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Doctoral Thesis

FINANCIAL MODELING FOR CREDIT RISK AND PORTFOLIO OPTIMIZATION

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Contents

List of Tables

List of Figures

Executive Summary

Financial Modeling for Credit Risk and Portfolio Optimization

The first research aims to establish a model in which the contribution of systemic risk can be evaluated using banks' structure inferred under the European Banking Authority Stress test exercise. The risk assessment is performed by a dynamic conditional correlation GARCH (DCC-GARCH) model with a spatial weight matrix based on the EU-wide stress test. Then, the model results are used to capture the spillover effect of the credit risk market through CoVaR. We show that the Student-t spatial DCC GARCH(1,1) model explains the best results on the credit risk market's contagion compared to other models.

The main goal of the second research is to investigate the impact of searching query data and volume on the portfolio optimization models. We consider the different portfolio strategies applied to the returns conditional on the Google Trends and volume information. Then, we optimize the corrected returns on the various portfolio optimization models. The results show that the proposed model can be used as a profitable strategy.

1. Introduction

This thesis consists of three research papers. The main focus is on building the financial models for credit risk on the first paper and portfolio optimization on the second and third papers. This chapter introduces the idea behind this thesis and reviews the essential articles that shed some light on the proposed models.

1.1. Credit Risk

In the second Chapter, we examine the banks' credit risk during the recent global financial crisis. The effect generated similar severe economic conditions with spillover effects of risk across the European countries. This interconnectedness of risk between the banks has become an increasingly hot topic. According to the linkage among the banks, the stand-alone measurement of a Value-at-Risk (VaR) of each bank cannot ensure an accurate risk measurement and cannot capture the risk among the banks. As a sequence, we focus on the multivariate GARCH model, which has a useful feature to capture the volatilities' interactions.

At the starting point, we look for the CCC-GARCH model introduced by Bollerslev (1990), which is computationally less complex than other multivariate models (see, among others, Bollerslev et al. (1988); Diebold and Nerlove (1989); Engle et al. (1990)). However, the CCC-GARCH model cannot allow the interactions between the volatilities. To overcome this drawback, we come up with the extended (E)CCC-GARCH model proposed by Jeantheau (1998). Until this stage, we draw a question of how to improve the estimated results among their interactions. The most interesting model is the spatial model (see, Borovkova and Lopuhaa (2012)), which can explain the interactions between geographic properties. The authors found that the spatial GARCH(1,1) can handle the better result of spillover effects than the GARCH(1,1). Considering the advantage of the multivariate GARCH and the spatial GARCH model, we decide to integrate the spatial weight into the multivariate model. Thus, we implement the similarity in credit exposure structure as the spatial components on the multivariate GARCH model.

Next stage, we apply the VaR test (see Zhang et al. (2018); Girardi and Ergün (2013)) to check whether the model can accurately predict the VaR. To better capture the contribution of systemic risk, we apply the conditional VaR (CoVaR) to a stressed situation considering the spillover of risk between a particular institution and financial system as documented by Adrian and Brunnermeier (2014); Girardi and Ergün (2013). Finally, we examine which model is the preferred model using the backtesting based on loss functions following Cesarone and Colucci (2016).

1.1.1. Credit Risk Model

In second chapter, we propose to consider the cosine similarity of exogenous information between the different issuers (banks). We collect the credit exposure information of each bank. Suppose two attribute vectors, $U_{i,L} = (u_{i,1}, u_{i,2}, \ldots, u_{i,L})$ and $U_{j,L} = (u_{j,1}, u_{j,2}, \ldots, u_{j,L})$ describe the credit exposure information of bank i and j with $i, j = 1, \ldots, N$. We define "the degree of similarity" by the dot product and the vector length between two different banks:

$$C_{ij} = \frac{\sum_{l=1}^{L} u_{i,l} \times u_{j,l}}{\sqrt{\sum_{l=1}^{L} u_{i,l}^2 \times \sum_{l=1}^{L} u_{j,l}^2}}, \; i, j = 1, \ldots, N \tag{1.1}$$

Then, we normalize the rows of the cosine similarity matrix $C = [C_{ij}]$ by dividing each row for each sum of the row. Doing so, we obtain the matrix $\boldsymbol{W} = [\boldsymbol{W}_{ij}]$ that is the spatial weight matrix $(N \times N)$. The most attractive feature of this spatial weight matrix is that the higher the cosine similarity provides, the stronger the connectedness. We then plug into the spatial Dynamic Conditional Correlation (DCC) GARCH model proposed by Borovkova and Lopuhaa (2012). This model allows systematic dependence between neighbors, and it can be expressed for the banks $i = 1, \ldots, N$ at time t as:

$$\boldsymbol{h}_t = \boldsymbol{A}_0 + \sum_{k=1}^{q} (\boldsymbol{A}_{1,k} + \boldsymbol{A}_{2,k}\boldsymbol{W}) \boldsymbol{r}_{t-k}^2 + \sum_{k=1}^{p} (\boldsymbol{B}_{1,k} + \boldsymbol{B}_{2,k}\boldsymbol{W}) \boldsymbol{h}_{t-k}, \tag{1.2}$$

where:

$\boldsymbol{h}_t$ is the vector of the univariate conditional variances, $\boldsymbol{h}_t = [h_{1,t}, h_{2,t}, \ldots, h_{N,t}]^\top$,

$\boldsymbol{r}_{t-k}^2$ is the vector of squared returns, $\boldsymbol{r}_{t-k}^2 = \left[r_{1,t-k}^2, r_{2,t-k}^2, \ldots, r_{N,t-k}^2\right]^\top$,

p, q are order of the GARCH model,

$\boldsymbol{A}_0, \boldsymbol{A}_{1,k}, \boldsymbol{A}_{2,k}, \boldsymbol{B}_{1,k}, \boldsymbol{B}_{2,k}$ are the parameters of model, which

- $\boldsymbol{A}_0$ is the vector, $\boldsymbol{A}_0 = [a_{0,1}, a_{0,2}, \ldots, a_{0,N}]^\top$,
- $\boldsymbol{A}_{1,k}$ and $\boldsymbol{B}_{1,k}$ are the $(N \times N)$ diagonal matrix,
- $\boldsymbol{A}_{2,k}$ and $\boldsymbol{B}_{2,k}$ are the $(N \times N)$ matrix.

From the idea of the time-varying correlation between the financial system and a particular institution, Girardi and Ergün (2013) suggested that the conditional CoVaR is covering more severe distress events than an ordinary CoVaR (see, Adrian and Brunnermeier (2014)). To measure the contribution of CoVaR to systematic risk, we first find the estimated parameters of the Spatial DCC-GARCH model. Then, we calculate the CoVaR from the acquired parameters. The results show that the Student-t spatial DCC GARCH(1,1) model explains the best results on the credit risk market's contagion compared to other models. This study focuses on Chapter 2, namely “The Spatial Multivariate GARCH Model on Credit Risk Application”.

1.2. Google Trends

In the third Chapter, the idea is raising a question about how the information impacts the financial portfolio as the study of Danah and Kate (2012). The usage of big data for access the human information has been found in various fields of studies. The data has also been investigated in the sophisticated human behaviors such as spatial location, public health, Twitter, internet stock message board, and others (see, González et al. (2008); Krings et al. (2009); Haklay (2010); Zheng et al. (2013), Haklay (2010), Bollen et al. (2011), Antweiler and Frank (2004)). Particularly, Google Trends data used to analyze in the field of economics, medical services, information systems, and several others, as documented by Jun et al. (2018). As Google Trends can provide data with respect to human behaviors, we focus on using it as useful information to implement a portfolio selection model. Further, the use of Google Trends from the economic point of view found the predictable behavior of the economic activity, the investment strategy, and the stock market (see, Choi and Varian (2012); Heiberger (2015); Vlastakis and Markellos (2012); Preis et al. (2013)). In particular, Rujirarangsan and Ortobelli (2019) suggested that the relationship between the stock returns and Google Trends seem to have a significant relation. This result can ascertain the link between financial data and human behavior. We apply conditional expectation. To evaluate the conditional stock return on Google Trends information, we apply the conditional expectation using the Gaussian and Epanechnikov kernel function. The bandwidth is set following the Scott (2015).

For enhancing the portfolio allocation, we apply the second stochastic dominance rule to penalized the returns. Recall that the stochastic dominance rule provides a precise decision method to order the return distribution. In recent studies, stochastic dominance has been applied in the portfolio application, for instance, market portfolio efficiency in Kopa (2010); Kopa and Post (2015), robustness analysis of optimal portfolios in Dupačová and Kopa (2014), and Portfolio Choice in Post and Kopa (2017).

Our analysis proposes four optimization models applied to penalized returns: Sharpe ratio, CVaR, Sortino ratio, and Rachev ratio. The Sharpe ratio calculates the return with risk-free compensates by the risk or the standard deviation, see Sharpe (1966). In the CVaR optimization, we use the coherent risk measure introduced by Rockafellar and Uryasev (2000) to overcome the limits of value at risk. The Sortino ratio is defined as the ratio between the expected active portfolio return and the semi-standard target deviation of the underperforming portfolio (see Sortino and Price (1994)). With this measure of risk, only the downside deviation can be quantified as risky. We use the quadratic optimization proposed by Stoyanov et al. (2007) in order to maximize the Sortino ratio. Last, the Rachev ratio introduced by Biglova et al. (2004) is the performance measure that compared the extreme positive returns to the extreme negative returns at a certain level of the quantile.

1.2.1. Portfolio Selection that account Google Trends Information

Chapter 3, entitled "Impact of Google Trends on Portfolio Optimization," considers the Google Trends information for enhancing portfolio optimization. In this study, we use the penalized returns in a portfolio analysis framework to examine portfolio performance. The penalization method considers two different kinds of penalizations: the first based on the Google Trends (GT) information and the second based on momentum strategy. After that, we optimize the Sharpe ratio, CVaR, mean-variance, mean-CVaR, Sortino ratio, and Rachev ratio models applied to penalized returns. We found that using approximated in the different portfolio models provides outstanding results with respect to the classical model.

In this framework, we first compute the GT returns by applying the logarithmic returns on the GT data, $GT_j = \ln\left(\frac{gt_j}{gt_{j-1}}\right)$, $j = 1, \ldots, N$. In this context, we penalized the return when it is not coherent with GT interests or the non-isotonic news (we say that news is isotonic with returns r when $r \cdot GT > 0$. In this Chapter, no short sales are allowed; thus, we apply the first penalization, called one-size penalization, to consider that we avoid short sales and speculation, $(r < 0 \,\&\, GT > 0)$. And the second subcase, called two-size penalization, penalized the non-isotonic behavior between return and GT $(r > 0 \,\&\, GT < 0 \text{ or } r < 0 \,\&\, GT > 0)$. On the other case, we approximate the return conditional GT return for all the other situations. For the k^{th} asset, we have these approximated returns:

subcase 1 (one-size penalization):

$$\widetilde{r}_{k,(j)} = \begin{cases} -1 & \text{, for } r_{k,(j)} < 0 \quad \& \quad GT_{k,j} > 0 \\ \mathbb{E}\left(r_{k,(j)} | GT_{k,j-1}\right) & \text{otherwise} \end{cases} \tag{1.3}$$

and,

subcase 2 (two-size penalization):

$$\widetilde{r}_{k,(j)} = \begin{cases} -1 & \text{, for } r_{k,(j)} > 0 \,\&\, GT_{k,j} < 0 \quad \text{or} \quad r_{k,(j)} < 0 \,\&\, GT_{k,j} > 0 \\ \mathbb{E}\left(r_{k,(j)} | GT_{k,j-1} \right) & \text{otherwise} \end{cases} \tag{1.4}$$

where k is the asset and j is the data series.

Next, in the second type of the penalized model, we evaluate the impact of conditional expectation considering the penalized GT based on momentum strategy. In particular, we penalized the case that the last two weeks $\left(r_{[(j-10),(j)]} \right)$ of return distribution are worse in the second stochastic dominance sense (SSD) with respect to the previous two weeks $\left(r_{[(j-20),(j-11)]} \right)$.

subcase 1 (historical returns penalization):

$$\widetilde{r}_{k,(j)} = \begin{cases} -1 & \text{, for } r_{k,[(j-20),(j-11)]} \overset{SSD}{>} r_{k,[(j-10),(j)]} \\ r_{k,(j)} & \text{otherwise} \end{cases} \tag{1.5}$$

and,

subcase 2 (conditional expectation penalization):

$$\widetilde{r}_{k,(j)} = \begin{cases} -1 & \text{, for } r_{k,[(j-20),(j-11)]} \overset{SSD}{>} r_{k,[(j-10),(j)]} \\ \mathbb{E}\left(r_{k,(j)} | GT_{k,j-1} \right) & \text{otherwise} \end{cases} \tag{1.6}$$

In this subcase 1, we penalize that the recent returns (last two weeks) are worse than the previous one (past two weeks), but we do not use the conditional returns on GT information. On the other hand, in subcase 2, we penalize recent returns, which are worst than the past (like in some momentum strategies), and we use conditional returns on GT information. Then, we turn all the cases into portfolio optimization.

We use the Sharpe ratio, global minimum CVaR, Sortino ratio, and Rachev ratio models for the optimization. To analyze the past performance, we use the different backtesting of in-sample/out-of-sample periods. The results show that the applied penalty-based correction gives a profitable strategy.

1.3. The Use of Volume in Portfolio Selection Model

In the last Chapter 4, entitled “Impact of Volume on Portfolio Optimization,” we implement a similar strategy using trading volume as information we get from Google

Trends. This assumption's motivation is that the volume explains the rate of information that flows into the stock market (see, Ying (1966); Westerfield (1977); Karpoff (1987); Gervais et al. (2001)). Moreover, there is an existing significant dynamic correlation between the stock return and volume, as documented by Lamoureux and Lastrapes (1990); Chen et al. (2001); Lee and Rui (2002).

In this context, we study how the conditional we study stock returns on volumes information impacts the portfolio performance. We approximate the conditional expectation using the Gaussian and Epanechnikov kernel density function. Even in this case, we use penalized stock returns as we have done for GT information. Then, we optimize the portfolio performance by using Sharpe Ratio, global minimum $\text{CVaR}_{5\%}$, and Rachev Ratio.

1.3.1. Portfolio Selection that account Volume Information

In the last Chapter, we examine how the volume impacts the portfolio selection scheme. Recall that Ying (1966) and Westerfield (1977) found positive relationships between the absolute value of price changes and volume. Similarly, Karpoff (1987) documented that the rate of information flow can explain the evidence of the price-volume relationship in the stock market. The results also provide the behavior of relations between the volume to absolute price ratio and the market trend. Moreover, Gervais et al. (2001) revealed that the large trading volumes tend to induce large changes in the stock prices in the next future period.

Next, in the dynamic relation scheme, the stock price and volume positively correlate to volume. The Granger causality tests also show the persistence of its lagged relations; see Chen et al. (2001). Considering the volatility, Lee and Rui (2002) showed that the return volatility reacts to a causal relationship to the trading volume. Moreover, if we consider the volume as additional information, the forecast volatility model can be explained appropriately by the behavior of the stock returns (Lamoureux and Lastrapes (1990); Gallant et al. (2015)).

The change in stock return tends to occur on a high-volume day than a low-volume day, as suggested by Campbell et al. (1993). The results underlying this work explained that the buying or selling volume is associated with the stock return changes. Thus, the basic idea of this work is to implement the effects of volume returns and stock returns in portfolio strategies based on conditional expectation. Inspired by taking the volume as information to return, we investigate how the stock returns conditional volumes information impacts the portfolio performance.

To sum up, Chapter 2 theoretical aspects of the financial modeling on credit risk will be scrutinized. Chapter 3 will analyze the impact of Google Trends on portfolio optimization. Chapter 4 is about financial modeling of volume on portfolio optimization. Moreover, the dynamic correlation analyses and results will be defined in each chapter. Finally, the thesis concludes with Chapter 5.

2. The Spatial Multivariate GARCH Model on Credit Risk Application

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2.1. Introduction

Following the recent global financial crisis, several countries have simultaneously faced similar severe economic conditions with spillover effects of risk across the EU. The interconnectedness of risk between the banks is an increasingly hot topic. According to the linkage among the banks, the stand-alone measurement of a Value-at-Risk (VaR) of each bank can not capture the effect of risk among the banks.

Recently, the multivariate time-varying variance model has been playing a crucial role in estimating the risk interconnectedness. The constant conditional correlation GARCH (CCC-GARCH) proposed by Bollerslev (1990), is computationally less complex than other multivariate models (see, among others, Bollerslev et al. (1988); Diebold and Nerlove (1989); Engle et al. (1990)). However, the CCC-GARCH model cannot allow the interactions between the volatilities. To overcome this drawback, Jeantheau (1998) introduced the extended (E)CCC-GARCH. The possibility to model the volatility interactions motivate the use of spatial components to enhance credit risk measures.

Keiler and Eder (2013) introduced the systematic risk that integrates the interaction between the micro and macro stress situations as spatial econometrics parameters. Borovkova and Lopuhaa (2012) introduced the spatial GARCH to handle the spillover effects. In particular, the spatial weights are obtained from the GDP and from the market capitalization of the US and European countries' stock market and embedded in the extended CCC-GARCH model. As a result, they better capture the high kurtosis of squared returns. As an alternative of multivariate GARCH model, a BEKK model proposed by Baba et al. (1991) provide the positive definite on the conditional covariance matrices. Thus, this model can ensure non-negative estimated variances. However, it needs a high computations due to the large numbers of parameters. Chen (2017) analogously showed that when the spatial weights are derived from credit rating downgrades, the multivariate spatial BEKK-GARCH model can capture the spillover effects among the southern European stock index:

Portugal, Italy, Ireland, Greece, and Spain (PIIGS). According to credit risk applications, the spillover effects have been paid less attention. Zhang et al. (2018) applied the multivariate GARCH with a dynamic panel of spatial weight matrices based on the GDP. The method encounters the countries' interconnectedness of returns and uses the estimated parameter to forecast the portfolio risk of six stock indices. Then an application for testing on VaR is used.

Our contribution is to extend the work of Borovkova and Lopuhaa (2012) intoducing a dynamic conditional correlation GARCH (DCC-GARCH) model with spatial weights based on the structural similarity between banks derived from the EU-wide stress test. Then, the model results are used to capture the spillover effect of the credit risk market through CoVaR. We add the spatial components into DCC-GARCH to improve the accuracy of capturing the spillover effects. We estimate the spatial DCC-GARCH model and construct a confidence interval using block bootstrap to assess whether differences between estimation parameters.

We then compute the VaR and pairwise CoVaR, from the given estimated parameters. Lastly, we perform Kupiec (1995); Christoffersen (1998) to evaluate the statistical accuracy of VaR estimates and Abad et al. (2014); Caporin (2008); Cesarone and Colucci (2016) to compare of VaR estimates by loss functions methods.

The remainder is organized as follows. In Section 2.2, the spatial DCC-GARCH model is defined with the extension of Student-t distribution on the standardized residuals. Section 2.3 is devoted to the financial application. The data and preliminary analysis of spatial weights are studied in Section 2.4, and the empirical results are described in Section 2.5. Section 2.6 concludes.

