Factor decomposition of the Eurozone sovereign CDS spreads

Frank J. Fabozzi (contact author), EDHEC Business School
(E-mail: frank.fabozzi@edhec.edu)

Rosella Giacometti, Department of Management, Economics and Quantitative Methods, University of Bergamo (E-mail: rosella.giacometti@unibg.it)

Naoshi Tsuchida, Institute for Monetary and Economic Studies, Bank of Japan
(E-mail: naoshi.tsuchida@boj.or.jp)

February 14, 2016

Abstract

In this paper, we examine the factors driving Eurozone sovereign credit default swap (CDS) spreads during the Eurozone sovereign debt crisis. For identifying factors we utilize independent component analysis (ICA), a technique similar to principal component analysis (PCA). We identify three factors that impact spreads and capture the features specific to the crisis such as the breakup risk of the Eurozone: peripheral factor, global factor, and Eurozone common factor. In contrast, when PCA is applied, only a single factor is identified. Moreover, using ICA with a GARCH model, we show that the source of volatility for CDS spreads shifted from the global factor in 2009 and the peripheral factor in 2010 to the Eurozone common factor in 2012, and that the dynamic correlation reflects the decoupling between low credit risk countries such as Germany and high credit risk countries such as Greece. We also show that the goodness-of-fit of the ICA-based model is better than other models used such as the Student-t copula model.

Keywords: independent component analysis;credit default swap;Eurozone sovereign debt crisis;redenomination risk.

JEL classification: C18; G01; G15.

1 Introduction

The *Eurozone sovereign debt crisis* is an ongoing crisis. The crisis is said to have started in late 2009, and remains a critical sector of the global credit market where there is concern about sovereign risk. Prior to the crisis the credit risk of Eurozone countries was considered to be very low; however, after the onset of the crisis, the credit risk, as indicated by the widening of credit default swap (CDS) spreads and bond yields, has increased dramatically. Moreover, the credit spreads between countries with lower risk, such as Germany, and those with higher risk, such as Greece, widened substantially.

In this paper, we investigate the factors driving Eurozone sovereign CDS spreads. While a default of a country ultimately depends on reasons specific to the country, preceding studies reported that sovereign CDS spreads exhibit co-movements across countries. Using principal component analysis (PCA), Longstaff et al. (2011) studied the spreads of 26 sovereign CDSs of monthly frequency from 2000 to 2010. They reported that co-movements are linked to US equity, equity volatility, and bond risk premium. Pan and Singleton (2008) studied the term structure of three countries (Korea, Mexico, and Turkey) seeking to estimate default and recovery rates. They propose a pricing model with stochastic default intensity, and decompose the variation of CDS into default intensity and risk premium. The risk premium is shown to have co-movements across countries, and are linked to the CBOE VIX volatility index, US corporate bond spreads, and the volatility of the country's foreign exchange rate. Ang and Longstaff (2013) studied systemic credit shock by comparing the US and the Eurozone using the CDS for US Treasuries, US states, and major Eurozone countries. In the pricing model, they proposed the use of two types of credit events, systemic and sovereign-specific. They found that the systemic risk component is more important in the Eurozone than in the US, and is related to global financial factors such as the VIX index.

In addition to the issue of co-movements of CDS spreads, we consider two issues specific to the period which is referred to as the Eurozone sovereign debt crisis. First, during this a strong relation was observed between sovereign CDS spreads and the stability of the Eurozone financial sector. According to Mody (2009), following the US government bail out of Bear Stearns in 2008 the Eurozone sovereign CDS spread began widening, reflecting the rescuing cost of stressed financial institutions. Since the onset of the US subprime mortgage crisis starting in 2007, banks and other financial institutions have encountered difficulties, some being forced into bankruptcy while others being fortunate to be bailed out by governments. The costs of such government bailouts were potentially so large that they were expected to increase the government debt level, resulting in increased sovereign credit risk. This is particularly the case in the Eurozone since the scale of the financial sector is larger than in the other countries.[1] Considerable theoretical and empirical research has been devoted to show the fundamental relation between sovereign credit and the financial industry. One recent example of such research is the paper by Acharya et al. (2014).

The other issue of the Eurozone sovereign debt crisis we investigate is what is referred to as the "risk of convertibility" or the "risk of redenomination." The President of the European Central Bank (ECB) Mario Draghi mentioned in a speech on July 26, 2012 that convertibility risk was charged on the Eurozone sovereign CDS spreads (Draghi, 2012). Before this speech, the crisis of the Eurozone became deepened, and the CDS spreads of countries such as Italy, Portugal and Spain increased to the level such that market participants started thinking of the possible breakup of the Eurozone into national currencies. President Draghi stated that "the euro is irreversible," and confirmed that the ECB would take whatever the action was in order to preserve the Eurozone. The word redenomination risk is also used in referring to this kind of risk (Draghi and Constâncio, 2012). De Santis (2015) showed that the redenomination risk serves as the systemic risk during the crisis period; Krishnamurthy et al. (2013) showed that it is to some extent through the reduction of redenomination risk that some monetary measures by the

[1]According to Table 2.3.1 in the report about the EU banking sector published by European Commission (2012), the ratio of the bank assets over GDP is 349% in the EU, 78% in the US, and 174% in Japan.

ECB (such as the Outright Monetary Transaction) work effectively during the Eurozone sovereign crisis.

As a consequence of this crisis, the decoupling of sovereign credits across the Eurozone countries was observed, as well as the co-movements of the sovereign credits. The countries with low credit risk such as Germany are often colloquially termed *core* countries, whereas those countries with high credit risk such as Greece termed *peripheral* countries. There are some who question whether these terms are appropriate (Kaletsky, 2012). However, these terms are commonly used, appearing in some prior studies about the Eurozone (e.g., Artis and Zhang, 2001; Stockhammer, 2011). We use these terms since we consider it is important to investigate the decoupling between the "core" countries and "peripheral" countries.

In this paper, our objective is to identify the factors driving the changes in CDS spreads, separating the decoupling factor and the co-movement factor. Following Kumiega et al. (2011) and García-Ferrer et al. (2012), in this paper we employ independent component analysis (ICA) for that purpose. Although similar to factor analysis (FA) and PCA in that it is a linear model, ICA differs from these two traditional models in that its objective is to find a linear representation for non-Gaussian variables so that the components identified are statistically independent from the other components identified.[2]

The non-Gaussianity plays an essential role in ICA. If the data obey the Gaussian distribution, then ICA produces the same components as PCA. In the case of asset returns for financial instruments, there is a preponderance of empirical evidence that asset returns violate the Gaussian assumption; for example, see Chapter 11 of Rachev et al. (2005). In fact, tail heaviness is the important feature of asset returns causing the catastrophic events realized in the financial market that began in 2007, as well as other documented market crashes (for example, see Kim et al., 2011). Consequently, the application of ICA to the analysis of asset returns seems to be more appropriate than other techniques that

[2]The theory and applications of ICA and algorithms for solving for the components are described in Hyvärinen and Oja (2000) and Hyvärinen et al. (2001). Recent advances in ICA are described in Hyvärinen (2013).

assume a Gaussian distribution.[3]

The ICA-based factor model has several advantages in analysis. As shown by Kumiega et al. (2011), the dynamic volatility and correlation can be calculated more easily by the ICA-GARCH model than by the other methods. Also, the factors obtained can be utilized for analysis detecting the effect of policy measures by the authorities such as the ECB. The introduction of these applications of the ICA is also our objective.

The paper is structured as follows. Section 2 briefly reviews the Eurozone sovereign CDS market during the crisis and describes the data we analyze. In Section 3 we discuss factor decomposition based on ICA and compare the ICA results with those found using PCA. Section 4 discusses the volatility of the ICs and the correlation between the CDS spreads based on the GARCH model, and compares the goodness-of-fit of the ICA-based model with the alternative models using copula functions. Section 5 provides our conclusions.

2 The data and the Eurozone sovereign CDS market during the crisis

2.1 CDS spread

In this paper, we analyze Eurozone sovereign CDS spreads from Jaunary 2009 to December 2013 (see Figure 1).[4] More precisely, we use data for CDS spreads denominated in US dollars with a maturity of five years obtained from Bloomberg. We use weekly data to avoid noise that may be contained in daily data. The observation period is January

[3]For example, Chapter 24 of Hyvärinen et al. (2001) shows several examples of ICA applications to financial data.

[4]Note that another source representing the sovereign credit is the yield of sovereign bonds. The CDS spread referencing an entity theoretically matches the spread of the bond yield issued by the entity over the risk free rate. In the real-world of financial markets, however, the market for these two credit products are very similar but not identical, reflecting the difference of some market features. For the Eurozone sovereign credit markets, Fontana and Scheicher (2010) showed that the difference between the CDS and cash bond markets arises because the cash market is less liquid than the CDS market, that is, the CDS market is more liquid. In this paper, therefore, we focus on the CDS spread.

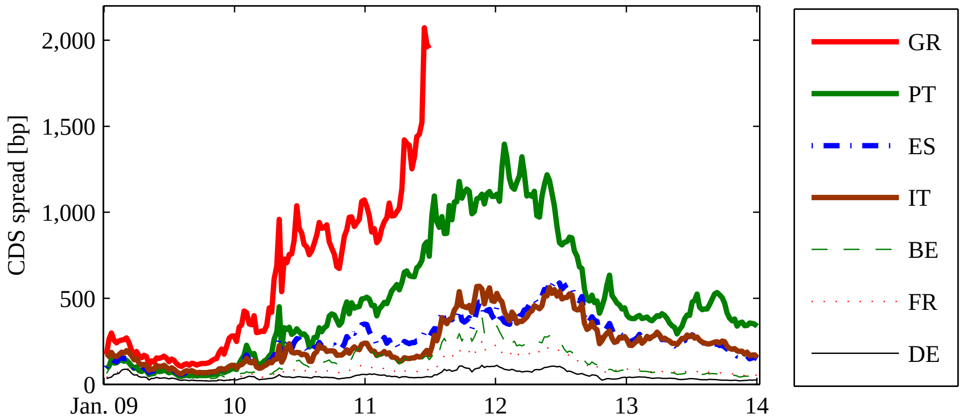
2009 to December 2013. We use the CDS spreads of seven Eurozone countries: Belgium (BE), France (FR), Germany (DE), Greece (GR), Italy (IT), Portugal (PT), and Spain (ES).[5]

From Figure 1, we can observe that CDS spreads started widening from the middle of 2009, soared toward the middle of 2012, and then decreased. It is also observed that the level of the CDS spreads in 2013 is generally much higher compared to the level in the middle of 2009. Moreover, the difference of CDS spreads between the Eurozone sovereign countries in 2013 is much larger than that in 2009. Figure 1b shows the CDS spreads in logarithmic scale, depicting this credit decoupling more clearly.

