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MPC for tracking of constrained nonlinear systems

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Abstract : This paper deals with the tracking problem for constrained nonlinear systems using a model predictive control (MPC) law. MPC provides a control law suitable for regulating constrained linear and nonlinear systems to a given target steady state. However, when the target operating point changes, the feasibility of the controller may be lost and the controller fails to track the reference. In this paper, a novel MPC for tracking changing constant references is presented. This controller extend a recently presented MPC for tracking for constrained linear systems to the nonlinear case. The main characteristics of this controller are: considering an artificial steady state as a decision variable, minimizing a cost that penalizes the error with the artificial steady state, adding to the cost function an additional term that penalizes the deviation between the artificial steady state and the target steady state (the so-called *offset cost function*) and considering an invariant set for tracking as extended terminal constraint. The properties of this controller has been tested on a constrained CSTR simulation model.

1 Introduction

Model predictive control (MPC) is one of the most successful techniques of advanced control in the process industry. This is due to its control problem formulation, the natural usage of the model to predict the expected evolution of the plant, the optimal character of the solution and the explicit consideration of hard constraints in the optimization problem. Thanks to the recent developments of the underlying theoretical framework, MPC has become a mature control technique capable to provide controllers ensuring stability, robustness, constraint satisfaction and tractable computation for linear and for nonlinear systems [1].
The control law is calculated by predicting the evolution of the system and computing the admissible sequence of control inputs which makes the system evolves satisfying the constraints and minimizing the predicted cost. This problem can be posed as an optimization problem. To obtain a feedback policy, the obtained sequence of control inputs is applied in a receding horizon manner, solving the optimization problem at each sample time. Considering a suitable penalization

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of the terminal state and an additional terminal constraint, asymptotic stability and constraints satisfaction of the closed loop system can be proved [2].

Most of the results on MPC consider the regulation problem, that is steering the system to a fixed steady-state (typically the origin), but when the target operating point changes, the feasibility of the controller may be lost and the controller fails to track the reference [3, 4, 5, 6]. Tracking control of constrained nonlinear systems is an interesting problem due to the nonlinear nature of many processes in industry mainly when large transitions are required, as in the case of changing operating point.

In [7] a nonlinear predictive control for set point families is presented, which considers a pseudolinearization of the system and a parametrization of the set points. The stability is ensured thanks to a quasi-infinite nonlinear MPC strategy, but the solution of the tracking problem is not considered.

In [8] an output feedback receding horizon control algorithm for nonlinear discrete-time systems is presented, which solves the problem of tracking exogenous signals and asymptotically rejecting disturbances generated by a properly defined exosystem. In [9] an MPC algorithm for nonlinear systems is proposed, which guarantees local stability and asymptotic tracking of constant references. This algorithm need the presence of an integrator preliminarily plugged in front of the system to guarantee the solution of the asymptotic tracking problem.

Another approach to the tracking of nonlinear systems problem are the so-called reference governors [10, 4, 11]. A reference governor is a nonlinear device which manipulates on-line a command input to a suitable pre-compensated system so as to satisfy constraints. This can be seen as adding an artificial reference, computed at each sampling time to ensure the admissible evolution of the system, converging to the desired reference.

In [12] the tracking problem for constrained linear systems is solved by means of an approach called dual mode: the dual mode controller operates as a regulator in a neighborhood of the desired equilibrium wherein constraints are feasible, while it switches to a feasibility recovery mode, whenever this is lost due to a set point change, which steers the system to the feasibility region of the MPC as quickly as possible. In [13, 14] this approach is extended to nonlinear systems, considering constraint-admissible invariant sets as terminal regions, obtained by means of a LPV model representation of the nonlinear plant.

In [15] an MPC for tracking of constrained linear systems is proposed, which is able to lead the system to any admissible set point in an admissible way. The main characteristics of this controller are: an artificial steady state is considered as a decision variable, a cost that penalizes the error with the artificial steady state is minimized, an additional term that penalizes the deviation between the artificial steady state and the target steady state is added to the cost function (the so-called *offset cost function*) and an invariant set for tracking is considered as extended terminal constraint. This controller ensures that under any change of the target steady state, the closed loop system maintains the feasibility of the controller and ensures the convergence to the target if admissible. In this paper, this controller is extended to the case of nonlinear constrained systems.

