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Abstract

In this paper we investigate a two-period Bertrand-Edgeworth oligopoly model in which two capacity-constrained firms (incumbents) compete facing future demand uncertainty as well as uncertainty about entry. These firms must choose between pricing low and secure sales in the first period or, alternatively, pricing high in the first period to sell later when their non-contestable demand — i.e., the demand which their capacity constrained rival cannot contest — may be greater. We find that, when each incumbent is able to meet all demand needs, firms randomize in the second period and set deterministic prices in the first period. We then show that the expected market price across both periods reacts relatively more to changes in the likelihood of entry than in the probability of positive future demand. We conclude by exploring the implications of our model for merger control and determine how demand- and supply-side uncertainty affects the compensating marginal cost reductions required for prices not to rise after a merger.

Keywords: Capacity constraints, compensating marginal cost reductions, demand and supply uncertainty, intertemporal pricing, mergers.

JEL Codes: D43, K21, L13

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1 Introduction

The role of production capacity constraints has received considerable attention in the IO literature since Edgeworth's extension of the canonical Bertrand price competition model.[1] Rivalry among firms that compete in prices over time, produce at constant marginal cost up to a capacity constraint, and supply a homogeneous product is typically referred to as Bertrand-Edgeworth (BE) competition. In these models, firms face an intuitive intertemporal trade-off: they must choose between pricing low and secure sales at the early stages of the game or, alternatively, pricing high in these stages to sell later when their non-contestable demand — i.e., the demand which their capacity constrained rival cannot contest — will be greater. For this reason, the equilibria of these games typically require firms to play mixed-strategies and, in contrast to what a standard Bertrand logic would mandate, feature price dispersion and positive profits: two phenomena widely observed in many real markets.[2]

Uncertainty about future market fundamentals is a key determinant of such trade-off: when setting prices at early stages of the game, firms must carefully evaluate how the competitive arena will evolve over time, and adjust their intertemporal strategies according to these expectations. Yet, existing models (see the literature review below) mainly focus on uncertainty on the demand side, and neglect the supply side, which is a major driver of firms behavior in sectors with modest barriers to entry. Understanding how these two different sources of uncertainty affect firms' equilibrium strategies in BE games is, therefore, a fundamental step to gain a deeper understanding of the relationship between capacity constraints, firms' competitive conduct and welfare in environments that are more realistic than the standard Bertrand competition model.

In this paper we consider a simple stylized two-period BE game in which firms selling under capacity constraints are informed about first-period demand but face uncertainty in the second period. We introduce two sources of uncertainty in our model. First, as in the earlier literature, we assume that demand changes over time. Second, we also consider uncertainty related to the threat of future entry by a non-pivotal rival, whose presence in the market drives the equilibrium to a traditional zero-profit Bertrand outcome. Such a distinction, allows us to disentangle and compare the effects on equilibrium prices of future demand uncertainty from those related to uncertainty in the supply side.

We characterize equilibrium prices and show that when no firm can meet all demand needs, they randomize in the second period and set a deterministic price in the first period. As in Sun (2017), firms in the first period must be indifferent between posting a low price, expecting to sell sooner, and posting a higher price, selling later. We find that the expected average price (across the two periods) is increasing in the so-called 'residual supply index' (hereafter RSI) of the industry for either of the pivotal firms, which equals for each of them the ratio of the capacity in the hands of the other pivotal firms and the demand to be served by the pivotal firms. As intuition suggests, given the RSI, the expected market price is growing when the probability of future entry falls and the probability of future demand grows. Interestingly, however, the expected market price is relatively more responsive to changes in the likelihood of entry than to changes in the probability of positive future demand — i.e., supply-side shocks have a greater impact than demand side-shocks on equilibrium prices.

From an applied angle, the simple structure of our model enables us to examine antitrust related questions that are ignored by the previous literature. Specifically, we characterize the effects of pivotality changes on (expected) equilibrium prices and determine the compensating marginal cost

[1]Edgeworth (1925)

[2]We refer to Varian (1980) for results on price dispersion in the presence of mixed strategies, and Padilla (1991) for results on price competition over two periods.

reductions — i.e., the reduction in marginal costs — that are required for these prices to remain constant when firms' pivotality becomes more relevant. We find that, as future entry becomes more likely — e.g., because of lower barriers to entry — and low demand becomes more likely — e.g., because of higher uncertainty in the market — lower compensating marginal cost reductions are required. We also find that in the case of asymmetric capacities, the effect of pivotality on equilibrium prices is non-monotone when the largest firm in the market gains more capacity — i.e., equilibrium expected prices may increase rather than fall — and hence the effect on compensating marginal cost reductions is non-monotone as well.

These results have important policy implications for the assessment of horizontal mergers in markets where firms operate under binding capacity constraints. In fact, the European Commission has relied on simplified versions of the BE model in several recent merger cases — e.g., *Outokumpu/Inoxum*, *Ineos/Solvay/JV* and *Novelis/Aleris*.[3] In the last case, the Commission concluded that Novelis was pivotal pre-merger ($RSI < 1$) and would become even more so post-merger. Yet, De Coninck and Fischer (2020) have criticised the Commission's analysis in this case claiming that it rests on three central assumptions which failed to hold in practice: certainty about demand, certainty about supply and product homogeneity. Our analysis shows that De Coninck and Fischer are correct in stating that uncertainty about demand and supply matters. However, the idea that pivotality is no longer important when demand and supply are uncertain, is incorrect. Other things equal, pivotality and expected prices are positively related except for extreme circumstances. Yet, the impact of an increase in pivotality on prices will be smaller when entry is likely and future demand is highly uncertain.

The distinction between these two sources of uncertainty enables us to derive novel comparative statics results and study how regulators dealing with merger control policy should quantify the compensating marginal cost reductions needed to ensure a merger is consumer welfare enhancing as well as the divestments required for prices not to raise in the post-merger scenario when the market features excess capacity.

Related literature. Our paper is related and contributes to an established bulk of literature dealing with BE competition models. Seminal game-theoretic analyses of static models of BE competition appeared in Beckman (1965) and Levitan and Shubik (1972). Allen and Hellwig (1986), Dasgupta and Maskin (1986), and Vives (1986), among others, extended the static, game-theoretic analysis of BE models in various ways — e.g., by showing existence and uniqueness of the mixed strategy equilibrium, introducing product differentiation, considering non-downward sloping demands, or allowing for the possibility of limit pricing.

Many other scholars, instead, have recognized the importance of repeated interactions and the resulting intertemporal trade-off between selling at the early stages of the game and storing capacity to gain market power in the future — e.g., Brock and Scheinkman (1985), Benoit and Krishna (1987), and Davidson and Deneckere (1990), among others. These models mainly focus on the role that the number of sellers and their capacity constraints play in enforcing collusive pricing schemes and were developed within deterministic environments, in which present and future market conditions are common knowledge.

Our paper builds on and contributes to this literature by proposing a tractable, albeit simple, model that allows us to disentangle the effects of demand and supply-side uncertainty on equilibrium prices in markets where firms are subject to capacity constraints and act in-cooperatively even if they interact over time.

[3] Case M.6471 *Outokumpu/Inoxum*, Case M.6905 *Ineos/Solvay/JV* and Case M.9076 *Novelis/Aleris*, respectively.

The paper that is closer to ours is Sun (2017) which developes a BE model where demand is uncertain. He finds that demand uncertainty induces price instability over time. Sun does not consider the interaction between demand and supply uncertainty and does not consider the policy implications, including on merger control, analysed here.

The rest of the paper is organised as follows. We present the model setup in Section 2 and characterise its equilibria in Sections 3 and 4. We use the analytical expressions derived for equilibrium to calculate the expected average prices predicted in the model in Section 5. We then exploit our model to perform comparative statics. First, we analyse the impact of changes of pivotality on expected average prices in Section 6, and then we establish the compensating marginal cost reductions that offset changes in pivotality in Section 7. Section 8 concludes.

2 Model setup

Players. Consider a market for a homogeneous good in which there are three firms (indexed by $i = 1, 2, 3$) interacting over two subsequent periods (each denoted by $t = A, B$). Firms 1 and 2 (the incumbents) are active in both periods and do not discount future profits. Firm 3 (the entrant) is modeled as a smaller competitor whose entry is not strategic, but probabilistic and exogenously determined with probability $1 - \rho$. As standard in the literature, we assume that firm 3's entry is resolved before period B takes place — i.e., each incumbent knows whether in period B it faces one or two rivals. Firms have the same marginal cost $c \geq 0$. We normalise $c = 0$ in Sections 3 to 6 in order to simplify exposition. Instead, in Section 7, we assume that $c > 0$ to calculate the compensating marginal cost for a market structure change that increases the pivotality of the incumbents.

Demand. Let D^A and D^B represent the respective *maximum* quantities that could be demanded for each period, and define total demand as $D \triangleq D^A + D^B$. In every period $t = A, B$, demand is inelastic and is equal to the maximum D^t at any price equal or lower than R, and zero for larger prices. Demand in period B is uncertain: it is equal to $D^B > 0$ with probability λ, and zero otherwise. This uncertainty is resolved after period A has taken place, but before period B begins — i.e., firms learn demand before pricing.

Capacity constraints and pivotality. All firms face capacity constraints. For simplicity, in the baseline model we posit that firms 1 and 2 have symmetric capacity (hereafter denoted by k) while firm 3 has capacity k_3. Firms decide how to allocate capacity across periods through their intertemporal pricing strategies. We also assume that firms 1 and 2 are larger than firm 3 — i.e.,

$$k \geq k_3,$$

so that the incumbents are pivotal relative to the entrant. We extend the model to allow for asymmetric capacities and only one pivotal firm in period B in Appendix B. When firm 3 enters the market, overall capacity is so large relative to D^B that no firm is required to serve demand (i.e., $k_3 \geq D - k$). In that scenario, market prices equal marginal costs.

