\hat{x}_s,\hat{u}_s,p)$ represents the gradient of V_{eco} w.r.t. (x,u), evaluated at the point $(\hat{x}_s,\hat{u}_s)$.

As it can be seen, this approximated optimal solution tries to optimize problem $P_N(x,p)$ by means of a simplified version of it. Notice that this solution is sub-optimal (in the transient) with respect to the optimal solution to the original problem $P_N(x,p)$ and hence its direct application into the MPC scheme does not guarantee convergence of the closed-loop system to the optimal solution to the original problem $P_N(x,p)$.

Let us denote the optimal solution to problem $P_N^{app}(x,p)$ (which we name *approximated optimal solution*) as $\mathbf{c}^* = \{c^*(0),\cdots,c^*(N-1)\}, x_s^*, u_s^*$ and $\mathbf{x}^* = \{x^*(0),\cdots,x^*(N)\}$

The cost function corresponding at the approximated optimal solution $(\mathbf{x}^*,\mathbf{c}^*,x_s^*,u_s^*)$ reads:

$$V_N(x,p;\mathbf{c}^*,x_s^*,u_s^*) = V_N^{dyn}(x^*;\mathbf{c}^*,x_s^*,u_s^*) + V_{eco}(x_s^*,u_s^*,p)$$

The idea now is to use a convex combination of the *feasible solution* and the *approximated optimal solution*,

$$\begin{aligned} \mathbf{c}(\lambda) &= (1-\lambda)\hat{\mathbf{c}} + \lambda\mathbf{c}^* \\ \mathbf{x}(\lambda) &= (1-\lambda)\hat{\mathbf{x}} + \lambda\mathbf{x}^* \\ u_s(\lambda) &= (1-\lambda)\hat{u}_s + \lambda u_s^* \\ x_s(\lambda) &= (1-\lambda)\hat{x}_s + \lambda x_s^*, \quad \text{with } \lambda \in [0,1], \end{aligned}$$

to obtain a new **suboptimal solution** that produces a decreasing MPC cost. Now, the following theorem can be established:

Theorem 1. *Let us consider Problem $P_N(x,p)$, with $x \neq x_s^{eco}$, and the aforementioned suboptimal solutions $\mathbf{c}(\lambda), x_s(\lambda), u_s(\lambda)$. Consider also that $(\hat{x}_s,\hat{u}_s) \neq (x_s^{eco},u_s^{eco})$. Then there exists a $\tilde{\lambda} \in (0,1]$ such that, for all $0 \le \lambda \le \tilde{\lambda}$*

$$V_N(x,p;\mathbf{c}(\lambda),x_s(\lambda),u_s(\lambda)) < V_N(x,p;\hat{\mathbf{c}},\hat{x}_s,\hat{u}_s). \quad (10)$$

The proof of theorem can be seen in [9].

According to the later result, we can define the **suboptimal solution** as the one that produces a (positive) decrement in the cost function. That is, $\mathbf{c}^{so}, x_s^{so}, u_s^{so} \triangleq \mathbf{c}(\tilde{\lambda}), x_s(\tilde{\lambda}), u_s(\tilde{\lambda})$. Associated to this solution is the suboptimal state sequence $\mathbf{x}^{so} \triangleq \mathbf{x}(\tilde{\lambda})$.

Remark 2. *The way to implement this suboptimal solution sequentially (i.e., at every time k the MPC control action must be implemented) is as follows. In the first place, sets $\mathcal{X}_j$ and $\mathcal{U}_j$ (necessary to define the feasible space of Problem $P_N(x,p)$) must be computed offline. Then*

1. *Compute the* feasible solution $(\hat{\mathbf{x}},\hat{\mathbf{u}})$ *to problem $P_N(x,p)$, using the shifted solution applied to the system at the sample time $k-1$. If the current time is $k=0$, compute the* feasible solution $(\hat{\mathbf{x}},\hat{\mathbf{u}})$ *by solving the reduced problem $P_N^{dyn}(x)$.*
2. *Compute the gradient of the economic cost function $V_{eco}(x,u,p)$ w.r.t. (x,u), $\nabla V_{eco}(x,u,p)$.*
3. *Compute the value of the parameter $\tilde{\lambda}$ that defines the* ***suboptimal solution*** *(this value can be computed heuristically in such a way that condition (10)). Note that to compute this value, the* approximated optimal solution *to problem $P_N(x,p)$, $(\mathbf{x}^*,\mathbf{u}^*)$, must be computed first, by minimizing the approximated problem $P_N^{app}(x,p)$.*
4. *From the suboptimal solution $\mathbf{c}^{so}, x_s^{so}, u_s^{so} = \mathbf{c}(\tilde{\lambda}), x_s(\tilde{\lambda}), u_s(\tilde{\lambda})$, take the first input of the sequence $\mathbf{c}^{so}$ to implement the implicit MPC control law, $\kappa_N(x,p) \triangleq u^{so}(0;x) = K(x-x_s^{so}) + u_s^{so} + c^{so}(0;x)$.*

5. Stability and convergence of the proposed controller

In this section some new results are presented regarding the convergence, the economic optimality and the stability of the proposed algorithm.