2.2. Modelling and Inference

2.2.1. Spatial DCC-GARCH

In a financial context, most of the markets follows the efficient market hypothesis in which the ex-post returns cannot predict the return of today. Besides, the high volatility of today may influence the high volatility of tomorrow. This behavior is defined as volatility clustering or time-varying conditional variance. The GARCH process is used to capture the volatility clustering. According to the ARCH model illustrated by Engle (1982), let r_t be the return discrete-time process with zero means. The standardized disturbances ε_t are independent and identically distributed (iid) with zero mean, $E(\varepsilon_t \mid \varepsilon_{t-1}, \ldots) = 0$, and unit variance, $Var(\varepsilon_t \mid \varepsilon_{t-1}, \ldots) = 1$. Then, the ARCH($q$) process for return r_t is defined as

$$r_t = \sqrt{h_t}\varepsilon_t, \quad t = 1, \ldots, T \tag{2.1}$$

and

$$h_t = \omega + \sum_{k=1}^{q} \alpha_k r_{t-k}^2, \tag{2.2}$$

where h_t is the conditional variance of return, $\omega > 0$ and $\alpha_k \geq 0$. In practical cases, the conditional variance of the ARCH(q) model often needs a high number of lags (q) to gain more persistence of the process. For solving this issue, the past values of the conditional variance (h_{t-k}) are added to the ARCH(q) process. This gives rise to the definition of GARCH(p, q) process introduced by Bollerslev (1986) as follows

$$h_t = \omega + \sum_{k=1}^{q} \alpha_k r_{t-k}^2 + \sum_{k=1}^{p} \beta_k h_{t-k}, \tag{2.3}$$

where $\omega > 0$, $\alpha_k \geq 0$ and $\beta_k \geq 0$.

In the portfolio perspective, when the comovement of assets is considered simultaneously, the covariance turns out to be a key element to be modeled. Bollerslev et al. (1988) introduced the multivariate GARCH (MGARCH) to estimate the conditional covariance of each asset of the portfolio. Consider a portfolio of N assets at time $t = 1, \ldots, T$. The following are the quantities of interest:

- $\boldsymbol{r}_t$ is the vector of returns of the asset i ($i = 1, \ldots, N$)
- $\boldsymbol{H}_t$ is the conditional covariance matrix
- $\boldsymbol{h}_t$ is the vector of the univariate conditional variances
- $\boldsymbol{R}_t$ is the positive definite conditional correlation matrix
- $\boldsymbol{Q}_t$ is the conditional covariance of standardized residuals
- $\bar{\boldsymbol{Q}}$ is the unconditional covariance matrix of the standardized residuals
- $\boldsymbol{D}_t$ is the conditional standard deviation matrix

Given the N assets Equation (2.3) becomes

$$\boldsymbol{r}_t = \sqrt{\boldsymbol{h}_t} \boldsymbol{\varepsilon}_t, \tag{2.4}$$

and

$$\boldsymbol{h}_t = \boldsymbol{\omega} + \sum_{k=1}^{q} \boldsymbol{\alpha}_k \boldsymbol{r}_{t-k}^2 + \sum_{k=1}^{p} \boldsymbol{\beta}_k \boldsymbol{h}_{t-k}, \tag{2.5}$$

where $\boldsymbol{h}_t = [h_{1,t}, h_{2,t}, \ldots, h_{N,t}]^\top$, $\boldsymbol{r}_{t-k}^2 = \left[r_{1,t-k}^2, r_{2,t-k}^2, \ldots, r_{N,t-k}^2\right]^\top$, $\boldsymbol{\omega}$ is the $N \times 1$ dimensional vector of unconditional variances with $\boldsymbol{\omega} \in \mathbb{R}^+$, $\boldsymbol{\alpha}_k$, and$\boldsymbol{\beta}_k$ are the N dimensional matrices of ARCH and GARCH parameters of order q and p with $\boldsymbol{\alpha}_k \in \mathbb{R}_0^+$, $\boldsymbol{\beta}_k \in \mathbb{R}_0^+$.

The correlation of errors among assets is a crucial part of the multivariate model. The constant conditional correlation model (CCC) proposed by Bollerslev (1990) assumes that the conditional covariance matrix, $\boldsymbol{H}_t$, can be factorized into

$$\boldsymbol{H}_t = \boldsymbol{D}_t \boldsymbol{R} \boldsymbol{D}_t, \tag{2.6}$$

where the correlation matrix is assumed to be constant throughout the time ($\boldsymbol{R}_t = \boldsymbol{R} \forall t$) and the conditional standard deviation matrix is given by

$$\boldsymbol{D}_t = diag\,(\boldsymbol{h}_t)\,.$$

Hence, the generic element of conditional covariance matrix $\boldsymbol{H}_t$ is constructed as

$$[\boldsymbol{H}_t]_{ij} = \sqrt{h_{it}}\rho_{ij}\sqrt{h_{jt}}, \quad i \neq j;\, i,j = 1,\ldots,N. \tag{2.7}$$

The multivariate GARCH model with a dynamic conditional correlation structure (DCC), introduced by Engle (2002), improves the dynamic relationship, assuming a time-varying correlation matrix as follows

$$\boldsymbol{H}_t = \boldsymbol{D}_t \boldsymbol{R}_t \boldsymbol{D}_t \tag{2.8}$$

The dynamic correlation model allows $\boldsymbol{R}_t$ to be time-varying, and its dynamics is modeled assuming a GARCH(1,1) process for the covariance of the standardized residuals. Hence $\boldsymbol{R}_t$ is decomposed into

$$\boldsymbol{R}_t = diag(\boldsymbol{Q}_t^{-1})\boldsymbol{Q}_t diag(\boldsymbol{Q}_t^{-1}), \tag{2.9}$$

where

$$\boldsymbol{Q}_t = \bar{\boldsymbol{Q}}(1 - \gamma - \delta) + \gamma(\boldsymbol{\epsilon}_{t-1}\boldsymbol{\epsilon}_{t-1}^\top) + \delta \boldsymbol{Q}_{t-1}, \tag{2.10}$$

where γ and δ are DCC parameters. By following the GARCH model from Equation (2.3), the generic element of the time-varying conditional covariance matrix of the standardized residuals $[\boldsymbol{Q}_t]_{ij} = q_{ij,t}$ can be expressed as

$$q_{ij,t} \;=\; \bar{q}_{ij}\,(1 - \gamma - \delta) + \gamma(\epsilon_{i,t-1}\epsilon_{j,t-1}) + \delta q_{ij,t-1}, \tag{2.11}$$

The process will be mean-reverting as long as $0 < \delta < 1$ and $\gamma + \delta < 1$. In the particular case of $\gamma + \delta = 1$, the process will follow the exponential smoother matrix of the standard residuals, as described in Engle (2002). Finally, the generic conditional correlation

$$\rho_{ij,t} = \frac{q_{ij,t}}{\sqrt{q_{ii,t} q_{jj,t}}}, \tag{2.12}$$

can be written into matrix form as in Equation (2.9). Substituting the conditional correlation matrix into Equation (2.8), the DCC is given by

$$\boldsymbol{H}_t = \boldsymbol{D}_t \boldsymbol{R}_t \boldsymbol{D}_t = \boldsymbol{D}_t diag(\boldsymbol{Q}_t^{-1}) \boldsymbol{Q}_t diag(\boldsymbol{Q}_t^{-1}) \boldsymbol{D}_t. \tag{2.13}$$

Following Borovkova and Lopuhaa (2012), in order to enrich the model with a spatial component, we consider the vector of the conditional variances $\boldsymbol{h}_t$ and introduce a spatial matrix $\boldsymbol{W}$. The conditional variance is

$$\boldsymbol{h}_t = \boldsymbol{A}_0 + \sum_{k=1}^{q} (\boldsymbol{A}_{1,k} + \boldsymbol{A}_{2,k} \boldsymbol{W}) \boldsymbol{r}_{t-k}^2 + \sum_{k=1}^{p} (\boldsymbol{B}_{1,k} + \boldsymbol{B}_{2,k} \boldsymbol{W}) \boldsymbol{h}_{t-k}, \tag{2.14}$$

where $\boldsymbol{A}_0 = (a_{0,1}, \ldots, a_{0,N})^\top$, $\boldsymbol{A}_{1,k}$, $\boldsymbol{A}_{2,k}$, $\boldsymbol{B}_{1,k}$, and $\boldsymbol{B}_{2,k}$ are diagonal matrices. The term $\boldsymbol{W} = [\boldsymbol{W}_{ij}]$ is the weight matrix for banks i and j, with generic element w_{ij} $(i, j = 1, \ldots, N)$ such that $\sum_{j=1}^{N} w_{ij} = 1$ and $w_{ii} = 0 \; \forall i$:

$$\boldsymbol{W} = \begin{bmatrix} 0 & w_{12} & \cdots & w_{1N} \\ w_{21} & 0 & \cdots & w_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ w_{N1} & w_{N2} & \cdots & 0 \end{bmatrix}$$

Given this specification i^{th} element of $\boldsymbol{h}_t$ becomes

$$h_{t,i} = a_{0,i} + a_{1,i} r_{t-1,i}^2 + a_{2,i} X_{t-1,i} + b_{1,i} h_{t-1,i} + b_{2,i} Y_{t-1,i}, \tag{2.15}$$

where $X_{t-1,i} = \sum_{j=1}^{N} w_{ij} r_{t-1,j}^2$ and $Y_{t-1,i} = \sum_{j=1}^{N} w_{ij} h_{t-1,j}$ are exogenous variables. The introduction of the spatial component results in two exogenous spatial variables in the conditional variance equation and two additional parameters $a_{2,i}$ and $b_{2,i}$, which measure the influence of the aggregated lagged variances and squared returns of all the other banks. These two new variables measure the aggregated spillover effects.

2.2.2. ML Estimation of the multivariate spatial GARCH(1,1) model

When the standardized error $\boldsymbol{\epsilon}_t$ has a multivariate Gaussian distribution, the log-likelihood function of $\boldsymbol{r}_t = \boldsymbol{H}_t^{\frac{1}{2}}\boldsymbol{\epsilon}_t$ is defined as

$$\begin{aligned}\ln\left(L\left(\boldsymbol{\theta}\right)\right) &= -\tfrac{1}{2}\sum_{t=1}^{T}\left(N\log\left(2\pi\right)+\log(|\boldsymbol{H}_t|)+\boldsymbol{r}_t^{\top}\boldsymbol{H}_t^{-1}\boldsymbol{r}_t\right)\\ &= -\tfrac{1}{2}\sum_{t=1}^{T}\left(N\log\left(2\pi\right)+\log(|\boldsymbol{D}_t\boldsymbol{R}_t\boldsymbol{D}_t|)+\boldsymbol{r}_t^{\top}\boldsymbol{D}_t^{-1}\boldsymbol{R}_t^{-1}\boldsymbol{D}_t^{-1}\boldsymbol{r}_t\right),\end{aligned} \tag{2.16}$$

where $\boldsymbol{\theta}$ is the vector of model's parameters. Let divide it into two sub vector $\boldsymbol{\theta} = (\boldsymbol{\xi}, \boldsymbol{\phi})$ where $\boldsymbol{\xi} = (\boldsymbol{A}_0, \boldsymbol{A}_1, \boldsymbol{A}_2, \boldsymbol{B}_1, \boldsymbol{B}_2)$ is matrix parameters of the spatial GARCH(1,1) and $\phi = (\gamma, \delta)$ are the parameters of the time-varying conditional correlation. The estimation of the correctly specified log likelihood is difficult, and hence the DCC model is designed to allow for two stage estimation.

We follow a two-steps procedure for the DCC-GARCH estimation, as described in Engle and Sheppard (2001) and Engle (2002). The first step is devoted to the estimation of (2.15) where the exogenous variable is not observable since it is a fuction of the conditional variance of the other assets. Hence, following Borovkova and Lopuhaa (2012) we first estimate the standard univariate GARCH(1,1) model to obtain the initial parameters $\left(a^0_{0,i}, a^0_{1,i}, b^0_{1,i}\right)$ and the estimated variances $\left(h^0_{1,i}, \dots, h^0_{T,i}\right)$. Then, given the weights (w_{ij}) and the initially estimated variances $\left(h^0_{2,i}, \dots, h^0_{T,i}\right)$ we compute a realizations of the exogenous variables $(Y_{t-1,i})$. Next, we estimate the complete set of parameters $\left(a^1_{0,i}, a^1_{1,i}, b^1_{1,i}, a^1_{2,i}, b^1_{2,i}\right)$ and the new estimated variances $\left(h^1_{2,i}, \dots, h^1_{T,i}\right)$ by following Equation (2.15).

We iterate this procedure till the estimated results percentage variation is less than a fixed value at 10^{-3}. The introduction of two exogenous variables allow to identify the presence of an aggregated spillover effect in the conditional variance equation. In the second step, we consider the correlation part by estimating the quasi log-likehood as follows

$$\begin{aligned}\ln\left(L_2(\phi|\hat{\xi})\right) &= -\tfrac{1}{2}\sum_{t=1}^{T}\left(N\log\left(2\pi\right)+2\log|\boldsymbol{D}_t|+\log(|\boldsymbol{R}_t|)+\boldsymbol{r}_t^{\top}\boldsymbol{D}_t^{-1}\boldsymbol{R}_t^{-1}\boldsymbol{D}_t^{-1}\boldsymbol{r}_t\right)\\ &= -\tfrac{1}{2}\sum_{t=1}^{T}\left(N\log\left(2\pi\right)+2\log|\boldsymbol{D}_t|+\log(|\boldsymbol{R}_t|)+\boldsymbol{\epsilon}_t^{\top}\boldsymbol{R}_t^{-1}\boldsymbol{\epsilon}_t\right),\end{aligned} \tag{2.17}$$

where ϕ is the parameters (γ, δ). Since $\boldsymbol{D}_t$ is constant, we can exclude it and maximize

$$\ln\left(L_2(\phi|\hat{\xi})\right) = -\tfrac{1}{2}\sum_{t=1}^{T}\left(\log(|\boldsymbol{R}_t|) + \boldsymbol{\epsilon}_t^\top \boldsymbol{R}_t^{-1}\boldsymbol{\epsilon}_t\right). \tag{2.18}$$

Moreover, we estimate the quasi log-likelihood function under the Student-t distribution that can be written as

$$\begin{aligned} L_2(\phi'\hat{\xi}) &= \sum_{t=1}^{T}\left(\log\left(\Gamma\left(\tfrac{\nu+N}{2}\right)\right) - \log\left(\Gamma\left(\tfrac{\nu}{2}\right)\right) - \tfrac{N}{2}\log\left(\pi\left(\nu-2\right)\right)\right) \\ &= -\tfrac{1}{2}\log\left(|\boldsymbol{D}_t\boldsymbol{R}_t\boldsymbol{D}_t|\right) - \tfrac{\nu+N}{2}\left(\log 1 + \tfrac{\boldsymbol{r}_t^T\boldsymbol{D}_t^{-1}\boldsymbol{R}_t^{-1}\boldsymbol{D}_t^{-1}\boldsymbol{r}_t}{\nu-2}\right), \end{aligned} \tag{2.19}$$

where ν is the degrees of freedom, $\Gamma(.)$ is the Gamma function, and ϕ' is the multivariate parameter of (γ, δ, ν). In this study, the BFGS[1] algorithm is employed to optimize the log-likelihood function.

2.2.3. Spatial matrix

To estimate the spatial DCC-GARCH describe in Section 2.2.1, we need to specify the weight matrix $\boldsymbol{W}$ which incorporates the spatial structure defined a priori. The most intuitive way to compute the weights is to consider the geographical distance between the issuers' market cities. However, according to Borovkova and Lopuhaa (2012), the obtained weight seems to be counter-intuitive after normalization, and so they consider a different set of information and compute distance in GDP and market capitalization as a measure of a system component among the individual returns. We propose to consider the cosine similarity between exogenous information relative to the issuers that the higher the cosine similarity, the stronger the connectedness. We collect the credit exposure information of each bank. Suppose two attribute vectors, $U_{i,L} = (u_{i,1}, u_{i,2}, \ldots, u_{i,L})$ and $U_{j,L} = (u_{j,1}, u_{j,2}, \ldots, u_{j,L})$ which describe the credit exposure information of bank i and j with $i, j = 1, \ldots, N$. We define "the degree of similarity" as follows:

$$C_{ij} = \frac{\sum_{l=1}^{L} u_{i,l}\cdot u_{j,l}}{\sqrt{\sum_{l=1}^{L} u_{i,l}^2 \cdot \sum_{l=1}^{L} u_{j,l}^2}},\ i, j = 1, \ldots, N. \tag{2.20}$$

[1]BFGS named after authors Broyden, Fletcher, Goldfarb, and Shanno (Broyden (1970); Fletcher (1970); Goldfarb (1970); Shanno (1970))

We set $C_{ii} = 0$ for $\forall_i$ and we normalize the rows of C by dividing each element by the sum of the row. Doing so, we obtain the matrix $\boldsymbol{W}_{N\times N}$ that is the spatial weight matrix. Then, we normalize the rows of the cosine similarity matrix $C = [C_{ij}]$ by dividing each element in a row for each sum of the row.

2.3. Financial application: CoVaR

In general, most financial institutions use VaR to measure the standalone risk where is implicitly defined as the q-quantile, i.e.,

$$\Pr\Big(r_{i,t} \leq VaR^{i}_{q,t}\Big) = q.$$

However, the measurement of individual risk is not able to explain the linkages with the financial system. During the financial crisis, the systematic risk spreads across the system and enlarges the massive spillover. The Conditional Value-at-Risk (CoVaR) denoted $CoVaR_q^{System|\mathbb{C}(r_i)}$ is implicitly defined by the q-quantile of the probability distribution of the financial system conditional on some event $\mathbb{C}(r_i)$ of the institution i, where r_i is the return of institution i and $q \in (0, 1)$ (see Adrian and Brunnermeier (2014))

$$\Pr\Big(r_{System}|\mathbb{C}\left(r_i\right) \leq CoVaR_q^{s|\mathbb{C}(r_i)}\Big) = q.$$

The CoVaR can capture the contribution of systemic risk by conditioning the VaR to a stressed situation considering the spillover of risk between a particular institution and the financial system. Inspired by this idea, we concentrate our attention on a CoVaR pairwise analysis between institutions.

From the time-varying variance models we compute the VaR and pairwise CoVaR. Girardi and Ergün (2013) argue that in this setting the conditioning on CoVaR can cover more severe distress events than the CoVaR of Adrian and Brunnermeier (2014). Following the same methodology we compute $CoVaR_{q,t}^{j|i}$ solving

$$\Pr\left(r_{j,t} \leq CoVaR_{q,t}^{j|i}, r_{i,t} \leq VaR_{q,t}^{i}\right) = q^2, \tag{2.21}$$

then, we assume alternatively a bivariate Gaussian and Student-t density of return denoted by *pdf* $(r_{j,t}, r_{i,t})$ for solving double integral

$$\int_{-\infty}^{CoVaR_{q,t}^{x|y}} \int_{-\infty}^{VaR_{q,t}^{y}} pdf(x,y)dydx = q^2. \tag{2.22}$$

Last, we numerically compute the integral on a grid of values (starting from -10 to 10 with 0.01 increment step) for $CoVaR_{q,t}^{j|i}$ to find the approximated solution.

2.3.1. Backtesting based on VaR and CoVaR

In order to determine the accuracy of model among the ones proposed, we consider two tests based on the number of violations. The first is the Kupiec test or unconditional coverage test (Kupiec (1995)). The observed failure rate equal to the failure rate suggested by the confidence level of VaR, is tested: the null hypothesis is given by the observed violation rate statistically equal to the expected violation rate. If the null hypothesis is rejected, the model is considered inaccurate with 95% significance level ($p > 0.05$).

Denote, with a slightly change of notation in favour of readability, $R_t^i(x) = r_{i,t+1}$ as the ex-post returns of institution i with $t = 1, \dots, N$, and $VaR_{q,t}^i$ is the ex-ante of Value-at-Risk forecasts, where q is the expected coverage. Let define the indicator function as follows

$$I_t^i = \left\{ \begin{array}{ll} 1, & \text{if} \quad R_t^i(x) \leq VaR_{q,t}^i \\ 0, & \text{if} \quad R_t^i(x) > VaR_{q,t}^i \end{array} \right\},$$

where I_t^i is a sequence of violation for a given interval of the Value-at-Risk forecast. In the case of the backtesting of CoVaR proposed by Girardi and Ergün (2013), the indicator function is constructed as a first hit sequence for the losses of each institution (I_t^i) and a second hit sequence for the losses of the institution j conditional to institution ($I_t^{j|i}$). We define the second hit sequence, $R_t^j(x)$, by the sub-sample in which $R_t^i(x) \leq VaR_{q,t}^i$. Thus, the number of observations of the second hit sequence is equal to the number of violations of the first hit sequence. The second hit sequence compares between the past ex-post returns of the financial system and the ex-ante of $CoVaR_{q,t}^{s|i}$ forecasts,

$$I_t^{j|i} = \left\{ \begin{array}{ll} 1, & \text{if} \quad R_t^j(x) \leq CoVaR_{q,t}^{j|i} \\ 0, & \text{if} \quad R_t^j(x) > CoVaR_{q,t}^{j|i} \end{array} \right\},$$

where $I_t^{j|i}$ is a second hit sequence of violation for a given interval of the CoVaR forecast. In this study, we will show only the CoVaR $\left(I_t^{j|i}\right)$ definition for the following description of unconditional and conditional coverage tests. For the tests on VaR, the sequence of violation I_t^i can be used instead of the $I_t^{j|i}$.

