Estimating the probability of a multiple default using CDS and bond data

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February 28, 2013

Abstract

The on-going EU sovereign debt crisis is causing great concern about the sustainability of national debt issued by the member states. In this paper, we propose a methodology to estimate the likelihood of the default of one or more countries in the Euro Area by extending the approach in Pianeti *et al.* (2012) to the case of multiple defaults. We provide an assessment of the marginal, the joint and the conditional default probabilities within the Euro Zone. The adopted measure of systemic risk is the probability of a joint default of the EU countries over a 5 years' time horizon. We find evidence of increasing systemic risk and danger of contagion from early-2007 and more significantly from late 2011 onwards. We show the forecasting capability of the estimated default probabilities through a comparison with the dynamics of a benchmark stock market index which marked the timeline of the recent sovereign debt crisis.

JEL Classification Numbers: G01, G32.

Keywords: Joint default probability, EU sovereign risk, Early warning signals, Counterparty risk, Credit default swaps.

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1 Introduction

The determination of joint default probabilities is of extreme importance for its implications in credit risk management as well as in the process of assessing systemic risk. In the light of the recent events, the financial literature has focused its attention on the latter aspect, in particular.

There are several methodological approaches to measure systemic risk. In a first line of research, a systemic event is induced by severe disequilibria within the banking sector (see for instance Lehar, 2005, Huang *et al.*, 2009 and 2012, Adrian and Brunnermeier, 2011). A second line of research focuses on global market dynamics (rather than limited to the financial sector) as primary source of financial instability (Schwaab *et al.*, 2011, De Nicolò and Lucchetta, 2011, Hollo *et al.*, 2012).

In this paper, we measure the systemic risk of a financial system, where systemic risk is defined as multiple simultaneous defaults of large financial institutions, consistently with the first line of research. In this context, different systemic measures such as the Stability Index (SI), the Distressed Insurance Premium (DIP), the Systemic Expected Shortfall (SES) and the Conditional Value at Risk (CoVaR) have been proposed. Segoviano and Goodhart (2009) introduce the Stability Index (SI), which reflects the expected number of institutions becoming distressed given that at least one institution has become distressed. Huang et al. (2009 and 2012) measure systemic risk by means of the Distress Insurance Premium (DIP), defined as the premium required to cover distressed losses for a given pool of banking institutions. Acharya *et al.* (2010) introduce the Systemic Expected Shortfall (SES) of a bank as the expected shortfall on the bank equity value, conditioned on the materialization of a loss triggered by a systemic event. Adrian and Brunnermeier (2011) propose the concept of CoVar, defined as the financial sector's Value at Risk (VaR) given that an institution has incurred in a VaR loss.

The literature has addressed the estimation of the joint default probability with several contributions. Zheng(2012) proposes a copula contagion mixture model for correlated default times. Goodhart and Segoviano (2009) extract from market data marginal probabilities of default for a pool of banks and employ the Consistent Information Multivariate Density Optimizing (CIMDO) methodology in Segoviano (2006) to derive the probability of all the banks in the pool going into default, providing at the same time a measure of the tail risk of the system. In the light of the ongoing European sovereign-debt crisis, much of the focus has been drawn on applications to the Euro Area. Zhang *et al.* (2012) propose a model to estimate the joint default of Euro Area countries. They model the differences between perceived costs and benefits of the single countries default and correlate them by means of macro-factors as well as by a risk factor common to all the countries. A single country defaults when such difference exceeds a threshold calibrated on CDS data. They estimate marginal, multivariate and conditional probability of default via simulation. Radev (2012) introduces the concept of "change in the conditional joint probability of default" for a pool of EU sovereign entities and banks, as a new systemic risk measure for the EU area, where the joint default probability is estimated via the CIMDO approach in Segoviano (2006).

Our methodology aims at extracting market-implied probabilities of joint default. Both bond and credit derivative markets convey information on the default process: the former provides information on the *marginal* default probabilities whilst the latter on the *joint* default probabilities. Applying a no-arbitrage argument, Pianeti *et al.* (2012) derive a formula for the joint default probability for couple of institutions operating in the CDS market. In particular, they extract the joint default probability of the reference entity α and the protection seller β of a CDS contract. Giglio (2011) goes a step further: he extracts from market data the joint default probabilities for the couple (α, β) and proposes a linear programming model to derive upper and lower bounds for the joint default probability of N banks belonging to a network. Both the methodologies are in line with the literature on the Credit Value Adjustment (CVA) of derivative contracts in presence of counterparty risk[1]. The importance of adjusting the value of a derivative contract to take into account the likelihood of default of a counterparty in financial derivatives, has also been emphasized within the new Basel III regulatory framework.

In this paper, we extend the methodology proposed in Pianeti *et al.* (2012) in a multivariate setting and apply it to measure the sovereign debt risk in the Euro Zone. Our results is particularly suitable to assess the systemic consequences of the recent period of financial instability in Europe. We measure systemic risk as the probability of a joint default of the EU countries over a 5 years' time horizon. We use the result in Pianeti *et al.* (2012) to infer information about the joint default correlations of the single states with the representative protection seller in the sovereign CDS market. The defaults of the states are then correlated through their common dependence on the protection seller, typically a top tier investment bank, which represents the financial sector, considered as the channel through which crises evolve and spread out. Our methodology is based on the hypothesis of absence of arbitrage, hence our estimates are produced under the risk-neutral probability measure. In our setup, risk-neutral probabilities offer a conservative indication on real-world probabilities and as such, can be used to identify periods of systemic risk and contagion. We consider Radev (2012) as a benchmark, as one of the contributions of that paper is to apply a consolidated methodology like CIDMO on our same problem.

The reminder is as follows. Section 2 explores the relation between the CDS-bond basis and counterparty risk. In Section 3 we set up a theoretical framework to estimate the joint default probabilities of multiple entities. Section 4 proposes the empirical application referred to the Euro Zone and Section 5 concludes.

2 The informational content of the CDS-bond basis

In this section, we analyse the relation between bond and CDS prices, providing a survey of the literature on the topic. Furthermore, we set up the model framework and present a methodology to infer information on the counterparty risk from the CDS-bond basis.

