Noname manuscript No.
(will be inserted by the editor)

Market Implied Volatilities for Defaultable Bonds

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Received: date / Accepted: date

Abstract Typically, implied volatilities for defaultable instruments are not available in the financial market since quotations related to options on defaultable bonds or on credit default swaps are usually not quoted by brokers. However an estimate of their volatilities is nedeed for pricing purposes.
In this paper, we provide a methodology to infer market implied volatilities for defaultable bonds using equity implied volatilities and CDS spreads quoted by the market in relation to a specific issuer. The theoretical framework we propose is based on the Merton's model under stochastic interest rates where the short rate is assumed to follow the Hull-White model. A numerical analysis is provided to illustrate the calibration process to be performed starting from financial market data. The market implied volatility calibrated according to the proposed methodology could be used to evaluate options where the underlying is a risky bond, i.e. callable bond or other types of credit-risk sensitive financial instruments.

Keywords Defaultable bonds' implied volatility · credit default swap (CDS) · Merton model · Hull and White model

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1 Introduction

Implied volatilities of financial instruments are derived from the prices of options having as underlying the financial instrument itself. Typical cases are represented by equity options (call, put) or derivatives having as underlying interest rates (caps, floors, swaptions). Market quotations are available for these types of instruments where the implied volatility is quoted. Such volatilities are widely used by practitioners in order to calibrate stochastic models to be used for evaluating non-vanilla derivatives or other more complex financial products.

However, implied volatilities where the underlying financial instrument is affected by credit risk (credit implied volatility) are not available in the market. In fact, options on defaultable bonds or options on credit default swaps (CDS) are usually not quoted by brokers. Still the market implied volatility is fundamental to evaluate instruments such as callable bonds where the underlying is a credit-risky bond or other types of credit-risk sensitive financial instruments.

Several academics and practitioners have focused on credit implied volatility. Zheng (1999) [12] was the first to address the problem of deriving the default implied volatility curve from the values of barrier options. Bayraktar (2008) [1] developed a stock option price approximation for a model which takes into account both the risk of default and the stochastic volatility. He also showed that it might be possible to infer the risk-neutral default intensity from stock option prices. Cao et al. (2010)[5] has showed that CDS are similar to out-of-the-money put options in that both offer a low cost and effective protection against downside risk. Bayraktar and Yang (2010) [2] proposed a model which can be jointly calibrated to the corporate bond term structure and equity option volatility surface of the same company. The purpose is to obtain explicit bond and equity option pricing formulas that can be calibrated to find a risk-neutral model that matches a set of observed market prices. Kelly et al. (2015) [9] introduced the concept of a credit implied volatility surface using CDS quotations in order to derive the asset volatility of the firm. Bao and Pan (2013) [4] study the excess volatility and its drivers in the corporate bond market. They consider the connection between the return volatilities of credit market securities, equities, and Treasuries using the Merton (1974) model [7] with stochastic interest rates. They assume a two-factor Vasicek model to calculate model-implied corporate bond using Treasury bond and equity return volatilities as inputs in the Merton model. The two processes are assumed to be uncorrelated They find that in the CDS market, empirical volatilities exceed model implied volatilities by an average of 1.92% and 2.84% when daily and monthly returns are used, respectively.

In this paper, we propose a methodology to infer the market implied volatilities for defaultable bonds using observed market equity implied volatilities and CDS spreads, quoted by the market in relation to a specific specific issuer. The theoretical framework we propose is based on the Merton model (1974) [7] under stochastic interest rates; the short rate is assumed to follow the model proposed by Hull and White (1990) [8] and the firm value process is a Geometric Brownian Motion. In this framework, the equity and the defaultable bond values are exposed to two types of risk: stochastic interest rates and the firm's asset value. We assume the two processes are correlated. Using Ito's lemma we can derive the dynamic for the equity and the defaultable bond values and their instantaneous volatility. The default probability is linked to the probability of the exercise of the underlying put and the instantaneous volatility of the equity is related to the market implied volatility derived under the classical Black-Scholes-Merton framework.[1] From the two last observations, we set up a calibration procedure to estimate the parameters of our model using market prices of CDS and market implied volatility related to equity options.