Assuming that $I_t^{j|i}$ is identically and independently distributed Bernoulli with parameter q, $I_t^{j|i} \sim Bernoulli(q)$. The null hypothesis of unconditional coverage test ($H_{0,uc}$) defines the number of observed violations is equal to the expected coverage, $q = \hat{q}$. The likelihood under this null hypothesis can be written as

$$L\left(I^{j|i}, q\right) = (1-q)^{N-V_I}\, q^{V_I},$$

where $V_I = \sum_{t=1}^{N} I_t^{j|i}$ is the number of violations of $CoVaR_{q,t}^{j|i}$. Then, the unconditional coverage test can be formulated as a likelihood ratio (LR) test,

$$LR_{uc} = 2\ln\left[L\left(I^{j|i}, \hat{q}\right) - L\left(I^{j|i}, q\right)\right],$$

where $\hat{q} = \frac{V_I}{N}$ is the maximum likelihood estimate of q, and LR_{uc} is asymptotically to $\chi^2(1)$. Next, the likelihood ratio test of independence proposed by Christoffersen (1998) is used to check whether violations are independently distributed over time. Let consider the indicator variable, $\left\{I_t^{j|i}\right\}_{t=1,\ldots,N}$, as a first-order Markov chain with transition probability matrix

$$\Pi = \begin{bmatrix} 1-\pi_{01} & \pi_{01} \\ 1-\pi_{11} & \pi_{11} \end{bmatrix},$$

where the $R_t^j(x)$ is the sub-sample in which $R_t^i(x) \leq VaR_{q,t}^i$:

- π_{01} is probability ($\Pr_{t-1}$) that the conditional on today being a non violation $\left(R_t^j(x) > CoVaR_{q,t}^{j|i}\right)$ next period is a violation $\left(R_t^j(x) \leq CoVaR_{q,t}^{j|i}\right)$.
- π_{11} is probability ($\Pr_{t-1}$) that the conditional on today being a violation $\left(R_t^j(x) \leq CoVaR_{q,t}^{j|i}\right)$ next period is a violation $\left(R_t^j(x) \leq CoVaR_{q,t}^{j|i}\right)$.
- $1-\pi_{01}$ is probability ($\Pr_{t-1}$) that the conditional on today being a non violation $\left(R_t^j(x) \leq CoVaR_{q,t}^{j|i}\right)$ next period is a non violation $\left(R_t^j(x) > CoVaR_{q,t}^{j|i}\right)$.
- $1-\pi_{11}$ is probability ($\Pr_{t-1}$) that the conditional on today being a violation $\left(R_t^j(x) \leq CoVaR_{q,t}^{j|i}\right)$ next period is a non violation $\left(R_t^j(x) > CoVaR_{q,t}^{j|i}\right)$.

The null hypothesis of the conditional coverage test, $H_{0,ind} : \pi_{01} = \pi_{11}$, is that the violation indicators do repeat over the period of losses. The approximate likelihood function under this hypothesis is

$$L\left(I^{j|i}; \pi_{01}, \pi_{11}\right) = \left(1-\pi_{01}\right)^{N_{00}} \pi_{01}^{N_{01}} \left(1-\pi_{11}\right)^{N_{01}} \pi_{11}^{N_{11}},$$

where N_{mn} is the number of observations that state m followed by n. From the null hypothesis, the previous observations do not affect the probability of considering a violation. The estimation of π_{01} and π_{11} can be written as $\hat{\pi}_{01} = \frac{N_{01}}{N_{00}+N_{01}}$ and $\hat{\pi}_{11} = \frac{N_{11}}{N_{00}+N_{11}}$. Then, the LR test statistic for independent test under the null of, $\hat{\pi}_{01} = \hat{\pi}_{11} = \hat{q}$, is given by

$$LR_{ind} = 2\ln\left[L\left(I^{j|i}, \hat{\pi}_{01}, \hat{\pi}_{11}\right) - L\left(I^{j|i}, \hat{q}\right)\right],$$

where $\hat{\kappa} = \frac{N_{01}+N_{11}}{N} = \frac{V_I}{N}$, and LR_{ind} is asymptotically to $\chi^2(1)$. From the combination of the unconditional coverage test and independence test, the joint test or conditional coverage test can be performed as documented by Christoffersen (1998). The null hypothesis of this test is $H_{0,cc} : \hat{\pi}_{01} = \hat{\pi}_{11} = q$. In case the null hypothesis of $H_{0,uc}$ or $H_{0,ind}$ is rejected, the $H_{0,cc}$ is also rejected. The likelihood ratio becomes

$$LR_{cc} = 2\ln\left[L\left(I^{j|i}, \hat{\pi}_{01}, \hat{\pi}_{11}\right) - L\left(I^{j|i}, q\right)\right],$$

where LR_{cc} is asymptotically to $\chi^2(2)$.

To assess the goodness of risk on a stand-alone basis and among the interconnectedness, we analyze the unconditional and conditional coverage test on VaR and CoVaR. We can apply the test to the CoVaR for those time periods for which the condition event $(R_t^i(x) \leq VaR_{q,t}^i)$ is true.

In the backtesting based on VaR analysis, we first iterate the estimated results by setting the backtest windows as 250-week in-sample and 1-week out-of-sample. We then count the number of data that fall outside the confidence level of VaR estimates. If the number is higher than the confidence level, we observe it as a violation. As for the unconditional coverage test proposed by Kupiec (1995), the number of violations must be equal to VaR's correct exceedance. In comparison, the conditional coverage test proposed by Christoffersen (1998) indicates that the number of violations must be independently distributed along the testing period or correct exceedance. This test can prevent the unusual frequency of consecutive exceedances.

2.3.2. Backtesting based on loss function

The backtesting based on the confidence level of VaR estimates shows the accuracy of an individual model. However, the comparison between the different models is limited. To overcome the drawback, Lopez (1999) proposed the backtesting based on a loss function. The method focuses on the magnitude of the failure when the violation occurs. Thus, the VaR estimates under the loss function can provide the model's performance as a numerical score. The loss function can be given as

$$l_t = \begin{cases} f\left(R_t, VaR_{t|t-1}\right) & \text{if } R_t \leq VaR_{t|t-1} \\ g\left(R_t, VaR_{t|t-1}\right) & \text{if } R_t > VaR_{t|t-1} \end{cases}$$

where $\sum_{t=1}^{N} l_t$ defines as the total loss. The best model can be classified by the lowest total loss. In this analysis, we use the comparison of loss functions methods as Abad et al. (2014), Caporin (2008), and Cesarone and Colucci (2016). The method defines the loss functions from the regulator and investors' point of view. In the regulator's view, we consider the size of loss only if the violation occurs

$\left(g\left(R_t, VaR_{t|t-1}\right) = 0 \text{ if } R_t > VaR_{t|t-1}\right)$. While, the investors' view considers both the loss and the market risk sides $\left(f\left(R_t, VaR_{t|t-1}\right) = g\left(R_t, VaR_{t|t-1}\right) \forall R_t\right)$, as shown in Table 2.1.

Table 2.1.: The list of regulator's and investors' loss functions.

	Regulator's view		Investors' view
	Loss side (if $R_t \le VaR_{t\|t-1}$)	Market risk side (if $R_t > VaR_{t\|t-1}$)	Loss & market risk sides ($\forall R_t$)
Loss Function	$f\left(R_t, VaR_{t\|t-1}\right)$	$g\left(R_t, VaR_{t\|t-1}\right)$	$f\left(R_t, VaR_{t\|t-1}\right) = g\left(R_t, VaR_{t\|t-1}\right)$
Lopez	$1 + \left(R_t - VaR_{t\|t-1}\right)^2$	0	-
Caporin 1	$\|1 - \|\frac{R_t}{VaR_{t\|t-1}}\|\|$	0	$\|1 - \|\frac{R_t}{VaR_{t\|t-1}}\|\|$
Caporin 2	$\frac{\left(\|R_t\| - \|VaR_{t\|t-1}\|\right)^2}{\|VaR_{t\|t-1}\|}$	0	$\frac{\left(\|R_t\| - \|VaR_{t\|t-1}\|\right)^2}{\|VaR_{t\|t-1}\|}$
Caporin 3	$\|R_t - VaR_{t\|t-1}\|$	0	$\|R_t - VaR_{t\|t-1}\|$

2.4. Data and preliminary analysis

2.4.1. CDS data

The credit default swap (CDS) is a kind of financial contract that allows protection against losses in the event of default. CDS enables trading on credit risk exposure to the reference entity. By the definition of International Swaps and Derivatives Association (ISDA), credit event includes as following: bankruptcy, obligation acceleration, obligation default, failure to pay, repudiation/moratorium, and restructuring.

In this study, we consider the modified-modified restructuring (MMR)[2] because, in the European CDS market, MMR shows the most volatile and complete time intervals on the data stream source.

We consider ten years of weekly data of seven representative banks of Italy, France, Germany, the United Kingdom, Netherlands, Spain, and Belgium. The data spam from April 10, 2008, to January 16, 2019, included 562 weeks. Table 2.2 reports the descriptive statistics and tests of the return of credit default swap (CDS) with a five-year term. In the first column, we provide the abbreviation for each bank:

- Intesa Sanpaolo S.p.A.-Italy (ISP)
- Crédit Agricole Group-France (ACA)
- Deutsche Bank AG-Germany (DB)

[2]According to the 2003 International Swaps and Derivatives Association (ISDA) Credit Derivatives Definitions, the modified-modified restructuring term explained that the remaining maturity of deliverable assets of the restructured obligations must be less than 60 months and other obligations must be less than 30 months.

- Barclays Plc-United Kingdom (BCS),
- Coöperatieve Rabobank U.A.-Netherlands (RAB)
- Banco de Sabadell S.A.-Spain (SAB)
- KBC Group N.V.-Belgium (KBC)

We test the normality by using the Jarque-Bera test. The null hypothesis of the test consists of the joint hypothesis that the skewness and the excess kurtosis is zero. The Ljung-Box (L-B) test checks whether the return is a white noise. The null hypothesis of the test is that the residuals are independently distributed. Table 2.2 shows the descriptive statistics of CDS returns for each bank.

All the bank returns show a p-values lower than 1%. For the white noise test, we investigate whether the returns are white noise and the squared returns are clustered, that the volatility in this period will influence the next period's volatility. The Ljung-Box test on returns (L-B[r]) shows that the p-values of ISP, ACA, DB, BCS, and SAB are less than the 5% significance rejects the null hypothesis of white noise. The p-value of L-B[r^2] are all less than 1%. Thus, the returns are containing volatility clustering. Next, we consider the correlation between the banks. We found that the returns are a highly significant positive correlated with each other, as shown in Figure 2.1. So, the returns of all banks tend to react in the same direction.

Table 2.2.: Descriptive statistics of CDS weekly returns.

Bank	Mean	Stdev	Skewness	Kurtosis	Normality	L-B [r]	L-B[r^2]
ISP	0.0014	0.1059	0.1087	3.8626	0.0020***	0.0098***	$5.55 \times 10^{-07***}$
ACA	-0.0013	0.1054	0.0874	3.9457	0.0005***	0.0632**	$4.76\times10^{-04***}$
DB	0.0013	0.1037	0.0780	6.1070	0.0000***	0.0001***	$1.34\times10^{-08***}$
BCS	-0.0004	0.1063	0.0416	8.3448	0.0000***	0.0014***	$1.08\times10^{-04***}$
RAB	-0.0009	0.0855	-0.0195	5.1053	0.0000***	0.1847	$9.44\times10^{-07***}$
SAB	-0.0005	0.0695	0.1520	5.3765	0.0000***	0.0021***	$5.83\times10^{-04***}$
KBC	-0.0022	0.0755	0.9436	18.0952	0.0000***	0.4542	$6.66\times10^{-14***}$

Note: *,**,*** indicates significance of p-value at the 10%, 5%, and 1% levels, respectively.

2.4.2. Spatial weight data

The methodology for the fitting of DCC-GARCH has been described extensively based on a two-step procedure, see section 2.2.

For the spatial weight approach, we analyze the data from the EU-wide stress testing under the European Banking Authority (EBA). This test aims to evaluate financial institutions' resilience to adverse market conditions. It also provides the overall assessment of systematic risk in the European banking system. In the EU-wide stress test analysis report, we consider the base scenarios for each bank. The credit

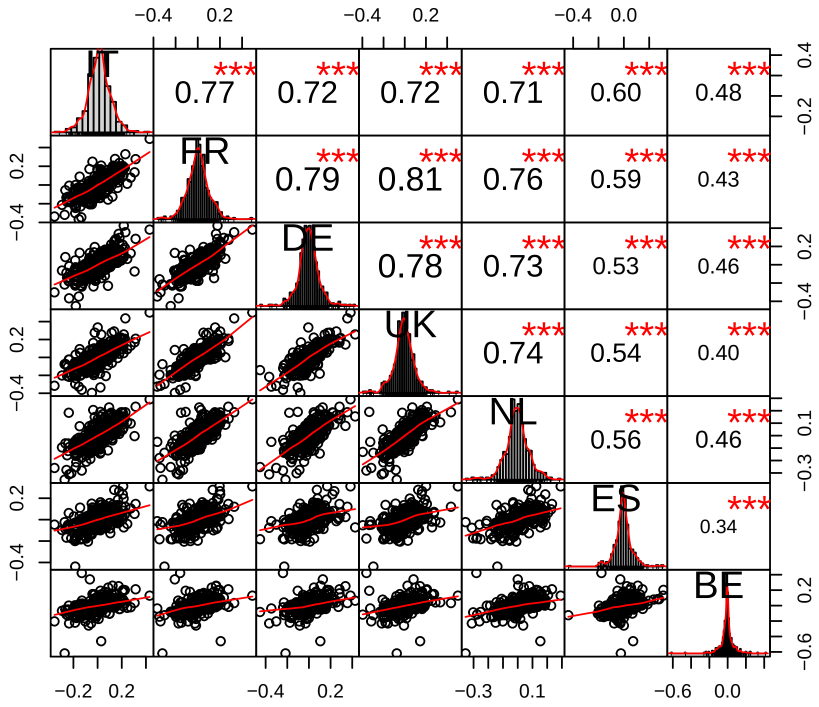

Figure 2.1.: Correlation and distribution of CDS weekly returns.

exposure information for each bank consists of four parts: exposure values, risk exposure amounts, a stock of provision, and leverage ratio under the internal ratings-based (IRB) approach or Standardized approach (STA) referred to credit exposure specific asset classes, such as: Central governments Institutions, Corporates, Retail, Equity, Securitization, and Other non-credit obligation assets, as presented on the EBA (2021)'s website. We use this information to compute the percentage values with respect to the total disclosure part.

We organize the pieces of information in a vector to compute the similarity between couples of banks. This indicator provides a broad view of the bank's credit structure and exposure. We then convert each row's values to the unity range $[0, 1]$, as shown in Table 2.3.

The spatial components of this study are computed using the EU-wide stress test of 2018. To ascertain the matrix weight's consistency, we test the equality of two matrices (Jennrich (1970)). The null hypothesis is $H_0 : \boldsymbol{W}_1 = \boldsymbol{W}_2$. We compare the normalized cosine similarity matrix weight from the EU-wide stress test of 2014 to 2016, 2016 to 2018, and 2018 to 2014. We do not reject the null hypothesis. The p-values are 0.9991, 0.9999, and 0.9999 that higher than the significant level at 0.05. Thus, this evidence indicates that the spatial components of the EU-wide stress test are not exogenous.

Table 2.3.: Normalized of cosine similarity matrix from the cosine similarity matrix.

Bank	ISP	ACA	DB	BCS	RAB	SAB	KBC
ISP	0	0.1764	0.1693	0.1256	0.1610	0.1834	0.1843
ACA	0.1614	0	0.1593	0.1523	0.1681	0.1823	0.1766
DB	0.1654	0.1700	0	0.1617	0.1584	0.1771	0.1674
BCS	0.1333	0.1766	0.1758	0	0.1652	0.1818	0.1673
RAB	0.1548	0.1765	0.1559	0.1495	0	0.1761	0.1871
SAB	0.1648	0.1790	0.1628	0.1538	0.1646	0	0.1750
KBC	0.1682	0.1761	0.1564	0.1438	0.1777	0.1778	0

2.5. Empirical Results

2.5.1. Block Bootstrap

The spatial DCC-GARCH(1,1) is estimated according to the procedure described in Section 2.2.2. To obtain a confidence interval, we use a block bootstrapping technique. We construct the 95% confidence intervals from 500 resamples by using block bootstrap. Firstly, this method requires cutting the CDS dataset sample into several blocks of equal dimension. To take into consideration the serial correlation of each bank data, we use four lagged returns for each block. Then all blocks are reconstructed into a new 500 resamples. After that, we estimate the Student-t spatial DCC GARCH(1,1) on all resamples.

Next, we estimate the Student-t spatial DCC GARCH(1,1) with the CDS dataset and compare them with its confidence intervals from the block bootstrap, as shown in Table 2.4. The results show that the estimated parameters are mostly specified within the confidence ranges.

Moreover, we estimate the student-t spatial DCC GARCH(1,1) model using the CDS weekly data, as shown in Table 2.5. We found that the estimated parameters on the first stage show all significance in GARCH terms while only significant in ARCH terms and not significant in unconditional variance terms. In the second stage, the parameter of the time-varying conditional correlation (δ) and the degree of freedom (ν) are significantly performed (p-value in brackets). The γ is relatively small, while the δ is large with a degree of freedom (ν) at 4.69. These results can reinforce the model's parameters before we apply them to the subsequent analysis. Considering the spatial components we observe that, apart from Intesa and Deutsche bank, the spatial volatility spillovers are indeed present among the credit risk of the considered banks. Generally, it seems that the spatial (G)ARCH and (G)ARCH parameters compensate each other: the greater is one, the lower is the other. Spatial ARCH coefficients are larger than spatial GARCH ones, indicating that the largest squared returns from other banks matter more for the future volatility levels than the previous values of other banks volatilities. For ISP, ACA, DB, BCS, and RAB the previous volatility of other banks matter more than the most recent innovations.

Table 2.4.: The Student-t spatial DCC GARCH(1,1) parameters and its confidence intervals from 500 samples of block bootstrap of CDS data.

