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CONTENTS

Insights for stochastic dominance extension in a multistage framework

Sebastiano Vitali [1]

Abstract

Stochastic dominance has proven to be an efficient tool to compare random variables. This is particularly true in financial application when one wants to compare the outcome of an optimal strategy with the outcome of a given benchmark. A classical example is the Asset and Liability Management problem that aims at defining the optimal allocation of the assets of a Pension Fund. The recent literature proposes univariate stochastic dominance constraints to guarantee that the optimal strategy is able to stochastically dominate a benchmark portfolio. The purpose of this work is (1) to provide an extended literature of the recent findings about stochastic dominance in pension fund management and (2) to propose some hints to investigate new formulations of stochastic dominance that could be particularly important when multivariate random variables are involved and when one of their dimension is the time.

Key words

Stochastic Programming, Pension Fund, Stochastic Dominance

JEL Classification: G11, C61

1. Introduction

One of the main area of application of the Asset and Liability Management (ALM) is the Pension Fund (PF) problem, meaning the capability of optimally managing the assets of the PF to guarantee the sustainability of the liabilities. Such topic has been widely analysed in the last decades. Pflug and Świetanowski (1999) proposed an ALM model for pension funds considering specific characteristics of both assets and liabilities. Consigli and Dempster (1998a,b) and Dempster et al. (2003) presented the CALM model that considers different types of pension contracts over a long-term horizon. For a review, we suggest Ziemba and Mulvey (1998) and Zenios and Ziemba (2006, 2007). More recent and advanced formulations of ALM models can be found in Consigli et al. (2011), Consigli and di Tria (2012), Consigli and Moriggia (2014), Consigli et al. (2017), and Moriggia, Kopa and Vitali (2019). Some applications focus not only on the pension fund manager's point of view, but also on the pension fund sponsor's (the issuer of the pension fund), see e.g. Vitali, Moriggia and Kopa (2017), and the pension fund investor's, see e.g. Consigli (2007), Consigli et al. (2012), Kopa, Moriggia and Vitali (2018) and Consigli, Moriggia and Vitali (2020).

In such contest, scholars focused on an efficient tool to compare the outcome of the optimal investment strategy with the outcome that the pension fund would produce with the business-as-usual allocation strategy. Of course, on the two random variables that represent the two outcomes, several statistics can be computed and compared. Still, recently, the researches in this area prefer not to rely simply on a set of relevant statistics, and prefer to compare the whole

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distributions of the random variables. In this perspective, the tool chosen by most of the researchers is Stochastic Dominance (SD).

The concept of SD was introduced more than 50 years ago and it was firstly applied to economics and finance in Quirk and Saposnik (1962), Hadar and Russell (1969) and Hanoch and Levy (1969). Two main formulations of SD are used in financial applications: First-order Stochastic Dominance (FSD) and Second-order Stochastic Dominance (SSD). As it will discussed in next section, the FSD turns out to be a too restrictive requirement and, thus, in many applications SSD is preferred. The SSD constraints were included in stochastic programs in Dentcheva and Ruszczynski (2003) and Luedtke (2008) and in portfolio efficiency analysis in Kuosmanen (2004), Dupačová and Kopa (2012) and Kopa and Post (2015). In multistage stochastic optimization, the stochastic sources evolve in times over a stochastic tree where each realization of the uncertain variables is contained in a node of the tree and the collection of consecutive nodes from the initial one (the root of the tree) and any of the last ones (the leaves of the tree) is called scenario. The SSD constraints in multistage ALM models have been widely investigated, see e.g. Yang, Gondzio and Grothey (2010), Kopa, Moriggia and Vitali (2018), Consigli, Moriggia and Vitali (2020) and Moriggia, Kopa and Vitali (2019). In all these papers, the SD constraints have been applied either on a single stage or on multiple stages simultaneously. The idea of applying SD constraints jointly on multiple stages is called Multistage Stochastic Dominance (MSD). Such approach requires the dominance on several stages, but, in all the mentioned works, the way in which a random variable dominates another random variable in a given stage is completely independent by the dominance relation in another stage. This means that applying SD constraints on multiple stages defines a dominance relation among nodes, while, in a multistage framework, it would be much more reasonable to have a dominance among scenarios so that we could be able to say “whichever scenario will happen, the one random variable dominates the other”.

