

Optimum Tuned Mass Dampers under seismic Soil-Structure Interaction

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Abstract

Tuned Mass Damper (TMD) devices are widely adopted as a valid mechanical solution for the vibration mitigation of structural systems and buildings under dynamic excitation. In the specific challenging context of seismic engineering, TMDs may represent a convenient option for both aseismic structural design and seismic retrofitting. However, the expectable efficiency rate of TMDs in that context is still debated. Besides, potential Soil-Structure Interaction (SSI) effects may become crucial in the mechanical system, and should be taken into account for the optimum TMD design, in order to avoid possible de-tuning. This work contributes to this framework, by investigating the effectiveness of an optimum TMD in reducing the linear structural response to strong-motion earthquakes of a given set of Multi-Degree-Of-Freedom (MDOF) low- and high-rise shear-type frame structures, by embedding SSI within the dynamic and TMD optimisation model. The TMD is seismically tuned through a dedicated two-variable optimisation procedure, for each specific case (primary structure, seismic event and soil type), therefore providing the optimum device setting for each given context. Average primary structure response indices are specifically targeted to that purpose, while maximum ones are monitored. A quite considerable range of optimisation cases is considered (eighty instances), to outline rather general considerations and average trends on TMD optimisation and effectiveness within the seismic SSI framework, for both low- and high-rise buildings. Such an investigation shall provide useful guidelines for a comprehensive tuning of TMDs in mechanical systems and specifically in the presence of seismic SSI, to be consulted in view of real-case applications.

Keywords: Optimisation of mechanical system; vibration control; Tuned Mass Damper (TMD); Soil-Structure Interaction (SSI); low- and high-rise frame structures; seismic TMD tuning.

1. Introduction

1.1. Framing on TMD tuning

The present work concerns a methodological optimisation approach of Tuned Mass Damper (TMD) devices towards vibration mitigation and control of structural mechanical systems under dynamic (seismic) excitation. Specifically, the reduction of the linear earthquake response of low- to high-rise frame buildings equipped with a seismic-tuned TMD added on top is sought, by accounting also for Soil-Structure Interaction (SSI) effects that may jeopardise the response of the whole mechanical system and spoil the TMD tuning. The optimum TMD setting is achieved through an original seismic tuning methodology [1], based on an optimisation process apt to provide the optimum TMD selection for a given structure-earthquake case, which is being developed within a wider research mainstream on TMD tuning [1, 2, 3, 4, 5].

The seismic tuning process here considers a complete and extensive analysis on TMD optimisation and performance in the presence of seismic SSI, **by focusing on the controlled (average) response of the primary structure**. Also, the present investigation is conceived in conjunction with companion work [6], where a separate refined Frequency Domain Decomposition modal dynamic identification approach is developed (see also [7, 8, 9, 10, 11, 12]), within the same seismic context explicitly considering SSI effects. Indeed, structural identification and TMD tuning may eventually be coupled into a single calibration process [9], on structures with unknown mechanical properties.

The need for seismic protection of buildings motivated, during the last decades, the research investigation on different vibration abatement and control devices. In this sense, TMDs appear to be as one of the most validated mechanical solutions for the reduction of unwanted or excessive structural vibrations, with many existing examples of application to high-rise buildings and different type of structures [13]. In this sense, the adoption of TMDs in the realm of earthquake engineering still constitutes a strategic and currently discussed

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research and engineering topic, with a wider relevance also in more general terms for different mechanical systems within the realm of structural dynamics.

The optimum TMD setting may be conveniently achieved through available tuning formulas, which however are strictly valid only for related benchmark ideal excitations. Contribution [4] provided a unified approach in that sense, by proposing convenient relationships for different loading cases, obtained from a common polynomial form. In [5], such trends for White-Noise base Acceleration have shown to provide an appropriate tuning for several cases of earthquakes and frame structures, as it will be here reconsidered under SSI effects.

Besides first fundamental studies on TMD adoption in the framework of earthquake engineering [14], the last decades count for various works on this topic, with tuning methods based either on the characteristics of the host structure [15, 16, 17, 18] or on the features of the external action, as a specific seismic excitation [19, 20, 21, 22, 23, 24].

The common point of all these studies concerns the hypothesis of a fixed-base structural system, i.e. by neglecting the potential influence of a flexible base into the optimum TMD tuning and the related seismic efficiency of the vibration reduction device. However, the seismic behaviour of structures may be considerably affected by the interaction of the surface or the embedded foundation with the connected soil layer [25, 26, 27, 28, 29], with potential viewable changes of the various modal parameters of the supported structure. This issue looks fundamental in light of achieving an appropriate TMD tuning process, since the effectiveness of this narrow-band passive vibration control device may be easily compromised by a misplaced evaluation of the underlying structural parameters (de-tuning).

1.2. TMD tuning under SSI

Hence, in recent research works, SSI has started to being considered within the TMD tuning problem, but seemingly mainly at a post-tuning stage. In [30], a frequency-independent structural modelling was adopted to test the TMD efficiency when SSI effects are included, with TMD parameters assumed from scratch, by finding that a strong SSI coupling could lead to a severe limitation of RMS response reduction. Work [31] showed that the co-existence of viscous dampers and of a TMD into a Multi-Degree-Of-Freedom (MDOF) primary structure allows for a sensible vibration reduction, even in the presence of remarkable SSI effects. Study [32] analysed the seismic behaviour of a Single-Degree-Of-Freedom (SDOF) primary structure with attached TMD, placed on a flexible base, within the frequency domain, by recovering a remarkable difference of dynamic response, as compared to the fixed-base case, with consequent loss of reliability of the TMD tuning. In [33], the wind-induced oscillation of a forty-storey frame building with a TMD added on top has been investigated, with the assumption of three different soil cases, as compared to the fixed-base case.

As a main outcome, it appeared that the presence of SSI would turn out to be more important for the case of a soft soil, while for the instance of a stiff soil its effect appeared to be smaller, also in terms of TMD tuning and efficiency. The crucial role played by the case of a soft soil was confirmed also in contributions [34, 35, 36], which however dealt with multiple passive and active TMDs, for asymmetric host buildings endowed of a torsional dynamic behaviour.

In [37], a novel optimum TMD tuning approach considering SSI within the optimisation process was developed through an Ant Colony Optimisation (ACO) method, adopted for a high-rise building (forty storeys), under two considered earthquake excitations (Kobe and Tabas), with a further confirmation of a significant influence of SSI effects on the achieved optimum TMD parameters and on the arising seismic response. There, a multi-objective optimisation criterion was stated, in the sense that a combined measure of maximum acceleration and displacement (maximum acceleration plus 10 times maximum displacement) was assumed as an objective function. A large TMD mass was considered, with a mass ratio referred to the first mode of vibration of the primary structure up to the rather high value of 6.5%, to emphasise the TMD effectiveness in terms of vibration mitigation on peak response indices. Optimisation was shown to be rather robust, on the assumed cost function, to consistently determine mass, stiffness and damping parameters of the optimum TMD, through a three-variable numerical optimisation process. Results for the two considered seismic excitations showed rather diverging outcomes where, at increasing soil stiffness, the optimum TMD was characterised by a lower natural frequency and damping ratio (almost vanishing for the Tabas instance), leading to a lighter, flexible and slightly damped optimum TMD. Moreover, further complementary results were also provided, in the very same context, by [38, 39], still with rather high values of TMD mass ratio in the order of 3.5% on the first mode of vibration, adopting and comparing different optimisation methods, analyzing wider sets of earthquake signals and building samples, and consistently evaluating the TMD performance on tall buildings under SSI. Though, this was achieved by again recording a wide spreading of optimum TMD parameters and response reduction for the various cases, which makes it difficult to outline general trends, as based on such a peak-performance optimisation process and monitoring, and motivates further complementary investigations.

An additional time-domain seismic TMD optimisation accounting for SSI effects was also recently derived in [40]. It provided an extensive investigation on seismic TMD tuning under SSI, by comparing the outcomes of two different metaheuristic optimisation methods, showing one of the two as being slightly more reliable with

respect to the other. Thus, focus was placed there on the optimisation methodology, which was again based on maximum index responses (top-storey displacement, with monitored total acceleration). The optimisation process was still a three-variable one, as in [37, 38, 39], including also the TMD mass ratio, which even in this case was pushed up to high upper-bound values of 5% on the first mode. Moreover, an interesting alternative TMD optimisation approach in the frequency domain, still within the seismic engineering scenario has also been recently carried out in [41], deriving results that look rather consistent with those obtained through the earlier-mentioned time-domain optimisation approaches.

The effectiveness of TMDs in suppressing excessive vibration of buildings under near-field ground motions accounting for nonlinear SSI effects has been recently discussed in [42], where TMDs were a priori tuned with the classical procedure outlined in [19] and relatively high values of mass ratio and TMD damping ratio of 5% have been assumed. It has been shown that the TMD helps in suppressing the primary structure response, more without SSI or with linear SSI, rather than with nonlinear SSI, thus leading to a possible underestimation of the structural response in the ideal assumption of a fixed-base behaviour (no SSI) or of a linear SSI behaviour.

Experimental studies on the response of MDOF frames with TMDs under dynamic SSI were recently investigated by [43, 44, 45], where interesting centrifuge tests were conducted in that context. In [43], it was shown that the TMD tuning based on the properties of the soil-structure system may lead to double the improvement gain in reducing the original peak response, while a great response magnification may be recorded for a detuned TMD neglecting SSI effects, thus pointing out to the importance of making a tuning on the soil-structure system, and with an appropriate description of the SSI effects. In [44], within the same context, the issue of storey TMD positioning was investigated, showing that this may play a role for the TMD effectiveness, in the SSI context, and possibly lead, if non-optimal, to detuning-similar effects. In [45], Structure-Soil-Structure Interaction (SSSI) of adjacent structures retrofitted by TMDs was investigated, showing that harmful effects linked to detuning may become even higher.

1.3. Present contribution

The present paper, aiming at providing crucial findings within such a latter research mainstream, concerns a TMD tuning optimisation methodology for a mechanical system where seismic structural response and SSI effects are indeed systematically embedded within a time-domain optimisation process, based on *average (RMS) response indices*, as applied to both *low- and high-rise buildings*. As a result, the so-achieved optimum TMD parameters are also affected by the characteristics of the soil, at variable height of the building, and therefore the recovered benefit in terms of primary structure response reduction shall become more reliable if the characteristics of the soil may be known or at least estimated. Hence, by taking into account the theoretical nature of the obtained results, it is however possible to detect the real rate of TMD effectiveness, and how this may vary for different primary structures, earthquake events and soil cases.

Moreover, the present study considers a rather wide seismic investigation on TMD tuning under SSI effects, with a significant number of structural and earthquake instances, for a total of *eighty tuning optimisation cases*, forming a comprehensive campaign of simulations, apt to outline general directives on TMD design in this context. Indeed, *average trends for optimum TMD design and TMD seismic effectiveness* are outlined, at variable soil type, in view of deciphering and interpreting the outcomes of the optimisation process achieved for a priori-known seismic input, to extrapolate possible trends and guidelines for a priori-unknown earthquake excitation, which may then reflect real application cases.

As a difference to the main corresponding contributions in [37, 38, 39] and [40], as above discussed, in the present paper, a standard, but also rather robust two-variable optimisation method targeted on RMS response indices is adopted, outlining new realistic results and conclusions about the effect of SSI on seismic TMD tuning and achieved seismic vibration mitigation, for low- and high-rise buildings, considering reasonable values of added TMD mass, with a mass ratio referred to the first mode in the order of 2%, attempting to recognise deviations and average trends, thus deriving useful and complementary information to that earlier provided in [37, 38, 39] and in [40].

Extensive *comparison to classical and recent TMD tuning formulas* is also foreseen and provided, showing how the dispersion of the optimum TMD results for the various earthquake instances and their average displaces with respect to them, at variable soil type, pointing out at rather significant differences for the case of a soft soil, especially for the considered case of a high-rise building.

The paper is organised as follows. In Section 2, main characteristics of the structural and seismic dynamic framework are reported, with focus on the implementation of SSI effects into the whole model. *Two* MDOF shear-type buildings are considered (a *low-rise five-storey* and a *high-rise forty-storey* frame structure), placed on *three different soil types* (plus on the reference *fixed-base case*, which is also considered for comparison purposes) and subjected to *ten different seismic excitation events*, for a total of *eighty seismic-optimisation TMD-SSI cases of analysis*. Section 3 outlines the key-point features of the proposed seismic tuning method, and presents a significant extract of the numerical results, pointing out average trends, at variable seismic excitation and soil type, for both the low- and the high-rise building, together with an extensive discussion

and an interpretation analysis, including on the features of the time-history seismic responses controlled by the TMD. Section 4 presents additional simulation results concerning the controlled response of the foundation of the source system, and of a specific SDOF case of seismic building-foundation interaction where building deformation and foundation rocking are suppressed, to explore the amount of achievable vibration control on the translational response of the foundation, for different mass ratios of the inserted TMD, at variable soil type. Finally, Section 5 summarises the salient closing considerations of the whole study on seismic TMD-SSI optimum tuning.

2. Structural and seismic dynamic context

2.1. Structural system and SSI model

A structural system comprising of a MDOF shear-type frame host (or primary) structure with a TMD added on top, lying on a Sway-Rocking (SR) foundation SSI model is depicted in Fig. 1, as earlier described in [37].

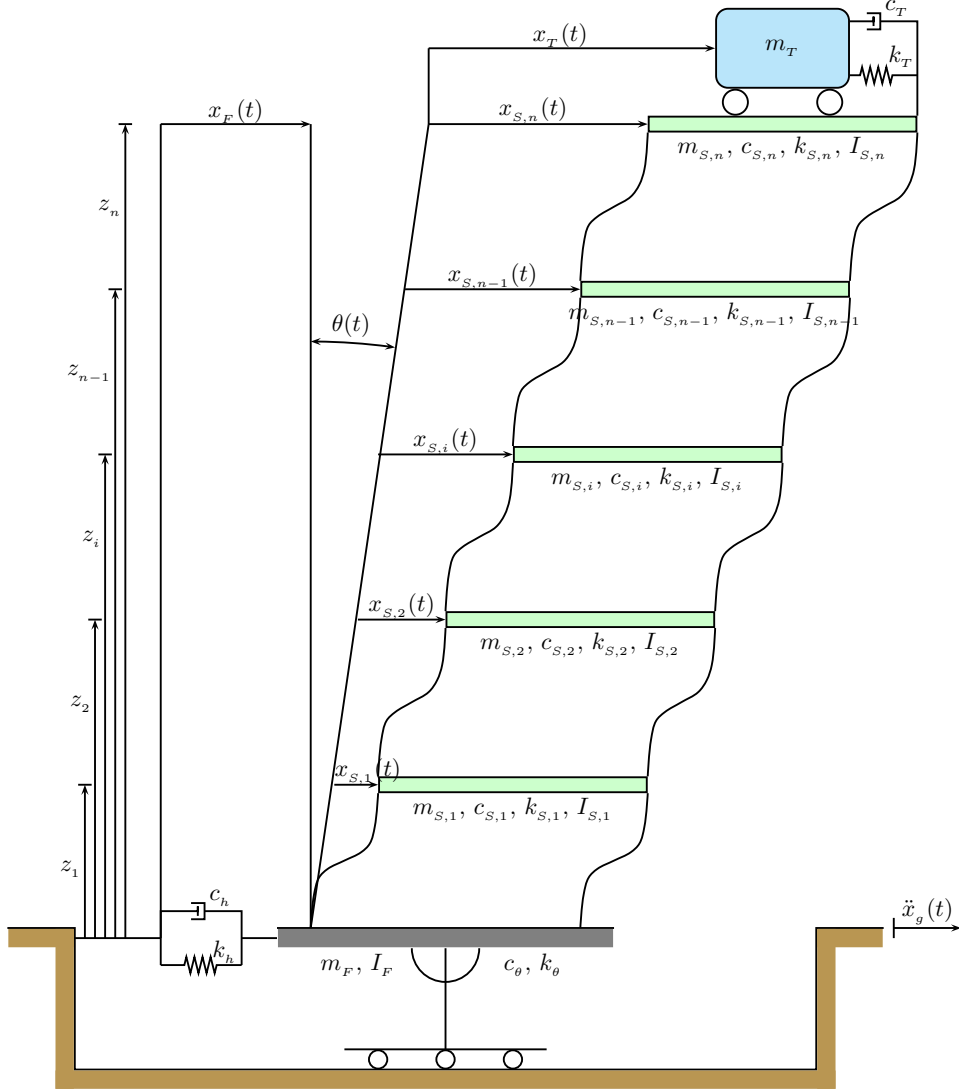


Figure 1: Structural parameters and absolute (relative to the ground) dynamic degrees of freedom of a super-structure system composed of a MDOF shear-type frame primary structure (subscript S) equipped with a TMD added on top (subscripts T) and placed on a Sway-Rocking foundation model (subscripts h, θ), undergoing base seismic excitation $\ddot{x}_g(t)$. Adapted from [26, 37].

In this regard, the *low-rise five- and high-rise forty-storey shear-type frame buildings*, presented in [7] and in [33], respectively, have been assumed as MDOF primary structures for this study, with main fixed-base modal parameters as resumed in Table 1.

These **primary structures are characterised (fixed-base condition)** by a diagonal mass matrix \mathbf{M}_S , a tridiagonal stiffness matrix \mathbf{K}_S and a viscous damping matrix \mathbf{C}_S , the latter being modelled through a classical Rayleigh damping assumption, as being simply proportional to the stiffness matrix [15]:

$$\mathbf{C}_S = \beta \mathbf{K}_S, \quad \beta = \frac{2\zeta_{S,I}}{\omega_{S,I}} \quad (1)$$

where $\zeta_{s,i}$ and $\omega_{s,i}$ are the given structural damping ratio and the angular frequency of the primary structure referred to its first mode of vibration, respectively.

