



# INSPIRING THE NEXT GENERATION

CONFERENCE PROCEEDINGS

The 7<sup>th</sup> International Conference on Spatial Structures  
and the Annual Symposium of the IASS

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UNIVERSITY OF  
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## **Table of Contents**

Foreword.....	ii
Acknowledgements.....	iii
Summary of Sessions.....	iv
Contents.....	vi
Papers.....	1
Authors Index.....	3546

## **Foreword:**

The present proceedings contain a collection of 330 papers on various aspects of analysis, design and construction of spatial structures such as gridshells, barrel vaults, domes, towers, retractable systems and tension structures. These papers were written by 763 authors, representing a total of 44 countries for presentation at the seventh International Conference on Spatial Structures.

The International Conference on Spatial Structures has been organised and held on five previous occasions by the Spatial Structures Research Centre of the University of Surrey in 1966, 1975, 1984, 1993 and 2002, and in 2011 in collaboration with the International Association for Bridge and Structural Engineering (IABSE) and International Association for Shell and Spatial Structures (IASS).

The seventh conference was hosted by the University of Surrey from the UK during 23rd – 27th August 2021; it was combined with the 2020 annual symposium of the IASS. The conference was named IASS 2020/21 – Surrey 7 and its strapline was “Inspiring the next generation”.

The planning and delivery of the conference had a long history: beginning in January 2016, and latterly very heavily affected by the worldwide Covid 19 pandemic emerging during late 2019/early 2020. The original planned dates were 24th – 28th August 2020, but as the impact of the pandemic became more profound, the mode of delivery evolved from fully in-person, through hybrid in-person/virtual to eventually becoming fully virtual throughout the revised dates during 23rd – 27th August 2021.

The Scientific Committee was formed in May 2018 and had a membership of a hundred and twenty one leading International Engineers and Architects, from both industry and academia; representing over thirty countries. After a worldwide call for abstracts, the Scientific Committee received 468 submissions involving 1061 authors and subsequently each abstract was reviewed at least twice by members of the Committee. Feedback on the abstracts was presented to almost all of the authors and afterwards, three hundred and thirty nine full papers were received. The full papers were, in turn, reviewed by members of the Scientific Committee, who generously gave their time to provide feedback to the corresponding authors. This proceedings include contributions submitted directly by the authors and the editors cannot accept responsibility for any inaccuracies, comments and opinions contained in the text.

The editors would like to take the opportunity to thank all authors for submitting their contributions, the Scientific Committee for reviewing the abstracts and full papers and the Organising Committee for their countless effort in making the conference a success.

**Alireza Behnejad, Gerard Parke and Omidali Samavati**

**University of Surrey, Guildford, UK, August 2021**

## Summary of Sessions (alphabetical order):

21st century fabrication and construction of shells (IASS WG 5).....	1
Active bending.....	92
Advanced manufacturing (3D Printing, Digital fabrication and etc.).....	165
Bamboo structures.....	363
Bio-based concepts inspiring the spatial structures and architecture of the next generation (IASS WG 12).....	395
Bridge structures.....	668
Computational form-finding methods.....	709
Conceptual design.....	821
Conservation and preservation of 20th century historic concrete shells (IASS WG 5).....	1001
Damping control and seismicity.....	1072
Deployable and foldable structures.....	1120
Design and realisation case studies: Lattice structures.....	1199
Design and realisation case studies: Membrane structures.....	1290
Dynamic response of metal spatial structures (IASS WG 8).....	1330
Formian in the design activity of architects and engineers.....	1514
Graphic statics.....	1567
High-performance membrane buildings and challenges.....	1691
Historical structures.....	1776
Inflatable structures.....	1807
Innovative engineering.....	1820
Life-cycle design and assessment of structures (IASS WG 18).....	1859
Metal gridshell structures, connections, and stability (IASS WG 8).....	1898
Morphology and configuration processing.....	2084
Next generation parametric design (WG13).....	2181
Novel micro and macro analysis and construction techniques.....	2274
Optimisation.....	2378
Optimisation methods for analysis and design of roof structures (IASS WG 13).....	2456
Origami.....	2658
Shell structures.....	2766
Symmetry and spatial structures.....	2818
Tactile strategies for teaching spatial structures (IASS WG 20).....	2854
Tensegrity systems.....	2942

<b>Tension and membrane structures: Recent developments</b>	
<b>in and around the United Kingdom.....</b>	<b>3001</b>
<b>The future of structural design - symbiosis in architecture and structural engineering.....</b>	<b>3043</b>
<b>The Milgo experiment.....</b>	<b>3149</b>
<b>Transformable structures (IASS WG 15).....</b>	<b>3203</b>
<b>WG21: Design competition.....</b>	<b>3302</b>
<b>Wind engineering and CFD simulation.....</b>	<b>3402</b>
<b>Hangai prize lectures.....</b>	<b>3436</b>

## Contents:

### 21st century fabrication and construction of shells (IASS WG 5)

A mechanism for the deployment of Kinetic Umbrellas for coastal hazard adaptation.....1	
<i>Shengzhe WANG, Maria GARLOCK, Branko GLISIC</i>	
Some experiments on flexible formworks for shell structures.....14	
<i>Murilo S.O. SOTO, Ruy M.O. PAULETTI, Leila C. MENEGHETTI</i>	
A path towards off-site automated fabrication of segmented concrete shells as building floors.....26	
<i>Robin OVAL, Eduardo COSTA, Mishael NUH, Diana THOMAS-McEWEN, John ORR, Paul SHEPHERD</i>	
What future for reinforced concrete shell construction in the UK in the 21st century? .....37	
<i>John CHILTON</i>	
Computational Design Strategies for Thin-tile Shells Manufacturing.....45	
<i>M. Wesam Al Asali, Antiopi Koronaki, Michael H. Ramage</i>	
Dual-Purpose Concrete Shells.....53	
<i>Alphose ZINGONI</i>	
Thin shell foundations: Historical review and future opportunities.....67	
<i>Kiley FEICKERT, Caitlin T. MUELLER</i>	
Transverse structural behaviour of doubly curved beam-like shells.....81	
<i>Max DOMBROWSKI, Paul MERZ, Juan P. OSMAN-LETELIER, Mike SCHLAICH</i>	

### Active bending

Pattern Recognition and Prediction of Bending-active Thin Sheets via Artificial Neural Networks.....92	
<i>Xinjie ZHOU, Xiang WANG, Dan LUO, Philip F. YUAN</i>	
Erection and shape control of a hybrid bending-active gridshell through cable activation.....102	
<i>Ioanna ANASTASIADOU, Marios C. PHOCAS</i>	
Hybrid formation: tension-based assembly system for bending-active plate structures.....112	
<i>Niloofar IMANI, Axel KÖRNER, Riccardo LA MAGNA, Jan KNIPPERS</i>	
Hybrid bending active systems: a novel application of carbon fiber in lightweight structures.....123	
<i>Maged S. GUERGUIS, Alex STILES, John-Michael WORSHAM</i>	
Grid shell sphere by the active bending weaving structure.....135	
<i>Weixin Huang, Tepyuthyea Sophoan, Hsin-Lun Li, Ziniu Luo, Zi'an Wang, Jingyuan Hu</i>	
Lightweight curved surface system composed of bending-active plate units: A study of materials and composition.....142	
<i>Yi-Hsuan TU, Tsung-Nan LIN</i>	

Design for manufacturing of a digital hollow section beam using interlocking connection.....	154
<i>Seyed Mobin MOUSSAVI, Felix KALLWIES, Arnold WALZ, Philipp RUMPF, Stefan BÖHM, Niklas SOMMER, Stephan VÖLKERS, Julian LIENHARD</i>	
<b>Advanced manufacturing (3D Printing, Digital fabrication and etc.)</b>	
Achieving elegant and efficient geometry in architectural frames with casting technology and advanced manufacturing: day's end project case study.....	165
<i>Justin BINDER, Jennifer PAZDON, Richard FETUGA, Carlos DE OLIVEIRA, Bolivar VILLAMARIN</i>	
HexBox Canopy: a segmented timber plate shell with hardwood wedge joints.....	176
<i>Christopher ROBELLER, Eduardo DE OLIVEIRA BARATA, Enrico Valentino TAGLIABOSCHI, Felix SCHMIDT-KLEESPIES</i>	
New calculation approach for selecting and orienting the reinforcing material for robotic concrete manufacturing.....	188
<i>Abtin BAGHDADI, Robin DOERRIE, Harald KLOFT</i>	
Adaptive cross-panel system: digital fabrication for freeform architecture.....	199
<i>Ruben AGOSTINO, Sergio PONE</i>	
Robotic construction of a self-balancing glass masonry vault: design and tessellation.....	211
<i>Alessandro BEGHINI, Masaaki MIKI, Samantha WALKER, Isla Xi HAN, Sigrid ADRIAENSSENS, Stefana PARASCHO</i>	
Towards a multifunctional 3DCP slab system.....	219
<i>Saqib AZIZ, Diego APELLÁNIZ, Max WEBER, Giovanni BETTI, Christoph GENGNAGEL</i>	
Rule-based generative design of translational and rotational interlocking assemblies.....	231
<i>Pierre GILIBERT, Olivier BAVEREL, Romain MESNIL</i>	
Tailored structures – parametrics for sustainable constructions.....	243
<i>Markus SCHEIN, Max Benjamin ESCHENBACH, Asko FROMM</i>	
Constructive design of double curved shells for 3D concrete printing.....	255
<i>Zlata TOŠIĆ, Martin Friedrich EICHENAUER, Egor IVANIUK, Daniel LORDICK, Sonja KRASIĆ, Viktor MECHTCHERINE</i>	
Evolution of design integration with digital fabrication.....	264
<i>Joseph BURNS</i>	
Basic research on stress deformation analysis and buckling analysis of thin chemically tempered bent glass.....	269
<i>Midori YUSA, Yutaka MISAWA, Takeshi ASAKAWA, Masamichi SASATANI</i>	
Repair it with additive manufacturing: an approach to the circular economy in architectural education....	278
<i>Dana SAEZ, Kevin MORENO GATA, Martin TRAUTZ</i>	

Rolled-up paper spatial structure.....	285
<i>Toshiaki KIMURA, Yosuke KOMIYAMA, Hiroshi TAKEYAMA, Hiroki TAKEUCHI, Yuto ISOMURA</i>	
Robotic 3D printing with earthen materials as a novel sustainable construction method.....	292
<i>Yelda GIN, Burcu Biçer SANER, Michael H. RAMAGE</i>	
Hybrid fabrication of discrete topology optimized structures using 3D printing.....	302
<i>Mauricio MORALES-BELTRAN, Kaan ÇETIN, Berk SELAMOĞLU, Halis Arda ÖZDEMİR, Fulya ÖZBEY</i>	
Robotic construction of a self-balancing glass masonry vault: DEM study of stability during the construction stages.....	314
<i>Vittorio PARIS, Nicola LEPORE, Edvard P.G. BRUUN, Giuseppe RUSCICA, Mario Daniele PICCIONI, Alessandro BEGHINI, Stefana PARASCHO, Sigrid ADRIAENSSENS</i>	
All glass, compression-dominant polyhedral bridge prototype: form-finding and fabrication.....	326
<i>Yao LU, Matthew CREGAN, Philipp A CHHADEH, Alireza SEYEDAHMADIAN, Mohammad BOLHASSANI, Jens SCHNEIDER, Joseph R YOST, Masoud AKBARZADEH</i>	
Scaffold-free robotic 3D printing of a double-layer clay shell.....	337
<i>Mahan MOTAMEDİ, Romain MESNIL, Robin OVAL, Malo CHARIER, Olivier BAVEREL</i>	
Multi-objective optimization of 3D printed shell toolpaths.....	350
<i>Alexander CURTH, Tim BRODESSER, Lawrence SASS, Caitlin MUELLER</i>	

## **Bamboo structures**

Creation of a pavilion with bamboo as the main material.....	363
<i>Romuald SAUVERVALD, François ZANINETTI, Marie SERANTONI, Justin PONTHENIER, Elora DESREUMAUX</i>	
Parametric simulation of bamboo structures in Mexico.....	371
<i>Edwin GONZALEZ, M. Guadalupe ESTRADA, Giancarlo DI MARCO</i>	
Development of an active bending temporary corridor based on bamboo strips: preliminary results.....	383
<i>Qiu ZHANG, Yiyi WANG, Haomin WANG, Cristoforo DEMARTINO, Francesco MARMO, Zhi LI, Yan XIAO</i>	

## **Bio-based concepts inspiring the spatial structures and architecture of the next generation (IASS WG 12)**

Timber elements with graded performances through digital forest to timber workflows.....	395
<i>Martin TAMKE, Tom SVILANS, Sebastian GATZ, Mette RAMSGAARD THOMSEN</i>	
Proposal of clamped joint for connecting wooden members and study of their application to spatial structures.....	408
<i>Yoshisato ESAKA, Tatsuki ISHIZAKI, Tokuma AZEMI</i>	
The Paper Brick Vault.....	419
<i>Hélène BARBIER, Théo FERRIEUX, Baptiste LACH, Kenza MAACHI, Mossaab SJAALI, Margaux</i>	

