A Modifier-Adaptation Approach to the One-Layer Economic MPC

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Abstract: In this paper, we address the problem of modeling error in economically optimal control. A single layer controller is proposed that integrates the economical part of the Real Time Optimization (RTO), the dynamic part of the Model Predictive Control (MPC) and the Modifier Adaptation strategy (MA), resulting in a controller with the following characteristics: a) recursive feasibility guarantee of the controller ; b) asymptotic closed-loop stability for any change in the economic cost function; c) convergence guarantee to the economic optimum of the real plant (offset-free) for any change in the cost function of the controller; and d) simple implementation of the controller. We show the behaviour of the proposal by means of a motivating example that highlights the performance of the proposed algorithm.

Keywords: Model predictive control (MPC). Real-time optimization (RTO). Economic design. Modifier-adaptation. Uncertainty. Nonlinear systems. One-Layer Control.

1. INTRODUCTION

Economic optimization and control schemes are in general proposed as a multi-layer hierarchical structure (Findeisen et al., 1980), which divides the optimization problem in order to simplify it. The first layer typically is an economic planner that determines the general parameters of the plant. The next layer uses a so-called Real-Time Optimization [1] (RTO) strategy, where the production setpoints are calculated, that is, the operating points that the system must reach to minimize a certain economic criterion, taking into account the information it receives from the planning layer. The setpoints calculated by the RTO are then sent to the advance control level, typically an MPC, which calculates the control actions required to drive the system to the setpoint provided by the RTO.

One of the problems with the hierarchical approach is that communication between the RTO and MPC layers can be inconsistent, mainly since the RTO is usually based on a complex nonlinear static plant model, while MPC takes into account a simplified (possibly linear) dynamic model. On the other hand, given the complexity of the

optimization problems that the RTO layer solves, it is common for its sampling time to be larger than the MPC one. One of the implications of this approach is the possibility that a given steady-state provided by the RTO is an unreachable point for the MPC, due to the significant differences between these two the optimization problems, such as the set of constraints, as well as due to model-plant mismatches.

To face this problem, the work of Muske (1997) suggests using an intermediate layer between RTO and MPC called Steady-State Target Optimizer (SSTO), which, given a reference provided by the RTO, calculates a feasible stationary point for the MPC, minimizing (typically) a quadratic cost function that penalizes the error with respect to the setpoint provided by the RTO.

One of the MPC formulations that faces model inconsistency issues and changes in the system operating points is the so-called MPC for Tracking (MPCT) (Limon et al., 2008). This predictive controller integrates the SSTO layer into its formulation, thus always guaranteing stability, for any change in the setpoint. That is, given a setpoint calculated by the upper RTO layer, MPCT finds a feasible reference for the controller, that is the closest (to that setpoint) steady-state that fulfills the constraints.

Another way to address the difficulties of hierarchical control is to consider the so-called Economic MPC formulation (EMPC) (Amrit, 2011; Angeli et al., 2012; Rawlings et al., 2012), which uses the RTO cost function as the

[1] RTO is a family of optimization methods that incorporate process measurements in the optimization framework to drive a real process (or plant) to optimal performance, while guaranteeing constraint satisfaction (Marchetti et al., 2016). Commonly, the acronym RTO is used to define the layer that makes use of some RTO strategy.

dynamic stage cost of the MPC formulation, solving the whole problem in one layer. This approach offers the advantage of calculating not only an optimal stationary point but the optimum trajectory to it, improving the economic optimality of the controller. In (Angeli et al., 2012; Rawlings et al., 2012), the authors prove that if the system is dissipative regarding the cost function, then there exists a Lyapunov function, which makes it possible to ensure that the closed-loop system will be asymptotically stable. Despite the advantage that EMPC offers in aggregating the functionality of RTO and MPC, stability and recursive feasibility may be lost in the event of changes in the cost-effective function (Limon et al., 2013; Ferramosca et al., 2014).

Based on the contributions of Souza et al. (2010) and Alamo et al. (2012), Limon et al. (2013) proposes an enhanced one-layer MPC strategy by adding a second-order approximation of the RTO cost function to the MPCT cost. Thus, the optimization problem is transformed into a convex problem, requiring only an evaluation of the gradient of the economic cost function at each sampling time. This approach has the advantage of ensuring recursive feasibility and convergence to achieve the economically optimal steady-state of the system.

