Nonlinear Model of a Servo-Hydraulic Shaking Table with Dynamic Model of Effective Bulk Modulus

Paolo Righettini[a], Roberto Strada[a], Shirin Valilou[a,*], Ehsan KhademOlama[a]

[a] *Università degli studi di Bergamo, Department of Engineering and Applied Sciences, Mechatronics Laboratory*

Abstract

In this paper, based on fluid mechanical expressions and a new modified effective bulk modulus model of hydraulic oil built upon IFAS model (developed at the Institute für Fluidtechnische Antriebe und Steuerungen, RWTH Aachen university), an empirical nonlinear model for a servo-hydraulic uni-axial shaking table is developed. This new model can precisely simulate the acceleration, velocity and position outputs of the system with respect to different kinds of inputs such as pulse and sinusoidal signals for a wide range of frequencies and different weights of the specimen. Therefore, it can be helpful for designing and optimizing the parameters of a model-based controller for tracking reference force or acceleration signals, which is the goal of the shaking table with only position sensor. In the new modified IFAS model, the effective bulk modulus of hydraulic oil on both sides of the piston has been considered as two nonlinear springs, which are connected serially. The minimum stiffness of the spring effect of the hydraulic oil in a symmetric double-acting hydraulic cylinder occurs, when the piston is in the center of its travel, which can be characterized with differential pressure on its both sides. When the differential pressure is less than a specific threshold pressure, these springs have the minimum stiffness and reserve energy in themselves. Based on the experimental observations, this effect has been modeled with a function, which multiplies the IFAS model. The experimental acceleration output of the system demonstrates the dynamic behaviors of the effective bulk modulus of

*Corresponding author.

Email addresses: paolo.righettini@unibg.it (Paolo Righettini), roberto.strada@unibg.it (Roberto Strada), shirin.valilou@unibg.it (Shirin Valilou), ehsan.khademolama@unibg.it (Ehsan KhademOlama)

Preprint submitted to Journal of Mechanical Systems and Signal Processing

hydraulic oil, which occurs in the center of the piston travel. The parameters of the simulated model are estimated with the nonlinear least square method in MATLAB. Finally, the accuracy of the proposed model for simulating the motion states of the shaking table, by comparing the experimental and simulated results in different ranges of amplitudes and frequencies with respect to the new and the previous model of the hydraulic servo-systems have been shown.

Keywords:
Shaking table, Servo-hydraulic actuator, Acceleration nonlinear model, Effective bulk modulus, IFAS model.

---

1. Introduction

Shaking table is an important experimental device for simulating dynamic response of complex mechanical systems to high natural dynamical forces. It has been widely used in many industrial applications such as aerospace, automotive and civil engineering. In recent years, its usage stands out in civil engineering for analyzing and simulating dynamic response of high rise buildings and structures to earthquakes and wind storms [1, 2, 3]. Due to requirement of large forces and displacements in the shaking tables, they are mostly driven by servo-hydraulic actuators.

Hydraulic actuators in comparison with other driving powers have many advantages, including high force to weight ratio, high durability and fast responses [4, 5, 6, 7]. In spite of these advantages, the dynamic features of these systems are highly nonlinear and designing a high accuracy tracking control for them is a difficult problem. Most nonlinearities of these systems arise from compressibility of the hydraulic fluid, the complex flow properties of the servo-valve, valve overlap and friction in the hydraulic cylinder [7]. Aside from the nonlinear nature of the hydraulic dynamics, there are many considerable model uncertainties, such as internal and external leakages and external disturbances, which cannot be modeled exactly. Therefore, in order to design a high performance model-based controller for simulating the acceleration behavior that is the goal of a servo-hydraulic shaking table, a suitable dynamical model of the system needs to be formulated.

Many researchers use the dynamic model of hydraulic actuator based on fluid mechanical expressions in [4, 5, 6, 7] to model the nonlinear behavior of the system (see for instance [8, 9, 10, 11, 12]). The results in these papers show that even with considering different nonlinear friction models, the simulated velocity for small ranges of amplitude and frequency is fairly accurate and so is the simulated acceleration. Additionally, in [4, 13, 14] have been shown that the acceleration output of the system to the sinusoidal input is distorted and it contains harmonics with the fundamental frequency and its integer multiplications. In [15, 16, 17], for simulating the acceleration response, a function consisting of sinusoidals with known frequencies have been considered. Then, the amplitudes and phases of each function are calculated based on the least mean square method [15], the Kalman filter [16] and the unscented Kalman filter [17]. However, the input frequency of the system in all these methods is assumed to be known and the acceleration sensor data are always needed for updating the identification parameters. Furthermore, as stated in these papers, it takes $0.5[s]$ for the parameters to converge to the actual amplitude and phase of harmonics which is suitable for the control process up to $1[Hz]$. Therefore, these methods are not suitable for the shaking table which has operating frequency range $(0-15)[Hz]$. In [18], the acceleration of the system based on feed-forward neural network method, has been simulated. This method needs high computation resources and it has $2[s]$ convergence window at the first run which in some cases, it causes instability in control feedback loop. Thus, in order to model the acceleration behavior to any kind of inputs precisely, it is needed to model the main nonlinear features of the system such as effective bulk modulus (E-Modulus) and friction, as accurately as possible.

In the hydraulic systems, the spring effect of a hydraulic oil is characterized by the value for the bulk modulus. It is a fundamental and inherent property of liquids, which indicates the stiffness of the system and the speed of transmission of pressure waves. Therefore, system performance with respect to positioning, power loss, response time and stability of hydraulic servo-systems is affected by the value of bulk modulus. Several researches have been done on the topic of the bulk modulus without considering the effect of entrained air [19, 20, 21]. However, the real bulk modulus with considering the effect of entrained air, temperature and mechanical compliance in the hydraulic systems is presented by the value for the E-Modulus. In recent years, several theoretical models have been proposed to simulate the dependency of the E-Modulus upon pressure and entrained air content [22, 23, 24, 25, 26]. In [27, 28], based on the experimental verification of these models, have been shown that the IFAS model in [24] can simulate the behavior of the E-Modulus with higher accuracy. However, as shown in these papers, the minimum time for maximizing the input pressure are $2.5[s]$, which in our study is almost the steady state case.