2. Multivariate stochastic dominance

In the next definitions of FSD and SSD, let’s assume that the stochastic tree is represented as follows. The set of stages is denoted $\mathbb{T} = \{t_h\}_{h=0,\dots,H}$, where t_0 represents the here-and-now stage, and t_H represents the final horizon. The scenario tree is represented with the nodal notation, then each node is denoted by a unique index n. For each node n we define $t(n)$ as the corresponding stage time.

The definition of FSD relation is as follows: A random variable A FSD dominates a random variable B if the cumulative distribution function of A is below that of B. Equivalently, the FSD holds if and only if no rational and insatiable decision maker prefers B to A. When random variables are discrete with equiprobable realizations (as for most of the stochastic optimization problems in a multistage environment) it is useful to formulate the FSD conditions using a permutation matrix as proposed in Kuosmanen (2004). In particular, if we define $\mathbf{A}_{t_h}$ the vector of the optimal portfolio wealth realizations occurring in all nodes at stage t_h and similarly we define $\mathbf{B}_{t_h}$ the vector of a benchmark portfolio wealth realizations occurring in all nodes at stage t_h then the optimal portfolio FSD dominates the benchmark portfolio at stage t_h if and only if

$$\mathbf{A}_{t_h} \geq \mathbf{Q}^{t_h} \cdot \mathbf{B}_{t_h}$$

for some matrix $\mathbf{Q}^{t_h}$ which is a permutation matrix, i.e. satisfies the following conditions:

$$\sum_i \mathbf{Q}_{i,j}^{t_h} = 1$$

$$\sum_j \mathbf{Q}_{i,j}^{t_h} = 1$$

and the elements of $\mathbf{Q}^{t_h}$ belong to the set $\{0,1\}$. This set of constraints is a mixed-integer optimization problem.

The definition of SSD relation is as follows: A random variable A SSD dominates a random variable B if the integrated cumulative distribution function of A is below that of B. Equivalently, the SSD holds if and only if no risk-averse rational decision maker prefers B to A. Again, considering random variables with discrete and equiprobable realizations, it is useful to formulate the SSD conditions using a double stochastic matrix as proposed in Kuosmanen (2004). In particular, if we define $\mathbf{A}_{t_h}$ the vector of the optimal portfolio wealth realizations occurring in all nodes at stage t_h and similarly we define $\mathbf{B}_{t_h}$ the vector of a benchmark portfolio wealth realizations occurring in all nodes at stage t_h then the optimal portfolio SSD dominates the benchmark portfolio at stage t_h if and only if

$$\mathbf{A}_{t_h} \geq \mathbf{Q}^{t_h} \cdot \mathbf{B}_{t_h}$$

for some matrix $\mathbf{Q}^{t_h}$ which is double stochastic, i.e. satisfies the following conditions:

$$\sum_i \mathbf{Q}^{t_h}_{i,j} = 1$$

$$\sum_j \mathbf{Q}^{t_h}_{i,j} = 1$$

and the elements of $\mathbf{Q}^{t_h}$ belong to the interval $[0, 1]$, so each row and each column represents a convex combination. This set of constraints is a linear optimization problem.

Since the vector $\mathbf{A}_{t_h}$ and $\mathbf{B}_{t_h}$ are uni-dimensional, the FSD and SSD defined above are called *univariate* SD. The simplest evolution to *multivariate* SD when the further dimension is the time, can be obtained applying jointly multiple univariate SD. Indeed, the multistage SSD (or multivariate SSD) is obtained just by selecting jointly more than one stage, i.e. defining a subset $\mathbb{T}^{SSD} \subseteq \mathbb{T}$, and then defining the constraints

$$\mathbf{A}_{t_h} \geq \mathbf{Q}^{t_h} \cdot \mathbf{B}_{t_h}, \qquad \forall t_h \in \mathbb{T}^{SSD}$$

The same can be done with FSD constraints. Notice that the matrices $\mathbf{Q}^{t_h}$ can differ from stage to stage. This means that the dominance relation at stage t_i is not related with the dominance relation at stage $t_j, i \neq j$. Clearly, this is in contrast with the idea that an optimal solution dominates another one *through* the stages. Indeed, let's imagine to walk through a scenario and assume that $\mathbf{Q}^{t_i}$ establishes a dominance relation between A and B such that a specific node $n^a, t(n^a) = t_i$ is *linked* with node $n^b, t(n^b) = t_i$. Now, let's move to the next stage and consider the successor $\tilde{n}^a$ of node n^a and the successor $\tilde{n}^b$ of node n^b. It is possible that the matrix $\mathbf{Q}^{t_i+1}$ does not establish a *link* between $\tilde{n}^a$ and $\tilde{n}^b$ which means that it is not possible to conclude that A dominates B *through* the stages, but only over a set of stages.