The previously presented control strategies allow one to optimize the performance in case of changing economic cost functions. However, the mismatches between the real plant and the prediction model may prevent convergence of the controlled plant to the optimal operation point of the real plant. A relevant approach that emerged in Real-Time Optimization is a method called Modifier Adaptation (MA), which addresses the problem of modeling error by making corrections to the optimization problem. This is done including modifier terms in the constraints and in the cost function (or in the model), thus allowing the outputs of the model to converge to the actual process (Marchetti et al., 2009).

Vaccari and Pannocchia (2017, 2018) discuss the model error problem by proposing an offset-free EMPC controller using the modifier adaptation methodology. Along the same lines of integrating the EMPC controller with the modifier adaptation method, recently Hernández and Engell (2019) presented a variation of the work of Vaccari and Pannocchia (2017, 2018) using the modifier adaptation methodology, which uses measures of the transient to calculate the modifiers.

The controller proposed in this paper goes in the same direction. It is proposes a design of an offset-free controller based on the MPC for Tracking controller. We extend the work of Limon et al. (2013), taking advantage of its main features: stability guarantees, convergence and recursive feasibility for any change of the controller cost function, even using a linear model of the process (which facilitates the resolution of the optimization problem). The paper is organized as follows: Section 2 states the main problem, and Section 3 presents the gradient-based one-layer MPC controller. Section 4 goes over modifier adaptation methodology and Section 5 explains how can it be used in the one-layer MPC scheme. Section 6 presents a case study analyzing the results. Section 7 draws some conclusions to this work.

2. PROBLEM DEFINITION

Let the following optimization problem be the one solved by the RTO

$$\min_{x_s, u_s} \quad J(x_s, u_s, p) \tag{1a}$$

$$\text{s.t.} \quad f(x_s, u_s) = 0 \tag{1b}$$

$$h(x_s, u_s) \leq 0, \tag{1c}$$

wherein x_s and u_s stand for the state and input that define the optimal stationary state of the plant; J is the cost function which depends on the RTO parameters p (e.g. the price of raw materials or production costs); (1b) represents the static model of the plant and (1c) gatherss the operational constraints of the plant.

In a hierarchical control structure, the RTO provides to the MPC an optimal operating point (x_s, u_s), given by (1). However, if the economic criterion changes, due to variations of the parameter p, the economically optimal admissible steady state where the controller drives the system may change, and the feasibility of the controller may be lost. To face this problem, Limon et al. (2013) proposed a controller that cope with such an issue and besides integrates the RTO into the MPC control layer.

3. GRADIENT-BASED STRATEGY FOR ONE-LAYER MPCT

Assuming that the plant output vector y_s uniquely defines a plant equilibrium point, it is possible to represent the equilibrium point as a function of the plant output with $x_s = g_x(y_s)$ and $u_s = g_u(y_s)$. Overriding these functions in the RTO optimization problem (1) can be rewritten as

$$\min_{y_s} \quad f_{eco}(y_s, p) \tag{2a}$$

$$\text{s.t.} \quad h_q(y_s) \leq 0, \quad q \in \mathbb{I}_{1:n_h}, \tag{2b}$$

where n_h is the number of constraints, f_{eco} and h_q are closely related to $J(x_s, u_s, p)$, $f(x_s, u_s)$ and $h(x_s, u_s)$, but represented by function of y_s. The feasible set of this optimization problem is denoted as $\mathcal{Y}_t$.

The following assumptions hold:

Assumption 1. f_{eco} and h_q are convex functions.

Assumption 2. The gradient of f_{eco} and the gradient of h_q are Lipschitz continuous in $\mathcal{Y}_t$.

Assumption 3. The solution to the RTO optimization problem is unique.