As a first approximation, CDS prices reflect the expected loss of the reference entity given by its default probability and the recovery rate. These factors are actually the same that influence bond spreads: theoretically bond spreads should be equal to CDS premia for the same reference entity (see Duffie, 1999). For the sake of illustration, consider two financial agents α and β and let:

- $r(t,T)$ be the risk-free rate in t for the maturity T.
- $y_\alpha(t,T)$ be the yield at time t on a zero-coupon bond (ZCB) issued by α with maturity date T.
- $s_\alpha(t,T) \equiv y_\alpha(t,T) - r(t,T)$ be the spread over the risk-free rate of the issuance cost of α, prevailing in t and referred to the maturity T.
- $w_{\alpha,\beta}(t,T)$ be the periodic CDS premium to insure against the default of α within the period $[t,T]$ with β as the protection seller.

[1] The literature on this topic is too wide to survey here. See the seminal work of Jarrow and Turnbull (1995), the contribution of Hull and White (2001).

In equilibrium, a portfolio composed by a ZCB with maturity T and a CDS on that same bond with the same maturity, should replicate a synthetic risk-free asset. Hence the ZCB yield $y_\alpha(t,T)$ minus the CDS premium $w_{\alpha,\beta}(t,T)$ should be exactly equal to the risk-free rate $r(t,T)$. The invoked equilibrium is ensured by the two following arbitrage strategies[2]:

Strategy 1. *Case* $w_{\alpha,\beta}(t,T) < s_\alpha(t,T)$: the arbitrage strategy in this case consists in buying the bond, financing at the risk-free rate $r(t,T)$ and then buying the CDS by paying the premium $w_{\alpha,\beta}(t,T)$. The portfolio return is $y_\alpha(t,T) - r(t,T) - w_{\alpha,\beta}(t,T) = s_\alpha(t,T) - w_{\alpha,\beta}(t,T) > 0$.

Strategy 2. *Case* $w_{\alpha,\beta}(t,T) > s_\alpha(t,T)$: the arbitrage strategy in this case consists in short selling the bond, investing the proceeds at the risk-free rate of return $r(t,T)$ and selling protection in the CDS market to get the premium $w_{\alpha,\beta}(t,T)$. The portfolio return is $w_{\alpha,\beta}(t,T)+r(t,T)-y_\alpha(t,T) = w_{\alpha,\beta}(t,T)-s_\alpha(t,T) > 0$.

However, in reality $w_{\alpha,\beta}(t,T) \neq s_\alpha(t,T)$. We then define as "basis" the difference $b_{\alpha,\beta}(t,T) \equiv w_{\alpha,\beta}(t,T) - s_\alpha(t,T)$. Multiple are the factors that may give rise to a basis different from zero. O'Kane and McAdie (2001) and De Wit (2006) identify a comprehensive list of factors which cause the basis to differ from zero, and distinguish the factors which are technical in nature from the fundamental ones. The theoretical and applied literature on the topic insists in particular on few of them, that are:

(1) The counterparty risk which affects CDS contracts, consisting in the possibility that the protection seller β might not respect its obligations (see inter alia Pianeti *et al.*, 2012, Bai and Collin-Dufresne, 2011),

(2) Liquidity reasons that cause investors to prefer a riskless bond to a corporate bond plus a CDS or vice versa (Trapp, 2009),

(3) Market frictions that cause the above Strategies 1 and 2 to be asymmetric, thus undermining the role of arbitrageurs in reverting the basis towards 0 (Bai and Collin-Dufresne, 2011).

The case of the basis between sovereign CDS and the corresponding government bonds makes no exception and the influence of counterparty risk, liquidity and funding on the basis has been documented for the EU by Fontana and Scheicher (2010)).

Other market imperfections might cause the basis to be positive or negative. As an example of these, we can mention the different reactivity of CDS and bond markets to new information on a issuer. A negative or positive basis can reflect a different degree of adjustment between the two markets that arbitrage strategies correct only in the long run. A large body of literature has shown that CDS corporate markets have a leading position in the price discovery process i.e. the CDS prices variation anticipate the variations in bond prices which react with a temporary lag (see Amadei at al., 2011 and for the case of EU sovereign risk Palladini and Portes, 2011).

In the following, we concentrate our attention on counterparty risk, which is not present in bond markets, but crucially affects CDS markets; when counterparty risk increases the CDS premium decreases. However, CDS contracts traded in the OTC markets are increasingly subject to collateralization agreements, which are intended to mitigate the effect of counterparty risk. We take this extent into explicit consideration in our modelling framework by extracting from the basis information referred to counterparty risk. In order to take

[2] We assume the possibility of borrowing and lending at the risk free rate. The portfolio payoffs are guaranteed for each strategy if and only if the positions are kept until bond maturity or until the credit event occurs. Otherwise the strategy faces a roll over risk in the financing/investing positions linked to the volatility of $r(t,T)$.

into account the influence of liquidity on the basis, instead of modelling an unobservable liquidity process as in Giglio (2011), we normalize the observed quotes by means of the bid ask spreads, which is a direct measure of the liquidity linked to these contracts. The impact of the cost of funding when implementing Strategy 2 above, is also taken into account. We derive an expression for the component of the basis which corresponds to the manifestation of counterparty risk (see Appendix A for details):

$$B_{\alpha,\beta}(t,T) \equiv \frac{1}{1-q_c}\left(w'_{\alpha,\beta}(t,T) - s'_{\alpha}(t,T) - \delta(t,T)\right)^{-} \tag{1}$$

where $q_c \in [0,1)$ is the average quota of the CDS notional assisted by collateralization, $w'_{\alpha,\beta}(t,T)$ and $s'_{\alpha}(t,T)$ are the liquidity adjusted CDS and bond spread, respectively. $\delta(t,T)$ represents the spread of the Libor rates over the depo rates and arises from the relaxation of the hypothesis that market participants can equally borrow and lend at the risk free rate. We apply the operator $(\cdot)^{-} \equiv \min(\cdot,0)$ to rule out possible positive values of the basis associated with market imperfections.

3 A formula for the joint probability of default

In this section, we recall the methodology introduced by Pianeti *et al.* (2012) for the estimation of the joint probability for couple of financial institutions (Section 3.1). Next, we extend their methodology to the multiple-entity case, with an application to the Euro Zone (Section 3.2).

3.1 *The bivariate case*

Consider two risky financial institutions and denote them as α and β. Imagine that at time $t=0$, a portfolio is built, according to the following uniperiodal strategy:

- Buy a 1-year ZCB issued by α,
- buy a 1-year CDS from β, the protection seller, on the reference entity α,
- finance the positions on the market with a 1-year loan.