In particular, the first-mode damping ratio has been here taken as $\zeta_{s,i} = 2\%$, which then implies higher modal damping values for the subsequent modes, according to the stiffness-proportional damping assumption in Eq. (1). It should then be noted that this poses a rather challenging scenario for the TMD effectiveness in the seismic engineering context, since TMD-induced vibration mitigation is expected to become lower at increasing structural damping (see e.g. [1, 5], and references quoted therein).

Table 1: Main reference fixed-base modal parameters (frequencies, periods, effective masses) of the assumed primary shear-type frame structures.

Building	Mode	$f_{s,i}$ [Hz]	$T_{s,i}$ [s]	$M_{eff,s,i}$ [%]
Five-storey [7]	I	1.112	0.900	71.1
	II	2.560	0.391	15.5
	III	4.047	0.247	7.2
	IV	5.665	0.177	3.6
	V	8.135	0.123	2.5
Forty-storey [33]	I	0.261	3.830	78.7
	II	0.731	1.368	10.6
	III	1.210	0.827	3.9
	IV	1.688	0.593	2.0
	V	2.164	0.462	1.2

The TMD mechanical parameters are mass m_T , constant stiffness k_T and viscous damping coefficient c_T . The TMD angular frequency and damping ratio are classically defined as follows:

$$\omega_T = \sqrt{\frac{k_T}{m_T}}, \quad \zeta_T = \frac{c_T}{2\sqrt{k_T m_T}} \quad (2)$$

Besides for TMD damping ratio ζ_T , the other free parameters useful for the optimum tuning process of the control device are mass ratio μ and frequency ratio f , defined as follows:

$$\mu = \frac{m_T}{\Phi_{s,i}^T \mathbf{M}_s \Phi_{s,i}}, \quad f = \frac{\omega_T}{\omega_{s,i}} \quad (3)$$

where $\Phi_{s,i}$ is the first mode shape of the primary structure (with a fixed base), normalised so as to exhibit a unit value at the level of the top storey.

The equations of motion of the system in Fig. 1, accounting for SR-SSI effects, read:

$$\mathbf{M}\ddot{\mathbf{x}}(t) + \mathbf{C}\dot{\mathbf{x}}(t) + \mathbf{K}\mathbf{x}(t) = -\mathbf{m}^* \ddot{x}_g(t) \quad (4)$$

where $((n+3) \times (n+3))$ global structural matrices \mathbf{M} , \mathbf{C} and \mathbf{K} , and $((n+3) \times 1)$ mass vector \mathbf{m}^* related to the earthquake excitation, are stated as follows [25] (see also complementary information in companion work [6]):

$$\mathbf{M} = \begin{bmatrix} \mathbf{M}_{SS} & \mathbf{M}_{SS} \mathbf{r} & \mathbf{M}_{SS} \mathbf{z} \\ \mathbf{r}^T \mathbf{M}_{SS} & m_F + \sum_{i=1}^n m_{s,i} + m_T & \sum_{i=1}^n m_{s,i} z_i + m_T z_n \\ \mathbf{z}^T \mathbf{M}_{SS} & \sum_{i=1}^n m_{s,i} z_i + m_T z_n & I_F + \sum_{i=1}^n (I_{s,i} + m_{s,i} z_i^2) + m_T z_n^2 \end{bmatrix} \quad (5)$$

$$\mathbf{K} = \begin{bmatrix} \mathbf{K}_{SS} & \mathbf{0}_{(n+1) \times 1} & \mathbf{0}_{(n+1) \times 1} \\ \mathbf{0}_{1 \times (n+1)} & k_h & 0 \\ \mathbf{0}_{1 \times (n+1)} & 0 & k_\theta \end{bmatrix} \quad (6)$$

$$\mathbf{C} = \begin{bmatrix} \mathbf{C}_{SS} & \mathbf{0}_{(n+1) \times 1} & \mathbf{0}_{(n+1) \times 1} \\ \mathbf{0}_{1 \times (n+1)} & c_h & 0 \\ \mathbf{0}_{1 \times (n+1)} & 0 & c_\theta \end{bmatrix} \quad (7)$$

$$\mathbf{m}^* = \begin{bmatrix} \mathbf{M}_{SS} \mathbf{r} \\ m_F + \sum_{i=1}^n m_{s,i} + m_T \\ \sum_{i=1}^n m_{s,i} z_i + m_T z_n \end{bmatrix} \quad (8)$$

where $\mathbf{x}(t) = [x_{s,1}(t), \dots, x_{s,n+1}(t); x_F(t), \theta(t)]^T$ is the $((n+3) \times 1)$ vector of degrees of freedom; \mathbf{M}_{SS} , \mathbf{C}_{SS} and \mathbf{K}_{SS} are the $((n+1) \times (n+1))$ mass, damping and stiffness matrices of the structural system (**host primary structure + added TMD, fixed-base condition**), respectively; I_i is the mass moment of inertia of the i -th floor; m_F , I_F are the mass and the moment of inertia of the foundation, respectively; $\ddot{x}_g(t)$ is the seismic ground acceleration; \mathbf{r} is a $((n+1) \times 1)$ unit horizontal rigid-body motion vector; \mathbf{z} is the $((n+1) \times 1)$ vector of absolute (referred to the ground) heights of the floors.

Tables 2 and 3 report the characteristics of the *three considered soil cases* (ν_s Poisson's ratio, G_s shear modulus, ρ_s mass density, $V_s = \sqrt{G_s/\rho_s}$ shear wave velocity), as stated in [33, 37], and the parameters of the circular foundation of the two host structures (R_F radius, m_F mass, I_F mass moment of inertia), respectively.

The values of soil stiffness and damping $k_{s,h}$, $k_{s,\theta}$, $c_{s,h}$, $c_{s,\theta}$ (h subscript = horizontal swaying effect, θ subscript = rocking effect), further reported in Table 4, are determined as indicated in [26], **namely**:

$$k_{s,h} = \frac{8G_s R_F}{2 - \nu_s}, \quad k_{s,\theta} = \frac{8G_s R_F^3}{3(1 - \nu_s)}; \quad c_{s,h} = \frac{4.6}{2 - \nu_s} \rho_s V_s R_F^2, \quad c_{s,\theta} = \frac{0.4}{1 - \nu_s} \rho_s V_s R_F^4 \quad (9)$$

The adopted formulas are here taken as a main reference, in common with companion work [6]. As a matter of fact, in the practical experience, factor 0.4 in the expression of $c_{s,\theta}$ may be doubled to 0.8 (ASCE Standard, [46]). The *reference fixed-base case* has also been considered for comparison purposes.

Table 2: Parameters of the considered soil cases [33].

Soil type	ν_s	G_s [N/m ²]	ρ_s [kg/m ³]	V_s [m/s]
Soft soil	0.49	1.80×10^7	1800	100
Medium soil	0.48	1.71×10^8	1900	300
Dense soil	0.33	6.00×10^8	2400	500

Table 3: Foundation parameters of the assumed host structures [33, 7].

Building	R_F [m]	m_F [kg]	I_F [kg m ²]
Five-storey [7]	10	3×10^5	7.5×10^6
Forty-storey [33]	20	1.96×10^6	1.96×10^8

Table 4: Stiffness and damping coefficients for the considered flexible-base cases [33, 7].

Building	Soil type	$k_{s,h}$ [N/m]	$k_{s,\theta}$ [N m]	$c_{s,h}$ [N s/m]	$c_{s,\theta}$ [N s m]
Five-storey [7]	Soft soil	9.54×10^8	9.41×10^{10}	5.48×10^7	1.41×10^9
	Medium soil	9.00×10^9	8.77×10^{11}	1.73×10^8	4.39×10^9
	Dense soil	2.87×10^{10}	2.39×10^{12}	3.31×10^8	7.16×10^9
Forty-storey [33]	Soft soil	1.91×10^9	7.53×10^{11}	2.19×10^8	2.26×10^{10}
	Medium soil	1.80×10^{10}	7.02×10^{12}	6.90×10^8	7.02×10^{10}
	Dense soil	5.75×10^{10}	1.91×10^{13}	1.32×10^9	1.15×10^{11}

Table 5: Main natural frequencies of the building-foundation system at variable soil properties.

Building	Mode	Soft soil	Medium soil	Dense soil	Fixed-base
		$f_{S,i}$ [Hz]			
Five-storey [7]	I	1.017	1.100	1.108	1.112
	II	2.469	2.550	2.557	2.560
	III	3.887	4.030	4.042	4.047
	IV	5.439	5.642	5.658	5.665
	V	7.434	8.088	8.120	8.135
Forty-storey [33]	I	0.172	0.245	0.255	0.261
	II	0.706	0.728	0.730	0.731
	III	1.177	1.205	1.208	1.210
	IV	1.636	1.683	1.686	1.688
	V	2.109	2.158	2.162	2.164

Table 5 further gathers the arising modified natural frequencies (first five modes) of the structure-foundation system with SSI, for the two considered buildings, at variable soil properties (in the last column, fixed-base frequencies are also repeated from Table 1, for the ease of comparison).

About the earlier normalisation on mass ratio and frequency ratio adopted in Eq. (3), which are referred to the first modal mass and first frequency of the fixed-base building, and the general philosophy of this study, such a normalisation, a priori fixed disregarding for the soil-foundation characteristics, constitutes a main reference in the present context, which also considers the reference case of no SSI effects. The present TMD tuning study develops a wide parametric investigation (eighty seismic optimisation cases of analysis). The fixed-base case is taken as a common reference, to set the parameters of the system. Thereby, in the seismic TMD tuning literature it is classical to refer to the first mode of the primary structure (with no SSI effects), for the definition of mass and frequency ratios, as done in Eq. (3). Then, the TMD is optimized for the various SSI conditions and the corresponding evaluation is assessed, at same TMD mass and stiffness normalisation, to inspect the recorded variation at possibly variable SSI conditions, which may be unknown. Thus, within the present investigation study seeking how a standard TMD tuning may be jeopardized by developing underlying SSI effects, it looks rational to state a common TMD normalisation in terms of the fixed-base condition, toward the stated purposes of such a wide parametric study. Nonetheless, once given soil properties may be assessed, for a given case instance, thus not within a comprehensive parametric investigation, a specific refined tuning for each considered case could instead be made. Therefore, since the first eigenmode and frequency can be changed when SSI effects are considered, mass and frequency ratios could be normalized with respect to the changed first modal mass and fundamental frequency, to make the tuning more specific on the considered system-foundation case.

2.2. Seismic input

The set of *ten seismic input signals* considered for the present TMD-SSI optimisation analysis is listed in Table 6. The selected earthquake events exhibit different characteristics, magnitude and duration (see also complementary information on time-frequency features in companion work [6], as there reported in Appendix A, Figure A.1), in order to exploit here possible consequences within the same TMD optimisation process. Also, records (I) and (J) in Table 6 have been corrected through a two-stage process, composed of a baseline removal (mean value) and a Butterworth filtering.

Table 6: Adopted ten seismic input signals main data and reference labels used in the subsequent plots.

Earthquake	Station	Comp.	M	Dur. [s]	PGA [g]
(A) Imperial Valley 1940	El Centro	S00E	6.9	40	0.359
(B) Tabas 1978	70, Boshrooyeh	WE	7.3	43	0.929
(C) Imperial Valley 1979	01260	NS	6.4	58	0.331
(D) Loma Prieta 1989	Corralitos	0	7.0	25	0.801
(E) Northridge 1994	24436	WE	6.7	60	1.778
(F) L'Aquila 2009	Valle Aterno	WE	5.8	50	0.676
(G) Chile 2010	Angle	WE	8.8	180	0.697
(H) New Zealand 2010	163541	NS	7.1	82	0.752
(I) Tohoku 2011	Sendai	NS	9.0	180	2.667
(J) Katmandu 2015	Kanti Path	NS	7.8	100	0.164

3. Optimum TMD parameters and response reduction

3.1. Seismic tuning of TMDs

The seismic tuning process adopted in this study concerns the optimisation of the TMD for a specific seismic input, by involving the earthquake signal within the optimisation routine, for a given structure and a specific SSI foundation model. The main task of such an operating way is the ideal achievement of the optimum TMD for each selected seismic event and structural-SSI case [1], thus at a priori known seismic input. This method has been developed within the time domain, i.e. the optimisation process is looped with a time solver based on classical Newmark's average acceleration method [26], which allows to numerically derive the mechanical system response under seismic excitation.

The tuning of TMDs can be easily stated and managed by a classical optimisation problem, where the free parameters of the control device play the role of optimisation variables:

$$\min_{\mathbf{p}} \mathbf{J}(\mathbf{p}), \quad \mathbf{l}_b \leq \mathbf{p} \leq \mathbf{u}_b \quad (10)$$

where \mathbf{p} , $\mathbf{J}(\mathbf{p})$, \mathbf{l}_b and \mathbf{u}_b represent the optimisation variables, the objective function, the lower and upper bounds on the optimisation variables, respectively.

The two-parameter approach often adopted in the literature has been followed here, where frequency ratio f and TMD damping ratio ζ_T are taken as optimisation variables, namely $\mathbf{p} = [f, \zeta_T]$, while *mass ratio* μ is taken

constant and set equal to a contained, feasible value of $\mu = 0.02$, which is potentially representative of real application cases. This shall constitute a main difference with respect to other strictly related contributions that have investigated pushing the value of the mass ratio to higher upper-bound values (e.g. of even 6.5% in [37] and of 3.5% in [38, 39], and of 5% in [40]), within a three-variable optimisation process, to inspect how the TMD effectiveness may be increased, at least in theoretical terms.

Indeed, alternative complementary numerical tuning approaches, already commented in the Introduction, have instead considered 3-parameter optimisation processes involving also the use of the mass ratio [23, 37, 38, 39, 40, 41], however typically leading to fall on the upper-bound value set for such an optimisation parameter (also taken as rather high, to push for the TMD effectiveness, as commented above), confirming that, within the physical optimisation process, the TMD might never have enough of TMD mass, in order to mitigate the primary structure vibration. This may also influence the dispersion of the optimisation results, at variable earthquake excitation, and likely add unnecessary disturbance in terms of robustness of the optimisation process, to fall on true same global optima, at variable adopted optimisation technique. Thus, here the mass ratio is excluded from the optimisation process and also set to a reasonable contained value ($\mu = 2\%$) that may truly be encountered in civil and earthquake engineering applications.

The lower and upper bounds on two optimisation variables f and ζ_x have been taken with the following values:

$$\mathbf{l}_b = [0.5, 0], \quad \mathbf{u}_b = [1.1, 0.1], \quad (11)$$

which consider a rather wide search interval for the optimisation algorithm. In this regard, such bounds allow for investigating whether the optimum TMD parameters could be affected by both the randomness of the seismic input and, most important, the characteristics of the soil-foundation model, in the present SSI framework of TMD optimisation.

Since here a main goal for [measuring the TMD effectiveness is that of reducing the global seismic response of the primary structure, along the whole time window of analysis](#), the *Root Mean Square (RMS) estimate of the displacement of the primary structure (top-storey displacement) has been taken into account as the assumed objective function*. Motivations of such a choice have been widely discussed in [1, 5]. Thus, the present analysis provides complementary seismic tuning results to strictly related ones based on peak response indices (e.g. [37, 38, 39, 40]).

Also, the present optimisation study is a single-objective one, as focused on the top-storey displacement of the primary structure, as the only variable included within the objective function. Indeed, here the analysis is more concerned with the seismic response of the building, as controlled by the insertion of the TMD, under SSI effects. Separate, multi-objective trials could as well include the horizontal displacement and the rocking rotation of the foundation, in order to also control the movement of the foundation and to prevent any failure in the soil and foundation. Although this is not truly attempted in the present study, the last section of the paper considers a specific optimisation analysis on a particular horizontal translational SDOF model, where the building is rigidly tight to the foundation, so that the total displacement of the building coincides with that of the foundation and thus the above stated optimisation indirectly acts on the foundation movement as an objective function (see Section 4). Moreover, different optimisation strategies, considering various objective functions could in principle be derived, within the same implant, given a specific target for a practical case under investigation. Here, the purposes of a wide parametric investigation are recalled.

The numerical optimisation takes advantage of a classical nonlinear gradient-based optimisation algorithm implemented within MATLAB, relying on Sequential Quadratic Programming (SQP), which ensures a fast convergence and a high level of accuracy, by the use of the *fmincon* function. A flowchart of the TMD optimisation algorithm has been made available in [1] [and is here reported \(Fig. 2\) for the sake of completeness](#).

The initial evaluation of the tuning variables necessary for launching the optimisation process has been set through well-known Den Hartog's tuning formulas [47], reported below, depending only on assumed mass ratio μ and independently from assumed inherent structural damping ratio ζ_s . Indeed, alternative start-point assumptions from different possible tuning formulas, accounting also for the consideration of inherent structural damping ratio ζ_s (e.g. those proposed in [4], also reported below), would not affect the final estimation of the optimum TMD parameters obtained by the present optimisation algorithm, as a consequence of a demonstrated well-posedness and robustness of the stated seismic TMD tuning process (see e.g. [1, 5]).

Alternative and rather valuable TMD optimisation approaches by different computational techniques have been proposed by different authors (see e.g. [20, 21, 23, 37, 38, 39, 40, 41]). Here, the focus is rather placed on the physical and technical outcomes of the optimisation process (TMD tuning) and on the engineering implications in terms of achieved seismic vibration mitigation by TMD insertion, under SSI effects, rather than on the specific adopted optimisation methodology and on its implications, which actually should not matter much on the achieved results, if the optimisation process really reveals to be fully robust and pointing out to the true global optima, in the constrained space of the design variables.