Thus, it is clear that in a multistage framework a new definition of SD is needed. Recently, some studies have been performed in Dentcheva and Wolfhagen (2015), Dentcheva and Wolfhagen (2016) and Dentcheva, Martinez and Wolfhagen (2016) where the authors propose to linearly aggregate the random variables over the stages. Then, the realizations $\mathbf{A}_{t_h}$ of the random variable A are aggregated with a convex combination to generate a unique vector $\mathbb{A}$, similarly, the realizations $\mathbf{B}_{t_h}$ of the random variable B are aggregated to generate $\mathbb{B}$ and, finally, they establish a SD between $\mathbb{A}$ and $\mathbb{B}$. Still, this formulation does not directly address the issue of imposing an SD relation among scenarios and we believe that it must be possible to define a specific multistage SD that impose a dominance among the scenarios, and not among the nodes. Probably, the best way is to work on the definitions of matrices $\mathbf{Q}^{t_h}$ that must be put into relation among themselves.

3. Conclusion

In this work, we provide an extended literature review for SD and its application in ALM models, and we remind the definition of SD for univariate random variables. The idea of SD, as it is known until now, does not provide an easy and effective tool to compare multivariate distributions, especially when one of the dimension is the time and the distributions are nested through the time. Some hints have been provided by in Dentcheva and Wolfhagen (2015) that propose to aggregate the realizations over the stages through a convex combination. Such procedure has several drawbacks among which the main two are: the difficulties in the implementation, and the too complex interpretation of the resulting dominance relation. We believe that working on the matrices $\mathbf{Q}^{t_h}$ it can be possible to define a dominance among scenarios which would be the best achievement in terms of SD when the optimal solution evolves over a multistage scenario tree.

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Out-of-Sample Performance Evaluation of Mean-Variance Model Extensions

Anlan Wang[1], Aleš Kresta[2]

Abstract

In the mean-variance model proposed by Harry Markowitz in 1952, the trade-off between the return and risk of a portfolio is realized by using the mean and variance of assets' returns. Although the framework of mean-variance analysis is straightforward, several shortcomings have been demonstrated by practitioners in the applications. In this paper, we apply 6 extensions of the mean-variance model in the empirical analysis and make evaluations of the out-of-sample performances. In our study, the chosen dataset of the empirical analysis is the daily adjusted closing prices of 27 components of Dow Jones Industrial Average covering the period from January 3, 2006, to April 30, 2021. By applying the rolling window approach, we find that the minimum-CVaR strategy outperforms the minimum-variance strategy in the case of risk-minimizing, however, the maximum Sharpe Ratio strategy is proved as not robust. Last but not least, the extensions of the mean-variance model which are designed to reduce the estimation errors make almost no difference in the out-of-sample performance compared to the classical mean-variance model.

Key words

Mean-variance model, performance measures, portfolio optimization, random-weights portfolios, risk measures.

JEL Classification: G11, G17

1. Introduction

Harry Markowitz formulated Modern Portfolio Theory (henceforth MPT) in 1952 (Markowitz, 1952). It explains how to construct investment portfolios in a quantitative way. In the core idea of MPT, the trade-off between the return and the risk of a portfolio is balanced by using the mean and variance of assets' returns, so, the model proposed by Markowitz is also named as the mean-variance (henceforth MV) model. In the past, the MV model has become the foundation of the research on portfolio selection problems, however, although its framework is straightforward to understand, in practice, several shortcomings of the MV model have been demonstrated. According to the review on literature of MV analysis, we list three main shortcomings.

1.1 Limitations of using variance as the risk measure

In the optimization procedures of the MV analysis, the variance is used as the risk measure. Variance reflects both the upside and downside deviations over the mean value. In this sense, outperformance, as well as underperformance of the investments, are treated just as the same. Although the assumption of normal/symmetric distribution of assets' returns makes the application of variance more reasonable, in empirical studies the historical returns can be fat-

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tailed, for this reason, more alternative risk measures which only observe the unfavorable downside risk have been proposed, such as the semivariance that measures the dispersion of observations that fall below the mean, see Estrada (2008) and Ben Salah *et al.* (2018), or the Conditional Value at Risk (henceforth CVaR) that quantifies the amount of tail risk, see Xu *et al.* (2016) and Rachev *et al.* (2005).