The other main nonlinearity of the hydraulic systems is friction in the hydraulic cylinder. Many researchers employ an explicit dynamic friction in order to model nonlinear effects of friction which some of them have been commonly used to express the friction of hydraulic cylinder [29, 30, 31, 32, 33]. The steady state friction in [29], is the combination of Coulomb friction, viscous friction and static friction. The dynamic LuGre friction [30], which is implemented in the AMESim (Advanced Modeling Environment for performing Simulation of engineering systems) software [34, 35, 36], defines dynamic features of friction such as pre-sliding displacement, lag, varying break-away force and stick-slip. But this model cannot describe precisely the dynamic behaviors of friction in the sliding regime. Therefore, in [31, 32, 33] has been proposed a modified LuGre model which can simulate the dynamic behaviors of friction in the sliding regime. However, in [10] has been shown that based on the experimental results, the simulated velocity with considering modified LuGre model is fairly correct. In addition, in [12] and [33] have been pointed out that these models are valid with the assumption of the frequency up to $2[Hz]$ and the velocity under $0.15[m/s]$, that is a limit in real practical shaking tables. Aside from these problems, obtaining real values for some parameters in LuGre model are very difficult and some experiments and sensors are needed which can be expensive and time consuming [33].

In this paper, a new empirical nonlinear model for simulating the acceleration, velocity and position behavior of a servo-hydraulic shaking table in dynamic and steady state flow situations is proposed. Based on the experimental observations, this model is attained by modifying the IFAS model for simulating the E-Modulus of the hydraulic oil. The bulk modulus of hydraulic oil inside two chambers behaves like a spring. In a symmetric double-acting hydraulic cylinder which is used for driving the shaking table, the minimum stiffness of the spring effect of the hydraulic oil occurs when the piston is in the center of its travel [5]. In this new model, the nonlinear spring effect of the hydraulic oil in two interactive chambers, which are connected serially, is modeled based on differential pressure of them. When the differential pressure on both sides is less than a threshold pressure, these springs have the minimum stiffness and reserve energy in themselves. The new modified IFAS model has demonstrated this effect by a function built upon differential pressure, which multiplies the IFAS model. The experimental acceleration output of the system implies the dynamic behaviors of the E-Modulus of hydraulic oil, which occurs in the center of the piston travel. Here, for simplicity and the limitation of the LuGre model, the steady state friction model is used for simulating the friction in the hydraulic cylinder. The parameters of the model are estimated based on Nonlinear Least Square Method (NLSM) in MATLAB. Finally, the comparisons of experimental results with simulated ones show that the model of the servo-hydraulic actuator with considering the modified IFAS can simulate accurately the behavior of the shaking table with respect to different kinds of inputs such as pulse and sinusoidal signals for wide range of frequencies and different weights of the specimen. This achievement would be helpful for following reasons:

1. Since the proposed model can predict precisely the behavior of the system, the designed controller based on this model can enhance the performances of the model-based force controller for tracking reference signal.
2. It reduces the cost of the hydraulic servo-systems with eliminating the acceleration and internal pressure sensors. In all the previous works, using the pressure or force sensors for controlling the force or acceleration of the servo-hydraulic actuators are necessary. While, with considering this new model the pressure and force output of the system can be estimated with high accuracy.
3. It is not always possible to measure the full state of the servo-hydraulic system due to the hardware limitations. Therefore, it is suitable for constructed old industrial systems (such as the paper case study) in which to insert pressure sensor without changing the structure of the system is impossible.
4. The computational simulation developed by model could serve as a powerful result-interpretation tool and parameter optimization of the designed controller with respect to any parameter identification.

This paper is organized as follows. In Section 2, a detailed description of experimental servo-hydraulic shaking table is outlined. In section 3, the model of hydraulic actuator with considering new model for the E-Modulus of hydraulic oil and simple friction in hydraulic cylinder is developed. Then, the comparison between simulation and experimental results are presented in section 4 and finally, the conclusions are drawn in Section 5.

2. System description

In this section, hardware of the experimental shaking table has been presented. A $3D$ model of the system is shown in figure 1. This figure points out the elements composing a single degree-of-freedom (DOF) shaking table with operation range of $(0-15)[Hz]$, driven by a servo-hydraulic actuator: (A) moving platform for specimens housing ($1200[mm] \times 2000[mm]$) (the platform can be considered rigid because as shown in the experimental section its first mode with components in the direction of the motion is Mode 3, characterized by a frequency of $296[Hz]$, which is very far from the maximum working frequency of the system ($15[Hz]$).); (B) rail

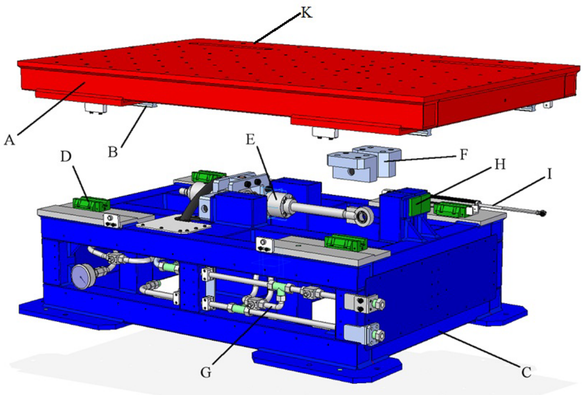

Figure 1: 3D model of the uni-axial servo-hydraulic shaking table

guides for coupling with the fixed base (C) by using linear ball type guide-ways (D); (E) hydraulic actuator for moving the mobile base attached to it, by using the connection (F); (G) hydraulic system for the actuator supply; (H) mechanical end-stroke; (I) displacement potentiometer transducer; (K) accelerometer transducer.

The main parts of the servo-hydraulic actuator are depicted in figure 2. The electro-hydraulic actuator, is a symmetric double rode type. This actuator is a Bosch-Rexroth double rod cylinder CGH2 series with a 260[mm] stroke, 50[mm] bore diameter and a 36[mm] rod. The other part of the hydraulic actuator is the four-way three-position proportional directional valve (Bosch-Rexroth 4WS.2E series) which has an open center configuration with 5% underlap and is characterized by a 20 [l/min] nominal flow and by a bandwidth of about 150[Hz].