Biobased textile reinforced concrete for the design of thin shells.....	431
<i>Giuliana SCUDERI, Liam FLANAGAN</i>	
Geodesic winding of minimal surfaces: the architecture, structure and geometry of an irregular grid-shell pavilion assembled from thin plywood strips.....	444
<i>Günther H. FILZ</i>	
Structural design of built-up timber beams with catenary curve shape.....	456
<i>Takaaki UDAGAWA, Satoru NAGASE, Kenji TODA</i>	
Comparative study of bracing patterns and materials for tall timber buildings.....	465
<i>Alexandra KAWAR, Gordana M. HERNING</i>	
Bending active plates with tensile elements: A biomimetic approach inspired by the Australian curly leaf spider ( <i>Phonognatha graeffae</i> ).....	475
<i>Cody TUCKER, Xiliu YANG, August LEHRECKE, Lior SKOURY, Manuela AUST, Friedrich HOLTSMANN, Axel KÖRNERb, Oliver BETZ</i>	
Made by material FRUSTRATION: generation of complex morphologies through controlled geometrical incompatibilities.....	487
<i>Arielle BLONDER, Eran SHARON</i>	
The Baya Nest pavilion project: braided pattern optimization for hanging shell structures by dynamic relaxation.....	500
<i>Marc LEYRAL, Quentin CHEF, Sylvain EBODE, Pierre GUEROLD</i>	
Flexible and sustainable building components through kerf patterns.....	514
<i>Ana GATÓO, Antiopi KORONAKI, M. Wesam AL ASALI, Aurimas BUKAUSKAS, Yelda GIN, Darshil U. SHAH, Eduardo WIEGAND, Michael H. RAMAGE</i>	
Utilising waste wood through reciprocal frame systems.....	524
<i>Xan BOWNE, Olga Popovic LARSEN, Caio CASTRIOTTO</i>	
An integrated architectural and structural design concept by using local, salvaged timber.....	536
<i>Gengmu RUAN, Günther H. FILZ, Gerhard FINK</i>	
Resilient adaptive structures of multifocal asymmetric shells.....	546
<i>Dmitri KOZLOV</i>	
Analyzing a fungal mycelium and chipped wood composite for use in construction.....	555
<i>Dana SAEZ, Denis GRIZMANN, Martin TRAUTZ, Anett WERNER</i>	
Re-use of formwork for a villa in Amsterdam.....	566
<i>Dr. ir. A.D.C. PRONK, R.R.L. van LOON, K.C.P. SANDERS</i>	
Material tests and execution methods of linear composite pykrete structural elements.....	578
<i>Dr. ir. A.D.C. PRONK, M.B.G. DE BOERA, J.M. HOEF, P.M.L.M. OTTERSPOOR, L.M.J. PIELOOR, O.D. VENS</i>	

Structural design and validation of a pykrete tower through form finding.....	590
<i>Dr. ir. A.D.C. PRONK, M.B.G. DE BOERA, J.M. VAN DE HOEF, P.M.L.M. OTTERSPOOR, L.M.J. PIELOOR, O.D. VENS</i>	
Leaf venation topologies for a ribbed floor system.....	598
<i>Daniel REYNOLDS, Naomi FRANGOS, Eugene LEE, Gustav FAGERSTRÖM</i>	
Research in temporary, biodegradable, thin-shell, cast paper structures at full scale.....	611
<i>Stephanie DAVIDSON</i>	
Load tests on ribbed timber shells for medium span roofs.....	618
<i>Alexander STAHR, Christian HEIDENREICH, André KILIAN, Cristoph DIJOUX, Lukas FRANKE, Ryan HALLAHAN</i>	
Design space exploration of materially efficient CLT structures.....	628
<i>Antiopi KORONAKI, Aurimas BUKAUSKAS, Ana GATÓO, Darshil U. SHAH, Michael H. RAMAGE</i>	
Three-dimensional fibre placement in wood for connections and reinforcements in timber structures.....	637
<i>Hans Jakob WAGNER, Dominga GARUFI, Tobias SCHWINN, Dylan Marx WOOD, Achim MENGES</i>	
Long-span timber gridshell design and analysis: the Taiyuan Domes.....	649
<i>Brandon SULLIVAN, Lucas EPP, Gerald EPP</i>	
Pragmatic design and fabrication of elastic timber gridshell dwellings.....	659
<i>Robin M. HARRISON, Mohammad BOLHASSANI</i>	
<b>Bridge structures</b>	
Saving a space structure bridge in Mexico.....	668
<i>Alejandro CALDERON-LANDAVERDE, Alejandro CALDERON-OLLIVER, Javier GONZALEZ-CANTU</i>	
The Tide – Steel monocoque interdisciplinary digital workflow and parametric structural analysis.....	679
<i>Wai PANG</i>	
Structural design iterations of tied arch shell bridges for Toronto’s waterfront revitalization.....	691
<i>Matthias PELTZ, Michael STEIN, Mathias NIER, Juan PORRAL, Michael MESCHINO</i>	
Physical modelling and design development of precast UHPC shell bridge.....	700
<i>Ahmet TOPBAS, Tarik Ateser, F. Omur TULEN, Muhammed MARASLI, Beni KOHEN</i>	
<b>Computational form-finding methods</b>	
A simulation process for implementation of knitted textiles in developing architectural tension structures.....	709
<i>Farzaneh OGHAZIAN, Paniz FARROKHSIAR, Felecia DAVIS</i>	
Applications of states of self-stress to the design of gridshells.....	723
<i>Alessandro BEGHINI, Max COOPER, Cameron MILLAR, William BAKER</i>	

Isogeometric finite elements for form finding and structural analysis using particle-spring methods within Kangaroo 2.....	732
<i>Laurin SCHÄFER, Anna M. BAUER, Philipp LÄNGST, Julian LIENHARD, Alexander MICHALSKI</i>	
A novel approach to the form finding of shells based on membrane theory.....	744
<i>F. MARMO, D. DI LIETO, S. GABRIELE, S. ADRIAENSSENS, M. PARADISO, L. ROSATI</i>	
Predicting the structural behavior of geometrical nonlinearity of shell structures by machine learning.....	756
<i>Yoshiya Moritomo, Shinnosuke Fujita</i>	
Computational hanging upside-down simulation to design axial-force-resistant stacked dome and tower structures.....	764
<i>Hiroyuki TAGAWA, Rino TAKEUCHI</i>	
The construction of new masonry bridges inspired by Paul Séjourné.....	772
<i>Emil ADIELS, Chris J. K. WILLIAMS</i>	
Towards trans-topological design exploration of reticulated equilibrium shell structures with graph convolution networks.....	784
<i>Kam-Ming Mark TAM, Vahid MOOSAVI, Tom VAN MELE, Philippe BLOCK</i>	
Deep Learning Algorithm to Speed up the process of Chebyshev Gridshell Structural Optimization.....	797
<i>Dai KANDIL, Enrique SORIANO, Dionís BOIXADER</i>	
Advanced form-finding with design constraints and objectives through constraint projection.....	809
<i>Kenryo TAKAHASHI</i>	
 <b>Conceptual design</b>	
Gridshell structures with discrete curvature lines:Modelling technique and evaluation of mechanical performance.....	821
<i>Yohei YOKOSUKA, Junichi INOBUCHI, Makoto OHSAKI, Toshio HONMA</i>	
An introduction to spatial structures for the practicing structural engineer – with case study.....	834
<i>Michael O'SHEA</i>	
Alternative conceptual design for hypar large-scale roof.....	846
<i>Odine MANFRONI, Lorenzo PEDERZOLI, Stefano SILVESTRI</i>	
Structural patterns in architecture.....	858
<i>Milica D. PETROVIĆ, Isidora D. ILIĆ</i>	
Exploring forms of masonry vaults built without centering .....	865
<i>Babita NEUPANE, Rui LIU</i>	
Re-defining arch structures through form-finding.....	879
<i>Wanda J. LEWIS</i>	

Performance of structural grids for tall buildings based on geometric Islamic patterns.....	890
<i>Carlos MERI-CELMA, David GALLARDO, Carlos LÁZARO</i>	
Conceptual design of three-dimensional structural and sun-shading façades supported by machine-learning.....	902
<i>Federico BERTAGNA, Pierluigi D'ACUNTO, Patrick Ole OHLBROCK</i>	
An application to generation of spatial structure using projection figures of rotated regular polyhedron based on group theory.....	913
<i>Toshifumi MAE, Yukihiro FUJIMOTO, Yuji MATSUMOTO</i>	
Grammar-based generation of bar networks in static equilibrium with bounded bar lengths.....	924
<i>Ioannis MIRTISOPOULOS, Corentin FIVET</i>	
Flow-informed topology design: Evaluating the conformity of structural topologies with vector fields....	936
<i>Demi FANG, Caitlin MUELLER</i>	
Experimental investigation on prestressed thin-walled concrete folded slabs with CFRP textiles .....	948
<i>María SERRANO-MESA, Juan P. OSMAN-LETELIER, Alex HÜCKLER, Mike SCHLAICH</i>	
Structure and fabrication-driven conceptual design of space-frame structures.....	958
<i>Antiopi KORONAKI, Paul SHEPHERD, Mark EVERNDEN</i>	
On the importance of global stability for discrete gridshells created by the Thrust Network Analysis form-finding method.....	966
<i>Sverre Magnus HAAKONSEN, June-Marie J. ESJEHOLM, Daniel May INSTANES, Steinar Hillersøy DYVIK, Anders RØNNQUIST</i>	
Postdigital ornamented architecture is not ornamented.....	976
<i>Ghali BOUAYAD</i>	
Generative structural design for embodied carbon estimation.....	989
<i>Ramon Elias WEBER, Caitlin MUELLER, Christoph REINHART</i>	
 <b>Conservation and preservation of 20th century historic concrete shells (IASS WG 5)</b>	
The High Life Factory: a 20th century historic concrete shell at risk of loss.....	1001
<i>Marisela MENDOZA, Juan Ignacio DEL CUETO RUIZ-FUNES</i>	
Morphological analysis of architectural work by F. Candela: the form and aperture shape of the HP shell structure.....	1009
<i>Masafumi TANAKA, Sumika INOUE</i>	
Smithfield Market: assessment of the poultry market roof.....	1016
<i>Christian TYGOER, James J. KINGMAN</i>	
Shrouded by time and tile: a structural investigation for preservation of Cuba's historic School of Ballet classroom and theatre domes.....	1029

*Moriah HUGHES, Camille HEUBNER, Sofia CELLI, Maria GARLOCK, Federica OTTONI, Davide DEL CURTO, Branko GLIŠIĆ*

A proposal for the structural preservation of Musmeci's Basento Bridge in Potenza.....1041

*Giulia BOLLER, Pierluigi D'ACUNTO, Lukas INGOLD, Aurelio MUTTONI, Joseph SCHWARTZ*

Saving the Green Lake Pool structure: The preservation discussion of Jack Christiansen's first shell project.....1051

*Tyler SPRAGUE*

Two replicas of Candela's Los Manantiales in Ecuador.....1059

*Mauricio LUZURIAGA*

## **Damping control and seismicity**

Experimental testing on semi-active vibration control through adaptive structural joints.....1072

*Qinyu WANG, Gennaro SENATORE, Kaspar JANSEN, Arjan HABRAKEN, Patrick TEUFFEL*

Design method and application of toggle brace viscous dampers in building renovation.....1085

*Daohang HU, Xin ZHAO*

Performance analysis and standardized design of base isolated modular steel structure in high seismicity area.....1094

*Xin ZHAO, Gang WANG*

Double damping knuckle damping system for high - rise steel structure.....1104

*Xin ZHAO, Huaikun CHEN*

Multi-level bi-directional evolutionary optimization for seismic design of SRC columns in super tall structure.....1112

*Jinlun CAI, Xin ZHAO*

## **Deployable and foldable structures**

Experimental characterisation of the nonlinear dynamics of bistable composite shell structures.....1120

*Christopher WILLETT, Robert DOREY, Andrew VIQUERAT*

Structural parameterization of Kinetic Umbrellas under hydrostatic inundation.....1132

*Shengzhe WANG, Vanessa NOTARIO, Maria GARLOCK, Branko GLISIC*

Numerical characterization and optimization of the anisotropy of foldcores under shear loading.....1145

*Fabian MUHS, Peter MIDDENDORF*

Design of operable petal roofs: The case of Mercedes-Benz Stadium.....1158

*Sudarshan KRISHNAN*

Numerical analysis of imperfections in Miura Ori sandwich cores using isogeometric analysis.....1164

*Simon THISSEN, Peter MIDDENDORF*

Procedure for designing and metrating three-dimensional structures based on foldings.....1175

*Alfredo Eulogio MUJICA YEPEZ*

Modeling and analysis of tripod-scissor deployable structures using mirrored assembly methods.....1187