1.2 Concentration of generated weights

According to the pioneering works on the MV analysis, the MV optimal portfolios have been proved as overly concentrated on few components with large expected returns, which go against the idea of diversification (Lin, 2013). To improve this shortcoming, several enhancements have been made to the classical MV model, for instance, except for the original constraints setting of MV portfolio optimization, other constraints targeting at the lower and upper bound of weights are considered.

1.3 Difficulty in estimating parameters

Another shortcoming of MV model is that there might be errors arisen in parameters' estimation. In the classical MV model, the weights of the components of a portfolio are generated by inputting the values of estimated parameters (i.e., mean and covariance matrix of the assets' returns), however, the input parameters in the MV model are estimated based on the historical trading data, which means even though the estimated parameters are descriptive, the estimation errors still might exist due to the differences between the history and the future actual status. So, to increase the robustness of the estimates, a wider range of parameter estimation methods are emerging, for instance, deep learning and machine learning models can be applied to predict future returns more effectively, see Ma *et al.* (2021). Moreover, the application of shrinkage estimators which shrink the unbiased vector of expected returns and the covariance matrix towards the biased targets can also improve the classical MV model, see Jorion (1986).

Various portfolio optimization models have been well developed in recent years, to compare the performance of the enhanced models to the classical MV model. DeMiguel *et al*. (2009) evaluate the out-of-sample performance of the MV model's extensions designed to reduce estimation error by using seven empirical datasets, while the evaluation results show that none of the extensions is consistently better than the naive portfolio in terms of the applied performance measures. Similarly, Martínez-Nieto *et al.* (2021) make a review of the diversification-based strategies and an experimental study with a complete repository of datasets with a total of 11 strategies. However, although there is already literature reviewing and evaluating the extensions of the classical MV model, there still lacks related works which systematically categorize the MV extensions and evaluate the performance by applying appropriate benchmarks, so, this paper contributes to these two knowledge gaps.

The goal of this paper is to categorize six extensions of the MV model according to the corresponding improvements, at the same time, the out-of-sample performance of the extensions is compared to that of the classical MV model as well as the generated random-weights portfolios.

This paper is structured as follows. In section 2, we describe the formulations of MV model and the studied extensions. In section 3, we introduce the performance evaluation methods which are applied in the empirical analysis. We show the empirical results in section 4. At last, the paper is concluded in section 5.

2. Portfolio Optimization Models

In this section, firstly we introduce the formulation of the minimum-variance (henceforth MVA) strategy generated from the classical MV model. Secondly, we introduce the other two strategies that are also generated from the mean-risk model in which the risk measure is replaced with CVaR and the mean absolute deviation (henceforth MAD).

2.1 MVA Strategy

In Markowitz (1952), the weights of the optimal portfolio are generated through the tradeoff between the return and risk. Under the framework of MV model, assuming that there are N assets in a portfolio, then the MV portfolio's expected return $E(R_p)$ and variance ${\sigma_p}^2$ can be calculated as follows:

$$E(R_p) = w^T \cdot E(R) = \sum_{i=1}^{N} w_i \cdot E(R_i) \tag{1}$$

$${\sigma_p}^2 = w^T \cdot \boldsymbol{Q} \cdot w = \sum_{i=1}^{N}\sum_{j=1}^{N} w_i \cdot \sigma_{i,j} \cdot w_j\,, (i\,, j\ =\ 1, \ldots, N) \tag{2}$$

where w is the vector of assets weights, $E(R)$ is the vector of expected returns of assets, $\boldsymbol{Q}$ is a $N \times N$ covariance matrix.

In this paper, since we aim at evaluating the performance of the extensions of MV model, so, we generate the MVA strategy as a benchmark. As it literally means, in MVA strategy the objective is to minimize the portfolio variance, so, its optimization can be formulated as follows:

$$\begin{aligned} &\min\ w^T \cdot \boldsymbol{Q} \cdot w, \\ &\text{s.t.}\ \ \textstyle\sum_{i=1}^{N} w_i = 1, \\ &\qquad w_i \geq 0, i = 1, \ldots, N. \end{aligned} \tag{3}$$

2.2 Alternative Minimum-Risk Strategies

However, the variance is not always a perfect risk measure, because it makes no distinction between gains and losses, so, to overcome this shortcoming, more alternatives to variance have been proposed in the later literature. In particular, in our study, we apply two alternative risk measures in the portfolio optimization, one is the CVaR which quantifies the amount of unfavorable tail risk, the other one is the MAD which measures the absolute deviation from the mean of portfolio return.