Concerning the present choice of optimisation TMD tuning process and novelties in the current investigation, it should be noted that the latter do not concern the optimisation technique, as basically taken from previous

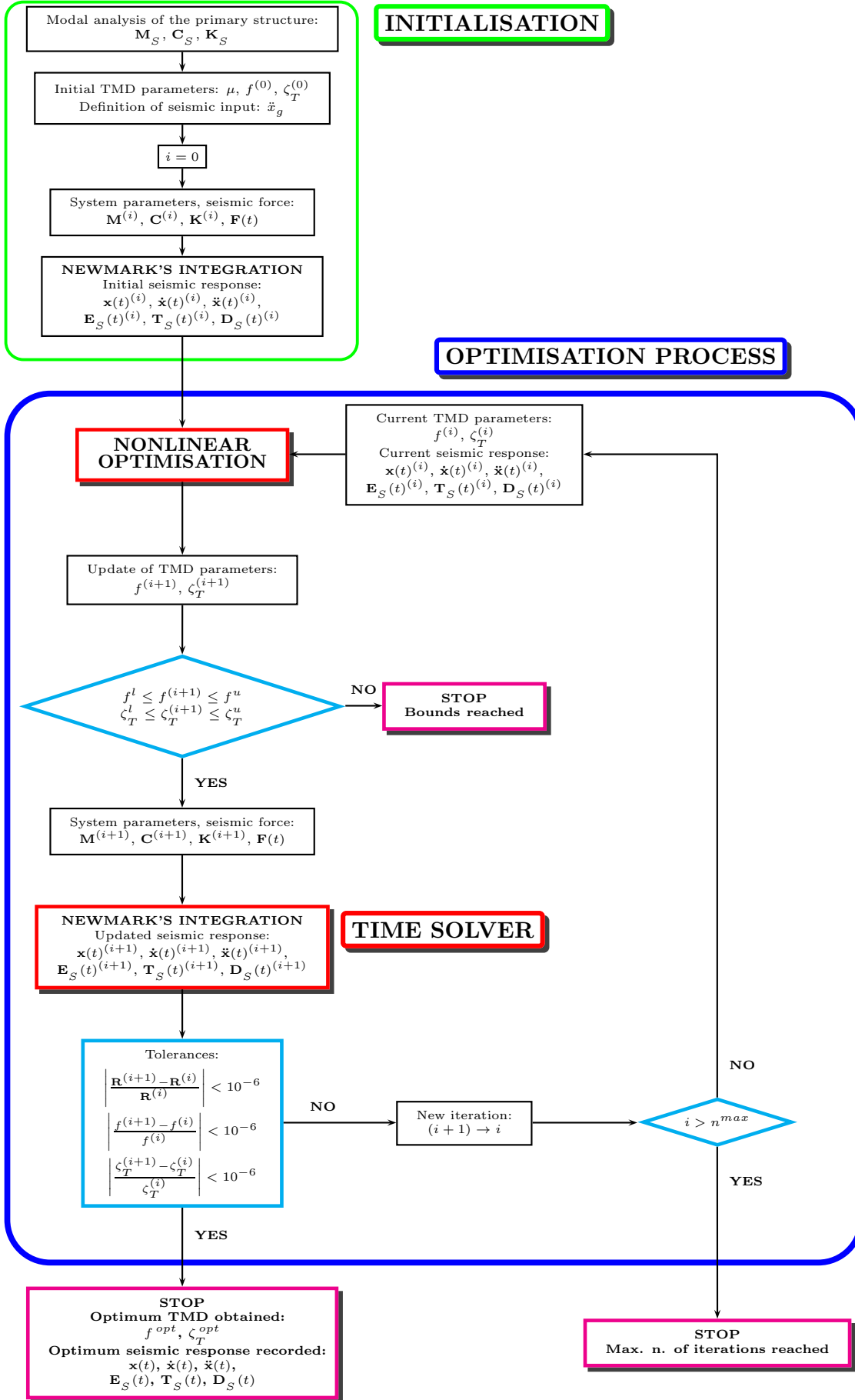


Figure 2: Flowchart of the optimisation algorithm for TMD tuning at seismic input [1].

work [1], rather its TMD tuning and effectiveness outcomes in the context of seismic SSI. Above, reference is made to several other very valuable approaches that have widely investigated different optimisation methodologies in the context of TMD tuning, and under concomitant SSI. Optimisation aspects and techniques have been deeply addressed in the TMD tuning literature, although they should not really matter much in the final outcomes, if the optimisation process becomes truly well posed, since different (robust-enough) optimisation techniques shall basically render the same optimum TMD parameters. Thus, the present paper does not focus on a new, additional optimisation methodology but shows that the previously implemented one confirms to be rather robust and affordable, even in the newly challenged SSI case, in deriving consistent estimates of the optimum TMD parameters, and allowing to inspect the associated controlled response. The paper develops a wide investigation on SSI effects (eighty optimisation cases), with several seismic input signals, and attempts to outline a reading and interpretation of the achieved results, in view of formulating useful design guidelines on TMD tuning under SSI, as outlined in the Conclusions.

3.2. Optimum TMD parameters

The optimum TMD parameters, achieved through the proposed numerical optimisation method, are displayed in Fig. 3, together with those obtained with analytic tuning formulas from (a) Den Hartog's work [47], which represent a known and effective universal reference, and (b) contribution [4]. Thus, this allows to outline average trends for the optimum TMD parameters at variable seismic excitation, for different structure-foundation systems, with useful comparisons to such reference tuning formulas, to appreciate dispersion and deviation of the tuning process.

The former celebrated tuning formulas by Den Hartog [47] constitute a classical reference tuning based on harmonic loading on an undamped primary structure, thus of a rather general validity:

$$f_{DH}^{opt} = \frac{1}{1 + \mu}, \quad \zeta_{T_{DH}}^{opt} = \sqrt{\frac{3}{8} \frac{\mu}{1 + \mu}} \quad (12)$$

whereas the latter formulas proposed in [4] take into account the presence of inherent structural damping, and consider different possible acting excitations.

Pertinent, easy-to-remember and convenient related formulas from [4], reported below, refer to the case of White-Noise base Acceleration (WNA), which may be assumed as a benchmark ideal excitation somehow similar to that of a general seismic input:

$$f_{WNA}^{opt} = 1 - \sqrt{3\mu} \left(\frac{2}{3} \sqrt{\mu} + \frac{3}{2} \zeta_s \right), \quad \zeta_{T_{WNA}}^{opt} = \frac{1}{2} \sqrt{\mu} \quad (13)$$

Notice that in Eq. (13) values of f^{opt} and ζ_T^{opt} are coupled to each other, at given structural damping ratio ζ_s , despite that the value of ζ_T^{opt} appears universal at variable structural damping.

The optimisation results recorded for the *low-rise five-storey building* (Fig. 3a) exhibit a spread but rather homogeneous distribution of optimum TMD parameters, with suitable values (i.e. close to those usually obtainable for benchmark excitations); it looks that no particular trends related to the soil type and the seismic event could be traced. Perhaps, clearer scenarios could be recovered by considering an even wider number of cases and partially compensating the randomness of the seismic excitation.

Fig. 3a displays similar trends for the optimum frequency ratio f^{opt} for the medium and the dense soils, and the fixed-base case, recovered for both each earthquake and mean value. On the other hand, the soft soil clearly provides a different picture, with an optimum frequency ratio shifted toward lower values, as reflected by the related displaced mean value. The optimum TMD damping ratio ζ_T^{opt} instead shows a massive concentration of values nearby after $\zeta_T^{opt} = 0.05$, with a spread as wider as the soil becomes stiffer.

The optimisation results recorded for the *high-rise forty-storey building* are reported in Fig. 3b. The optimum frequency ratio again shows more homogeneous trends for all the cases, except for that of the soft soil, with even lower values gathered within an interval of about $f = [0.6, 0.7]$. This emphasises the fact already detected for the five-storey case, showing results quite far from the usual values recovered for this parameter, for the soft-soil case. The optimum TMD damping ratio overall displays values that are lower than those obtained for the five-storey building, but still at around $\zeta_T^{opt} = 0.05$ as a global mean result, almost independently of the soil case.

Trends in Figs. 3a and 3b show a dispersion of results, depending on the considered earthquake, as visible also in similar previous representations in [37, 38, 39]. Here, average values at variable earthquake excitation are also determined and represented, and in comparison to classical optimum values from available TMD tuning formulas, in order to outline, on average, appropriate conclusions on the achieved TMD optimisation, at variable soil-foundation characteristics.

In general, for the two analysed shear frames and set of ten seismic excitations, the frequency ratio looks more sensitive to the characteristics of the soil than for the TMD damping ratio, and the presence of a soft soil

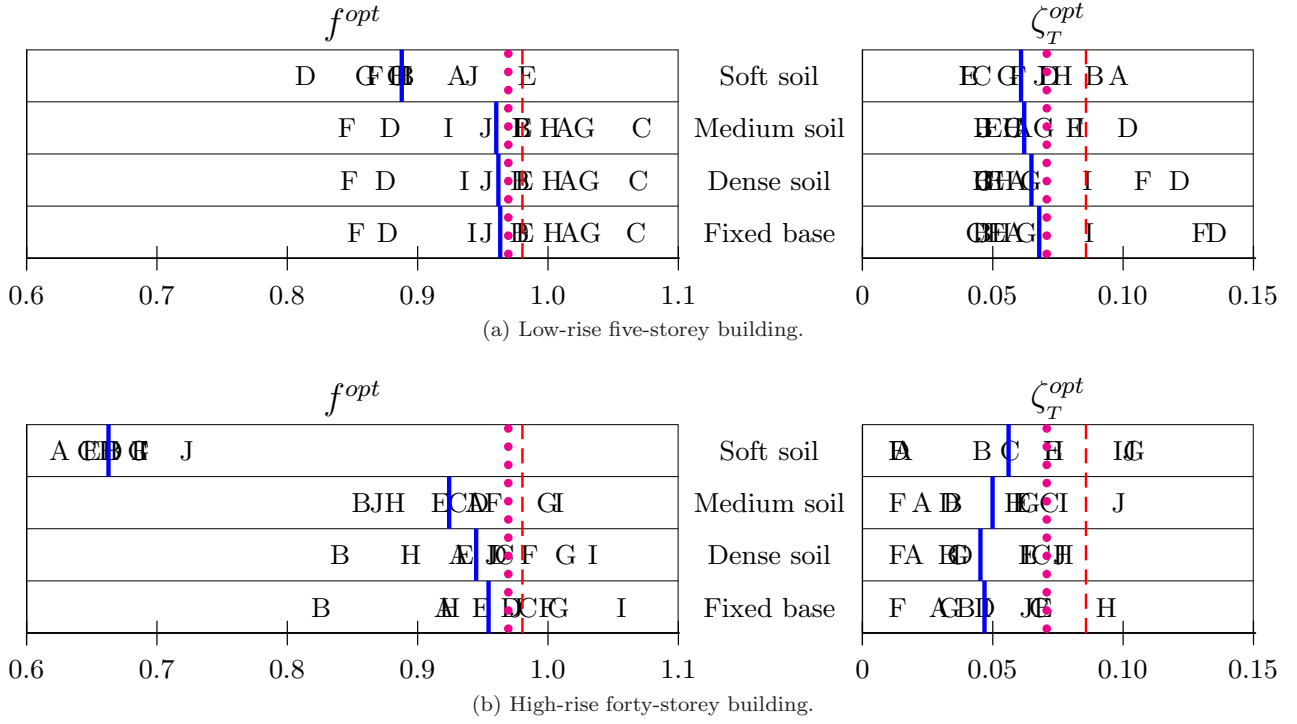


Figure 3: Optimum TMD parameters for (a) *low-rise five-storey building* and (b) *high-rise forty-storey building*, for each seismic event (reference letters A-J refer to the ten earthquake excitations in Table 6), with mean values for each type of soil (thick blue line), compared to the values achieved with reference tuning formulas from Den Hartog [47], Eq. (12) (dashed red line) and Salvi and Rizzi [4], Eq. (13) (dotted magenta line). mass ratio $\mu = 0.02$, first-mode inherent damping ratio $\zeta_{s,I} = 0.02$.

tends to modify the optimum setting of the TMD parameters, towards values that are far from those achievable from literature tuning formulas.

In this sense, notice that the proposed optimum tuning [4] for White-Noise base Acceleration always provides values closer to those (average ones) from the present optimum seismic tuning, with respect to classical results coming from Den Hartog’s formulas. This is likely due not only to the consistency of the dynamic loading (White-Noise base Acceleration being more representative of a seismic input than Harmonic Force acting on the primary structure), but also to the assumption of accounting for the presence of inherent structural damping within the evaluation of the optimum TMD parameters.

Finally, emphasised unusual results in terms of achieved optimum TMD parameters under SSI for the high-rise forty-storey building, especially on frequency ratio tuning for the soft soil case, suggest a remarkable influence of the modal parameters of the primary structure and the soil characteristics into the tuning process, expressed as a higher connection between structural and soil systems, in terms of challenging TMD tuning. This appears in line with the conclusions in [37], where it is overall stated that “*the soil characteristics greatly influence on the favorite TMD parameters*”, for a high-rise building.

3.3. Optimum seismic TMD performance under SSI

A significant extract of the numerical results related to the achieved seismic response reduction after the optimisation tuning based on an average response index (RMS top-storey displacement) has been reported in Figs. 4–5, referred to the low-rise five-storey and the high-rise forty-storey frame structure, respectively. Recall that, in terms of challenged potential TMD effectiveness, these results are achieved for a quite reasonable (contained) value of mass ratio $\mu = 0.02$ and of inherent structural damping ratio $\zeta_{s,I} = 0.02$ referred to the first mode of vibration (with stiffness-proportional damping, thus leading also to higher values for the subsequent modes), thus setting a realistic and challenging application framework for TMD benchmarking.

Each figure is composed of four panels, related to different recorded structural response indices, as described below:

- Figs. 4a and 5a: RMS top-storey displacement, which was selected as objective function within the optimisation process and therefore represents a targeted outcome;
- Figs. 4b and 5b: peak top-storey displacement;
- Figs. 4c and 5c: RMS kinetic energy of the primary structure;
- Figs. 4d and 5d: peak kinetic energy of the primary structure.

Such structural response indices have been evaluated at different levels of inspection:

- (i) for each of the eighty considered optimisation cases;
- (ii) as an average over all the four assumed soil instances, for a given earthquake event;
- (iii) as an overall average, for each primary structure (low-rise five-storey building and high-rise forty-storey building).

Hence, an exhaustive picture of the optimum TMD effectiveness for all the eighty considered tuning optimisation cases is outlined, allowing for several and diversified considerations.

Fig. 4, referring to the *low-rise five-storey frame*, is commented first. Fig. 4a exhibits a one-third suppression of RMS top-storey displacement (targeted objective function index), as an average value, but a sensible difference among the outcomes from the considered earthquake events. Indeed, for the majority of the seismic input signals, a response reduction higher than 30% is displayed, with the soft soil representing a worst case, whilst seismic events D and F (Table 6) lead to a lower TMD benefit, with the soft soil showing off as a best case. The peak top-storey displacement (Fig. 4b) is less reduced, with half of the mean value with respect to that of the RMS estimate. Again, seismic events D and F, here together with signals C and H, are those endowed with a lower TMD efficiency, while seismic input A, I and J show noticeable response decreases, as achieved for the RMS displacement.

The trends above outlined hold, but with higher values, for the kinetic energy reduction too, for both RMS (Fig. 4c) and peak (Fig. 4d) estimates, reduced by almost 40% and 30%, respectively. Notice that such remarkable results have been recorded even if kinetic energy indices were not accounted for within the tuning process. Moreover, the earthquakes from Table 6 where the TMD effect becomes more (A-C, G-J) or less (D, F) effective are the same as those detected for the displacement indices. This highlights the connection between the frequency content of the seismic input and the superstructure characteristics.

The abatement of both top-storey displacement and primary-structure kinetic energy for earthquake J (Table 6) looks rather outstanding. This is a very strong motion, and therefore calls for a considerable countermeasure of response reduction, which is indeed achieved by the present optimum-tuned TMD, pointing out to the beneficial insertion of the control device for such a “critical” earthquake event (see also considerations in [40]).

Fig. 5 outlines a comprehensive post-tuning picture related to the *high-rise forty-storey frame*. The RMS displacement (Fig. 5a) has been reduced of almost 35% as an average, with a homogeneous looking among all the events and soil cases: beside for a visible higher performance for earthquakes A and G (Table 6), other specific trends are not recovered. Once again, the peak displacement (Fig. 5b) is quite less decreased (average value of almost 12%), with several earthquake events showing a reduced TMD effect, while a high reduction for the soft soil at seismic event A and good results for signals G and J appear as isolated instances.

Earthquakes G, J are further confirmed to be those where the TMD produces the higher benefit of reduction of both RMS (Fig. 5c) and peak (Fig. 5d) kinetic energies. Also, the RMS estimate shows an average benefit (above 25%) which is twice that achieved for the peak kinetic energy (just above 10%). The latter displays an almost-negligible difference, after the optimum-TMD insertion, for several earthquake events, namely A, E, F, H and I.

Further general considerations on the recorded seismic TMD effectiveness may be traced down as follows:

- The RMS estimate of a response quantity, no matter whether adopted as objective function, always looks more reduced than for a peak evaluation on the same response index. This just confirms a known important result already recovered in past works on this research mainstream on TMD tuning for seismic engineering applications [1, 5].
- By overall comparing the trends for the low-rise (Fig. 4) and the high-rise (Fig. 5) building, it appears that, for the three untargeted response indices, the TMD provides a much compact effectiveness framework for the low-rise than for the high-rise frame, especially on the uncontrolled kinetic energy indices. Trends appear also much compact on the targeted RMS displacement index, though higher spikes for the high-rise building lead to a bit higher average percentage reduction.