Consider that a certain feasible equilibrium point, z is chosen to calculate an approximation of the $f_{eco}(y, p)$ function. Limon et al. (2013) suggests to use a second order Taylor approximation of the f_{eco} and $h_q(y)$ functions, that is

$$f_{eco}(y, p) \leq f_{eco}(z, p) + \nabla_y f_{eco}(z, p)^T (y - z) + \frac{\rho_f}{2} ||y - z||^2 \tag{3}$$

and

$$h_q(y) \leq h_q(z) + \nabla_y h_q(z)^T (y - z) + \frac{\pi_q}{2} ||y - z||^2, \quad q \in \mathbb{I}_{1:n_h} \tag{4}$$

for all $y \in \mathcal{Y}_t$ and $z \in \mathcal{Y}_t$, with $\mathcal{Y}_t$ being the feasible set of the optimization problem, being ρ_f and π_q Lipschitz constants.

Therefore, the approximated cost function for the proposed MPC can be defined as

$$V_N^a(x, \hat{d}, p; u, x_s, u_s) = \sum_{j=0}^{N-1} ||x_j - x_s||_Q^2 + ||u_j - u_s||_R^2 +$$
$$f_{eco}(z, p) +$$
$$\nabla_y f_{eco}(z, p)^T (y - z) + \frac{\rho_f}{2} ||y - z||^2 \quad (5)$$

with the convex set defined as

$$\mathcal{Y}_t^a(z) = \{y : \quad h_j(z) + \nabla_y h_j(z)^T (y - z)$$
$$+ \frac{\pi_j}{2} ||y - z||^2 \le 0, j \in \mathbb{I}_{1:n_h}\} \quad (6)$$

such that $\mathcal{Y}_t^a(z) \subseteq \mathcal{Y}_t$ for all $z \in \mathcal{Y}_t$.

The MPC has a prediction model given by

$$x_{k+1} = f_{mpc}(x_k, u_k) = Ax_k + Bu_k \quad (7a)$$
$$y_k = Cx_k, \quad (7b)$$

where $x \in \mathcal{R}^n$ is the system state, $u \in \mathcal{R}^m$ is the control vector and $y \in \mathcal{R}^p$ is the output variables of the plant, subject to constraints on state and input

$$(x_k, u_k) \in Z = \{z \in \mathbb{R}^{n+m} : A_z z \le b_z\}, \quad \forall k \ge 0, \quad (8)$$

where the set Z is assumed to be convex, closed and contains the origin in its interior.

Therefore, the optimization problem to be solved by the MPC $P_N^a(x, \hat{d}, p)$ is obtained by replacing the original cost function and constraints with the approximate ones, that is

$$\min_{u, x_s, u_s} \quad V_N^a(x, \hat{d}, p; u, x_s, u_s) \quad (9a)$$
$$s.t. \quad x_0 = x, \quad (9b)$$
$$x_{j+1} = Ax_j + Bu_j + \hat{d}, \quad (9c)$$
$$(x_j, u_j) \in Z, \quad j = 0, ..., N - 1, \quad (9d)$$
$$x_s = Ax_s + Bu_s + \hat{d}, \quad (9e)$$
$$y_s = Cx_s + Du_s, \quad (9f)$$
$$x_N = x_s \quad (9g)$$
$$y_s \in \mathcal{Y}_t^a(z). \quad (9h)$$

Solving this optimization problem will provide a feasible point for the MPC as close as possible to the RTO solution. Any estimated incompatibilities between the linear model and the plant can be taken into account by $\hat{d}$. Thus, the original optimization problem is transformed into a convex problem, requiring only an evaluation of the economic cost function gradient at each sampling period. The authors suggest to use the best reachable equilibrium point as a linearization point, that is, to use the value of y_s calculated at the previous instant. This approach has the advantage of ensuring recursive feasibility and convergence to achieve the economically optimal steady-state of the system (Limon et al., 2013).

However, it is known that there is a difference between the model used by the controllers and the actual plant process. If the controller had access to the actual plant model the solution would be optimal. Instead, the existence of mismatch-model causes that the solution of problem (9) presents an error (offset). This problem can be faced by using modifiers in the optimization problem strategy called Modifier Adaptation.

4. MODIFIER ADAPTATION METHODOLOGY

The modifier adaptation methodology (MA) originally used by Forbes et al. (1994), presents some variants in the literature, such as Gao and Engell (2005), Tatjeski (2002), B. Chachuat and Bonvin (2009) and Marchetti et al. (2009). The method seeks to match the necessary optimality conditions, also known as KKT (Karush-Kuhn-Tucker) conditions, of the real process and the prediction model by using modifiers in both cost and optimization problem constraints, allowing to compensate for both parametric and structural modelling errors.