2.2.1 Minimum-CVaR Strategy

CVaR is a risk measure that indicates the expected loss under the condition of exceeding Value at Risk (henceforth VaR). The CVaR of a portfolio can be calculated as follows:

$$CVaR_\alpha(R_p) = E\left(-R_p \middle| -R_p \geq VaR_\alpha(R_p)\right) \tag{4}$$

where α is the probability level which is usually set to 1% or 5%. Furthermore, the formulation of minimum-CVaR (henceforth MCV) strategy has the same framework as that of the MVA strategy, the only difference is that the risk measure in the objective function of MCV strategy formulation is replaced with CVaR.

2.2.2 Minimum-MAD Strategy

Comparing to the variance, MAD is more robust to the outliers of random returns. It can be calculated as follows:

$$MAD(R_p) = E(|R_p - E(R_p)|) \tag{5}$$

similarly, in the formulation of minimum-MAD (henceforth MMA) strategy, the risk measure in the objective function is changed to MAD.

2.3 Diversification Strategies

MV model is straightforward to understand, but it might generate efficient portfolios which are highly concentrated, in other words, the component assets with large expected returns might be overweighted, so, once there is an unexpected drop in the price of the overweighted asset, it could result in large losses. So, improving the diversification degree of the MV efficient portfolios is an important issue.

2.3.1 1/N Strategy

As it literally means, 1/N strategy is a strategy in which all the assets of the portfolio are invested at an equal weight $1/N$. In our study, we apply the 1/N strategy as a benchmark to compare its performance with that of the other MV extension designed to improve the diversification degree.

2.3.2 Maximum Sharpe Ratio Strategy

Under the framework of MV analysis, the optimal portfolio targeted at maximizing the expected portfolio return would invest all the wealth into the asset with the largest expected return, which means this maximum-return strategy would lose the advantage of the diversification of portfolio investment, so, in our study, instead of the maximum-return strategy, we apply a maximum-Sharpe Ratio (henceforth MSR) strategy which is also designed to maximize the performance of portfolio but expected to have a higher degree of diversification.

The Sharpe ratio (henceforth SR) is defined as the expected value of the difference between the portfolio return and the risk-free rate divided by the standard deviation of the portfolio return, it can be calculated as follows:

$$SR(R_p) = \frac{E(R_p - R_f)}{\sigma(R_p)} \tag{6}$$

where R_f is the risk-free rate. As for the formulation of the MSR strategy, it can be defined as follows:

$$\begin{aligned} &\max SR(R_p), \\ &\text{s.t.} \quad \textstyle\sum_{i=1}^{N} w_i = 1, \\ &\qquad\; w_i \geq 0, i = 1, \dots, N. \end{aligned} \tag{7}$$

2.4 Strategies Reducing Estimation Errors

In the MV model, the input parameters are estimated based on the historical data, however, the future actual trading data could be different from the history. Due to this reason, errors might exist in the estimated parameters. So, to increase the robustness of the estimates, in our study, we apply the Bayes-Stein shrinkage (henceforth BSS) estimation and the Fuzzy Probability (henceforth FP) estimation which are both designed to reduce the estimation errors of the classical historical sample method.

2.4.1 MVA Strategy in BSS Estimation

BSS estimation takes the subjective (a priori) assumption of the shape of the assets returns distribution into account, and the resulting (a posteriori) assumption is then a combination of the priori assumption and the probability distribution of the observed sample. In this paper, we apply the shrinkage estimation method suggested by Jorion (1986) in Bayesian portfolio selection problem, and by applying this method, the estimated expected returns of assets and the estimated covariance matrix are reformulated on the basis of historical sample estimation, see Kresta and Wang (2020).