We assume that the incumbents have enough capacity to serve the entire demand — i.e., $2k \geq D$. Moreover, we assume that each incumbent has enough capacity to serve the entire demand of period A — i.e., $k \geq D^A$ — but neither can serve both periods if the demand for period B realises — i.e.,

$k < D$. This means that neither firm 1 nor 2 can serve the entire maximum demand on their own, and that, absent firm 3's entry in period B, any allocation clearing would require both firms to serve the market. In other terms, firms 1 and 2 are pivotal in the market when entry does not occur. It is important to note that this implies that at least one of the two firms will reach period B being pivotal at maximum demand.

Absent firm 3's entry, firms 1 and 2 compete in each period by setting prices simultaneously. As it is common in models of price competition in markets with homogeneous goods, this means that the firm pricing lowest will capture all the demand it can serve given its capacity. In case of a tie, we assume that either firm will win with probabilities $s_1 \in [0, 1]$ and $s_2 \in [0, 1]$, and the other firm(s) will get the residual demand, if any.

Equilibrium concept. Since in period B players are fully informed about the entire history of the game, the equilibrium concept will be Subgame Perfect Nash Equilibrium (SPNE). Therefore, we solve the game by backward induction, considering each possible branch in which the game may fall.

3 Solution of the subgames in period B

We start by solving the subgames contained in period B. The specific subgame that materalises in this period will depend on period A's outcome, the realised uncertainty about demand and whether entry of firm 3 occurs. Let us focus on the most interesting case where high demand realises and no entry occurs, so that at least one of the firms remains pivotal in the market.[4]

Let $q_i^A \geq 0$ be firm i's production in period A, with $i = 1, 2$. Assume that $q_1^A + q_2^A = D^A$ and, without loss of generality, that $q_1^A \leq q_2^A$ so to capture any potential asymmetry that could arise in this period as a result of period A's equilibrium play.[5] We can define the amount of demand that each firm can absorb in period B to be

$$\kappa_j \triangleq \min\{D^B, k - q_j\},$$

and, as a consequence, $\kappa_1 \geq \kappa_2$. Since $2k \geq D$, it is the case that $\kappa_1 + \kappa_2 \geq D^B$ and thus that overcapacity characterises the market for this period as well.[6]

Under these conditions, firm 1 is pivotal. Whether firm 2 is pivotal too depends on the amount of demand that firm 1 can absorb in period B — i.e., κ_1. If $\kappa_1 < D^B$, then any market allocation clearing demand has to make use of firm 2's capacity, who is thus pivotal. On the contrary, if $\kappa_1 = D^B$, firm 1 can cover the entire demand of period B alone and firm 2 is not pivotal.

In the remainder of the section and in Appendix A, we characterise the equilibrium for this subgame in both cases. We refer to firm 1 as the pivotal firm for convenience and consider the different cases

[4] All the other subgames yield expected payoffs equal to zero. When firm 3 enters the market, the model reaches a subgame in which firms 1 and 2 are not pivotal. The same occurs when $D^B = 0$.

[5] In the asymmetric context of Appendix B, the assumption would entail a loss of generality. However, the results from this section are valid for the asymmetric case provided the indexes for the firms are interpreted contextually, so that firm 1 is understood to represent the firm with the most capacity remaining after period A (and the firm with the largest amount of capacity in the overall game).

[6] In the asymmetric case we define $\kappa_j = \min\{D^B, k_j - q_j\}$, and overcapacity still characterizes market for the period given that $k_1 + k_2 \geq D$.

for firm 2 separately whenever necessary.

3.1 No pure-strategy equilibria

The generic subgame described above has no pure-strategy equilibria. First, any asymmetric strategy profile where prices from both firms differ cannot be an equilibrium. Provided that both prices are below R, the firm setting the lowest price has an incentive to adjust it upwards, set it right below its competitor's price, and continue to produce at capacity. If the strategy of the firm pricing lowest is to set a price at or above R, the market is deserted. Therefore, both firms have incentives to choose a price of R and capture all or as many sales as they can serve with their available capacity. Moreover, it is apparent that any price $p^B > R$ will be strictly dominated by playing $p^B = R$ and selling only to the captive portion of demand for any pivotal firm. This means that prices in excess of the reservation value for any pivotal firm can be excluded from any equilibrium. The same is true for $p^B = 0$.

Second, consider a symmetric candidate equilibrium in which both incumbents price at $p_1^B = p_2^B = p^B \leq R$. For any $p^B > 0$, each firm will profitably undercut the rival's price and capture all the demand that its remaining capacity allows. The only price at which this is not the case is $p^B = 0$. However, expected profits are indeed zero at this point, but then the pivotal firm would be better off deviating to R, where it can serve the portion of residual demand for which it is pivotal at the highest possible margin.

3.2 A mixed-strategy equilibrium

A mixed-strategy Nash equilibrium necessarily exists in the subgames considered above.[7] In this equilibrium, each pivotal firm sets (positive) prices according to a distribution function in such a way that no competitor can profitably change the distribution of probability according to which its price is determined.

We characterise the unique mixed-strategy equilibrium of these subgames in the following subsections. First, we lay out the expected profit function that pivotal firms face when deciding their pricing strategies. Second, we characterise the common extremes of the strategy supports of the firms' distribution functions in equilibrium, and derive an analytical expression for the unique mixed strategy equilibrium of the subgame. All intermediate results that we use as a foundation for the equilibrium characterization are derived in Appendix A.

3.2.1 The expected profit function

In the following, whenever a mathematical expression applies to both pivotal firms, we denote the pivotal firm with $j = 1, 2$ and its rival with l $(j \neq l)$. The expected profits for firm j of setting a price $p_j^B \leq R$ conditional on firm l's price strategy is given by

$$\begin{aligned}\mathbb{E}\left[\pi_j^B(p_j^B, p_l^B) \middle| p_j^B\right] \triangleq & \left[\Pr(p_l^B > p_j^B) + \Pr(p_l^B = p_j^B)s_j\right] p_j^B \kappa_j \\ & + \left[\Pr(p_l^B < p_j^B) + \Pr(p_l^B = p_j^B)(1 - s_j)\right] p_j^B (D^B - \kappa_l),\end{aligned}$$

[7] Nash (1950) showed the general existence equilibria in games with continuous payoffs while Dasgupta and Maskin (1986) extended their existence to the context that is relevant here, in the presence of discontinuous payoff functions.

where the elements in the sum represent the profits of winning, drawing or losing in the tender, multiplied by the probability of each of these events happening. Recall that the parameter $s_j \in [0,1]$ is the fraction of demand served by the pivotal firm in case of tie. The expression above can also be written in terms of the distribution function that l is expected to employ to assign probabilities to prices in its support

$$\begin{aligned}\mathbb{E}\left[\pi_j^B(p_j^B, p_l^B) \middle| \, p_j^B\right] &\triangleq p_j^B \left[(1 - F_l(p_j^B))\kappa_j + F_l(p_j^B)\left(D^B - \kappa_l\right)\right] \\ &= p_j^B \left[\kappa_j - F_l(p_j^B) O^B\right], \quad \forall p_j \leq R.\end{aligned}$$

Where $O^B \triangleq \kappa_j + \kappa_l - D^B$ represents the overcapacity in the market up to $2D^B$ while $F_l(p^B)$ denotes the cumulative distribution function — hereafter CDF — characterising the randomized strategy that rival firm l is expected to follow in equilibrium (the same notation applies, *mutatis mutandis*, to j). The expression above ignores the event of ties occurring, which could exist if both firms assigned a positive mass of probability to the same price. In Appendix A we show that the probability of drawing is zero.

A mixed strategy equilibrium requires that $F_j(p^B)$ best-responds to $F_l(p^B)$ and vice-versa. A known property of mixed-strategy equilibria, on which we rely extensively for our results, is that firm j (resp. l) obtains the same expected profits for each of the prices p_j^B (resp. l) in the support of $F_j(p^B)$ (resp. $F_l(p^B)$). Indeed, if this was not the case, the strategy of firm j (resp. l) would not be a best response because it would be better off removing any probability assigned to prices with relatively low expected profits and assigning it to prices that yield higher expected profits.

3.2.2 Equilibrium derivations

We derive an analytical functional form for the equilibrium CDFs of the firms, $F_1(\cdot)$ and $F_2(\cdot)$, that describes the mixed-strategy Nash equilibrium of this generic subgame in two steps. First, we characterise the support of prices that both firms employ in equilibrium. Then, we use the method of indeterminate coefficients to obtain an expression for the CDFs.

Support. The price support for both firms in equilibrium is summarised in Proposition 1 below. We refer the reader to Appendix A for any auxiliary intermediate result and derivation leading to this result.

Proposition 1. *In equilibrium, both firms have a support with lower-bound at*

$$\underline{p}^B \triangleq R \frac{D^B - \kappa_2}{\kappa_1} > 0,$$

an upper-bound at $\overline{p}^B \triangleq R$. If $\kappa_1 > \kappa_2$, the strategy of firm 1 has a probability mass at R.

As intuition suggests, the equilibrium mixed strategy features a lower-bound $\underline{p}^B$ that is increasing in the reservation price R (the same obviously holds for the upper-bound $\overline{p}^B$) and in the second period demand D^B. By contrast, the lower-bound is decreasing in κ_1 and κ_2, which reflect a measure of the incumbents' effective capacity in period B.

