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## A cardinality-constrained approach for robust machine loading problems

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### Abstract

The Machine Loading Problem (MLP) refers to the allocation of operative tasks and tools to machines for the production of parts. Since the uncertainty of processing times might affect the quality of the solution, this paper proposes a robust formulation of an MLP, based on the cardinality-constrained approach, to evaluate the optimal solution in the presence of a given number of fluctuations of the actual processing time with respect to the nominal one. The applicability of the model in the practice has been tested on a case study.

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*Keywords:* machine loading problem; robust optimisation; production planning.

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### 1. Introduction

The Machine Loading Problem (MLP) consists of deciding (given a set of part types to produce, a set of machines, and a set of tools) the quantities to produce on each machine and the tools to be assigned to the machines, respecting technological and capacity constraints. It was firstly introduced by Stecke [1,2]: control policies for Flexible Manufacturing Systems (FMSs) were discussed in [1], while a mathematical programming model was

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introduced in [2]. The deterministic MLP has been deeply studied, and the number of publications on the subject is very large. Detailed surveys are available in Guerrero [3] and Grieco et al. [4]. Among the others, valuable contributions can be found in Shanker and Tzen [5], who compared a heuristic approach and the exact formulation, and in Abazari et al. [6], who compared two solution strategies based on a genetic algorithm. Another interesting contribution can be found in Das et al [7], where the authors analysed the problem in a comprehensive way, including machine loading, product part type grouping, and operations sequencing.

However, as far as the authors' knowledge, robust MLP models are missing in the literature despite processing times are subject to uncertainty in the practice due to a variety of reasons (e.g., failures, unexpected tool breaks, unplanned maintenance interventions). These perturbations on the input data might cause a substantial detriment of the objective function, or even make the MLP deterministic solution infeasible.

Robust Optimization (RO) allows including uncertain input parameters in optimization problems through the definition of uncertainty sets for the parameters, e.g., intervals in which parameter values can vary [8]. RO approaches formulate the problem on the basis of the uncertainty set of the input parameters, to find a solution that is feasible over all the set. There exist several RO approaches in the literature, based on the shape and the assumptions of the uncertainty set. Ben-Tal and Nemirovski [9] considered ellipsoidal-box uncertainty sets, while Bertsimas and Sim [10] considered a generic convex uncertainty set in which all uncertain parameters are assumed to belong to an interval.

RO approaches provide solutions that guarantee feasibility for all possible combinations of parameters within the uncertainty set. Thus, to avoid overconservative solutions that are unsatisfactory in practice, it is fundamental to control the level of robustness through a proper uncertainty set. In [10], the level of robustness is tuned by superiorly bounding the number of the parameters (cardinality) that assume the worst value of the interval instead of the nominal one. For this reason, such an approach is called *cardinality-constrained*.

#### Nomenclature

$I$	set of product part types
$J$	set of tools
$M$	set of machines
$T$	set of time periods
$E_{jt}$	set of processing times of products worked on tool $j \in J$ subject to uncertainty in period $t \in T$
$U_{jt}$	set of product types worked on tool $j \in J$ affected by disruptive events in period $t \in T$
$p_{jmt}$	time spent for production on the tool $j \in J$ of machine $m \in M$ in period $t \in T$
$q_{ijt}, r_{ij}$	auxiliary dual-variables
$S_i$	shortage of production of product type $i \in I$
$L_{jmt}$	Boolean variable that is 1 when tool $j \in J$ is loaded on machine $m \in M$ in period $t \in T$
$v_{ijt}$	time spent for production of product type $i \in I$ over tool $j \in J$ in period $t \in T$
$z_{ijt}$	auxiliary primal variable
$x_{it}$	quantity of product type $i \in I$ produced in period $t \in T$ .
$\alpha_j$	number of tool copies available for tool $j \in J$
$\Gamma_{jt}$	cardinality parameter of tool $j \in J$ in period $t \in T$
$A_{mt}$	available time for production on machine $m \in M$ in period $t \in T$
$C_i$	shortage cost per product type $i \in I$
$D_i$	total demand of product type $i \in I$
$k_j$	number of slots required by tool $j \in J$
$K_m$	total slots available in the slot magazine of machine $m \in M$
$O_{ij}$	processing time of one unit of product $i \in I$ on tool $j \in J$
$h_{it}$	holding cost per part of product type $i \in I$ in period $t \in T$ over the remaining time horizon
$w_i$	total earning per part for the production of product type $i \in I$