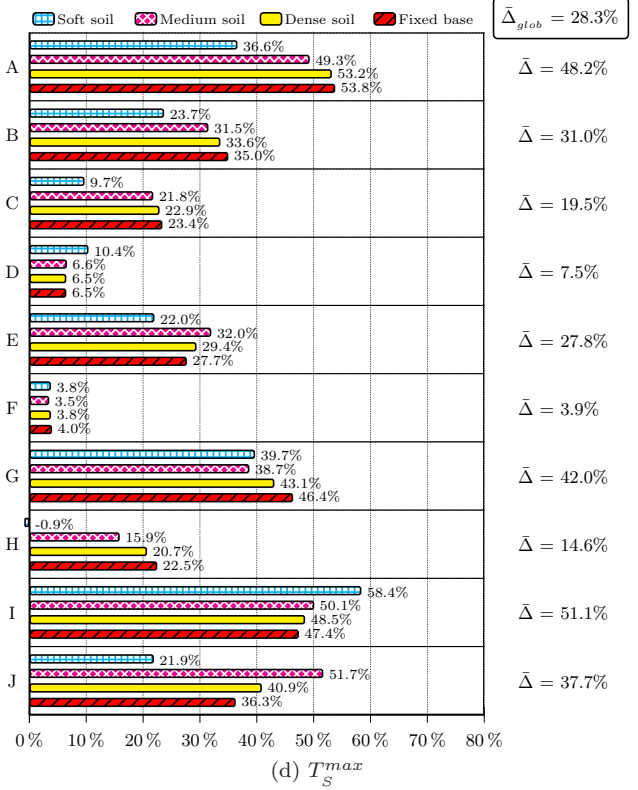
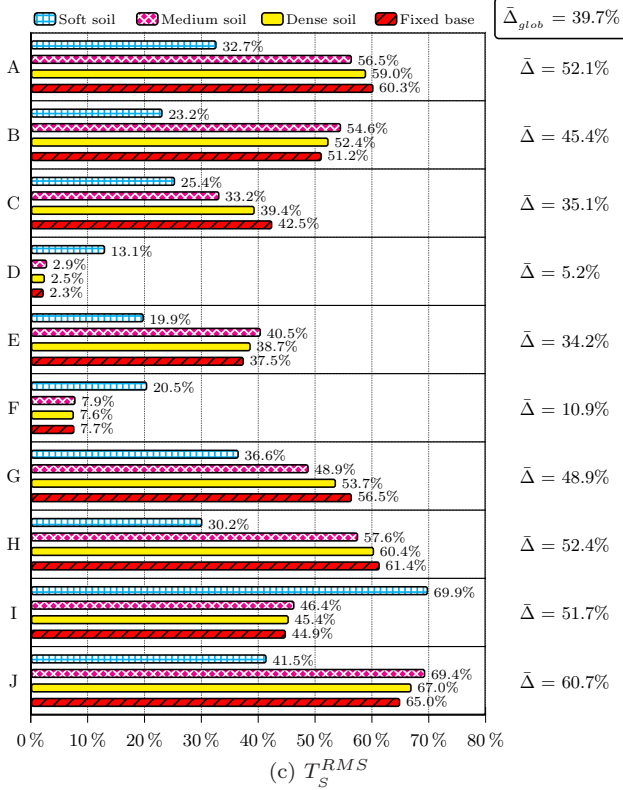
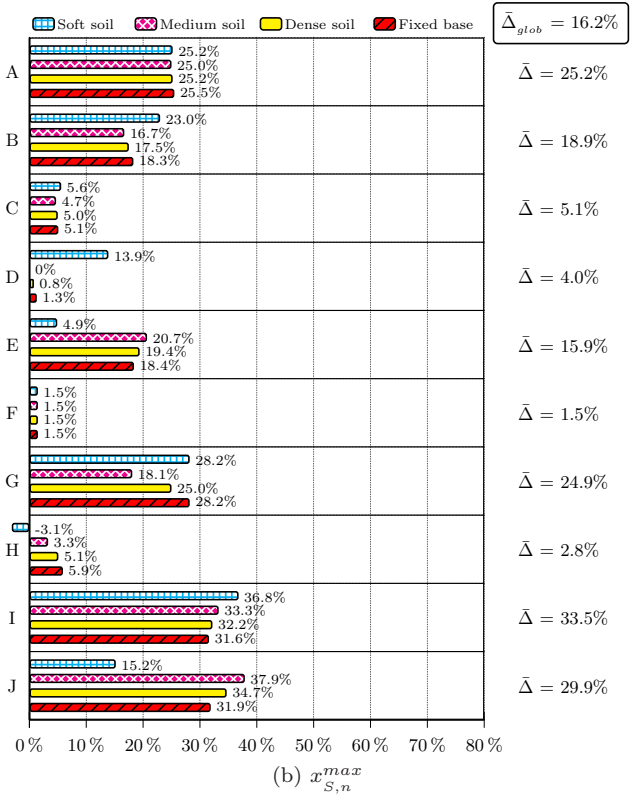
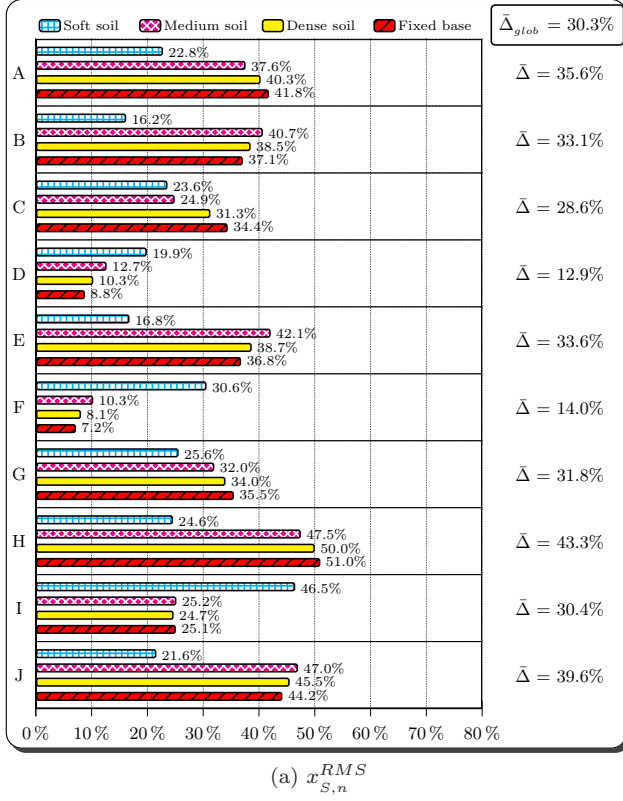


Figure 4: Seismic response reduction in terms of (a),(b) top-storey displacement (first row) and (c),(d) kinetic energy (second row), in terms of (a),(c) RMS (first column) and (b),(d) maximum (second column) indices, for the *low-rise five-storey building* (mass ratio $\mu = 0.02$, first-mode inherent damping ratio $\zeta_{S,1} = 0.02$). Frame on (a) highlights the targeted objective function.

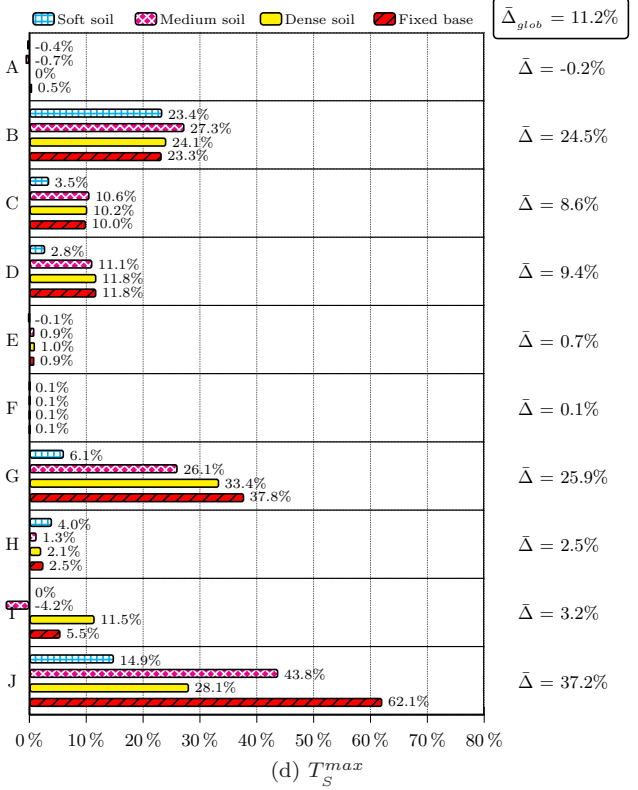
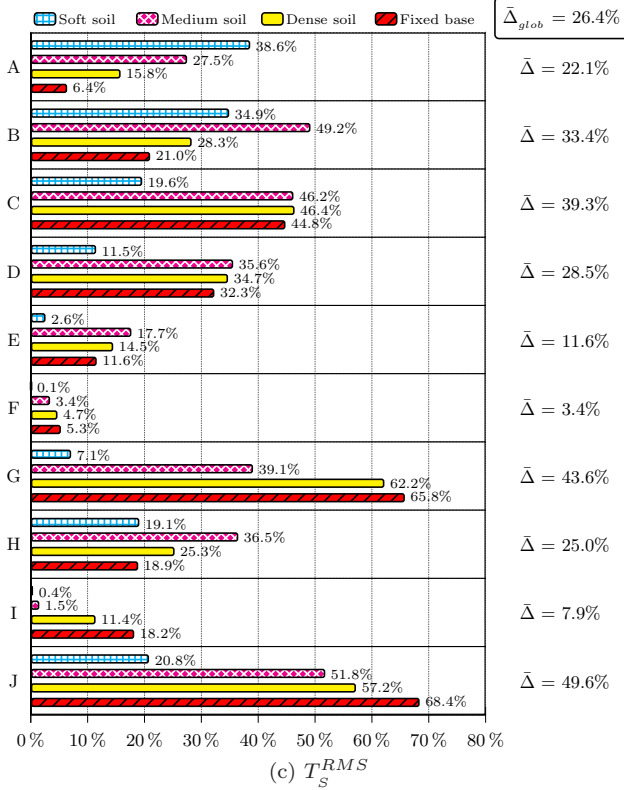
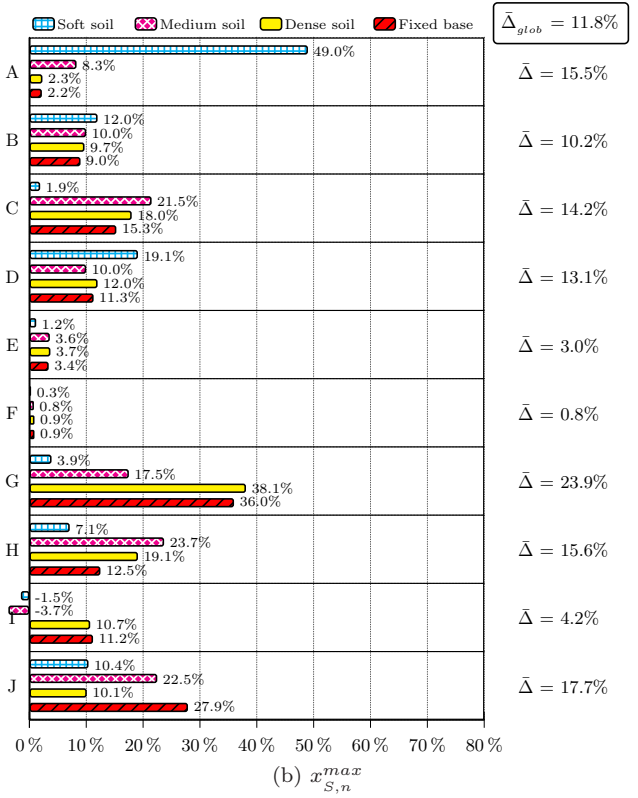
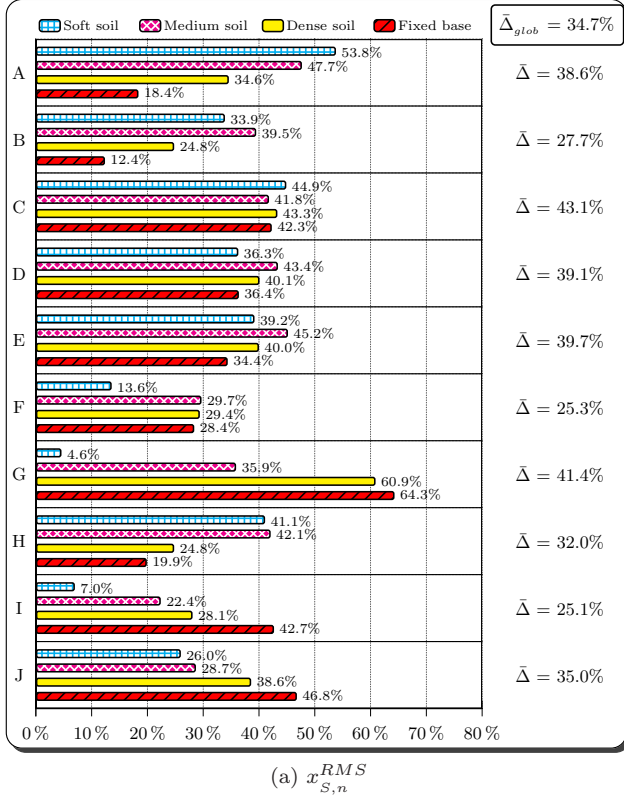


Figure 5: Seismic response reduction in terms of (a),(b) top-storey displacement (first row) and (c),(d) kinetic energy (second row), in terms of (a),(c) RMS (first column) and (b),(d) maximum (second column) indices, for the *high-rise forty-storey building* (mass ratio $\mu = 0.02$, first-mode inherent damping ratio $\zeta_{S,1} = 0.02$). Frame on (a) highlights the targeted objective function.

3.4. Time-history features of the TMD-controlled seismic response

Figs. 6–7 and 8–9, and Figs. 10–11 and 12–13, further display samples of the top-storey displacement and kinetic energy *time-domain responses* of the primary structure, for the low-rise five-storey frame and the high-rise forty-storey frame, respectively (see Figs. 4–5 for the related response indices), subjected to earthquakes A and J (Table 6), respectively. These sample response time histories provide further complementary and significant hints on the achieved TMD effectiveness and variation of recorded seismic response after TMD insertion.

These two selected earthquake input signals have been chosen as source representative ones, according to the TMD effectiveness results already reported in above Section 3.3 as:

- Earthquake A leading to a noticeable response decrease on all the inspected indices, for the low-rise five-storey building and for the targeted RMS displacement of the high-rise forty-storey frame, however with a much lower effectiveness on untargeted indices, specifically maximum ones and for the maximum kinetic energy index, despite for spikes of great TMD control for the soft-soil case (except again on maximum kinetic energy response), which constitutes a peculiar feature that shall be truly shown out from the various cases of analysis;
- Earthquake J leading to a rather compact and homogeneous TMD effectiveness, readable on all the inspected instances of earthquake and soil types, and for both the low-rise five- and high-rise forty-storey building, with specific unscheduled good outcomes also on the untargeted kinetic energy response indices, and on both (targeted) RMS and (untargeted) maximum values.

Overall, the majority of structural response reduction occurs after a first time interval that the TMD claims, as a typical passive control device, for its seismic “activation”, whose duration depends on the properties of the primary structure (shorter duration for the stiffer five-storey frame, longer duration for the softer forty-storey frame), beyond which the benefit of the TMD control device addition becomes visible, especially in terms of average values all along the considered excitation time window. This holds particularly true for the high-rise forty-storey frame, where the seismic response reduction becomes more visible from about one quarter of the excitation time, specifically for the seismic excitation due to earthquake A (see Figs. 8–9).

The soft-soil case for earthquake A (Figs. 6a–7a) looks as that where the seismic response mitigation is smaller for the five-storey frame, whereas it appears rather remarkable, similarly to the other soil cases, for the forty-storey frame (Figs. 8a–9a), while for earthquake J (Figs. 10a–11a and 12a–13a) the TMD effectiveness becomes lower than that for the other soil types, for both the considered buildings. However, this constitutes also the instance where the original response, i.e. that of the primary structure alone, was the lowest among that of the structure for all the soil cases, and therefore where the need for the TMD insertion appears smaller (the earthquake being not that “critical” in such a sense [40]).

A global picture of impressive seismic TMD effectiveness is wholly pointed out by the inspection of Figs. 10–11 and 12–13, for the earthquake J case, showing out in a sense this seismic excitation to be a “critical” one, for both the considered structures, in leading on one hand to high uncontrolled seismic structural response and then to a low TMD controlled seismic vibration. This really shows the potential effectiveness of the TMD insertion in this challenging vibration control context, possibly affected by SSI effects.

In general, when a good or even outstanding TMD performance is achieved, the case of a soft soil is often that displaying the lower TMD efficiency rate and, overall, this is the soil type that shows much different trends, with respect to those recovered for the other cases (also on the achieved TMD optimisation parameters). This fact could mean that a soft soil shall somehow be able to already provide an intrinsic response reduction by itself, bringing then to a “lower connection” to the structural system **and making somehow less “critical” a given earthquake excitation.**

Hence, the TMD seems to work just as a sort of complementary measure, in that situation, despite for the achieved TMD optimisation (namely, a lower performance for a soft soil is not linked here to a de-tuning, since the present optimisation anyway provides the optimum TMD performance, on average structural response indices, for each considered case).