*Yuan LIAO*

## **Design and realisation case studies: Lattice structures**

The Mero legacy – inspiring space structures design.....1199

*Jaime SANCHEZ-ALVAREZ, Herbert KLIMKE*

Geometry and stability: design and construction of a 115m span freeform roof in Kuala Lumpur.....1211

*Catherine POIRRIEZ, Yacine BOUZIDA*

Design and construction of a 300m long steel and ETFE roof in Malaysia.....1221

*Catherine POIRRIEZ, Yacine BOUZIDA*

Long-span free-form domes.....1229

*Alejandro ALGARA, Francisco CASTAÑO*

Design and construction of a large-span aluminum alloy free-form latticed shell located on 11 high-rise buildings.....1238

*Zhiqiang LI, Xiaowei LIU, Yuanwen OUYANG, Jian YIN, Zhenggen BI, Jian ZHOU, Qilin ZHANG*

Sliding construction methods and past works of Tomoe Corporation.....1251

*Yoshihiko KUROIWA, Hidekazu KONISHI*

Design for production - entrance canopies for the multi-functional building of the Lakhta Center in St. Petersburg.....1263

*Jaime SANCHEZ-ALVAREZ, Christian WOLKOWICZ, Kamil SCHWARNOWSKI, Mark FAHLBUSCH*

Yokkaichi City Gymnasium: structural design corresponding to architectural design.....1272

*Masari KAWAI, Tatsuya HANADA*

Case study: Roof truss structure with large cut out and elliptic glazing surface.....1280

*Zsolt NAGY, Zoltán KISS, Andrea KELEMEN, Károly BÁLINT, Annabella SÁNDULY*

## **Design and realisation case studies: Membrane structures**

Membrane roof structures of New Tochigi Stadium using PTFE membrane of different thickness.....1290

*EulSeok JEONG, Chikara KONISHI, Naonori HISHINUMA, Yuichi HASHIMOTO, Tomoya KOSHIO, Chikamasa OKUNO*

New challenges of membrane roof structures using ETFE film and PTFE membrane on Haneda Airport Passenger Terminal 2.....1301

<i>EulSeok JEONG, Chikara KONISHI, Naonori HISHINUMA, Yuichi HASHIMOTO, Ryunosuke INOUE, Mari USUI</i>	
Damping performance of arch structure with super elastic alloy.....	1310
<i>Ken'ichi MINOWA, Kaito SHIRASAKI, Atsuyuki NAKAYA</i>	
Refurbishment of the main grandstand and roof of Sepang International Circuit Malaysia.....	1318
<i>Migico SING, Kok Keong CHOONG, Azri Hariz CHE MALID, Chong Kiat NG, Tamrin HAMZAH, Diah TALIB</i>	
 <b>Dynamic response of metal spatial structures (IASS WG 8)</b>	
Elasto-plastic seismic response behavior and evaluation of permissible deformation of single layer lattice domes designed under various loads.....	1330
<i>Tomohiko KUMAGAI, Toshiyuki OGAWA</i>	
Optimal design of a large span arena with buckling restrained braces using generalized response spectrum analysis.....	1341
<i>Yuki TERAZAWA, Miho FUJISHIMA, Toru TAKEUCHI</i>	
Structural design of sports arena with vibration control mechanism.....	1353
<i>Yoshikazu SUGIURA, Yasuyoshi HITOMI, Daisuke NISHIKAWA</i>	
Seismic behavior factor of single-layer barrel vaults.....	1364
<i>Karim ABEDI, Reza KHALILI, Mehdi POURSHA</i>	
Seismic performance of building steel structures using equivalent linearization method.....	1372
<i>Masanori FUJITA, Shoji NAKAZAWA, Shiro KATO</i>	
Knowledge processing for determining seismic load of free-form reticulated shell structures based on two dominant vibrational modes.....	1383
<i>Yuji TAKIUCHI, Shoji NAKAZAWA, Yuki HIGASHIYAMA, Shiro KATO</i>	
Dynamic response research of aluminum foam-filled 6082-T6 aluminum circular tube under lateral impact.....	1393
<i>Ximei ZHAI, Lingzhao MENG, Zhaohui ZHOU</i>	
Study on equivalent static seismic force and seismic performance of arch structures.....	1404
<i>Shoji NAKAZAWA, Yuji TAKIUCHI, Naoki WAKAYAMA, Yoshiki TAKASHIMA, Naoya HIGUCHI, Shiro KATO</i>	
Influence of roof-substructure interaction on the seismic response of double-layered long-span domes....	1416
<i>Deepshikha NAIR, Toru TAKEUCHI</i>	
Spectrum-based mode selection method for spatial latticed structures.....	1426
<i>Qinglong HUANG, Jie BAI, Ning PAN, Yongfeng LUO, Nianduo WU</i>	
Nonlinear static analysis procedures for seismic response evaluation of single-layer latticed shells based on the equivalent modal stiffness.....	1436
<i>Qu YANG, Liu ZHIYONG, Ma HUAIZHANG, Chen BO, Li JIAPENG, Luo YONGFENG</i>	

A new seismic damage assessment method for single-layer steel latticed shells considering multi-modal combination.....	1449
<i>Yujian ZHANG, Yongfeng LUO, Qinglong HUANG, Yang QU</i>	
Dynamic failure analysis and simplified optimization for member sections of single-layer reticulated domes under severe earthquakes.....	1459
<i>Ming ZHANG, Junjie ZHANG, Xin XIE, Alireza BEHNEJAD, Gerry PARKE</i>	
Shake-table test of a partial model of a roller-supported steel roof, Part 1: Preliminary test with steel column.....	1471
<i>Akiko KISHIDA, Jun FUJIWARA, Yuki TERAZAWA, Toru TAKEUCHI, Tetsuo YAMASHITA, Yoshinao KONISHI, Ryunosuke KISHIZAWA, Koki NISHIKAWA</i>	
Shake-table test of a partial model of a roller-supported steel roof, part 2: Test for reproducing earthquake damage.....	1480
<i>Jun FUJIWARA, Akiko KISHIDA, Yuki TERAZAWA, Toru TAKEUCHI, Koshiro NISHIMURA, Tetsuo YAMASHITA, Yoshinao KONISHI, Ryunosuke KISHIZAWA, Koki NISHIKAWA, Ryota KAJIWARA</i>	
Responsive systems in seismic computational design.....	1089
<i>Tim FISCHER, Salvatore VISCUSO, Alessandra ZANELLI</i>	
Sensitivity analysis and optimal design for vibration mitigation of large span pedestrian bridge.....	1502
<i>Chenyun ZHANG, Yong HUANG, Xin ZHAO</i>	

## **Formian in the design activity of architects and engineers**

Formian in engineering and architectonic design and education.....	1514
<i>Janusz REBIELAK</i>	
Design and construction of reciprocal frames made of bamboo with connections by means of PVC pipe and rope.....	1523
<i>Koichiro ISHIKAWA, Sovannara SRUN, Sokol PHON</i>	
Connection orientation calculation in single-layer lattice space structures using formex algebra.....	1534
<i>Mohammad Reza CHENAGHLOU, Karim ABEDI, Hadi ESMAILNEJAD</i>	
Computational form exploration of branching columns using concepts of formex algebra and the ParaGen method.....	1546
<i>Peter VON BUELOW, Anahita KHODADADI</i>	
Data Generation for Prefabrication of Single Layer Spatial Structures.....	1557
<i>Hoshyar NOOSHIN, Omidali SAMAVATI, S. Alireza BEHNEJAD, Gerry PARKE</i>	

## **Graphic statics**

A note on tension-compression mixed membrane shell form-finding.....	1567
<i>Masaaki MIKI, Toby MITCHELL, William BAKER</i>	

Simple rankine gridshells.....	1579
<i>Allan McROBIE, Marina KONSTANTATOU, Georgios ATHANASOPOULOS, Giancarlo TORPIANO, Cameron MILLAR, William BAKER</i>	
Lightweight structures and the geometric equilibrium in dragonfly wings.....	1592
<i>Hao ZHENG, Marton HABLICSEK, Masoud AKBARZADEH</i>	
Saving appearances: treatment of anomalies in the projective geometry-graphic statics analogy.....	1604
<i>Thomas E. BOOTHBY, Annalisa CRANNELL, Nathan C. BROWN</i>	
On funicular gridshells and Airy stress functions.....	1616
<i>Cameron MILLAR, Toby MITCHELL, Arek MAZUREK, Ashpica CHHABRA, Alessandro BEGHINI, Allan McROBIE, William F. BAKER</i>	
Application of kinematics methods in the full 3D graphic-statics description for frames.....	1628
<i>Georgios-Spyridon ATHANASOPOULOS, Allan McROBIE</i>	
The force as a function: Towards analytical graphic statics for spatial structures.....	1637
<i>Tamás BARANYAI</i>	
On the large displacements of infinitesimal mechanisms using graphic statics.....	1647
<i>Cameron MILLAR, Allan McROBIE, William F. BAKER</i>	
Designing the geometry of auxetic materials using graphic statics.....	1658
<i>Márton HABLICSEK, Masoud AKBARZADEH</i>	
A waste-based structure for an emergency shelter through materialization of 3D graphic statics.....	1671
<i>Amirhossein AHMADNIA, Salvatore VISCUSO, Hamed BEHMANESH, Alessandra ZANELLI</i>	
Developing a polyhedral graphic statics formulation for tetrahedral truss analysis.....	1681
<i>Salma MOZAFFARI, Márton HABLICSEK, Masoud AKBARZADEH, Thomas VOGEL</i>	
<b>High-performance membrane buildings and challenges</b>	
Uniaxial central tearing behaviors of PVC coated fabric.....	1691
<i>Xubo ZHANG, Minger WU, Han BAO</i>	
Hysteretic behaviour in FM-FRP architected material.....	1702
<i>Arielle BLONDER, Maurizio BROCATO</i>	
Advantages of isogeometric B-Rep analysis for the parametric design of lightweight structures.....	1709
<i>Ann-Kathrin GOLDBACH, Anna M. BAUER, Kai-Uwe BLETZINGER</i>	
Sensitivity assessment of PVC coated PET woven fabrics under different weathering impacts.....	1717
<i>Hastia ASADI, Joerg UHLEMANN, Natalie STRANGHOENER, Mathias ULBRICHT</i>	
Degradation of PVDF-coated fabrics after engineering applications with emphasis on surface microstructure, mechanical properties and structural reliability index.....	1730

*Yingying SHANG, Bin YANG, Minger WU, Youji TAO, Jiayang QIN*

Measuring the aero-elastic movement of fabric structures: An experimental approach .....1742

*Arnaud DE COSTER, Maarten VAN CRAENENBROECK, Tine TYSMANS, Marijke MOLLAERT, Lars DE LAET*

Wrinkle-crease interaction in laminated membrane: effect of thermal loading.....1754

*Parth K. KAMALIYA, S H UPADHYAY, H.M.Y.C. MALLIKARACHCHI*

Computational sensitivity analysis in the design process of pre-stressed lightweight structures.....1763

*Martin FUSSEDER, Roland WÜCHNER, Kai-Uwe BLETZINGER*

## **Historical structures**

Discerning the evolution of Candela's Shells through parametric structural analysis.....1776

*Edwin GONZÁLEZ, Francisco MUSTIELES, Astrid PETZOLD, Sebastián NOVOA*

Early experimentations within the masonry barrel vault .....1785

*Valentina BEATINI, Alessandro TASORA*

Bauersfeld's concept for the subdivision of the first built geodesic dome structure.....1796

*Orsolya GÁSPÁR*

## **Inflatable structures**

Inflated cushions under a moving point force.....1807

*Slade GELLIN, Romuald TARCZEWSKI*

Parametric study of inflatable strut for Gossamer space antenna .....1814

*Vikas RASTOGI, S. H. UPADHYAY, Sammir SAKHARE, Kripa. S. SINGH*

## **Innovative engineering**

In\*Tension: nets and dance.....1820

*Sigrid ADRIAENSSENS, Tyler SPRAGUE, Janet ECHELMAN, Rebecca LAZIER*

C3 technology demonstration house: CUBE "from digital model to realisation".....1827

*Iurii VAKALIUK, Michael FRENZEL, Manfred CURBACH*

Innovative engineering inspired by visionary design.....1838

*Igor G. SIOTOR, Thomas HERMEKING, Christian SCHLOEGL*

Performative porosity – adaptive infills for architectural elements.....1846

*Mathias BERNHARD, Mohammad BOLHASSANI, Masoud AKBARZADEH*

## **Life-cycle design and assessment of structures (IASS WG 18)**

Exploratory study on a segmented shell made of recycled-HDPE plastic.....	1859
<i>Francesco LACCONI, Iason MANOLAS, Luigi MALOMO, Paolo CIGNONI</i>	
Carbon footprint benchmarking data for shell and spatial structure buildings.....	1871
<i>David COLLINGS</i>	
Fabrication-aware parametric design of segmented concrete shells.....	1879
<i>Eduardo COSTA, Robin OVAL, Paul SHEPHERD, John ORR</i>	
Computational tool for stock-constrained design of structures.....	1889
<i>Jonas WARMUTH, Jan BRÜTTING, Corentin FIVET</i>	