Our aim in this paper is to address the uncertainty of processing times in an MLP. Specifically, we integrate a deterministic model available in the literature with the cardinality-constrained approach. Thus, the solution of the robust MLP is the system configuration that covers from a certain number of disruptive events (we define disruptive

an event that perturbs a processing time from the nominal to the worst value). As deterministic model, we have chosen the one proposed by Sodhi et al. [14], because it includes all main parameters of the MLP and addresses the core issues identified in [4].

Production managers are likely able to estimate how many events might occur over a certain time horizon, and they can use this knowledge for making reasonable assessments on the most proper cardinality values to adopt for the uncertain parameters. This motivates the choice of the cardinality-constrained approach to develop a robust MLP, and our confidence in possible advantages from its application to the MLP.

Despite the cardinality-constrained approach has been widely applied in several fields [11], applications to manufacturing problems such as production planning are not common. Valuable contributions can be found in Moreira [12], who developed a robust assembly line balancing, and in Lu et al. [13], who proposed a robust single machine job scheduling problem with uncertain processing times.

This paper is organized as follows. Section 2 presents the deterministic MLP as in [14] and the proposed robust version. The tests conducted on a case study and the numerical results are presented in Section 3. Section 4 draws the conclusions of the work.

## 2. Model formulation

### 2.1. Deterministic model (MLP-DET)

Sodhi et al. [14] considered the short-term production plan and the allocation of tools to machines. The model is multi-period and considers a time horizon divided into a finite number of periods. The time horizon corresponds to the production planning period, while a single time period to the time interval between two consecutive tool changeovers. Within each time period  $t$ , each machine  $m$  processes the workpieces assigned to it with a given time availability  $A_{mt}$ . Workpieces are worked sequentially on their assigned machine using the selected tools; the processing time of product type  $i$  over tool  $j$  is denoted by  $O_{ij}$ . Thus, the number of products that is possible to produce in each time period is limited by the machine time availability. The model maximises the profit, while it does not consider any workload balancing among machines. Moreover, it is assumed that the batch of product types has been previously composed, that the tool storage capacity is limited, that no tool transportation system is present; thus, it is not possible to change the tools without stopping the system. Finally, production quantities are assumed to be continuous, to enable the partial production of certain product types over a time period. The model determines the production plan in terms of the variables  $x_{it}$ , i.e., the quantity of product type  $i$  to produce in period  $t$ . The machine loading is expressed by the variables  $L_{jmt}$ , Boolean variables equal to 1 if tool  $j$  is loaded on machine  $m$  in period  $t$ , and 0 otherwise. Finally, variables  $p_{jmt}$  represent the aggregate machining time spent on tool  $j$  of machine  $m$  in period  $t$ . The deterministic model MLP-DET is formulated as follows:

$$\text{MLP-DET:} \quad \max \quad \sum_i \sum_t w_i x_{it} - \sum_i C_i S_i - \sum_i \sum_t h_{it} x_{it} \quad (1)$$

$$\text{subject to:} \quad \sum_j L_{jmt} k_j \leq K_m \quad \forall m, t \quad (2)$$

$$\sum_t x_{it} = D_i - S_i \quad \forall i \quad (3)$$

$$\sum_i O_{ij} x_{it} \leq \sum_m p_{jmt} \quad \forall j, t \quad (4)$$

$$p_{jmt} \leq L_{jmt} A_{mt} \quad \forall j, m, t \quad (5)$$

$$\sum_j p_{jmt} \leq A_{mt} \quad \forall m, t \quad (6)$$

$$\sum_m L_{jmt} \leq \alpha_j \quad \forall j, t \quad (7)$$

$$L_{jmt} \in \{0, 1\} \quad \forall j, m, t$$

$$p_{jmt} \geq 0 \quad \forall j, m, t; \quad x_{it} \geq 0 \quad \forall i, t; \quad S_i \geq 0 \quad \forall i$$