The paper is organized as follows. Section 2 presents the theoretical framework. In Section 3 we derive the market implied volatility of the defaultable bond. Section 4 describes the model calibration using equity implied volatility and CDS spreads. Numerical analysis is provided in Section 5 and our conclusions follow in Section 6.

2 The theoretical framework

In order to infer market implied volatilities for defaultable bonds, we apply the Merton model [7], with stochastic interest rates under a risk-neutral probability measure Q. We assume that the short rate follows the Hull-White process. Our approach involves using equity implied volatilities and CDS spreads quoted by the market for a specific issuer to derive implied volatility for defaultable bonds under the proposed methodology.

According to the Merton model, we assume that the total value $A(t)$ of a firm's asset at time $t \geq 0$ follows a geometric Brownian motion under the risk-neutral measure Q,

$$\frac{dA(t)}{A(t)} = r(t)dt + \sigma_A dZ(t), \tag{1}$$

where $A(t) > 0$, σ_A is the asset's volatility, and $dZ(t)$ is a standard Brownian motion.[2] The short rate $r(t)$ follows the Hull-White process,

$$dr(t) = \big[\theta(t) - ar(t)\big]dt + \sigma_r dW(t), \tag{2}$$

[1] See Black and Scholes (1973) [3] and Merton (1974) [7].

[2] The Merton model assumes the follow: 1) there is an overly simple debt structure, 2) there are no bankruptcy costs (i.e., the liquidation value equals the firm value), and 3) the debt and equity are frictionless tradeable assets.

where $\theta(t)$ is a deterministic function of time, a and σ_r are constants parameters, and $dW(t)$ is a Brownian motion.[3] The two Brownian motions, $dZ(t)$ and $dW(t)$, are correlated such that $dZ(t)dW(t) = \rho dt$, where ρ is the instantaneous-correlation parameter between the asset's value and the short interest rate.

Under our proposed framework, the price of a risk-free zero-coupon bond at time t, with maturity in $T > t$, is denoted by $P(t,T)$ and satisfies the following stochastic differential equation under Q,

$$\frac{dP(t,T)}{P(t,T)} = r(t)dt - \sigma_r D_P(t,T)dW(t), \tag{3}$$

where $D_P(t,T)$ is the stochastic duration of the zero-coupon bond such that

$$D_P(t,T) = -\frac{1}{P(t,T)}\frac{\partial P(t,T)}{\partial r(t)} = \frac{1}{a}\left[1 - e^{-a(T-t)}\right]. \tag{4}$$

Consequently, $\sigma_P = \sigma_r D_P(t,T)$ is the volatility of the zero-coupon bond.

As in the Merton's model, we assume that the firm is funded by equity $E(t)$ and defaultable debt $\bar{P}(t,T)$. Debt consists of a single outstanding defaultable bond with face value K. At the maturity T, if the total value of the asset is greater or equal than the debt, the latter is paid in full and the remainder is distributed among shareholders. However, if $A(t) < K$ then default is deemed to occur: the bondholders exercise a debt covenant giving them the right to liquidate the firm and receive the liquidation value in lieu of the debt while shareholders receive nothing in this case.
From these simple assumptions, we see that shareholders have a cash flow at T equal to $\left[A(t) - K\right]^+$ and so equity can be viewed as a European call option on the firm's assets.
Under the proposed framework, the value at time t of the call option formula is,

$$E(t) = A(t)\Phi(d_1) - P(t,T)K\Phi(d_2), \tag{5}$$

where Φ denotes the cumulative distribution function of the standard Gaussian distribution and,

$$d_1 = \frac{\log\left[\frac{A(t)}{P(t,T)}\frac{1}{K}\right] + \frac{1}{2}\Sigma(t,T)^2}{\Sigma(t,T)}, \tag{6}$$

$$d_2 = \frac{\log\left[\frac{A(t)}{P(t,T)}\frac{1}{K}\right] - \frac{1}{2}\Sigma(t,T)^2}{\Sigma(t,T)}. \tag{7}$$

[3] An alternative formulation of the Hull-White model can be used where the short rate is expressed as $r(t) = \alpha(t) + x(t)$. See Russo and Fabozzi (2016) for further details.