Assume that all contracts have a face value of \$1 and let RR_α, RR_β be the recovery rates of α and β, respectively, so that $\$RR_\alpha$ and $\$RR_\beta$ are the amounts recovered when α and β default. When we exclude market imperfections, such as liquidity and funding issues, and in the presence of non collateralized counterparty risk, the portfolio value at time $t=0$ is given by $B_{\alpha,\beta}(0,1)$ (see Eq. (1)).

At time $t=1$, a non-zero cash flow equal to $(1-RR_\alpha)(1-RR_\beta)$ is generated only when both α and β default. Thus, in an arbitrage-free world, it must hold that:

$$P_{\overline{\alpha},\overline{\beta}}(0,1) = \frac{|B_{\alpha,\beta}(0,1)|}{(1-RR_\alpha)(1-RR_\beta)}e^{r(0,1)} \tag{2}$$

where $P_{\overline{\alpha},\overline{\beta}}(0,1)$ is the one-year joint default probability for α and β, with $B_{\alpha,\beta}(0,1) \leq 0$ by definition.

The extension of the formula in Eq. (2) over the generic time sequence $0 = t_0 < t_1 < \ldots < t_M$ is as follows:

$$P_{\overline{\alpha},\overline{\beta}}(t_k,t_{k+1}) = \begin{cases} \tilde{P}_{\overline{\alpha},\overline{\beta}}(0,t_1) & \text{for } k=0 \\ P_{\alpha,\beta}(t_{k-1},t_k)\tilde{P}_{\overline{\alpha},\overline{\beta}}(t_k,t_{k+1}) & \text{for } k=1,\ldots,M-1 \end{cases} \tag{3}$$

in which we define:

$$\tilde{P}_{\overline{\alpha},\overline{\beta}}(t_k, t_{k+1}) \equiv \frac{\Psi(t_k, t_{k+1})}{(1 - RR_\alpha)(1 - RR_\beta)} \quad k = 0, \dots, M-1 \tag{4}$$

where:

$$\Psi(t_k, t_{k+1}) \equiv |B_{\alpha,\beta}(t_k, t_{k+1})| \, (t_{k+1} - t_k) e^{r(t_k, t_{k+1})(t_{k+1} - t_k)} \tag{5}$$

where $B_{\alpha,\beta}(t_k, t_{k+1})$ is the basis expressed in forward terms. $P_{\overline{\alpha},\overline{\beta}}(t_k, t_{k+1})$ is the probability of joint default of α and β for the period (t_k, t_{k+1}), conditional on their joint survivor till time t_k. $\tilde{P}_{\overline{\alpha},\overline{\beta}}$ denotes the unconditional joint default probability, whereas $P_{\alpha,\beta}$ is the joint survivor probability.

All the variables entering the definition of joint default probability stated in Eq. (3) are directly observable on the market, with the only exception of the recovery rates RR_α and RR_β. A logistic transformation of $\Psi(t_k, t_{k+1})$ bounding the probabilities between 0 and 1, naturally leads to the dropping of the dependence of the formulae on arbitrary assumptions about the recovery rates.

The default correlation is estimated via the standard binomial correlation approach:

$$\rho_{\alpha,\beta}(t_k, t_{k+1}) = \frac{P_{\overline{\alpha},\overline{\beta}}(t_k, t_{k+1}) - P_{\overline{\alpha}}(t_k, t_{k+1}) P_{\overline{\beta}}(t_k, t_{k+1})}{\sqrt{P_{\overline{\alpha}}(t_k, t_{k+1}) \Big(1 - P_{\overline{\alpha}}(t_k, t_{k+1})\Big) P_{\overline{\beta}}(t_k, t_{k+1}) \Big(1 - P_{\overline{\beta}}(t_k, t_{k+1})\Big)}} \tag{6}$$

where $P_{\overline{\alpha}}$ and $P_{\overline{\beta}}$ are the marginal probability of default for α and β, which are extracted from bond prices.

3.2 *The multivariate case*

In this section we extend the methodology above to the case of multiple entities. The extension consists in formulating a credit risk model with a factor model structure (Schönbucher, 2001). This allows combining analytical tractability in the model with a realistic dependency structure.

Let us consider N defaultable entities, say $\alpha_1, \dots, \alpha_N$. Our goal is to estimate the joint default probabilities of such entities. Imagine that a protection seller β sells protection against the default of each of the N entities in the CDS market. In this case, we can apply the bivariate methodology above to estimate the pairwise default correlations between α_i and β, for $i = 1, \dots, N$ over the period (t_k, t_{k+1}), which we denote as $\rho_{\alpha_i,\beta}(t_k, t_{k+1})$.

We embrace the modelling setup of credit risk models of the factor model type, by assuming that the dependence of the single defaults is triggered by common factors. In a standard one-factor model, the value of the assets of the i-th reference entities, say V_i, is driven by a common systematic component Y, and an idiosyncratic component, here denoted as Z_i, according to the model:

$$V_i = \varrho_i Y + \sqrt{1 - \varrho_i^2} Z_i \tag{7}$$

where ϱ_i is a parameter correlating the single obligor i with the systematic trigger Y. Assuming that $Y, Z_1, \dots, Z_N$ are mutual independent with 0 mean and unit variance, the linear correlation between V_i and Y coincides with ϱ_i, whereas the correlation of reference entity i with $j \neq i$, is given by the product $\varrho_i \varrho_j$.

In our model framework, we identify the systematic component Y in Eq. (7) as being the protection seller β. From a financial and empirical point of view, this assumption leads to a realistic representation of the default correlation structure as long as:

1. the protection seller β is a representative institution for the considered reference entities (e.g. if the goal is to estimate the joint default probability for a given rating class, it is desirable that β itself is a representative institution of that rating class[3]) and/or

2. the protection seller β is an institution operating in a market/sector which arguably consists of the channel through which default propagation might occur (e.g. if the concern is the assessment of the likelihood of a joint default event which can possibly be triggered by a worsening in the macro-financial context, then β ought to be a representing institution operating in the financial sector and exposed to the considered entities).

In the light of the current debt sovereign crisis, the application proposed in Section 4 is referred to Euro Zone sovereign risk. In our empirical exercise, β is an EU bank exposed to the sovereign risk of the European area. The identification of the protection seller as the triggering factor for a joint default is justified from the point of view of both the extents listed above: β operates on the European markets and represents the financial sector, which was widely recognized as the channel through which the crisis evolved and spread out.