2.4.2 MVA Strategy in Fuzzy Probability Estimation

Tanaka *et al.* (2000) proposed a fuzzy probability model which helps to handle the uncertainty of the individual assets returns distribution by involving the fuzzy theory. In the proposed fuzzy probability model, the possibility grade is introduced to reflect the similarity degree between the future state of the asset return and the same asset's m-th historical return. For the formulations of estimated fuzzy expected returns of assets and the estimated fuzzy covariance matrix, see Kresta and Wang (2020).

For both the BSS estimation and the FP estimation, we input the reformulated parameters to the classical MV model to generate the optimal portfolios which are objective to minimize the portfolio variance, by this way, we can obtain the MVA-BSS strategy and MVA-FP strategy.

3. Approaches to Evaluate Portfolio Performance

To evaluate the performance of the strategies, we apply three performance measures and three risk measures in our empirical analysis. At the same time, we also generate 5,000,000 random-weights portfolios as a benchmark to compare the performances with that of the strategies.

3.1 Performance Measures

For the performance evaluations, we estimate the strategies' mean return (henceforth MR), SR and Calmar ratio (henceforth CR). We have already introduced the calculations of the portfolio's MR and SR in equations (1) and (6), respectively. As for the CR, it is a drawdown-based ratio which can be defined as follows:

$$CR\left(R_p, T\right) = \frac{E(R_p - R_f)}{MDD(T)} \tag{8}$$

where MDD is the maximum drawdown, to be more specific, it is the maximum of the drawdown of investment wealth evolutions during a specific period, its calculation can be shown as follows,

$$MDD(T) = \max_{\tau \in (0,T)} D(\tau) = \max_{\tau \in (0,T)} \left[\max_{t \in (0,\tau)} X(t) - X(\tau) \right] \tag{9}$$

where $D(\tau)$ is the drawdown at time τ and $X(t)$ is the investment wealth at time t.

3.2 Risk Measures

To evaluate the risk of the strategies, we apply three risk measures, which are the standard deviation (henceforth STD), CVaR and MDD. The calculations of CVaR and MDD has been introduced in equation (4) and (9), respectively. As for the STD of portfolio returns, it can be calculated as the square root of the variance.

$$\sigma_p = \sqrt{{\sigma_p}^2} \tag{10}$$

3.3 Random-Weights Portfolios

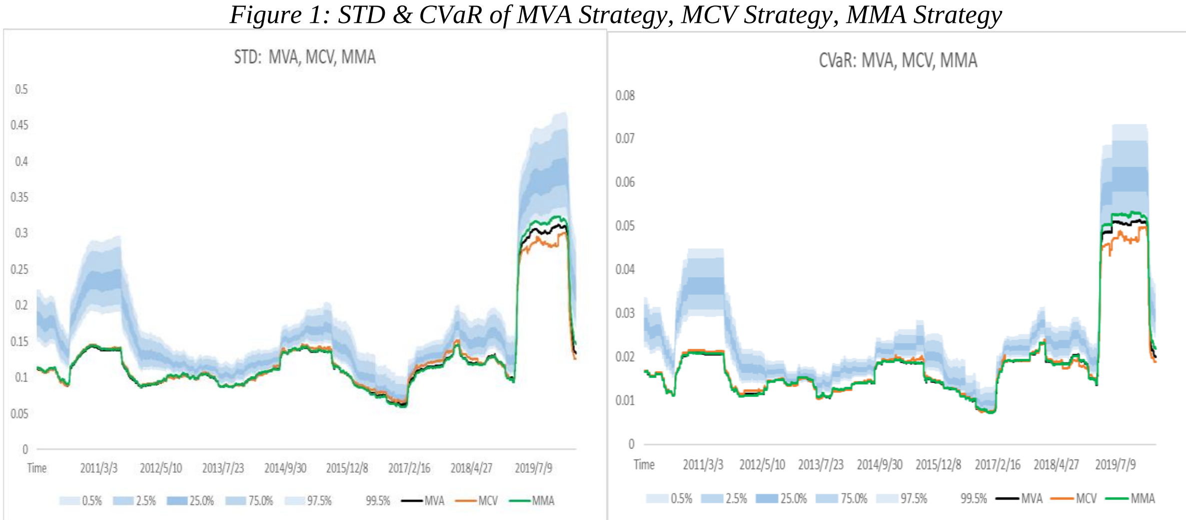
As it literally means, random-weights portfolios are the portfolios in which the assets are invested at random weights, and the method of generating the random weights can be referred to Kresta and Wang (2020). In our empirical study, we generate 5,000,000 random-weights portfolios as a benchmark, by comparing the performance of the random-weights portfolios with that of the strategies, we can verify the efficiency of the strategies.