Under stochastic interest rates, the variance to be considered in the option pricing formula above is calculated as follows,

$$\varSigma(t,T)^2 = \sigma_A^2(T-t) + \frac{\sigma_r^2}{a^2}\left[(T-t) + \frac{2}{a}e^{-a(T-t)} - \frac{1}{2a}e^{-2a(T-t)} - \frac{3}{2a}\right] + 2\rho\frac{\sigma_A\sigma_r}{a}\left[(T-t) - \frac{1}{a}\left(1 - e^{-a(T-t)}\right)\right]. \quad (8)$$

At maturity T, the bondholder receives the following payoff,

$$\min\big[A(T), K\big] = A(T) - \big[A(T) - K\big]^+ = K - \big[K - A(T)\big]^+. \quad (9)$$

The risky debt is equivalent to a portfolio composed by a risk-free bond with the same maturity and a long position in a put written on the value of the firm's asset with strike price equal to K,

$$\bar{P}(t,T) = P(t,T) - \big[P(t,T)K\varPhi(-\varPhi_2) - A(t)\varPhi(-d_1)\big], \quad (10)$$

where $P(t,T)$ is calculated according to the closed-form solution for the zero-coupon bond available under the Hull-White model[8].

3 The derivation of implied volatility for defaultable bonds

In this section, we describe how to infer market implied volatilities for defaultable bonds under the framework presented above. Consequently, calculations we perform are provided in a risk-neutral setting.

Under the stochastic dynamics for $A(t)$ and $r(t)$, applying Ito's lemma, we obtain the following differential equation for $G\big[A(t), r(t)\big]$,

$$dG(t) = \left[\frac{\partial G(t)}{\partial A(t)}A(t)r(t) + \frac{\partial G(t)}{\partial r(t)}\big[\theta(t) - ar(t)\big] + \frac{1}{2}\frac{\partial^2 G(t)}{\partial A(t)^2}A(t)^2\sigma_A^2 + \frac{1}{2}\frac{\partial^2 G(t)}{\partial r(t)^2}\sigma_r^2 + \frac{\partial^2 G(t)}{\partial r(t)\partial A(t)}A(t)\rho\sigma_A\sigma_r\right]dt + \left[\frac{\partial G(t)}{\partial A(t)}\sigma_A A(t)\right]dZ(t) + \left[\frac{\partial G(t)}{\partial r(t)}\sigma_r\right]dW(t). \quad (11)$$

Both equity and defaultable bond are functions of $A(t)$ and $r(t)$. Consequently, from (11),their differential equations are,

$$dE(t) = \mu_E(t)dt + \left[\frac{\partial E(t)}{\partial A(t)}\sigma_A A(t)\right]dZ(t) + \left[\frac{\partial E(t)}{\partial r(t)}\sigma_r\right]dW(t), \quad (12)$$

$$d\bar{P}(t,T) = \mu_{\bar{P}}(t)dt + \left[\frac{\partial \bar{P}(t,T)}{\partial A(t)}\sigma_A A(t)\right]dZ(t) + \left[\frac{\partial \bar{P}(t,T)}{\partial r(t)}\sigma_r\right]dW(t). \quad (13)$$

As the drift under the two stochastic processes above is unnecessary for our purpose, we neglect the calculations focusing on the diffusion components.