The objective function (1) aims at both maximizing the total profit related to the production of products and minimizing the shortage and stocking costs, where the stock is the quantity of products held in the warehouse before being sold. Constraints (2) limit the number of tools that can be loaded on machines, as the number of slots available for machine  $m$  is always constrained to  $K_m$  slots. Constraints (3) compute the production shortage  $S_i$  of each product  $i$ , which is defined as the difference between the production of part type  $i$  over all the time horizon  $\sum_t x_{it}$  and the demand  $D_i$ . Constraints (4) to (6) guarantee that all the production is made within the time availability of the machines. Specifically, constraints (4) limit the total production time on tool  $j$  in time period  $t$ . Constraints (5) guarantee that, for each tool  $j$ , machine  $m$ , and period  $t$ , the production can be done only if the necessary tool has been loaded. Constraints (6) guarantee that, over each time period  $t$ , the production time at machine  $m$  does not exceed the machine availability  $A_{mt}$ . Constraints (7) limit the maximum number of tool copies  $\alpha_j$  available for tool  $j$  in each period  $t$ . Finally, the other constraints define the domain of the variables.

### 2.2 Robust model formulation (MLP-ROBUST)

According to the cardinality-constrained approach, we consider the processing times as random variables  $\tilde{O}_{ij}$ , symmetrically distributed in the interval  $[\bar{O}_{ij} - \hat{O}_{ij}, \bar{O}_{ij} + \hat{O}_{ij}]$ , where  $\bar{O}_{ij}$  is the expected value of the processing time, and  $\hat{O}_{ij}$  is the maximum deviation. The deviation  $\hat{O}_{ij}$  represents a disruptive event that can cause the processing time to rise. Hence, constraints (4) are affected by the increase of total processing time, due to the disruptive events.

We denote as  $E_{jt}$  the set of the processing times over tool  $j$  at period  $t$ . Also, we define the cardinality matrix  $\Gamma = \{\Gamma_{jt}\}$ , where  $\Gamma_{jt}$  represents the number of product types subject to an increment of their processing time over tool  $j$  at period  $t$ . The increment is assumed to be up to the maximum value  $\bar{O}_{ij} + \hat{O}_{ij}$ . Set  $U_{jt} \subseteq E_{jt}$  represents any subset of  $E_{jt}$  constrained to have cardinality  $\Gamma_{jt}$  (i.e.,  $|U_{jt}| = \Gamma_{jt}$ ). Specifically, at the optimum of the robust formulation, set  $U_{jt}$  contains the worst combination that mostly affect the total usage time of tool  $j$  in period  $t$ . The selection of the processing times to include in set  $U_{jt}$  is represented by a specific set of auxiliary decision variables  $z_{ijt}$ . The robust counterpart of constraints (4) is:

$$\sum_i \bar{O}_{ij} x_{it} + \max_{U_{jt}} \left\{ \sum_{i \in U_{jt}} \hat{O}_{ij} x_{it} \right\} \leq \sum_m P_{jmt} \quad \forall j, t \quad (8)$$

where the selection of the worst  $U_{jt}$  is the inner maximization problem. The maximization problem in the above constraint, denoted as PRIMAL, can be written as:

PRIMAL:  $\max \sum_i \hat{O}_{ij} x_{it} z_{ijt} \quad (9)$

subject to:  $\sum_i z_{ijt} \leq \Gamma_{jt} \quad \forall j, t \quad (10)$

$$0 \leq z_{ijt} \leq 1 \quad \forall i, j, t \quad (11)$$

where  $\tilde{x}_{it}$  refers to a given solution of the problem. The dual of problem PRIMAL, denoted as DUAL, is:

DUAL:  $\min \Gamma_{jt} r_{jt} + \sum_i q_{ijt} \quad (12)$

subject to:  $r_{jt} + q_{ijt} \geq \hat{O}_{ij} \tilde{x}_{it} \quad \forall i, j, t \quad (13)$

$$r_{ij} \geq 0 \quad \forall i, j; \quad q_{ijt} \geq 0 \quad \forall i, j, t$$

where  $r_{jt}$  and  $q_{ijt}$  are dual variables referring to constraints (10) and (11), respectively. Since the problem PRIMAL is bounded and feasible, for the Strong Duality Theorem (cf. [15]) the optimal solution  $z_{ijt}^*$  of problem PRIMAL and the solutions  $r_{jt}^*$  and  $q_{ijt}^*$  of the problem DUAL are equivalent in terms of objective function in equations (9) and (12). The property holds for any  $\tilde{x}_{it}$ ; therefore, also for the optimal  $x_{it}^*$ :

$$\Gamma_{jt} r_{jt}^* + \sum_i q_{ijt}^* = \sum_i \hat{O}_{ij} x_{it}^* z_{ijt}^*$$

The objective function of problem DUAL can replace the maximization problem in (8). Together with its additional constraints, the robust counterpart of problem MLP-DET becomes:

MLP-ROBUST:                    max                     $\sum_i \sum_t w_i x_{it} - \sum_i C_i S_i - \sum_i \sum_t h_{it} x_{it}$                     (14)

subject to:                     $\sum_j L_{jmt} k_j \leq K_m \quad \forall m, t$                     (15)

$$\sum_t x_{it} = D_i - S_i \quad \forall i \quad (16)$$

$$p_{jmt} \leq L_{jmt} A_{mt} \quad \forall j, m, t \quad (17)$$

$$\sum_i \bar{O}_{ij} x_{it} + \Gamma_{jt} r_{jt} + \sum_i q_{ijt} \leq \sum_m p_{jmt} \quad \forall j, t \quad (18)$$

$$r_{jt} + q_{ijt} \geq \hat{O}_{ij} x_{it} \quad \forall i, j, t \quad (19)$$

$$\sum_j p_{jmt} \leq A_{mt} \quad \forall m, t \quad (20)$$

$$\sum_m L_{jmt} \leq \alpha_j \quad \forall j, t \quad (21)$$

$$L_{jmt} \in \{0, 1\} \quad \forall j, m, t$$

$$p_{jmt} \geq 0 \quad \forall j, m, t; x_{it} \geq 0 \quad \forall i, t$$

$$S_i \geq 0 \quad \forall i; r_{ij} \geq 0 \quad \forall i, j; q_{ijt} \geq 0 \quad \forall i, j, t$$

For practical applications, it is possible to reasonably estimate the time deviations  $\hat{O}_{ij}$  and to select the cardinality values  $\Gamma_{jt}$  from the information commonly available in industry. For instance, the estimation of the processing time interval  $[\bar{O}_{ij} - \hat{O}_{ij}, \bar{O}_{ij} + \hat{O}_{ij}]$  can be extracted from the historical data, or it can be computed using the expected Mean Time To Repair (MTTR<sub>j</sub>) of each tool  $j$  by assuming  $\hat{O}_{jt} \approx MTTR_j$ . Also, we can estimate the cardinality parameters based on the number of failures for each tool  $j$ , related to the expected Mean Time Between Failures (MTBF<sub>j</sub>) and the time availability at machine  $m$ . Furthermore, for a more exhaustive analysis, the values of  $\hat{O}_{jt}$  can be increased and different values of the cardinality  $\Gamma_{jt}$  can be investigated for the same instance.

### 3. Numerical results

The proposed robust model is tested using a real case from the literature [7], which is a multi-period single-machine MLP. The problem considers 12 product types to produce using 12 tools over 5 time periods, each one represented by a shift of 9 hours (i.e.,  $A_{mt} = A = 540$  min). The demand  $D_i$  of product type  $i$  and the nominal Production Time (PT<sub>i</sub>) per part of product type  $i$  (i.e.,  $PT_i = \sum_j \bar{O}_{ij}$ ) are in Table 1a. The slots  $k_j$  required by tool  $j$  are in Table 1b. All machines have a tool slots capacity  $K_m = K = 30$  slots, so that the capacity is not affecting the problem solution ( $\sum_j k_j \leq K$ ). Only one instance of each tool is available ( $\alpha_j = 1$ ); anyway, this does not affect the loading since we are in a single-machine scenario. For each product type, the weight, the shortage cost, and the holding cost are  $w_i = w = 30$  €/unit,  $C_i = C = 40$  €/unit, and  $h_{it} = h = 0$ , respectively. The nominal processing times