The paper is organized as follows. In section 2 the constrained tracking problem is stated. In section 3 the new MPC for tracking is presented. In section 4 an illustrative example is shown. Finally, in section 5 some conclusions are drawn.

2 Problem description

Consider a system described by a nonlinear invariant discrete time model

$$\begin{aligned} x^+ &= f(x,u) \\ y &= h(x,u) \end{aligned} \tag{1}$$

where $x \in \mathbb{R}^n$ is the system state, $u \in \mathbb{R}^m$ is the current control vector, $y \in \mathbb{R}^p$ is the controlled output and x^+ is the successor state. The function model $f(x,u)$ is assumed to be continuous. The solution of this system for a given sequence of control inputs $\mathbf{u}$ and initial state x is denoted as $x(j) = \phi(j,x,\mathbf{u})$ where $x = \phi(0,x,\mathbf{u})$. The state of the system and the control input applied at sampling time k are denoted as $x(k)$ and $u(k)$ respectively. The system is subject to hard constraints on state and control:

$$x(k) \in X, \quad u(k) \in U \tag{2}$$

for all $k \geq 0$. $X \subset \mathbb{R}^n$ and $U \subset \mathbb{R}^m$ are compact convex polyhedra containing the origin in its interior.

The steady state, input and output of the plant (x_s, u_s, y_s) are such that (1) is fulfilled, i.e. $x_s = f(x_s, u_s)$ and $y_s = h(x_s, u_s)$. Due to the relation derived from these equalities, it is possible to find a parameter vector $\theta \in \mathbb{R}^q$ which univocally defines each triplet (x_s, u_s, y_s), i.e., these can be posed as $x_s = g_x(\theta)$, $u_s = g_u(\theta)$ and $y_s = g_y(\theta)$. This parameter is typically the controlled output y_s although another parameter could be chosen for convenience.

For a (possible time-varying) target steady condition (x_t, u_t, y_t) given by θ_t, the problem we consider is the design of an MPC controller $\kappa(x,\theta_t)$ such that the system is steered as close as possible to the target while fulfilling the constraints.

3 MPC for tracking

In this section, the proposed MPC for tracking is presented. This predictive controller is characterized by the addition of the steady state and input as decision variables, the use of a modified cost function and an extended terminal constraint.

The proposed cost function of the MPC is given by:

$$V_N(x,\theta_t;\mathbf{u},\theta) = \sum_{i=0}^{N-1} \ell((x(i)-x_s),(u(i)-u_s)) + V_f(x(N)-x_s,\theta) + V_O(\theta-\theta_t)$$

where $x(j) = \phi(j,x,\mathbf{u})$, $x_s = g_x(\theta)$, $u_s = g_u(\theta)$ and $y_s = g_y(\theta)$. The controller is derived from the solution of the optimization problem $P_N(x,\theta_t)$ given by:

$$\begin{aligned} V_N^*(x,\theta_t) \quad = \quad & \min_{\mathbf{u},\bar{\theta}} V_N(x,\theta_t;\mathbf{u},\theta) \\ s.t. \quad & x(j) = \phi(j,x,\mathbf{u}), \quad j=0,\cdots,N \\ & x(j) \in X, u(j) \in U, \quad j=0,\cdots,N-1 \\ & x_s = g_x(\theta), u_s = g_u(\theta) \\ & (x(N),\theta) \in \Gamma \end{aligned}$$

Considering the receding horizon policy, the control law is given by

$$\kappa_N^{MPC}(x,\theta_t) = u^*(0;x,\theta_t)$$

Since the set of constraints of $P_N(x,\theta_t)$ does not depend on θ_t, its feasibility region does not depend on the target operating point θ_t. Then there exists a region $\mathcal{X}_N \subseteq X$ such that for all $x \in \mathcal{X}_N$, $P_N(x,\theta_t)$ is feasible. This is the set of initial states that can be admissibly steered to the projection of Γ onto x in N steps or less.