Functional form for $F_j(p)$. To derive the equilibrium CDFs we exploit the equilibrium condition that requires expected profits at each price in the support to be constant. Analyzing this condition reveals that the shape of the strategy is of the form

$$F_2(p^B) = \alpha - \frac{\beta}{p^B},$$

with α and β constants. Following Proposition 1, these constants can be determined computing the expected profits of firm 1 at $\underline{p}^B$ or R and equating them to the expected profits at any other point within the support. This is,

$$\begin{aligned} \mathbb{E}\left[\pi_1^B(p_1^B, p_2^B) \middle| p_1^B = R\right] &= \mathbb{E}\left[\pi_1^B(p_1^B, p_2^B) \middle| p_1^B = p^B\right] \\ \Leftrightarrow \quad R(D^B - \kappa_2) &= p^B\left(\kappa_1 - F_2(\cdot)O^B\right), \end{aligned}$$

yielding the expression

$$F_2(p^B) \triangleq \underbrace{\frac{\kappa_1}{O^B}}_{\alpha} - \underbrace{\frac{R(D^B - \kappa_2)}{O^B}}_{\beta}\frac{1}{p^B}. \tag{1}$$

The probability distribution for firm 1 is similar, although it contains an atom of probability and, hence, a discontinuity point at $p^B = R$. In this case, one can equate the expected profits at the infimum of the support to any other price within the support for $p^B < R$ and get a similar expression of the form

$$F_1(p^B) = \frac{\kappa_2}{\kappa_1}\left(\alpha - \frac{\beta}{p^B}\right),$$

with α and β being constants to identify. This is,

$$\begin{aligned} \mathbb{E}\left[\pi_2^B(p_1^B, p_2^B) \middle| p_2^B = \underline{p}^B\right] &= \mathbb{E}\left[\pi_2^B(p_1^B, p_2^B) \middle| p_2^B = p^B\right] \\ \Leftrightarrow \quad R\frac{\kappa_2}{\kappa_1}(D^B - \kappa_2) &= p^B\left(\kappa_2 - F_1(\cdot)O^B\right). \end{aligned}$$

Hence,

$$F_1(p^B) \triangleq \begin{cases} \underbrace{\frac{\kappa_2}{\kappa_1}\frac{\kappa_1}{O^B}}_{\triangleq \frac{\kappa_2}{\kappa_1}\alpha} - \underbrace{\frac{\kappa_2}{\kappa_1}\frac{R(D^B - \kappa_2)}{O^B}}_{\triangleq \frac{\kappa_2}{\kappa_1}\beta}\frac{1}{p^B} & \text{if } p^B < R, \\ 1 & \text{if } p^B = R, \end{cases} \tag{2}$$

which is similar to the one for the CDF of pivotal firm 2 except for the factor $\kappa_2/\kappa_1 \leq 1$ and the discontinuity at $p^B = R$.

The expression above also indicates that pivotal firm 1 prices below R with probability $\kappa_2/\kappa_1 \leq 1$, and that it distributes probability within this segment using a scaled version of the density function of firm 2. Pivotal firm 1 assigns a mass of probability $1 - \kappa_2/\kappa_1$ to a price equal to R where it exploits the maximum possible margin from its captive demand. Thus, firm 1 sets prices according to a distribution function that first-order stochastically dominates the equilibrium distribution function of firm 2 or, in simpler terms, firm 1 will charge, on average, higher prices in equilibrium when $\kappa_1 > \kappa_2$.

The difference in pricing strategies between firm 1 and firm 2 is proportional to the asymmetry in their available capacities at the beginning of period B. The larger the difference between the two capacities is, the larger the captive demand for pivotal firm 1 becomes, and therefore the probability mass set at $p^B = R$ becomes larger. Conversely, if the available capacities at period B for both firms are the same, then both firms play completely symmetric strategies.

It is apparent that this equilibrium is the unique one in these subgames. We prove this using the results in Appendix A, which are derived for any mixed-strategy equilibrium. Since the characterization of the equilibrium provided by these expressions is complete, this establishes the uniqueness of equilibrium.

4 Solution of the subgame in period A

Once having resolved the generic subgame in period B, we can proceed backward to solve for the equilibrium prices in the first period and thus complete the equilibrium characterisation for the entire game.

The expected profit function for either of firm 1 and firm 2 in period A is the sum of two components; the first (I) takes a similar form to the one for period B, while the second (II) corresponds to the expected profits in the following period, $\pi_j^B(\cdot)$. Let $\phi_j^A \triangleq \Pr(p_l^A > p_j^A) + \Pr(p_l^A = p_j^A)s_j$, then the expected profit function takes the form:

$$\mathbb{E}\left[\pi_j(p_j^A, p_l^A) \middle| p_j^A\right] \triangleq \underbrace{\phi_j^A p_j^A D^A}_{\text{I}} + \underbrace{\phi_j^A \mathbb{E}\left[\pi_j^B \middle| \kappa_j = k - D^A\right] + (1-\phi_j^A)\mathbb{E}\left[\pi_j^B \middle| \kappa_j = k\right]}_{\text{II}},$$

where the expectation operator in the continuation payoff is conditional on the capacity left after winning or losing the tender in period A, but integrates over all other random aspects, including the uncertainty about entry and demand in the second period, as well as the randomization produced by the mixed strategies. The expression for this continuation payoff takes the form:

$$\mathbb{E}\left[\pi_j^B \middle| \kappa_j\right] \triangleq \begin{cases} \rho\lambda R\left(D-k\right), & \text{if } \kappa_j = k, \\ \rho\lambda R\frac{k-D^A}{k}\left(D-k\right), & \text{if } \kappa_j = k - D^A. \end{cases} \tag{3}$$

It is interesting to observe analytically that λ and ρ mark the switches in the subgames reached in period B. With probability $\rho\lambda$, the demand in period B materialises to D^B without entry, at least one of the firms becomes pivotal, and they compete in the Bertrand-Edgeworth game described in the previous section. With probability $(1-\rho)\lambda$, high demand realises, but firms 1 and 2 are no longer pivotal because entry of firm 3 occurs and they compete à la Bertrand with profits being equal to zero. Finally, with probability $1-\lambda$, the bad state of demand realises: in this case profits in period B are always zero because there is no demand to be served.

Comparing $\mathbb{E}[\pi_j^B|\kappa_j]$ when $\kappa_j = k$ and when $\kappa_j = k - D^A$, we have that expected profits in period B are greater when $\kappa_j = k$, that is, when firm j lost the competition in period A.

4.1 Pure strategy equilibrium in the first period

Since $D^A \leq k$, any of the two firms, firm 1 and firm 2, suffices to absorb the entire demand, and the solution to this subgame resembles a standard Bertrand model of competition in prices with homogeneous goods. Mechanically, the equilibrium price is computed finding the indifference point between, on the one hand, winning in period A and selling D^A, and, on the other hand, waiting to compete with greater capacity in period B. This is, the price that ensures to capture the entire market in period A, and a reduced amount of capacity in period B, produces the same profits as when making no sales in period A, and moving to period B with all capacity in place. That is,

$$\begin{aligned}
&\mathbb{E}\left[\pi_j(p_j^A, p_l^A) \middle| p_j^A < p_l^A\right] \triangleq p_j^A D^A + \rho\lambda R \frac{k - D^A}{k} (D - k), \\
&\mathbb{E}\left[\pi_j(p_j^A, p_l^A) \middle| p_j^A > p_l^A\right] \triangleq \rho\lambda R(D - k).
\end{aligned}$$

Solving

$$\mathbb{E}\left[\pi_j(p_j^A, p_l^A) \middle| p_j^A > p_l^A\right] = \mathbb{E}\left[\pi_j(p_j^A, p_l^A) \middle| p_j^A < p_l^A\right]$$

for p_j^A, we obtain the following closed-form expressions solving for the first-period equilibrium price:

$$p^{A*} \triangleq \rho\lambda R \frac{D - k}{k} > 0. \tag{4}$$

The impact on the equilibrium price of an increase in the likelihood of positive demand in period B and a reduction in the probability of entry is identical: the greater $\lambda\rho$, the greater the equilibrium price is. The equilibrium price in period A is greater than zero because both firms are willing to sacrifise output and revenues in period A, when they are symmetric and competition is fierce given limited demand, to secure a pivotal (or more pivotal) position in period B when demand, if positive, would be large and individual capacity constraints binding.

To see why this is an equilibrium, in what follows we discard all potential deviations. First, if any firm deviated to prices lower than p^{A*}, it would win for sure in period A, make less margin than if they won in period A at p^{A*}, and secure the same expected profits in period B as if it won at p^{A*}. Because p^{A*} is defined so that profits are the same independently of whether the firm wins or waits when the tie is being resolved, it means the deviation is not profitable. Second, consider a potential deviation to price above p^{A*}. This deviation would ensure losing in period A, and produce the same profits than at p^{A*}, by definition of p^{A*}. This shows there are (weakly) no incentives to deviate upwards from p^{A**}.

4.2 Uniqueness

The expected profit of firm j can be written as follows:

$$\begin{aligned}
\mathbb{E}\left[\pi_j(p_j^A, p_l^A) \middle| p_j^A\right] &\triangleq \phi_j^A \left(p_j^A D^A + \rho\lambda R \frac{k - D^A}{\bar{k}} (D - k)\right) + \left(1 - \phi_j^A\right) \left(\rho\lambda R (D - k)\right), \\
&= \rho\lambda R (D - k) + \phi_j^A D^A \left(p_j^A - \rho\lambda R \frac{D - k}{\bar{k}}\right).
\end{aligned}$$

Thus, in equilibrium, where $p_j^A = p_l^A = p^{A*}$, the expected profits of each firm in the market equals:

$$\mathbb{E}\left[\pi_j(p^{A*}, p^{A*})\right] = \rho\lambda R(D-k).$$

The expression above serves to explain why p^{A*} is indeed the unique pure-strategy equilibrium price in period A. Consider any pure strategies equilibrium where the equilibrium price set by the winning firm p_j^A is below p^{A*}. In such an equilibrium, $\phi_j^A = 1$ and, hence, the winning firm would earn expected profits that are below the one of the losing firm, $\rho\lambda R(D-k)$, and would prefer to set a higher price in period A.

If, instead, the price set by the winning firm p_j^A was above p^{A*}, the firm losing would have incentives to undercut, since then $\phi_l^A = 1$ and, hence,

$$\mathbb{E}\left[\pi_l\left(p_l^A, p_j^A\right)\middle|\, p_j^A > p^{A*}\right] > \rho\lambda R(D-k).$$

In a nutshell, the equilibrium in period A has the form of the standard Bertrand equilibrium where prices are equal to costs, including the opportunity cost of not retaining greater idle capacity in period B.

5 Expected prices

In this section we derive analytical expressions for the equilibrium (expected) prices and profits. We define the expected average market price to be the average unit price that is expected to be paid in equilibrium.