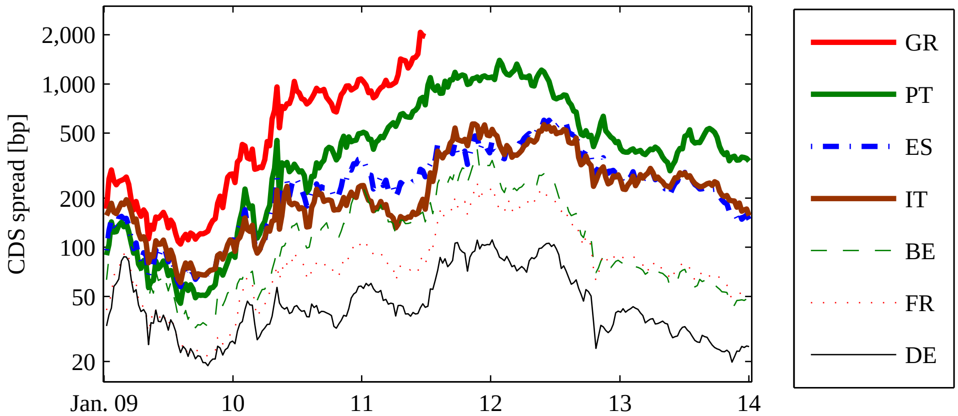
The period depicted in Figure 1 contains the major part of what is referred to as the Eurozone sovereign debt crisis. The onset of the crisis is not precisely defined, but is usually considered as the end of 2009, in which bad news about sovereign debt were reported and their CDS spreads started widening. For example, on November 5, 2009, Reuters reported that the budget deficit of Greece was more than the double of what was previously announced. This news increased the Greek CDS spread.

Figure 1 shows that the Greek CDS spread skyrocketed toward the summer of 2011. Since its economy and fiscal conditions are one of the most fragile among the Eurozone, the country has attracted attention from market participants. On May 2, 2010, in exchange for a bailout loan amounting to € 110 billion, the Greek government agreed about its austerity program with the ECB, the European Commission (EC), and the International Monetary Fund (IMF). However, its CDS spread increased during the week of May 3 to 7, from 722 basis points (at the closing on April 30, the end of the last week) to 965 basis points (at the closing on May 7). On May 10, the spread dropped to 590 basis points, reportedly because of the introduction of the Security Market Programme by the

[5]When selecting the seven countries, we considered their GDP share against the entire Eurozone, the attention that these countries attracted during the crisis, and the balance of numbers of core and peripheral countries. That is, the seven countries account for more than 80% of the Eurozone GDP. They include countries such as Germany and Greece, and they include about the same numbers (three and four) of the core and peripheral countries so that the numbers in these groups do not affect the result.



(a) Linear scale



(b) Log scale

Figure 1: CDS spreads of the seven Eurozone countries: Belgium (BE), France (FR), Germany (DE), Greece (GE), Italy (IT), Portugal (PT), and Spain (ES). The denomination currency is the US dollar, and the maturity is five years. Note that in the legend of the figure, the countries are in the descending order of the average values of CDS spreads. The data for Greece were discontinued in July 2011.

ECB on that day (European Central Bank, 2010). However, it kept increasing after that day. Finally, the International Swaps and Derivatives Association (ISDA) announced its resolution on March 9, 2012, that a restructuring credit event had occurred for the CDS contract referencing Greece (the Hellenic Republic; The International Swaps and Derivatives Association, 2012)

Obviously, Greece is the country we are most interested in. However, as already noted, the Greek CDS experienced a credit event, so it seems better to limit the data for Greece to the pre-2012 time period. On the other hand, we are also interested in investigating the CDS market around 2012, since the spreads for the other countries hit their maxima around that time. For these reasons, we partition the data into two parts. One part consists of all seven countries, covering the period from January 2009 to June 2011. The other part consists of all countries except Greece, covering the period from January 2009 to December 2013. We will refer to the former data as the "seven countries" data and the latter as the "six countries" data. The number of observations is 130 in the seven countries data and 261 in the six countries data.

Since we are interested in factors driving CDS spreads, we mainly focus on the changes of CDS spreads, not on the levels. We use the changes of log-returns of CDS spreads for easier understanding. From the Table 1 which provides descriptive statistics of the log-returns of CDS spreads, it can be seen that the standard deviations of log-returns are about the same (ranging from 10% to 15%) across the countries, and so are the values of factor loadings in a factor model. This facilitates comparison of the effects of the same factor on different countries.[6]

[6]The fundamental results in the following sections, such as the explanatory powers reported in Tables 3 and 4 in Section 3.3, do not change fundamentally if we use the changes in CDS spreads. However, the factor loadings shown in Table 4 vary if we use the changes in CDS spreads. Taking logarithm generally decreases the tail heaviness of the given data, but the values of kurtosis in Table 1 indicate that the log-returns of CDS spreads exhibit tail heaviness in this case.

Table 1: Descriptive statistics of log-returns of sovereign CDS spreads, their principal components (PCs), and their independent components (ICs). Columns "avg.," and "std.," show average and standard deviation, whose unit is 1% for the log-returns of CDS spreads (for the scales of the PCs and ICs, see Table 4). Column "skew." shows skewness. Column "kurt." shows kurtosis (not excess kurtosis; that is, 3.0 if the variable obeys the Gaussian distribution).

	Seven countries from January 2009 to June 2011 (130 datapoints)				**Six countries from January 2009 to December 2013 (261 datapoints)**			
	avg.	**std.**	**skew.**	**kurt.**	**avg.**	**std.**	**skew.**	**kurt.**
Logarithmic returns of CDS spreads								
Belgium	0.63	11.54	0.13	3.95	−0.13	10.11	0.07	4.47
France	0.28	11.50	−0.31	4.01	−0.02	10.12	−0.29	4.44
Germany	−0.07	11.53	−0.42	4.75	−0.25	10.42	−0.45	5.47
Greece	1.65	13.52	−0.28	6.09				
Italy	0.00	12.31	−0.43	6.11	−0.03	10.75	−0.32	6.26
Portugal	1.61	14.97	−0.92	10.31	0.50	11.94	−0.70	12.63
Spain	0.77	12.53	−0.55	6.17	0.15	10.54	−0.48	6.74
Principal components (PCs)								
PC1	0	2.34	−0.55	4.70	0	2.14	−0.43	5.07
PC2	0	0.82	1.52	9.15	0	0.86	0.68	9.77
PC3	0	0.54	−0.86	4.84	0	0.51	−0.25	4.88
Independent components (ICs)								
IC1	0	1	−1.52	21.02	0	1	−1.76	15.20
IC2	0	1	−0.01	7.54	0	1	−0.54	8.19
IC3	0	1	0.04	3.99	0	1	0.28	3.28

2.2 Financial market variables

The CDS spreads are supposed to reflect the other financial and economic variables. Here we consider the financial variables that may be possibly related to the factors impacting the CDS spreads. Longstaff et al. (2011) provide a comprehensive study about the relation between the sovereign CDS spread and financial and economic variables. They investigated the relation between monthly changes of 26 sovereign CDS spreads and 14 financial and economic variables. These variables include local stock market return, change in US corporate yield spread between BBB and AAA credits, change in US corporate yield spread between BB and BBB credits, change in volatility risk premium (measured as the difference between the implied volatility of the VIX and the realized volatility of S&P 100), and regional and global averages of CDS spreads.[7] These variables are related to the CDS spread. The significance of the other variables in the paper with the CDS spreads varies.

In the context of the systemic risk in the Eurozone, Ang and Longstaff (2013) investigated the relation between seven financial variables and what they refer to as the "intensity of the systemic risk," computed from their stochastic default intensity model.[8] They computed the intensities for the systemic risk for the US and the Eurozone using the CDS spreads for the US and Germany, respectively. They then investigated which financial variable drives weekly change in the systemic risk intensity. The variables they investigated are market stock returns (S&P500 for the US and DAX for the Eurozone), change in the 5-year constant maturity swap rates (denominated in the US dollar for the US and in the euro for the Eurozone), change in VIX (for both the US and the Eurozone), change in corporate CDS spreads (the CDX North American Investment Grade

[7] The other variables are: exchange rates, foreign reserves, US stock market return, change in 5-year US Treasury yields, equity premium, US term premium, and capital flows in bonds and equities.

[8] Ang and Longstaff (2013) proposed a model based on Duffie et al. (2003) that the default intensity of a specific country is the sovereign-specific intensity plus the intensity for the systemic risk (multiplied by a constant specific for the country representing its exposure to the systemic risk). They considered that the default intensity in the CDS spreads for the US Treasuries represents the intensity of the systemic risk for the CDS spreads of US states. Likewise, they regarded the intensity of Germany as the intensity of the systemic risk for the Eurozone sovereign CDS spreads.

Index for the US and the European iTraxx Index for the Eurozone), and changes in five foreign CDS spreads (Japan, China, and CDX Emerging Markets). They found that market stock returns, change in corporate CDS spreads, and the CDS spread change for the Chinese government are related to the intensity of systemic risk in both the US and the Eurozone. They also reported that for the US, change in the VIX is related to the systemic risk intensity.

Guided by the variables identified in these two studies, we selected six variables for our analysis. First, since both studies reported that stock returns impact the sovereign CDS spreads, we include two series of stock index returns. The first is the average of logarithmic returns across the three core countries (Belgium, France and Germany). We use the euro-denominated MSCI[9] for the stock index return for each country, and the equally-weighted average for these countries is labeled "stock index of core countries." The second series is the average of the peripheral countries (Greece, Italy, Portugal and Spain for the seven countries data and Italy, Portugal and Spain for the six countries data). The equally-weighted average for these countries is labeled "stock index of peripheral countries."

Second, since it was reported that volatility was related to sovereign CDS spreads, we introduced two volatility indices. One is the VSTOXX, an index of volatility implied by the EUROSTOXX 50 options, an index comprised of the most liquid Eurozone stocks representing the risk aversion within the Eurozone. We label this volatility index as the "Eurozone stock implied volatility (VSTOXX)." The other volatility index is the CBOE VIX, an index of volatility implied by the S&P 500 options. Since the US stock market is more global than the Eurozone stock market, we label the VIX "global stock implied volatility (VIX)."

Both Longstaff et al. (2011) and Ang and Longstaff (2013) observed that corporate credit condition is related to the CDS spreads. More specifically, Longstaff et al. (2011) reported that changes in US corporate yield spreads were related to changes in the CDS spreads, and Ang and Longstaff (2013) reported that changes in corporate CDS indexes

[9]Designed by the Morgan Stanley Capital International before, and by the MSCI Barra today.

were related to the systemic risk intensity in CDS spreads both in the US and the Eurozone. As a proxy for credit condition, in this paper we use the Markit iTraxx Europe Index, referring to it as the "Eurozone corporate CDS index."

Ang and Longstaff (2013) reported that China's CDS spread was related to the systemic risks of the US and the Eurozone. Longstaff et al. (2011) showed that systemic risks defined as the first PC of the 26 sovereign CDSs explained more than 60% of their variation. These findings indicate that the Eurozone sovereign CDS market and that of the rest of the world are related, multilaterally affecting each other. In order to represent this relation, we consider five major economies outside of the Eurozone: China, Japan, Switzerland, the United Kingdom, and the United States.[10] We use their equally-weighted average of the CDS spreads, labeling it "global sovereign CDS index."

In addition to these six variables, we consider four variables representing the risk characterizing the Eurozone sovereign debt crisis. The first variable is the one related to the cost for a sovereign state government to maintain the financial system by means such as bail-outs, as reported by Mody (2009). In this paper, we use the Markit iTraxx Europe Senior Financial Index for its continuity and simplicity. We label it "Eurozone financial CDS index."

The second, third and fourth variables are those representing the risk of redenomination of the Eurozone countries. One of the approaches to measure this risk is to compare the bond yields in different currencies based on the notion that the expected exchange rate changes due to redenomination are reflected in to the yield difference in different currencies. Krishnamurthy et al. (2013) introduced this idea by using bond yields of Italy, Portugal and Spain denominated in the US dollar and the euro. De Santis (2015) used CDS spreads of France, Italy, Spain and Germany since the USD-denominated bond issued by these countries is scarce. We use the CDS spreads, following De Santis (2015).

[10] The US CDS spread is denominated in the euro, while the others are in the US dollar. We take a simple average of these five series.