Consider the following assumption on the controller parameters:

Assumption 1

1. *Let $\theta \in \Theta$ be a parametrization variable of the steady state, input and output, with Θ convex set.*

2. *Let g_x, g_u and g_y be the defining functions of the steady state, input and output of system (1), i.e., $x_s = g_x(\theta)$, $u_s = g_u(\theta)$ and $y_s = g_y(\theta)$. Assume that g_x is a Lipschitz function.*

3. *Let $k(x,\theta)$ be a continuous control law such that for all $\theta \in \Theta$, system $x^+ = f(x,k(x,\theta))$ has $x_s = g_x(\theta)$ and $u_s = g_u(\theta)$ as steady state and input, and it is asymptotically stable.*

4. *Let $\Gamma \subset \mathrm{I\!R}^{n+q}$ be a set such that for all $(x,\theta) \in \Gamma$, $x \in X$, $\theta \in \Theta$, $k(x,\theta) \in U$ and $(f(x,k(x,\theta)),\theta) \in \Gamma$.*

5. *Let $V_f(x - g_x(\theta),\theta)$ be a Lyapunov function for system $x^+ = f(x,k(x,\theta))$:*

   $$V_f(f(x,k(x,\theta)) - g_x(\theta),\theta) - V_f(x - g_x(\theta),\theta) \leq -l(x - g_x(\theta), k(x,\theta) - g_u(\theta))$$

   for all $(x,\theta) \in \Gamma$. Moreover, there exist $b > 0$ and $\sigma > 1$ which verify $V_f(x_1 - x_2,\theta) \leq b\|x_1 - x_2\|^\sigma$ for all (x_1,θ) and (x_2,θ) contained in Γ.

6. *Let $l(x,u)$ be a positive definite function and let the offset cost function $V_O : \mathrm{I\!R}^p \to \mathrm{I\!R}$ be a convex, positive definite and subdifferentiable function.*

The following theorem proves asymptotic stability and constraints satisfaction of the controlled system.

Theorem 1 (Stability) *Consider that assumption 1 holds and consider a given target operation point parametrization θ_t, such that $x_t = g_x(\theta_t)$, $u_t = g_u(\theta_t)$ and $y_t = g_y(\theta_t)$. Then for any feasible initial state $x_0 \in \mathcal{X}_N = Proj_x(\Gamma)$, the system controlled by the proposed MPC controller $\kappa(x,\theta_t)$ is stable, fulfils the constraints along the time and, besides*

(i) If $\theta_t \in \Theta$ then the closed loop system asymptotically converges to a steady state, input and output (x_t,u_t,y_t), that means $\lim_{k\to\infty} \|x(k) - x_t\| = 0$, $\lim_{k\to\infty} \|u(k) - u_t\| = 0$ and $\lim_{k\to\infty} \|y(k) - y_t\| = 0$.

(ii) If $\theta_t \notin \Theta$, the closed loop system asymptotically converges to a steady state and input $(\tilde{x}_s,\tilde{u}_s)$, such that $\lim_{k\to\infty} \|x(k) - \tilde{x}_s\| = 0$ and $\lim_{k\to\infty} \|u(k) - \tilde{u}_s\| = 0$, where $\tilde{x}_s = g_x(\tilde{\theta}_s)$, $\tilde{u}_s = g_u(\tilde{\theta}_s)$ and

$$\tilde{\theta}_s = \arg\min_{\theta\in\Theta} V_O(\theta - \theta_t)$$

Proof.