$\bar{O}_{ij}$  are in Table 1c. Each product requires at least 2 and maximum 5 tools. The model has been solved on a computer equipped with processor Intel Core i7 @2.5Ghz and 8GB of installed RAM, using IBM ILOG CPLEX v12.5. The computational times are very short, ranging between 5.13 s and 5.82 s for all runs.

In the deterministic case, the optimal solution is  $\mathbf{x}^* = \{124.8, 4, 8, 8, 40, 0, 4, 20, 20, 8, 0, 0\}$ , where each element  $x_i^*$  is the quantity of product  $i$  to produce over the whole time horizon, and it achieves an objective function of 7103 €, as shown in Table 2a. The selection of product types is based on a ranking of the nominal production time  $PT_i$ , since the products that require less production time are firstly selected. The machine availability is saturated, i.e., the Total Production Time  $TPT = \sum_i x_i^* PT_i$  coincides with the time availability  $\sum_t A_{mt} = 2700$  min. Since holding costs are zero, the arrangement of production quantities over the time periods does not follow any pattern, and they are therefore aggregated over the time horizon in the following analyses. Achieved results are aligned with [7].

Table 1. (a) Product demand  $D_i$  and nominal production time per part  $PT_i$ ; (b) slot occupancy  $k_j$ ; and (c) nominal processing time  $\bar{O}_{ij}$ .

(a) Product data			(b) Tool slots		(c) Processing times data (rows: product types, columns: tools)												
Product	$D_i$ [part]	$PT_i$ [min/part]	Tools	$k_j$	Nominal processing times ( $\bar{O}_{ij}$ ) [min/part]												
1	160	14	1	4	4.61	2	2.74	4.73	0	0	0	0	0	0	0	0	0
2	4	11.9	2	2	2.46	2.51	0	4.19	2.7	0	0	0	0	0	0	0	0
3	8	11.9	3	2	4.4	0	0	2.86	4.62	0	0	0	0	0	0	0	0
4	8	7.7	4	2	0	0	0	0	0	3.12	1.95	2.64	0	0	0	0	0
5	40	7.9	5	2	0	0	0	0	0	0	0	0	4.29	3.68	0	0	0
6	4	14.8	6	1	0	0	0	0	0	0	0	0	3.15	5.39	2.18	4.14	0
7	4	11.3	7	1	0	0	0	0	0	0	0	0	0	2.7	2.44	6.18	0
8	20	7.8	8	1	0	0	1.92	2.59	3.3	0	0	0	0	0	0	0	0
9	20	8.3	9	1	0	0	3.04	5.28	0	0	0	0	0	0	0	0	0
10	8	6.5	10	1	4.45	2.1	0	0	0	0	0	0	0	0	0	0	0
11	8	20.3	11	3	0	0	0	0	5.42	4	0	0	0	0	5.48	5.47	0
12	4	15.4	12	3	0	0	0	0	0	0	0	0	4.22	3.62	3.92	3.72	0

For the robust analysis, we consider the production time deviations as proportional to the nominal times, i.e.,  $\hat{O}_{ij} = \delta \bar{O}_{ij}$  where  $\delta$  is a parameter. We investigate the behaviour of the model varying the variability of processing times: specifically, we analyse three different scenarios:  $\delta = 0.1$ ,  $\delta = 0.5$ , and  $\delta = 1$ . Moreover, we consider six different cardinality matrices  $\Gamma = \{\Gamma_{jt}\}$  with equal elements, i.e.,  $\Gamma_{jt} = \Gamma = 0, 1, 2, 3, 4, 5$  where the case  $\Gamma = 0$  represents the deterministic case. As for example, with  $\Gamma = 1$  we are limiting the failures to a maximum of 60, one failure per tool (12 tools) at each period (5 periods).