Hence, the default correlation between α_i and α_j, with $i = 1, \ldots, N$, $j = 1, \ldots, N$ and $i \neq j$, is given by:

$$\rho_{\alpha_i,\alpha_j}(t_k, t_{k+1}) = \rho_{\alpha_i,\beta}(t_k, t_{k+1})\rho_{\alpha_j,\beta}(t_k, t_{k+1}) \tag{8}$$

Using Eq. (8), we can recover the full default correlation matrix for the system $(\alpha_1, \ldots, \alpha_N)$, which we denote as:

$$R(t_k, t_{k+1}) \equiv \Big[r_{i,j}(t_k, t_{k+1})\Big]_{i=1,\ldots,N;\ j=1,\ldots,N} \tag{9}$$

where:

$$r_{i,j}(t_k, t_{k+1}) \equiv \begin{cases} \rho_{\alpha_i,\beta}(t_k, t_{k+1})\rho_{\alpha_j,\beta}(t_k, t_{k+1}) & \text{if } i \neq j \\ 1 \quad \text{otherwise} \end{cases} \tag{10}$$

We estimate the joint default probability via simulation. We set up the simulation design as follows default contagion.. Let $\mathbf{1}_i(t_k, t_{k+1})$ be a random variable taking the value 1 if α_i defaults in the time period (t_k, t_{k+1}) and 0 otherwise. We assume that the system $\Big(\mathbf{1}_1(t_k, t_{k+1}), \ldots, \mathbf{1}_N(t_k, t_{k+1})\Big)$ follows a multivariate binomial distribution (see Davis and Lo [2001], Cousin et al.[2012]) with correlation equal to $R(t_k, t_{k+1})$:

$$\mathbf{1}_i(t_k, t_{k+1}) \sim Bin\Big(P_{\overline{\alpha}_i}(t_k, t_{k+1})\Big) \tag{11}$$

$$corr\Big(\mathbf{1}_1(t_k, t_{k+1}), \ldots, \mathbf{1}_N(t_k, t_{k+1})\Big) = R(t_k, t_{k+1}) \tag{12}$$

We simulate the system $\mathbf{1}_1(t_k, t_{k+1}), \ldots, \mathbf{1}_N(t_k, t_{k+1})$ by means of dynamic copula functions. We adopt as a baseline a standard Gaussian copula approach and we contrast it with a Gumbel Archimedian copula, which is the only copula function allowing for positive tail dependence with the advantage of parameter parsimony (Cherubini *et al.*, 2004). The generator function of the Gumbel copula is given by:

$$\phi(u) = (-\ln(u))^{\theta} \tag{13}$$

[3] Assuming homogeneity between reference entities and protection sellers is something not far from reality. Indeed, credit derivatives markets tend to be very concentrated among few players, both in terms of sellers and reference entities. This increases the interconnectedness of the system while posing a concrete threat for the global financial stability. The issue is well documented inter alia by ECB (2009).

where $\theta \in [1, +\infty)$ is the dependence parameter. The Gumbel N-dimensional copula is then given by:

$$C(u_1, \ldots, u_N) = \exp\left\{ - \left[\sum_{i=1}^{N} (-\ln(u_i))^{\theta} \right]^{\frac{1}{\theta}} \right\} \tag{14}$$

We denote the dependence parameter referred to the period (t_k, t_{k+1}) as $\theta(t_k, t_{k+1})$ and estimate it as:

$$\theta(t_k, t_{k+1}) = \frac{1}{1 - \overline{r}(t_k, t_{k+1})} \tag{15}$$

where $\overline{r}$ is the average default correlation across the N countries:

$$\overline{r}(t_k, t_{k+1}) = \frac{2}{N(N-1)} \sum_{i=1}^{N} \sum_{j>i}^{N} r_{i,j}(t_k, t_{k+1}) \tag{16}$$

The next section presents an application of this methodology to the sovereign risk in the Euro Area.

4 Empirical application

The on-going EU sovereign debt crisis is causing great concern about the sustainability of national debt issued by the member states. This empirical application is devoted to the estimation of the likelihood of the default of one or more countries in the Euro Zone.

4.1 *The data*

We consider bond and CDS data for the following countries: Austria, Belgium, Finland, Germany, France, Italy, Netherlands, Portugal, Slovakia, Slovenia and Spain[4]. This country selection is dictated by data availability. The data provider is Bloomberg^{TM}. Our sample consists of daily observations form 01-Jan-2004 to 11-Oct-2012 (2291 observations). We choose 5 and 10 years as reference maturities[5]. Our goal is to estimate risk-neutral joint default probabilities for the periods $[0, 5]$ and $[5, 10]$. We then fix $t_0 = 0$, $t_1 = 5$, $t_2 = 10$. In Tab. 1 we report descriptive statistics for the variables employed in the estimation. Data series are plotted in Fig. 1-4.

[Tab. 1 and Fig. 1-4 about here]

Sovereign bond spreads are estimated as the difference between the national bond yields and the German bond yields, being Germany considered as the EU benchmark country. Risk-free rates are the EU swap rates. We use 1, 5 and 10 year EU swap data series and we do interpolation for the intermediate maturities, when needed. In order to take into account the impact of funding issues on the basis we consider the spread between the 6 months EU depo rates and the Euribor over the same maturity. We apply a Hodrick-Prescott filter to the rate series, in order to rule out excess volatility and get a trend measure of the funding asymmetry. We specify the Hodrick-Prescott smoothing parameter according to the rule proposed in Ravn and Uhlig (2002)

[4]Our methodology requires the estimate of the probability of default of the protection seller β. For this sake, we consider CDS data for BNP Paribas, Deutsche Bank and Société Générale, which can be thought as the representative EU dealers for the over-the-counter CDS market (see ECB, 2009).

[5]The CDS quotes are expressed in annual term. We derive the 5 and 10 years equivalent premiums according to the methodology in Pianeti *et al.* (2012)

applied at the daily frequency. We apply the same filtering procedure to the bid/ask quotes of bond and CDS, in order to estimate a trending measure of the liquidity adjustment. We compute the basis and adjust it for liquidity and funding effects. The adjusted and the unadjusted bases are displayed in Fig. 5-8.