## **Metal gridshell structures, connections, and stability (IASS WG 8)**

Buckling strength of latticed domes with grid-purlins and I-beams.....	1898
<i>Toru TAKEUCHI, Koichi SUMA, Yuki TERAZAWA, Masanobu IWANAGA</i>	
Progressive collapse mechanism of large-span spatial structures and prevention measures.....	1907
<i>Xianzhong ZHAO, Ruifeng LUO, Shen YAN</i>	
Experimental investigation on mechanical performance of Temcor and Box-I section hybrid gusset joints.....	1917
<i>Ying ZHANG, Xinhang ZHI, Yuanqing WANG, Zhongxing WANG, Qin ZHANG</i>	
Progressive collapse analysis of single-layer latticed domes with assembled hub joints.....	1928
<i>Ying XU, Xiaoning ZHANG, Qinghua HAN</i>	
Stability analysis of transmission line under wind load.....	1938
<i>K. ABEDI, H. SARMASTI, M.R. CHENAGHLOU</i>	
Investigation into the instability and collapse behaviour of cable-stiffened single-layer latticed barrel vaults with different forms of out-of-plane cables.....	1946
<i>Karim ABEDI, Mohammad Hossein MATINPOUR, Behzad SHEKASTEHBAND</i>	
Investigation into the stability behavior and progressive collapse of double dome double layer free form space structures.....	1958
<i>Karim ABEDI, Yavar AHMADNIA, Mohamad Reza CHENAGHLOU</i>	
Parametric design of aluminium alloy joint systems applied in a grid shell roof case.....	1967
<i>Xinye LI, Yanke TAN, Qilin ZHANG</i>	
Experimental and numerical investigation on the mechanical performance of improved aluminium alloy joint.....	1976
<i>Yanke TAN, Xinye LI, Qilin ZHANG</i>	
Modelling and experimental tests of spatial structure focused on slip joints.....	1989

<i>Maroš MOJTO, Ján BRODNIANSKÝ, Prof. Ján BRODNIANSKÝ, Jozef RECKÝ, Miroslav TILINGER</i>	
Application of buckling restrained braces for upgrading earthquake resistance of single layer grid dome.....	1998
<i>Yuji TAKIUCHI, Shiro KATO, Yoichi MUKAIYAMA, Keisuke ABE, Shoji NAKAZAWA</i>	
Experimental investigation on the stability of aluminum foam-filled 6082-T6 aluminum alloy circular tube under axial compression.....	2012
<i>Ximei ZHAI, Lingzhao MENG, Xiaoxue SHEN, Guangming CUI</i>	
New shells, new drivers.....	2020
<i>Karly BAST, Michael HOEHN, Cristobal CORREA, Craig SCHWITTER</i>	
Experimental and numerical study on compressive behavior of welded hollow spherical joints with external stiffeners.....	2030
<i>Tingting Shu, Xian Xu</i>	
Influence of edge valencies on the overall resistance of single-layer reticulated cylindrical shells.....	2041
<i>Ranjith KOLAKKATTIL, Konstantinos Daniel TSAVDARIDIS, Arul Jayachandran SANJEEVI</i>	
The investigation into the stability behaviour of single-layer Braced domes with bolt-column joints.....	2053
<i>Majid TAHERI, Mohammad Reza CHENAGHLOU, Karim ABEDI</i>	
The study on equivalent stiffness of aluminium alloy honeycomb panel-beam composite grid structure.....	2063
<i>Wang GANG, Zhao CAIQI, Ma JUN</i>	
The stability of a new single-layer combined lattice shell based on aluminium alloy honeycomb panels.....	2072
<i>Caiqi ZHAO, Jun MA, Haoyue LI, Yating WANG</i>	
 <b>Morphology and configuration processing</b>	
Finding the shape of translation surfaces and gridshells.....	2084
<i>Juan Gerardo OLIVA, Kevin Uriel MORALES, Gisela Fernanda CHÁVEZ</i>	
An efficient method for design of discrete developable surfaces by Ricci flow.....	2096
<i>Jingyao ZHANG, Makoto OHSAKI</i>	
Creating novel non-periodic patterns for flat and curved surfaces from a single pentagonal element.....	2102
<i>Anooshe REZAEI JAVAN, Ting-Uei LEE, Yi Min XIE</i>	
Shells of circular elements from circular meshes design.....	2111
<i>Alice CLUZEAU-TOMATIS, Maxime BRUNOIS</i>	
Connection geometry evaluation in free form space structures.....	2122
<i>Mohammad Reza CHENAGHLOU, Karim ABEDI, Hadi ESMAILNEJAD</i>	

Adapting computational protein folding logic for growth-based, assembly-driven spatial truss design....	2134
<i>Keith J. LEE, Caitlin T. MUELLER</i>	
The Role of the sphere in the generation of Platonic solids: Greek and Chinese cross-views.....	2147
<i>S. MONNOT</i>	
Spatial structures based on solid tessellations.....	2159
<i>Vera VIANA</i>	
Curated deformation - dynamic shape change of tessellated surfaces.....	2171
<i>Mona MÜHLICH, David HORVATH, Axel KÖRNER, Riccardo LA MAGNA, Jan KNIPPERS</i>	

### **Next generation parametric design (WG13)**

Implementation of constructability in conceptual design.....	2181
<i>Katrine HØBJERG, Poul Henning KIRKEGAARD, Valentina BEATINI</i>	
Development of OpenSees component for Grasshopper for various types of computational morphogenesis.....	2194
<i>Shinnosuke FUJITA, Makoto OHSAKI</i>	
Immersive design of exposed and optimized structural systems.....	2205
<i>Mohamed A. ISMAIL, Gabriele SORRENTO, Colin DANIEL, Caitlin T. MUELLER</i>	
Web-based, Interactive Platform for Polyhedral graphic Statics.....	2214
<i>Hua CHAI, Masoud AKBARZADEH</i>	
Deep learning in early-stage structural performance prediction: assessing morphological parameters for buildings.....	2226
<i>Seyed Hossein ZARGAR, Nathan C. BROWN</i>	
Holistic layout optimization of building structures.....	2239
<i>Matthew GILBERT, Linwei HE</i>	
Machine learning for human design: Sketch interface for structural morphology ideation using neural networks.....	2249
<i>Bryan W. X. ONG, Renaud DANHAIVE, Caitlin T. MUELLER</i>	
Reinforcement learning for optimal topology design of 3D trusses.....	2261
<i>Kazuki HAYASHI, Makoto OHSAKI</i>	

### **Novel micro and macro analysis and construction techniques**

A novel construction method of stretching knitted metallized mesh in a deployable rib-membrane structure.....	2274
<i>Di WU, Minger WU, Ping XIANG, Zhongxi YAN</i>	

Numerical and experimental verification of thin-walled sandwich facade panels.....	2283
<i>Martin MAGURA, Prof. Ján BRODNIANSKY</i>	
Demolition construction technology of steel gird roof in the existing buildings based on birth and death element method.....	2291
<i>Nianduo WU, Nuo XU, Zhanjun WAN, Fan SUN, Yongfeng LUO</i>	
Application of VDC/BIM technology in Pudong Art Museum project.....	2297
<i>Yuken YING, Qilin ZHANG, Ran TAO, Pengfei WANG</i>	
Approximate analysis of dense spaceframes.....	2307
<i>Mehdi M. Khabbazan</i>	
StructuralComponents 6: An early-stage design tool for flexible topologies of mid-rise concrete buildings.....	2319
<i>Leah DIERKER VIK, Jeroen COENDERS, Sander PASTERKAMP</i>	
Evaluation of concrete floor systems for embodied carbon, air-borne, and structure-borne acoustic performance.....	2331
<i>Jonathan M. BROYLES, Micah R. SHEPHERD, Nathan C. BROWN</i>	
Efficiency-based exploration of externally post-tensioned structures.....	2343
<i>Javier CAÑADA PÉREZ-SALA, Leonardo TODISCO</i>	
F.A.B. Shell (Fabric – Arch – Base Shell): Concrete shell building using fabric membranes and telescopic arches as formwork.....	2355
<i>Krittika WALIA, Robin OVAL, Olivier BAVEREL</i>	
Design method for non-sequential assembly.....	2367
<i>Julien GLATH, Tristan GOBINb, Romain MESNIL, Marc MIMRAM, Olivier BAVEREL</i>	
 <b>Optimisation</b>	
Design optimization of two-way filler slabs: lightweight concrete floor systems for affordable urban construction.....	2378
<i>Ashley J. HARTWELL, Caitlin T. MUELLER</i>	
Displacement-based seismic assessment of Heinz Isler’s shell structures.....	2393
<i>Abtin BAGHDADI, Mahmoud HERISTCHIAN, Harald KLOFT</i>	
Parametric design of in-plane concrete dry joints by FE method and Fuzzy logic toward utilising additive manufacturing technique.....	2405
<i>Abtin BAGHDADI, Annahita MESHKINI, Harald KLOFT</i>	
Application of different optimisation techniques for material minimisation of individual structural elements.....	2417
<i>Abtin BAGHDADI, Robin DOERRIE, Harald KLOFT</i>	

A discrete topology optimisation method with feature size control for partial double-layer gridshell design.....	2430
<i>Yongpeng HE, Paul SHEPHERD, Jie WANG</i>	
Structural robustness analyses of concrete tall and supertall buildings.....	2443
<i>Julio GARZÓN-ROCA, Karl MICALLEF, Juan SAGASETA</i>	
 <b>Optimisation methods for analysis and design of roof structures (IASS WG 13)</b>	
Structural shape optimization with parametric square-root functions.....	2456
<i>Stavros TSERANIDIS</i>	
Case study: optimization of envelope tertiary structure.....	2468
<i>Viktoria HENRIKSSON, Nicholas MUNDELL</i>	
Non-parametric shape design of free-form shells using extended Gauss map and discrete differential geometry.....	2474
<i>Makoto OHSAKI, Kentaro HAYAKAWA</i>	
Parametric grid mapping design tool for freeform surfaces using a genetic algorithm.....	2480
<i>Lasse W. RAHBEK, Poul H. KIRKEGAARD, Umberto ALIBRANDI</i>	
An innovative hybrid topology optimization method for truss structures based on a combination of continuous and discrete topology optimization.....	2492
<i>Xianzhong ZHAO, Ruifeng LUO</i>	
A novel triangular grid generation approach for the design of free-form grid structure.....	2505
<i>Ruoqiang FENG, Fengcheng Liu</i>	
Arch-based form finding technique in design of optimum shell structures.....	2513
<i>Abtin BAGHDADI, Mahmoud HERISTCHIAN, Harald KLOFT</i>	
Morphological research on developable free-form surface structure composed of plates.....	2525
<i>Jinglan CUI, Yaozhi LUO, Makoto OHSAKI</i>	
Optimizing support locations in structural design.....	2532
<i>Ting-Uei LEE, Xianchuan MENG, Yi Min XIE</i>	
Penn station entrance canopy - optimization with integrated design and engineering workflow.....	2540
<i>Jingwen WANG, Yen-Ju Timothy TAI, Neil KATZ, Preetam BISWAS</i>	
Shells' shape optimization based on R-Funicularity.....	2548
<i>Gloria R. ARGENTO, Francesco MARMO, Valerio VARANO, Stefano GABRIELE</i>	
Topology optimisation of gridshell structures using a density-based approach.....	2557
<i>Yongpeng HE, Paul SHEPHERD, Jie WANG</i>	
Application of a Gaussian filter to smooth optimization of free form grid shells.....	2568

*Hiroki TAMAI*

Topology optimization of multidirectional link elements for CLT shells.....2578

*Ken NODA, Toshiaki KIMURA*

D-Nets on rotational surfaces: equilibrium gridshell layout, symmetric to the principal stress directions.....2589

*Eike SCHLING, Jonas SCHIKORE, Thomas OBERBICHLER*

Simultaneous optimization of the column arrangement and shape of the shell roof using grid point coordinates.....2602

*Shuichiro KOMINAMI, Shinnosuke FUJITA*

Simultaneous optimization of member cross-section and brace topology to solve as a continuous problem.....2611

*Keita TAKESHITA, Shinnosuke FUJITA*

Simultaneous optimization of shell shape, plate thickness and topology using sigmoid function.....2619

*Teppei WATANABE, Shinnosuke FUJITA*

Structural optimization of latticed shells considering strain energy and collapse load factor.....2627

*Taisei NISHEI, Shinnosuke FUJITA*

Design method for modular shells in TRC: A combined geometrical - structural study .....2636

*Arnaud DE COSTER, Marie HENNEMANN, Lars DE LAET, Tine TYSMANS*

Building shells with leaning arches.....2648

*Amandine CERSOSIMO, Marcin MARCH, Devon RYAN*

## **Origami**

Local rigidity analysis of rigid origami.....2658

*Zeyuan HE, Simon D. GUEST*

Active control of 3D printed pill-bug inspired adaptive origami.....2671

*Angshuman C. BARUAH, Ann C. SYCHTERZ*

Description of the origami waterbomb cell kinematics as a basis for the design of thin-walled oricrete shells.....2681