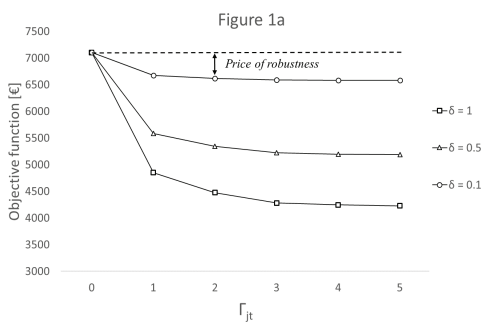


Fig. 1a. Results – Objective function value for each given cardinality vector and values of  $\delta$ . Point  $\Gamma_{jt}=0$  corresponds to the deterministic case. The distance between the robust solution and the deterministic one is the price of robustness.

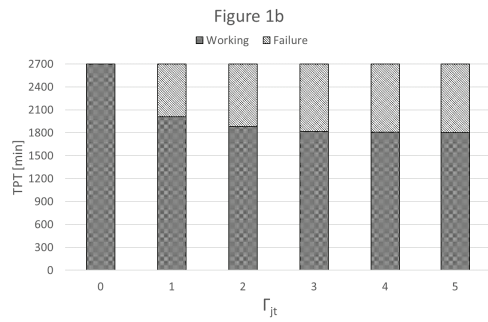


Fig. 1b. Results – Actual TPT with  $\delta = 0.5$  and ratio between working and failure time. It can be noticed that the machine availability  $\sum_t A_{mt} = 2700$  min is always saturated, and that the failure time increases with the cardinality.

The objective function is shown in Fig. 1a by varying the cardinality  $\Gamma$  and the variability  $\delta$ . Given a certain  $\delta$ , the objective function decreases as the cardinality  $\Gamma$  increases. The TPT is composed by both the nominal processing times and the failures. Fig. 1b shows the ratio between the working and failure time over all parts produced in the system. Since the machine availability is limited and saturated, the total number of parts produced decreases together with the objective function. The gap between the objective functions for  $\Gamma = 0$  and for any other scenario represents the “price of robustness”, i.e., the price required for covering from unfortunate events. Since the most disruptive events are selected first, the marginal price of robustness decreases in the cardinality.

Table 2. Results – (a) Deterministic case; (b) Robust solution with  $\delta = 0.1$ ; (c) Robust solution with  $\delta = 0.5$ ; (d) Robust solution with  $\delta = 1$ .

<i>i</i>	(a) Deterministic case			(b) $x_i^*(\delta = 0.1)$					(c) $x_i^*(\delta = 0.5)$					(d) $x_i^*(\delta = 1)$				
	$D_i$	$PT_i$	$x_i^*$	$\Gamma=1$	$\Gamma=2$	$\Gamma=3$	$\Gamma=4$	$\Gamma=5$	$\Gamma=1$	$\Gamma=2$	$\Gamma=3$	$\Gamma=4$	$\Gamma=5$	$\Gamma=1$	$\Gamma=2$	$\Gamma=3$	$\Gamma=4$	$\Gamma=5$
1	160	14	124.8	103.8	108.6	107.6	107.4	107.3	63.3	58.5	62	61.2	60.8	37.2	27.4	30.6	29.4	28.8
2	4	11.9	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4
3	8	11.9	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8
4	8	7.7	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8
5	40	7.9	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40
6	4	14.8	0	4	0	0	0	0	4	4	0	0	0	4	4	0	0	0
7	4	11.3	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4
8	20	7.8	20	20	20	20	20	20	20	20	20	20	20	20	20	20	20	20
9	20	8.3	20	20	20	20	20	20	20	20	20	20	20	20	20	20	20	20
10	8	6.5	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8
11	8	20.3	0	0	0	0	0	0	2.7	0	0	0	0	4.5	1.8	0	0	0
12	4	15.4	0	2.6	0	0	0	0	4	3.5	0	0	0	4	4	0	0	0