4. Empirical Analysis

In this section, we make the empirical analysis. The chosen dataset of the analysis is the daily adjusted closing prices of 27 components of Dow Jones Industrial Average index (henceforth DJIA) which starts from January 3rd, 2006 to April 30th, 2021. Three DJIA stocks were excluded due to the incomplete data in the chosen period. In order to test the robustness of the strategies to the changes of the periods, the analysis is made by applying the rolling window approach, in particular, we always take 3 years (750 days) as the in-sample period and 1 year (250 days) as the out-of-sample period, then we move the start of the window day by day.

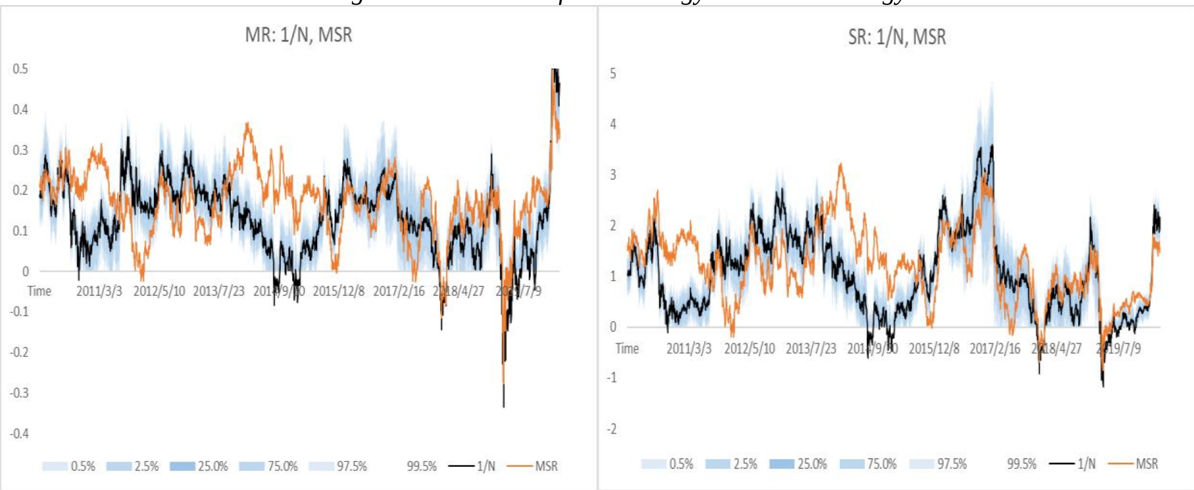
To evaluate the performance of the alternative mean-risk models, we generate the MCV strategy and the MMA strategy. In the meanwhile, to make the evaluation results more convincing, we apply the performance of MVA strategy and the performance of the random-weights portfolios as the benchmark. In Figure 1, the risks of all the strategies are evaluated by STD and CVaR, respectively. The blue shadow area is constructed by the different quantiles of the values of the corresponding risk measurement for all the 5,000,000 generated random-weights portfolios. We can see no matter for STD or CVaR, the risks of the three minimum-risk strategies are lower than that of the random investments, and by comparison, we find that the MCV strategy, intending to minimize the CVaR in the in-sample period, has the best out-of-sample performance.