*Rostislav CHUDOBA, Alice C. NIEMEYER, Homam SPARTALI, Daniel ROBERTZ, Wilhelm PLESKEN*

Analysis of buckling behaviour of tubular braces based on non-rigid origami patterns.....2690

*Ya ZHOU, Yuhang ZHOU, Jian FENG, Jianguo CAI*

Energy absorption behaviour analysis of origami metamaterial based on self-locking mechanism.....2699

*Ruijun MA, Jianguo CAI, Jian FENG*

Lattice kirigami for the development of a deployable catenary arch.....	2710
<i>Florian LARRAMENDY, Loïs TAVERNIER</i>	
How to use parametric curved folding design methods- a case study and comparison.....	2720
<i>Riccardo FOSCHI, Robby KRAFT, Rupert MALECZEK, Klara MUNDILOVA, Tomohiro TACHI</i>	
Generating developable and rigidly foldable origami surfaces with arbitrary Gaussian curvatures.....	2731
<i>Sree Chandana MADABHUSHI, Kishore Sreekumar SHENOY, Phanisri Pradeep PRATAPA</i>	
Form generation of rigid origami by multiobjective optimization for approximating curved surface.....	2743
<i>Kentaro HAYAKAWA, Makoto OHSAKI</i>	
Structural Characteristics of Curved Surface Composed of Tubular Origami.....	2755
<i>Yuya ISHIZAWA, Tomohiro TACHI</i>	
<b>Shell structures</b>	
Geometry optimization and laminate layup design of CFRP boom with lenticular cross section.....	2766
<i>Zhongxi YAN, Minger WU, Ping XIANG, Di WU</i>	
Vaults from Buildings: Thin-tile Vaulting from Stabilised Rubble Tiles .....	2776
<i>M. Wesam Al ASALI, Will HAWKINS, Michael H. RAMAGE</i>	
Assembly aware design and construction of interlocking masonry structures.....	2788
<i>Rison Prasad KARAYIL THEKKOOT, Romain MESNIL, Jean-François CARON, Olivier BAVEREL</i>	
Segmentation and assembly strategy for lamella roof shell structures.....	2800
<i>Hannes LÖSCHKE, Alexander STAHR, Tim Henrik SCHRÖDER, Felix SCHMIDT-KLEESPIES, Ryan HALLAHAN</i>	
Design and study of a fractal shell (based on dry leaf anatomy) using Lindenmayer system algorithm...	2808
<i>Aman UPADHAYAY, Savita MARU</i>	
<b>Symmetry and spatial structures</b>	
Symmetry in design and analysis of some spatial structures.....	2818
<i>Janusz REBIELAK</i>	
‘Quintuplet’ Phenomenology of 3D Space and the Exhaustive Enumeration of 3D Symmetry Space Groups.....	2828
<i>Michael BURT</i>	
Symmetry-generated space grids.....	2840
<i>Alphose ZINGONI</i>	

## **Tactile strategies for teaching spatial structures (IASS WG 20)**

Teaching lightweight and tensile structures: a pedagogy for beginning design students.....	2854
<i>Charles MACBRIDE</i>	
Learning by building: physical vs. numerical form-finding.....	2865
<i>Jelena VUKADIN, Dominik VIDOVIĆ, Josip VUCO, Elizabeta ŠAMEC, Petra GIDAK, Krešimir FRESL</i>	
Zero Gravity: radical creativity by multidisciplinary collaboration.....	2873
<i>Günther H. FILZ, Serenay ELMAS, Athanasios A. MARKOU, Katja HÖLTTÄ-OTTO, Saurabh DEO</i>	
Visualizing structures: integrative methodology for teaching structural principles to architecture students.....	2886
<i>Maged S. GUERGUIS, Kristin K. PITTS</i>	
Elastica project: dynamic relaxation for post-formed elastic gridshells.....	2897
<i>Marc LEYRAL, Sylvain EBODE, Pierre GUEROLD, Clément BERTHOU</i>	
Designing intuitive experiences for responsive architecture.....	2913
<i>Christoph GENGNAGEL, Saqib AZIZ, Elena Francesca AMBACHER, Lukas UTZIG,, Jörg RÄDLER</i>	
A student perspective on teaching the design of shell and spatial structures through project based learning.....	2925
<i>Miriam GRAHAM, Richard HARPIN, John CARR, Paul HULBERT</i>	
Internationalisation of the Curriculum in Spatial Structures .....	2934
<i>Ramsha SALEEM, S. Alireza BEHNEJAD</i>	

## **Tensegrity systems**

The effect of friction coefficient of the structural behavior of String Crescent Structure.....	2942
<i>Akira TANAKA</i>	
Analysis of novel adaptable tensegrity towers.....	2953
<i>Aguinaldo FRADDOSIO, Andrea MICHELETTI, Gaetano PAVONE, Mario D. PICCIONI</i>	
Design and analysis of deployable clustered tensegrity cable domes .....	2964
<i>Shuo MA, Muhao CHEN, Xingfei YUAN, Robert E. SKELTON</i>	
A full-scale plate-based tensegrity structure as a bike parking canopy.....	2977
<i>Heather GATHMAN, Ann C. SYCHTERZ</i>	
Actuation of an adaptive tensegrity-based roof structure using continuous cables.....	2989
<i>Sagnik PAUL, Ann C. SYCHTERZ</i>	

## **Tension and membrane structures: Recent developments**

Maintaining membrane structures – experiences from Ashford.....	3001
---	------

*Katja BERNERT*

The analytic and numerical form-finding of minimal surfaces and their application as shell structures.....3008

*Alexander SEHLSTOM, Chris J. K. WILLIAMS*

X-Madrid roof.....3019

*Guillermo CAPELLÁN, Santiago. GUERRA, Julio GONZÁLEZ, Javier MARTÍNEZ*

Stability of stayed columns.....3029

*Sudarshan KRISHNAN*

The carbon footprint of long span structures: review of the Millennium Dome and subsequent tensile systems.....3033

*Tim C. R. FINLAY*

## **The future of structural design - symbiosis in architecture and structural engineering**

Geometry-based teaching of structures through computational graphic statics.....3043

*Juney LEE, Lluís ENRIQUE, Tom VAN MELE, Philippe BLOCK*

Teaching structures to architects – think, experience, communicate, and design.....3051

*Yvonne WONG, Iori KANAO, Nachamma SOCKALINGAM, Sam C. JOYCE*

Integration of parametric structural analysis and architectural design using a novel approach to enhancing timber compression members.....3060

*Matthew TAM, Florian FEND, Clemens PREISINGER*

Homogeneity versus heterogeneity in architect-engineer design teams.....3071

*Stephanie BUNT, Nathan C. BROWN*

Equilibrium without statics: The modern Müller-Breslau method.....3083

*Edmond SALIKLIS*

Teaching structural design to architecture students. Experiences at the Academy of Architecture in Mendrisio, Switzerland.....3096

*Stefano MICCOLI, Andrea FRANGI, Roberto GUIDOTTI*

Blended design studio in teaching architectural students about spatial structures.....3108

*Jelena MILOŠEVIĆ*

Dialogic design process: an analysis of an irregular shell with polyurethane foam.....3120

*Jun CHEN*

Decoding complex geometry for craftsmanship.....3127

*Urvi SHETH, Shylaja REGUNATHAN*

Zero Gravity: a novel cantilever beam utilizing elastic torsion for structures and architecture.....3139

*Serenay ELMAS, Günther H. FILZ, Athanasios A. MARKOU, Jani ROMANOFF*

## **The Milgo experiment**

Form-making and industrial production: algorithms, Milgo experiment 1 (1997-2004).....3149

*Haresh LALVANI*

Expanded surfaces: Xurf, Milgo experiment 2 (1998-2010) .....3163

*Haresh LALVANI*

X-structures, Milgo experiment 3 (2008-2014).....3177

*Haresh LALVANI*

Hypersurfaces, Milgo experiment 4 (2006-2012) .....3190

*Haresh LALVANI*

## **Transformable structures (IASS WG 15)**

Vertex pavilion: deployable arches structure with triangular section.....3203

*O. AVELLANEDA, M. MENDOZA, D. PEÑA*

On the kinematics of rigid bar-linkage system typologies.....3211

*Niki GEORGIU, Marios C. PHOCAS*

Geometric strategies to design a bistable deployable structure with straight scissors using flexible rods.....3222

*Carlos J. GARCÍA-MORA, Jose SÁNCHEZ-SÁNCHEZ*

Using flexible trapezoidal quad-surfaces for transformable design.....3236

*Kiumars SHARIFMOGHADDAM, Georg NAWRATIL, Arvin RASOULZADEH, Jonas TERVOOREN*

Active control and energy assessment of a prototype adaptive structure that adapts to loading through large shape changes.....3249

*Arka P. REKSOWARDOJO, Gennaro SENATORE, Ian F.C. SMITH*

Alignable nets: compacting elastic gridshells..... 3262

*Xavier TELLIER*

3D Expandable origami network.....3274

*Anastasia IOANNIDI, Katherine LIAPI, Vasileios SPITAS, Christos PAPALEXIS*

Hybrid deployable structures consisting of miura-ori folded surfaces and translational scissors expandable frames .....3283

*Evangelia VLACHAKI, Katherine A. LIAPI*

Star system: A reconfigurable structure for architectural applications .....3294

*Katherine A. LIAPI, Despoina KARAMPELA, Myrto LADA, Nikolaos LEVENTIS*

## WG21: Design competition

A study on algorithm-generated assembly of curved I and Y shaped branches for temporary shelters.....	3302
<i>Anton D. KEREZOV, Mikiro KOSHIHARA</i>	
Designing lightweight structures from recyclable and organic materials: the rethinking lightweight pavilion.....	3316
<i>Maren ZYWIETZ, Karsten SCHLESIER, Annette BÖGLE</i>	
Knots and bars kit to construct translation gridshells.....	3326
<i>Juan Gerardo OLIVA SALINAS, Magdalena TRUJILLO BARRAGÁN</i>	
Innovative multi-culm bamboo spatial lattice structures system.....	3337
<i>Xin ZHUO, Shilin DONG</i>	
A computational framework for the design and fabrication of spatial structures with mycelium-based composites .....	3344
<i>Ali GHAZVINIAN, Arman KHALILBEIGI, Esmail MOTTAGHI, Benay GURSOY</i>	
WAUXI (wall auxetic installation).....	3356
<i>Hanna M. GLETHOFER, BA Kilian HOFFMANN, Simon WINTER</i>	
Design and analysis of moment-resisting nature-inspired-design structure using graphic statics methods.....	3367
<i>Georgios-Spyridon ATHANASOPOULOS, Timothy IRELAND, Howard GRIFFIN, Kevin SMITH, Colin CRESSER, Julien SOOSAIPILLAI, Mohammed FAWAZ, Hasin ZAHIN</i>	
The Geldeford Riband Pavillion .....	3378
<i>Dewitt GODFREY, Jeg DUDLEY, Joel HILMERSSON</i>	
A retractable roof pavilion - future and precedent.....	3390
<i>Leo Gamborg HEINZL, Valentina BEATINI, Poul Henning KIRKEGAARD</i>	
<b>Wind engineering and CFD simulation</b>	
Aerodynamic shape optimization of helical super high-rise buildings combined with parametric modelling.....	3402
<i>Zheng HE, Tian LIANG, Xiao LAI, Yi LU, Fan FENG, Guohui HUANG</i>	
CFD numerical simulation of typical shape stadiums.....	3413
<i>Lei GU, Xinxin ZHANG, Hongtuo QI, Hongjun LIU, Fang WU</i>	
The effect of design parameters on the fluid-structure interaction of hyperbolic paraboloid membrane structures.....	3425
<i>Maarten VAN CRAENENBROECK, Marijke MOLLAERT, Lars DE LAET</i>	

## Hangai prize lectures

Adaptive framework for structural pattern optimization.....	3436
<i>Diego A. RIVERA, Renaud A. DANHAIVE, Caitlin T. MUELLER</i>	
Parametric study of non-periodic and hybrid auxetic bending-active gridshells.....	3449
<i>Yusuke SAKAI, Makoto OHSAKI</i>	
Coupled sizing, shape and topology optimisation of bistable deployable structures.....	3461
<i>Liesbeth I.W. ARNOUITS, Thierry J. MASSART, Niels DE TEMMERMAN, Péter Z. BERKE</i>	
Voss surfaces: A design space for geodesic gridshells.....	3473
<i>Nicolas MONTAGNE, Cyril DOUTHE, Xavier TELLIER, Corentin FIVET, Olivier BAVAREL</i>	
A graphical method for determining truss stability.....	3484
<i>Cameron MILLAR, Allan McROBIE, William BAKER</i>	
Self-folding rigid origami based on auxetic kirigami.....	3496
<i>Kotaro SEMPUKU, Tomohiro TACHI</i>	
Study on thermal damage of large-span ice shell structure based on thermal-mechanical coupling.....	3508
<i>Xiuming LIU, Yue WU, Shizhao SHEN</i>	
From ruled surfaces to elastica-ruled surfaces: New possibilities for creating architectural forms.....	3524
<i>Ting-Uei LEE, Yi Min XIE</i>	
Programmable and reconfigurable surfaces with kirigami-inspired bistable elements.....	3536
<i>Qian ZHANG, Ning PAN, Jianguo CAI, Jian FENG</i>	

## Papers:

## **Robotic construction of a self-balancing glass masonry vault: DEM study of stability during the construction stages**

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### **Abstract**

Self-balancing construction technologies were used for centuries in the building of masonry domes and vaults. Such construction techniques were made possible through the careful design of the block tessellation and the structural form, which enabled construction of complex geometries that remain stable without falsework. These historical masonry technologies have a disruptive potential for today's construction industry when coupled with emerging innovations such as novel computational form-finding approaches and robotic fabrication. This paper presents a structural analysis of the construction stages for a doubly curved, compression-only, 2-metre-tall masonry vault inspired by traditional construction technologies and built with a cooperative human-robot fabrication process. Two ABB-IRB 6400 industrial robotic arms were precisely sequenced to alternate placing a masonry block and providing temporary support to the unfinished structure. As a result, no form- or falsework was needed during any construction stage. The paper reports an iterative procedure based on the Limit State Analysis (LSA) and Discrete Element Method (DEM) to numerically study the equilibrium of the masonry vault during all construction stages.