Feasibility. The first part of the proof is devoted to prove the feasibility of the controlled system, that is $x(k+1) \in \mathcal{X}_N$, for all $x(k) \in \mathcal{X}_N$ and θ_t. Assume that $x(k)$ is feasible and consider the optimal solution of $P_N(x(k), \theta_t)$, $\mathbf{u}^*(x(k), \theta_t)$, $\theta^*(x(k), \theta_t)$. Define the following sequences:

$$\begin{aligned} \mathbf{u}(x(k+1), \theta_t) &\triangleq [u^*(1; x(k), \theta_t), \cdots, u^*(N-1; x(k), \theta_t), \\ &\quad K(x^*(N; x(k), \theta_t), \theta^*(x(k), \theta_t))] \\ \bar{\theta}(x(k+1), \theta_t) &\triangleq \theta^*(x(k), \theta_t) \end{aligned}$$

Then, due to the fact that $x(k+1) = f(x(k), u^*(0; x(k), \theta_t))$ and to condition 4 in assumption 1, it is easy to see that $\mathbf{u}(x(k+1), \theta_t)$ and $\bar{\theta}(x(k+1), \theta_t)$ are feasible solutions of $P_N(x(k+1), \theta_t)$. Consequently, $x(k+1) \in \mathcal{X}_N$.

Convergence. Consider the feasible solution at time $k+1$ previously presented. Following standard steps in the stability proofs of MPC [2], we get that

$$V_N^*(x(k+1), \theta_t) - V_N^*(x(k), \theta_t) \leq -l(x(k) - g_x(\theta^*(x(k), \theta_t)), u(k) - g_u(\theta^*(x(k), \theta_t)))$$

Due to the definite positiveness of the optimal cost and its non-increasing evolution, we infer that $\lim_{k\to\infty} \|x(k) - g_x(\theta^*(x(k), \theta_t)\| = 0$ and $\lim_{k\to\infty} \|u(k) - g_u(\theta^*(x(k), \theta_t)\| = 0$.

Optimality. Define $x_s^*(x(k),\theta_t) = g_x(\theta^*(x(k),\theta_t)$ and $u_s^*(x(k),\theta_t) = g_u(\theta^*(x(k),\theta_t)$. Let $\tilde{\Theta}$ be the convex set such that $\tilde{\Theta} = \{\tilde{\theta} : \tilde{\theta} = \arg\min_{\theta\in\Theta} V_O(\theta - \theta_t)\}$.

We proceed by contradiction. Consider that $\theta^* \notin \tilde{\Theta}$ and take a $\tilde{\theta} \in \tilde{\Theta}$, then $V_O(\theta^* - \theta_t) > V_O(\tilde{\theta} - \theta_t)$. Due to continuity of the model and the control law, there exists a $\hat{\lambda} \in [0, 1)$ such that, for every $\lambda \in [\hat{\lambda}, 1)$, the parameter $\bar{\theta} = \lambda\theta^* + (1-\lambda)\tilde{\theta}$ fulfils $(x_s^*, \bar{\theta}) \in \Gamma$.
Defining as $\mathbf{u}$ the sequence of control actions derived from the control law $k(x, \bar{\theta})$, it is inferred that $(\mathbf{u}, x_s^*, \bar{\theta})$ is a feasible solution for $P_N(x_s^*, \theta_t)$. Then from assumption 1 and using standard procedures in MPC, we have that

$$\begin{aligned} V_N^*(x_s^*, \theta_t) = V_O(\theta^* - \theta_t) &\leq V_N(x_s^*, \theta_t; \mathbf{u}, \bar{\theta}) \\ &= \sum_{i=0}^{N-1} \ell((x(i) - \bar{x}), (k(x(i), \theta) - \bar{u})) \\ &\quad + V_f(x(N) - \bar{x}, \theta) + V_O(\bar{\theta} - \theta_t) \\ &\leq V_f(x_s^* - \bar{x}, \theta) + V_O(\bar{\theta} - \theta_t) \\ &\leq L_{V_f}\|\theta^* - \bar{\theta}\|^\sigma + V_O(\bar{\theta} - \theta_t) \\ &= L_{V_f}(1-\lambda)^\sigma\|\theta^* - \tilde{\theta}\|^\sigma + V_O(\bar{\theta} - \theta_t) \end{aligned}$$