Figure 1: STD & CVaR of MVA Strategy, MCV Strategy, MMA Strategy



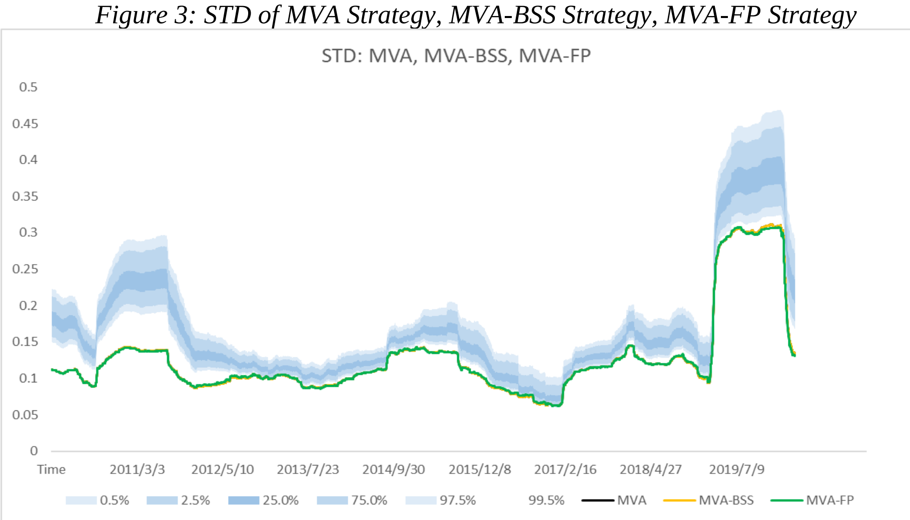
The other category of the strategies is composed of *1/N* strategy and MSR strategy. To evaluate the strategies' performance, we apply the performance measures MR and SR. From Figure 2, for both MR and SR, we can see that the performance of *1/N* strategy is on the average level of the performance of random investments, however, the performance of MSR strategy is not always better than that of the other portfolios, which means the MSR strategy aiming at maximizing the SR in the in-sample periods is not robust in the out-of-sample periods.

Figure 2: MR & SR of 1/N Strategy and MSR Strategy



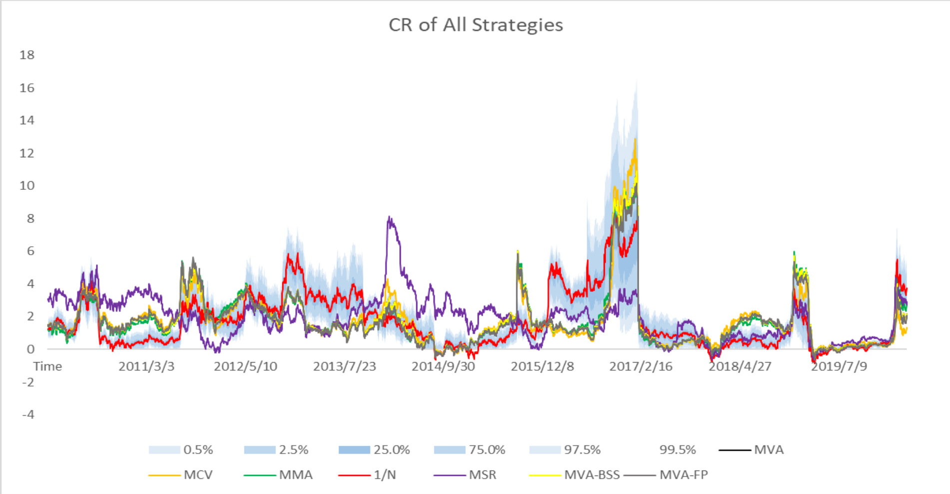
In section 2, we have introduced two extensions to the classical MV model which are designed to reduce the estimation errors. In Figure 3, we show the evolutions of STD of the applied strategies, while the results show that there is no big difference between the baseline MVA strategy and its extensions (MVA-BSS strategy and MVA-FP strategy) in variance-minimizing in the out-of-sample periods.

Figure 3: STD of MVA Strategy, MVA-BSS Strategy, MVA-FP Strategy



In Figure 4, the evolutions of CR of all the generated strategies (include the baseline MVA strategy) are presented, we can see that the values of CR of most strategies are on the average level of the random-weights portfolios, while the CR value of MSR strategy is most floating comparing to the others.

Figure 4: CR of All the Generated Strategies



5. Conclusion

In this paper, based on the review of the literature of MV portfolio optimization, we list three main shortcomings of the classical MV model. Since different extensions designed to improve the MV model have been developed in recent years, so, we choose six improved strategies and make the evaluations on the performance. In our empirical analysis, we apply the trading data of DJIA from January 3rd, 2006 to April 30th, 2021, furthermore, to verify the efficiency of the generated strategies, we compare the performance of the strategies with that of the baseline MVA strategy as well as the random-weights portfolios. From the empirical results, we make the following conclusions. Firstly, we find that when the portfolio risk is measured by STD and CVaR, the MCV strategy outperforms the MVA strategy in risk-minimizing in the out-of-sample periods. Secondly, no matter the portfolio performance is measure by MR, SR, or CR, we find the MSR strategy is not robust. Last but not least, the extensions of MV model which are designed to reduce estimation errors make almost no difference to the baseline MVA strategy in variance-minimizing in the out-of-sample periods.

Acknowledgments

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