Accordingly, the redenomination risk I underlying the CDS spread is defined as follows:

$$I_t^{c,b,h} = (S_t^{c,h,\mathrm{USD}} - S_t^{c,h,\mathrm{EUR}}) - (S_t^{b,h,\mathrm{USD}} - S_t^{b,h,\mathrm{EUR}}), \tag{1}$$

where S denotes a CDS spread, the superscript c denotes the country referenced by the CDS, b denotes Germany, USD denotes denomination in US dollar, EUR denotes denomination in the euro, h denotes the maturity, and the subscript t denotes time.

In order to compute I in equation (1), we use the US dollar-denominated CDS spreads explained in Section 2, and the euro-denominated 5-year CDS spreads downloaded from the Bloomberg Financial Markets.[11] We consider the redenomination risk I for Belgium, France, Greece, Portugal and Spain.[12] We compute the equally-weighted average of the values of I for Belgium and France and label it "redenomination risk of core countries." We also compute the average of I for Greece, Portugal, and Spain (for the seven countries data) and for Portugal and Spain (for the six countries data), labeling it "redenomination risk of peripheral countries."

Finally, the volatility of the exchange rate between the euro and the US dollar would be expected to be related to the redenomination risk because it is related to the valuation of the euro. We use the volatility implied by the three-month at-the-money option of the exchange rate obtained from Bloomberg, labeling it "EURUSD implied volatility."

Table 2a lists the 10 variables we use in this study. Also, the correlation coefficients of these variables from 2009 to 2013 are shown in Table 2b. From the table, we notice the following three points. First, two "redenomination risk" variables are almost uncorrelated with the other variables. Second, the correlation coefficient between these two redenomination risks seems weak (0.26). Third, the variables other than "redenomination risk" are to some extent correlated with each other, whose (absolute) values of correlation

[11] There are many missing values of the euro-denominated CDS spreads in 2012. We complement these values from the sovereign bond yields, under the assumption that the weekly change of CDS spread is a linear function of that of bond yield. Note that both changes match theoretically since both are denominated in the same currency, the euro.

[12] Germany is excluded since it is used as the basis in equation (1). Italy is excluded for the scarcity of the available data.

coefficients range from 0.29 to 0.91. The third point suggests the existence of a common risk factor affecting the entire market. In addition, the first point suggests the existence of risk factors independent of the common risk factor. The second point implies that there is more than one factor that is independent of the common risk factor.

3 Factor Decomposition of CDS based on ICA

In this section, we discuss factor decomposition of the data we presented in Section 2.1. For that purpose, we introduce the idea of ICA in Section 3.1, present the model based on ICA in Section 3.2, show the result based on the model in Section 3.3. In Section 3.4, we discuss the relation between the factors obtained in Section 3.3 and the financial market variables described in Section 2.2.

3.1 Definition of ICA and PCA

Suppose a q-dimensional random vector $\mathbf{X} = (X_1, \dots, X_q)^\top$ representing the observable variables, and assume $\mathrm{E}[\mathbf{X}] = 0$. ICA is the following model of $\mathbf{X}$:

$$\mathbf{X} = A\mathbf{S}, \tag{2}$$

where $\mathbf{S} = (S_1, \dots, S_q)^\top$ is a set of q independent random variables and A is a q-by-q constant matrix. The random variable S_j, $j = 1, \dots, q$, is referred to as the *independent component* (IC). The independence is equivalent to the factorization of the probability density function (PDF) as $f_{\mathbf{S}}(s) = f_{S1}(s_1) \cdots f_{Sq}(s_q)$ where $f_{\mathbf{S}}$ and f_{Sj} are the joint and the marginal PDF of $\mathbf{S}$ and S_j, respectively.

An algorithm for ICA decomposes $\mathbf{X}$ into $\mathbf{S}$ by finding the matrix A such that S_j is as *independent* from the other $S_{j' \neq j}$ as possible.[13] There are several algorithms for finding

[13]Note that the components obtained by an ICA algorithm are not completely independent of each other. For example, if each component of $\mathbf{X}$ is completely dependent on each other, such decomposition is impossible. However, practically, we apply ICA to the non-correlated data obtained by PCA, as discussed later. While no correlation does not mean independence, it is close to independence. Therefore

Table 2: Definition and correlation of financial variables in this study

(a) Definition

	Name	Definition
1	Stock index of core countries:	Weekly log-return of the EUR-denominated MSCI, equally-weighted average of Belgium, Germany and France. The unit is one percent.
2	Stock index of peripheral countries:	Weekly log-return of the EUR-denominated MSCI, equally-weighted average of Greece, Italy, Portugal, and Spain. The unit is one percent.
3	Global stock implied volatility (VIX):	Weekly changes of the CBOE VIX. The unit is one percent.
4	Eurozone stock implied volatility (VSTOXX):	Weekly changes of the VSTOXX. The unit is one percent.
5	Eurozone corporate CDS index:	Weekly change of the Markit iTraxx Europe. The unit is one basis point.
6	Global sovereign CDS index:	Weekly change of 5-year sovereign CDS spreads, equally-weighted average of China, Japan, Switzerland, the United Kingdom (denominated in USD), and the United States (denominated in EUR). The unit is one basis point.
7	Eurozone financial CDS index:	Weekly change of the Markit iTraxx Europe Senior Financials. The unit is one basis point.
8	Redenomination risk of core countries:	Weekly change of "redenomination risk" defined in equation (7) in section 3.3, equally-weighted average of France and Belgium. The unit is one basis point.
9	Redenomination risk of peripheral countries:	Weekly change of "redenomination risk" defined in equation (7) in section 3.3, equally-weighted average of Greece, Portugal, and Spain. The unit is one basis point.
10	EURUSD implied volatility:	Weekly change of the volatility implied by the at-the-money option of EURUSD with three month maturity. The unit is one percent.

(b) Correlation

	1	2	3	4	5	6	7	8	9	10
1	1	0.87	–0.69	–0.74	–0.78	–0.50	–0.72	–0.14	–0.03	–0.48
2	0.87	1	–0.65	–0.69	–0.80	–0.52	–0.78	–0.16	0.03	–0.53
3	–0.69	–0.65	1	0.82	0.65	0.39	0.57	0.07	0.06	0.58
4	–0.74	–0.69	0.82	1	0.67	0.41	0.60	0.03	0.01	0.57
5	–0.78	–0.80	0.65	0.67	1	0.61	0.91	0.12	–0.02	0.55
6	–0.50	–0.52	0.39	0.41	0.61	1	0.50	0.07	–0.06	0.29
7	–0.72	–0.78	0.57	0.60	0.91	0.50	1	0.21	0.04	0.56
8	–0.14	–0.16	0.07	0.03	0.12	0.07	0.21	1	0.26	0.02
9	–0.03	0.03	0.06	0.01	–0.02	–0.06	0.04	0.26	1	–0.01
10	–0.48	–0.53	0.58	0.57	0.55	0.29	0.56	0.02	–0.01	1

A; among them, the *fastICA* algorithm introduced by Hyvärinen and Oja (2000) realizes ICA by maximizing the non-Gaussianity of each independent component based on the fixed point method.

Equation (2) has the same form as PCA. Although PCA and ICA provide for linear transformation of multidimensional data, they differ as to how to determine the matrix A. PCA diagonalizes the covariance matrix of $\mathbf{X}$ in equation (2) by a unitary matrix A, usually denoted as U, that is, $\mathbf{X} = U\mathbf{P}$ where $D = \mathrm{cov}\mathbf{P} = \mathrm{diag}(\lambda_1, \ldots, \lambda_q)$, $UU^\top = U^\top U = I_q$ and $\mathbf{P} = (P_1, \ldots, P_q)^\top$. Component P_j, which has no correlation with the other $P_{j' \neq j}$, is referred to as the *principal component* (PC), and its variance is equal to the diagonal element λ_j. Conventionally, λ_j is in the descending order as $\lambda_1 \geq \cdots \geq \lambda_q \geq 0$, and P_1 is referred to as the first PC since its variance is the largest.

ICA is often used in combination with PCA. ICA cannot determine the scales of the ICs, since A can be a non-unitary matrix. Consequently, ICA cannot reduce the dimension of data based on their scales. In contrast, PCA can reduce the data dimension based on the variances of the PCs, since the variances are determined with the unitarity of U. Therefore, for the data with large dimension d $(> q)$, the dimension is reduced from d to q by PCA at first, and then ICA is applied to the q-dimensional data obtained by PCA.

3.2 The return model based on ICA

Our principal interest is applying ICA to the multivariate time series of the CDS spreads. Let $\mathbf{r}_t = (r_{1t}, \ldots, r_{dt})^\top$, $t = 1, \ldots, T$ denote the log-return of d sovereign CDS spreads on

we postulate that each component obtained using ICA is independent.

day t. Let us assume that the returns $\mathbf{r}_t$ obeys a factor model with q ($\leq d$) factors:[14]

$$\begin{cases} \mathbf{r}_t = \mu + A\mathbf{s}_t + \epsilon_t, \\ \mathbf{s}_t = (s_{1t}, \ldots, s_{qt})^\top\text{: source of variation,} \\ \epsilon_t = (\epsilon_{1t}, \ldots, \epsilon_{dt})^\top\text{: noise term,} \\ \mu\text{: constant vector, and} \\ A\text{: } d \times q\text{-constant matrix.} \end{cases} \tag{3}$$

Note that $\mathbf{s}_t$ and ϵ_t can be interpreted as the sources of variation of the CDS spreads and the noise term, respectively. We assume that these vectors are independent of each other. We employ both PCA (PCA only) and ICA (ICA combined with PCA) to determine the matrix A and the series $\{s_{jt}\}_{1\leq t\leq T}$, $j = 1, \ldots, q$, in equation (3). For PCA, we use the decomposition based on the correlation matrix. For ICA, we use the fastICA algorithm.[15]

The model in equation (3) assumes the existence of q latent factors, and we utilize PCA or ICA to find these factors statistically. The factors found are not necessarily the factors with meaningful interpretations, which is a major disadvantage common to similar purely statistical methods. However, if the number of factors is known theoretically as q, these statistical methods can be the clue to identifying these q factors.

We set $q = 3$. Since we observe decoupling of CDS spreads, we assume more than one decoupling factors in addition to the common factor. Based on the correlation of the financial variables shown in Table 2b in Section 2.2, financial variables are characterized by at least three different risk factors: The redenomination risk of the peripheral, the redenomination risk of the core, and the risk common to the other variables. However, based on the principle of parsimony, a small value of q is better. Based on the PCA anal-

[14] Inversely, we can derive the formula $\mathbf{s}_t = (D_r^{-1/2}A)^+ D_r^{-1/2}(\mathbf{r}_t - \mu)$ where $X^+ = (X^\top X)^{-1}X^\top$ is the Moore-Penrose pseudoinverse matrix and $D_r = \mathrm{diag}((\sigma_1^{(r)})^2, \ldots, (\sigma_d^{(r)})^2)$ is the variances of the original CDS returns $\mathbf{r}_t$.

[15] We use the MATLAB code distributed by the authors at their webpage:
`http://www.cs.helsinki.fi/u/ahyvarin/` .
In the fastICA algorithm, we select the hyperbolic tangent (tanh) function as the nonlinearity function, and set the scale parameter as unity.

Table 3: The explanatory powers of the first three PCs and ICs. The table includes the results for two different data: the "Seven countries" data, from January 2009 to June 2011 including Greece, and the "Six countries" data, from January 2009 to December 2013 excluding Greece.