For each combination of  $\Gamma$  and  $\delta$ , the optimal solution  $x^*$  is detailed in Tables 2a-2d. Results show that the demand of some product types is always satisfied ( $i = 2, 3, 4, 5, 7, 8, 9, 10$ ), while the optimal quantity of other product types changes from case to case. In fact, when uncertainty is included, the actual processing times are considered instead of the nominal ones, a product can be more affected by failures than the others, and the selection might change compared to the deterministic case. Let us consider the case with  $\Gamma = 1$  and  $\delta = 1$  (Table 2d). The nominal  $PT_i$  (Table 2a) for product  $i = 6$  is 14.8 min/part and it would suggest the selection of type 6 only after product 1 is completely satisfied ( $PT_1 = 14$  min/part). Since machine availability is saturated, product 6 is not produced in the deterministic case.

However, the actual processing times at the optimum with  $\Gamma = 1$  and  $\delta = 1$  become 28.2 min/part and 14.8 min/part for  $i = 1$  and  $i = 6$ , respectively, so that product 6 is chosen firstly. Furthermore, it is noteworthy that the robust solution might not complete the demand of two product types. The actual processing times (28.2 min/part for  $i = 1$  and 25.8 min/part for  $i = 11$ ) would suggest the selection of type 1 only after type 11 is completed. However, with  $\Gamma = 1$  and  $\delta = 1$ , type 11 is produced up to 4.5 parts only. This holds because the ranking of part types  $i$  also changes with the produced quantity. Indeed, producing a higher quantity of product type 11 would change the ranking, making the actual processing time of  $i = 11$  equal to 31.1 min/part, in such a way that product type 1 should be selected. Understanding how the actual processing times are affected is not trivial and depends on the combination of events that is selected. For this reason, the proposed mathematical formulation is very effective in finding the optimal.

For a practical application, the proposed approach allows to evaluate the products that are mostly subject to risk in a specific layout. As for example, Fig. 2 shows that, with  $\Gamma = 1$  and  $\delta = 0.5$ , product type 1 is affected by the highest number of failures. This analysis can be useful for the production planning process and for the implementation of maintenance interventions. Indeed, the user is supposed to pay particular attention to the production of product 1, for example by providing multiple copies of the used tools (in this case 1, 2, 3, and 4) and by maintaining all these tools in a good state.

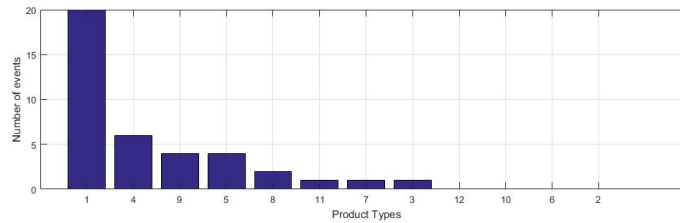


Fig. 2. Number of failures affecting product types ( $\Gamma = 1, \delta = 0.5$ ).

#### 4. Conclusions and future developments

In this work we have developed a robust version of an existing MLP model. The proposed model provides a robust solution against a given number of unfortunate events that the production planner is expecting to happen or against which he/she wishes to cover. The behaviour of the model has been also analysed by means of a case study. The achieved results can be very useful in terms of impact analysis at the first stages of a production planning process. Indeed, the robust model allows to evaluate the maximum performance we can expect from the system once the cardinality and the entity of the disruptions are set.

The analysis on the test case does not consider the cost of holding parts. An additional analysis could include the inventory cost, e.g., to avoid or delay the production of product types associated with higher holding. Further, a sensitivity analysis will provide more insights on the effect of other model parameters, and the application to real cases will allow us to better evaluate the applicability of the model. As for the model, future developments will be devoted to include a broader cardinality sets, e.g., by considering failures that affect single work-pieces. The proposed model may be also used under a rolling horizon approach: once a plan is implemented, if less events occur than expected, the remaining time can be used to schedule both the remaining products and the newly arrived orders. Finally, additional analyses will include a comparison with other existing MLP models, by applying the cardinality-constrained approach to them, and with other robust approaches.

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