5.1 Expected prices in period B

Let $p^{B*}\left(p_1^B, p_2^B\right)$ denote the average price that is charged across all sales in period B conditional on the (actual) prices charged by firms 1 and 2, when D^B materialises and in the absence of entry by firm 3. Formally,

$$p^{B*}\left(p_1^B, p_2^B\right) \triangleq \begin{cases} p_1^B \frac{\kappa_1}{D^B} + p_2^B \frac{D^B - \kappa_1}{D^B}, & \text{if } p_1^B \leq p_2^B, \\ p_2^B \frac{\kappa_2}{D^B} + p_1^B \frac{D^B - \kappa_2}{D^B}, & \text{if } p_1^B > p_2^B. \end{cases}$$

Computing the expected average price of $p^{B*}(\cdot)$ over the strategic randomisation of both firms requires distinguishing between two kinds of events: (i) events in which firm 1 prices below R; and (ii) others where firm 1 prices at R.

Firm 1 prices below R. Let j and l be the winning and losing firms, respectively. The integral over the price of the losing and winning firms conditional on the one for the winning firm being smaller than that of the losing firm takes the form:

$$\int_{\underline{p}^B}^{R}\int_{p_j^B}^{R} p^{B*}\left(p_j^B, p_l^B\right) dF_l(p_l^B) dF_j(p_j^B) = \int_{\underline{p}^B}^{R}\int_{p_j^B}^{R} \left(p_j^B \frac{\kappa_j}{D^B} + p_l^B \frac{D^B - \kappa_j}{D^B}\right) dF_l(p_l^B) dF_j(p_j^B).$$

Firm 1 prices at R. The integral that covers the events in which firm 1 sets a price equal to R is similar to the one above, but in this case integration over the prices of the losing firm (firm 1) is not required, and the expression takes the form:

$$\int_{\underline{p}^B}^{R} p^{B*}\left(p_2^B, R\right)(1-\bar{F}_1)dF_2(p_2^B) = \int_{\underline{p}^B}^{R}\left(p_2^B\frac{\kappa_2}{D^B} + R\frac{D^B-\kappa_2}{D^B}\right)\left(1-\bar{F}_1\right)dF_2(p_2^B).$$

where $\bar{F}_1 \triangleq F_1\left(R-\varepsilon\right)$.

Expected price in period B with positive demand and no entry. The first integral above can be computed when each of the firms wins yielding two terms, while the second integral can be computed to yield a third term. We add the three terms to obtain a simplified expression for the average prices in period B when there is high demand and firm 3 does not enter:

$$\mathbb{E}\left[p^{B*}\left(\cdot\right)\right] = R\left(1-\frac{\kappa_2}{\kappa_1}\frac{O^B}{D^B}\right). \tag{5}$$

This expression is decreasing in the amount of relevant overcapacity O^B, converging to the monopolist price (R) when there is none. Since $0 \leq \kappa_2/\kappa_1 \leq 1$ with the ratio taking the value of 1 when there is symmetry and taking the value of 0 when the asymmetry is maximal, the expression is also decreasing in the relevant degree of symmetry (κ_2/κ_1) in period B, also converging to the monopolist price when asymmetry becomes extreme ($\kappa_2/\kappa_1 \to 0$). This expression can be interpreted as a fraction of R that is reduced as: (i) the degree of capacity symmetry increases, and (ii) the amount of overcapacity relative to demand increases.

The price in period B is equal to zero if firm 3 enters the market, since then the market will move from a situation of excess demand to a scenario of excess capacity where none of the firms is pivotal. It follows that a reduction in the likelihood of entry — an increase in ρ — will result in higher expected prices in period B when $\lambda > 0$. Likewise, an increase in the probability of $D_B > 0$ will increase prices when $\rho < 1$.

5.2 Expected average market price across periods

The expected average market price equals the weighted average of the prices expected in both periods, with weights given by the amount of sales in each period. They integrate across the various uncertainty scenarios. Formally, we have the expression,

$$\mathbb{E}[p^*\left(\cdot\right)] = \underbrace{\lambda\rho\left(\mathbb{E}\left[p^{B*}\left(\cdot\right)\right]\frac{D^B}{D} + p^{A*}\frac{D^A}{D}\right)}_{\substack{\text{Positive demand, but} \\ \text{no entry in period } B}} + \underbrace{\lambda(1-\rho)\left(p^{A*}\frac{D^A}{D}\right)}_{\substack{\text{Positive demand and} \\ \text{entry in period } B}} + \underbrace{(1-\lambda)p^{A*}}_{\substack{\text{No demand} \\ \text{in period } B}},$$

which can be rewritten as

$$\mathbb{E}[p^*\left(\cdot\right)] = \lambda\rho R\left(1-\frac{O}{D}+(1-\lambda)\frac{D-k}{D}\frac{D^B}{k}\right), \tag{6}$$

where $O = 2k - D$. This expression converges to

$$\mathbb{E}[p^*(\cdot)] = R\left(1 - \frac{O}{D}\right),$$

when the context is least competitive — i.e., when it is certain demand will materialise in period B ($\lambda = 1$) and that firm 3 will *not* enter ($\rho = 1$).

5.3 Equilibrium price and the RSI

The expression above for the expected average prices can be rewritten in terms of the residual supply index (RSI, henceforth) of the industry for either one of the pivotal firms, which equals k/D in this setup. A firm's RSI is a measure of its market power. The coefficient $1 - RSI$ equals the percentage of market demand that is captive to the firm under consideration and cannot be served by any of its rivals. The greater that percentage, the higher the price (and margin) the pivotal firm will be able to charge in equilibrium for a given level of demand and capacity uncertainty.

Rearranging terms, the expected average price takes the following form

$$\mathbb{E}[p^*(\cdot)] = \lambda\rho R(1 - RSI)\left(2 + (1-\lambda)\frac{D^B}{k}\right). \tag{7}$$

This expression is monotonically increasing in the presence of overcapacity and converges to $2R(1 - RSI) < R$ when the context is least competitive — i.e., when it is certain that demand will materialise in period B ($\lambda = 1$) and that firm 3 will *not* enter ($\rho = 1$) — and is equal to zero when $RSI = 1$ (i.e., when firms 1 and 2 are not pivotal).[8]

Moreover, the following can also be established for expected average prices in the market:

$$\frac{\partial \log \mathbb{E}\left[p^*(\cdot)\right]}{\partial \log \rho} = 1 > \frac{2k - \lambda D^B}{2k + (1-\lambda)D^B} = \frac{\partial \log \mathbb{E}\left[p^*(\cdot)\right]}{\partial \log \lambda}.$$

That is, the expected market price reacts relatively more to changes in the probability of entry, ρ, than to an increase in expected demand, λ. Both an increase in ρ and an increase in λ increase period A and (expected) period B prices in the same way. However, for given period A and B prices, an increase in ρ shifts probability from states of nature with lower average market prices across periods towards the state of nature with higher average market prices, whereas an increase in λ only does so when ρ is high enough.

6 Changes in pivotality

In this section, we study the relationship between changes in the industry overall capacity and equilibrium prices. Consider an increase in the RSI. Such increase could happen, for instance, if a competitive fringe, including one or more price-taking firms, had available capacity to absorb some

[8] $RSI \leq 1$ in our model since $k \leq D$. In practice, k may be greater than D and so the RSI may be greater than one, in which case market prices will converge to marginal costs.

demand in period A. The partial derivative of (7) over this change in the RSI is

$$\frac{\partial \mathbb{E}[p^*(\cdot)]}{\partial RSI} = -\lambda \rho R \left(2 + (1-\lambda)\frac{D^B}{k}\right) < 0.$$

That is, an increase in the RSI — or, equivalently, a reduction in the amount of captive demand for the pivotal firms — reduces the expected level of prices.[9]

We use the derivative above to perform comparative statics of the effect that changes in market structure have on expected average prices. Let ψ be the aggregate capacity of the competitive fringe in period A, then the RSI for firms 1 and 2 equals $RSI(\psi) \triangleq \frac{k}{D-\psi}$, where the function $RSI(\psi)$ is increasing in ψ. A change in ψ will have an impact on expected prices:

$$\begin{aligned}\frac{\partial \mathbb{E}[p^*(\cdot)]}{\partial \psi} &= \underbrace{\frac{\partial \mathbb{E}[p^*(\cdot)]}{\partial RSI}}_{<0}\underbrace{\frac{\partial RSI(\psi)}{\partial \psi}}_{>0} \\ &= -\lambda \rho R \left(2 + (1-\lambda)\frac{D^B}{k}\right)\frac{k}{(D-\psi)^2} < 0. \end{aligned} \tag{8}$$

That is, a reduction in the aggregate capacity of the competitive fringe will result in an increase of the expected level of prices. Or, equivalently, if firms 1 and 2 acquired a fraction of the capacity held by the competitive fringe, that would result in an increase in the average price across periods.

The price impact of a reduction in ψ, or a merger, will be smaller when λ and ρ are small — i.e., demand is uncertain and entry is relatively more likely. The more uncertain demand is and/or the more likely entry in period B, the smaller the impact on prices of reductions in the aggregate capacity of the competitive fringe in period A. An implication of this last result is that no capacity divestment may be needed when demand is uncertain and entry likely despite the existence of capacity constraints and high market shares.

7 Compensating marginal cost reductions

We can use a slightly modified version of equation (7) to assess whether a reduction in the marginal costs of firms 1 and 2 can offset the increase in prices resulting from a reduction in ψ or, in other words, an increase in the incumbents' pivotality.

We first develop a normalisation of the model that allows us to modify equation (7) to include a positive marginal cost, $c > 0$. Then, we determine the reduction in marginal costs that offsets the price impact of a reduction in ψ. This is the so-called compensating marginal cost reduction (CMCR, henceforth) as in Werden and Froeb (1994).

[9] We focus on the prices charged by the strategic firms, abstracting from the price the competitive fringe may charge for the sales it makes.

7.1 Normalisation

In this subsection, we show that the conclusions of the model are robust to any arbitrary constant marginal cost $c > 0$, and use this result to rewrite (7) for the CMCRs.