All this confirms a specificity of SSI effects on TMD tuning, effectiveness and control of seismic time-history response for the soft-soil case. Moreover, the evaluation of the features displayed by the time-history seismic responses suggests that the flexibility of the primary structure within the SSI effect indeed plays a key role, first by importantly affecting the optimum TMD parameters and performance, and second in terms of recorded seismic response variation.

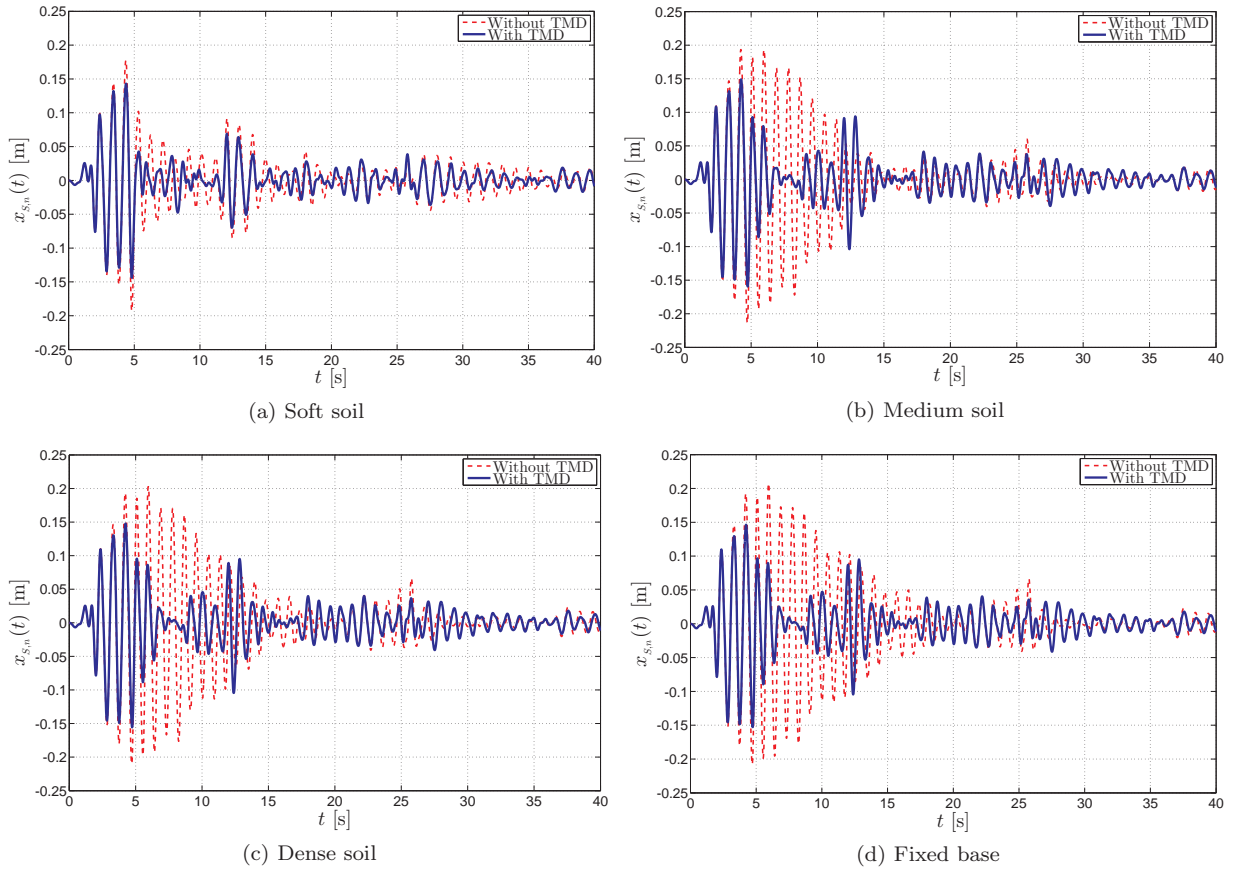


Figure 6: Time-domain displacement of the top storey $x_{S,n}(t)$ for the *low-rise five-storey frame* (mass ratio $\mu = 0.02$, first-mode inherent damping ratio $\zeta_{S,I} = 0.02$), subjected to *earthquake A* (Table 6), for the four different considered soil cases (Tables 2–4).

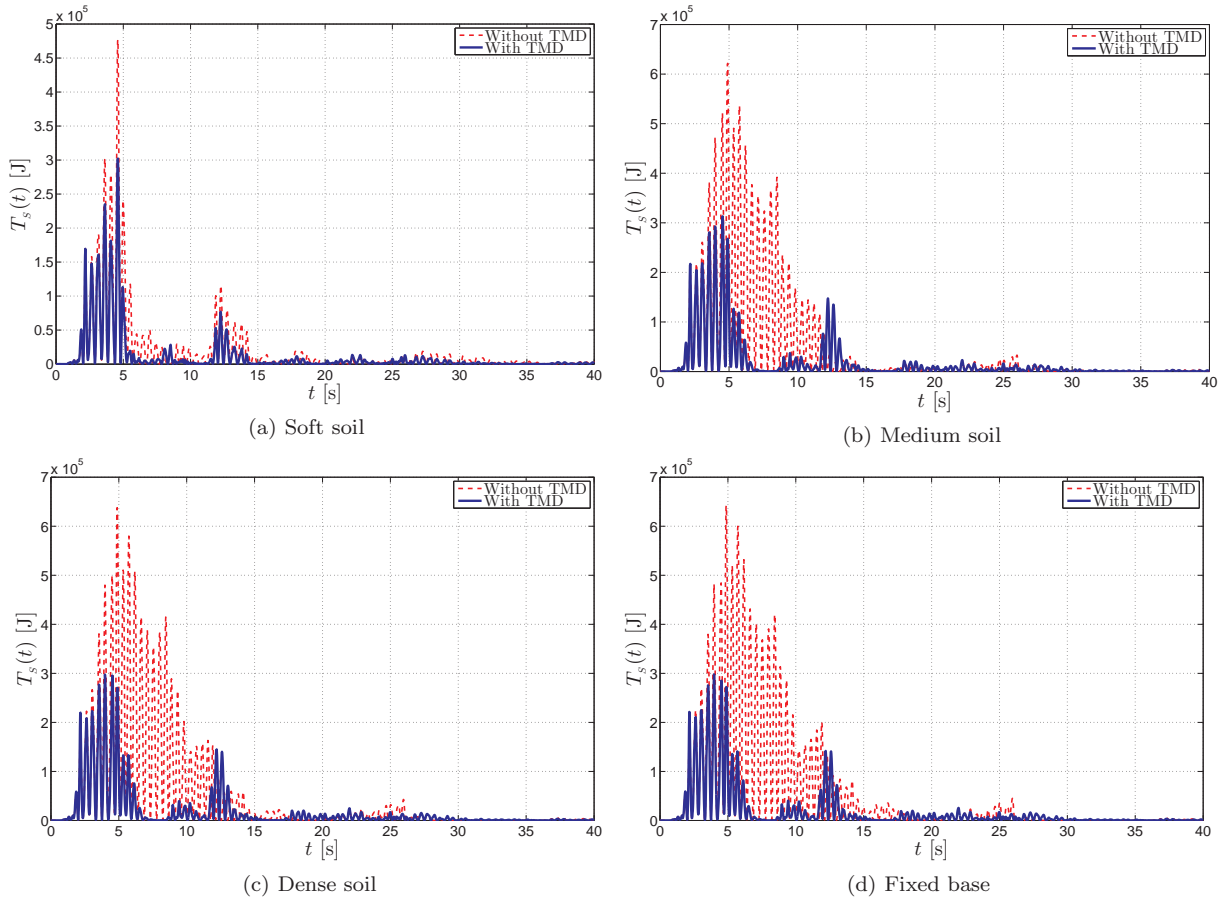


Figure 7: Time-domain kinetic energy of the primary structure $T_S(t)$ for the *low-rise five-storey frame* (mass ratio $\mu = 0.02$, first-mode inherent damping ratio $\zeta_{S,I} = 0.02$), subjected to *earthquake A* (Table 6), for the four different considered soil cases (Tables 2–4).

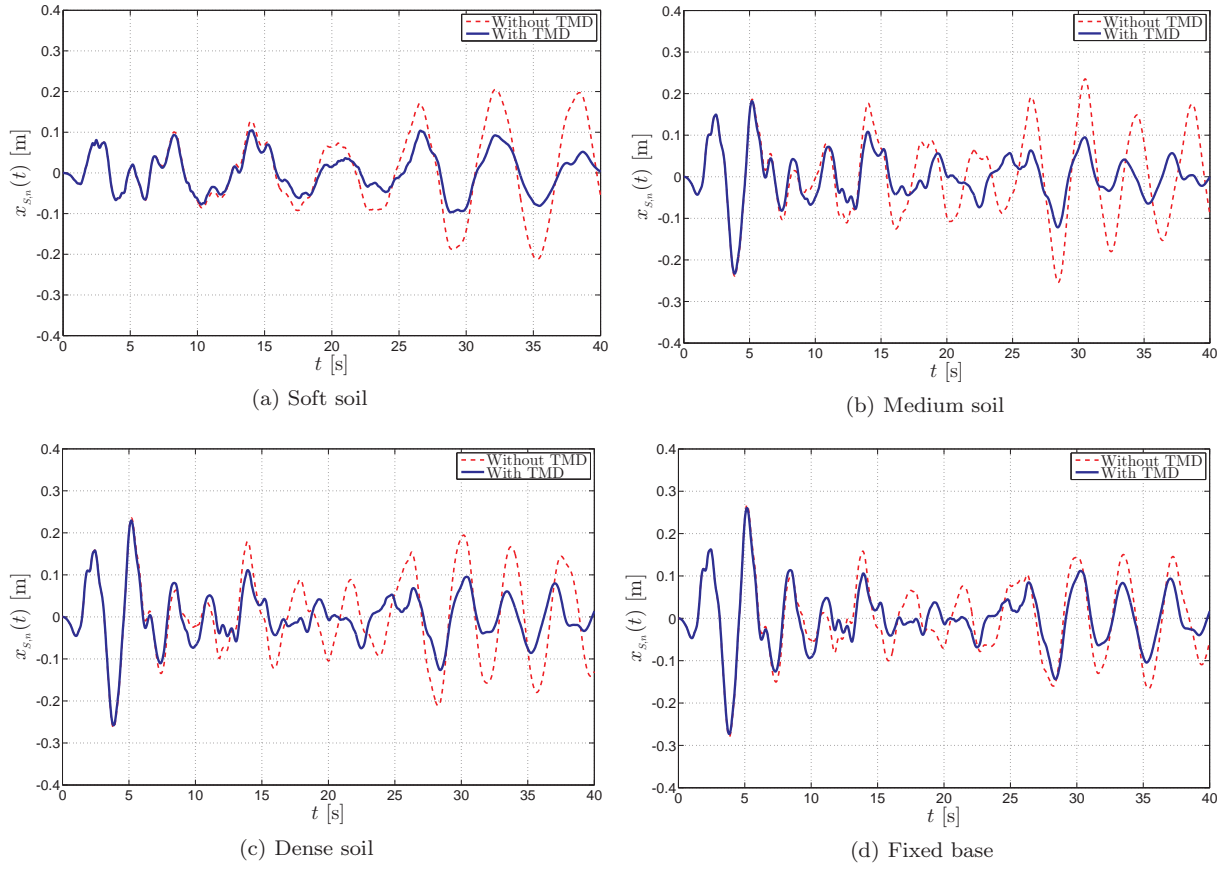


Figure 8: Time-domain displacement of the top storey $x_{s,n}(t)$ for the *high-rise forty-storey frame* (mass ratio $\mu = 0.02$, first-mode inherent damping ratio $\zeta_{s,I} = 0.02$), subjected to *earthquake A* (Table 6), for the four different considered soil cases (Tables 2–4).

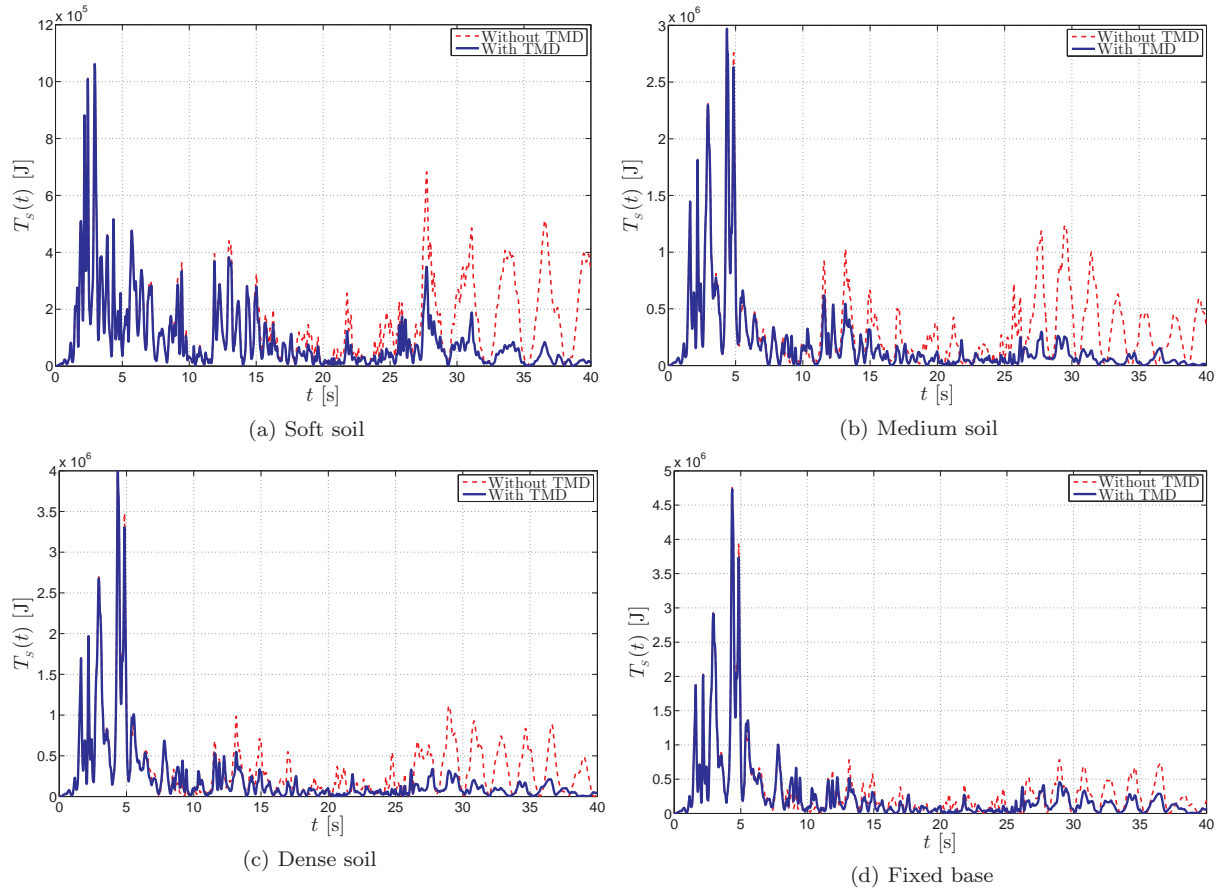


Figure 9: Time-domain kinetic energy of the primary structure $T_s(t)$ for the *high-rise forty-storey frame* (mass ratio $\mu = 0.02$, first-mode inherent damping ratio $\zeta_{s,I} = 0.02$), subjected to *earthquake A* (Table 6), for the four different considered soil cases (Tables 2–4).

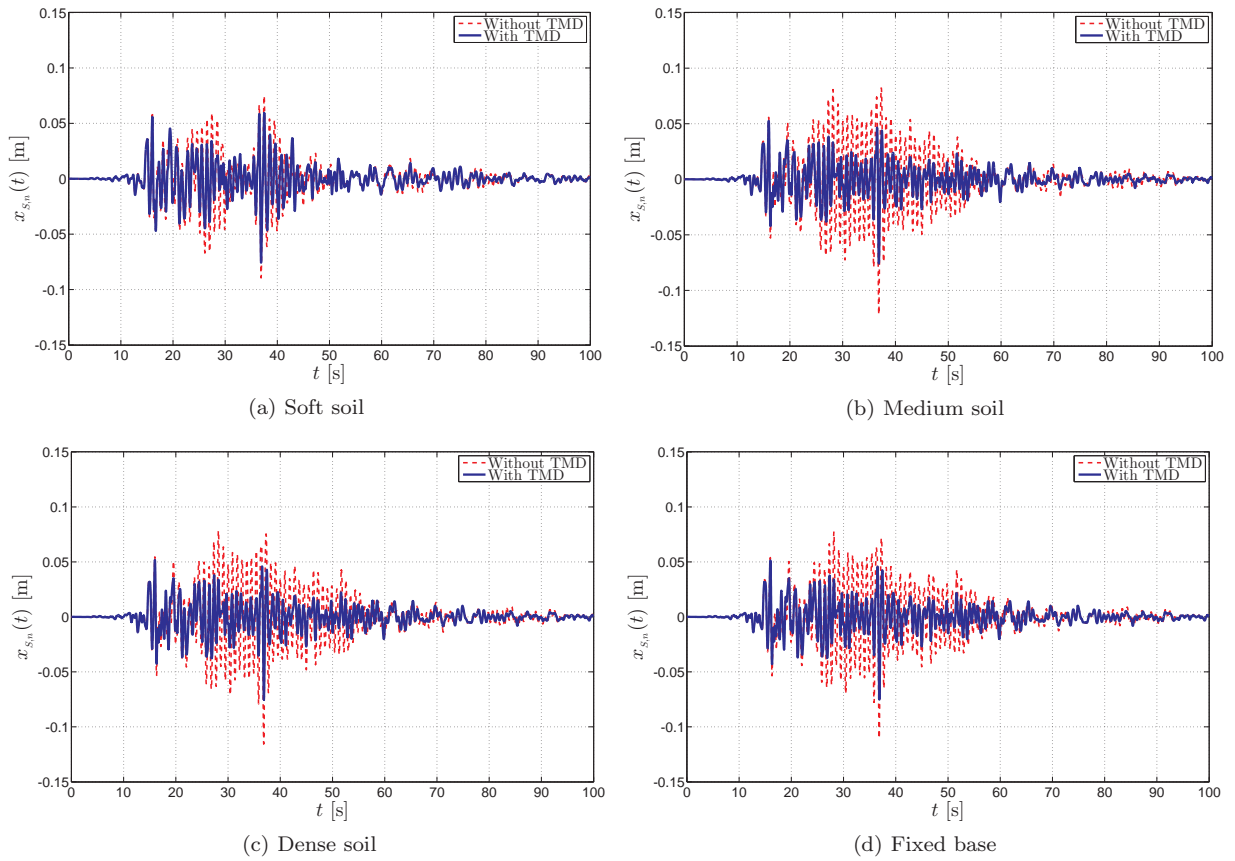


Figure 10: Time-domain displacement of the top storey $x_{s,n}(t)$ for the *low-rise five-storey frame* (mass ratio $\mu = 0.02$, first-mode inherent damping ratio $\zeta_{S,I} = 0.02$), subjected to *earthquake J* (Table 6), for the four different considered soil cases (Tables 2–4).