	PCA Results			ICA Results		
	PC1	PC2	PC3	IC1	IC2	IC3
Seven countries	78%	10%	4%	29%	26%	37%
Six countries	76%	12%	4%	29%	26%	38%

ysis, 92% of the variation of the CDS spreads can be explained by the linear combination of factors shown in equation (3) at maximum, and it does not improve so much when $q > 3$.[16] Therefore, we set $q = 3$.

3.3 Decomposition results

Table 3 shows the explanatory powers of the first three PCs and ICs.[17] PC1, PC2 and PC3 in the table are the first, second, and the third PCs. IC1, IC2 and IC3 in the table are the ICs. The ICs are in the descending order of non-Gaussianity.

It can be seen from Table 3 that the explanatory powers of the first PC is 78% or 76%, which is dominant over the other two PCs because they have much lower explanatory power. In stark contrast to the PC for PCA, the first three ICs when ICA is used have similar values for the explanatory power.

Table 4 shows the entries of the constant matrix A in equation (3) for both PCA and ICA, where the unit is one percent point per unit value of the PC or the IC. The R^2 values of the regression for each factor for each country is also shown in the parentheses in the table. For example, PC1 in the seven countries data explains 77% of the CDS spread

[16]This value (92%) is the summation of the top three PCs' explanatory power for the CDS spreads, shown in Table 3 in Section 3.3.

[17]Here, the explanatory power is defined in the following way: For each country and each component, a simple regression is estimated by setting the CDS spread returns of the country as the explained variable and the component as the explanatory variable, and obtain the coefficient of determination R^2. The explanatory power of the component is defined as the average of the R^2 across the countries. Note that for PCA, the explanatory power by this definition matches its usual definition; that is, it is the ratio of the eigenvalue corresponding to the component over the total of the eigenvalues.

Table 4: The factor loadings of the first three PCs and ICs. The unit of factor loadings is 1% increase of CDS spreads per one point increase of each PC or IC. The figures in parentheses are the R^2 values of the regression for each component on each country. The parenthesized figures in the "Tot." rows show the explanatory powers shown in Table 3.

	PCA Results						ICA Results					
	PC1		PC2		PC3		IC1		IC2		IC3	
Seven countries from January 2009 to June 2011												
Tot.		(78%)		(10%)		(4%)		(29%)		(26%)		(37%)
BE	4.32	(77%)	2.19	(2%)	−6.92	(10%)	2.91	(6%)	3.93	(12%)	9.70	(71%)
FR	4.28	(76%)	5.58	(16%)	2.92	(2%)	2.09	(3%)	8.81	(59%)	6.40	(31%)
DE	4.10	(69%)	6.93	(25%)	1.06	(0%)	0.70	(0%)	8.58	(56%)	7.04	(38%)
GR	5.00	(75%)	−3.59	(5%)	9.24	(13%)	9.93	(54%)	7.56	(32%)	3.62	(7%)
IT	4.76	(82%)	−3.52	(6%)	−3.42	(2%)	7.71	(40%)	2.99	(6%)	8.12	(44%)
PT	5.79	(82%)	−5.09	(8%)	0.39	(0%)	10.58	(50%)	4.82	(10%)	8.00	(29%)
ES	5.00	(87%)	−3.93	(7%)	−1.84	(1%)	8.52	(47%)	3.64	(8%)	7.82	(39%)
Six countries from January 2009 to December 2013												
Tot.		(76%)		(12%)		(4%)		(29%)		(26%)		(38%)
BE	4.22	(80%)	2.03	(3%)	−5.25	(7%)	2.33	(5%)	4.28	(18%)	8.22	(66%)
FR	4.22	(80%)	3.99	(11%)	1.78	(1%)	2.78	(8%)	7.31	(52%)	5.71	(32%)
DE	3.98	(67%)	6.09	(25%)	3.14	(2%)	1.54	(2%)	8.67	(69%)	4.94	(23%)
IT	4.51	(80%)	−3.45	(8%)	−3.23	(2%)	6.47	(36%)	1.83	(3%)	7.67	(51%)
PT	4.60	(68%)	−5.76	(17%)	8.07	(12%)	10.62	(79%)	3.51	(9%)	3.54	(9%)
ES	4.48	(83%)	−3.75	(9%)	−2.62	(2%)	6.76	(41%)	1.78	(3%)	7.39	(49%)

return of Belgium, and the CDS of Belgium increases by 4.32% when PC1 increases by one.

For PC1, the factor loadings range from 3.98 to 5.00, and the explanatory powers range from 67% to 87%. This PCA result suggests that PC1 can be considered as the common factor whose changes affect all the countries. In contrast, PC2 has much less value in terms of its explanatory power. Looking more closely at the PCA results, we see that the factor loadings for PC2 are positive for Belgium, France, and Germany, but negative for the other countries. The three countries for which the PC2 is positive are considered as the core countries, while the other countries are considered as the peripheral countries. From these observations, it can be argued that PC2 may be related to the difference between core countries and peripheral countries, while the difference explained by PC2 is much smaller than the common factor explained by PC1 from the viewpoint of explanatory power. As for PC3, it is difficult to find some tendency or interpretation.

As seen here, PCA has the advantage that it can reduce the dimension of data into the minimum number of factors needed to explain the data. The large value for the R^2 of PC1 suggests that only PC1 should be considered and the other PCs can be ignored. However, decoupling of the credit spreads suggests adding an additional component in order to explain the decoupling. PC2 can be a candidate for such a component, and the values of factor loading reported in Table 4 show that to some extent PC2 fulfills that role. However, the small R^2 values of PC2 show the difficulty of such interpretation. Therefore, we apply an alternative method of decomposition, ICA.

Turning to the results for the factor loadings obtained from ICA, we see that the factor loading for IC3 for all but Greece in the seven countries data and Portugal in the six countries data ranges from 4.94 to 9.70. For Greece in the seven countries data, IC3's factor loading is 3.62, and for Portugal in the six countries data, IC3's factor loading is 3.54. The explanatory powers of IC3 range from 23% to 71% for all countries except these two cases. For Greece in the seven countries, it is 7%, and for Portugal in the six countries, it is 9%. These ICA results suggest that IC3 is related to the changes of all countries except these two cases. Therefore it seems that IC3 can be viewed as the common factor.

The other two components, IC1 and IC2, have different characteristics. IC1 and IC2 have large factor loadings and explanatory powers for peripheral countries and core countries, respectively. Like PC2, IC1 and IC2 are related to the distinctive country groups in the Eurozone. However, different from PCA, ICA can separate the factor found by PC2 into the two factors reflecting the characteristics of the two country groups. Moreover, the explanatory powers of IC1 and IC2 are much larger than those of PC2, showing the significance of the factors.

These results show the relative advantage of ICA. ICA can find different multiple factors and these independent factors have an interesting interpretation concerning credit decoupling between countries. In addition, the role of the ICA-obtained factors are comparable with respect to explanatory power, whereas the second and the third factors obtained from PCA are negligible compared to the first PC. However, in order to identify the ICs as meaningful factors, additional evidence would be required. For this purpose, we consider the relation of the ICs with the variables observed in the financial markets in Sections 2.2 and 3.4.

The descriptive statistics of the PCs and the ICs are shown in Table 1. The PCs are in the descending order of their standard deviation values. In contrast, the ICs are in the descending order of their kurtosis values, since ICs are obtained by maximizing non-Gaussianity, and non-Gaussianity is measured by higer-order moments of distribution such as skewness and kurtosis.

It can be seen from Table 1 that IC1, the factor related to the peripheral countries, has values for kurtosis that exceeds 15.0. Considering the fact that the value of kurtosis is 3.0 for the Gaussian distribution and 12.0 for the Student-t distribution with degrees of freedom 4.5, which is often used in order to describe tail heaviness of return distribution, a kurtosis value exceeding 15 is extremely large, strongly suggesting that the tail risk in the peripheral countries is high. This result suggests the importance of using tail-heavy models for CDS spreads. Such a tail-heavy model includes not only a tail-heavy distribution but also volatility clustering effect which is discussed in Section 4.1.

3.4 Link between ICs and financial variables

Here we investigate whether the ICs found in our analysis are related to the financial variables selected in Section 2.2. We estimate a simple regression whose the explanatory variable (X) is one of the financial variables we selected, and whose explained variable (Y) is one of the PCs or ICs.[18] Since the explained variable (a PC or an IC) has large kurtosis as can be seen from Table 1, it is possible that outlier observations influence the regression result. Therefore, we adopt a robust regression using the Tukey's biweight function.[19] Table 5 shows the results of the simple regressions for the PCs and the ICs,

[18]Therefore, both X and Y are variables showing returns or changes.

[19]Robust regressions incorporate a weighting function for residuals in order to mitigate the effect of outliers. The Tukey's biweight function is $[\max\{0, 1 - r^2/k^2\}]^2$ where r is a standardized residual. We

respectively. It shows regression coefficients, the significance of these coefficients ('*' and '**' denote the levels of 1% and 5%, respectively), and R^2 values.[20]

For PC results reported in Table 5, all financial variables except the redenomination risk of peripheral countries are found to be significantly related to PC1. The signs of the regression coefficients are negative for the stock index returns and are positive for the other variables, which is an intuitive result. As for R^2, the values for the stock index of peripheral countries and for the Eurozone financial and corporate CDS indexes range from 0.42 to 0.53. These results show that PC1 is an important component.

In contrast, the regression results for PC2 and PC3 are less informative. PC2 is significantly related to several financial variables. However, the R^2 values are less than 0.05 for all the cases, showing that these variables cannot explain the variation of PC2. As for PC3, the regression coefficient is not significant for most cases, and their signs are difficult to interpret.

Table 5 shows the result for the ICs. Observing the results for each IC, we see that all ICs have more than one financial variable significantly related to the ICs with large R^2 values. For example, for both datasets IC1 can be related to the stock index of peripheral countries, the Eurozone financial CDS index, and the redenomination risk of peripheral countries. IC2 can be related to the stock indices (of the core and the peripheral countries), the stock implied volatilities (of the global and the Eurozone), and the CDS indices (of the Eurozone corporate, the global sovereign, and the Eurozone financial). IC3 can be related to the stock indices, the Eurozone stock implied volatility (VSTOXX), the CDS indices, the redenomination risk of core countries, and the EURUSD implied volatility.

In turn, the results for each financial variable indicate that some of the financial variables have a selective effect on ICs. The stock index of core countries is significant for IC2 and IC3 but not for IC1 in both datasets. The global stock implied volatility

obtained similar results with the Huber weighting function (1 for $|r| < 1$ and $1/r$ otherwise). The ordinary least squares method gives different results due to the outliers.

[20] The R^2 values here are those based on the ordinary least squares.

Table 5: Simple linear regression results when each PC or IC is set as the explained variable (Y) and each financial variable as the explanatory variable (X). For estimation of the regression coefficients, robust regression with Tukey's biweight function is adopted. The single asterisk (*) means that the regression coefficient deviates from zero using a significance level of 5%, and the double asterisk (**) means that the regression coefficient deviates from zero using a significance level of 1%. The R^2 values are those obtained from applying ordinary least squares.