Theorem 2. *Consider that assumptions 1-4 hold, and consider a given parameter p for the economic cost $V_{eco}(x,u,p) = f_{eco}(x,u,p)$. Then, for any initial state $x \in \mathcal{X}_N$, the optimization problem $P_N(x,p)$ is recursively feasible and steers the disturbed system 1 to $(x_s^{eco},u_s^{eco}) \oplus (\mathcal{R}_\infty \times K\mathcal{R}_\infty)$.*

Sketch of Proof According to previous results presented in [10], it is possible to show that the contracting constraints based on the reachable sets, make that shifted suboptimal solution corresponding to time step k could be used as feasible solution for the same problem at time step $k+1$. This feasible solution produce in turn a MPC cost function smaller than the suboptimal solution at time k. Then, given that Theorem (1) ensures that the suboptimal solution at time $k+1$ produces a smaller MPC cost than the suboptimal solution at the same time, then, a decreasing cost along the solution is obtained.

However, given that true state at time $k+1$ is given by $\bar{x}^+ = A_K(x - x_s^{so}) + u_s^{so} + Bc^{so}(0;x) + w$, and not by the nominal prediction $x^+ = A_K(x - x_s^{so}) + u_s^{so} + Bc^{so}(0;x)$, then the decreasing cost is obtained whenever the state is outside the set $x_s^{eco} \oplus \mathcal{R}_\infty$, since otherwise, the disturbance w avoid the property.

This last fact, together with the fact that $c^{so}(0;x(k)) \to 0$ as $k \to \infty$, is the one that allows us to ensure the convergence of $(x(k), u(k))$ to a the set $(x_s^{so}(k), u_s^{so}(k)) \oplus (\mathcal{R}_\infty \times K\mathcal{R}_\infty)$, first, and given that the only one closed loop equilibrium is given by (x_s^{eco}, u_s^{eco}), then, the convergence to $(x_s^{eco}, u_s^{eco}) \oplus (\mathcal{R}_\infty \times K\mathcal{R}_\infty)$.

6. Example

In this section some simulations results will be presented, to evaluate the proposed control strategy. First, a description of the system is shown. Then we present the results of dynamic simulations.

6.1. System description

In order to demonstrate the benefits and the properties of the proposed controller, we consider a constrained sampled double integrator:

$$x^+ = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} x + \begin{bmatrix} 0 & 0{,}5 \\ 1 & 0{,}5 \end{bmatrix} u + w$$

where the set $\mathcal{W}$ of possible disturbances realization is given by $\mathcal{W} = \{w \in \mathcal{R}^2 : \|w\|_\infty \leq 0{,}1\}$ and the system must fulfill the following constraints:
$\bar{\mathcal{X}} = \{x \in \mathbb{R}^2 : \|x\|_\infty \leq 5\}$ and $\bar{\mathcal{U}} = \{u \in \mathbb{R}^2 : \|u\|_\infty \leq 0{,}5\}$ Matrix K has been chosen as the LQR for $Q = 0{,}5\, I_2$ and $R = 2\, I_2$, and it is given by:

$$K = \begin{bmatrix} -0{,}1201 & -0{,}5843 \\ -0{,}2854 & -0{,}5776 \end{bmatrix}$$

The prediction horizon has been chosen as $N = 3$, and corresponding economic function reads:

$$f_{eco}(x, u, p) = (u_1^2 + p_1 u_2^2) + p_2/(5 - x_1)$$

It is important to note that this function is strictly convex in (x, u) and twice differentiable. Figure 1 shows the different sets of robust economic MPC for tracking. As can be seen in the Figure, the sets meet Assumption 2.