The position and acceleration measurements of the system are carried out by a linear potentiometer transducer (PC-M-300) and an accelerometer (ADXL05 EM-EM-3) separately. The Ethernet sends acquired data that needs to be displayed or saved on the host computer. The signal conditioner contains analog-to-digital (AD), digital-to-analog (DA), and digital I/O conditioners for the analog transducer, servo-valve control and digital I/O signals, respectively.

In the next section, the model of the hydraulic actuator based on fluid mechanical expressions is developed.

3. Hydraulic actuator model

A hydraulic servo-system is composed of separate components, interconnected to provide a desired form of hydraulic transfer (figure 2). In this section, in or-

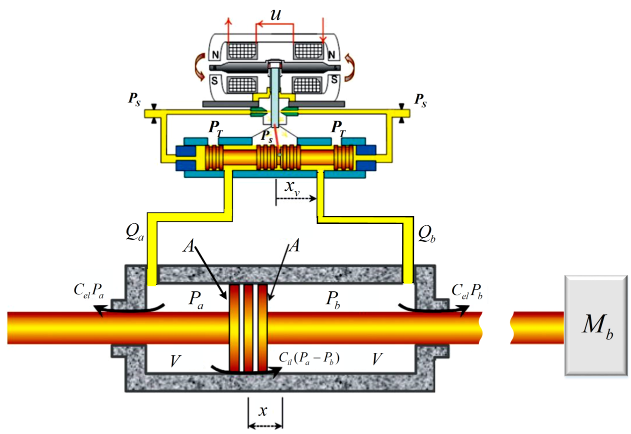

Figure 2: Servo-hydraulic actuator representative.

der to obtain structural insight into the system with respect to relevant dynamics as well as relevant nonlinearities, a mathematical model of the main features of each component based on basic physical laws such as Newton's laws and continuity equations is constructed. Then, for simulating the dynamic behavior of the acceleration, a new nonlinear model for E-Modulus of hydraulic oil is developed.

3.1. Servo-valve

A servo-valve is a component which regulates the rate of hydraulic oil flow from the supply circuit to the hydraulic cylinder. Flow rate through valve orifices can be expressed as the orifice equation with a linear relationship between the valve spool position and the flow area. Thus, the flow in and out of the cylinder are given by

$$Q_a = R\Big(sgt(x_v, U)sign(P_s - P_a)\sqrt{|P_s - P_a|} - sgt(-x_v, U)sign(P_a - P_T)\sqrt{|P_a - P_T|}\Big) \tag{1}$$

$$Q_b = R\Big(sgt(-x_v, U)sign(P_s - P_b)\sqrt{|P_s - P_b|} - sgt(x_v, U)sign(P_b - P_T)\sqrt{|P_b - P_T|}\Big) \tag{2}$$

where the function $sgt(x_v, U)$ is defined as

$$sgt(x_v, U) = \begin{cases} 0 & x_v \le -U \\ x_v + U & -U < x_v \end{cases} \tag{3}$$

and the signum function $sign(\cdot)$ is defined as

$$sign(z) = \begin{cases} -1 & z < 0 \\ 0 & z = 0 \\ 1 & z > 0 \end{cases} \tag{4}$$

P_a and P_b are the pressures in the two chambers of hydraulic cylinder, P_s is the supply pressure, P_T is the reservoir pressure, R is the flow coefficient which is determined experimentally, or calculated using the catalog data of the valve manufacturer, U is normalized spool underlap region and x_v is normalized valve spool position. In the ideal case, the valve dynamics can be neglected and x_v can be the control command of the system. However, in practice, x_v is the response of the valve to an input signal. Thus, based on the valve catalog the relation between the servo-valve spool position x_v and the input voltage u can be approximated as a second order system as follows:

$$x_v = \frac{w_0^2}{s^2 + 2\zeta w_0 s + w_0^2} u$$

$$w_0{}^2 u = \ddot{x}_v + 2\zeta w_0 \dot{x}_v + w_0{}^2 x_v \tag{5}$$

where w_0 is the undamped natural frequency and ζ is the damping ratio of the system.

3.2. Cylinder

Cylinder is one of the components of hydraulic systems which converts hydraulic power into linear mechanical force or motion. It consists of two oil chambers separated by a piston. The resulting oil flows Q_a and Q_b moving into and out of the chambers drive the piston, thereby generating the required pressures P_a and P_b separately, to move the load of the actuator. Therefore, the behavior of the cylinder can be described by piston and pressure dynamics.

The dynamics of the pressure in each chamber of cylinder are defined based on the continuity flow equations for both chambers. Thus, for the hydraulic cylinder under study which is a symmetric double rod cylinder, the dynamic pressures P_a and P_b with respect to the flow rates Q_a and Q_b are defined as:

$$\dot{P}_a = \frac{E_a}{(\frac{V_t}{2} + Ax)} (-Q_{ea} - Q_{ia} - A\dot{x} + Q_a) \tag{6}$$

$$\dot{P}_b = \frac{E_b}{(\frac{V_t}{2} - Ax)} (-Q_{eb} - Q_{ib} + A\dot{x} + Q_b) \tag{7}$$

where V_t is the total volume of the cylinder, x and $\dot{x}$ are the position and velocity of the cylinder piston, respectively. A is the piston area, E_a and E_b are the E-Modulus of the hydraulic oil in chamber a and b which they will be modeled in the next section, Q_{ia} and Q_{ib} denote the internal leakage flow and Q_{ea} and Q_{eb} denote the external leakage flow in chamber a and b respectively. The equation of the internal and the external leakage flow for both chambers are given by:

$$Q_{ea} + Q_{ia} = C_{il}\left(|P_a - P_b|\right) + C_{el}\left(P_a\right) \tag{8}$$

$$Q_{eb} + Q_{ib} = C_{il}\left(|P_a - P_b|\right) + C_{el}\left(P_b\right) \tag{9}$$

where C_{el} and C_{il} are the coefficient of the external and internal leakage flow, respectively.