[Fig. 5-8 about here]

The time series for the unadjusted bases fluctuate around the arbitrage-free value of 0. For some of the countries analysed, the divergence from 0 is systematic, with Portugal, Slovenia and Slovakia showing a persistent negative basis, whereas a positive basis is typically observed for Austria and France in particular. This evidence is in accordance with the applied literature on the topic (see inter alia Fontana and Scheicher (2010). After adjusting for the liquidity and the cost of funding asymmetries, most of the observations have a negative sign, which we attribute to the presence of counterparty risk, whereas the positive observations are considered as manifestation of market imperfections and then, they are averaged out.

The CDS quotes we considered can be assumed to be the average price on a CDS written by a top tier investment banks, which typically operate worldwide. We want to infer from these quotes what is the price of a CDS offered by an EU bank exposed to the EU sovereign risk. Let us define $0 < q_{EU} \leq 1$ the proportion of investment banks exposed to the sovereign risk in the Euro Zone. Thus, the market average quote of a CDS on α_n, provided by the representative dealer β, can be seen as the weighted average of a contract provided by an investment bank exposed to the sovereign risk in Europe and a contract written by an institution completely immune to that risk. We adjust the basis in total analogy with what has been done in Section 5 for partial collateralization. This leads to a multiplicative adjustment which is analogous to the one applied in the passage from Eq. (28) to Eq. (29). We denote the resulting basis as $B^{EU}_{\alpha_i,\beta}$, which is then given by:

$$B^{EU}_{\alpha_i,\beta}(t_k, t_{k+1}) = \frac{1}{q_{EU}} B_{\alpha_i,\beta}(t_k, t_{k+1}) \tag{17}$$

with $k = 1, \ldots, M$ and $i = 1, \ldots, N$.

4.2 *The simulation and systemic risk measurement*

We use Eq. (3) to (6), together with the bases $B^{EU}_{\alpha_i,\beta}(0, 5)$ and $B^{EU}_{\alpha_i,\beta}(5, 10)$ to estimate the pairwise default correlations between each of the countries in the sample and a representative EU investment bank, which we have denoted as β. The defaults are then correlated via the common dependence of the single states to the European financial sector, represented by β. We then estimate via simulation the joint probability of defaults of the considered EU countries. We run our estimation procedure for the following parameter space: $q_c = 0.8, 0.9, 0.95$ and $q_{EU} = 0.15, 0.25, 0.35$ (see Appendix B for details).

The results of the estimations are reported in Fig. 9-10. We observe the 5-year and the 5-to-10-year probability having quite a similar dynamic evolution. In particular they are almost null up to 2006. This situation is indicative of a well-functioning economic and financial system, with no tangible threat of sovereign risk contagion. Afterwards, the estimated probabilities feature remarkable peaks in correspondence of some notable facts of the recent financial history. In particular, we observe a surge in correspondence to the harshest period of the 2008-09 economic and financial crisis. Later, other peaks are recorded in correspondence of the first EU intervention for the Greece bailout in May-2010 and since mid-2011 onwards, to mark the spreading of the sovereign crisis throughout and outside Europe, a process which we are still witnessing. As expected the highest figures are recorded for the least conservative case $q_c = 0.95, q_{EU} = 0.15$.

For the 5-year case the figures referred to the recent crisis exceeds the figure referred to the late 2000s crisis in 5 cases out of 9. On the contrary, for the case of the forward estimates in Fig. 10, much higher probabilities were recorded during 2008, as to anticipate the worsening of the crisis in Europe. The current estimate of a breakdown of the Euro caused by a joint default of the member states in the next 5 years is estimated to happen with a probability between 0.01% and 0.14%, a remote event, but with catastrophic impact. The magnitude of our estimates are in line with what estimated by other applied paper on the topic, such as Radev (2012) and Zhang *et al.* (2012). We adopte the 5 years joint default probability as our measure of systemic risk for the Euro Area.

[Fig. 9-10 about here]

In Fig. 11 and 12 we report the estimated probabilities of at least $1, 2, \ldots, N$ defaults to happen, contrasting the performance of the Gumbel copula with the Gaussian copula. The estimates of the probability of observing at least 1, 2, 3 and 4 default(s) are plotted in the top panel of Fig. 11. The dynamics of these probabilities witnesses an increasing risk of default from early 2010 onwards, with the current estimate of observing at least another default after Greece in the next 5 years of the order of 35%, down from a peak recorded at the end of 2011 of about 60%. In this case, the estimation with the Gumbel copula and the Gaussian copula are quite similar. On the contrary, in the middle panel and, even more evidently, in the bottom panel, we document the inaccuracy of the Gaussian copula to capture the extreme event of a default of more than 7 countries. In this respect, similar conclusions can be drawn from the forward probabilities in Fig. 12, which however show a more marked trending behaviour than the 5-year estimate for the case of the default of a small number of countries (top panel).

[Fig. 11-12 about here]

The simulated realizations of the multivariate default process $\left\{\mathbf{1}_i(t_k, t_{k+1})\right\}_{i=1,\ldots,N}$ defined in Section 3.2 offer an insight into conditional default probabilities, too. In particular, we look at the probability that at least one country defaults given that one default has been observed within a group of selected countries. We compute joint default probabilities conditional on the default of Italy, Portugal and Spain respectively for the period 2007-2012. Results are reported in Fig. 13. Overall, the conditional probability estimates for the period 2007-2009 exceed the 2011-2012 figure. This corresponds to the idea that the market impact of a default of any of the named countries would have been bigger during the late 2000s crisis than in the current sovereign crisis. A Spanish default in 2007 would have led to at least another default with a likelihood almost twice as much as in the case of Italy and Portugal, whose default in more recent times would have had a smaller effect than a default of the other two (see top panel left of Fig. 13). In a similar way, Radev (2012) shows that the contribution of the default of Greece to systemic risk is limited, when compared to the impact of a German default.

[Fig. 13 about here]

In Fig. 14 we compare our estimates of the 5-year joint default probability in the case $q_c = 0.9, q_{EU} = 0.25$ with the EuroStoxx50, that we consider as a good indicator for the phases of the EU sovereign crisis. It is clearly specular behaviour of the joint default probability with respect to the dynamic of the stock index. In early 2007 the joint default probabilities started reacting, anticipating the subsequent fall in the stock index, and more remarkably, in mid-2011 our estimates surges sharply well before the index plummets amid concerns on the spreading of the crisis.

[Fig. 14 about here]

To corroborate these claims with empirical evidence, the next section proposes a forecasting exercise.