Parameter/Bank		ISP	ACA	DB	BCS	RAB	SAB	KBC
A_0	5% CI	-5.10e-05	-3.24e-05	-3.60e-05	-7.40e-06	-1.98e-05	-8.67e-04	-7.52e-05
		5.62e-14	**1.02e-15**	**7.20e-14**	**4.66e-14**	**2.66e-16**	**2.52e-16**	**2.86e-16**
	95% CI	3.35e-05	2.36e-05	2.39e-05	5.28e-06	1.35e-05	6.97e-04	5.88e-05
A_1	5% CI	2.13e-06	-2.23e-06	-5.12e-03	-1.61e-03	-1.40e-02	-7.59e-02	4.96e-01
		2.86e-06	**2.69e-06**	**3.74e-06**	**3.96e-06**	**7.78e-06**	**9.23e-06**	**9.87e-01**
	95% CI	6.93e-06	9.48e-06	3.87e-03	1.23e-03	1.14e-02	6.01e-02	2.49
B_1	5% CI	-2.37e-01	-3.40e-01	-2.98e-01	-3.06e-01	-2.86e-01	-1.03e-01	2.31e-02
		5.04e-01	**4.78e-01**	**4.93e-01**	**5.15e-01**	**4.91e-01**	**4.66e-01**	**4.25e-01**
	95% CI	7.87e-01	8.31e-01	7.29e-01	8.50e-01	8.61e-01	8.82e-01	1.14
A_2	5% CI	-6.37e-01	-4.82e-01	-4.27e-01	-1.92e-01	-2.40e-01	-8.45e-01	-1.39e+01
		6.07e-07	**1.17e-06**	**3.30e-07**	**4.75e-06**	**1.04e-06**	**1.28e-07**	**1.01e-08**
	95% CI	3.35e-01	3.10e-01	2.94e-01	1.55e-01	1.93e-01	3.50e-01	7.20
B_2	5% CI	5.99e-02	4.42e-02	4.51e-02	3.73e-02	3.11e-02	2.16e-02	-8.66e-01
		6.51e-02	**6.39e-02**	**6.02e-02**	**6.51e-02**	**5.71e-02**	**4.78e-02**	**1.11e-08**
	95% CI	1.70e-01	1.66e-01	1.57e-01	1.70e-01	1.41e-01	1.30e-01	3.93e-01

γ	5% CI	5.45e-03
		1.58e-02
	95% CI	4.30e-02
δ	5% CI	4.36e-01
		8.58e-01
	95% CI	1.52
ν	5% CI	2.32
		4.69
	95% CI	5.78

2.5.2. Backtesting Results

In the application, we apply the estimated parameter results under different specifications of time varying volatility multivariate GARCH(1,1) models, which consists of Gaussian DCC (GaussDCC), Student-t DCC (tDCC), Gaussian spatial DCC (GaussSpDCC), and Student-t spatial DCC (tSpDCC).

The $VaR_{5\%}$ and $CoVaR_{5\%}$ are computed under the different model specifications, setting a rolling window of 250 data points to calculate the one data point ahead $VaR_{5\%}$ forecasts. The relative $CoVaR_{5\%}$ is computed numerically, according to (2.22) using the time varying covariance matrices.

We report the descriptive statistics of weekly CDS data for $VaR_{5\%}$ and $CoVaR_{5\%}$ in Tables 2.6, 2.7, 2.8, 2.9, and 2.10. We observed that the mean, standard deviation, and skewness of the $VaR_{5\%}$ and $CoVaR_{5\%}$ are relatively similar among different models except for the kurtosis.

We apply the two-sample Kolmogorov-Dmirnov test between the $VaR_{5\%}$ and $CoVaR_{5\%}$. The null hypothesis of the test is that $VaR_{5\%}$ and $CoVaR_{5\%}$ are drawn from a similar continuous distribution. The results show that the p-value of the GaussDCC, tDCC, GaussSpDCC, and tSpDCC models are all close to zeros. Thus, we reject the null hypothesis that there is no difference between the $VaR_{5\%}$ and $CoVaR_{5\%}$ for all models. The CDF function between VaR and CoVaR, as shown in Figures 2.2, 2.3, 2.4 and 2.5 is similarly found consistent with the Kolmogorov–Smirnov test.

Figure 2.2.: The cdf funtion between $VaR_{5\%}$ and $CoVaR_{5\%}$ of the Gaussian DCC model.

For the backtesting based on $VaR_{5\%}$, the analysis examines the accuracy of each model. The $VaR_{5\%}$ forecasts obtained on a rolling window of 250 data points of in-sample are compared with the one data point ahead out-of-sample. Tables 2.11 provides the p-value of the unconditional coverage (UC) and conditional coverage

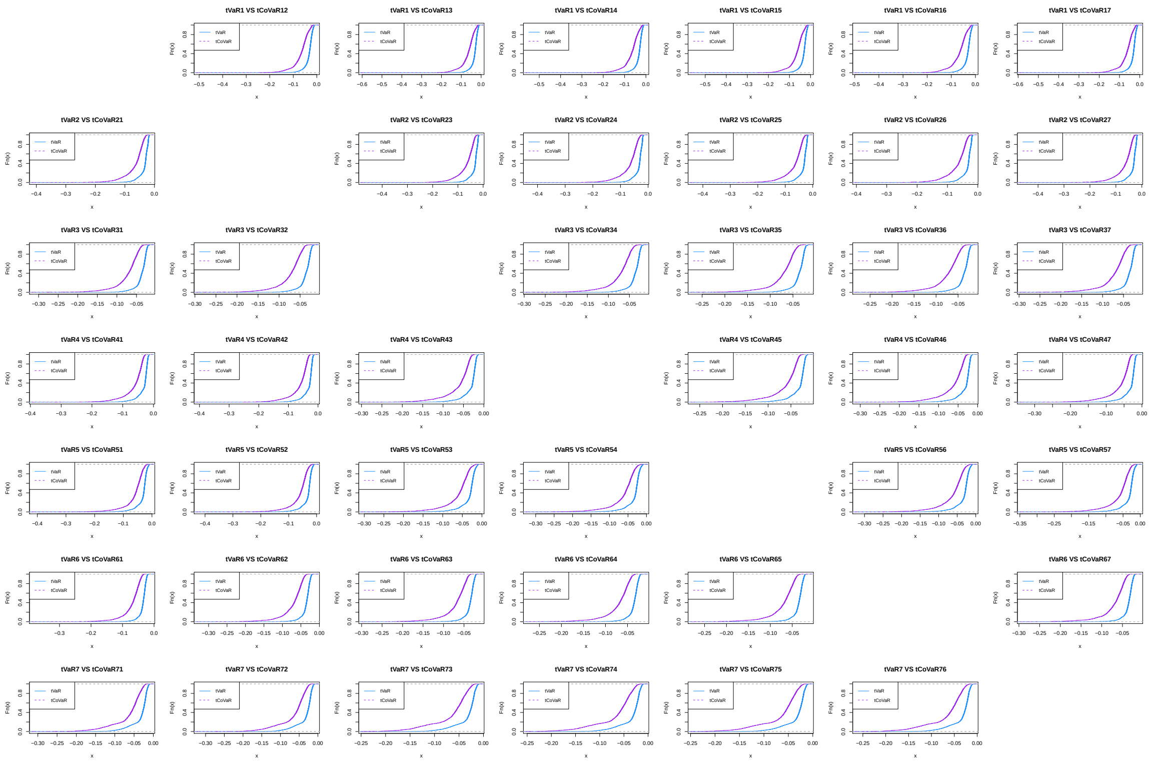

Figure 2.3.: The cdf funtion between $\text{VaR}_{5\%}$ and $\text{CoVaR}_{5\%}$ of the Student-t DCC model.

(CC) tests. We define the bold as accepted at a 95% significance level and the highlighted light-gray as accepted of model at a 99% significance level but rejected at 95% significance level.

In Table 2.11, we test the violations of $\text{VaR}_{5\%}$ on weekly data. The GaussDCC and GaussSpDCC models show within the UC test's acceptance range at a 99% significant level but one rejection case on the CC test at a 95% significant level. Inversely, the tDCC and tSpDCC models show within the CC test's acceptance range at a 99% significant level but one rejection case on the UC test at a 95% significant level.

To specify which model is the preferred model, we perform the backtesting based on loss functions following Cesarone and Colucci (2016). The procedure proposes that the model with the lowest total loss is the best. The $\text{VaR}_{5\%}$ backtesting in Table 2.12 where the best results are marked as bold, the tSpDCC model performs the best result from three out of four regulator's loss functions. For the investors' loss functions in Table 2.13, the tSpDCC model provides the best result.

According to the limitation of weekly data that strictly required samples from the $\text{VaR}_{5\%}$ violations $\left(R_t^i(x) \leq VaR_{q,t}^i\right)$, we extend the backtesting based on the $\text{CoVaR}_{5\%}$ test on the daily equity data of each bank instead. We obtain the daily equity data at 2610 data points by considering the same period of CDS data. There-

Figure 2.4.: The cdf funtion between $\text{VaR}_{5\%}$ and $\text{CoVaR}_{5\%}$ of the Gaussian spatial DCC model.

fore, the expected violations of the backtesting based on $\text{CoVaR}_{5\%}$ can provide around 5 data points from the $\text{VaR}_{5\%}$ violations of 118 data points, which are large enough to allow the test. We repeated the analysis on daily equity data. The main findings are confirmed, as highlighted in Appendix A.

For the backtesting based on $\text{CoVaR}_{5\%}$ of equity data, we first test the violations of $\text{VaR}_{5\%}$ forecast on each bank as same as the weekly CDS data, as described in Appendix A. Then, Table 2.14, we pairwise test the violation of $\text{CoVaR}_{5\%}$ forecast between the institution j and the institution i, for instance, if we consider a pair of the ACA (as institute j^{th}) to the ISP (as institution i^{th}), we obtain the p-value of the UC and CC test at 0.0054 and 0.0034 for the Gaussian DCC model.

Table 2.14 shows that the GaussDCC model presents only 15 acceptance cases of the UC test and 17 acceptance cases of the CC test at a 99% significant level. In contrast, at a 95% significant level, we found 3 and 3 rejection cases for the UC and CC tests. Further, the tDCC model found 28 and 28 acceptance cases of the UC and CC tests at a 99% significant level, but at a 95% significant level, we observe 5 and 3 rejection cases of the UC and CC tests. The results show evidence that the tDCC model is better than GaussDCC model as consistent with Girardi and Ergün (2013). Considering the GaussSpDCC model, we found 34 and 30 cases of the UC and CC test at a 99% significant level, while we found 4 and 5 rejection cases of the

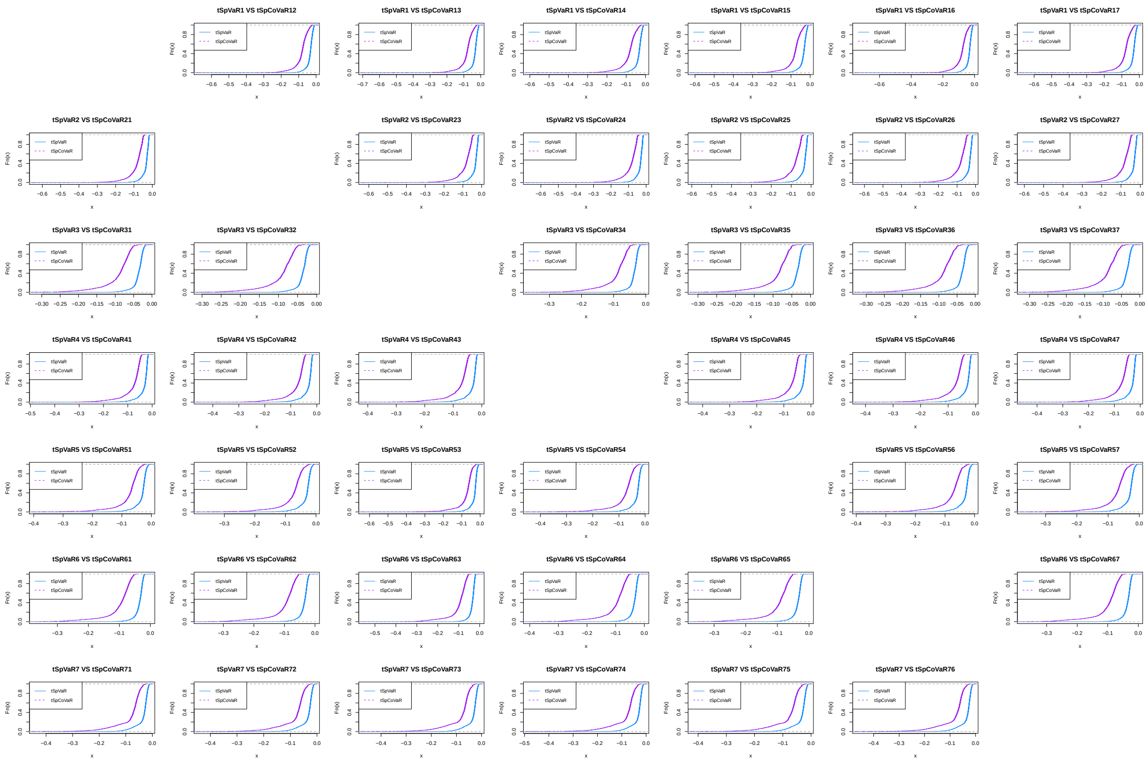

Figure 2.5.: The cdf funtion between $\text{VaR}_{5\%}$ and $\text{CoVaR}_{5\%}$ of the Student-t spatial DCC model.

UC and CC test at a 95% significant level. The tSpDCC model gives all accepted cases of the CC test at a 99% significant level and only 3 out of 42 cases of the UC test that rejected, Still, we observe 2 and 5 rejection cases of the UC and CC test at a 95% significant level. The tSpDCC model, moreover, presents the highest amount of the accepted model. Overall, from the four consideration models, the tSpDCC model shows the best performance.

For the $\text{CoVaR}_{5\%}$ backtesting based on loss functions, in Tables 2.15 and 2.16 we observe that the tSpDCC model shows the best result for all cases of the regulator's and investers' loss functions. Moreover, we observed that the differences between the GaussDCC and tSpDCC models are relatively large compared to the other models, as shown in Figures 2.17 and 2.18.

To sum up, we observe that the result of backtesting based on $\text{VaR}_{5\%}$ of weekly CDS data accepts all models at a 99% significant level for the UC and CC tests. While the backtesting based on $\text{CoVaR}_{5\%}$ of the daily equity data provides the applied spatial models (GaussSpDCC and tSpDCC) show a higher amount of acceptance cases than the ordinary models (GaussDCC and tDCC). Moreover, the tSpDCC model shows all acceptance cases (42 cases) for the CC test at a 99% significant level. To justify the preferred model, the backtesting based on loss functions of $\text{CoVaR}_{5\%}$ for both the regulator's and investors' loss functions report the tSpDCC model as the best.

2.6. Conclusions

As the spillover effects of risk become a problem across the interconnected banks, this study investigates the spatial multivariate GARCH model to provide more accuracy of risk measures. In particular, we propose a cosine similarity that appertains under the spatial multivariate GARCH(1,1) model.

We then apply to financial applications. We investigate the presence of the spatial volatility spillovers among the credit risk of the considerd banks. Next, we disentangle the pairwise contributions via the CoVaR analysis.

First, we examine the accuracy of the spatial multivariate GARCH model on credit risk application using the UC and CC tests. The result shows that the Student-t spatial DCC GARCH(1,1) model explains the highest amount of the accepted model on $\text{CoVaR}_{5\%}$ compared to other models.

Second, we investigate the preferred model using the backtesting based on loss functions. We find that the Student-t spatial DCC GARCH(1,1) model on $\text{CoVaR}_{5\%}$ are the most preferred model compared to other models.

In summary, the multivariate GARCH model with proposed cosine similarity can improve the assessment of credit risk profiles. The Student-t spatial DCC GARCH(1,1) model provides the best results on the credit risk market's spillover.

Table 2.5.: Estimated results of Student-t spatial DCC GARCH(1,1) model (p-value in brackets).

Panel A: The First Stage of Student-t Spatial DCC GARCH(1,1) Model

$$
\begin{bmatrix} h_{1,t} \\ h_{2,t} \\ h_{3,t} \\ h_{4,t} \\ h_{5,t} \\ h_{6,t} \\ h_{7,t} \end{bmatrix}
=
\begin{bmatrix} \underset{(1.0000)}{5.62\times10^{-14}} \\ \underset{(1.0000)}{1.02\times10^{-15}} \\ \underset{(1.0000)}{7.20\times10^{-14}} \\ \underset{(1.0000)}{4.66\times10^{-14}} \\ \underset{(1.0000)}{2.66\times10^{-16}} \\ \underset{(1.0000)}{2.52\times10^{-16}} \\ \underset{(1.0000)}{2.86\times10^{-16}} \end{bmatrix}
+
\left(
\begin{bmatrix}
\underset{(0.9999)}{2.86\times10^{-6}} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \underset{(1.0000)}{2.69\times10^{-6}} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \underset{(0.9999)}{3.74\times10^{-6}} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \underset{(0.9999)}{3.96\times10^{-6}} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \underset{(0.9999)}{7.78\times10^{-6}} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \underset{(0.9999)}{9.23\times10^{-6}} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \underset{(0.0000)}{\mathbf{9.87\times10^{-1}}}
\end{bmatrix}
+
\begin{bmatrix}
\underset{(1.0000)}{6.07\times10^{-7}} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \underset{(1.0000)}{1.17\times10^{-6}} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \underset{(1.0000)}{3.30\times10^{-7}} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \underset{(0.9998)}{4.75\times10^{-6}} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \underset{(1.0000)}{1.04\times10^{-6}} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \underset{(1.0000)}{1.28\times10^{-7}} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \underset{(0.9992)}{1.01\times10^{-8}}
\end{bmatrix} W
\right)
\begin{bmatrix} r^2_{1,t-1} \\ r^2_{2,t-1} \\ r^2_{3,t-1} \\ r^2_{4,t-1} \\ r^2_{5,t-1} \\ r^2_{6,t-1} \\ r^2_{7,t-1} \end{bmatrix}
+
$$

$$
\left(
\begin{bmatrix}
\underset{(0.0000)}{\mathbf{5.04\times10^{-1}}} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \underset{(0.0000)}{\mathbf{4.78\times10^{-1}}} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \underset{(0.0000)}{\mathbf{4.93\times10^{-1}}} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \underset{(0.0000)}{\mathbf{5.15\times10^{-1}}} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \underset{(0.0000)}{\mathbf{4.91\times10^{-1}}} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \underset{(0.0000)}{\mathbf{4.66\times10^{-1}}} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \underset{(0.0000)}{\mathbf{4.25\times10^{-1}}}
\end{bmatrix}
+
\begin{bmatrix}
\underset{(0.0034)}{\mathbf{6.51\times10^{-2}}} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \underset{(0.0174)}{\mathbf{6.39\times10^{-2}}} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \underset{(0.0000)}{\mathbf{6.02\times10^{-2}}} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \underset{(0.0000)}{\mathbf{6.51\times10^{-2}}} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \underset{(0.0044)}{\mathbf{5.71\times10^{-2}}} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \underset{(0.1190)}{4.78\times10^{-2}} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \underset{(0.9988)}{1.11\times10^{-8}}
\end{bmatrix} W
\right)
\begin{bmatrix} h_{1,t-1} \\ h_{2,t-1} \\ h_{3,t-1} \\ h_{4,t-1} \\ h_{5,t-1} \\ h_{6,t-1} \\ h_{7,t-1} \end{bmatrix}
$$

Panel B: The Second Stage of Student-t Spatial DCC GARCH(1,1) Model

$$
\begin{bmatrix} Q_{1,1,t} & \cdots & Q_{1,7,t} \\ Q_{2,1,t} & \cdots & Q_{2,7,t} \\ \vdots & \ddots & \vdots \\ Q_{7,1,t} & \cdots & Q_{7,7,t} \end{bmatrix}
=
\left(1 - \underset{(0.2540)}{1.58\times10^{-2}} - \underset{(0.0002)}{\mathbf{8.58\times10^{-1}}} \right)\bar{Q} + \underset{(0.2540)}{1.58\times10^{-2}}
\begin{bmatrix} \epsilon_{1,t-1} \\ \epsilon_{2,t-1} \\ \epsilon_{3,t-1} \\ \epsilon_{4,t-1} \\ \epsilon_{5,t-1} \\ \epsilon_{6,t-1} \\ \epsilon_{7,t-1} \end{bmatrix}
\begin{bmatrix} \epsilon_{1,t-1} \\ \epsilon_{2,t-1} \\ \epsilon_{3,t-1} \\ \epsilon_{4,t-1} \\ \epsilon_{5,t-1} \\ \epsilon_{6,t-1} \\ \epsilon_{7,t-1} \end{bmatrix}^{\top}
+ \underset{(0.0002)}{\mathbf{8.58\times10^{-1}}}
\begin{bmatrix} Q_{1,1,t-1} & \cdots & Q_{1,7,t-1} \\ Q_{2,1,t-1} & \cdots & Q_{2,7,t-1} \\ \vdots & \ddots & \vdots \\ Q_{7,1,t-1} & \cdots & Q_{7,7,t-1} \end{bmatrix}, \nu = \underset{(0.0000)}{\mathbf{4.69}}
$$

Table 2.6.: The descriptive of statistics of $\text{VaR}_{5\%}$.