We derive the sensitivities of equity and risky debt with respect to the underlying asset and to the interest rate. Using the terminology of the the option *Greeks* we have that *Delta* and *Rho* respectively are as follows,

$$Delta_E = \frac{\partial E(t)}{\partial A(t)} = \Phi(d_1), \tag{14}$$

$$Rho_E = \frac{\partial E(t)}{\partial r(t)} = K(T-t)P(t,T)\Phi(d_2), \tag{15}$$

$$Delta_{\bar{P}} = \frac{\partial \bar{P}(t,T)}{\partial A(t)} = \Phi(-d_1), \tag{16}$$

$$Rho_{\bar{P}} = \frac{\partial \bar{P}(t,T)}{\partial r(t)} = -P(t,T)D_P(t,T) - \big[-K(T-t)P(t,T)\Phi(-d_2)\big]. \tag{17}$$

On the base of the equation (12) and using the *Greeks*, we can calculate σ_E as:

$$\begin{aligned}\sigma_E^2 &= \left\{\left[\frac{\partial E(t)}{\partial A(t)}\sigma_A A(t)\right]dZ(t) + \left[\frac{\partial E(t)}{\partial r(t)}\sigma_r\right]dW(t)\right\}^2 = \\ &= \big[\Phi(d_1)\sigma_A A(t)\big]^2 + \big[\sigma_r K(T-t)P(t,T)\Phi(d_2)\big]^2 \\ &+ \rho\big[\Phi(d_1)\sigma_A A(t)\big]\big[\sigma_r K(T-t)P(t,T)\Phi(d_2)\big].\end{aligned} \tag{18}$$

Easily available from the market and commonly used from practitioners is the implied volatility computed according to the well-known Black-Scholes-Merton formula. According to this formula, the process for the equity is

$$dE(t)^M = \mu_E(t)E(t)^M dt + \sigma_E^M E(t)^M dZ(t). \tag{19}$$

If the firm's equity is traded in the markets and European options on equity are quoted according to the standard Black-Scholes-Merton formula, we can observe the market value $E(t)^M$ and the equity's implied volatility σ_E^M. Consequently, combining (18) and (19), the following relation must hold,

$$E(t)^M \sigma_E^M = \sigma_E. \tag{20}$$

In establishing the relation above, we refer to the approach proposed by KMV Corporation.[4] However, in a different manner with respect to the KMV approach, the solution we propose is derived in a stochastic interest rates environment. Moreover, under the proposed model the equity value is an output of the model as it is derived across the calibration process while under the KMV

[4] See Crosbie and Bohn (1997) for further details.

model $E(t)^M$ is observed in the market.

We apply the same approach from (14) to (19) in order to derive the market implied volatility for defaultable bonds. Denoting by $\sigma_{\bar{P}}$ the volatility of the defaultable zero-coupon bond, we have that,

$$\begin{aligned}\sigma_{\bar{P}}^2 &= \left\{ \left[\frac{\partial \bar{P}(t,T)}{\partial A(t)} \sigma_A A(t) \right] dZ(t) + \left[\frac{\partial \bar{P}(t,T)}{\partial r(t)} \sigma_r \right] dW(t) \right\}^2 = \\ &= \big[\Phi(-d_1)\sigma_A A(t)\big]^2 + \big[-\sigma_r P(t,T) D_P(t,T) + \sigma_r K(T-t) P(t,T) \Phi(-d_2)\big]^2 + \\ &+ \rho\big[\Phi(-d_1)\sigma_A A(t)\big]\big[-\sigma_r P(t,T) D_P(t,T) + \sigma_r K(T-t) P(t,T) \Phi(-d_2)\big].\end{aligned} \tag{21}$$

On the basis of the results obtained in the previous section, we also are able to calculate the market implied volatility for defaultable bonds. Denoting such volatility by $\hat{\sigma}_{\bar{P}}(t,T)$, we have that,

$$\bar{P}(t,T)\hat{\sigma}_{\bar{P}} = \sigma_{\bar{P}}, \tag{22}$$

where $\bar{P}(t,T)$ is the value of a defaultable zero-coupon bond for the maturity T. Finally, the market implied volatility for defaultable bonds can be calculated as

$$\hat{\sigma}_{\bar{P}} = \frac{\sigma_{\bar{P}}}{\bar{P}(t,T)}. \tag{23}$$

It is important to highlight the fact that σ_E^M, the equity's implied volatility, is quoted by the market while $\hat{\sigma}_{\bar{P}}(t,T)$ is the market implied volatility of a defaultable bond derived according to the proposed approach. In order to calculate $\hat{\sigma}_{\bar{P}}(t,T)$, it is needed to perform the calibration procedure described in the following section.