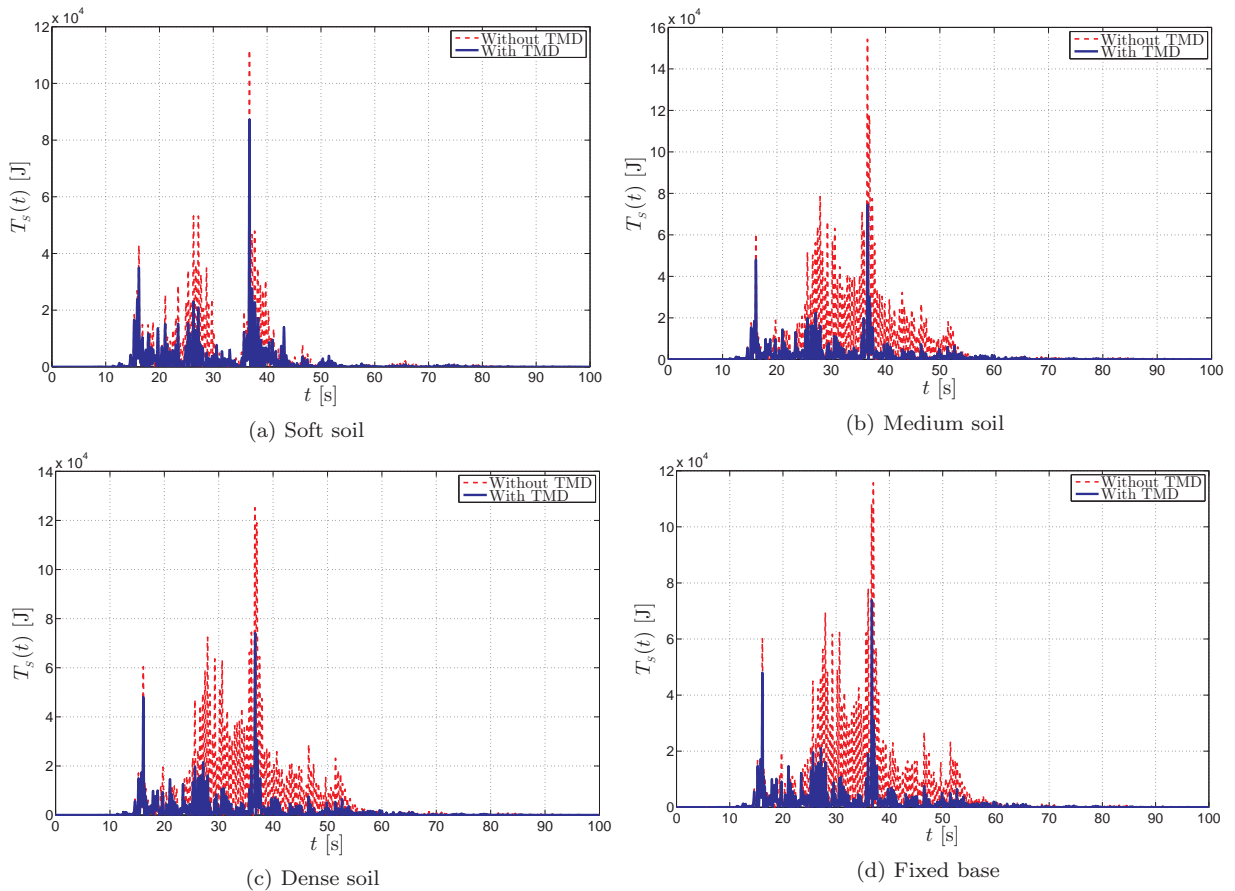


Figure 11: Time-domain kinetic energy of the primary structure $T_s(t)$ for the *low-rise five-storey frame* (mass ratio $\mu = 0.02$, first-mode inherent damping ratio $\zeta_{S,I} = 0.02$), subjected to *earthquake J* (Table 6), for the four different considered soil cases (Tables 2–4).

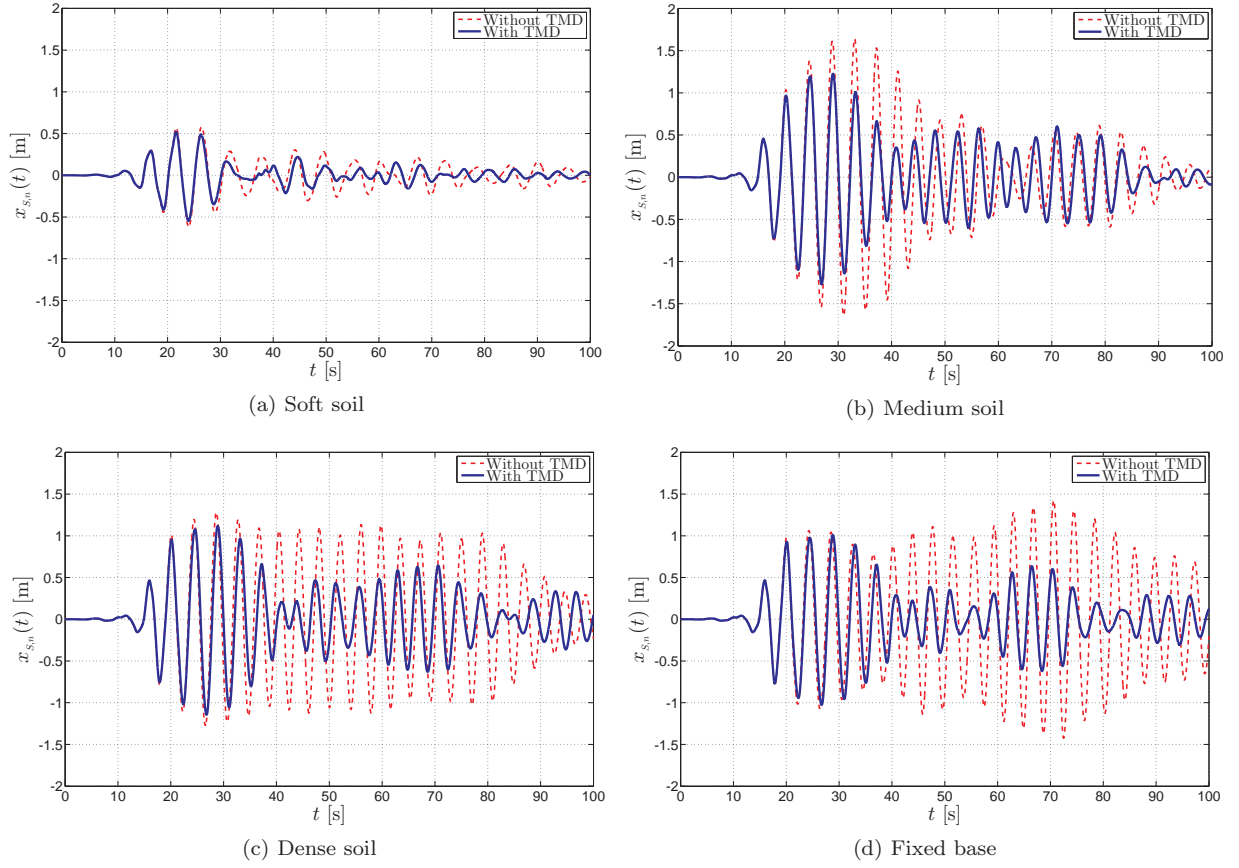


Figure 12: Time-domain displacement of the top storey $x_{S,n}(t)$ for the *high-rise forty-storey frame* (mass ratio $\mu = 0.02$, first-mode inherent damping ratio $\zeta_{S,I} = 0.02$), subjected to *earthquake J* (Table 6), for the four different considered soil cases (Tables 2–4).

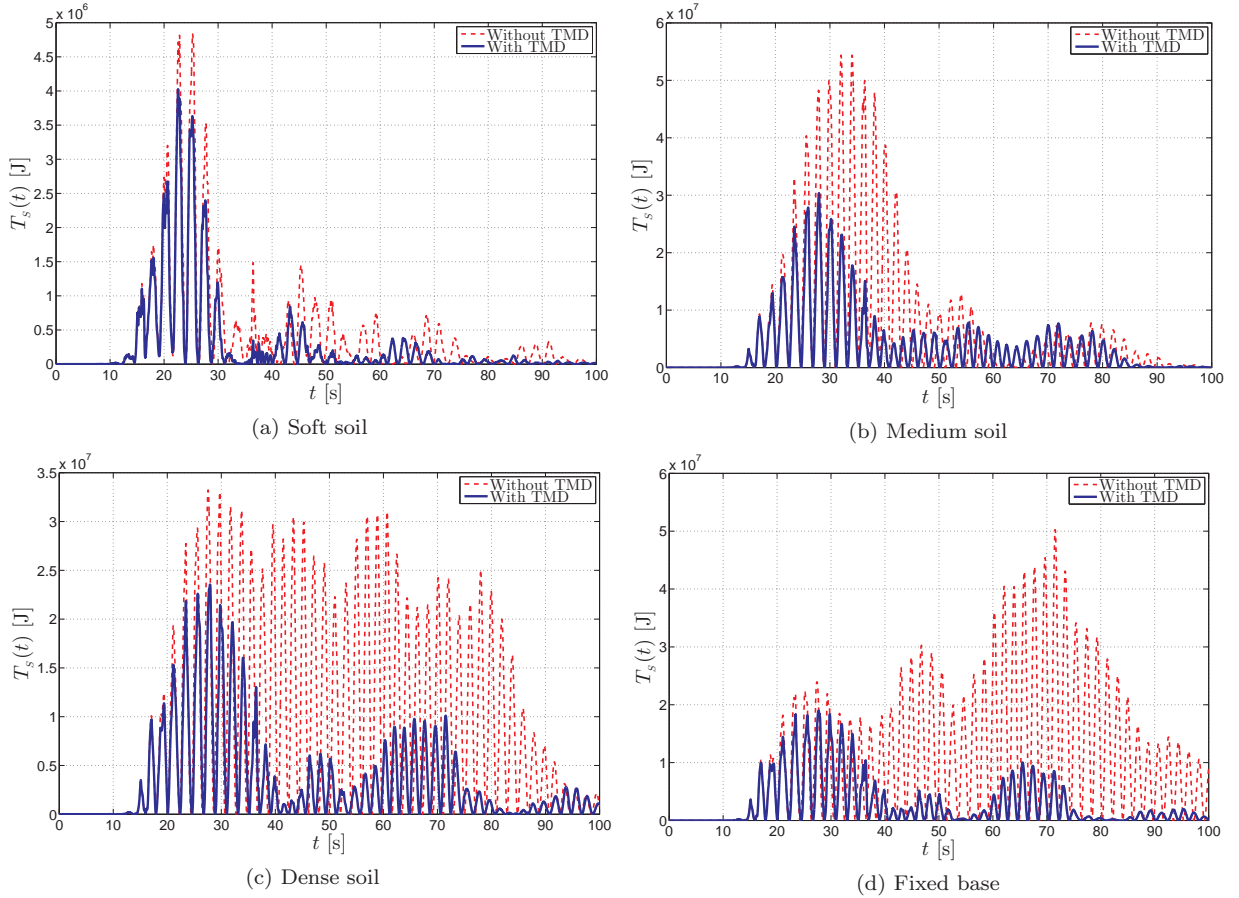


Figure 13: Time-domain kinetic energy of the primary structure $T_S(t)$ for the *high-rise forty-storey frame* (mass ratio $\mu = 0.02$, first-mode inherent damping ratio $\zeta_{S,I} = 0.02$), subjected to *earthquake J* (Table 6), for the four different considered soil cases (Tables 2–4).

4. Additional results on the control of the foundation movement

¹This section provides additional numerical results concerning the achievable control of foundation movement by TMD insertion. Further output on the recorded foundation vibration reduction and relevant time histories is first provided in Section 4.1. Then, a specific particular case of the considered SSI model in Fig. 1, and attached TMD optimisation study, is further developed in Section 4.2.

4.1. Controlled foundation response

The response of the foundation, for the three deformable soil types, is further inspected for a single earthquake instance, namely earthquake J (Katmandu, 2015) in Table 6, for the low-rise five-storey building. Optimum TMD parameters and vibration reduction results are gathered in Table 7; time-histories sway/rocking responses of the foundation are depicted in Fig. 14.

Obviously, the foundation response decreases at stiffening soil type (going from first row to third row in Fig. 14). A few orders of magnitudes less (up to three) are recorded, with respect to the primary structure displacement response earlier depicted in Fig. 10. Thus, the foundation response is much contained with respect to that. So, what it looks crucial, also for the optimisation process, is the primary structure displacement relative to the foundation, which then confirms to be a feasible target choice for the definition of the objective function. In fact, optimisation and control results do not change much if terms from the foundation response were included within the objective function. Indeed, a few separate runs have been performed, targeting either sway, rocking, or total structure displacement, without appreciable differences to the outcomes here reported, as obtained from the relative structural displacement taken as only response index within the objective function.

Despite for the lower recorded magnitude of the foundation movements, the benefit of the added TMD is still visible, for the considered case. Quantitative results on response reduction can be read on Table 7, with percentage reductions that are of a similar amount to that achieved for the targeted primary structure displacement of the top storey relative to the foundation. Reductions are rather similar for primary structure relative displacement and foundation rocking, and a bit smaller for foundation sway, ranging from about 47% (on targeted RMS) and 22%, to 38% and 17%, respectively. Effectiveness in terms of peak response is also visible, ranging from about 13% to 42%. A lower performance appears to be obtained for the soft soil case; similar achievements are recorded for the medium and dense soil cases. However, notice that, in terms of structural response of the Structure-TMD system, the TMD is apt to flatten out the kinematic response (especially on u_{ST}^{max} , but also on u_{ST}^{RMS}), at variable soil type; thus, despite for showing less percentage reductions of response, for the soft soil, the achieved amount of control is wholly the same, the soil already playing as a sort of isolation device in that sense, making the specific earthquake excitation “*less critical*”, as already above commented.

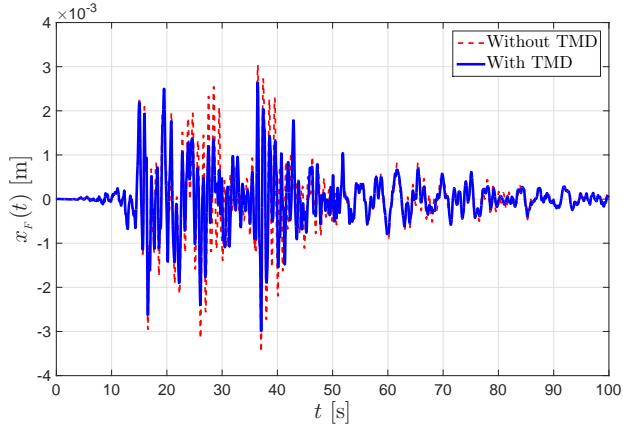
4.2. Study of a specific translational SSI SDOF model

Now, further consider a stiff building installed on a swaying deformable soil. The structure displays an infinite stiffness. Thus, no relative to the ground displacements of the building are produced. The rocking stiffness of the soil is assumed to be infinite; thus, rocking motion of the foundation is suppressed. Then, the only degree-of-freedom of the whole mechanical system is constituted by the horizontal sway displacement of the foundation, as common to the total displacement of each floor of the supported building. Such a SSI system is then represented by a translational SDOF system under earthquake excitation. Its mass is the total mass of the building and of the foundation. Its stiffness and damping are the horizontal stiffness and damping of the soil, respectively. An added TMD on the building + foundation system can be employed to abate the horizontal vibration of such a considered system. The TMD may display different mass ratios.