	Statistic	Gaussian DCC	Student-t DCC	Gaussian spatial DCC	Student-t spatial DCC
ISP	Mean	-0.0375	-0.0367	-0.0386	-0.0341
	Stdev	0.0214	0.0193	0.0153	0.0195
	Skewness	-3.8830	-2.7021	-1.9555	-3.2096
	Kurtosis	35.6294	15.1189	8.2145	19.0384
ACA	Mean	-0.0342	-0.0347	-0.0372	-0.0323
	Stdev	0.0189	0.0175	0.0147	0.0183
	Skewness	-3.7520	-3.2055	-2.0628	-2.9855
	Kurtosis	35.0690	21.5338	8.6644	16.7722
DB	Mean	-0.0371	-0.0372	-0.0388	-0.0348
	Stdev	0.0216	0.0208	0.0152	0.0201
	Skewness	-3.5304	-3.2996	-1.9771	-3.0138
	Kurtosis	30.6122	23.4637	8.4824	16.7051
BCS	Mean	-0.0364	-0.0366	-0.0374	-0.0343
	Stdev	0.0205	0.0200	0.0148	0.0214
	Skewness	-3.2389	-2.7469	-2.0785	-2.8781
	Kurtosis	26.5118	14.9644	9.0081	13.7518
RAB	Mean	-0.1223	-0.1241	-0.1234	-0.1252
	Stdev	0.0203	0.0225	0.0263	0.0259
	Skewness	-1.1163	-0.9566	-0.9961	-0.7046
	Kurtosis	4.3861	3.0589	4.3574	3.6742
SAB	Mean	-0.1025	-0.0997	-0.1026	-0.0970
	Stdev	0.0124	0.0163	0.0143	0.0171
	Skewness	0.0179	-0.9904	-0.5249	-0.7225
	Kurtosis	3.7766	6.2165	5.7543	5.5554
KBC	Mean	-0.1064	-0.0756	-0.1100	-0.0750
	Stdev	0.0353	0.0381	0.0407	0.0398
	Skewness	-0.1997	-0.1023	-0.5154	-0.0812
	Kurtosis	3.2110	2.9495	3.4938	2.8279

Table 2.7.: The descriptive statistics of $CoVaR_{5\%}$ of the Gaussian DCC model.

	Statistic	ISP	ACA	DB	BCS	RAB	SAB	KBC
ISP	Mean	-	-0.0638	-0.0623	-0.0627	-0.0622	-0.0618	-0.0638
	Stdev	-	0.0380	0.0371	0.0387	0.0369	0.0346	0.0386
	Skewness	-	-2.1108	-2.2279	-2.5509	-3.0419	-1.9831	-2.7267
	Kurtosis	-	12.1748	13.9587	19.1574	31.9169	12.2626	23.3569
ACA	Mean	-0.0629	-	-0.0617	-0.0617	-0.0618	-0.0612	-0.0630
	Stdev	0.0312	-	0.0315	0.0329	0.0314	0.0297	0.0331
	Skewness	-2.6814	-	-2.1085	-2.4583	-2.7155	-2.0527	-2.7946
	Kurtosis	18.4454	-	11.7975	15.3471	22.0371	11.4476	19.9384
DB	Mean	-0.0689	-0.0685	-	-0.0672	-0.0674	-0.0667	-0.0667
	Stdev	0.0294	0.0288	-	0.0302	0.0288	0.0275	0.0275
	Skewness	-2.1290	-1.9330	-	-2.0981	-2.1133	-1.8023	-1.8023
	Kurtosis	10.8771	8.9475	-	9.4354	10.2073	7.6881	7.6881
BCS	Mean	-0.0606	-0.0607	-0.0593	-	-0.0595	-0.0587	-0.0610
	Stdev	0.0316	0.0323	0.0306	-	0.0303	0.0292	0.0330
	Skewness	-2.9904	-2.4868	-2.1922	-	-2.3265	-2.2493	-2.7021
	Kurtosis	20.7334	13.9317	10.1063	-	11.5667	11.6996	15.8104
RAB	Mean	-0.0583	-0.0587	-0.0573	-0.0575	-	-0.0570	-0.0586
	Stdev	0.0345	0.0365	0.0352	0.0365	-	0.0336	0.0368
	Skewness	-2.7556	-2.5436	-2.4635	-2.5895	-	-2.4245	-2.6901
	Kurtosis	15.1586	11.9539	11.0846	11.9458	-	11.0958	13.2697
SAB	Mean	-0.0706	-0.0698	-0.0685	-0.0685	-0.0692	-	-0.0699
	Stdev	0.0339	0.0323	0.0318	0.0336	0.0326	-	0.0337
	Skewness	-3.1337	-2.5190	-2.2350	-2.5898	-2.5858	-	-2.7867
	Kurtosis	22.8768	14.0589	11.1340	14.0751	14.8425	-	16.2146
KBC	Mean	-0.0586	-0.0591	-0.0578	-0.0579	-0.0573	-0.0570	-
	Stdev	0.0349	0.0375	0.0358	0.0365	0.0346	0.0342	-
	Skewness	-2.0684	-2.0394	-1.9066	-1.9635	-1.9119	-1.9486	-
	Kurtosis	8.7128	7.4498	6.9544	7.2010	6.8844	7.2752	-

Table 2.8.: The descriptive of statistics of $CoVaR_{5\%}$ of the Student-t DCC model.

	Statistic	ISP	ACA	DB	BCS	RAB	SAB	KBC
ISP	Mean	-	-0.0673	-0.0672	-0.0682	-0.0668	-0.0680	-0.0666
	Stdev	-	0.0347	0.0359	0.0366	0.0353	0.0355	0.0358
	Skewness	-	-2.1981	-2.5295	-2.2573	-2.4245	-1.9112	-2.5733
	Kurtosis	-	16.1119	24.1635	18.1575	21.0975	13.0569	23.2009
ACA	Mean	-0.0662	-	-0.0674	-0.0678	-0.0672	-0.0680	-0.0667
	Stdev	0.0348	-	0.0359	0.0363	0.0357	0.0359	0.0359
	Skewness	-2.3977	-	-2.2722	-2.1906	-2.1822	-2.0128	-2.3570
	Kurtosis	13.1027	-	13.5072	11.3372	11.5261	9.5968	13.3964
DB	Mean	-0.0725	-0.0734	-	-0.0731	-0.0726	-0.0734	-0.0728
	Stdev	0.0323	0.0316	-	0.0321	0.0313	0.0317	0.0322
	Skewness	-2.1685	-2.0242	-	-2.0972	-2.0756	-1.9158	-2.1153
	Kurtosis	10.2865	9.1376	-	9.3394	9.0262	8.2519	9.5733
BCS	Mean	-0.0619	-0.0621	-0.0618	-	-0.0619	-0.0625	-0.0619
	Stdev	0.0340	0.0330	0.0333	-	0.0332	0.0336	0.0344
	Skewness	-2.5397	-2.4856	-2.1827	-	-2.2117	-2.1258	-2.3560
	Kurtosis	13.4578	13.0468	9.1314	-	9.0939	9.0054	10.4148
RAB	Mean	-0.0605	-0.0614	-0.0610	-0.0616	-	-0.0623	-0.0603
	Stdev	0.0351	0.0346	0.0347	0.0351	-	0.0357	0.0353
	Skewness	-2.6001	-2.4945	-2.2045	-2.2263	-	-2.1401	-2.2963
	Kurtosis	13.9952	13.8396	9.8423	9.9284	-	9.5205	10.4852
SAB	Mean	-0.0704	-0.0711	-0.0704	-0.0707	-0.0708	-	-0.0701
	Stdev	0.0314	0.0307	0.0308	0.0312	0.0310	-	0.0313
	Skewness	-2.5910	-2.3327	-2.2451	-2.3660	-2.2313	-	-2.3900
	Kurtosis	14.6196	11.5999	10.2738	11.1519	10.1821	-	11.2993
KBC	Mean	-0.0642	-0.0645	-0.0652	-0.0656	-0.0642	-0.0655	-
	Stdev	0.0396	0.0386	0.0404	0.0407	0.0397	0.0405	-
	Skewness	-1.8246	-1.7982	-1.6454	-1.6819	-1.6879	-1.6660	-
	Kurtosis	6.6293	6.6441	5.5019	5.6431	5.5863	5.6092	-

Table 2.9.: The descriptive of statistics of $CoVaR_{5\%}$ of the Gaussian spatial DCC model.

	Statistic	ISP	ACA	DB	BCS	RAB	SAB	KBC
ISP	Mean	-	-0.0731	-0.0724	-0.0742	-0.0722	-0.0754	-0.0724
	Stdev	-	0.0401	0.0379	0.0398	0.0385	0.0453	0.0371
	Skewness	-	-4.5070	-3.9739	-3.9535	-3.7228	-6.9596	-3.9343
	Kurtosis	-	44.0858	35.4756	35.3029	30.5360	107.4077	35.9073
ACA	Mean	-0.0748	-	-0.0744	-0.0753	-0.0742	-0.0771	-0.0737
	Stdev	0.0372	-	0.0361	0.0372	0.0365	0.0397	0.0339
	Skewness	-2.8397	-	-2.8025	-2.8078	-2.5954	-3.0814	-2.7622
	Kurtosis	16.6131	-	16.4537	16.4834	13.7778	21.2445	17.3850
DB	Mean	-0.0807	-0.0811	-	-0.0811	-0.0801	-0.0831	-0.0811
	Stdev	0.0369	0.0368	-	0.0369	0.0365	0.0382	0.0356
	Skewness	-1.8289	-1.8628	-	-1.7832	-1.9404	-1.8694	-1.7408
	Kurtosis	6.8883	7.3251	-	6.7642	8.1330	7.7427	6.8175
BCS	Mean	-0.0700	-0.0695	-0.0686	-	-0.0688	-0.0715	-0.0692
	Stdev	0.0388	0.0383	0.0367	-	0.0372	0.0425	0.0365
	Skewness	-2.5450	-2.7335	-2.6215	-	-2.6061	-4.4468	-2.5573
	Kurtosis	12.0772	14.2683	13.1840	-	12.8705	50.5984	12.8248
RAB	Mean	-0.0662	-0.0667	-0.0657	-0.0669	-	-0.0693	-0.0653
	Stdev	0.0381	0.0388	0.0364	0.0375	-	0.0417	0.0349
	Skewness	-2.4870	-2.8613	-2.6630	-2.5818	-	-3.4649	-2.3409
	Kurtosis	11.2325	17.4978	14.1112	13.0757	-	29.1653	11.6928
SAB	Mean	-0.0731	-0.0732	-0.0728	-0.0735	-0.0732	-	-0.0730
	Stdev	0.0267	0.0264	0.0255	0.0266	0.0259	-	0.0262
	Skewness	-5.3891	-5.5006	-5.5087	-5.6171	-4.9732	-	-4.9948
	Kurtosis	78.9265	79.3116	82.0662	84.6955	68.8182	-	67.3002
KBC	Mean	-0.0714	-0.0710	-0.0713	-0.0721	-0.0703	-0.0736	-
	Stdev	0.0449	0.0443	0.0438	0.0447	0.0436	0.0481	-
	Skewness	-2.4695	-2.7323	-2.5331	-2.5591	-2.3890	-3.1081	-
	Kurtosis	10.6740	14.0362	11.6030	11.7622	10.2190	20.2187	-

Table 2.10.: The descriptive of statistics of $\text{CoVaR}_{5\%}$ of the Student-t spatial DCC model.

	Statistic	ISP	ACA	DB	BCS	RAB	SAB	KBC
ISP	Mean	-	-0.0851	-0.0852	-0.0875	-0.0827	-0.0847	-0.0854
	Stdev	-	0.0432	0.0443	0.0457	0.0421	0.0438	0.0437
	Skewness	-	-3.5616	-4.0196	-3.3594	-3.3666	-3.7599	-3.3963
	Kurtosis	-	31.6903	37.5554	26.6726	27.5719	36.8625	29.0922
ACA	Mean	-0.0885	-	-0.0890	-0.0904	-0.0868	-0.0889	-0.0886
	Stdev	0.0448	-	0.0458	0.0477	0.0443	0.0459	0.0444
	Skewness	-3.5095	-	-3.6041	-3.5329	-3.1185	-3.4123	-3.2438
	Kurtosis	25.9678	-	26.5636	26.1069	20.7697	24.6521	22.5953
DB	Mean	-0.0927	-0.0929	-	-0.0938	-0.0904	-0.0929	-0.0940
	Stdev	0.0417	0.0413	-	0.0444	0.0404	0.0413	0.0420
	Skewness	-2.1244	-2.0372	-	-2.1726	-2.1042	-2.0373	-2.0327
	Kurtosis	8.5433	7.9759	-	8.7642	8.3066	8.2291	8.0306
BCS	Mean	-0.0805	-0.0796	-0.0791	-	-0.0777	-0.0801	-0.0806
	Stdev	0.0430	0.0415	0.0423	-	0.0406	0.0429	0.0426
	Skewness	-2.9191	-2.8671	-2.9640	-	-2.8196	-2.8069	-2.7900
	Kurtosis	14.9903	14.1603	14.7086	-	13.7479	13.5377	13.5715
RAB	Mean	-0.0777	-0.0781	-0.0781	-0.0797	-	-0.0792	-0.0777
	Stdev	0.0446	0.0440	0.0455	0.0466	-	0.0456	0.0437
	Skewness	-2.8771	-2.6807	-3.3759	-2.8621	-	-2.7376	-2.6199
	Kurtosis	14.4010	12.6643	22.0884	14.3974	-	13.3848	12.4734
SAB	Mean	-0.1000	-0.1005	-0.1006	-0.1019	-0.0990	-	-0.1008
	Stdev	0.0471	0.0469	0.0476	0.0486	0.0463	-	0.0472
	Skewness	-2.5373	-2.5913	-2.8085	-2.5913	-2.5026	-	-2.5823
	Kurtosis	10.7255	10.8972	13.6563	11.3596	10.5028	-	11.0298
KBC	Mean	-0.0826	-0.0821	-0.0838	-0.0849	-0.0801	-0.0833	-
	Stdev	0.0515	0.0499	0.0527	0.0544	0.0502	0.0531	-
	Skewness	-2.5646	-2.5517	-2.6933	-2.7149	-2.4567	-2.6528	-
	Kurtosis	11.3026	11.2016	12.5515	12.8700	10.4649	12.0970	-

Table 2.11.: The p-value of backtesting based $VaR_{5\%}$ tests of weekly CDS data.

Bank	Gaussian DCC		Student-t DCC		Gaussian spatial DCC		Student-t spatial DCC	
	UC	CC	UC	CC	UC	CC	UC	CC
ISP	**0.6819**	**0.8310**	**0.6819**	**0.3111**	**0.8856**	**0.9398**	**0.3367**	**0.3888**
ACA	**0.0652**	0.0122	**0.3367**	**0.1270**	**0.1234**	0.0310	**0.2125**	**0.0674**
DB	**0.3367**	**0.3888**	**0.0652**	**0.1395**	**0.9072**	**0.4156**	**0.0652**	**0.1395**
BARC	**0.4953**	**0.6693**	**0.4953**	**0.6693**	**0.8856**	**0.0920**	**0.6819**	**0.3111**
ING	**0.6819**	**0.3111**	**0.6819**	**0.3111**	**0.6819**	**0.3111**	**0.4953**	**0.2106**
SAB	**0.9072**	**0.9743**	**0.8856**	**0.9398**	**0.8856**	**0.9398**	**0.9072**	**0.9743**
KBC	**0.8856**	**0.9398**	0.0233	**0.0763**	**0.8856**	**0.9398**	0.0233	**0.0763**

Note: The UC and CC stand for the unconditional coverage and conditional coverage tests. The bold defines as the acceptance at a 95% significance level and the highlighted light-gray defines as the acceptance at a 99% significance level.

Table 2.12.: The backtesting based the loss functions of $VaR_{5\%}$ under the regulator's view of weekly CDS data.

Regulator's view				
Bank	Lopez			
	Gaussian DCC	Student-t DCC	Gaussian spatial DCC	Student-t spatial DCC
ISP	308.61	308.56	**307.67**	310.14
ACA	312.93	**309.66**	311.81	310.92
DB	309.33	312.05	**305.04**	312.08
BCS	309.47	307.98	307.32	**306.90**
RAB	**303.53**	303.68	303.73	304.84
SAB	299.11	299.94	300.13	**298.80**
KBC	300.49	**288.78**	300.84	288.80
Bank	Caporin1			
	Gaussian DCC	Student-t DCC	Gaussian spatial DCC	Student-t spatial DCC
ISP	188.76	188.26	188.19	**181.45**
ACA	186.28	**184.35**	186.01	185.20
DB	189.86	190.79	**187.57**	191.28
BCS	196.90	196.51	196.76	**196.32**
RAB	180.48	180.68	**180.04**	181.66
SAB	201.52	199.71	202.03	**196.01**
KBC	**239.62**	438.35	240.65	424.44
Bank	Caporin2			
	Gaussian DCC	Student-t DCC	Gaussian spatial DCC	Student-t spatial DCC
ISP	23.13	22.70	23.05	**18.79**
ACA	22.57	**21.78**	22.41	22.39
DB	22.02	22.09	**21.20**	22.26
BCS	25.52	24.25	25.85	**24.12**
RAB	**16.36**	16.73	16.50	16.85
SAB	16.42	15.85	16.46	**15.13**
KBC	**23.45**	60.44	24.36	56.97
Bank	Caporin3			
	Gaussian DCC	Student-t DCC	Gaussian spatial DCC	Student-t spatial DCC
ISP	49.88	49.52	49.87	**45.43**
ACA	50.42	**49.34**	50.08	50.13
DB	47.81	47.86	**46.62**	47.86
BCS	50.13	48.07	50.27	**47.86**
RAB	**38.24**	38.74	38.54	38.99
SAB	31.31	30.45	31.36	**29.58**
KBC	32.40	23.20	33.44	**22.98**

Note: The bold defines as the lowest total loss among different models.

Table 2.13.: The backtesting based the loss functions of $VaR_{5\%}$ under the investors' view of weekly CDS data.

Investors' view				
Bank	Caporin1			
	Gaussian DCC	Student-t DCC	Gaussian spatial DCC	Student-t spatial DCC
ISP	193.89	193.93	193.77	**184.32**
ACA	190.86	189.97	190.93	**190.22**
DB	193.25	193.87	**191.77**	194.38
BCS	201.26	**200.96**	201.37	201.10
RAB	186.51	186.16	**185.77**	186.32
SAB	208.80	206.68	209.19	**202.71**
KBC	**252.73**	505.51	252.93	486.99
Bank	Caporin2			
	Gaussian DCC	Student-t DCC	Gaussian spatial DCC	Student-t spatial DCC
ISP	23.63	23.26	23.59	**18.97**
ACA	23.05	**22.40**	22.93	22.95
DB	22.42	22.52	**21.59**	22.62
BCS	26.02	24.99	26.45	**24.95**
RAB	**16.85**	17.16	16.98	17.27
SAB	16.95	16.32	16.99	**15.56**
KBC	25.99	66.80	26.77	63.11
Bank	Caporin3			
	Gaussian DCC	Student-t DCC	Gaussian spatial DCC	Student-t spatial DCC
ISP	50.54	50.21	50.56	**45.86**
ACA	51.14	**50.15**	50.83	50.89
DB	**48.33**	48.36	47.21	48.36
BCS	50.77	48.65	50.90	**48.47**
RAB	**38.91**	39.36	39.17	39.53
SAB	31.95	31.08	31.99	**30.18**
KBC	33.49	24.70	34.48	**24.43**

Note: The bold defines as the lowest total loss among different models.