Assuming that the constraints (1b)-(1c) are not active at a given operating point (x_0, u_0) and that the functions $J(x_s, u_s, p)$ and $h(x_s, u_s)$ are differentiable in (x_0, u_0), there will be a single vector of Lagrange multipliers $\lambda \in \mathbb{R}^{nh}$ whose KKT conditions are met in point (x_0, u_0), and they are

$$f(x_s, u_s) = 0 \quad (10a)$$
$$h(x_s, u_s) \le 0 \quad (10b)$$
$$\lambda_f f(x_s, u_s) = 0 \quad (10c)$$
$$\lambda_h h(x_s, u_s) = 0 \quad (10d)$$
$$\lambda_f \ge 0 \quad (10e)$$
$$\lambda_h \ge 0 \quad (10f)$$
$$\nabla L(x_s, u_s, p, \lambda) = 0, \quad (10g)$$

being $L(x_s, u_s, p, \lambda) = J(x_s, u_s, p) + \lambda_f f(x_s, u_s) + \lambda_h h(x_s, u_s)$ the Lagrangian of the optimization problem (1).

According to Marchetti et al. (2009), there is another possibility of implementing the modifier adaptation methodology by directly modifying the model, rather than the cost function. Considering that the to input and state constraints (1c) are given by

$$\overline{u} \le u \le \underline{u} \quad (11a)$$
$$\overline{x} \le x \le \underline{x}, \quad (11b)$$

being $(\overline{x}, \overline{u})$ the high limits and $(\underline{x}, \underline{u})$ the low limits, then (1) can be augmented by modifiers resulting in the following augmented optimization problem:

$$\min_{u} \quad J(x, u, p) \quad (12a)$$
$$s.t.$$
$$x = f(x, u) + \lambda'_{u,k}(u - u_k) + \lambda'_{x,k}(x - x_k) + \epsilon_k \quad (12b)$$
$$\overline{u} \le u \le \underline{u} \quad (12c)$$
$$\overline{x} \le x + \epsilon_k \le \underline{x} \quad (12d)$$

with the modifiers being

$$\lambda_{u,k} = \left.\frac{\partial f_p(x, u)}{\partial u}\right|_{u_k} - \left.\frac{\partial f(x, u)}{\partial u}\right|_{u_k} \quad (13a)$$

$$\lambda_{x,k} = \left.\frac{\partial f_p(x, u)}{\partial x}\right|_{x_k} - \left.\frac{\partial f(x, u)}{\partial x}\right|_{x_k} \quad (13b)$$

$$\epsilon_k = f_p(x_k, u_k) - f(x_k, u_k), \quad (13c)$$

where f_p refers to the plant model. Solving (12) results in the next optimal operation point (x_{k+1}, u_{k+1}).

To ensure feasibility of the optimization problem, such modifiers can be filtered with first-order filters of the form

$$\hat{\lambda}_k = [I - \beta]\lambda_{k-1} + \beta\lambda_k \quad (14)$$

with $\beta \in (0,1]$, for the previously calculated modifiers.

5. *OFFSET-FREE ONE-LAYER* MPCT-MA CONTROLLER

The MPC (9) can take into account any modelling errors between the actual plant and the model used by the controller by estimating $\hat{d}$. However, this approach requires knowing exactly the nature of the uncertainties in order to create a disturbance estimation model, which difficulties its application (Vaccari and Pannocchia, 2017). An alternative to this problem is to integrate the Modifiers in the MPC prediction model, allowing a controller design capable to reach the plant optimum, i.e. an offset-free controller.

The contribution of this paper is the reformulation of the controller presented in Section 3 to integrate the modifier adaptation technique. This will allow us to inherit the good properties of both approaches: constraint satisfaction, stability and convergence to the economically optimal operation point of the real plant under any change of the economic criterion.