**Keywords:** Shell, vault, masonry, robotic construction, discrete element model, limit state analysis, self-balancing.

### **1. Introduction**

Ancient master builders have built arches, vaults, and domes using masonry for millennia. These ancient forms create impressive architectural spaces that in some cases surpass the longevity of contemporary structures [1]. From the 2<sup>nd</sup> millennia BCE onwards, self-balancing construction techniques have been in use to realize many such complex structures [2]. The pitched vault [3] is perhaps the first self-balancing building system developed, which dates back to as early as the 21<sup>st</sup> century BCE [4] and is still applied in a few parts of North Africa [5]. As shown by various studies [3] [6] [7], the self-balanced state under construction is due to several factors all related with the construction work such as: the properties of building materials [6], the orientation of bed joints (as shown in figure 1) [3], and the tessellation [7].

Considering all of these factors, through history, several self-balancing construction technologies have also been developed [7] [8] [9], even the ones built in dry-stone masonry, widespread in the vernacular architecture of all the Mediterranean area [10]. Among all, the herringbone [11] and tile vaulting [12] constitute the most influential systems [9]. Santa Maria del Fiore's dome in Florence (1418-1436) [13] represents a notable example of the herringbone technique applied on a large scale.

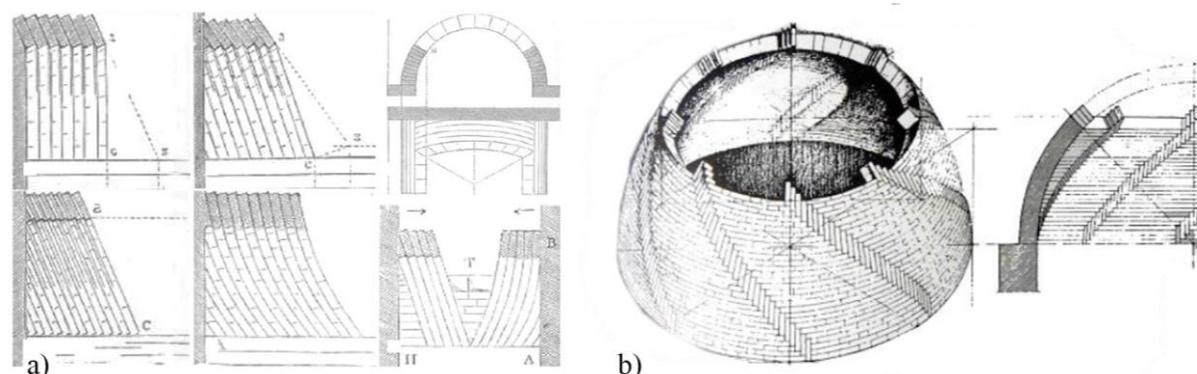


Figure 1: a) The pitched vaulting technique, different scheme to lay bricks. Drawing of A. Choisy [14]. b) Revolution dome and herringbone spiralling tessellation. Drawing of F. Gurrieri [15].

As illustrated in figure 1b), the herringbone technique gets its name from its characteristic tessellation pattern: the bricks are arranged in an alternating horizontal and vertical manner [16]. Due to this tessellation and a precise sequence of laying bricks, self-supporting structural actions are formed in the dome even during its construction [17]. On the other hand, tile vaulting methods, originating in Spain around the 13<sup>rd</sup>-14<sup>th</sup> century CE [12], allows the construction of doubly-curved structures without the aid of any temporary supports through the use of fast setting gypsum mortar coupled with light tiles [18]. This technique has been a topic of recent research [6] [19] since its application to the construction of form-found vault geometries is more manageable than other self-balanced technologies. Despite specific differences in geography or time period, all the historic self-balancing masonry construction techniques mentioned share an important choice of laying sequence and structural form the technique is being applied to. The potential of these historic technologies in the construction of complex geometries without falsework is being increasingly rediscovered in the context of emerging innovations such as novel computational form-finding approaches and robotic construction technologies.

Different approaches have been developed from the structural point of view to analyse masonry curved structures. A significant milestone toward understanding the structural behaviour of voussoir arches and vaults was made by Heyman [20], who reinterpreted the geometrical and equilibrium-based rules used by the ancient master builders through the Limit State Analysis (LSA) framework, reclaiming the power of graphic static (GS) approaches [21]. The success of this approach was due to the possibility of obtaining an accurate estimation of collapse loads using a straightforward constitutive model known as the no-tension model [22]. This model does not require the calibration of any parameters for the masonry's mechanical characterisation, which overcomes the high level of uncertainty that is usually associated with masonry structures, especially in historical constructions. More recent LSA methods based on the Thrust Line Analysis (TLA), have further reworked and extended to the analysis of complex three-dimensional structures [23] [24] [25]. In these approaches, the search for a thrust line entirely contained within the vault's thickness was extended to the three-dimensional space, as a search for discrete compression-only network or continuum unilateral membrane. Thus, according to the Safe Theorem [21], the masonry structure is stable if the thrust network [26] or surface [27] lies entirely between the intrados and the extrados surface of the vault. From a numerical point of view, literature [28] [29] [30] shows that Discrete Element Modelling (DEM) is particularly suitable to analyse the equilibrium of masonry structures. Indeed, Lemos and other researchers have proven its validity to simulate the static and dynamic behaviour of masonry structures [28] [31]. Although the potential of the

methods mentioned above to analyse complete structures is well known, evaluating the equilibrium of masonry structures under construction is scarcely explored; few works that address this, are presented in the literature [17] [32].

This paper extends the existing literature on the structural analysis of masonry structures by applying LSA and DEM methods to calculate the structural performance of a doubly-curved compression-only glass brick shell during its construction stages. The main challenge is how to investigate the vault's structural behaviour in each construction stage, while also accounting for the fact that the entire building process is carried out through a human-robot fabrication process without temporary form-or-falsework. To perform this analysis, an iterative procedure was developed that accounts for the action of the robotic arms on the structure under construction.

This research culminated in the robotic fabrication of a full-scale vault prototype at the “Anatomy of Structure” exhibit hosted at the Ambika P3 Gallery in London [33], and shown in figure 2. Different aspects of this research, from the development of the cooperative robotic fabrication strategy to the analysis of different construction sequences, are further detailed in recent works [33] [34] [35] [36].

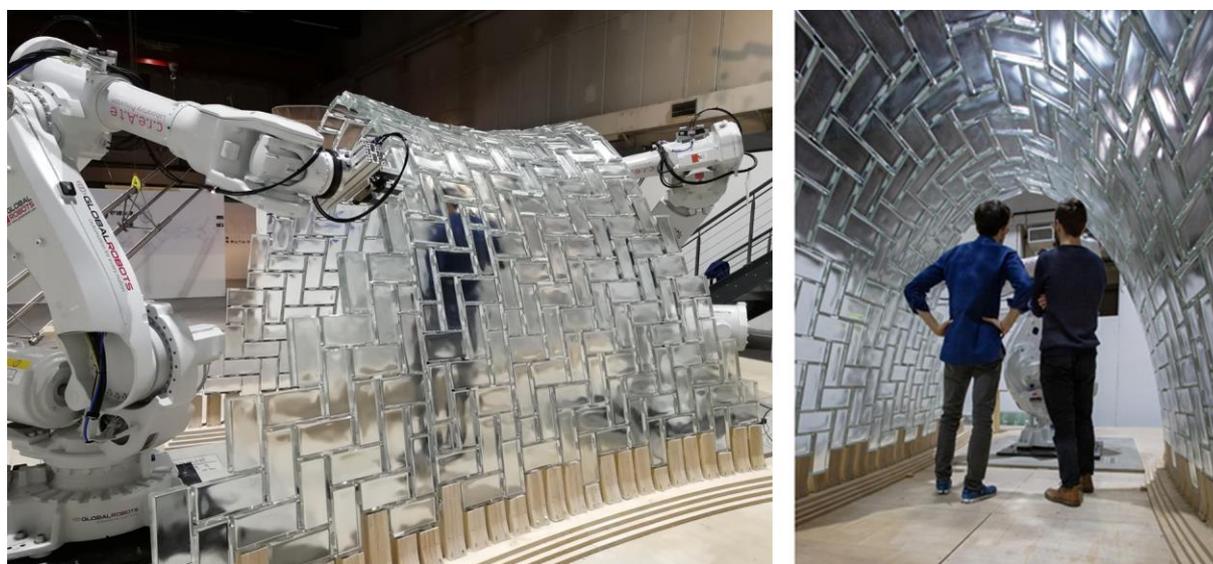


Figure 2: Robotic fabrication of a masonry vault for the “Anatomy of Structure” exhibit.

The paper is organised as follows: section 2 describes the geometry of the vault and the material system; section 3 provides an overview on the construction process adopted and illustrates the analogies with historical self-balancing technologies; section 4 describes the methodology adopted to assess the structural state of the vault during its construction; section 5 presents the structural analysis conducted and the results; and section 6 concludes the paper and presents a discussion.

## **2. Vault geometry and material**

The motivation for the current research comes from the exploration of ultra-light, yet efficient and strong, timber vault structures. This work was inspired by historical construction methods that relied on inclined courses to build vaults with the aid of only light falsework [37]. Starting from this idea, the current project was then developed with the goal of building a geometrically complex self-balancing shell without the aid of any temporary falsework. To achieve this goal, industrial robotic arms were used to both place bricks in precise complex spatial orientations and to act as temporary support to the unfinished structure [34]. As shown in figure 3, the vault has a form-found geometry, characterised by a saddle shape with a catenary profile. The span of the vault is 2.7 meters, with a length of 4.4 meters, and a rise about 2.2 meters on the outside edges and 1.9 meters for the central arch. Glass bricks were the primary elements used for the structure. As is common in masonry constructions, the geometry of

the vault had to be defined so as to minimise any tensile forces. A form-finding approach, based on the Airy's stress function [38], enabled the calculation of a doubly-curved compression-only shell. The geometry was found by an iterative process, which took into account the real distribution of the self-weight of the structure [39].

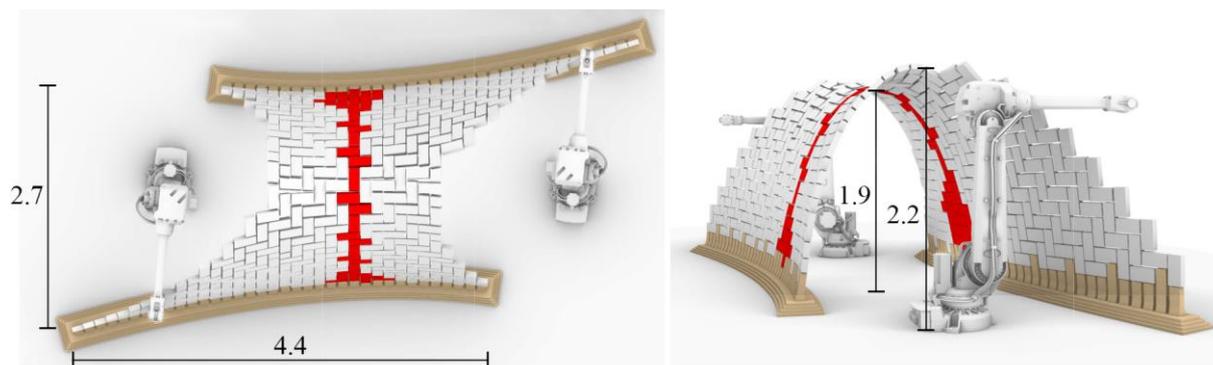


Figure 3: Geometry of the form-found shell. The central arch (red) corresponds to the first portion of the vault built. The two sides of the vault (white) are built only after completing the central arch.