4 Calibration of the model

In order to perform the calibration process of the proposed modelfor a specific date, the following information must be obtained:[5]

- term structure of risk-free interest rates;
- quotation of swaption to be used for the calibration of the Hull-White model;
- at-the-money (ATM) equity implied volatility (σ_E^M);
- CDS par spread related to the maturity T_n denoted by $S^M(t,T_n)$;
- market consensus for the recovery rate (RR) embedded in the CDS quotations and the relative loss given default (LGD) with $RR = 1 - LGD$.

[5] For all quantities observed in the market we use the suffix M.

In the first step, the Hull-White model parameters (a, σ_r) are calibrated. The objective of calibration is to choose the model parameters in such a way that the model prices are consistent with the market prices of simple instruments. In order to derive the volatility parameters, we use at-the-money (ATM) swaptions quoted by the market. The calibration to swaptions is performed by choosing the values of the parameters so as to minimize the square root of the sum of the squares of the relative differences between market and model swaption prices,

$$\arg\min_{a,\sigma_r} \sqrt{\sum_{i=1}^{N} \left(\frac{Swpt_i - Swpt_i^M}{Swpt_i^M} \right)^2}, \tag{24}$$

where $Swpt_i^M$ is the value of the swaption quoted by the market and $Swpt_i$ represents the swaption's theoretical price under the Hull-White model. The number of calibrated instruments is N.

The second step involves calibrating the Merton model parameters ($A(t)$ and σ_A) and the correlation coefficient (ρ).
An important result under the Merton model is that the probability of default of the firm (PD), fixed at time t for the maturity T_n, is such that

$$PD(t, T_n) = \Phi(-d_2), \tag{25}$$

where d_2 is calculated according to (7). Therefore, under the Merton model, it is possible to derive the firm's probability of default by looking at the asset's volatility and other balance sheet items.
In order to calibrate the model, we need to derive an estimate of the probability of default implied by the market on the time horizon T_n. This probability, denoted by $PD^M(t, T_n)$, is derived using CDS quotations. For this scope, we need to establish a CDS pricing model using as input the term structure of the risk-free interest rates, the value of the quoted CDS spread, and the recovery rate.
Consider a CDS written on a single name with a set of n annual payments at the discrete times $T_1, T_2, ..., T_i, ...T_n$. Assuming (1) the fair value of the CDS is the difference between the premium leg and the protection leg and (2) the CDS spread is quoted at par, we have that,

$$\begin{aligned} &\sum_{i=1}^{N} P^M(t, T_i)\big[1 - PD^M(t, T_i)\big] S^M(t, T_n)\tau(T_{i-1}, T_i) - \\ &\sum_{i=1}^{N} P^M(t, T_i)\big[PD^M(t, T_i) - PD^M(t, T_{i-1})\big] LGD = 0, \end{aligned} \tag{26}$$

where $P^M(t, T_i)$ is the value of a risk-free zero-coupon bond quoted in the market and $\tau(T_{i-1}, T_i)$ denotes the time measure between T_{i-1} and T_i computed as a fraction of the year. For simplicity, we have assumed that payments by the CDS protection seller are postponed to the first discrete time T_i.

Assuming a time-inhomogeneous Poisson process,[6] we are able to compute numerically the deterministic but time-varying default intensity for the maturity T_n denoted by $\lambda^M(t,T_n)$. Such simple calculation is established on the basis of the following relationship,

$$PS^M(t,T_n) = e^{-\lambda^M(t,T_n)\tau(t,T_n)}, \tag{27}$$

where $PS^M(t,T_n)$ is the survival probability such that $PS^M(t,T_n) = 1 - PD^M(t,T_n)$.
A numerical optimization procedure allows us to derive the value of the default intensity by simply imposing that the fair value of a CDS contract has to be equal to zero at the valuation date. This procedure also allows us to derive the default probability $PD^M(t,T_n)$.