where $L_{V_f} = L_g^\sigma b$ and L_g is the Lipshitz constant of $g_x(\cdot)$.
Define $W(x_s^*, \theta_t, \lambda) \triangleq L_{V_f}(1-\lambda)^\sigma\|\theta^* - \tilde{\theta}\|^\sigma + V_O(\bar{\theta} - \theta_t)$. Notice that $W(x_s^*, \theta_t, \lambda) = V_N^*(x_s^*, \theta_t)$ for $\lambda = 1$. Taking the partial of W about λ, we have that

$$\frac{\partial W}{\partial \lambda} = -L_{V_f}\sigma(1-\lambda)^{\sigma-1}\|\theta^* - \tilde{\theta}\|^\sigma + g^T(\theta^* - \tilde{\theta})$$

where $g^T \in \partial V_O(\bar{\theta}-\theta_t)$, defining $\partial V_O(\bar{\theta}-\theta_t)$ as the subdifferential of $V_O(\bar{\theta}-\theta_t)$. Evaluating this partial for $\lambda = 1$ we obtain that:

$$\left.\frac{\partial W}{\partial \lambda}\right|_{\lambda=1} = g^{*T}(\theta^* - \tilde{\theta})$$

where $g^{*T} \in \partial V_O(\theta^*-\theta_t)$, defining $\partial V_O(\theta^*-\theta_t)$ as the subdifferential of $V_O(\theta^*-\theta_t)$. Taking into account that V_O is a subdifferentiable function, from convexity [16] we can state that

$$g^{*T}(\theta^* - \tilde{\theta}) \geq V_O(\theta^* - \theta_t) - V_O(\tilde{\theta} - \theta_t)$$

Considering that $V_O(\theta^* - \theta_t) - V_O(\tilde{\theta} - \theta_t) > 0$, it can be derived that

$$\left.\frac{\partial W}{\partial \lambda}\right|_{\lambda=1} \geq V_O(\theta^* - \theta_t) - V_O(\tilde{\theta} - \theta_t) > 0$$

This means that there exists a $\lambda \in [\hat{\lambda}, 1)$ such that $W(x_s^*, \theta_t, \lambda)$ is smaller than the value of $W(x_s^*, \theta_t, \lambda)$ for $\lambda = 1$, which equals to $V_N^*(x_s^*, \theta_t)$.

This contradicts the optimality of the solution and hence the result is proved, finishing the proof.

Remark 1 *The problem of computing the terminal conditions is not easy to solve. In literature, this problem is handled in many ways, such as LDI [17] or LPV [13, 12] model representations of the system. In [10] the authors state that the command governors strategy ensures the viability property, which implies the existence of such a not trivial invariant set.*

Remark 2 *The local nature of the terminal controller and the difficulty of computing set Γ makes this set potentially small. In fact, a sensible choice of Γ is as level sets of the local Lyapunov function. In order to minimize the effect of the conservative nature of the terminal ingredients, a formulation with a prediction horizon larger that the control horizon [18] can be used. This provides an enhanced closed loop performance and a larger domain of attraction maintaining the stabilizing properties.*

4 Example

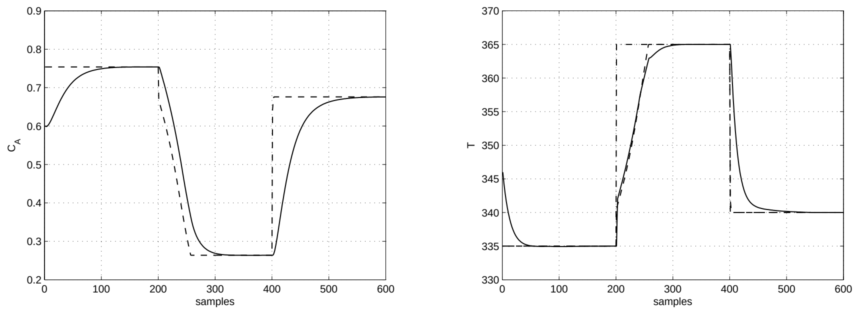
This section presents the application of the proposed controller to the highly nonlinear model of a continuous stirred tank reactor (CSTR), [18]. Assuming constant liquid volume, the CSTR for an exothermic, irreversible reaction, $A \rightarrow B$, is described by the following model:

$$\begin{aligned} \dot{C}_A &= \frac{q}{V}(C_{Af} - C_A) - k_o e^{(\frac{-E}{RT})} C_A \\ \dot{T} &= \frac{q}{V}(T_f - T) - \frac{\Delta H}{\rho C_p} k_o e^{(\frac{-E}{RT})} C_A + \frac{UA}{V \rho C_p}(T_c - T) \end{aligned} \quad (3)$$

where C_A is the concentration of A in the reactor, T is the reactor temperature and T_c is the temperature of the coolant stream. The objective is to regulate $y = x_2 = T$ and $x_1 = C_A$ by manipulating $u = T_c$. The constraints are

$0 \leq C_A \leq 1$ mol/l, $280K \leq T \leq 370K$ and $280K \leq T_c \leq 370$. The nonlinear discrete time model of system (3) is obteined by defining the state vector $x = [C_A - C_A^{eq}, T - T^{eq}]^T$ and $u = T_c - T_c^{eq}$ and by discretizing equation (3) with $t = 0.03$ min as sampling time. We considered an MPC with $N_c = 3$ and $N_p = 10$ and with $Q = diag(1/0.5, 1/350)$ and $R = 1/300$ as weighting matrices.

The output $y = x_2$ has been chosen as the parameter θ. To illustrate the proposed controller, three references has been considered, $Ref_1 = 335$ K, $Ref_2 = 365$ K and $Ref_3 = 340$ K. In figures 1(a) and 1(b) the evolutions of the states (solid lines), the artificial references (dashed lines) and the real one (dashed-dotted line) are showed. See how the controller leads the system to track the artificial reference when the real one is unfeasible. The artificial reference represents the feasible trajectory determined by the value of $\tilde{\theta}_s$ that minimizes $V_O(\theta - \theta_t)$.



(a) Evolution of C_A.

(b) Evolution of T.

Figure 1: Evolutions of the states.

The terminal region and cost function have been computed in an explicit form, depending on θ. Linearizing the system around θ, the control gain has been computed explicitly, depending on θ, $K(\theta)$. Defining $A_K(\theta) = A + BK(\theta)$ and $Q(\theta)^* = Q + K(\theta)^T RK(\theta)$, $P(\theta)$ has been found as solution of $A_K(\theta)^T P(\theta) A_K(\theta) - P(\theta) = -Q(\theta)^*$. Then, $V_f(x - g_x(\theta), \theta) = (x - g_x(\theta))^T P(\theta)(x - g_x(\theta))$ and $\Gamma = \{(x, \theta) \in \mathbb{R}^{n+q} : V_f(x - g_x(\theta), \theta) \leq \alpha\}$, where $\alpha > 0$ is such that for all $(x, \theta) \in \Gamma$, $x \in X$, $u = K(\theta)(x - g_x(\theta)) + g_u(\theta) \in U$ and

$$V_f(f(x, k(x, \theta)) - g_x(\theta), \theta) - V_f(x - g_x(\theta), \theta) \leq -(x - g_x(\theta))^T Q(\theta)^* (x - g_x(\theta)).$$

5 Conclusion

In this paper a novel MPC controller for tracking changing references for constrained nonlinear systems has been presented, as extension, to the nonlinear case, of the one presented in [15]. This controller ensures feasibility by means of adding an artificial steady state and input as decision variable of the optimization problem. Convergence to an admissible target steady state is ensured by using a modified cost function and a stabilizing extended terminal constraint. Optimality is ensured by means of an offset cost function which penalizes the difference between the artificial reference and the real one. The proposed controller can be formulated with a prediction horizon larger than the control horizon.

This formulation provides an enhanced closed loop performance and a larger domain of attraction maintaining the stabilizing properties. The properties of the controller have been illustrated in an example.

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