Table 2.14.: The p-value of backtesting based $\text{CoVaR}_{5\%}$ tests of equity data.

	Institution $i \rightarrow$	ISP		ACA		DB		BARC		ING		SAB		KBC	
	System $j \downarrow$	UC	CC	UC	CC	UC	CC	UC	CC	UC	CC	UC	CC	UC	CC
Gaussian DCC	ISP	-	-	0.0002	0.0009	0.0000	0.0000	**0.0604**	0.0370	0.0000	0.0000	0.0002	0.0001	0.0006	0.0023
	ACA	0.0054	0.0034	-	-	**0.0731**	**0.2007**	**0.7465**	**0.1208**	0.0020	0.0082	0.0331	**0.0758**	**0.0731**	**0.0680**
	DB	0.0017	0.0008	0.0294	**0.0695**	-	-	**0.0659**	**0.1844**	0.0046	0.0162	**0.0659**	**0.1844**	**0.0659**	**0.1834**
	BARC	0.0000	0.0000	0.0027	0.0070	0.0000	0.0000	-	-	0.0001	0.0001	**0.0891**	**0.1354**	0.0073	0.0252
	ING	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	**0.1507**	**0.3522**	-	-	0.0003	0.0008	0.0026	0.0008
	SAB	0.0000	0.0000	0.0084	0.0125	0.0205	**0.0666**	**0.8424**	**0.5790**	**0.0979**	**0.1003**	-	-	0.0205	**0.0559**
	KBC	0.0000	0.0000	0.0001	0.0005	0.0000	0.0000	**0.2640**	**0.2081**	0.0000	0.0000	0.0001	0.0005	-	-
Student-t DCC	ISP	-	-	**0.0699**	**0.1921**	**0.1348**	**0.3263**	**0.2953**	**0.1877**	**0.1348**	**0.1887**	0.0065	0.0012	**0.1348**	**0.0525**
	ACA	**0.0667**	**0.0582**	-	-	**0.2464**	**0.2308**	**0.4471**	**0.6188**	**0.0667**	**0.1858**	**0.6602**	**0.5505**	**0.2464**	**0.2308**
	DB	0.0293	0.0232	**0.4085**	**0.6235**	-	-	**0.9376**	**0.6797**	**0.1278**	**0.3114**	**0.4085**	**0.6235**	**0.4085**	**0.3735**
	BARC	0.0000	0.0003	0.0452	0.0080	0.0000	0.0000	-	-	0.0088	0.0053	**0.1761**	**0.1972**	**0.0925**	**0.2429**
	ING	0.0000	0.0000	0.0183	**0.0532**	0.0011	0.0018	**0.6638**	**0.4412**	-	-	0.0407	**0.1206**	**0.0845**	**0.1427**
	SAB	0.0000	0.0001	**0.2915**	**0.5173**	**0.0845**	**0.2253**	**0.6638**	**0.7291**	0.0077	0.0256	-	-	0.0183	0.0051
	KBC	0.0002	0.0001	0.0043	0.0166	0.0000	0.0001	**0.4842**	0.0392	0.0001	0.0003	0.0006	0.0005	-	-
Gaussian spatial DCC	ISP	-	-	**0.3968**	**0.1997**	**0.2295**	**0.1970**	**0.7629**	**0.0536**	**0.7629**	**0.4828**	**0.1224**	**0.1628**	**0.6311**	**0.0667**
	ACA	**0.3742**	**0.3703**	-	-	**0.3742**	**0.3703**	**0.4177**	**0.1918**	**0.9325**	**0.7160**	**0.6200**	**0.5616**	**0.7273**	**0.0293**
	DB	**0.0561**	**0.0520**	**0.0561**	**0.1014**	-	-	**0.6671**	**0.5861**	**0.2314**	**0.4533**	**0.1190**	**0.2920**	**0.9832**	0.0000
	BARC	0.0011	0.0047	0.0440	0.0316	0.0000	0.0000	-	-	**0.1829**	**0.1784**	**0.3297**	**0.1912**	**0.3297**	0.0000
	ING	0.0009	0.0033	0.0366	0.0331	0.0009	0.0016	**0.2073**	**0.1009**	-	-	0.0067	0.0046	**0.2730**	0.0000
	SAB	0.0001	0.0001	**0.0979**	0.0291	**0.1902**	**0.3959**	**0.2802**	**0.5073**	**0.3405**	**0.5371**	-	-	**0.8424**	0.0002
	KBC	0.0000	0.0000	**0.0561**	**0.1014**	0.0000	0.0000	**0.4116**	0.0191	0.0244	**0.0774**	0.0244	0.0228	-	-
Student-t spatial DCC	ISP	-	-	**0.1013**	0.0320	**0.2487**	**0.4542**	0.0051	0.0196	**0.4813**	**0.2849**	**0.6166**	**0.1661**	**0.2487**	**0.4542**
	ACA	**0.5556**	**0.6847**	-	-	**0.1254**	**0.2875**	0.0380	**0.1125**	**0.1254**	**0.2875**	**0.2971**	**0.5099**	**0.1254**	**0.2875**
	DB	**0.5315**	**0.4800**	**0.5556**	**0.6847**	-	-	**0.1254**	**0.2875**	**0.2971**	**0.5099**	**0.2971**	**0.5099**	0.0380	**0.1125**
	BARC	**0.0934**	**0.1393**	**0.8264**	**0.1157**	0.0078	0.0139	-	-	**0.5426**	**0.3136**	**0.8541**	**0.5152**	**0.2886**	**0.5005**
	ING	**0.3205**	**0.5695**	**0.5198**	**0.6915**	**0.3205**	**0.5695**	**0.0678**	**0.1770**	-	-	**0.6239**	**0.4227**	**0.9237**	**0.6113**
	SAB	**0.1631**	**0.1649**	**0.4816**	**0.1660**	**0.1949**	**0.3841**	0.0211	**0.0689**	**0.3942**	**0.5788**	-	-	**0.1949**	**0.0951**
	KBC	**0.0891**	0.0366	**0.5556**	**0.3195**	**0.3191**	**0.1895**	**0.1254**	0.0397	**0.8104**	**0.6521**	**0.8104**	**0.6521**	-	-

Note: The UC and CC stand for the unconditional coverage and conditional coverage tests. The bold defines as the acceptance at a 95% significance level and the highlighted light-gray defines as the acceptance at a 99% significance level.

Table 2.15.: The backtesting based the loss functions of $\text{CoVaR}_{5\%}$ under the regulator's view of equity data.

Bank	Institution $i\rightarrow$	ISP				ACA				DB				BCS				RAB				SAB				KBC			
System $j\downarrow$	Model	L	C1	C2	C3	L	C1	C2	C3	L	C1	C2	C3	L	C1	C2	C3	L	C1	C2	C3	L	C1	C2	C3	L	C1	C2	C3
ISP	Gaussian DCC	-	-	-	-	2375	1886	136	201	2374	1886	136	202	2376	1895	141	207	2373	1872	131	196	2375	1882	135	200	2373	1886	137	202
	Student-t DCC	-	-	-	-	2364	1808	110	173	2362	1807	109	172	2364	1816	113	176	2360	1801	108	171	2366	1819	115	179	2362	1807	109	172
	Gaussian spatial DCC	-	-	-	-	2355	1759	98	160	2353	1753	97	159	2354	1759	100	162	2352	1752	97	159	2353	1760	99	161	2354	1749	96	158
	Student-t spatial DCC	-	-	-	-	**2344**	**1705**	**91**	**152**	**2338**	**1690**	**88**	**148**	**2338**	**1687**	**88**	**149**	**2338**	**1693**	**88**	**148**	**2340**	**1692**	**86**	**147**	**2346**	**1704**	**91**	**152**
ACA	Gaussian DCC	2375	1914	144	209	-	-	-	-	2375	1917	146	210	2377	1920	149	214	2374	1905	141	205	2375	1915	145	210	2376	1915	145	209
	Student-t DCC	2361	1836	115	177	-	-	-	-	2365	1838	115	177	2363	1841	116	178	2361	1833	113	176	2363	1848	120	182	2361	1836	112	174
	Gaussian spatial DCC	2350	1762	97	157	-	-	-	-	2353	1770	99	160	2355	1773	100	161	2352	1770	99	159	2354	1775	100	161	2350	1764	98	158
	Student-t spatial DCC	**2343**	**1736**	**90**	**149**	-	-	-	-	**2341**	**1720**	**88**	**146**	**2344**	**1717**	**88**	**146**	**2341**	**1725**	**88**	**147**	**2339**	**1721**	**87**	**145**	**2345**	**1733**	**90**	**149**
DB	Gaussian DCC	2376	1908	152	218	2376	1910	152	218	-	-	-	-	2378	1910	154	220	2374	1898	147	212	2375	1909	152	218	2378	1914	155	221
	Student-t DCC	2361	1850	127	190	2361	1852	128	191	-	-	-	-	2360	1852	128	190	2359	1846	126	188	2364	1862	133	195	2360	1853	128	190
	Gaussian spatial DCC	2351	1787	108	170	2352	1795	110	172	-	-	-	-	2351	1794	109	172	2353	1792	108	170	2352	1797	110	172	2352	1791	109	171
	Student-t spatial DCC	**2345**	**1763**	**101**	**162**	**2340**	**1762**	**100**	**161**	-	-	-	-	**2341**	**1748**	**97**	**158**	**2340**	**1753**	**98**	**158**	**2340**	**1749**	**96**	**157**	**2340**	**1749**	**96**	**157**
BCS	Gaussian DCC	2370	1910	132	190	2370	1906	130	188	2369	1903	128	186	-	-	-	-	2368	1897	125	183	2370	1907	131	189	2370	1911	132	190
	Student-t DCC	2356	1839	109	165	2357	1837	108	164	2358	1833	107	163	-	-	-	-	2356	1832	106	162	2358	1845	112	169	2356	1838	107	163
	Gaussian spatial DCC	2349	1777	92	146	2349	1782	92	147	2352	1776	91	146	-	-	-	-	2351	1780	92	146	2350	1782	93	147	2350	1776	92	146
	Student-t spatial DCC	**2347**	**1770**	**89**	**143**	**2344**	**1766**	**89**	**143**	**2343**	**1755**	**86**	**140**	-	-	-	-	**2348**	**1761**	**87**	**141**	**2347**	**1753**	**85**	**139**	**2346**	**1768**	**90**	**144**
RAB	Gaussian DCC	2373	1883	124	184	2374	1886	125	185	2375	1886	124	185	2375	1892	128	188	-	-	-	-	2375	1891	127	187	2373	1883	123	184
	Student-t DCC	2359	1803	99	157	2358	1808	100	158	2361	1806	99	156	2361	1812	100	158	-	-	-	-	2362	1823	105	164	2357	1803	97	155
	Gaussian spatial DCC	2354	1752	87	143	2353	1763	89	145	2354	1756	88	144	2355	1762	89	146	-	-	-	-	2356	1767	90	147	2356	1748	86	143
	Student-t spatial DCC	**2348**	**1727**	**82**	**138**	**2351**	**1725**	**83**	**139**	**2342**	**1710**	**80**	**136**	**2344**	**1711**	**81**	**136**	-	-	-	-	**2347**	**1714**	**79**	**135**	**2355**	**1723**	**83**	**139**
SAB	Gaussian DCC	2381	1944	170	235	2382	1947	171	236	2383	1946	172	236	2385	1950	175	239	2380	1940	168	232	-	-	-	-	2381	1947	172	236
	Student-t DCC	2356	1822	112	171	2359	1825	112	171	2359	1823	112	171	2357	1826	112	172	2354	1825	112	172	-	-	-	-	2357	1823	111	171
	Gaussian spatial DCC	2346	1789	106	165	2349	1795	107	167	2346	1791	106	165	2345	1792	107	166	2346	1792	107	166	-	-	-	-	2347	1786	106	164
	Student-t spatial DCC	**2346**	**1785**	**107**	**165**	**2345**	**1779**	**105**	**164**	**2340**	**1769**	**102**	**161**	**2341**	**1766**	**102**	**161**	**2343**	**1775**	**104**	**162**	-	-	-	-	**2344**	**1780**	**105**	**164**
KBC	Gaussian DCC	2377	1884	131	196	2377	1884	130	195	2379	1891	134	199	2380	1894	136	201	2376	1869	125	190	2379	1886	133	197	-	-	-	-
	Student-t DCC	2363	1808	107	170	2366	1808	106	169	2363	1810	107	170	2367	1816	108	171	2363	1800	104	167	2368	1820	112	175	-	-	-	-
	Gaussian spatial DCC	2354	1742	92	153	2352	1750	92	153	2354	1749	94	155	2355	1754	95	156	2352	1743	92	153	2355	1753	94	156	-	-	-	-
	Student-t spatial DCC	**2345**	**1693**	**80**	**140**	**2346**	**1693**	**81**	**141**	**2340**	**1680**	**78**	**138**	**2348**	**1678**	**79**	**138**	**2339**	**1679**	**77**	**137**	**2346**	**1676**	**77**	**136**	-	-	-	-

Note: The loss functions, Lopez, Caporin1, Caporin2, and Caporin3, are represented as L, C1, C2, and C3. The bold defines as the lowest total loss among different models.

Table 2.16.: The backtesting based the loss functions of $\text{CoVaR}_{5\%}$ under the investors' view of equity data.

Bank	Institution $i\rightarrow$	ISP			ACA			DB			BCS			RAB			SAB			KBC		
System $j\downarrow$	Model	C1	C2	C3	C1	C2	C3	C1	C2	C3	C1	C2	C3	C1	C2	C3	C1	C2	C3	C1	C2	C3
ISP	Gaussian DCC	-	-	-	1888	136	202	1888	136	202	1897	141	207	1875	131	196	1885	135	201	1888	137	202
	Student-t DCC	-	-	-	1814	111	174	1812	110	172	1821	113	176	1806	109	171	1825	116	179	1812	109	172
	Gaussian spatial DCC	-	-	-	1767	98	160	1761	98	160	1768	100	162	1761	97	159	1768	100	162	1758	97	159
	Student-t spatial DCC	-	-	-	**1718**	**92**	**152**	**1705**	**88**	**149**	**1702**	**89**	**150**	**1706**	**88**	**149**	**1705**	**87**	**147**	**1717**	**92**	**152**
ACA	Gaussian DCC	1917	145	209	-	-	-	1919	146	211	1922	149	214	1907	141	205	1918	146	210	1917	145	210
	Student-t DCC	1843	115	177	-	-	-	1844	115	177	1847	116	178	1839	114	176	1855	120	183	1841	113	175
	Gaussian spatial DCC	1772	97	157	-	-	-	1780	100	160	1784	100	161	1780	99	160	1785	101	162	1774	98	159
	Student-t spatial DCC	**1745**	**90**	**150**	-	-	-	**1731**	**88**	**147**	**1728**	**88**	**147**	**1737**	**88**	**147**	**1731**	**87**	**146**	**1743**	**91**	**150**
DB	Gaussian DCC	1909	152	218	1911	152	218	-	-	-	1911	154	220	1899	147	212	1910	152	218	1915	155	221
	Student-t DCC	1863	133	190	1866	134	191	-	-	-	1866	134	191	1860	131	188	1875	138	196	1867	133	191
	Gaussian spatial DCC	1793	108	171	1801	110	173	-	-	-	1800	110	172	1798	109	171	1803	110	173	1798	109	171
	Student-t spatial DCC	**1771**	**101**	**162**	**1772**	**100**	**162**	-	-	-	**1758**	**98**	**159**	**1762**	**98**	**159**	**1759**	**96**	**157**	**1759**	**96**	**157**
BCS	Gaussian DCC	1914	132	190	1911	130	188	1908	129	187	-	-	-	1902	126	183	1911	131	189	1915	132	190
	Student-t DCC	1848	118	166	1846	116	164	1843	116	163	-	-	-	1842	115	163	1854	121	169	1847	116	164
	Gaussian spatial DCC	1785	92	147	1791	93	147	1785	92	146	-	-	-	1788	92	146	1790	93	148	1785	92	147
	Student-t spatial DCC	**1778**	**89**	**143**	**1775**	**90**	**144**	**1766**	**87**	**141**	-	-	-	**1772**	**87**	**141**	**1763**	**85**	**139**	**1778**	**90**	**144**
RAB	Gaussian DCC	1885	124	184	1889	125	185	1888	125	185	1893	128	188	-	-	-	1893	127	187	1885	124	184
	Student-t DCC	1808	99	157	1813	100	158	1811	99	157	1816	100	159	-	-	-	1827	106	164	1809	97	155
	Gaussian spatial DCC	1759	87	143	1769	89	146	1763	88	145	1769	89	146	-	-	-	1773	91	148	1755	87	143
	Student-t spatial DCC	**1734**	**83**	**139**	**1733**	**83**	**139**	**1719**	**80**	**136**	**1721**	**81**	**137**	-	-	-	**1723**	**80**	**135**	**1730**	**83**	**139**
SAB	Gaussian DCC	1947	171	235	1950	172	236	1950	172	236	1954	175	239	1944	168	233	-	-	-	1951	172	237
	Student-t DCC	1830	112	172	1832	112	172	1831	113	172	1834	113	173	1833	112	172	-	-	-	1831	112	172
	Gaussian spatial DCC	1800	107	166	1805	108	167	1801	107	166	1804	107	166	1802	108	167	-	-	-	1797	106	165
	Student-t spatial DCC	**1797**	**108**	**166**	**1793**	**106**	**164**	**1784**	**103**	**161**	**1782**	**103**	**161**	**1789**	**105**	**163**	-	-	-	**1793**	**106**	**165**
KBC	Gaussian DCC	1886	131	196	1885	130	195	1893	134	199	1896	136	201	1871	126	190	1888	133	198	-	-	-
	Student-t DCC	1813	107	170	1811	106	169	1815	107	170	1820	108	171	1805	105	167	1826	112	175	-	-	-
	Gaussian spatial DCC	1748	92	153	1757	93	154	1756	94	155	1761	95	156	1750	92	153	1760	95	156	-	-	-
	Student-t spatial DCC	**1703**	**80**	**140**	**1704**	**81**	**141**	**1692**	**79**	**138**	**1690**	**79**	**138**	**1691**	**78**	**137**	**1687**	**77**	**136**	-	-	-

Note: The loss functions, Lopez, Caporin1, Caporin2, and Caporin3, are represented as L, C1, C2, and C3. The bold defines as the lowest total loss among different models.

Table 2.17.: The differences comparison between the Gaussian DCC and other models of backtesting based the loss functions of $\text{CoVaR}_{5\%}$ under the regulator's view of equity data.