Considering the MPC (9), the correction of the modelling error between the prediction model and the real plant can be done by adding modifiers to the prediction model, and the optimal real stationary operation point (x_s, u_s) can be calculated by solving the following optimization problem

$$\min_{u, x_s u_s} \quad V_N^{ma}(x, \hat{d}, p; u, x_s, u_s) \tag{15a}$$

$$s.t. \quad x_0 = x, \tag{15b}$$

$$x_{j+1} = Ax_j + Bu_j + \lambda_{u,k}^T(u_j - u_k) + \tag{15c}$$
$$\lambda_{x,k}^T(x_j - x_k) + \epsilon_k,$$

$$(x_j + \epsilon_k, u_j) \in Z, \quad j = 0, ..., N-1, \tag{15d}$$

$$x_s = Ax_s + Bu_s + \lambda_{u,k}^T(u_s - u_k) + \tag{15e}$$
$$\lambda_{x,k}^T(x_s - x_k) + \epsilon_k,$$

$$(x_s + \epsilon, u_s) \in Z, \tag{15f}$$

$$x_N = x_s \tag{15g}$$

with the pair $V_N^{ma}(x, \hat{d}, p; u, x_s, u_s)$ defined by

$$V_N^{ma} = \sum_{j=0}^{N-1} ||x_j - x_s||_Q^2 + ||u_j - u_s||_R^2 + \tag{16a}$$

$$J(x_{s,k-1}, u_{s,k-1}, p) + \tag{16b}$$

$$\nabla_x J(x_{s,k-1}, u_{s,k-1}, p)^T(x_s - x_{s,k-1}) + \tag{16c}$$

$$\nabla_u J(x_{s,k-1}, u_{s,k-1}, p)^T(u_s - u_{s,k-1}) + \tag{16d}$$

$$\frac{\rho_x}{2}||x_s - x_{s,k-1}||^2 + \tag{16e}$$

$$\frac{\rho_u}{2}||u_s - u_{s,k-1}||^2 \tag{16f}$$

with $x_{s,k-1}$ and $u_{s,k-1}$ being the stationary operation point at instant $k-1$. The modifiers $\lambda_{u,k}^T$, $\lambda_{x,k}^T$, ϵ_k from the equations (15c-15e) are given by

$$\lambda_{u,k}^T = \nabla_u f_p(x_k, u_k) - \nabla_u f(x_k, u_k) \tag{17a}$$

$$\lambda_{x,k}^T = \nabla_x f_p(x_k, u_k) - \nabla_x f(x_k, u_k) \tag{17b}$$

$$\epsilon_k = f_p(x_k, u_k) - f(x_k, u_k). \tag{17c}$$

The operators $\nabla_u f$ and $\nabla_x f$ are Jacobians with respect to u and x, calculated at equilibrium point (x_k, u_k).

5.1 One-Layer Economic MPC Properties

The controller (15) maintains the same structure as an MPC for tracking controller (Limon et al., 2008), inheriting its main properties such as stability, feasibility and convergence guarantee. Moreover, it ensures convergence to the optimum real plant due to MA compensation, resulting in an offset-free controller. These properties are detailed next:

(1) *Recursive Feasibility*: since the One-Layer Economic MPC uses the so-called artificial variables (optimization variables that represents the best admissible equilibrium point for the MPC model, inheritance of the MPCT formulation (Limon et al., 2013)), and a relaxed terminal equality constraint in such a way that the terminal predicted state is forced to be any equilibrium point (not the setpoint), the controller ensures recursive feasibility, even in case of changes in the economic cost function J, which implies that it is unnecessary to recalculate the controller in case of a change of the economic objective.

(2) *Simple implementation*: thanks to the approximations (3)-(4) the optimization problem (15) results in a Quadratic Programming.

(3) *Asymptotic Stability*: It is possible to demonstrate, following the same arguments used by Limon et al. (2013), that the proposed controller guarantees asymptotic stability of the closed loop

(4) *Offset-Free*: By using the modifier adaptation strategy (Marchetti et al., 2009), the proposed controller eliminates the mismatch between the model used for predictions and the actual plant, so that the solution found by the controller always coincides with the optimal operation point of the real plant.