Using standardised glass bricks, simplified construction by avoiding the need to field cut complex brick shapes to resolve unique geometric conditions. The material used in the joints needed to be fast-setting, rigid and with enough strength to hold the brick in place as it was placed. As reported in [34], this material was also used to compensate for the gap between different bricks courses and to deal with construction tolerances. For all these reasons, fast-setting epoxy putty [40] was selected as the joint mortar.

### 3. The construction phases

Two robotic arms (ABB-IRB 6400) were used to build the vault. A combination of historical technologies, such as herringbone and pitched techniques, allowed for the design of a precise construction process, with two distinct phases [34]. Phase **I** consists of all the construction stages related to the central arch, which was the first portion of the structure to be built. The central arch's construction is started from one side of the vault and bricks are then laid sequentially until the opposite side of the vault has been reached. The process of developing a robotic fabrication sequence that maintains stability during the construction is discussed in detail in recent works [34] [36]. Phase **I** ends when the last brick at the base of the arch is laid, this specific construction stage is denoted in this paper as **I-h**. Phase **II** follows phase **I**; here, the robot builds outward from the central arch to complete the remaining portion of the vault.

These two phases are characterised by the manner in which the two robotic arms were used. During phase **I**, the robots work cooperatively to build the central arch, one supports the incomplete arch while the second places the new glass brick. In phase **II**, the two robotic arms work separately, building two different portions of the vault; these portions are shown as the white bricks in figure 3.

This construction process incorporates several characteristics of the historical self-balancing technologies discussed in the introduction. In phase **I**, similar to the tile vaulting technique, the fast-setting property of the epoxy allows the glass bricks to maintain their position as they are laid. Meanwhile in phase **II**, the vault's tessellation is similar to the traditional herringbone technique, linking together two consecutive brick courses. Finally, the overall construction sequence is inspired by the pitched vault technology, where several consecutive self-balancing substructures are created before the full structure is completed. These substructures are shown in figure 4, where in each construction stage, **II-a**, **II-b**, **II-c**, a new arch is built.

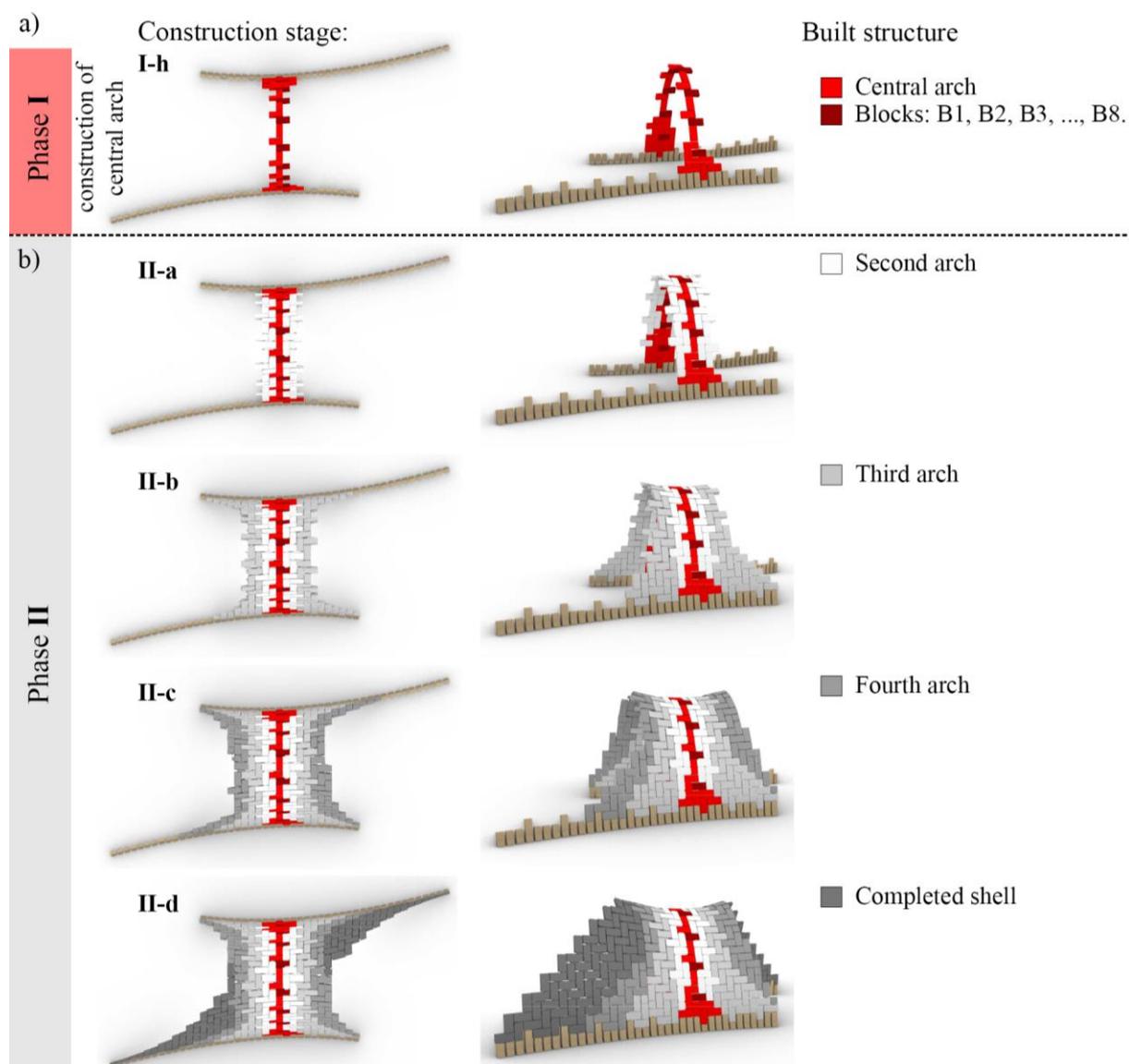


Figure 4: Construction process for the masonry vault. a) Final stage of phase I and the closing of the central arch (construction stage I-h) b) phase II, where each stage is defined by the construction of a new inclined arch substructure.

#### 4. Methodology for structural analysis

An evaluation of phase I was performed to assess the balanced state during construction. First a detailed TLA was carried out, dividing the central arch into individual blocks equivalent to the number of glass bricks. Thus, the equilibrium of the full structure was investigated by simulating the sequential placement of the bricks. As a result, for each construction stage related to phase I, an estimation of the external forces required to achieve stability was determined. These forces represent the static interaction between the structure and the robotic arms. According to LSA, no-tension material (i.e., any tensile strength provided by the epoxy putty is neglected) is adopted to perform TLA, therefore this analysis is a conservative investigation.

The epoxy putty's realistic material properties were considered in a detailed numerical analysis conducted by DEM, which was carried out for both phases in order to investigate the balanced state and

to estimate the displacements associated with each construction stage. The vault was modelled in the commercially available DEM software, 3DEC (Itasca, Minneapolis, MN, USA) [41], which allows to analyse masonry structures as a complex system of blocks [28]. These blocks can slide along their interfaces, collide or even detach [29] [30]. Through the explicit integration of Newton's laws of motion, using the finite-difference method and assuming small time-steps, the algorithm permits the evaluation of the structural behaviour of a system of bodies (either deformable or rigid) subjected to static or dynamic loads.

In the analyses performed, the vault's bricks are modelled as rigid blocks, whose size corresponds to that of the actual glass brick, which is then attached to a layer of epoxy putty surrounding it. Adopting this discretisation, the system of rigid bodies represents a good approximation of the form-found shell's structural behaviour, since the deformation is lumped at the joints. The material properties of the epoxy putty are used to model the interfaces, where a Mohr-Coulomb model with a tensile cut-off is assumed. As illustrated in figure 5, this nonlinear interface behaviour is ruled by the joint parameters:  $JK_n$ ,  $JK_s$ ,  $J_{ten}$ ,  $J_{fric}$  and  $J_{coh}$  [42]. The  $JK_n$ ,  $JK_s$  terms represent the normal and the shear stiffness of the rigid body's interfaces, respectively, while  $J_{ten}$  is the tensile strength,  $J_{fric}$  is the friction angle, and  $J_{coh}$  is the cohesive capacity. These parameters are determined based on the epoxy material's specifications [40] and verified by experiment tests, incorporating a safety coefficient.

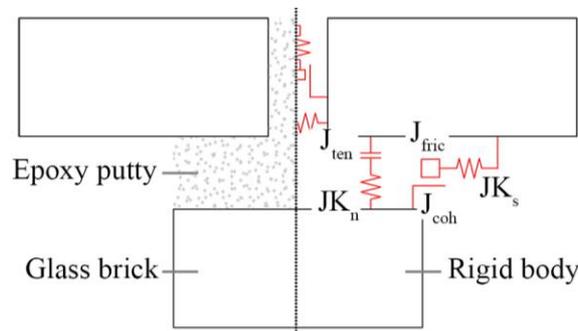


Figure 5: Joint parameters for the rigid bodies interface (adapted from [43]).

## 5. Analysis and results

Both phases of construction for the masonry vault were analysed, with the purpose of verifying the self-balancing state during the different stages. Sections, 5.1 (phase **I**) and 5.2 (phase **II**) describe the structural behaviour of the partially completed structure throughout the construction: arch behaviour is expected in phase **I**, and a compression-only shell behaviour in phase **II**

### 5.1. Phase I

As mentioned in section 4, TLA is executed to assess the equilibrium state of the central arch and determine the external forces that the robots must apply to guarantee this state of equilibrium. Therefore, all TLA were performed assuming that the robotic arms act as a support to the arch during construction. Although the final form-found shell was designed to achieve a fully-compressed membrane stress state, it was found that during construction the initial central arch was unable to achieve optimal structural behaviour. This is shown in figure 6a: the thrust line (red) of the central arch under construction does not entirely lie in the cross-section of initial form-found shell geometry. For this reason, the form of the complete shell had to be re-designed to ensure stability during construction.

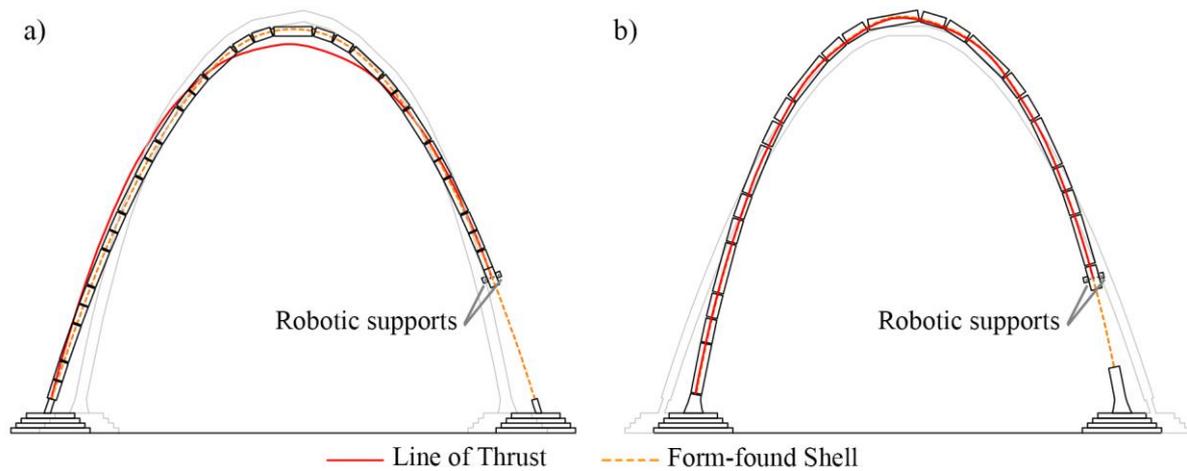


Figure 6: TLA for central arch under construction. a) the thrust line (red) of the arch under construction falls outside of the midline of the shell geometry b) the thrust line (red) and midline of the re-designed arch coincide

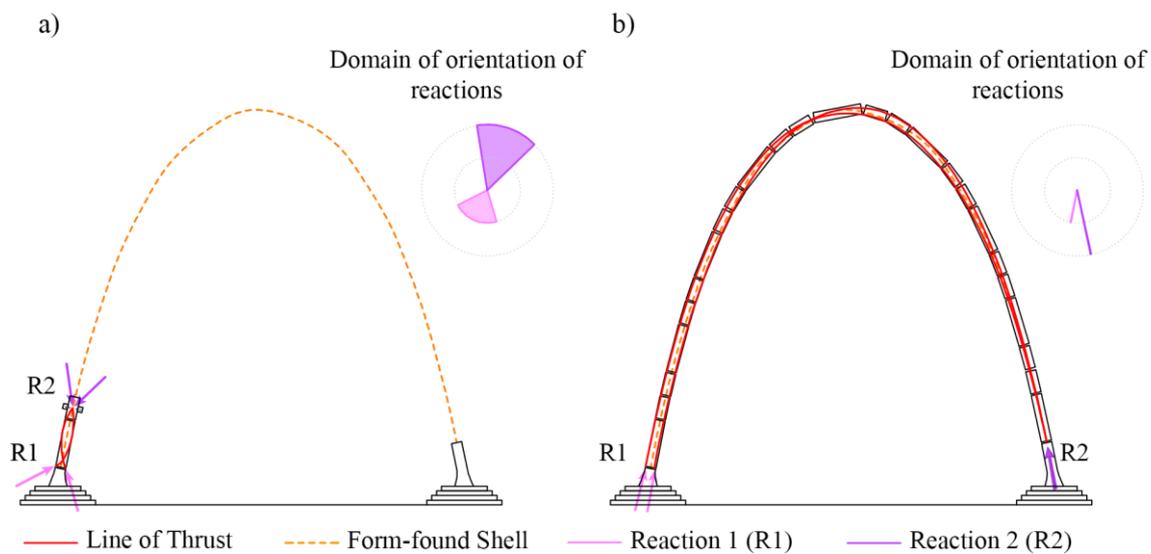


Figure 7: Domain of orientation of the reactions R1 and R2. a) first construction stage b) last construction stage of the central arch.