Bank System $j\downarrow$	Institution $i\rightarrow$ Model	ISP	ACA	DB	BCS	RAB	SAB	KBC
ISP	Student-t DCC	-	0.1066	0.1226	0.1344	0.1170	0.1594	0.0987
	Gaussian spatial DCC	-	0.0419	0.0581	0.0656	0.0545	0.0729	0.0325
	Student-t spatial DCC	-	**0.2365**	**0.2613**	**0.2810**	**0.2338**	**0.2640**	**0.2387**
ACA	Student-t DCC	0.1317	-	0.1505	0.1537	0.1396	0.1811	0.1195
	Gaussian spatial DCC	0.0361	-	0.0641	0.0686	0.0601	0.0774	0.0399
	Student-t spatial DCC	**0.2805**	-	**0.3075**	**0.3218**	**0.2789**	**0.3141**	**0.2809**
DB	Student-t DCC	0.1221	0.1309	-	0.1474	0.1344	0.1744	0.1528
	Gaussian spatial DCC	0.0352	0.0483	-	0.0603	0.0531	0.0692	0.0626
	Student-t spatial DCC	**0.2360**	**0.2439**	-	**0.2717**	**0.2343**	**0.2699**	**0.2813**
BCS	Student-t DCC	0.1054	0.0986	0.1127	-	0.1057	0.1483	0.0925
	Gaussian spatial DCC	0.0145	0.0169	0.0294	-	0.0264	0.0435	0.0106
	Student-t spatial DCC	**0.2245**	**0.2125**	**0.2289**	-	**0.2087**	**0.2491**	**0.2192**
RAB	Student-t DCC	0.0958	0.0969	0.1122	0.1176	-	0.1525	0.0833
	Gaussian spatial DCC	0.0262	0.0338	0.0480	0.0515	-	0.0656	0.0217
	Student-t spatial DCC	**0.2333**	**0.2331**	**0.2583**	**0.2721**	-	**0.2744**	**0.2292**
SAB	Student-t DCC	0.0267	0.0361	0.0507	0.0531	0.0417	-	0.0333
	Gaussian spatial DCC	-0.0011	0.0135	0.0203	0.0230	0.0158	-	0.0026
	Student-t spatial DCC	**0.2797**	**0.2961**	**0.3165**	**0.3302**	**0.2901**	-	**0.2967**
KBC	Student-t DCC	0.1552	0.1461	0.1717	0.1775	0.1634	0.2080	-
	Gaussian spatial DCC	0.0668	0.0668	0.0924	0.0960	0.0864	0.1061	-
	Student-t spatial DCC	**0.2914**	**0.2787**	**0.3217**	**0.3344**	**0.2845**	**0.3290**	-

Table 2.18.: The differences comparison between the Gaussian DCC and other models of backtesting based the loss functions of $\text{CoVaR}_{5\%}$ under the investors' view of equity data.

Bank System $j\downarrow$	Institution $i\rightarrow$ Model	ISP	ACA	DB	BCS	RAB	SAB	KBC
ISP	Student-t DCC	-	0.1346	0.1547	0.1698	0.1478	0.2039	0.1248
	Gaussian spatial DCC	-	0.0518	0.0720	0.0820	0.0680	0.0925	0.0402
	Student-t spatial DCC	-	**0.3036**	**0.3351**	**0.3605**	**0.2993**	**0.3395**	**0.3074**
ACA	Student-t DCC	0.1713	-	0.1941	0.1993	0.1804	0.2357	0.1548
	Gaussian spatial DCC	0.0477	-	0.0831	0.0893	0.0778	0.1006	0.0528
	Student-t spatial DCC	**0.3651**	-	**0.3995**	**0.4186**	**0.3619**	**0.4086**	**0.3657**
DB	Student-t DCC	0.1781	0.1888	-	0.2114	0.1947	0.2468	0.2186
	Gaussian spatial DCC	0.0451	0.0608	-	0.0772	0.0677	0.0886	0.0804
	Student-t spatial DCC	**0.3067**	**0.3157**	-	**0.3525**	**0.3038**	**0.3504**	**0.3649**
BCS	Student-t DCC	0.1700	0.1601	0.1792	-	0.1706	0.2270	0.1520
	Gaussian spatial DCC	0.0189	0.0216	0.0367	-	0.0341	0.0564	0.0133
	Student-t spatial DCC	**0.2928**	**0.2765**	**0.2971**	-	**0.2715**	**0.3241**	**0.2852**
RAB	Student-t DCC	0.1253	0.1261	0.1447	0.1512	-	0.1991	0.1095
	Gaussian spatial DCC	0.0338	0.0435	0.0609	0.0652	-	0.0848	0.0285
	Student-t spatial DCC	**0.3047**	**0.3035**	**0.3351**	**0.3530**	-	**0.3576**	**0.2997**
SAB	Student-t DCC	0.0326	0.0429	0.0617	0.0648	0.0511	-	0.0398
	Gaussian spatial DCC	-0.0023	0.0154	0.0241	0.0279	0.0189	-	0.0018
	Student-t spatial DCC	**0.3619**	**0.3816**	**0.4076**	**0.4251**	**0.3740**	-	**0.3830**
KBC	Student-t DCC	0.2019	0.1883	0.2216	0.2296	0.2106	0.2707	-
	Gaussian spatial DCC	0.0863	0.0862	0.1185	0.1243	0.1111	0.1378	-
	Student-t spatial DCC	**0.3795**	**0.3616**	**0.4170**	**0.4347**	**0.3681**	**0.4278**	-

3. Impact of Google Trends on Portfolio Optimization

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3.1. Overview

This study investigates the impact of Google search queries on portfolio optimization. We gather the daily stock prices and Google Trends indexes of 30 selected companies components of S&P100 index. In particular, we propose a methodology to use Google Trends information in portfolio selection problems. We enhance portfolio performance by implementing the optimization of several portfolio strategies applied to corrected log returns. We examine two different penalty-based log-return corrections that account only for the useful Google Trends investors' interests when no shorts sell are allowed. Finally, we show that portfolio strategies applied to corrected log-returns perform better than the same strategies applied to historical return series.

3.2. Introduction

In the Information Age, the usage of big data for access the human information has been investigated in various fields of studies, as suggested by Danah and Kate (2012). The data sources, for example, spatial location (González et al. (2008); Krings et al. (2009); Haklay (2010); Zheng et al. (2013)), public health (Haklay (2010)), Twitter (Bollen et al. (2011)), internet stock message board (Antweiler and Frank (2004)), and others, have been specifically used to model the sophisticated human behaviors. In particular, Jun et al. (2018) documented that over the past decade, the data based Google Trends (referred to as GT henceforth) were analyzed in the field of economics, medical services, information systems, and several others. From the economic point of view, the data analysis becomes a successful tool to quantify the predictable behavior. For instance, Choi and Varian (2012); Heiberger (2015); Vlastakis and Markellos (2012); Preis et al. (2013) documented the GT information impacts of economic activity, investment strategy, and the stock market. Moreover, the evidence of searching pattern data and financial data seems to be related, as highlighted in Rujirarangsan and Ortobelli (2019). Thus, this gives us an idea to enhance portfolio optimization using Google Trends for the portfolio optimal choices.

In particular, we propose to optimize several portfolio strategies starting from the fundamental mean-variance model Markowitz (1952) to more recent ones such as mean-CVaR (Rockafellar and Uryasev (2002)), Sortino (Sortino and Price (1994)), and Rachev (Stoyanov et al. (2007)) type of strategies applied to the conditional return on GT information.

First, we chose 30 assets from the components of the S&P100 index, taking into account the case of the sensitivity from the Google searching query. In particular, we use specific stock tickers that cannot be confused with other products. For instance, if the stock ticker, say IBM, can be products, the number of chipsets, shops nearby, recruitments, or several others that cover a broad meaning than an investment, it will be excluded from our choices. With this selection procedure, our information perceived solely for an investment reason rather than the others: see, Da et al. (2011); Vlastakis and Markellos (2012).

Second, we discuss how to consider the innovations of GT using two different penalizations of log returns. We want to avoid the speculators' GT interests (that grows when returns go down) since we assume no short sales are allowed. Moreover, we want to avoid counterintuitive information from GT that we have when returns are growing while the GT interests are decreasing. Thus, we penalize this information suggesting two alternative penalization procedure. In the first, we use conditional return on GT information only when the return and GT interest grows jointly, and then we penalize the other situations. In the conditional expectation estimator, we use the Gaussian, Epanechnikov, and Student-t kernel function, and the bandwidth selection follows Scott (2015). In the second, we penalize the negative return when a momentum condition is applied. In the momentum condition, we consider when the return distribution of the last two weeks are worse (respect to the second stochastic dominance order) than the previous two weeks.

For enhancing the portfolio allocation, stochastic dominance plays a pivotal role in decision making. In particular, when we compared to mean-risk approaches, the stochastic dominance provides a more precise decision because the entire distribution of returns is used instead of the mean returns and the risk of returns: see (Levy (2016)). Recently, the stochastic dominance has been applied in the portfolio application, for instance, market portfolio efficiency in Kopa (2010); Kopa and Post (2015), robustness analysis of optimal portfolios in Dupačová and Kopa (2014), and Portfolio Choice in Post and Kopa (2017).

The principal contribution of this paper consists of observing that GT useful information can be used to portfolio problems with profit and this results is based on more of 500 portfolio models used during the same period.

The paper is structured as follows. First, Section 3.3 describes the preparation of Google Trends data and assets data. Then, we define the portfolio optimization models, the penalization strategies, and the conditional expectation framework. Section 3.4 presents the empirical results of portfolio performance by obtained models Sharpe ratio, CVaR, mean-variance, mean-CVaR, Sortino ratio, and Rachev ratio. Last, the paper concludes with Section 3.5.

3.3. Portfolio Selection with Penalized Returns

In this section, we discuss the portfolio selection problem taking into account GT data. In particular, we first examine the different optimization models. Second, we introduce the definition of the GT dataset. Third, we consider two different ways to account for the GT information and SSD on momentum strategy. Finally, we apply the portfolio selection model to the penalized the return.

3.3.1. Portfolio Optimization

Let us recall different portfolio selection models which we use in our empirical analysis. In particular, we optimize the following portfolio models on penalized returns, where the penalization takes into account the main information from GT. We point out the vector of portfolio weights $\mathbf{w} = [w_1, \ldots, w_n]^{'}$. We assume that no short sales are allowed (i.e. $\mathbf{w} \geq 0$, $\mathbf{w}^\top \mathbf{1} = 1$). We denote by $r = [r_1, \ldots, r_n]^{'}$ the vector return, and by $\boldsymbol{\mu}$ the vector of mean. We suppose to have J observations; thus, we refer to $r_{(j)}$ as the j^{th} observation of the vector r.

In particular, we examine two different risk-reward portfolio problems (mean-variance and mean-CVaR) and alternative portfolio strategies obtained with the maximization of three gained-risk ratios (Sharpe, Sortino and Rachev). All portfolio strategies will be applied either to historical returns, conditional returns on GT information (see Appendix B), or two penalized returns. Next, we list the four portfolio problems which will be used in our analysis.

3.3.1.1. Mean-Variance

Modern portfolio theory was born by the fundamental mean-variance analysis, developed by Markowitz (1952). According to Markowitz, the risk-averse investors solve the following optimization problem,

$$\begin{aligned} &\underset{\mathbf{w}}{Minimize} && -\lambda \cdot \mathbf{w}^\top \boldsymbol{\mu} + (1-\lambda) \cdot \mathbf{w}^\top \Sigma \mathbf{w} \\ &subject\,to && \mathbf{w}^\top \mathbf{1} = 1, \\ & && \mathbf{w} \geq 0, \end{aligned} \tag{3.1}$$

For a given $\lambda \in [0, 1]$, where Σ is the variance-covariance matrix of the return vector r, and $\boldsymbol{\mu}$ is the mean return of assets. By varying λ between 0 and 1, we will obtain all the Pareto optimal mean-variance portfolios.

3.3.1.2. Mean-CVaR

According to the risk-metric (Longerstaey and Zangari (1996)), the Value-at-Risk (VaR) is a measurement tool that assesses a financial position with a random return. Consider a random variable X, the VaR at level α, $\alpha \in (0, 1)$ is the opposite of quantile function F_X^{-1} valued at the level α, i.e. $VaR_\alpha(X) = -F^{-1}X(\alpha)$ (F_X is the distribution function of X). To overcome the limits of VaR, Artzner et al. (1999) proposes to use coherent risk measure. In particular, Rockafellar and Uryasev (2000) introduce the conditional Value-at-Risk, $CVaR_\alpha(X) = \frac{1}{\alpha}\int_0^\alpha VaR_\varepsilon(X)d\varepsilon$. We recall the linearizable mean-CVaR problem:

$$\begin{aligned} \underset{(\mathbf{w},\theta,z_j)}{Minimize} \quad & -\lambda \cdot \left(\tfrac{1}{J}\textstyle\sum_{j=1}^{J} \mathbf{w}^\top r_{(j)}\right) + (1-\lambda) \cdot \left(\theta + \tfrac{1}{(\alpha)J}\textstyle\sum_{j=1}^{J} z_j\right) \\ subject\,to \quad & z_j \geq -\mathbf{w}^\top r_{(j)} - \theta,\ j = 1, 2, \ldots, J \\ & \mathbf{w}^\top \mathbf{1} = 1, \\ & \mathbf{w} \geq 0,\ z_j \geq 0 \end{aligned} \tag{3.2}$$

For a given λ, the resulting optimal θ is the VaR_α of the optimal portfolio and $z_{j=1,2,\ldots,J}$ are auxiliary variables. In our optimization problem, we set the α equal to 0.05.

3.3.1.3. Sharpe Ratio

One of the extensively used criteria for assessing the portfolio's performance is the Sharpe ratio developed by Sharpe (1966). This ratio calculates the return with risk-free compensations.

$$\begin{aligned} \underset{\mathbf{w}}{Maximize} \quad & \frac{\mathbf{w}^\top \boldsymbol{\mu} - r_f}{\sqrt{\mathbf{w}^\top \sum \mathbf{w}}} \\ subject\,to \quad & \mathbf{w}^\top \mathbf{1} = 1, \\ & \mathbf{w} \geq 0, \end{aligned} \tag{3.3}$$

where $\sum$ is variance-covariance matrix of returns.

3.3.1.4. Sortino Ratio

The Sortino ratio $\frac{\bar{r}-\tau}{\sqrt{\frac{1}{J}\sum_{j=1}^{J}\left(\mathrm{Min}\left(0, r_j-\tau\right)\right)^2}}$ is defined as the ratio between the expected active portfolio return and the semi-standard deviation of return $r_{j=1,\ldots,J}$ with the target τ of the underperforming portfolio (see Sortino and Price (1994)). With this measure of risk, only the downside deviation can be quantified as risky. We use the quadratic optimization problem proposed by Stoyanov et al. (2007) in order to maximize the Sortino ratio as follows:

$$
\begin{array}{ll}
\underset{(\mathbf{w},\tau,d_j,t)}{Minimize} & \sum_{j=1}^{J} d_j^2 \\
\\
subject\,to & d_j \geq -\mathbf{w}^\top r_{(j)} + t\tau,\ j=1,2,\ldots,J \\
& \frac{1}{J}\sum_{j=1}^{J} \mathbf{w}^\top r_{(j)} - t\tau \geq 1, \\
& \mathbf{w}^\top \mathbf{1} = t, \\
& d_j \geq 0,\ \mathbf{w} \geq 0,\ t \geq 0,
\end{array}
\tag{3.4}
$$

where τ is the target rate of return, t is an additional variable, and d_j is the downside risk of the portfolio defined by a lower semi-absolute deviation $|\tau - \mathbf{w}^\top r_{(j)}|_-$. In our empirical analysis, we set the the target τ equal to 0.1. Thus, the returns below the target rate are considered as losses of the portfolio.

3.3.1.5. Rachev Ratio

The Rachev ratio introduced by Biglova et al. (2004) is the performance measure that compared the extreme positive returns to the extreme negative returns at a certain level of the quantile. It can be defined as $RR_{\alpha,\beta}(X) = \frac{CVaR_\alpha(-X)}{CVaR_\beta(X)}$, where α is the upper tail probability, β is the lower tail probability. In the portfolio optimization, we use the mixed-integer linear programming by setting the binary variables into the optimization as shown by Stoyanov et al. (2007) for a symmetric case $\alpha = \beta$:

$$
\begin{array}{ll}
\underset{(\mathbf{w},y_j,\lambda,z_j,\theta,t)}{Maximize} & \frac{1}{(\alpha)J}\sum_{j=1}^{J} y_j \\
subject\,to & y_j \le B\gamma_j,\ j=1,2,\ldots,J \\
 & y_j \ge \mathbf{w}^\top \boldsymbol{r}_{(j)} - B\left(1-\lambda_j\right), \\
 & y_j \le \mathbf{w}^\top \boldsymbol{r}_{(j)} + B\left(1-\lambda_j\right), \\
 & \gamma^T \mathbf{1} = [\alpha J],\ \gamma_j \in \{0,1\}, \\
 & \theta + \frac{1}{(\alpha)J}\sum_{j=1}^{J} z_j \le 1, \\
 & z_j \ge -\mathbf{w}^\top \boldsymbol{r}_{(j)} - \theta, \\
 & \mathbf{w}^\top \mathbf{1} = t, \\
 & z_j \ge 0, \mathbf{w} \ge 0, t \ge 0,
\end{array}
\tag{3.5}
$$

where z_j and y_j are auxiliary variables, γ_j is a vector of binary variables, α is set at 0.01, B is a very large number, such that $|\mathbf{w}^\top \boldsymbol{r}_{(j)}| \le B$, and t is the additional variable. In our empirical, we consider the symmetric case $\alpha = \beta = 0.01$.

3.3.2. Google Trends data

The GT data is used to recognize the investors' information from the searching queries. To perceive the investors' information, we focus on the searching pattern data on the Google search engine. The Google search analysis called Google Trends provides the query search on a specific geographic location and category. To define the shifts in gathering information, we consider SV_j as a search volume for a specific keyword with amount $j = 1, \ldots, N$. The relative search volume (RSV_j) can be calculated as

$$
RSV_j = \frac{SV_j}{\sum_{k=1}^{N} SV_{j-k}}
$$

Then, the RSV_n is scaled on the ranges between 0 to 100 by standardized its maximum value at a specific interval. The GT data is

$$
gt_j = \frac{RSV_j}{max(RSV_j, ..., RSV_{j-N})} \times 100 \tag{3.6}
$$

where N defines a specific temporal interval; in our analysis, we use $N = 30$ days. As described in Equation 3.6, the Google Trends data provides only a query index instead of raw data. As a sequence, our data will be dynamically adjusted based on every new query search. In particular, in our analysis, we calculate the GT return by applying the logarithmic return on the GT data, $GT_j = \ln\left(\frac{gt_j}{gt_{j-1}}\right)$.

We carefully select the 30 assets components of S&P100 index,[1] avoiding the sensitive cases of search queries. For instance, the International Business Machines (IBM), which trades on the market, is similar to the company's name IBM. The investors may intend to search for products, the number of chipsets, shops nearby, recruitments, etc., which are not relevant to the trading proposed. In the Google queries data, we retrieved the upper case of each asset name as a query search, "United States" as a geographic location, and "Finance" as a category. As a limitation of the Google server, we can download only a limited length of daily data. Thus, we split the download of daily data length into a monthly basis and then aggregate them back into a single file. With this method, however, we need to normalize every piece of data to ascertain the real value of Google Trends. After we gather the daily closing stock prices and the daily Google Trends index, we propose two alternative return penalization to account for GT information.

3.3.3. Penalization

A relationship between Google searching information and asset price is revealed in several studies: see Da et al. (2011); Vlastakis and Markellos (2012); Vozlyublennaia (2014). For this reason, we propose a penalty-based correction by using GT return (GT) and asset return (r). We distinguish two types of penalization which consist of the GT interest and the momentum strategy-based SSD. In this framework, we compute the GT returns by applying the logarithmic returns on the GT data, $GT_j = \ln\left(\frac{gt_j}{gt_{j-1}}\right)$, $j = 1, \dots, N$.

Before we consider the penalization cases, we apply the returns conditional GT return using three alternative kernel functions, Gaussian, Epanechnikov, and Student-t (see Appendix B), when the return grows with GT interest ($r > 0 \,\&\, GT > 0$) is observed.