6. CASE STUDY

To illustrate how the proposal works, the so-called four-tank process (Johansson, 2000) is used as case study. The system consists of four tanks, two upper tanks and two lower tanks, and a reservoir located under these lower ones. The upper tanks have free flow through a hole for the lower tanks, as shown in Figure 1. Water is pumped from the reservoir by two centrifugal pumps and, through two three-way valves, water is directed to each of the tanks. The manipulated variables are the pump flow rates (q) and the states (h) are the tank levels. The system has output consisting of two states $y = (h1, h2)$ and input by $q = (q1, q2)$.

The following One-Layer Economic MPC used is

$$J = ||y_s - y_{sp}||^2 \tag{18}$$

leading the system to an optimal setpoint y_{sp}.

To demonstrate the existence of the so-called *offset* resulting from a modeling error, Figures (2) and (3) show the result of the controller *One-Layer* MPCT with modeling error (9). For this controller a prediction horizon $N = 5$ was used and weighting matrices $Q = I$, $R = 0.01 \times I$. In this test the following references were used for the One-Layer MPCT controller: $y_{sp1} = (1.5, 1.5)$, $y_{sp2} = (1.6, 1.4)$,
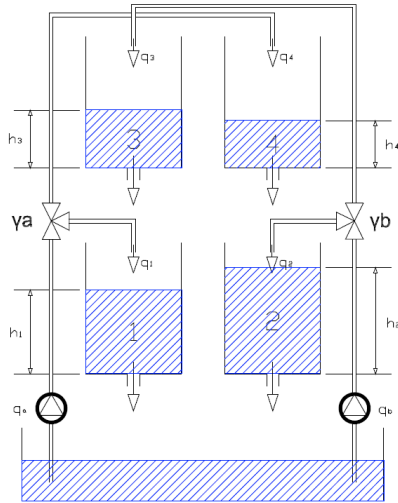
Fig. 1. Four-tank process.

$y_{sp3} = (1.7, 1.8)$, $y_{sp4} = (1.0, 1.0)$ e $y_0 = (h_1^0, h_2^0)$, where y_0 is the starting point of the system.

Note, in Figures 2 and 3, that the system does not converge to the references, remaining an offset due to the inconsistency between model and plant. Figures 4 and
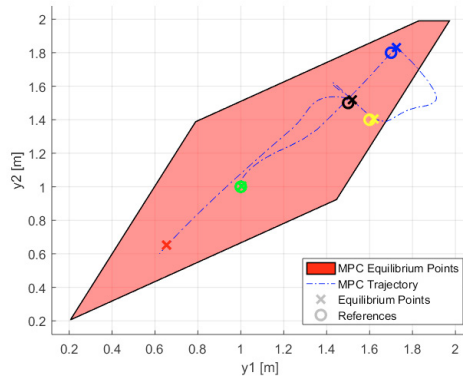

Fig. 2. Output space plot - system controlled by One-Layer MPCT with modeling error: real nonlinear plant and linearized model for the controller.

5 show the result of the one-layer MPCT-MA controller using MA (15), where there is a clear improvement with respect to the previous simulation. It can be seen that the controller has offset-free error, that is, the controller corrects the modelling error between plant and model through modifiers.

As can be seen from the equation (15c), model modification is accomplished by linear compensation made by the addition of terms composed by modifiers in the original model. One can see in Figures 6 and 7 the evolution of the terms $\lambda_{u,k}^T(u - \bar{u}_k)$ and ϵ in each sampling period. Note that: a) Figure 7 shows that first-order modifiers (ϵ) are disturbed with each reference change, but always converge over time; b) in Figure 6 the second-order modifiers ($\lambda_{u,k}^T$), which make up the plots given by $\lambda_{u,k}^T(u - \bar{u}_k)$, complement the value of the ϵ modifier only during the transient,
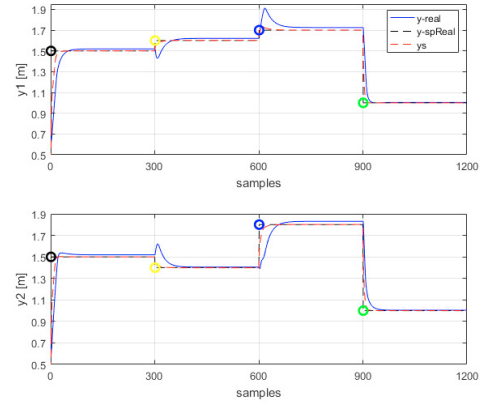