For every $t = A, B$, let us redefine p_j^t to be the margin of firm j; and $p_{j,\text{abs}}^t$, the actual price charged by firm j, so that $p_j^t = p_{j,\text{abs}}^t - c$. Solving the two-period game set out in Section 2, where firms 1 and 2 compete in periods A and B setting prices p_j^t with a reservation price of r is equivalent to solving the same game where they compete setting margins p_j^t and the reservation price is $R = r - c$. This is because there is a one-to-one mapping between the demands that firm j faces when choosing margins and absolute prices, respectively, as we demonstrate in Appendix B.

Therefore, equation (7) can be rewritten accounting for a non-zero marginal cost as

$$\mathbb{E}[p_{\text{abs}}^*(\cdot)] = c + \lambda\rho(r-c)(1-RSI)\left(2+(1-\lambda)\frac{D^B}{k}\right), \tag{9}$$

where $\mathbb{E}[p_{\text{abs}}^*(\cdot)]$ is the actual expected average market price across periods.

7.2 The CMCRs

It follows from (9) that

$$\begin{aligned}\frac{\partial\mathbb{E}[p_{\text{abs}}^*(\cdot)]}{\partial\psi} &= -\lambda\rho(r-c)\left(2+(1-\lambda)\frac{D^B}{k}\right)\frac{k}{(D-\psi)^2} < 0 \\ &\leq 1-\lambda\rho(1-RSI)\left(2+(1-\lambda)\frac{D^B}{k}\right) = \frac{\partial\mathbb{E}[p_{\text{abs}}^*(\cdot)]}{\partial c}.\end{aligned}$$

where the second inequality is strict in the presence of overcapacity ($2k > D$) — i.e., when $RSI < 1$. Then, the CMCR is calculated as the change in the marginal cost c, dc, such that $d\mathbb{E}[p_{\text{abs}}^*(\cdot)] = 0$ when $d\psi \neq 0$. That is,

$$dc = -\frac{\partial c}{\partial\psi}d\psi, \tag{10}$$

where

$$\frac{\partial c}{\partial\psi} = \frac{\lambda\rho\theta(\lambda)}{1-\lambda\rho(1-RSI)\theta(\lambda)}\left(\frac{k}{(D-\psi)^2}\right) > 0,$$

and

$$\theta(\lambda) = 2+(1-\lambda)\frac{D^B}{k}.$$

It follows that a reduction in ψ, $d\psi < 0$, requires a drop in marginal costs given by equation (10) so that the increase in pivotality leaves market prices unchanged for the competing firms.

Note that dc is increasing in both ρ, i.e., a lower probability of entry, and in λ, i.e., a higher probability of future demand. In the extreme, $dc = 0$ when λ and ρ are zero — i.e., when the likelihood of positive demand in period B is zero and/or the probability of entry is large. in other words, a reduction in the capacity of the fringe due, for example, to a merger will require some cost efficiency to be pro-competitive, but the magnitude of such efficiency needed to that end is decreasing when future demand is more uncertain and future entry more likely.

8 Concluding remarks

We have shown that capacity constrained oligopolists' ability to price above costs is limited by uncertainty in demand and supply. Of course, other things equal, equilibrium prices are greater when firms' pivotality is increased. Yet, unlike in the previous literature, the relationship is mediated by the probability of high future demand and, especially, the likelihood of future entry. These findings have implications for the design of merger control policy in cases involving markets with excess capacity and, in particular, the balancing of efficiencies, the weight given to the structural presumption, and the scope of any divestiture remedies.

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A Proof of Proposition 1

A.1 Auxiliary results on $F_j(\cdot)$ in equilibrium

In this appendix, we derive a set of auxiliary results that are useful to characterise the equilibrium and establish its uniqueness. We present these results in a series of lemmata. The main insights that can be drawn from these results are that: (*i*) the strategy support of both pivotal firms is convex and their extrema coincide, and (*ii*) the CDF for pivotal firms is continuous in the interior of the support.

Lemma 1. *In equilibrium, the supremum of the support of the equilibrium strategies of both firms is at or below R.*

Proof. Assume a firm assigned probability mass to prices above R. Then, its expected profits would have to be zero because of the equilibrium condition that requires the expected profits to be the same across all prices in the support. We show that this cannot be the case in equilibrium.

First, and by contradiction, if the firm were pivotal it would have incentives to deviate and price equal to R, sell to its captive demand, and make positive profits.

Second, if the firm were not pivotal, the model assumptions imply that its competitor would be pivotal. Let the non-pivotal firm be indexed by j, and the pivotal competitor firm be indexed by l, with the supremum of the support of its equilibrium strategy denoted as $\overline{p}_l^B$. From the first part of the proof we know that $\overline{p}_l^B \leq R$. Then, firm j would have incentives to move some or all of the probability mass in its equilibrium strategy to a price below $\overline{p}_l^B$ and obtain positive profits, provided that $\overline{p}_l^B > 0$. It then follows immediately that $\overline{p}_l^B > 0$ because if it were zero, firm l would make zero profits, and it could make positive profits by pricing purely at R and capturing its captive sales. □

Lemma 2. *The probability of a tie in equilibrium is zero.*

Proof. For a tie to have positive probability in equilibrium, it needs to be the case that both firms have a mass of probability at some common point in the support of their strategies. Lemma 1 rules out prices above R immediately. In what follows, we show ties have no positive probability in equilibrium for prices at or below R.

By contradiction, assume that both firms set strategies such that they include a mass of probability at some price $p^B \leq R$. That is, they assign non-zero probabilities to this price and distribute the remaining probability mass across the rest of prices in their support.

Then, there exists $\varepsilon > 0$ small enough such that assigning the mass of probability at p^B to $p^B - \varepsilon$ would be profitable to either $j = 1, 2$. Denote $F_j(\cdot)$ and $F_j'(\cdot)$ the distribution functions with the mass at p^B and $p^B - \varepsilon$ and the rest of probability distributed identically over the support. Formally

$$\begin{aligned}
&\lim_{\varepsilon\to 0^+} \left(\mathbb{E}\left[\pi_j^B\left(p_j^B, p_l^B\right)\middle| F_j\right] - \mathbb{E}\left[\pi_j^B\left(p_j^B, p_l^B\right)\middle| F_j'\right]\right)\\
&\quad= \lim_{\varepsilon\to 0^+} \Pr\left(p_j^B = p^B\middle| F_j\right)\left(F_l(p^B) - F_l(p^B-\varepsilon)\right)(p^B-\varepsilon)\kappa_j\\
&\qquad - \Pr\left(p_l^B = p^B\right)\Pr\left(p_j^B = p^B\middle| F_j\right)p^B\left[s_j\kappa_j + (1-s_j)(\kappa_j - (D^B - \kappa_l))\right]\\
&\quad= \Pr\left(p_j^B = p^B\middle| F_j\right)\Pr\left(p_l^B = p^B\right)p^B\kappa_j\\
&\qquad - \Pr\left(p_l^B = p^B\right)\Pr\left(p_j^B = p^B\middle| F_j\right)p^B\left[s_j\kappa_j + (1-s_j)\left(\kappa_j - (D^B - \kappa_l)\right)\right]\\
&\quad= \Pr\left(p_j^B = p^B\middle| F_j\right)\Pr\left(p_l^B = p^B\right)(1-s_j)p^B O^B,
\end{aligned}$$

which is positive because of relevant overcapacity. □

In intuitive terms, the result above reflects that the profits for all prices other than $p^B - \varepsilon$ are virtually the same, while the profits at $p^B - \varepsilon$ increase. The probability of winning the tender and producing at capacity increases while the profits for each unit decrease by a negligible amount ε.

This result is similar in nature to the one showing that firms have incentives to undercut symmetric pure strategy profiles. However, in this case, the undercut involves moving a probability mass to lower prices instead of pricing purely at them.

Lemma 3. *In equilibrium, the support of the distribution function characterising the equilibrium strategies for the pivotal firms has no gaps, i.e., it is convex.*

Proof. Lemma 1 rules out gaps with a supremum above R, since such kind of gap would imply some probability mass would be located above the supremum of the gap. Below, we show there are no gaps in the support of the strategies of either firm when the supremum of the gap is at or below R.

First, notice that a gap in the support of one of the firms would imply that its rival firm would also have the same gap in its support. If it didn't, the strategy of the rival firm would be strictly dominated by a similar one where the probability mass allocated along the gap would be relocated to its supremum. This reallocation would increase margins under some events without reducing the probability of winning in any of the events. Hence, it would be profitable.

Second, in order for an equilibrium to exist in which both firms have a gap in their support, it would need to be the case that, for both firms, the expected profits at the infimum of the segment were equal to those at its supremum. This is implied by the fact that any mixed strategy in equilibrium should have equal expected profits at each price of the support. Let $\underline{g} < \overline{g} \leq R$ within the support denote extrema of the gap. Then,

$$\begin{aligned}
\mathbb{E}\left[\pi_j^B\left(p_j^B, p_l^B\right)\middle| p_j^B = \underline{g}\right] &= \underline{g}\left[\kappa_j - F_l\left(\underline{g}\right)O^B\right]\\
&= \overline{g}\left[\kappa_j - F_l\left(\overline{g}\right)O^B\right]\\
&= \mathbb{E}\left[\pi_j^B\left(p_j^B, p_l^B\right)\middle| p_j^B = \overline{g}\right].
\end{aligned}$$

However, this condition would imply:

$$F_l\left(\overline{g}\right) - F_l\left(\underline{g}\right) = \frac{\overline{g} - \underline{g}}{\underline{g}}\frac{1}{O^B}\left[\kappa_j - F_l\left(\overline{g}\right)O^B\right] > 0.$$

Since the presence of a gap would imply $F_j(\cdot)$ and $F_l(\cdot)$ are constant in the interior of the gap (i.e., in the segment $(\underline{g}, \overline{g})$), the above condition could only hold if there was a discontinuity in both $F_j(\cdot)$ and $F_l(\cdot)$ at $\overline{g}$. This discontinuity means both firms would have to place a mass of probability at $\overline{g}$, which is not possible in equilibrium, as shown in Lemma 2. □

Lemma 4. *In equilibrium, the CDFs of the strategies of both firms are continuous in the interior of their support.*

Proof. Lemma 1 rules out discontinuities in the CDFs at or above R. If there was one, given it would have to be in the *interior* of the support by assumption, it would imply some mass of probability would have to be located above R, contradicting the lemma. Below, we show there are no discontinuities in the CDFs of either firm below R.