4.3 *The forecast*

Our forecasting exercize is designed as follows. We consider the weekly returns of the EuroStoxx50, denoted with y_t, and the weekly changes in the 5-year estimate of the joint default probabilities, say ΔJDP_t, over the period 2007–2012, where most of the variability of ΔJDP_t is observed. We set up a simple regression model, which we use to forecast y_t by means of lagged values of ΔJDP_t. The model is as follows:

$$y_t = \omega + \sum_{l=1}^{L} \phi_l \Delta JDP_{t-l} + \epsilon_t \tag{18}$$

where ω is the constant term, ϕ_l is the coefficient referred to ΔJDP_{t-l}, L is the number of lags and ϵ_t is assumed to be a white noise normally distributed error term. In the application we consider different forecasting time horizon, setting $L = 1, 4, 12, 26, 52$, which corresponds to the case in which the forecasting power of the default probabilities is evaluated with a lag of one week, one month, one quarter, one semester and one year, respectively. The simple model in Eq. (18) cannot forecast equity markets: this small exercise is intended to show the capability of our default probability estimates to anticipate the most relevant episodes of stress during the EU sovereign crisis epitomized by extreme price movements in the stock markets.

Pre-2011 data are considered as in-sample. We perform an out-of-sample forecasting exercise over the period 2011-2012, which roughly corresponds to 30% of the period 2007–2012. We estimate model (18) for every out-of-sample data point, following a standard walking-forward optimization procedure and perform a one step ahead forecast analysis. On the basis of the estimates of model (18), we compute $P(y_{t+1} < 0)$, the probability of a negative equity return for the week ahead. The results are reported in Tab. 2.

[Tab. 2 about here]

We focus on the biggest stock market drawdowns in the period 2011–2012, by reporting the results referred to the first decile of the out-of-sample equity returns distribution. These observations span the second semester of 2011, when the EU sovereign debt crisis spread from the peripheral countries in Europe to the rest of the continent and ultimately affected the US, too. The biggest slump in the EU stock market was recorded in the last week of July 2011, with a weekly negative return of 13%. For this data point, on the basis of our simple regression model, we forecast a fall in the equity market, with a probability of 60 to 90%, when the forecasting time horizon is at least one-quarter long. Similar figures are recorded for the other observations in August-2011. Negative stock return probabilities above 50% are in general observed earlier in the same year (first two rows in Tab. 2), whereas more contrasted evidence emerges from the last three rows. The average drawdown probability across all the cases reported in Tab. 2 equates to more than 55%, providing evidence, albeit not strong, for the ability of the estimated default probabilities to forecast the materialization of a tail event on the equity market.

5 Conclusions

In this paper, we derived a multivariate extension of the methodology introduced by Pianeti *et al.* (2012) for the estimation of the joint default probability. The defaults of the single institutions are correlated through their common dependence on the financial cycle, considered as the channel through which systemic instabilities propagate. The application to the EU sovereign risk provided evidence of increasing systemic risk and danger of contagion from early-2007 and more significantly from late 2011 onwards. The estimates show to be very reactive to changes in market conditions and their magnitude is coherent with the analysis in Radev (2012) and Zhang *et al.* (2012). We documented the total inaccuracy of the Gaussian Copula in capturing the extreme event of a joint default. Marginal and conditional default probability estimates were also provided. We historically validated the forecasting capability of the proposed estimates through a comparison with the dynamics of a benchmark stock market index, which marked the timeline of the recent sovereign debt crisis.

There are a number of areas for future research. In Section 2, we extracted information on counterparty risk from the basis and excluded the positive values, due to manifestation of market imperfections. It would be interesting to further explore the issue and model the determinants of this phenomenon. Also, alternative less parsimonious copula functions might be worth considering once an appropriate estimation method consistent with our methodology is derived. Developments on the model to test for forecasting capability of our indicator might also be interesting to consider. We leave these developments to future research.

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Appendix A: Derivation of the adjusted basis

The scope of this Appendix is to derive the formula for the adjusted basis, which has been introduced in Eq. (1).

The concept of basis is built upon the comparison of two contracts, a bond and a CDS, which typically differ from the point of view of the liquidity. We apply a liquidity adjustment to facilitate a direct comparison between CDS and bond spreads. We treat the liquidity as an exogenous phenomenon. Liquidity impacts the price of a bond by lowering it, leading to a liquidity premium, which is thought to be a constituent of the bond spread (Perraudin and Taylor, 2004, Driessen, 2005). Let α be the issuer of a bond, with a spread on the risk free rate denoted as $s_\alpha(t,T)$. We denote the liquidity premium on such bond as $\lambda_\alpha(t,T)$, so that:

$$s_\alpha(t,T) = s'_\alpha(t,T) + \lambda_\alpha(t,T) \tag{19}$$

where $s'_\alpha(t,T)$ is the liquidity adjusted bond spread. We use the bid ask spread as a liquidity proxy, to infer information about the liquidity components of the bond spread. We first rewrite the above equation as:

$$s_\alpha(t,T) = s'_\alpha(t,T)\,(1 + l_\alpha(t,T)) \tag{20}$$

where $l_\alpha(t,T)$ is a proportionality constant linking the bond spread with its liquidity adjusted counterpart. We estimate $l_\alpha(t,T)$ as the bid ask spread expressed in percentage term, that is:

$$l_\alpha(t,T) = \frac{s^{bid}_\alpha(t,T) - s^{ask}_\alpha(t,T)}{s^{ask}_\alpha(t,T)} \tag{21}$$

This corresponds to the assumption that the liquidity premium $\lambda_\alpha(t,T)$ is proportional to the bid-ask spread. In the limiting case that $s^{bid}_\alpha(t,T) = s^{ask}_\alpha(t,T)$, the market is characterized by infinite liquidity, and no adjustment is performed. As the bid price deviates from the ask, we assume that investors demand an additional premium for holding the bond, to compensate the liquidation risk. Such a premium is thought to be proportional to the deviation of the bid price from the ask price. The liquidity adjusted bond spread then becomes:

$$s'_\alpha(t,T) = \left(1 + \frac{s^{bid}_\alpha(t,T) - s^{ask}_\alpha(t,T)}{s^{ask}_\alpha(t,T)}\right)^{-1} s_\alpha(t,T) \tag{22}$$

We apply the same concept to CDS spread and perform the following adjustment[6]:

$$w'_{\alpha,\beta}(t,T) = \left(1 + \frac{w^{ask}_{\alpha,\beta}(t,T) - w^{bid}_{\alpha,\beta}(t,T)}{w^{bid}_{\alpha,\beta}(t,T)}\right)^{-1} w_{\alpha,\beta}(t,T) \tag{23}$$

The liquidity adjusted basis is then given by:

$$b'_{\alpha,\beta}(t,T) \equiv w'_{\alpha,\beta}(t,T) - s'_\alpha(t,T) \tag{24}$$

We also explicitly take into account funding issues linked to the implementation of arbitrage Strategies 1 and 2, outlined in Section 2. We show how the basis might be affected and propose an adjustment. We relax the hypothesis that all the market participants can borrow and lend money at the risk free rate $r(t,T)$.