The other dynamic of the hydraulic cylinder is the dynamic of the piston which describes the forces acting on the piston itself. Based on the Newton's second law of motion, since on the shaking table there are no other external actions besides friction force, it is expressed as:

$$M\ddot{x} = (P_a - P_b)A - F_{fric} \tag{10}$$

where $\ddot{x}$ is the acceleration of the cylinder piston, $M = M_b + M_s$ is total mass moved by actuator (mobile base mass M_b plus the specimen mass M_s) and F_{fric} denotes the friction force in the hydraulic cylinder. The steady state friction model of F_{fric} is defined as:

$$F_{fric} = (F_C + (F_s - F_C)e^{-(\frac{\dot{x}}{v_s})^2})sign(\dot{x}) + F_v\dot{x} \tag{11}$$

where F_C is coulomb friction force, F_s is static friction force, F_v is the viscous friction coefficient and v_s is the Stribeck velocity. In this model, for avoiding discontinuity of the $sign(\dot{x})$ function, it can be replaced by the $tanh(\beta\dot{x})$. This function has the property that, as β approaches infinity, it essentially becomes the *sign* function:

$$\lim_{\beta\to\infty} \tanh(\beta\dot{x}) = sign(\dot{x}). \tag{12}$$

3.3. E-Modulus of hydraulic oil

The E-Modulus is a physical characteristic of the hydraulic oils which mainly affects dynamic characteristics of the hydraulic systems. Mathematically, it defines the reciprocal of volume compressibility. Several researchers have been studied the nonlinear behavior of the bulk modulus with considering the effect of pressure, temperature and entertained air in a single-acting hydraulic cylinder. Among these studies, based on the experimental verification in [27, 28], the IFAS model can simulate the behavior of the E-Modulus accurately. The formula of the IFAS E-Modulus is expressed as:

$$E = \frac{(1-\alpha)\left(1+\frac{m(P-P_0)}{E_0}\right)^{-\frac{1}{m}} + \alpha\left(\frac{P_0}{P}\right)^{\frac{1}{\kappa}}}{\frac{1}{E_0}(1-\alpha)\left(1+\frac{m(P-P_0)}{E_0}\right)^{-\frac{m+1}{m}} + \frac{\alpha}{\kappa P_0}\left(\frac{P_0}{P}\right)^{\frac{\kappa+1}{\kappa}}} \tag{13}$$

where α is the volumetric content of entrained air at initial pressure (entrained air content), P_0 is the initial pressure, P is the absolute pressure, κ is the polytropic constant of air, E_0 is the constant term and m is the pressure related term in the bulk modulus of oil. However in a double-acting cylinder, there is oil on both sides of the piston. In [5], Akers et al. have proved the minimum stiffness value of hydraulic oil in a general condition which the two sides of the cylinder are not matched for piston area, has been occurred when

$$\frac{A_a^2}{V_a} = \frac{A_b^2}{V_b} \tag{14}$$

where V_a and V_b are volumes of trapped oil in two chambers. A_a and A_b are the respective piston areas. This condition for the shaking table, which drives with a symmetric double rod hydraulic actuator, occurs when the piston is in the center of its travel. Thus, based on this result and the experimental observation, a new modified IFAS model is proposed for simulating the E-Modulus of the hydraulic oil in the center of piston travel. In this model, the effective bulk modulus of hydraulic oil on both sides of the piston has been considered as two nonlinear springs, which are connected serially. The minimum stiffness of the spring effect of hydraulic oil in a symmetric double-acting hydraulic cylinder occurs when the piston is in the center of its travel, which characterizes with differential pressure on its both sides. When the differential pressure is less than a threshold pressure, these springs have the minimum stiffness and reserve energy in themselves. In the new model, this effect is simulated with a function based on the load pressure that multiplies the IFAS model. The modified IFAS E-Modulus of two chambers are defined as:

$$E_a = E_{ai}h(P_a, P_b) \tag{15}$$

$$E_b = E_{bi}h(P_a, P_b) \tag{16}$$

where

$$E_{ai} = \frac{(1-\alpha)\left(1+\frac{m(P_a-P_0)}{E_0}\right)^{-\frac{1}{m}} + \alpha\left(\frac{P_0}{P_a}\right)^{\frac{1}{\kappa}}}{\frac{1}{E_0}(1-\alpha)\left(1+\frac{m(P_a-P_0)}{E_0}\right)^{-\frac{m+1}{m}} + \frac{\alpha}{\kappa P_0}\left(\frac{P_0}{P_a}\right)^{\frac{\kappa+1}{\kappa}}} \tag{17}$$

$$E_{bi} = \frac{(1-\alpha)\left(1+\frac{m(P_b-P_0)}{E_0}\right)^{-\frac{1}{m}} + \alpha\left(\frac{P_0}{P_b}\right)^{\frac{1}{\kappa}}}{\frac{1}{E_0}(1-\alpha)\left(1+\frac{m(P_b-P_0)}{E_0}\right)^{-\frac{m+1}{m}} + \frac{\alpha}{\kappa P_0}\left(\frac{P_0}{P_b}\right)^{\frac{\kappa+1}{\kappa}}} \tag{18}$$

and

$$\begin{aligned} h(P_a, P_b) = \lambda_1 - \lambda_2\Big(&\tanh(\mu(P_a - P_b + P_{Lt})) \\ &-\tanh(\mu(P_a - P_b - P_{Lt}))\Big). \end{aligned} \tag{19}$$

In this equation λ_1, λ_2 and μ are constant coefficients and P_{Lt} is a threshold pressure. When the differential pressure in two sides of the piston is less than this value, the compressibility of the hydraulic oil inside two chambers are minimum. This spring effect of hydraulic oil is modeled with equation (19).

In the next section, the model of the system by considering the modified IFAS model and the IFAS E-Modulus is simulated.

4. Simulation and experimental result

In this section, the model of the hydraulic system with considering the steady state friction and the modified IFAS model has been validated experimentally. The

Figure 3: Physical shaking table

shaking table that has been used for the experiments, is located at the Department of Engineering and Applied Sciences of the University of Bergamo (figure 3).

During the design phase of the shaking table, both the fixed base and the mobile base of the system are analyzed using the FEM code ([3, 12]). As shown in figure 4, the first mode with components in the direction of the motion is Mode 3, characterized by a frequency of $296[Hz]$, which is very far from the maximum working frequency of the system ($15[Hz]$).