By contradiction, assume in equilibrium one of the firms played a probability distribution that yielded a discontinuous CDF in the interior of the support. That is, suppose there was some $g < R$ within the support such that:

$$F_j(g) - \lim_{\varepsilon \to 0^+} F_j(g - \varepsilon) > 0.$$

Following Lemma 3, if that were the case, for an arbitrarily small $\varepsilon > 0$, $g - \varepsilon$ would have to be part of the support of the strategy. However, the equilibrium condition requiring equality of expected profits at all prices in the support implies that it would also need to be the case that:

$$F_j(g) - \lim_{\varepsilon \to 0^+} F_j(g - \varepsilon) = \lim_{\varepsilon \to 0^+} \left(\frac{\varepsilon}{g - \varepsilon} \frac{1}{O^B} \left[\kappa_j - F_l(g)\, O^B \right] \right) = 0,$$

which is a contradiction with the inequality above. □

Lemma 5. *In equilibrium, the extremes of the support coincide for both firms.*

Proof. Lemma 1 rules out case in which the infima and/or the suprema of the supports are above R. Below, we show that the extremes of the support coincide for both firms when they are at or below R. Let $\underline{p}_j^B$ and $\overline{p}_j^B$ denote the infimum and supremum of the support of the strategies for firm j in equilibrium, respectively.

For the infima, consider strategies for the two firms such that $\underline{p}_j^B < \underline{p}_l^B \leq R$. First, notice that firm j would have a profitable deviation from this strategy, which is to move the mass of probability originally placed at prices between $\underline{p}_j^B$ and $\underline{p}_l^B$, $F(\underline{p}_l^B) - F(\underline{p}_j^B)$, to a price equal to $\underline{p}_l^B$. This form of iterative elimination of dominated strategies would work for firm j because it would provide it with the same probability of winning the tender at a higher margin for all the mass of probability previously assigned below $\underline{p}_l^B$. This argument mirrors, in a mixed-strategy context, one of the arguments used to rule out asymmetric pure-strategy profiles. However, in the mixed-strategy context the argument does not rule out equilibria, but instead shows that $\underline{p}_j^B$ coincides for both firms.

For the suprema, consider strategies for the two firms in which $\overline{p}_l^B < \overline{p}_j^B \leq R$. First, if firm j were pivotal, it would have an incentive to move all the mass of probability it currently assigns to prices between $\overline{p}_l^B$ and $\overline{p}_j^B$ to R. This would maximise the margin on each unit of its captive demand, without changing the expected amount of sales captured under any event.

Second, if firm j were not pivotal, its expected profits would have to be zero because they would have to be the same across all prices played with positive probability, and they would be zero for the prices with positive probability located above $\overline{p}_l^B$. Therefore, firm j would have incentives to move some or all mass in its strategy to a price below $\overline{p}_l^B$ and obtain positive profits, provided that $\overline{p}_l^B > 0$. That $\overline{p}_l^B > 0$ follows from the same argument as in Lemma 1. The assumptions of the model imply that firm l would be necessarily pivotal if firm j were not. Thus, firm l could make positive profits based on its captive sales, and would make zero profits if $\overline{p}_l^B$ were zero, which would preclude the latter from being part of a strategy in equilibrium. □

Given that the extremes of the supports coincide, in what follows we characterise the extremes of the support using the following reduced notation: $\underline{p}^B = \underline{p}_j^B = \underline{p}_l^B$, and $\overline{p}^B = \overline{p}_j^B = \overline{p}_l^B$.

Lemma 6. *In equilibrium, there is no probability mass at $\underline{p}^B$ for either of the two firms.*

Proof. Lemma 1 states that $\underline{p}^B < R$, while Lemma 2 implies that only one of the two firms could have a mass at $\underline{p}^B$. Therefore it suffices that no individual firm would have incentives to place a mass of probability at a price $\underline{p}^B$.

We show this by means of a contradiction. Assume that $\underline{p}^B < R$, that firm j had no mass at $\underline{p}^B$, and that firm l assigned some probability $P_l > 0$ to $\underline{p}^B$, and distributed its probability using F_l' over the remainder of the support. If this were the case, firm j would have higher profits at $\underline{p}^B$ than it would at price $\underline{p}^B + \varepsilon$ for some arbitrarily small $\varepsilon > 0$:

$$\begin{aligned}\lim_{\varepsilon\to 0^+} \mathbb{E}\left[\pi_j^B\left(p_j^B, p_l^B\right)\middle|\, p_j^B = \underline{p}^B + \varepsilon\right] &= \lim_{\varepsilon\to 0^+} (\underline{p}^B + \varepsilon)\left[\kappa_j - \left(P_l + F_l'\left(\underline{p}^B + \varepsilon\right)\right) O^B\right] \\ &= \underline{p}\left[\kappa_j - P_l O^B\right] \\ &< \underline{p}\left[\kappa_j - P_l(1 - s_j)O^B\right] \\ &= \mathbb{E}\left[\pi_j^B\left(p_j^B, p_l^B\right)\middle|\, p_j^B = \underline{p}^B\right].\end{aligned}$$

This would either violate the equality of expected profits within the support, or entail that $\underline{p}^B + \varepsilon$ were not part of the support of the strategy for firm j, implying a gap in the support of its strategy, contradicting Lemma 3. □

Lemma 7. *In equilibrium, the expected profits for both firms are equal to those they obtain if they capture as much demand as they can at a price of $\underline{p}^B$.*

Proof. First, Lemma 6 states that there is no probability mass at $\underline{p}^B$ for either player, which entails that the probability of winning the tender tends to one for both firms as prices approach this point. Second, in equilibrium, the expected profits a firm makes at any price included in the support of its strategy, including the neighborhood of $\underline{p}^B$, are the same. Together, these two results imply that the expected profits that each firm makes in equilibrium are equal to $\underline{p}^B\kappa_j$. Formally,

$$\lim_{\varepsilon\to 0^+} \mathbb{E}\left[\pi_j^B\left(p_j^B, p_l^B\right)\middle|\, p_j^B = \underline{p}^B + \varepsilon\right] = \underline{p}^B\kappa_j.$$

□

A.2 The support of the strategies

In this subsection, we characterise the common extremes of the support that both firms have in equilibrium. We describe the support for each of the firms in Proposition 1.

Lemma 8. *In equilibrium, the supremum of the support of the strategies of both firms is R.*

Proof. Consider an equilibrium where both firms use strategies with a supremum $\overline{p}^B < R$. The definition of the common supremum entails that $F_j(\overline{p}^B) = 1$. However, it is the case that at this point

$$\begin{aligned}\mathbb{E}\left[\pi_j^B\left(p_j^B, p_l^B\right)\middle| p_j^B = \underline{p}^B\right] &= \overline{p}^B(\kappa_j - O^B)\\ &\leq R(\kappa_j - O^B)\\ &= \mathbb{E}\left[\pi_j^B\left(p_j^B, p_l^B\right)\middle| p_j^B = R\right],\end{aligned}$$

which means any pivotal firms, of which there is at least one by the model assumptions, would have incentives to deviate to a price $p^B = R$. □

Lemma 9. *In equilibrium, if $\kappa_1 > \kappa_2$ there is a probability mass at R in the strategy of firm 1.*

Proof. Given that firm 1 is the largest of the two firms, it is necessarily pivotal. Its pivotality implies that, in order for its strategy to be part of an equilibrium, in has to yield profits at least as high as those firm 1 would make if it focused on its captive sales, $R(D^B - \kappa_2)$, because those profits can be secured independently of the actions of firm 2. If we combine this restriction with the result from Lemma 7 we obtain:

$$\underline{p}^B\kappa_1 \geq R(D^B - \kappa_2).$$

The inequality above implies $\underline{p}^B \geq R\frac{D^B-\kappa_2}{\kappa_1}$. We note here that if firm 2 were pivotal, a symmetric restriction on $\underline{p}^B$ would apply based on the expected profits firm 2 could secure by focusing on its captive sales. However, this second restriction is not binding given that $\kappa_1 > \kappa_2$.

The restriction derived above combined with the result from Lemma 7 has implications for the limit of F_1 as it approaches R from the left:

$$\begin{aligned}\lim_{\varepsilon\to 0^+}\mathbb{E}\left[\pi_2^B\left(p_2^B, p_1^B\right)\middle| p_2^B = R-\varepsilon\right] &= \lim_{\varepsilon\to 0^+}(R-\varepsilon)\left[\kappa_2 - F_1\left(R-\varepsilon\right)O^B\right]\\ &= \underline{p}^B\kappa_2 \geq R\left(D^B - \kappa_2\right)\frac{\kappa_2}{\kappa_1},\end{aligned}$$

which implies:

$$\lim_{\varepsilon\to 0^+}(R-\varepsilon)\left[\kappa_2 - F_1\left(R-\varepsilon\right)O^B\right] - R\left(D^B-\kappa_2\right)\frac{\kappa_2}{\kappa_1} \geq 0$$

The inequality above implies $\lim_{\varepsilon\to 0^+} F_1\left(R-\varepsilon\right) < 1$, since $\kappa_2 < \kappa_1$ by assumption. Given that R is the supremum of the support of the strategy of firm 1 by Lemma 8, it follows that $F_1(R) = 1$. Both assertions combined imply F_1 has a discontinuity at R, i.e. the strategy of firm 1 has a probability mass at that point. □

Lemma 10. *In equilibrium, the infimum of the support of the strategies of both firms is* $\underline{p}^B = R\frac{D^B - \kappa_2}{\kappa_1}$.