	# of	Y=PC1				Y=PC2				Y=PC3			
X=	obs.	coeff.	p		R2	coeff.	p		R2	coeff.	p		R2
Seven countries from January 2009 to June 2011													
Stock index of core countries	130	−0.42	0.00	**	0.30	−0.03	0.10		0.00	−0.01	0.50		0.01
Stock index of peripheral countries	130	−0.46	0.00	**	0.49	−0.02	0.18		0.00	0.00	0.78		0.01
Global stock implied volatility (VIX)	130	0.21	0.00	**	0.13	0.02	0.11		0.03	0.01	0.38		0.02
Eurozone stock implied volatility (VSTOXX)	130	0.24	0.00	**	0.15	0.02	0.12		0.02	0.01	0.31		0.02
Eurozone corporate CDS index	130	0.18	0.00	**	0.48	0.02	0.00	**	0.00	0.00	0.55		0.00
Global sovereign CDS index	120	0.29	0.00	**	0.43	0.05	0.00	**	0.09	0.00	0.54		0.00
Eurozone financial CDS index	130	0.13	0.00	**	0.53	0.01	0.03	*	0.01	−0.01	0.03	*	0.00
Redenomination risk of core countries	130	0.20	0.00	**	0.06	−0.01	0.64		0.00	−0.03	0.01	*	0.03
Redenomination risk of peripheral countries	130	0.05	0.00	**	0.07	0.00	0.39		0.02	0.00	0.47		0.00
EURUSD implied volatility	130	1.25	0.00	**	0.18	0.03	0.54		0.04	−0.08	0.13		0.00
Six countries from January 2009 to December 2013													
Stock index of core countries	261	−0.41	0.00	**	0.32	−0.04	0.01	**	0.00	0.02	0.04	*	0.00
Stock index of peripheral countries	261	−0.41	0.00	**	0.42	−0.02	0.23		0.00	0.02	0.02	*	0.00
Global stock implied volatility (VIX)	261	0.28	0.00	**	0.17	0.03	0.01	**	0.01	0.00	0.59		0.00
Eurozone stock implied volatility (VSTOXX)	261	0.26	0.00	**	0.17	0.03	0.00	**	0.00	−0.01	0.08		0.00
Eurozone corporate CDS index	261	0.15	0.00	**	0.45	0.01	0.01	*	0.00	0.00	0.14		0.00
Global sovereign CDS index	251	0.28	0.00	**	0.38	0.04	0.00	**	0.06	0.00	0.53		0.00
Eurozone financial CDS index	261	0.09	0.00	**	0.49	0.00	0.21		0.00	−0.01	0.00	**	0.01
Redenomination risk of core countries	261	0.07	0.00	**	0.04	−0.01	0.12		0.01	−0.01	0.02	*	0.02
Redenomination risk of peripheral countries	261	0.01	0.10		0.01	0.00	0.03	*	0.03	0.00	0.81		0.00
EURUSD implied volatility	261	1.33	0.00	**	0.21	0.02	0.68		0.01	−0.03	0.38		0.00

	# of	Y=IC1				Y=IC2				Y=IC3			
X=	obs.	coeff.	p		R2	coeff.	p		R2	coeff.	p		R2
Seven countries from January 2009 to June 2011													
Stock index of core countries	130	−0.04	0.07		0.11	−0.11	0.00	**	0.11	−0.11	0.00	**	0.09
Stock index of peripheral countries	130	−0.07	0.00	**	0.17	−0.10	0.00	**	0.17	−0.12	0.00	**	0.16
Global stock implied volatility (VIX)	130	0.02	0.22		0.13	0.06	0.00	**	0.03	0.04	0.07		0.01
Eurozone stock implied volatility (VSTOXX)	130	0.02	0.11		0.13	0.07	0.00	**	0.05	0.05	0.05	*	0.01
Eurozone corporate CDS index	130	0.01	0.08		0.17	0.03	0.00	**	0.15	0.05	0.00	**	0.16
Global sovereign CDS index	120	0.02	0.04	*	0.01	0.07	0.00	**	0.27	0.09	0.00	**	0.24
Eurozone financial CDS index	130	0.01	0.02	*	0.22	0.01	0.01	**	0.10	0.05	0.00	**	0.22
Redenomination risk of core countries	130	0.02	0.14		0.00	−0.01	0.51		0.00	0.08	0.00	**	0.09
Redenomination risk of peripheral countries	130	0.01	0.00	**	0.07	0.01	0.08		0.01	0.01	0.13		0.02
EURUSD implied volatility	130	0.02	0.81		0.15	0.09	0.24		0.01	0.26	0.01	*	0.05
Six countries from January 2009 to December 2013													
Stock index of core countries	261	−0.04	0.03	**	0.05	−0.08	0.00	**	0.08	−0.18	0.00	**	0.20
Stock index of peripheral countries	261	−0.05	0.00	**	0.12	−0.06	0.00	**	0.07	−0.16	0.00	**	0.25
Global stock implied volatility (VIX)	261	0.03	0.02	**	0.09	0.05	0.00	**	0.03	0.10	0.00	**	0.06
Eurozone stock implied volatility (VSTOXX)	261	0.02	0.08		0.05	0.06	0.00	**	0.05	0.10	0.00	**	0.08
Eurozone corporate CDS index	261	0.02	0.00	**	0.12	0.03	0.00	**	0.10	0.05	0.00	**	0.24
Global sovereign CDS index	251	0.03	0.00	**	0.04	0.09	0.00	**	0.26	0.08	0.00	**	0.15
Eurozone financial CDS index	261	0.01	0.00	**	0.12	0.01	0.00	**	0.06	0.04	0.00	**	0.31
Redenomination risk of core countries	261	0.02	0.03	*	0.01	−0.01	0.55		0.00	0.03	0.00	**	0.06
Redenomination risk of peripheral countries	261	0.01	0.01	**	0.03	0.00	0.61		0.01	0.00	0.39		0.00
EURUSD implied volatility	261	0.16	0.01	**	0.12	0.17	0.00	**	0.03	0.37	0.00	**	0.07

(VIX) is significant only for IC2. The Eurozone stock implied volatility (VSTOXX) and the CDS indices (Eurozone corporate CDS index, Global sovereign CDS index, and Eurozone financial CDS index) are significant for both IC2 and IC3. The redenomination risk of core countries is significant for IC3, while the redenomination risk of peripheral countries is significant for IC1. The EURUSD implied volatility is significant only for IC3.

These results support the interpretation of ICs that we proposed in Section 3.3. There we reported that IC1 can be interpreted as the factor related to the risk of the peripheral countries. The regression results above support this idea, since IC1 is related to peripheral variables such as the stock index return of peripheral countries and the redenomination risk of peripheral countries. IC3 is exclusively related to the redenomination risk of core countries and the EURUSD implied volatility, in addition to the stock returns and the CDS indices. This supports the view that IC3 represents the risk common to the Eurozone.

In Section 3.3 we observed that IC2 can be considered to be a factor of core countries. However, the above results indicate that IC2 is exclusively related to the global stock implied volatility (VIX), and also is related to the global sovereign CDS spreads. Since the markets of core countries such as Germany and France are connected to global financial markets in the same way that the US market is, it is not surprising that the factor representing core countries is related to global financial market variables. Therefore, we can interpret IC2 as the global risk factor.

In summary, based on the factor loadings and the regression by financial variables, we can interpret the three ICs as the peripheral risk factor, the global risk factor, and the Eurozone common risk factor.[21] Figure 2 provides a comparison between the cumulative sum of each IC and the level of one of the variables significantly related to the IC. We selected the stock index of peripheral countries for the cumulative sum of IC1 $\times(-1)$,[22]

[21]Note that the effects of these regional factors are not limited to the regions. These names are just labels used for the regions affecting the factors the most.

[22]We multiplied (-1) in order that the stock returns and IC1 change in the same direction.

the global sovereign CDS index for IC2, and the Eurozone financial CDS index for IC3. From the figure the relations between the ICs and the financial variables are observed.

4 Analyses utilizing the ICA model

In this section, we propose possible analyses based on the ICA model given by equation (3). We first introduce the conditional volatility of the ICs, and consider the dynamic correlation based on that, showing the decoupling of the Eurozone sovereign CDS marketsand some policy effect. We also show the PDF of the ICA-based models and propose an example of model comparison based on the likelihood.

Hereafter, we assume that the components of $\mathbf{s}$ are independent and therefore each component of $\mathbf{s}$ can be modeled by a one-dimensional distribution. This is a major advantage compared to the other multivariate models because it reduces a good deal of computational burden tied to multivariate models.

4.1 Volatility of the ICs

Because the changes of the CDS spreads in the crisis period are volatile, the log-returns of CDS are expected to have a volatility clustering effect, and so are the ICs. In fact, by using the statistical test proposed by Engle (1982), IC1 and IC2 are expected to exhibit a volatility clustering effect.[23] Therefore, we apply the exponential GARCH (EGARCH)

[23]The null hypothesis of no serial correlation in the squared ICs is rejected for IC1 up to at least 8 lags for both datasets with 99% significance level. For IC2, the null is rejected for most lags up to 10 with 90% significance level, many of which can be rejected with 99% significance. However, the null for IC3 cannot be rejected for most cases.

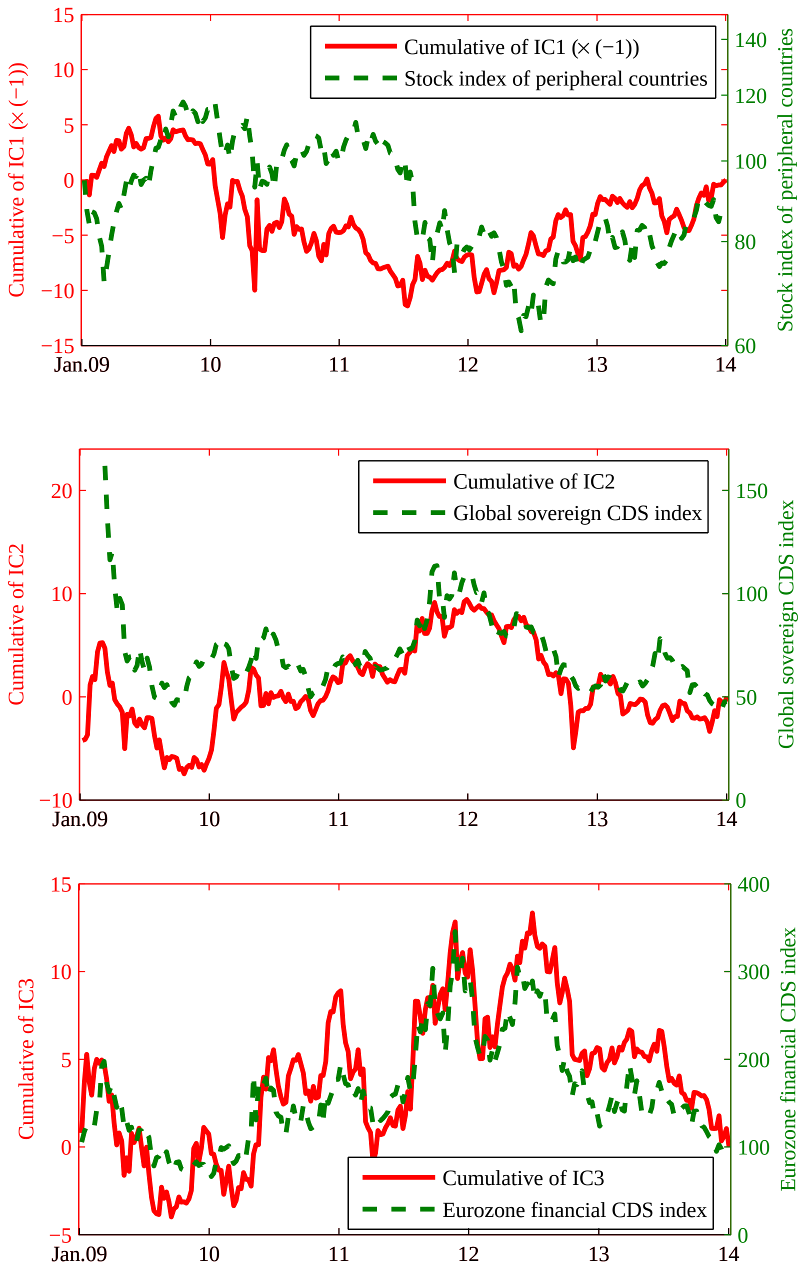


Figure 2: Comparison between the cumulative sum of each IC and the levels of variables in relation with the IC. Note that the cumulative sum of IC1 is inverted since the regression coefficients between IC1 and the stock index of peripheral countries is negative.