Consequently, we can calculate the price of the defaultable zero-coupon bond denoted by $\bar{P}^M(t,T_n)$ and implied by the CDS market such that,[7]

$$\bar{P}^M(t,T_n) = P^M(t,T_n)PS^M(t,T_n) + P^M(t,T_n)PD^M(t,T_n)RR. \tag{28}$$

At this point, the remaining model parameters can be calibrated: the asset's value $A(t)$, the asset's volatility (σ_A), and the correlation coefficient (ρ). The optimization procedure involves minimizing the sum of the squares of (1) the difference between the equity volatility implied by the market and the equity volatility according to the proposed model and (2) the difference between the default probability implied by the market (by means of CDS quatations) and the default probability according to the Merton model with stochastic interest rates.

In order to perform the model calibration, the accounting equation has to hold,

$$\hat{A}(t) = \bar{P}^M(t,T_n) + \hat{E}(t), \tag{29}$$

where $\hat{A}(t)$ represent the total asset's value of the firm while the liability side is composed by the sum of $\bar{P}^M(t,T_n)$ and $\hat{E}(t)$. As balance sheet amounts considered in our model are conventional, we standardize both asset and liability items at the valuation date. Consequently, we assume that $\hat{A}(t) = 100\%$ while both $\bar{P}^M(t,T_n)$ and $\hat{E}(t)$ are represented as a percentage of the total asset.

[6] See Lando (1998).

[7] An alternative approach could be adopted using, directly, prices of zero-coupon defaultable bonds (if they exist for the relevant issuer) in place of CDS quotations. Our choice to use CDS in place of bonds derive by the fact that CDS are quoted in a standardized and liquid market while defaultable bond quotations are not always available for the desired maturity and sometimes they are not liquid.

Finally, the optimization procedure is performed according to the following expression,

$$\arg \min_{A(t),\sigma_A,\rho} \left\{ \left[\sigma_E^M - \sigma_E\right]^2 + \left[PD^M(t,T) - PD(t,T)\right]^2 \right\}. \qquad (30)$$

Once we have estimated $\hat{A}(t)$, $\hat{\sigma}_A$, and $\hat{\rho}$, we can calculate $\hat{\sigma}_{\bar{P}}(t,T_n)$ according to the equation (23).

5 Numerical results

The model calibration is performed looking at the market data quoted as of 15 June 2017 and applying our model to three issuers: Bank of America (BoA), Microsoft, and General Electric (GE).[8]

First, we derive the initial term structures of interest rates from USD traded instruments quoted in the cash, forward rate agreement/futures, and swap markets. We applied the bootstrapping technique to derive the spot rates from the traded market instruments for the reference date.

The Hull-White model calibration is performed using USD swaption prices quoted at the same date. We consider the 10-year ATM co-terminal swaptions. This is a common practice adopted to calibrate interest rate models (in part due to hedging reasons) which consists of selecting swaptions with a fixed co-terminal (maturity and tenor). The algorithm proposed by Russo and Fabozzi (2016) was adopted for calibration purposes. The calibration results for the two parameters of the Hull-White model are $\hat{a} = 0.0577$ and $\hat{\sigma}_r = 0.0104$.

In the second step, for each of the three issuers (BoA, Microsoft, and GE) we collect market quotations for equity implied volatilities and CDS. We refer to maturities of 5 and 10 years. In addition, in all cases considered, we assume $K = 1$ and $RR = 40\%$ on the base of the market conventions related to the CDS quotations. Using the Hull-White parameters calibrated in the first step, we performed the second calibration step. Tables 1 and 2 show the results of the calibration procedure.

In Table 1, we report the simplified balance sheet for the three names considered at the valuation date. Table 2 shows market data used as input for the calibration and the market implied volatility of defaultable zero-coupon bonds, denoted by $\hat{\sigma}_{\bar{P}}(t,T_n)$, respectively with maturity 5 and 10 years.

[8] Market data were obtained from Datastream.

Table 1 Simplified balance sheet for each issuer considered in the analysis.