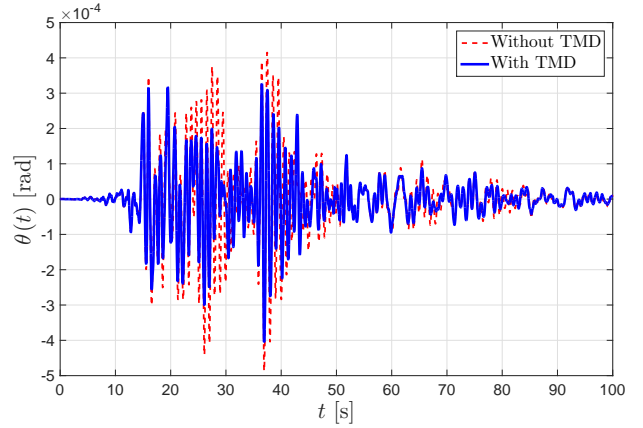
Here, the mass of the superstructure is represented by that of the low-rise five-storey building, namely $m_{S,tot} = 5 \times 3 \cdot 10^5 \text{kg} = 15 \cdot 10^5 \text{kg}$ [6]; the corresponding mass of the foundation is taken as $m_F = 3 \cdot 10^5 \text{kg}$ (Table 3); for a total mass of the Structure-Foundation system of $m_{SF} = m_{S,tot} + m_F = 18 \cdot 10^5 \text{kg}$. The soil takes properties from the three considered instances of deformable soil, namely stiffness $k_{s,h}$ and damping $c_{s,h}$ in Table 4 (for the five-storey building).

However, about damping, this would lead to very heavy (approaching critical) apparent damping ratios of the soil-foundation system of: $\zeta_{SF} = c_{s,h} / (2\sqrt{k_{s,h} m_{SF}}) = 66.18\%$, 67.76% , 72.65% , for the three soil cases, which would be quite unrealistic, for the considered investigation analysis, especially in terms of TMD applicability and effectiveness (TMDs are expected to provide benefit for abating the vibration of lowly-damped mechanical systems). Indeed, according to ASCE standard [46], for low expected shear strains in the soil “*realistic damping values range from 0.5% to 2%. At very low strains, damping should be limited to a maximum of 2%. At large strains, damping is limited to 15%*”. Thus, the above horizontal damping coefficients of the soil have been reduced by two orders of magnitude, i.e. by multiplying the values of $c_{s,h}$ in Table 4 by factor 10^{-2} . Also, a

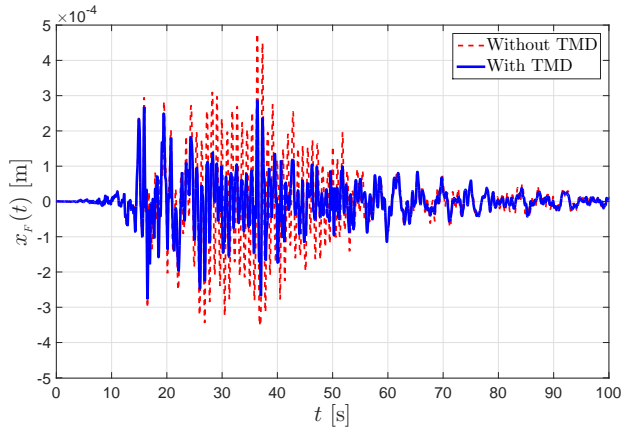
¹This section has been inspired by useful hints emerged from an anonymous reviewer during the review process.



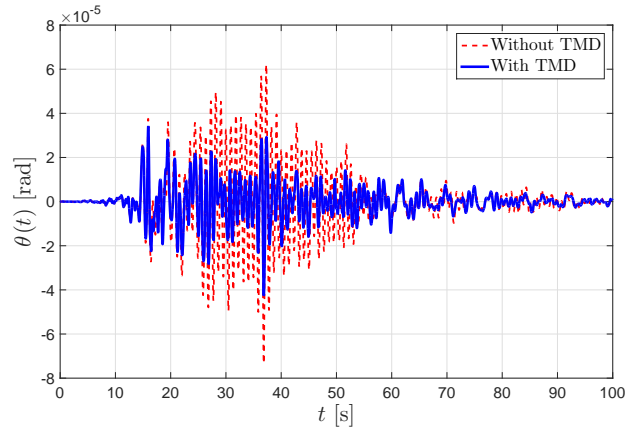
(a) Soft soil - Sway



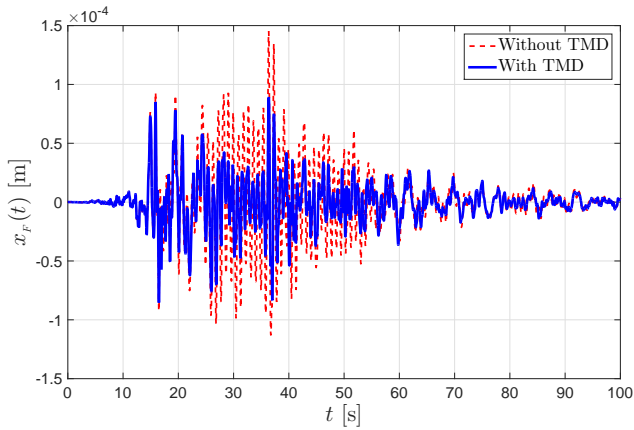
(b) Soft soil - Rocking



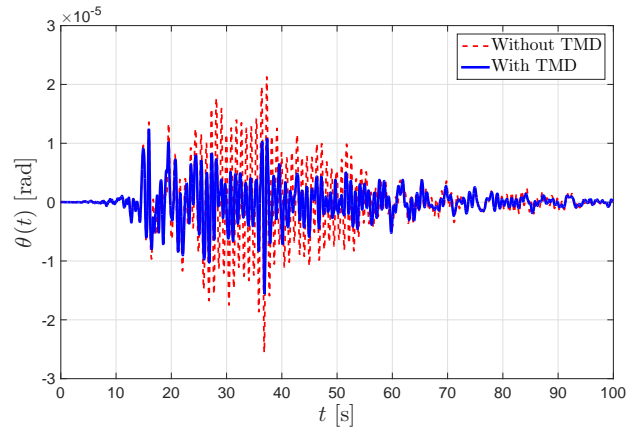
(c) Medium soil - Sway



(d) Medium soil - Rocking



(e) Dense soil - Sway



(f) Dense soil - Rocking

Figure 14: Time-domain foundation movement $x_F(t)$ (Sway, left column) and $\theta(t)$ (Rocking, right column) for the *low-rise five-storey frame* (mass ratio $\mu = 0.02$, first-mode inherent damping ratio $\zeta_{S,I} = 0.02$), subjected to *earthquake J* (Table 6), for the three considered deformable soil (Soft soil - first row; Medium soil - second row; Dense soil - third row) cases (Tables 2–4). Vertical axes are differently scaled for the various soil types, to appreciate responses with different magnitudes. Corresponding top-floor primary structure response in Fig. 10.

Table 7: Optimum TMD parameters and recorded vibration control of primary structure and foundation, for the *low-rise five-storey frame* (mass ratio $\mu = 0.02$, first-mode inherent damping ratio $\zeta_{S,I} = 0.02$), subjected to *earthquake J* (Table 6), for the three considered deformable soil cases (Tables 2–4) and the fixed-base reference. Reference tuning values of TMD parameters: $f_{DH}^{opt} = 0.980392$, $\zeta_{DH}^{opt} = 0.0857493$, from Eq. (12); $f_{WNA}^{opt} = 0.969558$, $\zeta_{WNA}^{opt} = 0.0707107$, from Eq. (13) (see also global results reported in Fig. 3a).

Variable	Soft soil	Medium soil	Dense Soil	Fixed-base
TMD				
f^{opt}	0.936241	0.946888	0.947241	0.947269
ζ_T^{opt}	0.0653374	0.0440916	0.0439379	0.0438136
m_{TMD}^{opt} [kg]	10688.5	10688.5	10688.5	10688.5
k_{TMD}^{opt} [N/m]	456975	467427	467776	467803
c_{TMD}^{opt} [Ns/m]	9132.63	6233.05	6213.64	6196.24
Structure				
u_S^{max} [m]	0.0892631	0.121745	0.115441	0.110413
u_{ST}^{max} [m]	0.0757166	0.0755974	0.0753280	0.0752408
u_S^{RMS} [m]	2.25461	3.05096	2.93591	2.84439
u_{ST}^{RMS} [m]	1.76662	1.61835	1.59906	1.58802
Δ^{max} [%]	15.18	37.91	34.75	31.86
Δ^{RMS} [%]	21.64	46.96	45.53	44.17
Sway				
u_F^{max} [m]	0.00346694	0.000476079	0.000145053	-
u_{FT}^{max} [m]	0.00299725	0.000285999	0.0000890023	-
u_F^{RMS} [m]	0.0957417	0.0135140	0.00411333	-
u_{FT}^{RMS} [m]	0.0796717	0.00828259	0.00258218	-
Δ^{max} [%]	13.55	39.93	38.64	-
Δ^{RMS} [%]	16.78	38.71	37.22	-
Rocking				
θ_F^{max} [rad]	0.000487206	0.0000733010	0.0000255571	-
θ_{FT}^{max} [rad]	0.000404377	0.0000426060	0.0000155859	-
θ_F^{RMS} [rad]	0.0141569	0.00201535	0.000711476	-
θ_{FT}^{RMS} [rad]	0.0110448	0.00107625	0.000390288	-
Δ^{max} [%]	17.00	41.88	39.02	-
Δ^{RMS} [%]	21.98	46.60	45.14	-

further reference analysis within the theoretical condition of zero damping is performed, in order to investigate the maximum ideal effectiveness of TMD control achievable in the present setting. The single earthquake excitation of Katmandu, 2015 (earthquake J, Table 6) is still considered.

The following shows how the proposed TMD tuning optimisation approach can be applied to such a considered SSI system, in order to further and simultaneously address the following issues:

- Further inspect and control the amount of sway displacement of the foundation at variable SSI effects. Since all along the paper the primary structure displacement relative to the foundation has been taken (as an average norm) as a target within the objective function, the previous optimisation process was focused just on the building movement drift from the foundation, rather than on the foundation vibration. Now, given the rigid link between floor displacement and foundation displacement, a similar optimisation model focused on the common horizontal sway displacement in the objective function may account for the optimisation of the foundation horizontal displacement, by the addition of a TMD controller on such a building + foundation system.
- Further investigate the possibility to set different TMD mass ratios. The whole preceding extensive parametric study has been set with a reasonable, feasible value of mass ratio, namely $\mu = 2\%$. Here, two different values of mass ratio are considered, namely $\mu = 2\%$, 5% , to also appreciate how this may change the outcomes of the vibration control process, at variable (three) soil characteristics (soft, medium and dense soil), in terms of horizontal sway displacement of the building + foundation system.

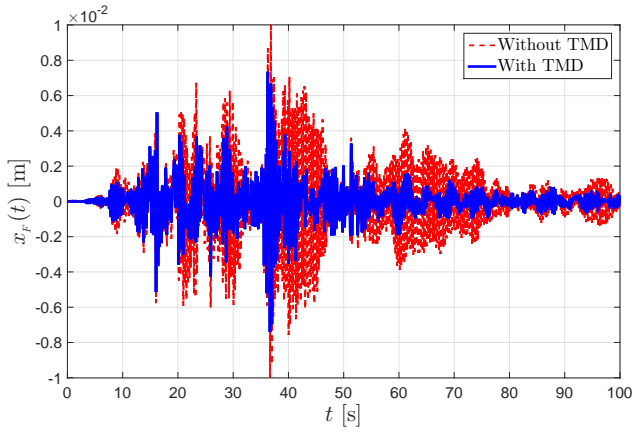
- Further compare the outcomes of the TMD seismic optimisation process, with target formulas from the literature (Eqs. (12) and (13)), and relevant achieved TMD performance, in also reading and interpreting the movement of the foundation.

Table 8: Optimum TMD parameters and recorded vibration control of the considered *lightly damped* horizontal sway SDOF building-foundation system with TMD.

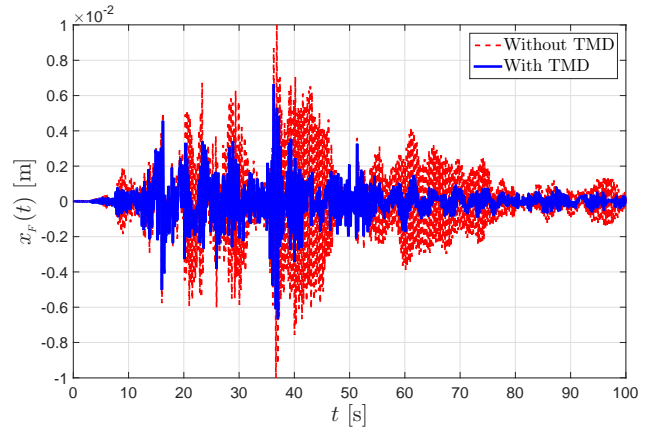
Variable	Soft soil	Medium soil	Dense Soil
$\mu = 0.02$			
TMD			
f^{opt}	0.951823	0.947575	0.964502
ζ_T^{opt}	0.0775688	0.0745865	0.0638461
m_{TMD}^{opt} [kg]	36000	36000	36000
k_{TMD}^{opt} [N/m]	17279354	161621632	534762384
c_{TMD}^{opt} [Ns/m]	122358	359825	560268
Structure-Sway			
u_{SF}^{max} [m]	0.0101795	0.000333000	0.000117650
u_{SFT}^{max} [m]	0.00740315	0.000316839	0.0000994131
u_{SF}^{RMS} [m]	0.269158	0.00837131	0.00262648
u_{SFT}^{RMS} [m]	0.143157	0.00780429	0.00237135
Δ^{max} [%]	27.27	4.85	15.50
Δ^{RMS} [%]	46.81	6.77	9.71
$\mu = 0.05$			
TMD			
f^{opt}	0.893619	0.900167	0.931830
ζ_T^{opt}	0.112971	0.136506	0.0911027
m_{TMD}^{opt} [kg]	90000	90000	90000
k_{TMD}^{opt} [N/m]	38076777	364635101	1247866539
c_{TMD}^{opt} [Ns/m]	418260	1563979	1930930
Structure-Sway			
u_{SF}^{max} [m]	0.0101795	0.000333000	0.000117650
u_{SFT}^{max} [m]	0.00667664	0.000332306	0.000101742
u_{SF}^{RMS} [m]	0.269158	0.00837131	0.00262648
u_{SFT}^{RMS} [m]	0.130816	0.00802020	0.00241971
Δ^{max} [%]	34.41	0.21	13.52
Δ^{RMS} [%]	51.40	4.19	7.87

The additional results for the horizontal building + foundation SDOF system are reported in Table 8 and in Fig. 15. Table 8 first gathers the values of the optimum seismic TMD parameters, as compared to the reference ones coming from Eqs. (12) and (13), for the three soil types (J_{WNA}^{opt} takes three values, depending on the above apparent damping ratio of the lightly damped SSI system):

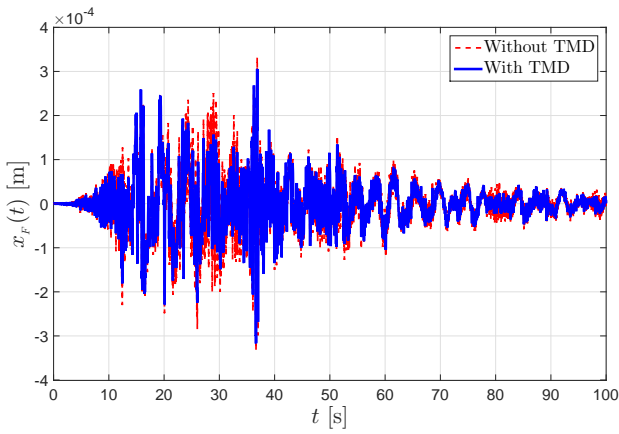
- for $\mu = 0.02$:
 $f_{DH}^{opt} = 0.980392$ and $\zeta_{DH}^{opt} = 0.0857493$;
 $f_{WNA}^{opt} = 0.974475, 0.974416, 0.974236$ and $\zeta_{WNA}^{opt} = 0.070711$;
- for $\mu = 0.05$:
 $f_{DH}^{opt} = 0.952381$ and $\zeta_{DH}^{opt} = 0.133631$;
 $f_{WNA}^{opt} = 0.938421, 0.938328, 0.938044$ and $\zeta_{WNA}^{opt} = 0.111803$.