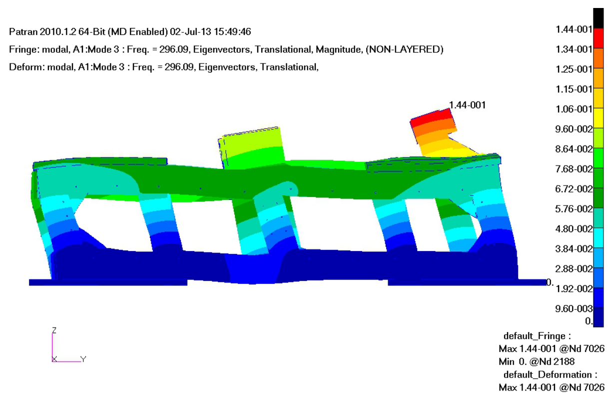

Figure 4: Fixed base modal analysis: Mode 3, 296 [Hz].

The nonlinear model of the system has been developed in the previous section. In this model, the parameters of the actuator which are known from equipment specifications, are listed in the table 1 and the others (the parameters of the friction and the modified IFAS model) have to be experimentally studied and estimated.

The estimation of the system is carried out by NLSM in MATLAB. NLSM

Table 1: Values of the shaking table parameters

Parameters	Value	SI unit
A	9.4562×10^{-4}	$[m^2]$
M_b	650	$[kg]$
U	0.005	$[-]$
$R = \frac{C_d w}{\sqrt{\rho}}$	1.28×10^{-7}	$[\frac{m}{\sqrt{kg/m^3}}]$
P_s	206	$[bar]$
w_0	625	$[\frac{rad}{s}]$
ζ	0.73	$[-]$

is the form of least squares analysis used to fit a set of m observations with a model that is nonlinear in n unknown parameters ($m > n$). It is used in some forms of nonlinear regression. The basis of the method is to approximate the model by a linear one and to refine the parameters by successive iterations. There are many similarities to linear least squares, but also some significant differences. Consider a set of m data points, $(x_1, y_1), (x_2, y_2), \ldots, (x_m, y_m)$ where the data points are defined as:

$$\begin{aligned} x_i &= inputs\ of\ the\ system, \quad i = 1, 2, ..., m \\ y_i &= outputs\ of\ the\ system, \quad i = 1, 2, ..., m \end{aligned} \tag{20}$$

a curve (model function) $y = f(x, \boldsymbol{\beta})$, that in addition to the variable x also depends on n unknown parameters, $\boldsymbol{\beta} = (\beta_1, \beta_2, \ldots, \beta_n)$, with $m \geq n$. It is desired to find the vector $\boldsymbol{\beta}$ of parameters such that the curve fits best the given data in the least squares sense. Then, the cost function of the estimation which have to be minimized is defined as:

$$S = \textstyle\sum_{i=1}^{m} r_i^2 \tag{21}$$

where the residuals (errors) r_i are given by $r_i = y_i - f(x_i, \boldsymbol{\beta})$ for $i = 1, 2, \ldots, m$. The minimum value of S occurs when the gradient is zero ([37]).

In order to estimate the unknown parameters, the data acquisition has been performed in open loop configuration by measuring position and acceleration signals. Here, due to the open loop data gathering procedure, all the acquired position data of the system have drift which are not suitable for estimation process. Therefore, velocity, $\dot{x}$, of the system has been calculated by an approximate differentiation of the measured system position. The noise in the calculated velocity signal has been filtered through a butter-worth low-pass filter with order 8 and a bandwidth of $40[Hz]$. In order to estimate the unknown parameters of the system based on NLSM, the input and output signals to the estimation algorithm is considered as:

$$\begin{aligned} x_i &= input\ voltage\ of\ the\ valve = u \\ y_i &= \begin{bmatrix} Velocity \\ Acceleration \end{bmatrix}. \end{aligned} \tag{22}$$

The unknown parameters of the system by considering the modified IFAS and the IFAS E-Modulus model have been estimated and listed in Table 2.

Comparison between the experimental and simulation results are shown in figures 5-11. First, with considering the specimen mass $M_s = 200[Kg]$, the comparisons are done for different sinusoidal frequencies 2, 8, $14[Hz]$. In figure 5,

Table 2: Estimated parameters of the system with considering the modified IFAS and the IFAS E-Modulus model

Parameters	Value (Modified IFAS)	Value (IFAS)
C_{il}	4.343×10^{-13}	4.343×10^{-13}
C_{el}	4.05×10^{-14}	4.05×10^{-14}
F_c	573.72	531.85
F_s	1254.16	952.6
F_v	429.4	153.4
v_s	0.014	0.08
β	7.33	5.8
E_0	4.73×10^8	4.7×10^8
P_0	36.85	8.7
m	9.02	7.88
α	7.9×10^{-7}	6.8×10^{-7}
κ	0.49	0.7224
λ_1	1.08	
λ_1	0.64	
μ	8.80×10^{-6}	
P_{Lt}	9.177×10^5	

the experimental and the simulated velocity and acceleration signals by considering the modified IFAS model in equations (15), (16) and the IFAS E-Modulus in response to the input voltage $u = 0.3\sin(2\pi \times 2t)$, are shown. As shown in this figure, the new model can accurately simulate the experimental acceleration and velocity. However, the model with IFAS E-Modulus cannot simulate the acceleration behavior of the system. The corresponding modified IFAS E-Modulus of hydraulic oil and the friction of hydraulic cylinder are shown in figure 6.