Proof. From Lemma 9 we know that $\underline{p}^B \geq R\frac{D^B - \kappa_2}{\kappa_1}$, so it suffices to show that the inequality is binding. By contradiction, assume $\underline{p}^B > R\frac{D^B - \kappa_2}{\kappa_1}$. This restriction combined with the result from Lemma 7 has implications for the limit of F_2 as it approaches R from the left:

$$\begin{aligned}\lim_{\varepsilon \to 0^+} \mathbb{E}\left[\pi_1^B\left(p_1^B, p_2^B\right) \middle| p_1^B = R - \varepsilon\right] &= \lim_{\varepsilon \to 0^+} (R - \varepsilon)\left[\kappa_1 - F_2(R - \varepsilon) O^B\right] \\ &= \underline{p}^B \kappa_1 > R\left(D^B - \kappa_2\right),\end{aligned}$$

which would imply:

$$\lim_{\varepsilon \to 0^+} F_2(R - \varepsilon) < 1.$$

If $\kappa_1 > \kappa_2$, Lemma 9 implies that there would be a mass at R for firm 1, while the inequality above implies that there would be one for firm 2 as well. This means that there would be mass at R in the strategy of both firms, contradicting Lemma 2. Similarly, if $\kappa_1 = \kappa_2$, the inequality above applies symmetrically, implying once more that there would be a probability mass in the strategy of both firms at R, contradicting Lemma 2. $\square$

B Further material

In this appendix we provide the characterization of the equilibrium with positive and asymmetric marginal cost.

B.1 Normalisation

The demand that firm j absorbs in period t when prices are $(p^t_{j,\text{abs}})$, for $j, l = \{1, 2\}$ and $j \neq l$ and the reservation price is r takes the form:

$$\begin{aligned} q^t_j\left(p^t_{\text{abs}}, r\right) =& \min\{D^t, k - q^A_j \mathbb{1}(t=B)\} \\ & \mathbb{1}\left(p^t_{j,\text{abs}} < r\right)\left[\mathbb{1}\left(p^t_{j,\text{abs}} < p^t_{l,\text{abs}}\right) + \mathbb{1}\left(p^t_{j,\text{abs}} = p^t_{l,\text{abs}}\right)\mathbb{1}\left(\text{Win}_j\right)\right] \\ & + \min\{D^t - q^t_l, k - q^A_j \mathbb{1}(t=B)\} \\ & \mathbb{1}\left(p^t_{j,\text{abs}} < r\right)\left[\mathbb{1}\left(p^t_{j,\text{abs}} > p^t_{l,\text{abs}}\right) + \mathbb{1}\left(p^t_{j,\text{abs}} = p^t_{l,\text{abs}}\right)\mathbb{1}\left(\text{Lose}_j\right)\right], \end{aligned} \tag{B.1}$$

where Win_j and Lose_j represent the cases in which the two firms draw and the tie breaks in favour of firm j (Win_j) or against it (Lose_j), respectively.

This expression can be rewritten in terms of $R = r - c$ so that R represents the maximum margin with positive demand:

$$\begin{aligned} q^t_j\left(p^t, R\right) =& \min\{D^t, k - q^A_j \mathbb{1}(t=B)\} \\ & \mathbb{1}\left(p^t_j < R\right)\left[\mathbb{1}\left(p^t_j < p^t_l\right) + \mathbb{1}\left(p^t_j = p^t_l\right)\mathbb{1}\left(\text{Win}_j\right)\right] \\ & + \min\{D^t - q^t_l, k - q^A_j \mathbb{1}(t=B)\} \\ & \mathbb{1}\left(p^t_j < R\right)\left[\mathbb{1}\left(p^t_j > p^t_l\right) + \mathbb{1}\left(p^t_j = p^t_l\right)\mathbb{1}\left(\text{Lose}_j\right)\right]. \end{aligned} \tag{B.2}$$

Given that there is a one-to-one relationship between the demands in (B.1) and (B.2), we have that for any other parameters (D^A, D^B, k, s):

$$\begin{aligned} \mathbb{E}\left[\pi_{\text{j,abs}}(p^t_{\text{j,abs}}, p^t_{\text{l,abs}})\right] &= \sum_t \mathbb{E}\left[(p^t_{\text{j,abs}} - c) q^t_j(p^t_{\text{j,abs}}, r)\right] \\ &= \sum_t \mathbb{E}\left[p^t_j q^t_j(p^t, R)\right] = \mathbb{E}\left[\pi_j(p_j, p_l)\right]. \end{aligned}$$

It follows, therefore, that the solution to the game where firms 1 and 2 set margins p^t_j for $t = \{A, B\}$ with a reservation margin R is equivalent to the solution of that game where instead they set prices $p_{\text{j,abs}}$, $t = \{A, B\}$, with a reservation price r.

It is worth noticing that this result can also be used to show that the expected average prices derived in equation (7) can be interpreted as percentage price increases relative to an arbitrary initial situation. In that case p^t could be rewritten as a percentage price increase and the relationship between (B.1) and (B.2) would be of proportionality instead of equality.

B.2 Asymmetric capacities

In the remainder of this appendix we assume that firms have different capacities. Yet, we impose that this asymmetry is not too large — i.e., $k_1 > k_2 \geq k_1 - D^A$.

Clearly, in this case, the equilibrium characterized for the symmetric case does not hold, since the indifference point for both firms differ. We show that the asymmetric setup has no pure strategy equilibrium, while it features infinitely many equilibria in mixed strategies. However, in spite of this multiplicity, the market outcome that materialises in all of these equilibria is the same.

B.2.1 No pure strategies equilibrium

A symmetric equilibrium in pure strategies does not exist when the capacities of the competing firms are asymmetric. This happens because the indifference points between winning and losing in period A are different for each firm. The two expressions that define these indifference points are:

$$\mathbb{E}[\pi_j(p_j^A, p_l^A)|p_j^A < p_l^A] = p_j^A D^A + \mathbb{E}\left[\pi_j^B(\cdot) \mid \kappa_j = k_j - D^A\right],$$
$$\mathbb{E}[\pi_j(p_j^A, p_l^A)|p_j^A > p_l^A] = \mathbb{E}\left[\pi_j^B(\cdot) \mid \kappa_j = \overline{k}_j\right].$$

The indifference price of each firm, which we denote $p_j^{A^*}$, can be obtained by equating the expressions above and solving for p_j^A.

The differential in profits for period B We denote the differential in profits for period B between winning and waiting in period A as

$$\begin{aligned}
\Delta\mathbb{E}\left[\pi_j^B(\cdot)\right] &= \mathbb{E}\left[\pi_j^B(\cdot) \mid \kappa_j = \overline{k}_j\right] - \mathbb{E}\left[\pi_j^B(\cdot) \mid \kappa_j = k_j - D^A\right] \\
&= \rho\lambda R\left(D - k_l - \frac{k_j - D^A}{\overline{k}_l}(D - k_j)\right). \\
&= \rho\lambda R(D - k_j)\left(\frac{D - k_l}{D - k_j} - \frac{k_j - D^A}{\overline{k}_l}\right).
\end{aligned}$$

The analysis of period B in Section 3 shows that the differential is positive in the symmetric case when $k < D$. In contrast, in the asymmetric case, its sign can vary depending on the term

$$\frac{D - k_l}{D - k_j} - \frac{k_j - D^A}{\overline{k}_l}.$$

For both firms, the term $(k_j - D^A)/\overline{k}_l$ is smaller than or equal to one because the asymmetry between firms is small (i.e., $k_1 > k_2 \geq k_1 - D^A$). For firm 1, this condition ensures that its differential is positive since $\frac{D-k_2}{D-k_1} > 1$. For firm 2, the sign of $\Delta\mathbb{E}\left[\pi_2^B(\cdot)\right]$ is ambiguous and becomes positive when k_2 is close to k_1.

When its profits differential in period B is negative, firm 2 prefers to be the winning bid in period A independently of the demand in this period. This situation can arise when the capacity of firm 1 increases because the profits from waiting become very small, while the profits from winning remain unchanged. Additional restrictions would be required to ensure $\Delta\mathbb{E}\left[\pi_2^B(\cdot)\right]$ is always positive. However, we do not impose them since the equilibrium characterisation that we develop only requires $\Delta\mathbb{E}\left[\pi_1^B(\cdot)\right]$ to be positive.

The order of indifference points for the small and large firm We can use the term $\Delta\mathbb{E}[\pi_j^B]$ to express the indifference point between selling at period A and B for firms 1 and 2 as

$$p_j^{A*} = \frac{\Delta\mathbb{E}[\pi_j^B]}{D^A}.$$

If we evaluate this expression for firms 1 and 2, take their difference, and normalize it by $\rho\lambda R$, we can establish the indifference point is larger for firm 1 than for firm 2:

$$\frac{p_1^{A*} - p_2^{A*}}{\rho\lambda R} = \begin{cases} \frac{O(k_1-k_2)\left(O+D^B\right)}{D^A k_1 k_2} & \text{if } k_2 < k_1 \leq D^B, \\ \frac{D^B O(k_1-k_2)+\left(k_2-D^A\right)\left(\left(D^B-k_2\right)\left(k_2-D^A\right)+D^B\left(k_1-D^B\right)\right)}{D^A D^B k_2} & \text{if } k_2 < D^B < k_1, \\ \frac{(k_1-k_2)\left(k_1+k_2-2D^A\right)}{D^A D^B} & \text{if } D^B \leq k_2 < k_1. \end{cases}$$

It is immediate that the expressions are positive since all factors in the differences are positive by assumption.

Asymmetric indifference points preclude any possible equilibrium in pure strategies The asymmetry in the indifference points for the two firms rules out any possible symmetric equilibrium. An equilibrium where both firms set their prices symmetrically at p_1^{A*} does not exist because at this price firm 2 would strictly prefer to win at period A over waiting until period B and thus would have incentives to undercut firm 1. Similarly, a symmetric equilibrium where both firms price at p_2^{A*} cannot exist because firm 1 would prefer to wait until period B instead of selling in period A. Therefore, it would deviate to a higher price. It is worth noting that: (i) the first argument applies also to prices above p_1^{A*} although in that case firm 1 would also have incentives to undercut; (ii) the second argument applies also to prices below p_2^{A*}, although in that case firm 2 might also have incentives to wait depending on parameters; and (iii) both arguments apply simultaneously to prices in the interval $\left(p_2^{A*}, p_1^{A*}\right)$. This shows that there are no symmetric pure-strategy equilibria.