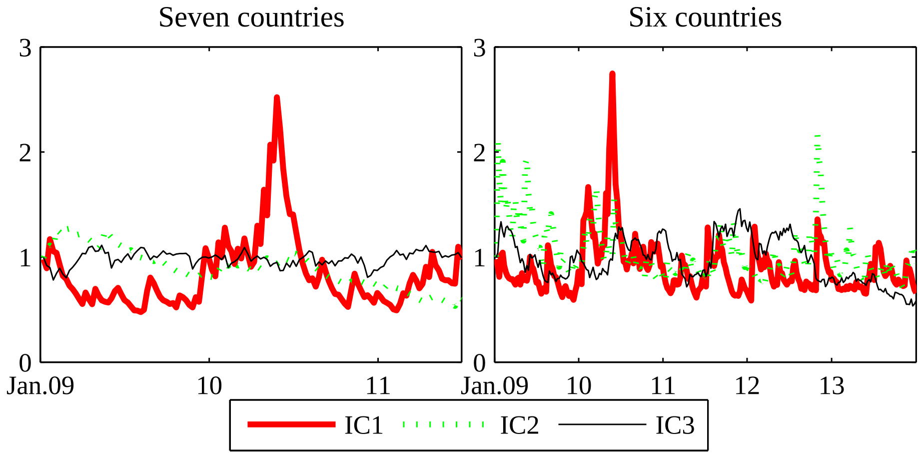


Figure 3: Volatilities of the ICs obtained. All series are obtained by the EGARCH(1,1) model with standardized Student-t residuals.

model. More specifically, we apply EGARCH(1,1) model[24] to s_{jt}:

$$\begin{cases} s_{jt} = \sigma_{jt}^{(s)} \varepsilon_{jt}, \\ \ln(\sigma_{j,t+1}^{(s)})^2 = \omega_j + \beta_j \sigma_{jt}^2 + \gamma_j \varepsilon_{jt} + \alpha_j(\varepsilon_{jt} - |\varepsilon_{jt}|), \\ \mathrm{E}\,\varepsilon_{jt} = 0, \mathrm{var}\varepsilon_{jt} = 1. \end{cases} \tag{4}$$

Figure 3 shows the volatility of the ICs obtained from the EGARCH(1,1) model with the standardized Student-t distribution.[25]

From the figure, the volatility of IC1, the peripheral factor, spiked to a level more than 4.0 in May 2010. Before this spike, the volatility of IC1 started increasing around the beginning of 2010, showing the increasing concern for the peripheral countries. The spike in May 2010 shows the magnitude of the concern for the events in Greece at that time (see Section 2), which invoked the following crisis that the Eurosystem confronted. After May 2010, the volatility of IC1 had calmed down as indicated by a decline in its level to around 1.0, showing that the concern shifted from the risk in periphery to the

[24] We apply EGARCH assuming the leverage effect, same with the preceding studies such as Kumiega et al. (2011). Note that the direction of the leverage in the CDS case is supposed to be opposite to the stock case; that is, CDS volatility is expected to extend more after the CDS spread increases than before it decreases.

[25] Here the Student-t distribution is multiplied by $\sqrt{\nu - 2/\nu}$ in order to set the variance to unity, where ν is the number of degrees of freedom.

other source of risk.

The volatility of IC2, the global factor, was high at the beginning of 2009. This is related to the remaining effect from the US financial crisis in 2008, which is considered to have a global effect. The volatility of IC2 during the Eurozone sovereign debt crisis post-2010 was relatively stable compared to IC1.

As for IC3 in the six countries data, its volatility widened around the end of 2011 and the middle of 2012. This widening becomes clearer by looking at its historical volatility. In contrast, the volatilities of IC1 and IC2 remained lower around these times compared with their peaks. So IC3, the Eurozone common factor, can be considered as the major source of volatility during the period around 2012. As we discussed in Section 1, the risk specific to this period was the risk of redenomination in the Eurozone. The result shows the possibility that the redenomination risk comes from the common factor to the Eurozone, not from the reasons specific to peripheral countries.

These different peaks for the volatilities indicates the shift of the volatility source of the Eurozone sovereign CDS spreads. That is, the volatility source was the global factor in 2009, but then shifted to the peripheral factor in 2010, and to the Eurozone common factor in 2011 to 2012. These different risk sources would correspond to different stages of the Eurozone sovereign debt crisis.

Note that the sharp increase for IC1 volatility can be observed in the six countries data which do not contain the information about Greece. This suggests that ICA can reconstruct the risk of the peripheral countries including Greece from the information excluding Greece.

4.2 Effect of policies

The residual term ε_{jt} in equation (4) can be considered the shocks occurring in the ICs considered. There are many sources driving these ICs. One of the important sources is the decision and execution of policies by authorities. We consider two cases of policy effects.

At the beginning of July 2012, the turmoil of the sovereign CDS market was still going on, and the spreads of the peripheral countries were around their peaks. On July 5, the ECB lowered the interest rate of its main refinancing operations from 1.00% to 0.75%.[26] The average of the IC3 before this date in the seven countries data is 0.09 (in the direction of widening the CDS spreads), while the average after this date is -0.11 (in the direction of tightening the CDS spreads). The Student-t test[27] shows that this difference of the averages is significant with 95% level, indicating that the ECB rate change lowered the sovereign CDS spreads via reducing the IC3, the Eurozone common factor. On contrary, the IC1, the peripheral factor, does not change its average level statistically at around that ECB rate cut. This statistical fact is consistent with the economic fact that the ECB rate change is an Eurozone-wide shock, not a periphery-specific one. Instead, it is supposed that the IC1 is more sensitive to the events specific to the peripheries.

In March 2012, it was the risk that the Greek government failed the scheduled redemption of the bond. On March 14, 2012, the Second Economic Adjustment Programme for Greece was approved by the Eurozone finance ministers,[28] and was ratified by the Hellenic Parliament on March 20,[29] reducing the default risk of Greece. The average of the IC1 before March 20 is 0.10 (widening direction), while that after March 20 is -0.13 (tightening direction). The p-value of this difference based on the Student-t test is 0.06, significant with 90% level. This result shows that the Greek bailout lowered the CDS spreads of the peripheral countries via reducing the IC1, the peripheral factor.

We see in Section 4.1 that the risk of the peripheral countries comes first in 2010 and the risk of the Eurozone comes in 2011 or 2012. The results of the analysis in this section indicates that these risks are reduced by different policies, corresponding to the range of

[26] `https://www.ecb.europa.eu/press/pr/date/2012/html/pr120705.en.html`

[27] For unequal sample sizes and equal variance case; Equal variance can be shown by the F test, consistent with the assumption of the unit variance of the residuals in the EGARCH model.

[28] `http://ec.europa.eu/economy_finance/assistance_eu_ms/greek_loan_facility/index_en.htm`

[29] `http://www.hellenicparliament.gr/en/`

the effect of each policy.

4.3 Dynamic correlation

Following Kumiega et al. (2011) and García-Ferrer et al. (2012), we derived our model's dynamic correlation. We use the variances of the ICs given by equation (4), and assume the covariance $\text{cov}\epsilon_t$ in equation (3) is constant.[30] Then we obtain $\text{cov}_t(\mathbf{r}_t) = A \times \text{cov}_t(\mathbf{s}_t) \times A^\top + \text{cov}_t\varepsilon_t$, and the dynamic correlation between r_{tj_1} and r_{tj_2} is then defined as $\rho_{t,j_1,j_2} = \text{cov}_t(r_{t,j_1}, r_{t,j_2})/\sqrt{\text{var}_t(r_{t,j_1})\text{var}_t(r_{t,j_2})}$.

Figure 4 shows the pairwise dynamic correlations for the seven countries. The red bold line labeled "ICA" shows the dynamic correlation in this way. The black dashed line labeled "rolling" shows the historical correlation during the rolling-window period consisting of 26 weeks prior to the designated date. In addition to these correlations, for the purpose of comparison, we also show the dynamic correlation based on the conditional Gaussian copula model proposed by Patton (2006), which is shown by the blue thin sold line labeled "Conditional copula".

From Figure 4 it can be observed that the ICA correlations between the peripheral countries and Germany (DE-IT, DE-ES, DE-PT, DE-GR) decreased toward the middle of 2010. The ICA correlation between the peripheral countries and Greece (IT-GR, ES-GR, and PT-GR) increased toward the middle of 2010. These correlations strongly suggest the increasing disparity between the Eurozone sovereign CDS spreads during the period investigated.

Figure 5 enlarges the chart of the dynamic correlation between Germany and Greece from Figure 4. From the figure, the rolling correlation between Germany and Greece is close to unity in 2009, while it slides down to around 0.5 since 2010 as the Greek crisis deepened. In the ICA correlation, the correlation stays in the range 0.6 to 0.8 in 2009, and since 2010 descends gradually to the range 0.4 to 0.6 since 2010. In May 2010, the ICA correlation drops down to 0.4, which is not evident in the rolling correlation. As

[30] The terms var_t and cov_t mean that the variance and covariance are measured at time t.