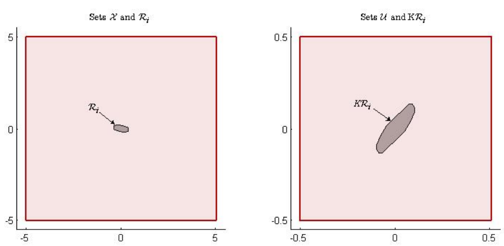

Figura 1: Different sets of the robust economic MPC for tracking

In this case, the nominal system has the following contracting constraints on the inputs and states (for $N = 3$):
u_{min}=$(-0{,}3984; -0{,}3660)$ and u_{max}=$(0{,}3984; 0{,}3660)$
x_{min}=$(-4{,}6283; -4{,}8181)$ and x_{max}=$(4{,}6283; 4{,}8181)$

6.2. Dynamic simulations

The results of the simulation are presented in Figures 2, 3, 4 and 5. In particular, Figure 2 shows the sets $\mathcal{X}$, the domain of attraction $\mathcal{X}_N$, the set of equilibrium $\mathcal{X}_s$ and how the uncertain system evolves, starting at point $x_0 = (-0{,}5; -1)$ with a given economic parameter $p = (10; 200)$. The green dot represents the economic optimal point, x_s^{eco}, while the red sequence represents the artificial state variables x_s^{so}. Figure 2 clearly shows that $\bar{x} \to x_s^{eco} \oplus \mathcal{R}_\infty$ as $x \to x_s^{eco}$. Notice that the sate evolution never leaves $x_s^{eco} \oplus \mathcal{R}_\infty$, once it is inside this set.

Figures 4 and figure 5 shows the time evolution of outputs and inputs, respectively.

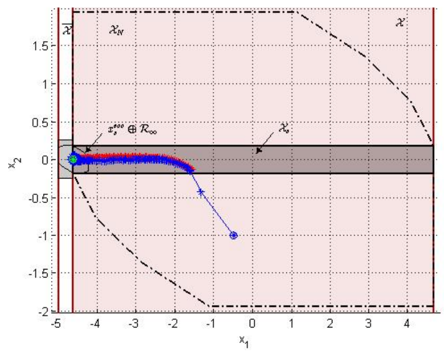

Figura 2: System evolution in set $\mathcal{X}$

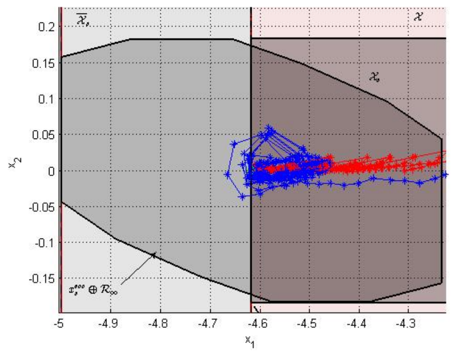

Figura 3: System evolution in set $\mathcal{X}$. Another view

As shown, the controller brings the system to the point economically feasible operation, obtained from the solution of problem $P_N(x, p)$. That is the controller proposed in this work satisfies the economic objective.

7. Conclusions

In this work a new robust economic MPC controller is presented. The overall idea is to robustly consider the stationary economic optimization cost into the controller

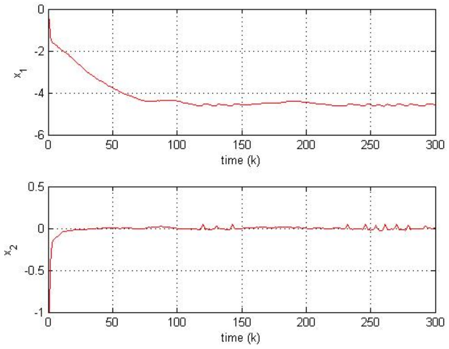

Figura 4: Time state evolution

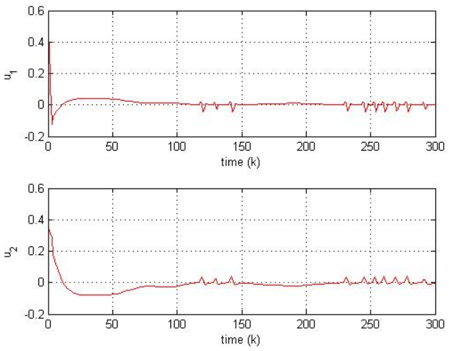

Figura 5: Time input evolution

formulation. By means of a suboptimal solution to the MPC optimization problems and the use of artificial targets variable stability and convergence to the optimum are ensured. The main benefits of the proposed controller are:

- The closed loop robustly converge to the optimal economic point that minimizes f_{eco}.
- The controller implementation requires the solution of just one QP, even when the economic cost is nonlinear.
- There is no need to compute the Hessian of f_{eco}, provided that an heuristic procedure is used to compute $\tilde{\lambda}$ holds.
- The controller remains feasible under any change of the economic objective and any disturbance realization.
- The use of artificial variables allows as the direct substitution of optimal economic points by optimal economic regions (zone control).

Several simulation results shown that the strategy could be useful from an application point of view.

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