The same arguments used to rule out asymmetric pure-strategy equilibria in the case of symmetric capacities can be applied to rule out most candidates to an asymmetric pure-strategy equilibrium in the asymmetric case. First, consider any pure strategies equilibrium where firm 1 wins and prices below p_1^{A*}, or in which firm 2 wins and prices below p_2^{A*}. In such equilibria the winning firm would expect profits that are below those it could make if it loses, and would prefer to deviate to a price above the one set by firm 2. Second, if firm 1 wins with a price above p_2^{A*}, or if firm 2 wins with a price above p_1^{A*}, the losing firm would have incentives to undercut since in those regions the price that needs to be immaterially undercut would not be low enough for the losing firm to be indifferent between winning and losing, undercutting would provide larger profits.

There exists, though, the remaining possibility not covered above of an asymmetric equilibrium in which firm 2 wins with a price in the interval (p_2^{A*}, p_1^{A*}). The reason such an equilibrium does not exist is standard and of a technical nature. Given that both firms set prices $(p_1^A, p_2^A) \in \mathbb{R}^2$, then for any $p_2^A < p_1^{A*}$ there exists some $\varepsilon > 0$ such that $p_2^A + \varepsilon < p_1^{A*}$ to which firm 2 would want to deviate. The argument that discards these potential equilibria is weaker than those ruling out other cases, in the sense that it does not resist the application of a coarser equilibrium notion, concretely that of an epsilon equilibrium. Based on this intuition a mixed strategies equilibrium in which firm 2 prices purely at $p_1^{A^*}$ and always wins can be derived, which we do in the following subsection.

B.2.2 Infinitely many mixed-strategy equilibria

In this section, we characterise the infinitely many mixed-strategy equilibria of this subgame. From a game theoretic perspective, we have identified a plethora of equilibria where firm 2 prices at $p_1^{A^*}$ and firm 1 mixes. However, the economically relevant outcomes are all equivalent because in all of them firm 2 wins with certainty, and the price charged for all sales in period A is $p_1^{A^*}$. Firm 1 randomizes above $p_1^{A^*}$, with the infimum at that point and without any mass on it, but the exact form of firm 1's strategy is not determined beyond these properties and a functional inequality.

We develop this equilibrium characterisation in two steps. We start from a situation similar to the epsilon equilibrium in pure strategies and argue that, in equilibrium, it has to be the case that the support of both firms coincide in their infimum. Then, we use the equilibrium condition of indifference between period A and B to characterise a non-degenerate support of prices in equilibrium for firm 1. Finally, we derive the functional inequality in equilibrium so that firm 2 does not have incentives to deviate.

B.2.3 The infima of the supports coincide

Consider an equilibrium in mixed strategies where only one of the firms prices using a non-degenerate probability distribution. In this equilibrium it has to be the case that the infimum of the support is the same for both firms. If the infimum of the support of the firm mixing were above its competitor's price, then the competitor would have incentives to deviate and price higher, but still below that infimum.

Alternatively, if the infimum of the support of the firm mixing were below the point at which its competitor prices purely, it would have an incentive to concentrate all the mass below the point at which its competitor prices to one slightly below it, and obtain a larger profit in expectation.

Formally, let F_j^A denote the non-degenerate strategy and $p_l^A \leq R$ the price at which the competitor prices. We focus on prices at or below R because if the competitor prices above it the best response of firm j would be to price purely at R without mixing, and hence deviating from a non-degenerate F_j^A. Therefore, if $p_l^A \leq R$, there exists an arbitrarily small $\varepsilon > 0$ that defines a profitable deviation strategy F'^A_j such that $F'^A_j = 0$ if $p < p_l^A - \varepsilon$ and $F'^A_j = F_j^A(p)$ for all $p \geq p_l^A - \varepsilon$. We now show F'^A_j provides a profitable deviation. To this effect, notice that the average expected profits for F_j^A and F'^A_j are equal conditional on $p > p_l^A - \varepsilon$, while they differ when $p \leq p_l^A - \varepsilon$. In both cases, the continuation payoffs in period B and the sales in period A are the same, but the average price for F_j^A is lower than that of F'^A_j $(p_j - \varepsilon)$ and thus firm j would prefer to deviate to the latter. That is,

$$\mathbb{E}\left[\pi(p_j^A, p_l^A)\middle|\, p_l^A, F'^A_j\right] - \mathbb{E}\left[\pi(p_j^A, p_l^A)\middle|\, p_l^A, F_j^A\right] = \left[(p_l^A - \varepsilon) - \int_0^{p_l^A - \varepsilon} p dF_j(p)\right] D^A > 0.$$

The above argument shows there are no equilibria with one firm pricing purely in which the infimum of the support of the firm mixing and the price set by the firm playing a pure strategy differ. Nevertheless, there are equilibria in which the pure strategy and infimum of the support coincide, which we discuss in the next subsection.

Equilibrium characterisation We start by considering the incentives of the largest firm. Firm 1 loses with certainty at any price $p_1^A > p_1^{A^*}$ and is indifferent between any of these prices. Moreover, it is not a profitable deviation for firm 1 to price at $p_1^A = p_1^{A^*}$ since, at this point, its profits are the same regardless of how the tie from pricing equally is resolved (i.e. independently of s_1) and thus yields the same expected profits as $p_1^A > p_1^{A^*}$. This shows that any set of prices composed exclusively by $p_1^A \geq p_1^{A^*}$ could form a support for firm 1's strategy in equilibrium, since any of these prices yield equal expected profits and firm 1 has no strict incentives to deviate from them.

Next, we analyse the incentives of firm 2 to deviate and in the process determine the requirements for F_1^A that ensure there is an equilibrium. Firm 2 has no incentive to price lower than $p_1^{A^*}$ because it would still win with certainty, capture the same sales, but make a smaller margin on them. However, it may have an incentive to price higher than $p_1^{A^*}$. Whether this is profitable or not depends on how the trade-off between a larger price and an increased probability of losing resolves. The rate at which the probability of losing increases depends on the strategy of firm 1, which we need to determine in order to characterise the equilibria.

Let firm 1 randomise following a strategy F_1^A. Then, an equilibrium exists if, for all $p \geq p_1^{A^*}$, F_1^A meets the following condition:

$$\mathbb{E}\left[\pi_2(p_2^A, p_1^A) \middle| \, p_2^A = p_1^{A^*}, F_1^A\right] \geq \mathbb{E}\left[\pi_2(p_2^A, p_1^A) \middle| \, p_2^A = p, F_1^A\right].$$

If we substitute the corresponding expressions in the above inequality it is straightforward to obtain:

$$F_1^A(p) \geq \frac{p - p_1^{A^*}}{p - p_2^{A^*}} \text{ for all } p \geq p_1^{A^*}.$$

The restrictions that this condition imposes on F_1^A do not preclude it from being a probability measure with the required properties. First, the right-hand side of the expression vanishes when $p \to p_1^{A^*}$, allowing the infimum of the support to be located at that point as long as it has no mass. Second, even if increasing in p, the expression is smaller than one because $p_1^{A^*} > p_2^{A^*}$.

Finally, it is interesting to note that this equilibrium is a generalization of the pure-strategy one derived under symmetric capacities, it converges to it: when $k_1 = k_2$ so that $p_1^{A^*} = p_2^{A^*}$, the probability distribution degenerates and reaches $F_1^A(p) = 1$ at $p_1^{A^*} = p_2^{A^*} = p^{A^*}$.

B.2.4 Expected average market prices across periods under asymmetry

The expressions that pin down the expected average price in the market for the model with asymmetric capacities can be expressed as a generalization of the ones in the main text. This is,

$$\mathbb{E}[p] = R\lambda\rho \left[1 - \frac{O}{D}(1-\omega_1) + (1-\lambda)\left(\frac{D-k_1}{D}\frac{D^B-\omega_2}{k_2-\omega_2} + \omega_3\right)\frac{D^B}{D}\right], \tag{11}$$

where ω_1, ω_2 and ω_3 help accommodate the expression in the body of the article to allow for (i) asymmetry between the competing firms and (ii) that either one or both firms are pivotal in period B.

$$\omega_1 = \begin{cases} \frac{\left(O+D^B\right)(k_1-k_2)}{k_1 k_2}, & \text{if } D^B \geq k_1 \geq k_2 \\ \frac{k_1-D^A}{k_2} - \frac{\left(k_2-D^A\right)^2}{OD^B}, & \text{if } k_1 > D^B \geq k_2 \\ \frac{\left(k_1-D^A\right)^2}{OD^B} - \frac{\left(k_2-D^A\right)^2}{OD^B}, & \text{if } k_1 \geq k_2 > D^B, \end{cases}$$

$$\omega_2 = \begin{cases} 0, & \text{if } D^B \geq k_2 \\ k_2 - D^A, & \text{if } D^B < k_2, \end{cases}$$

$$\omega_3 = \begin{cases} \frac{k_1-k_2}{D^A} \frac{O}{D} \frac{D^B}{k_2}, & \text{if } D^B \geq k_2 \\ \frac{k_1-k_2}{D^A} \frac{O}{D} \left(1 - \frac{k_2-D^B}{O}\right), & \text{if } D^B < k_2. \end{cases}$$

It is interesting to notice that (11) converges to (6) if $k_1 = k_2 < D^B$.

B.2.5 Pivotality changes, market prices, compensating marginal cost reductions, and mergers under asymmetry

The analyses conducted in Sections 6 and 7 produce complex analytical expressions when capacity is asymmetric. The reason for this relates to the way in which D^A enters the expected prices expression (11). Nevertheless, even if the expressions are more complex, the analysis can be conducted in an analogous way as the one in the sections mentioned above.