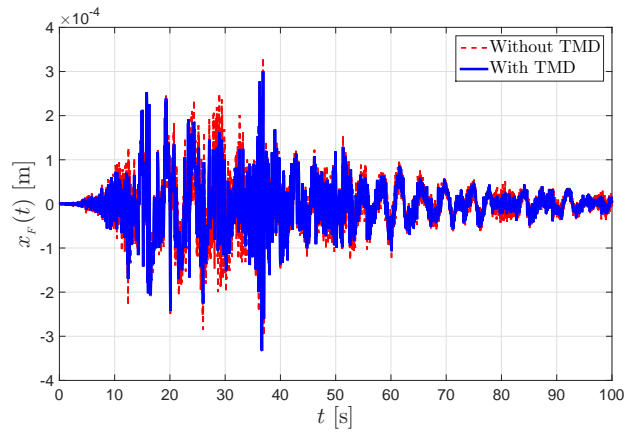
(a) Soft soil - $\mu = 0.02$



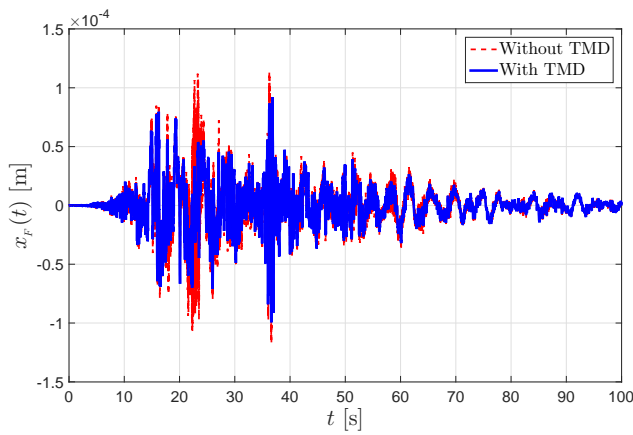
(b) Soft soil - $\mu = 0.05$



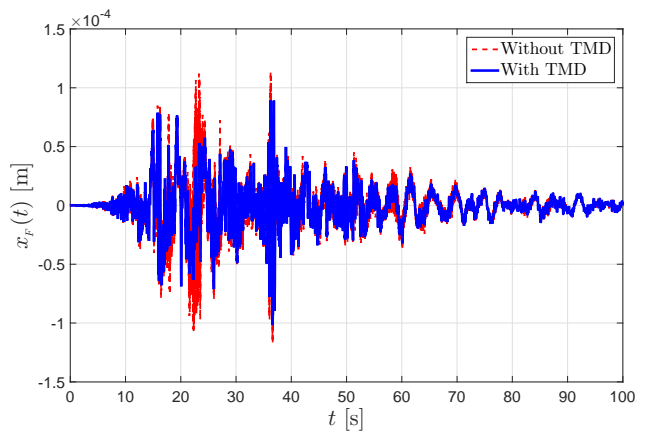
(c) Medium soil - $\mu = 0.02$



(d) Medium soil - $\mu = 0.05$



(e) Dense soil - $\mu = 0.02$

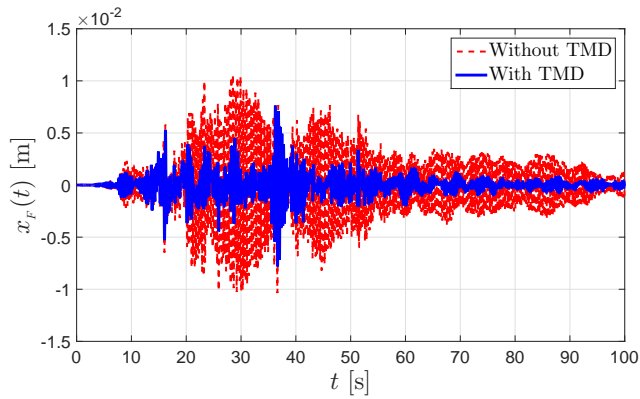


(f) Dense soil - $\mu = 0.05$

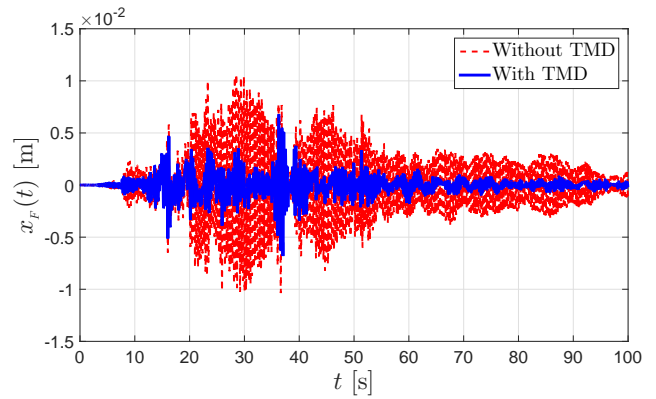
Figure 15: Time-history response of the considered *lightly damped* horizontal sway SDOF building-foundation system with TMD, for the three considered deformable soil (Soft soil - first row; Medium soil - second row; Dense soil - third row) cases and assumed TMD mass ratios ($\mu = 0.02$ - left column, $\mu = 0.05$ - right column).

Table 9: Optimum TMD parameters and recorded vibration control of the considered *undamped* horizontal sway SDOF building-foundation system with TMD.

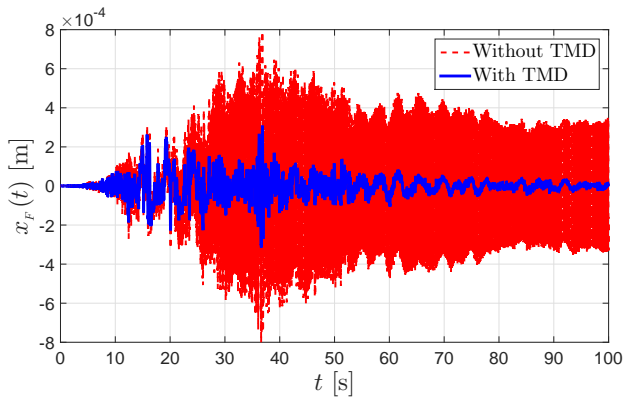
Variable	Soft soil	Medium soil	Dense Soil
$\mu = 0.02$			
TMD			
f^{opt}	0.958728	0.952598	0.968705
ζ_T^{opt}	0.0759823	0.0745327	0.0646579
m_{TMD}^{opt} [kg]	36000	36000	36000
k_{TMD}^{opt} [N/m]	17530977	163339714	539433072
c_{TMD}^{opt} [Ns/m]	120725	361472	569864
Structure-Sway			
u_{SF}^{max} [m]	0.0106618	0.000810846	0.000220832
u_{SFT}^{max} [m]	0.00780412	0.000314964	0.0000998698
u_{SF}^{RMS} [m]	0.411862	0.0357515	0.0120805
u_{SFT}^{RMS} [m]	0.149755	0.00782850	0.00238097
Δ^{max} [%]	26.80	61.16	54.78
Δ^{RMS} [%]	63.64	78.10	80.29
$\mu = 0.05$			
TMD			
f^{opt}	0.901142	0.906856	0.935304
ζ_T^{opt}	0.112968	0.132510	0.0920743
m_{TMD}^{opt} [kg]	90000	90000	90000
k_{TMD}^{opt} [N/m]	38720602	370074168	1257188136
c_{TMD}^{opt} [Ns/m]	421773	1529476	1958798
Structure-Sway			
u_{SF}^{max} [m]	0.0106618	0.000810849	0.000220832
u_{SFT}^{max} [m]	0.00680789	0.000332795	0.000101950
u_{SF}^{RMS} [m]	0.411862	0.0357515	0.0120805
u_{SFT}^{RMS} [m]	0.134457	0.00803194	0.00242323
Δ^{max} [%]	36.15	58.96	53.83
Δ^{RMS} [%]	67.35	77.53	79.94



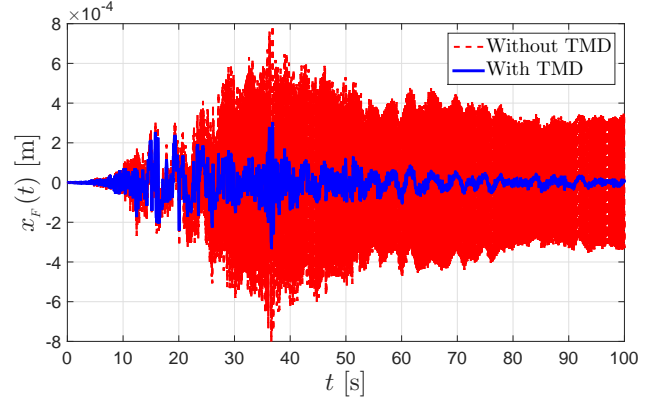
(a) Soft soil - $\mu = 0.02$



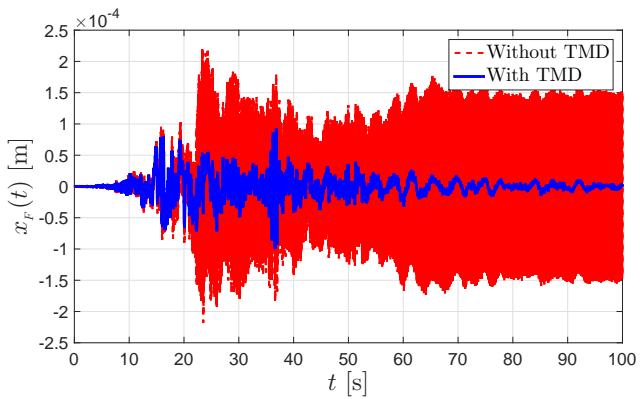
(b) Soft soil - $\mu = 0.05$



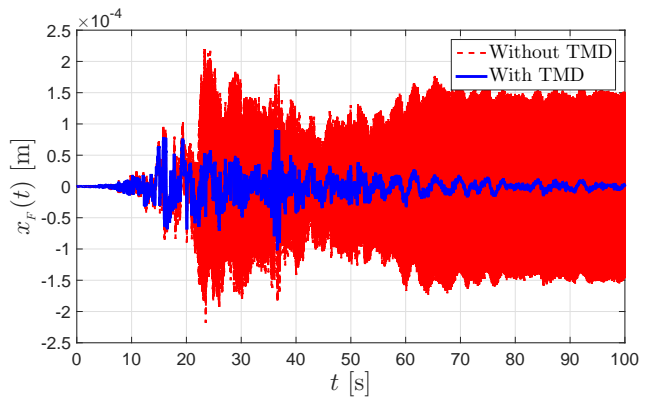
(c) Medium soil - $\mu = 0.02$



(d) Medium soil - $\mu = 0.05$



(e) Dense soil - $\mu = 0.02$



(f) Dense soil - $\mu = 0.05$

Figure 16: Time-history response of the considered *undamped* horizontal sway SDOF building-foundation system with TMD, for the three considered deformable soil (Soft soil - first row; Medium soil - second row; Dense soil - third row) cases and assumed TMD mass ratios ($\mu = 0.02$ - left column, $\mu = 0.05$ - right column).

Then, Table 8 reports the amount of achieved horizontal sway displacement response reduction (RMS and max values). Fig. 15 depicts the time-history variation of the seismic horizontal displacement, to further appreciate the achieved amount of TMD effectiveness, for the two assumed mass ratios. The outcomes globally confirm the ability of the TMD device to effectively control the movement of the building-foundation system, with RMS horizontal displacement reductions up to 47% for mass ratio $\mu = 2\%$ and 51% for mass ratio $\mu = 5\%$, for the considered values of system damping ratios. However, effectiveness is here rather visible only for the soft soil case, which leads to a conspicuous amount of vibration; indeed, vibration for the medium and dense soil cases is much more contained, and TMD effectiveness reduces down even below 10%. The worse case of TMD performance is recorded for the medium soil, non-monotonically setting in between the soft and dense soil cases. In practice, the outcomes are overturned, with respect to the earlier sway case, accounting also for building drift (and higher soil damping), where less gain of vibration reduction was obtained for the soft soil. Anyway, the case of soft soil always plays a different role with respect to those manifested by the medium and dense soils, which keep leading to similar responses. Moreover, no big differences are recorded for the two considered values of the mass ratio, showing the TMD being not that prone to effectively act on an infinitely stiff building, which does not develop an additional drift from the foundation movement.

Further results are also reported for the ideal instance of no inherent damping of the mechanical system (here potentially coming only from the soil), in view of exploring the maximum amount of achievable TMD effectiveness. In such an undamped case, the reference optimum TMD values for frequency ratio f_{WNA}^{opt} modify as follows (the other being unchanged): for $\mu = 0.02$, $f_{WNA}^{opt} = 0.976906$; for $\mu = 0.05$, $f_{WNA}^{opt} = 0.942265$. Final results are newly reported in Table 9 and in Fig. 16. Indeed, percentage reductions raise to rather high values, not only in terms of RMS but also on peak values, with a much visible effectiveness over the whole time windows of analysis, for all the considered soil cases, and two inspected mass ratios. Again, no much differences are recorded for the two different mass ratios. Monotonic trends of percentage response reduction are observed, on RMS indices, at variable soil type, raising from about 64-67% to 80%. Peak response reductions are also high, with maximum values in the order of 60%, for the medium soil, which leads to a better performance on peak norm. The mechanical system displays a high capability of control, despite for being in the absence of structure drift from the foundation (stiff building), although at zero inherent damping overall involved into the source mechanical system.

5. Conclusions

In the present paper, the influence of SSI effects on the seismic performance of optimum-tuned TMDs has been extensively investigated, by exploiting a specific TMD seismic tuning numerical optimisation methodology based on average (RMS) response indices of the primary structure, and by considering different primary MDOF structures (low- and high-rise buildings), soil cases (including a comparative analysis with respect to the fixed-base instance) and earthquake events, on a rather wide analysis of *eighty seismic TMD-SSI optimisation instances*. This has been scrutinised in view of outlining general and average trends on TMD optimisation and achieved seismic TMD effectiveness in the presence of SSI.

First, the optimum TMD parameters have been obtained for each considered case, as a result of an extensive *ad hoc* two-variable optimisation process targeted on the top-storey RMS superstructure displacement. Related optimisation results pointed out several important indications on TMD tuning in this context, which may be briefly summarised as follows:

- *Primary structure and soil case*: the assumption of SSI produces a much more visible effect on the frequency ratio, rather than on the TMD damping ratio. In particular, for the former parameter, at soil stiffness softening the value sensibly decreases, for both the adopted primary structures but especially for the high-rise forty-storey frame, by taking an average value (among all the assumed earthquake events) even lower than 0.7, thus quite far from a traditional expected value at around 1. Thus, the presence of a soft soil may require a dedicated TMD frequency ratio tuning, based on the specific soft-soil properties;
- *Inherent structural damping and TMD reference values*: besides from the exception above (on TMD frequency ratio for the soft soil case, markedly more for the high-rise frame), the achieved optimum TMD parameters turn out rather close to those coming from known tuning formulas, much for those also contemplating the presence of structural damping. This holds specifically true for the TMD damping ratio, as an average among the various considered earthquake instances, which also appears as just slightly sensitive to the characteristics of the soil. Mean values always turn out lower than those obtained by classical Den Hartog's formulas and nearer to a tuning proposal in [4] deduced for White-Noise base Acceleration. A wider spread is obtained for the high-rise vs. the low-rise building case.

Second, the response reduction rate after the achieved optimum-tuned TMD insertion has been detected by specifically measuring four different kinematic response indices, namely top-storey displacement and primary

structure kinetic energy, recorded as either RMS or peak evaluation (after a TMD optimisation targeted on RMS top-storey displacement only). The following indications may be highlighted from a comprehensive analysis of the achieved output:

- *Primary structure*: the low-rise five-storey frame (natural periods in the range of 0.90-0.12 s) displayed a higher benefit from the TMD insertion than that recorded for the high-rise forty-storey building (natural periods in the range of 3.83-0.46 s), as a stiffer, low-rise, shear-type frame;
- *Index estimate*: in all response records, including those that were not adopted as objective function (kinetic energy of the primary structure), the RMS estimate always displays a sensibly larger reduction than for the peak evaluation. This denotes a better TMD efficiency on the global response than on a single peak of oscillation, which confirms a known outcome from previous TMD seismic studies [1, 5] that did not take into account SSI effects so far;
- *Earthquake event*: the influence of the seismic input into the TMD behaviour appears quite evident, since the majority of the considered earthquake signals thoroughly provides high rates of TMD effectiveness, while a few events (especially D and F, Table 6) correspond to a reduced TMD effect;
- *Soil case*: SSI effects clearly influence the effectiveness outcome of the TMD. Specifically, the soft soil remarkably represents a very different case with respect to all the other instances, independently of the achieved rate of TMD efficiency.

Third, in general terms, from the comprehensive TMD-SSI optimisation methodology and computational analysis it is revealed that:

- The TMD effectiveness is rather appreciable, on average, over all the considered cases, for the considered realistic values of mass ratio and structural damping ratio referred to the first mode, both of 2%, thus quite appropriate to truly represent real cases. Average reductions of 30% vs. 35% are recorded for the low-rise vs. the high-rise building, on the targeted RMS top-storey displacement response index; lower values of 16% vs. 12% are recovered for the maximum displacement index. Kinetic energies show remarkable reductions for the low-rise vs. the high-rise case, of 40% vs. 26%, on RMS, and 28% vs. 11% on peak value.
- The TMD generally appears to provide more benefit on the seismic response reduction in case of a stiffer superstructure and of a medium and a hard soil, as clearly pointed out by the obtained outcomes. This fact strongly motivated the introduction of SSI effects within both the present tuning process and the post-tuning seismic performance analysis, and may lead to unexpected modifications of the best setup of TMDs. The control device shall be characterised by lower stiffnesses in the case of a soft soil, thus far from the resonance conditions for the fixed-base model. Indeed, for the latter case the host structure looks somehow more “connected” to the soil, which acts like an implicit damper, therefore making the TMD just as a sort of complementary control device in such a case;
- Another important issue, already detected in previous works [1, 5], not considering SSI effects, is represented by the connection between the modal characteristics and the features of the earthquake events, as proven by the large difference of the modal parameters, followed by a respective difference of TMD effect between the two considered primary structures. The main example in this sense concerns the more flexible forty-storey building, with high natural periods, less sensitive (on non-objective function indices) to the TMD insertion, especially when earthquakes with low-period PGA occur (e.g. like F, Table 6), able to induce just a small seismic vibration.

The present TMD-SSI optimisation analysis has been basically focused on the monitoring and control of the seismic response of the primary superstructure. The additional analysis in Section 4 has firstly investigated and inspected the effective possibility to also control the movement of the foundation, within the SSI context. Further attention on the response and role of the soil, in participating and affecting the overall picture could be placed on additional trials that may also systematically consider the soil response within the TMD tuning optimisation process (namely within the assumed objective function), possibly by a multi-optimisation task. This may constitute the subject of subsequent research investigations.

In closing, it could be globally affirmed that the TMD appears able to provide a remarkable seismic response reduction *when it really appears necessary towards that*, concerning the seismic effect on the host structure and the contribution of the soil characteristics in affecting the whole seismic response under SSI effects. Thus, this should generally support the adoption of such an easily accessible passive control device to achieve a basic seismic protection and vibration mitigation in this context, despite for the presence of potential SSI effects, which may require a dedicated TMD tuning, specifically in the case of a soft soil-foundation system.

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