According to Heyman's theory [21], and due to the modest dimensions of the structure examined, geometrical instability is the most probable cause for collapse. Therefore, investigations on the variation of minimum and maximum lines of thrust during the construction works were carried out. As reported in Figure 7a), at the first stage of construction, with only two bricks positioned, a balanced state is found if the horizontal thrust comprised in the range of 0.00-0.001kN. Despite that, the domain of orientation of the reactions, denoted by R1 and R2, is wide: about  $79.5^\circ$  for R1, and  $54.0^\circ$  for R2. This domain expresses the number of possible safe solutions that can be found, i.e., the narrower it is, the fewer number of solutions are possible. With the progress of the construction works, see figure 7b), the minimum and maximum horizontal thrust increases, to 0.09-0.10 kN, while the domain of orientation of reactions drastically narrows down to,  $1.1^\circ$  for R1 and  $0.8^\circ$  for R2.

From a numerical point of view, the robotic arm's interaction with the structure was evaluated as a kinematic constraint: the gripping points of the robotic arms allow no displacement. The first numerical simulations showed that the equilibrium state is influenced by the position of the robotic gripping point,

highlighting that an incorrectly positioned grip could lead to out-of-plane displacement. Thus, as described in [34] [36], a particular robotic sequencing construction method has been developed in order to avoid out-of-plane twisting displacement. The simulation of the construction stages conducted with DEM confirms the results obtained by TLA: the central arch reaches an equilibrium state, with the robotic arm as support, in each construction stage of phase **I**. As shown in figure 8, the arch shows a maximum displacement of  $4.27 \cdot 10^{-3}$  mm.

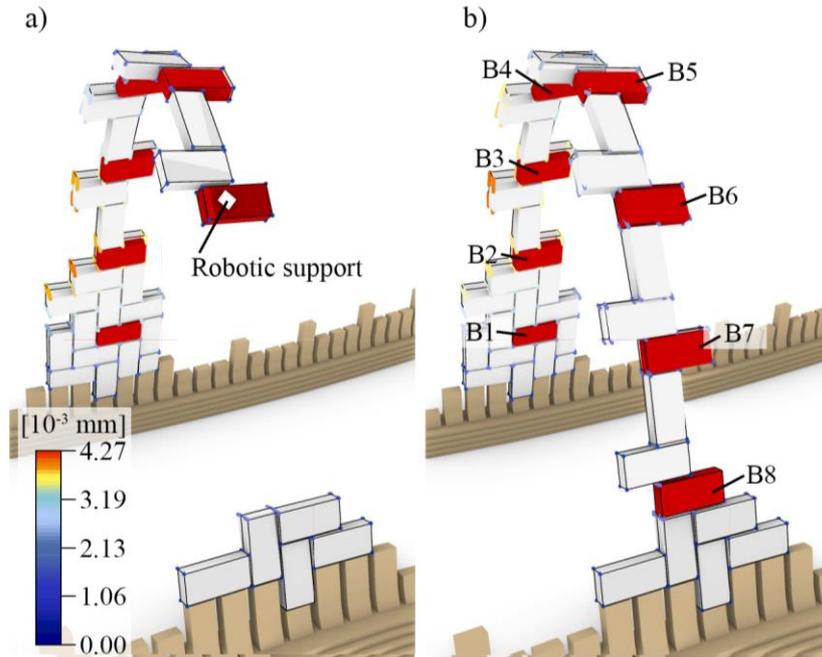


Figure 8: Construction stages of the central arch. a) incomplete arch showing the position of the robotic support b) finished arch showing the position of control blocks (red) for the monitoring of displacements during the construction. The colour scale is associated with the displacement vectors of each node.

## 5.2. Phase II

The structural behaviour of phase **II** has been investigated by DEM analysis. Here, the robotic arms work separately without supporting the shell during its construction, thus, the structure is required to reach a self-balanced state at each construction stage. The numerical analysis simulated the as-built construction process for the shell. Similar to phase **I**, the structural behaviour is investigated at each construction stage; the influence on the equilibrium is established as each brick is placed.

The analysis confirms the stability of the form-found shell; furthermore, as shown in figure 9, immediately after the completion of each of the construction stages **II-a**, **II-b**, **II-c** the magnitude of the displacement decreases. As the arch substructures are completed, it allows for a redistribution of the forces, consequently, improving the structural behaviour of the partially completed structure and reducing the overall displacements. Subsequently as the construction of the next part of the structure begins, the displacements again begin to increase. This phenomenon highlights the importance of the construction sequence and the masonry tessellation, inspired by the pitched vault and herringbone technique, respectively.

The surface illustrated in figure 9, denoted by  $\zeta$ , shows the variation in the magnitude of the displacements in relation to the construction stages. The surface  $\zeta$  is plotted based on monitoring the behaviour of eight control points: the centroids of the blocks  $B_i$  ( $B_1, B_2, \dots, B_8$ ), shown in figure 8b. For example, referring to figure 9, the curve starting from  $B_1$  in the centroid axis, expresses the variation of the magnitude of displacement recorded at the control point  $B_1$  through each construction stage. The surface  $\zeta$  displays peaks and valleys orientated in the direction perpendicular to the construction stage

axis, showing that as stages are completed the displacements decrease uniformly. The maximum displacement recorded is  $4.6 \cdot 10^{-2}$  millimetres located near the intersection of the construction stage **II-c** and control point  $B_3$ .

The surface  $\zeta$  also describes the magnitude of the residual displacement of the completed final shell (**II-d**); at this construction stage the displacement seen is lower than the maximum displacements recorded during phases **II**. The reason for this decrease is that when the structure is completed, the geometry corresponds to the final form-found shell, for which the structural behaviour has been optimised.

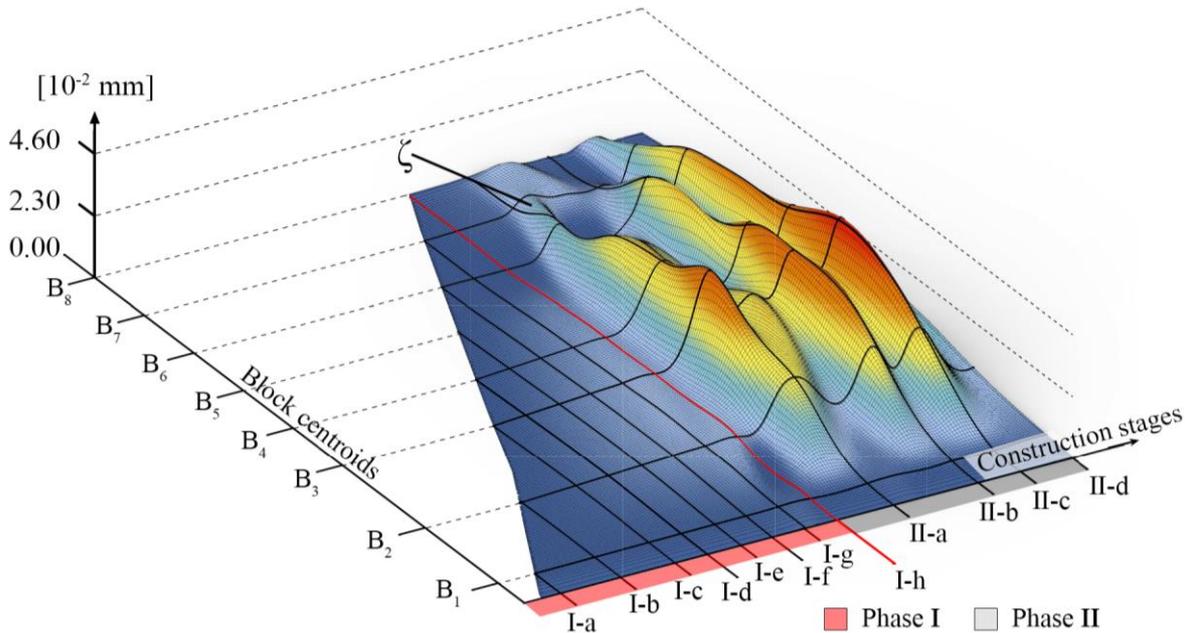


Figure 9: Displacement magnitudes surface ( $\zeta$ ). The vertical axis reports the magnitude of displacements. The other two axes represent the location of block centroids where the displacements have been recorded ( $B_1, B_2, B_3, \dots, B_8$ ) and the construction stages divided by the two phases **I** and **II**. Here, **I-a, I-b, I-c, \dots, I-h** are subdivisions in phase **I**, while **II-a, II-b** and **II-c** are the construction stage as described in figure 4.

## 6. Conclusion

The potential of historical construction techniques, adopted in the context of emerging innovations in novel computational form-finding approaches and robotic construction technologies, is emphasised in the current case study. Similar to the construction of pitched vaults and herringbone vaults, factors such as, the construction process and masonry tessellation, assume primary relevance. These construction factors play a fundamental role in achieving an equilibrium state of the structure under construction. For example, in the current case, the presence of the central arch has a large influence on the stability of the construction works referenced as phase **II**, i.e., it supports the construction of the other portions of the vault, providing external reactions needed to reach an equilibrium state. Keeping this aim in mind, the knowledge of the structural state under construction is crucial; only through studies conducted in this manner can possible failures be identified. The analysis reported in sections 5.1 and 5.2 highlights the displacement and forces present in the structure under construction. TLA carried out at the end of the phase **I**, shows the existence of a narrow domain of orientation of support reactions, emphasizing that even if a balanced state could be found, a slight variation in the geometry of the arch or in the location and orientation of the supports, could lead to overturning or collapse of the shell.

Furthermore, the introduction of a temporal variable in the structural analysis allows mapping of variations in the structural behaviour throughout the construction process. As illustrated in section 5.2,

the study of the surface  $\zeta$ , shows the variation in the structural behaviour between phases **I** and **II**. In phase **I**, the balanced state can be reached only by arch behaviour, here, the surface  $\zeta$  is regular and the maximum displacement recorded in the central arch is significantly lower than that of phase **II**, where the rest of the vault is constructed, and a compressed membrane behaviour occurs. In phase **II**, the surface  $\zeta$  is described by peaks and valleys. Between the construction stage **I-h** and **II-a**,  $\zeta$  shows a strong variation: the magnitude of displacement of all blocks of the central arch increases by an order, defining the first peak that corresponds to **II-a**. This temporal interval (between **I-h** and **II-a**) starts when the central arch is completed and ends when the substructure corresponding to the second arch is closed. Immediately after its completion (after **II-a**), a new and more stable structure exhibiting compression-shell behaviour occurs. With this altered structural behaviour, the magnitude of displacements decreases, which coincides with the first valley in  $\zeta$ , as seen in figure 9. Further, the valleys correspond to construction stages which follow the completion of arch substructures, and where redistribution of forces occurs leading to a more stable shell behaviour. This phenomenon is even more evident at the completed vault stage (end of **II-d**), where the pure compression shell behaves as designed.

The paper originates from a study of the historical construction technologies. The analysis and the discussion reported here are extremely relevant to the field of contemporary self-balancing technologies and ultimately for the task of construction cost-optimisation. Only by considering the state of the structure during construction works, is it possible to assess the self-balanced state and thus, drastically decrease the material cost of construction [44]. Recent works [45] [46] have shown potential in this direction, using cooperative robotics and other non-traditional fabrication systems to build complex structures without formwork. These developments have the potential to impact the building industry, but first require a better framework for evaluating a structure's behaviour during all phases of construction. As research [17] also shows, this investigation is one of the first bricks placed along the path towards establishing such an analysis-framework.

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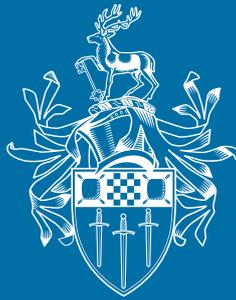
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