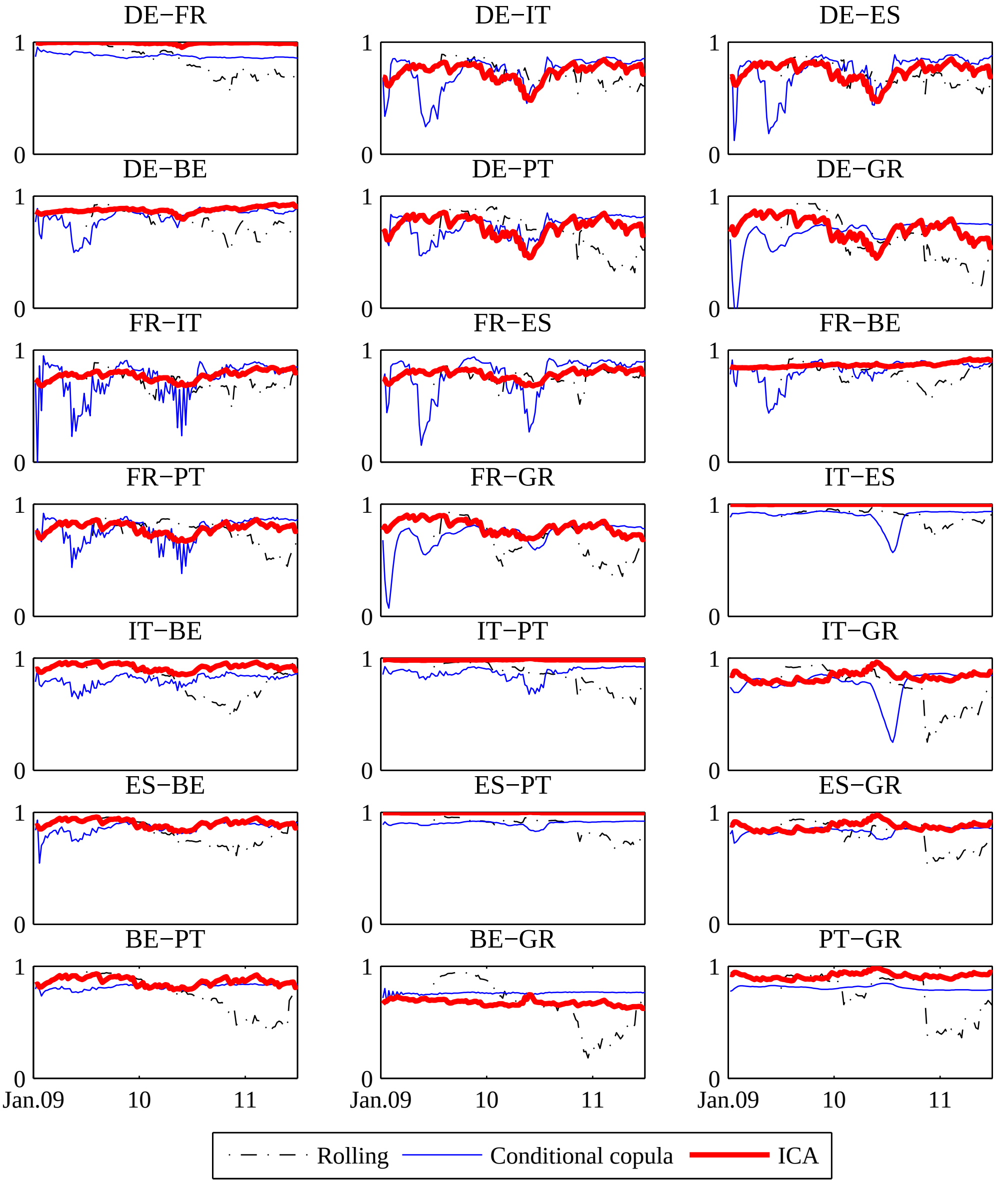


Figure 4: Dynamic correlations between the seven countries. The dynamic correlations are obtained by the three different methods: the historical correlation during the rolling-window (black dashed lines labeled “Rolling”), the correlation based on conditional copula model (blue thin lines labeled “Conditional copula”), and the correlation based on the ICA model combined with the GARCH model (red bold lines labeled “ICA”).

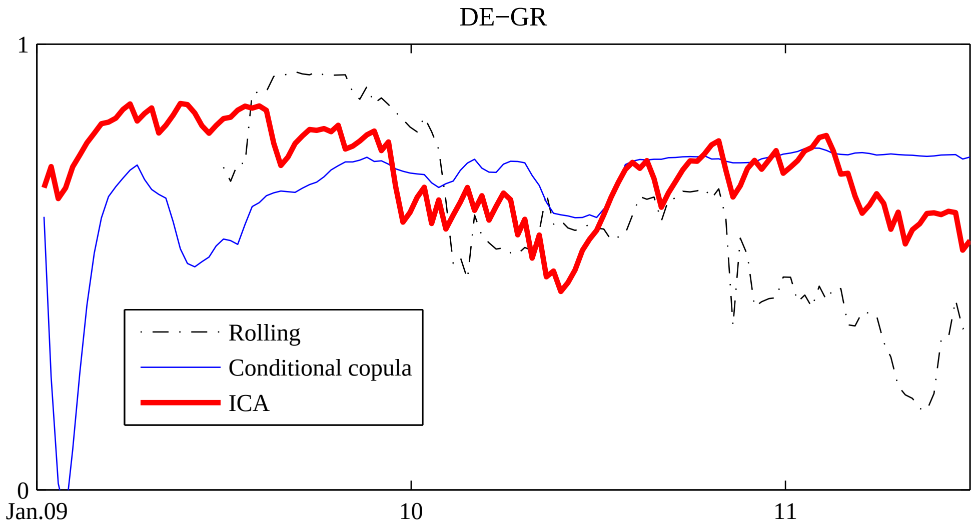


Figure 5: Dynamic correlations between Germany and Greece (based on the seven countries data). The dynamic correlations are obtained by the three different methods: the historical correlation during the rolling-window (black dashed line labeled as "Rolling"), the correlation based on conditional copula model (blue thin line labeled "Conditional copula"), and the correlation based on the ICA model combined with the GARCH model (red bold line labeled "ICA").

we indicated in Section 2, at that time the hidden debt of the Greek government was revealed and, as a result, the solvency of the country came into question. The decline in ICA correlation can be considered to be the market's immediate response, which is not evident by simply looking at the rolling correlation.

From Figure 5, the correlation based on the conditional Gaussian copula model also drops in May 2010, indicating that both models can detect the decrease in the correlation. It seems that the ICA correlation decreased earlier than the conditional Gaussian copula did. From these observations, it is fair to say that the ICA correlation provides a dynamic correlation whose feature is different from that provided by the dynamic copula model. An advantage of the ICA correlation compared to the rolling correlation or the dynamic copula model is that we can distinguish the factor of high volatility and high risk, as explained in Section 4.1.

Table 6: Log likelihood and Akaike information criteria. The first column gives the name of each model. The second column shows the number of parameters in the model. The parenthesized numbers are the ranks of the models among the five models studied. Commas denote thousand separators.

ID	Models and numbers of parameters		Log-likelihood		AIC		BIC	
	Seven countries from January 2009 to June 2011							
A	Multivariate Gaussian	35	1,134	(7)	-2,199	(7)	-2,098	(7)
B	Student t marginal and copula	43	1,234	(3)	-2,383	(3)	-2,259	(2)
C	EGARCH-Gaussian marginal and copula	49	1,178	(5)	-2,259	(5)	-2,118	(6)
D	GARCH-Gaussian marginal and copula	42	1,171	(6)	-2,258	(6)	-2,138	(5)
E	ICA-EGARCH-T dimension reduction	54	1,217	(4)	-2,327	(4)	-2,172	(4)
F	ICA-EGARCH-T no dimension reduction	70	1,273	(1)	-2,405	(2)	-2,204	(3)
G	ICA-T no dimension reduction	42	1,245	(2)	-2,406	(1)	-2,286	(1)
	Six countries from January 2009 to December 2013							
A	Multivariate Gaussian	27	2,087	(7)	-4,120	(7)	-4,024	(7)
B	Student t marginal and copula	34	2,242	(2)	-4,417	(2)	-4,295	(1)
C	EGARCH-Gaussian marginal and copula	39	2,194	(5)	-4,310	(5)	-4,171	(6)
D	GARCH-Gaussian marginal and copula	33	2,186	(6)	-4,306	(6)	-4,188	(5)
E	ICA-EGARCH-T dimension reduction	45	2,228	(3)	-4,365	(4)	-4,205	(4)
F	ICA-EGARCH-T no dimension reduction	57	2,294	(1)	-4,473	(1)	-4,270	(3)
G	ICA-T no dimension reduction	33	2,227	(4)	-4,389	(3)	-4,271	(2)

4.4 Model comparison based on likelihood

Next we consider the goodness-of-fit of the ICA-based model. For purposes of comparison, we apply seven models including the ICA-based models. A detailed description of these models is provided in A.1. We consider the likelihood of these models.[31] In addition, to penalize for the number of parameters, we use the Akaike and the Bayesian information criteria (the AIC and the BIC). Table 6 shows the result for the values of the log-likelihood, the AIC, and the BIC. The number in parentheses is the rank among the seven models.

The ICA-EGARCH-T dimension reduction model (ID "E" in Table 6; hereafter shown by a letter in parenthesis) exhibits a better fitting than the multivariate Gaussian (A) or the EGARCH-Gaussian marginal and copula (C) models, demonstrating the advantage of the ICA model in comparison to the Gaussian model. In contrast, the ICA-EGARCH-T dimension reduction model (E) is always worse than the ICA-EGARCH-T no dimension

[31] A.2 discusses how to compute the likelihood of the ICA-based models.

rediction model (F), the ICA-T no dimension reduction model (G), and the Student-t marginal and copula model (B). Therefore, it can be seen that dimension reduction reduces the goodness-of-fit, indicating that dimension reduction results in a lot of information loss.

Among these no-dimension-reduction models, the ICA-EGARCH-T no dimension reduction model (F) is shown to provide a better fit than the Student-t marginal and copula model (B) based on the log-likelihood and the AIC. Also, the ICA-T no dimension reduction model (G) is shown to provide a better fit than the Student-t marginal and copula model (B) for the seven countries data. Considering that the copula-based model can incorporate nonlinear dependence between variables, it is surprising that the linear models based on ICA show better fits than the copula model.

The comparison between the ICA-EGARCH-T no dimension reduction model (F) and the ICA-T no dimension reduction model (G) shows that the model's fit is improved by considering the volatility clustering effect of the ICs. The leverage effect can be observed since the ranks of the EGARCH-Gaussian marginal and copula model (C) are generally higher than those of the GARCH-Gaussian marginal and copula model (D).

Based on these observations, it is fair to conclude that the ICA model provides a good fit to the CDS spread data. Dimension reduction reduces the goodness-of-fit, but the reduced model is still better than popular models such as the EGARCH model, and comparable to the complicated dependence model such as Student-t copula model. As for volatility clustering, it is better to incorporate the effect into the model.

5 Conclusion

In this paper, we investigated the behavior of Eurozone sovereign CDS spreads during the period of the Eurozone sovereign debt crisis. For that purpose, we introduced a novel technique of factor decomposition, independent component analysis (ICA). We first showed that ICA can find multiple factors impacting CDS spreads, while principal component analysis (PCA) finds one factor. We identified three factors based on ICA, and we found interpretations for these three factors by combining the factor loadings and the regression analysis with financial variables. The first IC was interpreted as the factor related to the risk associated with the peripheral countries such as Greece and Portugal. The second IC was interpreted as the factor related to global risk. The third IC was interpreted as the factor related to the risk common to the Eurozone. More specificially, we showed that there were two risks specific to the Eurozone sovereign CDS spreads, that is, the risk of the financial sector and the risk of redenomination, are related to these three factors.

Next, we considered several analyses based on ICA. By applying GARCH, we found that the three factors identified had different peaks during the crisis. These different peaks indicated that the source of risk was the global factor in 2009, but it shifted to the peripheral factor in 2010, and finally shifted to the Eurozone common factor in 2012. Utilizing the GARCH residual terms, we show the effect of two policies conducted in 2012. Utilizing this GARCH volatility, we found that the dynamic correlation based on the ICA model can explain the decoupling between the core and peripheral countries, such as between Germany and Greece. Finally, we compared the ICA model to alternative models by using the likelihood, the AIC and the BIC, showing that the goodness-of-fit of the ICA model is generally better than the other models such as the GARCH and Gaussian marginal and copula model and the Student-t marginal and copula model.

The contributions of the findings of this paper are threefold. Our principal contribution is the finding that there were multiple factors impacting the Eurozone sovereign CDS spreads. This multiplicity is important for explaining the CDS spreads during the Eurozone crisis since it can reasonably explain the reported decoupling of the Eurozone sovereign countries. Second we provided a reasonable interpretation of these factors based on our statistical analysis (regression analysis with financial variables, volatility analysis, and dynamic correlation analysis). Third, our empirical analysis demonstrated why ICA is a more effective statistical tool to employ compared to the more commonly used tool, PCA.

References

References

Acharya, V., Drechsler, I., Schnabl, P., 2014. A pyrrhic victory? bank bailouts and sovereign credit risk. Journal of Finance 69 (6), 2689–2739.

Ang, A., Longstaff, F. A., 2013. Systemic sovereign credit risk: Lessons from the u.s. and europe. Journal of Monetary Economics 60 (5), 493–510.