In order to verify the model at frequencies 8 and 14[Hz], the simulation and experimental results are shown for the input voltage $u = 0.3\sin(2\pi \times 8t)$ and $u = 0.9\sin(2\pi \times 14t)$ in figures 7 and 8, respectively. As shown in these figures, also in high frequencies the new model can precisely simulate the velocity and acceleration of the experimental system. Whereas, the model with considering the IFAS E-Modulus cannot simulate the experimental acceleration in frequency 8[Hz]. In frequency 14[Hz], as indicated in figure 8, the model of the system with considering the modified IFAS model and the IFAS E-Modulus can simulate the experimental velocity and acceleration signals. However, the values of mean squared error (MSE) in equation (21) which are listed in table 3 demonstrate that

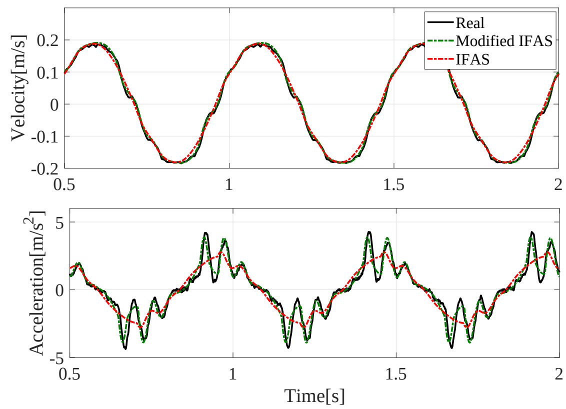

Figure 5: Simulated and experimental velocity and acceleration signals with considering the modified IFAS and the IFAS E-Modulus model, the specimen mass $M_s = 200[Kg]$, and the input voltage of the valve $u = 0.3\sin(2\pi \times 2t)$.

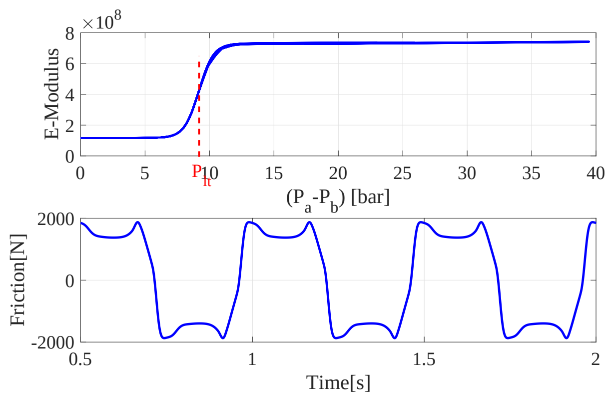

Figure 6: Top: the modified IFAS E-Modulus in chamber a with respect to $(P_a - P_b)$. bottom: the steady state friction of hydraulic cylinder in equation (11).

the error between experimental results and the simulated ones with considering modified IFAS model has less error than the IFAS E-Modulus.

Next, with changing the specimen mass of the table to $M_s = 500[kg]$, the experimental and simulation results are shown in figures 9 and 10. In figure 9, the

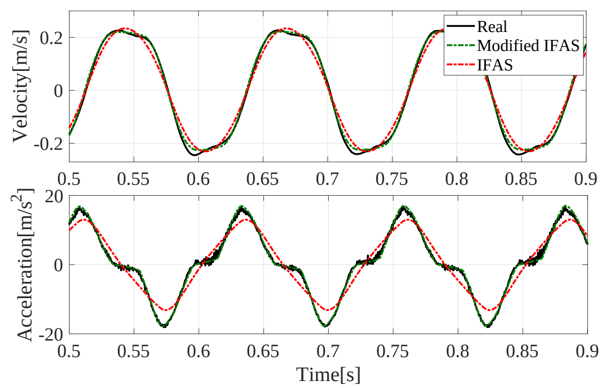

Figure 7: Simulated and experimental velocity and acceleration signals with considering the modified IFAS and the IFAS E-Modulus model, the specimen mass $M_s = 200[Kg]$ and the input voltage of the valve $u = 0.3\sin(2\pi \times 8t)$.

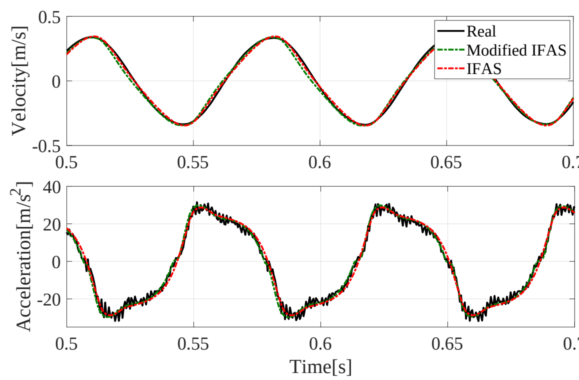

Figure 8: Simulated and experimental velocity and acceleration signals with considering the modified IFAS and the IFAS E-Modulus model, the specimen mass $M_s = 200[Kg]$ and the input voltage of the valve $u = 0.9\sin(2\pi \times 14t)$.

simulation and experimental results with considering the input voltage as:

$$u = \begin{cases} 0.1\sin(2\pi \times 2t), & 0 < t < 4.35s \\ 0.2\sin(2\pi \times 2t), & 4.35 < t < 6s \end{cases} \tag{23}$$

are shown. This figure displays that, even when the input voltage is in transition between two sinusoidal signals with different amplitudes, the new model can simulate the transient time behavior precisely. In contrary, the system model with

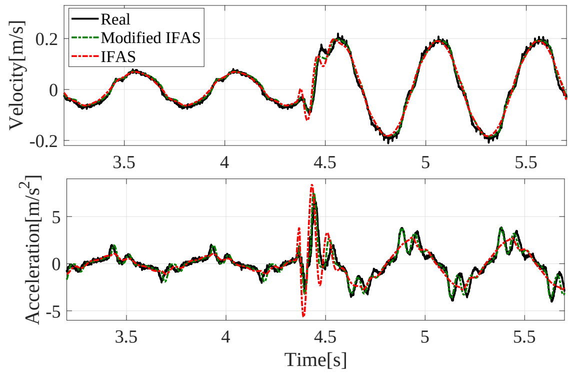

Figure 9: Simulated and experimental velocity and acceleration signals with considering the modified IFAS and the IFAS E-Modulus model, the specimen mass $M_s = 500[Kg]$ and the input voltage in equation (23).