Fig. 3. RTO optimal values and system output - system controlled by One-Layer MPCT with modelling error: real nonlinear plant and linearized model for the controller.
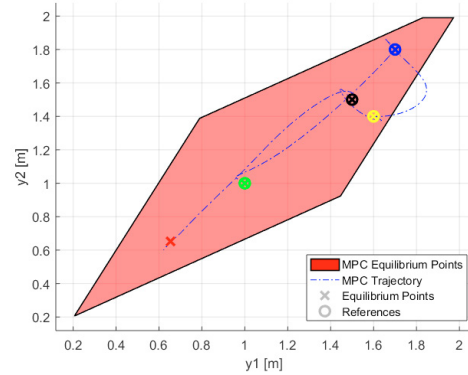

Fig. 4. Output space plot - system controlled by One-Layer Economic MPC: controller has no offset for the modelling error system (real nonlinear plant and linearized model for the controller).
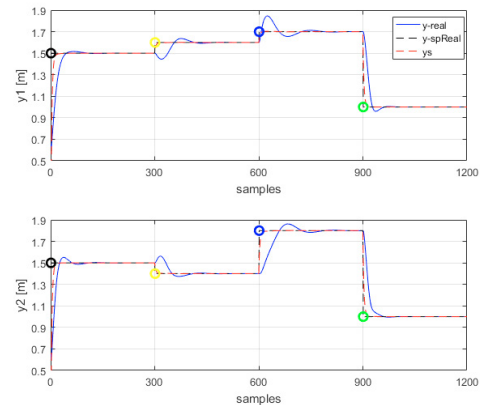

Fig. 5. RTO optimal values and system output - system controlled by One-Layer Economic MPC: controller has no offset for the modelling error system (real nonlinear plant and linearized model for the controller)

tending to zero whenever ϵ converges. The sum of the values of these two plots equals the error between the model and the plant at each time k. Therefore, the One-

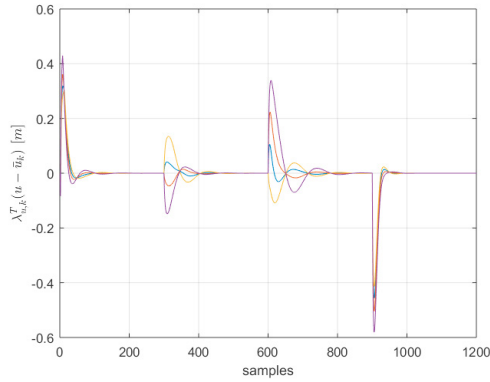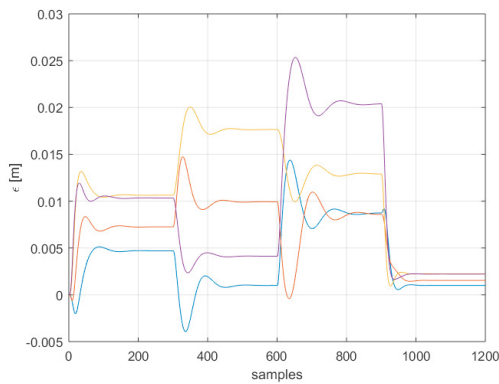Fig. 6. Compensation value given by $\lambda_{u,k}^T(u - \bar{u}_k)$

Fig. 7. Compensation value given by ϵ_k

Layer Economic MPC controller is an offset-free controller with all the advantages of the so-called MPC for tracking such as stability, recursive feasibility, and convergence, including in the case of changes in the economic cost function.

7. CONCLUSIONS

The controller presented here combines the advantages and characteristics of MPC for Tracking with the possibility of correcting, on line, any modelling errors through the inclusion of modifiers (MA) in the optimization problem. The union of MPCT with MA resulted in a controller with guarantee of feasibility and stability and zero offset, that is, with guarantee of convergence to the economical optimum of the real plant for any case of change in the economic cost function. Also, it is noteworthy that the calculation of the economic optimal point of the plant is made by the predictive controller making the solution of the RTO problem unnecessary.

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