Artis, M. J., Zhang, W., 2001. Core and periphery in emu: A cluster analysis. Economic Issues Journal Articles 6 (2), 47–58.

De Santis, R., April 2015. A measure of redenomination risk. Working Paper 1785, European Central Bank.

Demarta, S., McNeil, A. J., 2005. The t copula and related copulas. International Statistical Review 73 (1), 111–129.

Draghi, M., July 2012. Verbatim of the remarks made by mario draghi. Speech at the Global Investment Conference in London 26 July 2012, https://www.ecb.europa.eu/press/key/date/2012/html/sp120726.en.html.

Draghi, M., Constâncio, V., December 2012. Introductory statement to the press conference (with q&a). https://www.ecb.europa.eu/press/pressconf/2012/html/is121206.en.html.

Duffie, D., Pedersen, L. H., Singleton, K. J., 2003. Modeling sovereign yield spreads: A case study of russian debt. Journal of Finance 58 (1), 119–159.

Engle, R. F., 1982. Autoregressive conditional heteroscedasticity with estimates of the variance of united kingdom inflation. Econometrica: Journal of the Econometric Society 50 (4), 987–1007.

European Central Bank, May 2010. 10 may 2010 - ecb decides on measures to address severe tensions in financial markets. Press release, https://www.ecb.europa.eu/press/pr/date/2010/html/pr100510.en.html.

European Commission, October 2012. High-level expert group on reforming the structure of the eu banking sector: Final report. Chaired by Erkki Liikanen, http://ec.europa.eu/internal_market/bank/docs/high-level_expert_group/report_en.pdf.

Fontana, A., Scheicher, M., November 2010. An analysis of euro area sovereign cds and their relation with government bonds. Working Paper 1271, European Central Bank. URL http://ssrn.com/abstract=1715483

García-Ferrer, A., González-Prieto, E., Peña, D., 2012. A conditionally heteroskedastic independent factor model with an application to financial stock returns. International Journal of Forecasting 28 (1), 70–93.

Hyvärinen, A., 2013. Independent component analysis: Recent advances. Philosophical Transactions of the Royal Society A: Mathematical, Physical and Engineering Sciences 371 (1984), 20110534.

Hyvärinen, A., Karhunen, J., Oja, E., 2001. Independent Component Analysis. John Wiley & Sons.

Hyvärinen, A., Oja, E., 2000. Independent component analysis: Algorithms and applications. Neural Networks 13 (4), 411–430.

Joe, H., 1997. Multivariate models and multivariate dependence concepts. Chapman & Hall/CRC Press.

Kaletsky, A., August 2012. Column: Italy refutes the idea it's on europe's 'periphery'. http://uk.reuters.com/article/email/idUSBRE87L0X920120822.

Kim, Y. S., Rachev, S. T., Bianchi, M. L., Mitov, I., Fabozzi, F. J., 2011. Time series analysis for financial market meltdowns. Journal of Banking & Finance 35 (8), 1879–1891.

Krishnamurthy, A., Nagel, S., Vissing-Jorgensen, A., 2013. Ecb policies involving government bond purchases: Impact and channels. Tech. rep., Mimeo Northwestern University, University of Michigan, and University of California Berkeley.

Kumiega, A., Neururer, T., Van Vliet, B., 2011. Independent component analysis for realized volatility: analysis of the stock market crash of 2008. Quarterly Review of Economics and Finance 51 (3), 292–302.

Longstaff, F. A., Pan, J., Pedersen, L. H., Singleton, K. J., 2011. How sovereign is sovereign credit risk? American Economic Journal: Macroeconomics 3 (2), 75–103.

Mody, A., 2009. From bear stearns to anglo irish: How eurozone sovereign spreads related to financial sector vulnerability. IMF Working Paper WP/09/108, International Monetary Fund.

Nelsen, R. B., 2007. An Introduction to Copulas, 2nd Edition. Springer.

Pan, J., Singleton, K. J., 2008. Default and recovery implicit in the term structure of sovereign cds spreads. Journal of Finance 63 (5), 2345–2384.

Patton, A. J., 2006. Modelling asymmetric exchange rate dependence. International Economic Review 47 (2), 527–556.

Rachev, S. T., Menn, C., Fabozzi, F. J., 2005. Fat-tailed and Skewed Asset Return Distributions: Implications for Risk Management, Portfolio Selection, and Option Pricing. John Wiley & Sons.

Stockhammer, E., 2011. Peripheral europe's debt and german wages: the role of wage policy in the euro area. International Journal of Public Policy 7 (1), 83–96.

The International Swaps and Derivatives Association, March 2012. Isda emea determinations committee: Restructuring credit event has occurred with respect to the hellenic republic. Press release, `http://www2.isda.org/greece-sovereign-debt-faq/`.

Appendix

A.1 Details of models for comparison

The first model is the *multivariate Gaussian* model:

$$\mathbf{r}_t \sim \mathrm{N}(\mu, \Sigma), \tag{5}$$

where μ is a vector representing mean and Σ is a matrix representing covariance.

The second model is based on the theory of *copula* distribution, which is a popular statistical concept to describe the dependency structure of multiple variables. A copula function has the advantage that it can describe the nonlinear structure of dependence that would be impossible to express by linear models such as ICA, PCA or FA (Joe, 1997; Nelsen, 2007). A Gaussian copula is popular since it is easy to implement. In this model we use a Student-t copula as a dependency structure, which can describe important dependence structures between variables such as tail dependence (Demarta and McNeil, 2005). Because marginal distributions must be specified when using a copula, we use Student-t distributions for the marginals. We refer to the following model as the *Student-t marginal and copula* model:

$$\begin{cases} r_{jt} = \mu_j + \sigma_j F^{-1}_{\mathrm{tstd}(\nu_j)}(u_j), \\ \mathbf{U} = (u_1, \ldots, u_d)^\top \sim C_t(\Sigma, \nu), \end{cases}$$

where $F^{-1}_{\mathrm{tstd}(\nu_j)}(u_j)$, $j = 1, \ldots, d$, is the inverse cumulative density function of the standard Student-t distribution with the degree of freedom ν_j, and $C_t(\Sigma, \nu)$ is the Student-t copula distribution with the correlation matrix parameter Σ and the degree of freedom ν.

The third and fourth models are the EGARCH(1,1) and GARCH(1,1) model with Gaussian marginal and copula. The volatility of the third model is specified by EGARCH(1,1) given in equation (4), and the GARCH(1,1) volatility of the fourth model is specified by $(\sigma^{(s)}_{j,t+1})^2 = \omega_j + a_j s^2_{jt} + b_j \sigma^2_{jt}$. In addition, for these two models,

$$\begin{cases} r_{jt} = \mu_j + s_{jt}, \\ \varepsilon_{jt} = \Phi^{-1}(u_{jt}), \\ \mathbf{U}_t = (u_{1t}, \ldots, u_{dt})^\top \sim C_G(\Sigma), \end{cases} \tag{6}$$

where Φ^{-1} is the inverse cumulative distribution function of the standard Gaussian distribution and $C_G(\Sigma)$ denotes the Gaussian copula with correlation parameter matrix Σ. We label the third and the fourth models the *EGARCH-* and *GARCH-Gaussian marginal and copula* model.

The other models are the ICA-based models. The fifth model is the model which has three ICs ($q = 3$) in Section 3.2. These ICs are assumed to obey the EGARCH(1,1) model in equation (4), as discussed in Section 4.1. We refer this model as the *ICA-GARCH-T dimension reduction* model.

In the sixth and seventh models, we consider the ICs without dimension reduction in order to evaluate the loss of information due to the dimension reduction ($q = 3$) described in Section 4.1. In the sixth model we apply EGARCH(1,1) with Student-t residuals to each IC. We label this model the *ICA-EGARCH-T no dimension reduction* model. In the seventh model, we do not apply GARCH but apply the standardized Student-t distribution for each IC in order to check the difference between homoscedastic and heteroscedastic models for the ICs. We label this model the *ICA-T no dimension reduction* model.

A.2 ICA model likelihood

In this paper, we compare model fitness by using likelihood function. We derive a formula for the likelihood of models based on equations (3) and (4). Let $f_X(\cdot)$ denote the PDF of a random scalar or vector X, hereafter.

For simplicity, we first consider the case that the number of components q is equal to the number of the data dimension d. In this case the noise term ϵ_t in equations (3) satisfies $\epsilon_t \equiv 0$, and we obtain $\mathbf{r}_t = \mu + A\mathbf{s}_t$. Using the formula for the change of variables (Jacobian transformation) we obtain $f_{\mathbf{r}_t}(\mathbf{r}_t) = f_{\mathbf{r}_t}(A\mathbf{s}_t) = f_{\mathbf{s}_t}(\mathbf{s}_t)/|\det A|$. Then, using the factorization of he PDF of the ICs (see Section 3.1), we obtain $f_{\mathbf{s}_t}(\mathbf{s}_t) = f_{s_{1t}}(s_{1t}) \cdots f_{s_{dt}}(s_{dt})$. For the unconditional S_j, the distribution of $\mathbf{r}_t$ is

$$f(\mathbf{r}_1, \ldots, \mathbf{r}_T) = \frac{1}{|\det A|^T} \prod_{j=1}^{d} \prod_{t=1}^{T} f_{Sj}(s_{jt}). \tag{7}$$

For the GARCH-type conditional distribution in equation (4), $f_{s_{j1}, s_{j2}, \ldots, s_{jT}}$ is factorized as $f_j(s_{j1}, s_{j2}, \ldots, s_{jT}) = f_{j1}(s_{j1}) f_{j2|1}(s_{j2}) \cdots f_{jT|T-1}(s_{jT})$ where $f_{jt|t-1}(s_{jt})$ is the PDF of s_{jt} conditional to the information up to time $t-1$, $f_{jt|t-1}(s_{jt}) = f_\varepsilon(\varepsilon_{jt})\sigma_{jt}$, and $f_\varepsilon = f_\varepsilon(\varepsilon)$ is the PDF of the residuals ε independent of t. Therefore, we can derive the PDF as

$$f(\mathbf{r}_1, \ldots, \mathbf{r}_T) = \frac{1}{|\det A|^T} \prod_{j=1}^{d} \prod_{t=1}^{T} \frac{f_\varepsilon(\varepsilon_{jt})}{\sigma_{jt}}. \tag{8}$$

Equations (7) and (8) are the formula for computing the likelihood of an ICA-based model. Together with these equations, we compute the likelihood of the data, allowing us to compare the model with other models that we review in Section 4.4.

For the general case $q < d$, we adopt two assumptions. First, factors $\mathbf{s}_t$ and noises ϵ_t are independent of each other. Second, the noise term ϵ_t is a linear combination of $(d-q)$ independent noise sources $\mathbf{p^b}_t = (p_{q+1,t}, \ldots, p_{dt})^\top$. Let U denote the matrix diagonalizing $\mathrm{corr}(\mathbf{r}_t)$ in PCA and U_a and U_b are the left q and right $(d-q)$ columns of U, and we assume $\epsilon_t = D_r^{1/2} U_b \mathbf{p^b}_t$ for noise term ϵ_t. Then we obtain the following formula for the

PDF of equation (3):

$$f_{\mathbf{r}_t}(\mathbf{r}_t) = \frac{1}{|\det D_r^{1/2} \det A_{\mathrm{IC}}|} \prod_{j=1}^{q} f_{S_{jt}}(s_{jt}) \prod_{j=q+1}^{d} f_{p_{jt}}(p_{jt}), \tag{9}$$

where $A_{\mathrm{IC}} = (D_r^{1/2} U_a)^+ A$.