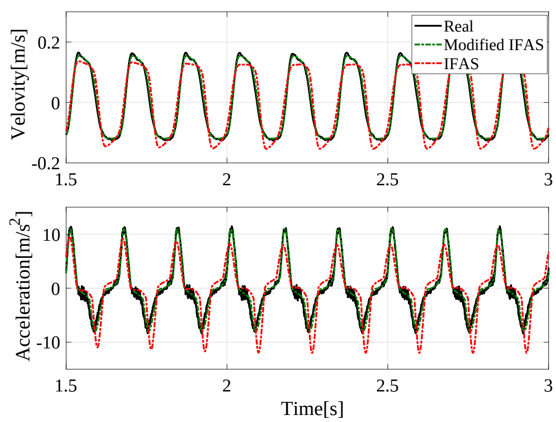

Figure 10: Simulated and experimental velocity and acceleration signals with considering The modified IFAS and the IFAS E-Modulus model, the specimen mass $M_s = 200[Kg]$ and the input voltage $u = 0.2sin(2\pi \times 6t)$.

considering the IFAS E-Modulus model cannot simulate the acceleration of the system. In figure 10, the acceleration and velocity of the system in response to the input voltage $u = 0.2sin(2\pi \times 6t)$ is displayed. As shown in this figure, the performance of the new model for simulating the experimental results with asymmetric acceleration response in frequency $6[Hz]$ is better than the IFAS E-Modulus model.

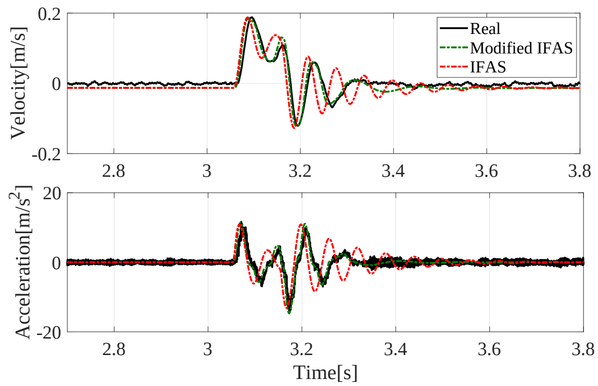

Figure 11: Simulated and experimental velocity and acceleration signals with considering the modified IFAS and the IFAS E-Modulus model, the specimen mass $M_s = 200[Kg]$ and a pulse signal as input voltage.

Then, for verifying the model with respect to other kinds of input signal, the experiment has been done with a pulse signal as input voltage. The comparison results between the experimental and the simulated velocity and acceleration with the specimen mass $M_s = 200[Kg]$, are shown in figure 11. This figure illustrates that, with different kinds of input signal the new model can reproduce the experimental velocity and acceleration results accurately. While, the model of the system with considering the IFAS E-Modulus cannot simulate the behavior of the system, as correctly as the new model.

Finally, the values of the MSE function in equation (21), for aforementioned different inputs and weights of specimen have been depicted in table 3. As indi-

Table 3: The values of the MSE function in equation (21) for the simulate model with the modified IFAS and the IFAS E-Modulus model

Input	Mass	MSE Value (Modified IFAS)	MSE Value (IFAS)
$u = 3sin(4\pi t)$	$200[kg]$	31.4	119.1
$u = 3sin(16\pi t)$	$200[kg]$	53.88	310.92
$u = 9sin(28\pi t)$	$200[kg]$	132.71	141.51
u in equation (23)	$500[kg]$	17.6	50.57
$u = 2sin(10\pi t)$	$200[kg]$	54.49	188.1
u in figure 11	$200[kg]$	41.69	125.8

cated in this table and conforming the figures, the MSE values for the simulated model based on the modified IFAS model are less than the simulated model with the IFAS E-Modulus.

5. Conclusion and Future Work

In this paper, a new empirical nonlinear model for simulating the acceleration behavior of a servo-hydraulic shaking table in dynamic and steady state flow situations has been proposed. Based on the experimental observations, this model is attained by considering a new modified IFAS model for simulating the E-Modulus of the hydraulic oil. In this model, the effective bulk modulus of hydraulic oil on both sides of the piston has been considered as two nonlinear springs, which are connected serially. The minimum stiffness of the spring effect of hydraulic oil in a symmetric double-acting cylinder occurs when the piston is in the center of its travel, which characterizes with differential pressure on its both sides. When the differential pressure is less than a specific threshold pressure, these springs have the minimum stiffness and reserve energy in themselves. Based on the experimental observations, this effect has been modeled with a function, which multiplies the IFAS model. Due to the limitation of the LuGre model for the velocities and frequencies higher than $0.15[m/s]$ and $2[Hz]$, the steady state friction model has been used for simulating the friction in the hydraulic cylinder. Then, the parameters of the model have been estimated by using NLSM method in MATLAB. The experimental results show that the new model can simulate accurately the acceleration and velocity of the system with respect to different kinds of inputs such as pulse and sinusoidal signals and different weights of the specimen. Whereas, based on the figures and the values of the MSE function in table 3, the model of the system with considering the IFAS E-Modulus model cannot reproduce the behavior of the acceleration signal as accurate as the new model.

As a future work, the new model of the E-Modulus will be verified on a specific test bench for measuring the E-Modulus with considering air content and different cylinder conditions.

[1] E. Yu, D. Whang, J. Conte, J. Stewart, J. Wallace, Forced vibration testing of buildings using linear shaker seismic simulation (lsss) testing method, Earthquake Engineering and Structural Dynamics 34 (7) (2005) 737–761.

[2] S. Pagano, R. Russo, S. Strano, M. Terzo, Modelling and control of a hydraulically actuated shaking table employed for vibration absorber testing,

in: ASME 2012 11th Biennial Conference on Engineering Systems Design and Analysis, 2012, pp. 651–659.

[3] P. Righettini, R. Strada, V. Lorenzi, A. Oldani, M. Rossetti, Modeling, control and experimental validation of a device for seismic events simulation, Proceeding of second international Conference on Advances In Civil, Structural and Mechanical Engineering (2014) 1–6.

[4] H. Merritt, Hydraulic control systems, John Wiley & Sons, 1967.

[5] A. Akers, M. Gassman, R. Smith, Hydraulic power system analysis, CRC press, 2006.

[6] D. Maneetham, N. Afzulpurkar, Modeling, simulation and control of high speed nonlinear hydraulic servo system, Journal of Automation Mobile Robotics and Intelligent Systems 4 (2010) 94–103.

[7] M. Jelali, A. Kroll, Hydraulic servo-systems: modeling, identification and control, Springer Science & Business Media, 2012.

[8] L. Márton, S. Fodor, N. Sepehri, A practical method for friction identification in hydraulic actuators, Mechatronics 21 (1) (2011) 350–356.