[6] In the notation we follow the market practice according to which, in the case of the bond, the bid and the ask quotes are referred to the bond price so that $s^{bid}_\alpha \geq s^{ask}_\alpha(t,T)$, whereas in the case of the CDS they are directly referred to the spread, so that $w^{ask}_{\alpha,\beta}(t,T) \geq w^{bid}_{\alpha,\beta}(t,T)$.

We more realistically assume that they borrow money at the Libor rate $\mathcal{L}(t,T)$ and they have the possibility to deposit money at the depo rate, denoted as $d(t,T)$. Let us denote with $\delta(t,T)$ the spread of the Libor rates over the depo rates, that is $\delta(t,T) \equiv \mathcal{L}(t,T) - d(t,T)$. When a negative basis is observed, the payoff of Strategy 1 is given by $y_\alpha(t,T) - \mathcal{L}(t,T) - w_{\alpha,\beta}(t,T)$, which, in the case the bond spread is measured over $\mathcal{L}(t,T)$, is still equal to $s_\alpha(t,T) - w_{\alpha,\beta}(t,T)$. On the other hand, in the case of a positive basis, the payoff of Strategy 2 changes to $w_{\alpha,\beta}(t,T) + d(t,T) - y_\alpha(t,T)$, which is equal to $w_{\alpha,\beta}(t,T) - s_\alpha(t,T) - \delta(t,T)$. Such a spread stems from the asymmetry of the strategies in relation to the process of funding[7]. Thus, Strategy 2 will be implemented as long as $b_{\alpha,\beta}(t,T) > \delta(t,T)$. Hence, it might well be that arbitrageurs can not revert a positive basis to 0 because of the presence of funding asymmetries in the implementation of the two strategies. We correct the basis as:

$$b''_{\alpha,\beta}(t,T) \equiv w'_{\alpha,\beta}(t,T) - s'_\alpha(t,T) - \delta(t,T) \tag{25}$$

where $b''_{\alpha,\beta}(t,T)$ is the basis adjusted by funding and liquidity.

The basis $b''_{\alpha,\beta}(t,T)$ consists in the materialization of counterparty risk in the underlying CDS contract, as any funding and/or liquidity effects has been accounted for. However, as stated in the ISDA Margin Survey (2012), the practice of requiring the counterparty in OTC derivatives to post a collateral against the possibility of a default, has become more and more common. We adapt our formulas to take this effect into account. Realistically, the quote of a CDS is the average of the prices provided by different dealers. We can then imagine that a quota $0 \le q_c \le 1$ of such contracts are assisted by a collateral agreements covering the entire nominal amount[8]. Then, the market average quote of a CDS on α, provided by the representative dealer β, can be seen as the weighted average of a contract assisted by a total collateralization and a contract without collateral, so that the liquidity adjusted CDS quote $w'_{\alpha,\beta}(t,T)$ can be written as:

$$w'_{\alpha,\beta}(t,T) = (1-q_c)w''_{\alpha,\beta}(t,T) + q_c\tilde{w}''_{\alpha,\beta}(t,T) \tag{26}$$

where $w''_{\alpha,\beta}(t,T)$ is the quote referred to an uncollateralized CDS contract on α, whereas $\tilde{w}''_{\alpha,\beta}(t,T)$ is the value of a CDS on α, fully assisted by collateral. In absence of market frictions, the latter coincides with the spread on the debt of α, that is $s_\alpha(t,T)$. In the case of no collateralization ($q_c = 0$), the liquidity adjusted CDS spread $w'_{\alpha,\beta}(t,T)$ coincides by definition with $w''_{\alpha,\beta}(t,T)$, so that no adjustment for collateralization is needed. On the other hand, when the counterparty risk in the CDS is fully collateralized ($q_c = 1$), the CDS quote is perfectly in line with the spread on the corresponding bond, leading to a basis equal to 0. This extent is in clear contrast with what we observe in reality, as market data show in Section 4. Let us exclude the possibility of full collateralization by letting $0 \le q_c < 1$.

Including the effect of funding, we have that $\tilde{w}''_{\alpha,\beta}(t,T) = s_\alpha(t,T) + \delta(t,T)$, so that from the previous equation we get:

$$w''_{\alpha,\beta}(t,T) = \frac{w'_{\alpha,\beta}(t,T) - q_c\left(s_\alpha(t,T) + \delta(t,T)\right)}{1-q_c} \tag{27}$$

Now we define $b'''_{\alpha,\beta}(t,T)$ as the basis which cumulates adjustments for liquidity, funding and partial collateralization, which is given by:

$$b'''_{\alpha,\beta}(t,T) \equiv w''_{\alpha,\beta}(t,T) - s'_\alpha(t,T) - \delta(t,T) \tag{28}$$

[7]In normal market conditions $\delta(t,T)$ can be assumed to be positive, however the proposed adjustment doesn't necessarily requires this hypothesis to hold.

[8]Alternatively, we can assume that the contracts are assisted by partial collateralization, so that the average quota of nominal amount covered is equal to q_c. From the modelling point of view, the two cases are equivalent.

Substituting in the definition of $w''_{\alpha,\beta}(t,T)$, we get:

$$b'''_{\alpha,\beta}(t,T) = \frac{1}{1-q_c}\left(w'_{\alpha,\beta}(t,T) - s'_{\alpha}(t,T) - \delta(t,T)\right) \tag{29}$$

Hence $b'''_{\alpha,\beta}(t,T)$ results in a multiplicative adjustment of $b''_{\alpha,\beta}(t,T)$, defined in Eq. (25).