Issuer	$\hat{A}(t)$	$\bar{P}^M(t,T)$	$\hat{E}(t)$
Bank of America	100,0%	34,1%	65,9%
Microsoft	100,0%	77,9%	22,1%
General Electric	100,0%	83,3%	16,7%

a Maturity of 5 years

Issuer	$\hat{A}(t)$	$\bar{P}^M(t,T)$	$\hat{E}(t)$
Bank of America	100,0%	26,1%	73,9%
Microsoft	100,0%	70,6%	29,4%
General Electric	100,0%	73,4%	26,6%

b Maturity of 10 years

Table 2 Merton's model calibrated parameters as of 15 June 2017.

Issuer	$S^M(t,T_n)$	σ_E^M	$\hat{\sigma}_A$	$\hat{\rho}$	$\hat{\sigma}_{\bar{P}}$
Bank of America	0,54%	36,0%	24,8%	-50,2%	5,0%
Microsoft	0,29%	18,9%	6,7%	-78,7%	4,6%
General Electric	0,31%	14,6%	5,5%	-89,9%	4,6%

a Maturity of 5 years

Issuer	$S^M(t,T_n)$	σ_E^M	$\hat{\sigma}_A$	$\hat{\rho}$	$\hat{\sigma}_{\bar{P}}$
Bank of America	0,95%	36,0%	28,6%	-50,0%	9,0%
Microsoft	0,49%	18,9%	8,1%	-72,1%	7,9%
General Electric	0,71%	14,6%	8,7%	-87,5%	8,1%

b Maturity of 10 years

6 Conclusion

In this paper, we propose a methodology to infer the market implied volatilities for defaultable bonds using observed market equity implied volatilities and CDS spreads quoted by the market in relation to a certain issuer. The theoretical framework we propose is based on the Merton's model under stochastic interest rates where the short rate is assumed to follow the Hull-White model which is correlated with the asset.

In this framework, the equity and the defaultable bond are functions of two sources of risk: the stochastic interest rates and the value of asset of the firm. Using Ito's lemma we can derive the dynamic for the equity and the defaultable bond and their instantaneous volatility.
The default probability is linked to the probability of the exercise of the underlying put and the instantaneous volatility of the equity is related to the market implied volatility derived under the classical Black-Scholes-Merton framework. From the two last observations, we set up a calibration procedure to estimate the parameters of our model using market prices of CDS and market implied volatility related to equity options. The estimated parameters are finally used to derive the implied volatility of defaultable bonds. Such market implied volatility, calibrated according to the proposed model, could be used to evaluate options having as underlying risky bonds, i.e. callable bonds or other types of credit-risk sensitive financial instruments.

References

1. Bayraktar E., Pricing Options on Defaultable Stocks, Applied Mathematical Finance, 15, 277-304 (2008).
2. Bayraktar E., Yang B., A Unified Framework for Pricing Credit and Equity Derivatives, Mathematical Finance, 21, 493-517 (2011).
3. Black F., Scholes M., The pricing of options and corporate liabilities, Journal of Political Economy, 81, 637-54 (1973).
4. Bao J., Pan J.,Bond Illiquidity and Excess Volatility, The Review of Financial Studies, 26(12), 30683103, (2013).
5. Cao C., Yu F., Zhong Z., The information content of option-implied volatility for credit default swap valuation, Journal of Financial Markets, 13, 321-343 (2010).
6. Crosbie P.J., Bohn J.R., Modeling Default Risk, KMV Corporation, (1997).
7. Merton R.C., On the pricing of corporate debt: the risk structure of interest rates, Journal of Finance, 29, 449-70 (1974).
8. Hull J., White A., Pricing Interest Rate Derivative Securities, Review of Financial Studies, 3, 573-592 (1990).
9. Kelly B.T., Manzo G., Palhares D., Credit-Implied Volatility, Chicago Booth Research Paper, No. 17-12, (2017).
10. Lando D., On Cox Processes and Credit Risky Securities, Review of Derivatives Research, 2, 99-120 (1998).
11. Russo V., Fabozzi F.J., Pricing Coupon Bond Options and Swaptions under the One-Factor HullWhite Model, Journal of Fixed Income, 25, 76-82 (2016).
12. Zheng C.K., Default Implied Volatility for Credit Spread, available at SSRN: https://ssrn.com/abstract=163848, (1999).