[9] H. YANADA, W. H. KHAING, X. B. TRAN, Effect of friction model on simulation of hydraulic actuator, The 3rd International Conference on Design Engineering and Science, ICDES 2014.

[10] X. B. Tran, W. H. Khaing, H. Endo, H. Yanada, Effect of friction model on simulation of hydraulic actuator, Proceedings of the Institution of Mechanical Engineers, Part I: Journal of Systems and Control Engineering (2014) 0959651814539476.

[11] A. Cologni, M. Mazzoleni, F. Previdi, Modeling and identification of an electro-hydraulic actuator, in: Control and Automation (ICCA), 2016 12th IEEE International Conference on, IEEE, 2016, pp. 335–340.

[12] P. Righettini, R. Strada, S. Valilou, E. Khademolama, Nonlinear modelng and experimental validation of uni-axial servo-hydraulic shaking table, Proceedings of the 2016 Bath/ASME Symposium on Fluid Power and Motion Control (2016) 1–6.

[13] Y. Jianjun, Research on acceleration harmonic cancellation of electro-hydraulic servo shaking table, Harbin Institute of Technology, PhD Thesis.

[14] J. J. Yao, S. H. Hu, W. Fu, J. W. Han, Impact of excitation signal upon the acceleration harmonic distortion of an electro-hydraulic shaking table, Journal of Vibration and Control 17 (7) (2011) 1106–1111.

[15] Y. Jianjun, J. Guilin, D. Duotao, L. Sheng, Acceleration harmonic identification for an electro-hydraulic servo shaking table based on the normalized least-mean-square adaptive algorithm, Journal of Vibration and Control 19 (1) (2013) 47–55.

[16] J. Yao, D. Di, G. Jiang, S. Gao, H. Yan, Real-time acceleration harmonics estimation for an electro-hydraulic servo shaking table using kalman filter with a linear model, IEEE Transactions on Control Systems Technology 22 (2) (2014) 794–800.

[17] J. Yao, R. Xiao, S. Chen, D. Di, S. Gao, H. Yu, Acceleration harmonic identification algorithm based on the unscented kalman filter for shaking signals of an electro-hydraulic servo shaking table, Journal of Vibration and Control 21 (16) (2015) 3205–3217.

[18] P. Righettini, R. Strada, S. Valilou, E. Khademolama, Gray-box acceleration modeling of an electro hydraulic servo shaking table with neural network, in: Advanced Intelligent Mechatronics (AIM), 2017 IEEE International Conference on, IEEE, 2017, pp. 1388–1392.

[19] K. Witt, Druckflüssigkeiten und thermodynamisches Messen, Ingenieur Digest Verlag-Ges., 1974.

[20] S. Eggerth, Beitrag zur messung von volumenströmen viskoser flüssigkeiten in druckleitungen, Ph.D. thesis, Diss, Tech University of Dresden (1980).

[21] M. Spilker, Druck-und temperaturabhängige Eigenschaften von Schmierstoffen und Hydraulikflüssigkeiten, Institute for Fluid Power Drives and Controls, RWTH Aachen, Germany., 1981.

[22] E. B. Wylie, V. L. Streeter, L. Suo, Fluid transients in systems, Vol. 1, Prentice Hall Englewood Cliffs, NJ, 1993.

[23] T. Nykanen, S. Esque, A. Ellman, Comparison of different fluid models, in: Bath Workshop on Power Transmission and Motion Control (PTMC 2000), Bath, UK, 2000, pp. 101–110.

[24] H. Murrenhoff, Grundlagen der fluidtechnik, teil 1: Hydraulik (2011).

[25] H. Gholizadeh, D. Bitner, R. Burton, G. Schoenau, Modeling and experimental validation of the effective bulk modulus of a mixture of hydraulic oil and air, Journal of Dynamic Systems, Measurement, and Control 136 (5) (2014) 051013.

[26] S. SAKAMA, Y. TANAKA, Mathematical model for bulk modulus of hydraulic oil containing air bubbles, Mechanical Engineering Journal 2 (6) (2015) 15–00347.

[27] S. Kim, H. Murrenhoff, Measurement of effective bulk modulus for hydraulic oil at low pressure, Journal of Fluids Engineering 134 (2) (2012) 021201.

[28] A. M. Hurst, J. VanDeWeert, A study of bulk modulus, entrained air, and dynamic pressure measurements in liquids, Journal of Engineering for Gas Turbines and Power 138 (10) (2016) 101601.

[29] B. Armstrong-Helouvry, Control of machines with friction, Vol. 128, Springer Science & Business Media, 2012.

[30] C. C. De Wit, H. Olsson, K. J. Astrom, P. Lischinsky, A new model for control of systems with friction, Automatic Control, IEEE Transactions on 40 (3) (1995) 419–425.

[31] H. Yanada, Y. Sekikawa, Modeling of dynamic behaviors of friction, Mechatronics 18 (7) (2008) 330–339.

[32] H. Yanada, K. Takahashi, A. Matsui, Identification of dynamic parameters of modified lugre model and application to hydraulic actuator, JFPS International Journal of Fluid Power System 3 (1) (2010) 1–8.

[33] X. B. Tran, N. Hafizah, H. Yanada, Modeling of dynamic friction behaviors of hydraulic cylinders, Mechatronics 22 (1) (2012) 65–75.

[34] X. Z. Kong, J. Liang, P. Kong, L. H. Gai, Z. G. Guan, W. H. Si, Combined simulation based on amesim and simulink for hydraulic cylinder controlled by servo valve, in: Applied Mechanics and Materials, Vol. 470, Trans Tech Publ, 2014, pp. 316–319.

[35] E. Botellé, M. Lebrun, Amesim–simulationsumgebung für motorradentwicklung environment for conceptual design of motorcycles using amesim, Entwicklungstendenzen im Motorradbau 1 (2003) 29.

[36] H. Li, Z. Yu, L. Xiong, W. Han, Hydraulic control of integrated electronic hydraulic brake system based on lugre friction model, Tech. rep., SAE Technical Paper (2017).

[37] S. Van Huffel, P. Lemmerling, Total least squares and errors-in-variables modeling: analysis, algorithms and applications, Springer Science & Business Media, 2002.