Eq. (29) isolates the component of the basis motivated by counterparty credit risk. We thus expect $b'''_{\alpha,\beta}(t,T) \leq 0$. To exclude the effect of market imperfections which might cause the basis to be positive, we define:

$$B_{\alpha,\beta}(t,T) \equiv \left(b'''_{\alpha,\beta}(t,T)\right)^{-} \tag{30}$$

where $(\cdot)^{-} \equiv \min(\cdot, 0)$. Now, combining Eq. (30) with (29), gives (1).

Appendix B: Parameter estimate

The simulation engine employed in the empirical application in Section 4 requires the specifications of two parameters: the quota of collateralized trades on the OTC credit derivate market q_c and the quota of investment banks exposed to the sovereign EU risk q_{EU}.

Regarding the specification of the first parameter, the ISDA Margin Survey (ISDA, 2012) states that 93% of the OTC transactions that took place in 2012 in the credit derivative markets have been assisted by collateralization. We fix q_c to the following set of values: 0.8, 0.9, 0.95. The consideration of values of q_c less than the ISDA estimates are motivated as follows. A smaller value of q_c leads to a smaller estimate of the collateralization-adjusted basis (see Eq. (29)) and thus to a more conservative estimate of the probability of default. Furthermore, collateralization is a practice become widespread only after the credit crunch crisis of late 2007, and thus we might consider the ISDA estimate for 2012 to be bigger than the estimate to be referred to our sample size.

We provide an estimate of the parameter q_{EU} as follows. We assume that the global equity markets convey information on the exposures to local risk factors of a representative investment bank operating worldwide. In this perspective, q_{EU} represents the relative importance of the EU market within the global financial markets. Thus, for the sake of estimating q_{EU}, we consider MSCI data for benchmark stock indices for US, Europe, UK, Japan and Emerging markets, offering a wide coverage of the international stock markets. Using standard principal component analysis, we extract the first component of the weekly equity returns of the benchmark indices listed above, in order to estimate the global equity factor, common to the set of the considered financial markets. The estimate of q_{EU} is given by the contribution of the European index in the estimated global factor. The application of this procedure to weekly data from 2001 to 2012, leads to the extraction of a principal component accounting for more than 75% of the total variance and a point estimate for q_{EU} of 0.25. Along with this estimate, we consider the alternative values of 0.15 and 0.35 for robustness check.

CDS - 5 years

Country	***Mean***	***Standard deviation***	***Min***	***Max***
Austria	0.517%	0.616%	0.010%	2.690%
Belgium	0.689%	0.896%	0.021%	4.061%
Finland	0.228%	0.248%	0.030%	0.908%
France	0.461%	0.615%	0.015%	2.496%
Italy	1.157%	1.509%	0.056%	5.915%
Netherlands	0.634%	0.321%	0.105%	1.398%
Portugal	2.287%	3.624%	0.040%	15.269%
Slovakia	0.706%	0.786%	0.053%	3.282%
Slovenia	0.932%	1.278%	0.043%	5.111%
Spain	1.221%	1.585%	0.026%	6.410%

CDS - 10 yers

Country	***Mean***	***Standard deviation***	***Min***	***Max***
Austria	0.591%	0.683%	0.018%	2.611%
Belgium	0.733%	0.912%	0.024%	3.975%
Finland	0.278%	0.293%	0.027%	1.103%
France	0.522%	0.680%	0.029%	2.581%
Italy	1.194%	1.435%	0.093%	5.696%
Netherlands	0.631%	0.405%	0.075%	1.574%
Portugal	2.089%	3.034%	0.061%	10.694%
Slovakia	0.814%	0.838%	0.071%	3.403%
Slovenia	1.242%	1.420%	0.073%	5.414%
Spain	1.181%	1.490%	0.031%	5.754%

BOND - 5 years

Country	***Mean***	***Standard deviation***	***Min***	***Max***
Austria	3.035%	0.873%	0.615%	4.897%
Belgium	3.280%	0.723%	1.065%	5.404%
Finland	2.442%	1.063%	0.526%	4.930%
France	2.960%	0.889%	0.734%	4.929%
Italy	3.745%	0.832%	2.562%	7.703%
Netherlands	2.869%	0.982%	0.558%	4.883%
Portugal	5.281%	3.609%	2.517%	21.748%
Slovakia	3.403%	0.734%	1.675%	4.973%
Slovenia	3.968%	1.051%	2.080%	5.671%
Spain	3.713%	0.790%	2.435%	7.498%

BOND - 10 years

Country	***Mean***	***Standard deviation***	***Min***	***Max***
Austria	3.676%	0.623%	1.850%	4.921%
Belgium	3.895%	0.496%	2.394%	5.814%
Finland	3.283%	0.833%	1.322%	4.870%
France	3.637%	0.572%	2.046%	4.844%
Italy	4.453%	0.693%	3.217%	7.244%
Netherlands	3.542%	0.724%	1.521%	4.856%
Portugal	5.516%	2.686%	3.160%	16.605%
Slovakia	3.991%	0.464%	2.573%	5.062%
Slovenia	4.993%	0.975%	3.492%	6.123%
Spain	4.363%	0.790%	3.005%	7.566%

Table 1: Descriptive statistics for CDS spreads and bond yields.

		Probability of a drawdown				
Date	***Observed stock return***	***L = 1***	***L = 4***	***L = 12***	***L = 26***	***L = 52***
04-Jul-2011	-5.029%	63.853%	57.623%	48.183%	59.151%	63.953%
25-Jul-2011	-5.526%	55.498%	51.980%	51.562%	57.924%	63.279%
01-Aug-2011	-12.967%	47.306%	42.771%	62.227%	81.906%	88.602%
15-Aug-2011	-6.596%	53.102%	59.478%	72.009%	79.830%	78.393%
29-Aug-2011	-4.980%	52.405%	39.249%	76.220%	80.351%	74.457%
05-Sep-2011	-4.209%	51.121%	49.762%	60.718%	70.487%	78.801%
14-Nov-2011	-6.270%	52.859%	52.443%	50.076%	27.601%	50.673%
05-Dec-2011	-4.385%	53.170%	51.698%	44.528%	24.973%	48.615%
02-Apr-2012	-4.163%	50.887%	48.717%	48.615%	23.189%	15.696%

Table 2: Results of the forecasting analysis. Reported are the observations in the first decile of the out-of-sample return distribution, with the probability of observing a negative return as forecast by the regression model (18) for different value of L.