In this context, we penalized the return when it is not coherent with GT interests or the non-isotonic news (we say that news is isotonic with returns r when $r \cdot GT > 0$. In this Chapter, no short sales are allowed. Thus, we apply the first penalization, called one-size penalization, to consider that we avoid short sales and speculation, ($r < 0 \,\&\, GT > 0$). And the second subcase, called two-size penalization, penalized the non-isotonic behavior between return and GT ($r > 0 \,\&\, GT < 0$ or $r < 0 \,\&\, GT > 0$). On the other case, we approximate the return conditional GT return for all the other situations. For the k^{th} asset, we have these approximated returns:

[1] The list of selected assets components of S&P100 index consists of AAPL, ADBE, AMGN, AMZN, AXP, BDX, BMY, BRKB, CMCSA, CSCO, CVX, INTC, ISRG, JNJ, JPM, KSS, MCD, MRK, MSFT, NVDA, ORCL, QCOM, SBUX, SLB, TXN, UTX, VLO, WFC, WMT, and XOM.

subcase 1 (one-size penalization):

$$\widetilde{r}_{k,(j)} = \begin{cases} -1 & \text{, for } r_{k,(j)} < 0 \quad \& \quad GT_{k,j} > 0 \\ \mathbb{E}\left(r_{k,(j)} | GT_{k,j-1}\right) & \text{otherwise} \end{cases} \tag{3.7}$$

In this subcase 1, we penalize interests for short-sales and speculation.

subcase 2 (two-size penalization):

$$\widetilde{r}_{k,(j)} = \begin{cases} -1 & \text{, for } r_{k,(j)} > 0 \, \& \, GT_{k,j} < 0 \quad \text{or} \quad r_{k,(j)} < 0 \, \& \, GT_{k,j} > 0 \\ \mathbb{E}\left(r_{k,(j)} | GT_{k,j-1}\right) & \text{otherwise} \end{cases} \tag{3.8}$$

In this subcase 2, we want to penalize the non-isotonic behavior of return and GT.

Following the investment decision rules (see, Hanoch and Levy (1969)), the second-order stochastic dominance (SSD) can be used as a comparison of prospects ranking. We provide the SSD decision rules by the following condition that let two investments be X and Y, whose cumulative distributions are F and G, respectively. Then, X dominates Y by SSD for if and only if:

$$\mathrm{I}_2(x) \equiv \int_{-\infty}^{x} [\mathrm{G}(t) - \mathrm{F}(t)] \, dt \geq 0$$

for all x, and $\mathrm{G} \neq \mathrm{F}$ for some x_0. In our analysis, F and G define as two series of returns.

Next, in the second type of the penalized model, we evaluate the impact of conditional expectation considering the penalized GT based on momentum strategy. In particular, we penalized the case that the last two weeks $\left(r_{[(j-10),(j)]}\right)$ of return distribution are worse in the second stochastic dominance sense (SSD) with respect to the previous two weeks $\left(r_{[(j-20),(j-11)]}\right)$.

The first subcase is called historical returns penalization. We will be considered when the returns between the last weeks SSD dominates the last two week. In the second subcase, called conditional expectation penalization, we use the conditional expectation when the past two weeks SSD dominates the previous two week. Thus, for the k^{th} asset, we get:

subcase 1 (historical returns penalization):

$$\widetilde{r}_{k,(j)} = \begin{cases} -1 & \text{, for } r_{k,[(j-20),(j-11)]} \overset{SSD}{>} r_{k,[(j-10),(j)]} \\ r_{k,(j)} & \text{otherwise} \end{cases} \tag{3.9}$$

In this subcase 1, we penalize that the recent returns (last two weeks) are worse than the previous ones (past two weeks), but we do not use the conditional returns on GT information.

subcase 2 (conditional expectation penalization):

$$\widetilde{r}_{k,(j)} = \begin{cases} -1 & , \text{for } r_{k,[(j-20),(j-11)]} \overset{SSD}{>} r_{k,[(j-10),(j)]} \\ \mathbb{E}\left(r_{k,(j)} | GT_{k,j-1}\right) & \text{otherwise} \end{cases} \tag{3.10}$$

On the other hand, in subcase 2, we penalize recent returns, which are worst than the past (like in some momentum strategies), and we use conditional returns on GT information. Then, we turn all the cases into portfolio optimization.

Example: Let us consider AAPL stock returns during the period from 02/04/2020 till 30/04/2020. In subcase 2, we separate daily returns from 17/04/2020 till 30/04/2020 and from 02/04/2020 till 16/04/2020. Under this condition we have:

$$r_{AAPL,[17/04/2020,...,30/04/2020]} = [-0.013,-0.020,-0.0313,0.028,-0.003,0.028,0.001,-0.016,0.032,0.020],$$

$$r_{AAPL,[02/04/2020,...,16/04/2020]} = [0.016,-0.014,0.083,-0.011,0.025,0.007,0.019,0.049,-0.0091,0.007].$$

Observe that $r_{AAPL,[02/04/2020,...,16/04/2020]} \overset{SSD}{>} r_{AAPL,[17/04/2020,...,30/04/2020]}$, thus, we penalized the returns by -1, i.e.,

$$\widetilde{r}_{AAPL,(30/04/2020)} = -1 .$$

Therefore, substituting into equation (3.10), we get

$$r_{AAPL,[30/04/2020]} = \begin{cases} -1 & , \text{for } r_{AAPL,[02/04/2020,...,16/04/2020]} \overset{SSD}{>} r_{AAPL,[17/04/2020,...,30/04/2020]} \\ \mathbb{E}\left(r_{k,(j)} | GT_{k,j-1}\right) & \text{otherwise} \end{cases}$$

3.4. Ex-Post Empirical Analysis

In this section, we apply the returns conditional GT information with different penalization cases. In particular we classifies the different in-sample/out-of sample. Then, we measure the ex-post performance of optimum portfolio models using the rolling backtesting analysis and the ex-post performance returns.

First of all, we retrieved the assets data series from Thomson Reuter DataStream. We download the daily adjusted closing prices of 30 selected assets, avoiding the sensitive cases of GT, as shown in Footnote 1. Then, we calculate the logarithmic returns from the asset prices series. To synchronize the GT with asset data, we retrieve the data from January 01, 2004, to December 31, 2018, excluding the weekends and holidays from the GT data. In particular, we also compute the logarithmic returns from the GT data (see Section 3.3.2). Next, we apply the returns conditional GT return using three alternative kernel functions, Gaussian, Epanechnikov, and Student-t. The condition will observe when the return grows with GT interest ($r > 0 \,\&\, GT > 0$). After that, we use two types of penalization: the GT interest and the momentum strategy-based SSD, as shown in Section 3.3.3. In particular we classify in Table 3.1 the different models, we get according to the penalization and conditional expectation definition used in this empirical analysis.

Table 3.1.: Description of the different penalization and conditional expectation definition.

Model	Description
Historical	We use historical returns.
Gauss	We use conditional expectation based on Gaussian kernel (see Appendix B)
Gauss1side	We use conditional expectation based on Gaussian kernel applied to penalized return according to equation 3.7
Gauss2side	We use conditional expectation based on Gaussian kernel applied to penalized return according to equation 3.8
GaussSSD	We use conditional expectation based on Gaussian kernel applied to penalized return according to equation 3.10
Epa	We use conditional expectation based on Epanechnikov kernel (see Appendix B)
Epa1side	We use conditional expectation based on Epanechnikov kernel applied to penalized return according to equation 3.7
Epa2side	We use conditional expectation based on Epanechnikov kernel applied to penalized return according to equation 3.8
EpaSSD	We use conditional expectation based on Epanechnikov kernel applied to penalized return according to equation 3.10
Student	We use conditional expectation based on Student-t kernel (see Appendix B)
Student1side	We use conditional expectation based on Student-t kernel applied to penalized return according to equation 3.7
Student2side	We use conditional expectation based on Student-t kernel applied to penalized return according to equation 3.8
StudentSSD	We use conditional expectation based on Student-t kernel applied to penalized return according to equation 3.10
SSD	We use penalized historical returns according to equation 3.9

We examine the optimum portfolio models of Sharpe ratio, $\text{CVaR}_{5\%}$, Sortino ratio, and Rachev ratio as mentioned in equations 3.3, 3.2, 3.4, and 3.5, respectively. To minimize $\text{CVaR}_{5\%}$, we give the weight λ in the mean-CVaR equation (3.2) equal to zero. We analyze the ex-post results by the percentage of annual return, maximum drawdown, and Sharpe ratio. And we additionally use stochastic dominance to pairwise compare the ex-post returns. Moreover, we perform the optimum portfolio models by varying the weight parameters (λ) on mean-variance and mean-$\text{CVaR}_{5\%}$.

In particular, we summarize the main steps of our procedure of optimization as follows, i.e., at k^{th} optimization, four main steps are used to compute the ex-post wealth:

Step 1 Compute the return conditional GT information based on different kernel functions with different penalization cases as described in Section 3.3.3.

Step 2 Determine the optimal portfolio according to Section 3.3.1. At this stage, to assess the portfolio optimization problem, we account for the computationally complexity. In fact, we find that:

- We need to solve a quadratic problem to maximize either the Sharpe ratio (3.3) and Sortino ratio (3.4)
- A linear optimization problem is used for the minimization of $CVaR_{5\%}$.
- A mixed interger linear program (3.5) is used to maximize the Rachev ratio.

Step 3 Compute the ex-post wealth.

Step 4 Repeat the previous steps for all models, for different kernel functions, and for different in/out sample windows till observations are finished.

The final results of this procedure are illustrated in Tables 3.2, 3.3, 3.4, 3.5, 3.6, 3.7, 3.8, and Figures 3.1, 3.2.

In Tables 3.2, 3.3, and 3.4, we summarize respectively the ex-post annual return, the Sharpe ratio, and the maximum drawdown[2] valued on the ex-post results obtained by the optimization models with different penalizations, kernel functions, and in/out sample windows. For the out-of-sample analysis, we propose to recalibrate the portfolio every week, or every month, or every two months. Moreover, we consider three possible lengths of in-samples before each ex-ante analysis: 125 daily trading observations (6 months), 250 daily trading observations (1 year), and 500 daily trading observations (2 years). Thus, considering all these parameters, we analyze 504 ex-post sample paths of wealth in total using conditional returns and conditional penalized returns for different in-sample and out-of-sample length periods. In comparison, the symbol little star ($*$) underlines the highest ex-post annual return, ex-post Sharpe ratio, and ex-post maximum drawdown for each in-sample and out-of-sample length period among all penaliztions (one-side, two-side), models (SR, $CVaR_{5\%}$, Sortino, Rachev), and conditional expectation definitions (Gauss, Epa, Student, see Appendix B). The plus symbol ($+$) points out the highest ex-post annual return (3.2), the ex-post Sharpe ratio (Table 3.3), and the minimum ex-post maximum drawdown (Table 3.4) for each model among all in-sample/out of sample periods. Last, the big star sign ($\bigstar$) points out the highest ex-post annual return (3.2), the ex-post Sharpe ratio (Table 3.3), and the minimum ex-post maximum drawdown (Table 3.4).

To refine all optimization models' performance with the length periods, we examine which model gives the best analysis of the percentage of annual return, maximum drawdown, and Sharpe ratio. Tables 3.2, 3.3, and 3.4 show that

[2]We use the maxdrawdown formula proposed by Matlab version R2020a for maximum drawdown.

- In view of backtesting length periods, the ex-post annual return of 2-year in-sample and 1-week out-of-sample shows 17 more times better results for any fixed model, conditional expectation kernel definition, and penalization model (see Table 3.2). We have got similar results in Table 3.3 where, the Sharpe ratio of 2-year in-sample and 1-week out-of-sample shows 14 times better results. However, in terms of risk, Table 3.4 shows that the best in/out of sample period performance is given by 1-year in-sample and 1-month out-of-sample, while for the 2-year in-sample and 1-week out-of-sample, we get 9 times better results with respect to other models (that is anyway good enough).
- In view of the models, the Sortino ratio optimization model performs 5 times better ex-post annual return and Sharpe ratio compared to the other in-sample/out-of-sample length periods (see Tables 3.2, 3.3). Instead, the optimization Sharpe ratio model presents the most conservative strategies with 7 times the lowest ex-post maximum drawdown compared to the other in-sample/out-of-sample length period.
- At 1-year in-sample and 1-month out-of-sample, the Sortino ratio optimization model applied the penalized SSD with the conditional expectation using Epanechnikov kernel functions shows the best ex-post annual return (see Table 3.2) and the best ex-post Shape ratio (see Table 3.3). While the Rachev ratio model represents the lowest maximum drawdown applied the penalized conditional expectation using Gaussian kernel function at 6-month in-sample and 1-week out-of-sample.
- For the Sharpe ratio model, the conditional expectation using Gaussian and Epanechnikov kernel functions with any possible penalizations we obtain very good results (in term of ex-post annual return and Shape ratio) generally better than the ones obtained with historical returns. We obtain similar results for the Sortino ratio and $CVaR_{5\%}$ models. However, the Rachev ratio gives the best performance when we use historical returns. This phenomenon is justifiable because the Rachev ratio is based on both right and left tails definition. Therefore we have a much higher impact on the Rachev ratio definition when we correct the returns taking into account the tail behavior returns and GT information clearly.

Moreover, the ex-post annual return of S&P500 during that period was -0.24%, that is 499 times lower than the analyzed models.

Furthermore, we perform stochastic dominance tests on the Sharpe ratio model, $CVaR_{5\%}$, Sortino ratio, and Rachev ratio to compare the portfolios' performance (see, e.g. Davidson and Duclos (2000), Müller and Stoyan (2002), Ortobelli et al. (2015), and Castellano and Cerqueti (2016)). We use the performance returns from different penalization models and in-sample/out-of-sample length periods. We then pairwise test for the first-order stochastic dominance (FSD), second-order stochastic dominance (SSD), and increasing-convex-order (ICX). The FSD is used to find the investors who prefer more wealth to less (non-satiable). The SSD requires more assumptions that the investors who do not like risk but still prefer more wealth to less (non-satiable risk-averse). The ICX inversely requires that the investors prefer risk and more wealth to less (non-satiable risk-seeking). We find only fours case of first-order stochastic dominance and for this reasons we do not insert FSD test in the following tables. From the results of this analysis, the wealth seems to have characteristics for coving investors' preferences of non-satiable risk-averse and non-satiable risk-seeking. In Tables 3.5, 3.6, 3.7, and 3.8, we observed that

- From these Tables we deduce that the best results in terms of stochastic dominance test for when we optimize the Sharpe ratio, $\text{CVaR}_{5\%}$, Sortino ratio, and Rachev ratio models are respectively obtained for SSD, GaussSSD, Epa2side, and historical returns for ICX order. Therefore, we confirm that Rachev ratio works better with historical returns while the other models perform better using particular penalization or conditional return definitions.
- These stochastic dominance tests justify the use of conditional penalized returns in particular for models like Sharpe ratio, Sortino Ratio and $\text{CVaR}_{5\%}$ based models.
- When we optimize Sharpe ratio and $\text{CVaR}_{5\%}$ applied to historical returns, we often obtain good performance for non satiable risk averse investors respect to other strategies.

Finally, we evaluate how the weight (λ) of mean-variance and mean-$\text{CVaR}_{5\%}$ optimization models (see Subsection 3.3.1.1 and 3.3.1.2) influence all optimum mean choices. For the mean-variance model, we use SSD penalization (see Table 3.1) and 6-month in-sample and 1-week out-of-sample length period, because we obtain the best performance of the Sharpe ratio with this length period and penalization model. For the mean-$\text{CVaR}_{5\%}$ we use Gaussian conditional expectation (see Table 3.1) and 1-year in-sample/1-month out-of-sample length period because it was the most performing in the minimization of $\text{CVaR}_{5\%}$.

The mean-variance in Figure 3.1 shows a peak for $\lambda = 0.88$. The ex-post wealth for mean-$\text{CVaR}_{5\%}$ is minimum when we consider the global minimum $\text{CVaR}_{5\%}$ (corresponding to $\lambda = 0$) or the maximum mean strategy ($\lambda = 1$). The mean-$\text{CVaR}_{5\%}$ model gives the highest ex-post wealth for $\lambda = 0.45$, as displayed in Figure 3.2.

To sum up, after we analyzed 504 ex-post samples, we observed that most of the models present better performance than the S&P500 index during the same period. In particular, the ex-post annual return of the Sharpe ratio, $\text{CVaR}_{5\%}$, Sortino ratio, and Rachev ratio are higher 77, 63, 74, and 8 times than the historical. Moreover, all the best results of each optimization model dominate in the sense of ICX order the S&P500 index.

Our finding indicates that the portfolio optimizations using the conditional expectation with penalty-based correction models can apply as a profitable strategy. From the results mentioned above, there is evidence of using the searching information to

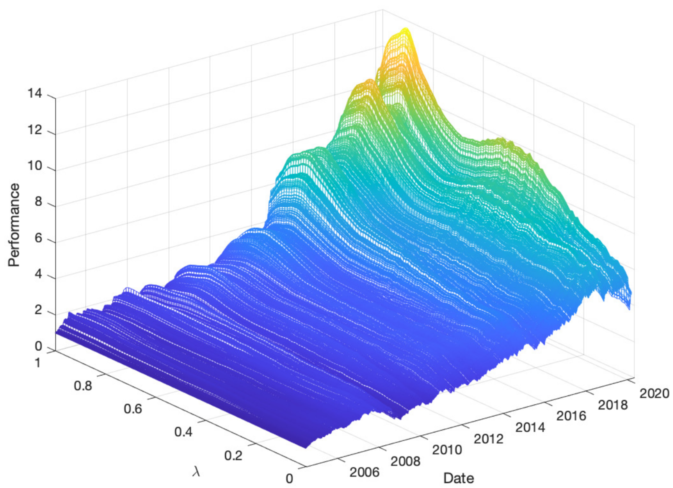

Figure 3.1.: The ex-post performance of mean-variance of penalization with SSD at 6-month in-sample and 1-week out-of-sample performs by varying the weight (λ) from 0 to 1.

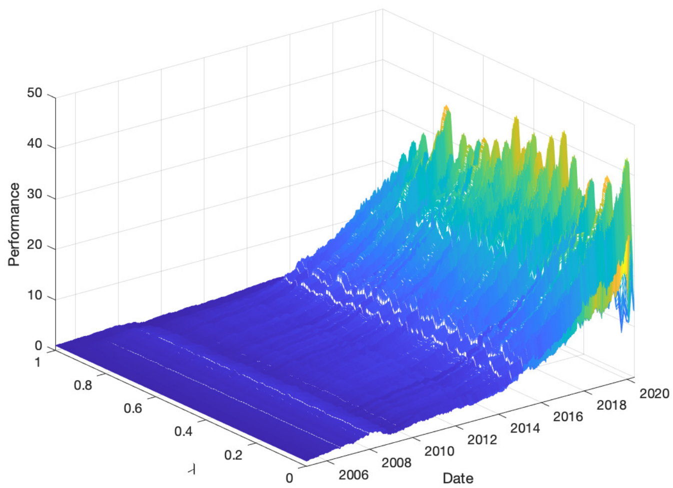

Figure 3.2.: The ex-post performance of mean-CVaR$_{5\%}$ of conditional expectation using the Gaussian kernel function at 1-year in-sample and 1-month out-of-sample performs by varying the weight (λ) from 0 to 1.

predict the financial data as consistent with Da et al. (2011); Vlastakis and Markellos (2012); Vozlyublennaia (2014); Ortobelli et al. (2015).

3.5. Conclusions

As data becomes a new source for financial prediction, we investigate the impact of searching query data on the portfolio optimization models in this study. In particular, we propose a penalty-based correction with conditional expectation and second-order stochastic dominance. We then perform different portfolio optimization problems.

First, we examine the impact of penalization models using portfolio optimization. The result shows the highest ex-post annual return and Sharpe ratio when using EpaSSD with 1-year in-sample and 1-month out-of-sample. As with the dominance comparison, the best results of each optimization model seem to have characteristics for coving investors' preferences of non-satiable risk-seeking (ICX).

Second, we suggest adding the mean-variance and mean-$\text{CVaR}_{5\%}$ optimization models by varying its weight (λ) to evaluate the performance because the results are related to the λ parameter's change.

In summary, the proposed penalty-based correction with conditional expectation using portfolio optimization models can provide a profitable return on investment.

Table 3.2.: The summary of the ex-post annual return using the Sharpe ratio, $CVaR_{5\%}$, Sortino ratio, and Rachev ratio optimization models with different penalizations and kernel function over the backtesting periods.

Penalization	Model	In-sample/Out-of-sample: Annual Return% (Number of times we recalibrate the optimal portfolios) 6mth/1wk (796)	6mth/1mth (199)	6mth/2mth (99)	1yr/1wk (771)	1yr/1mth (192)	1yr/2mth (96)	2yr/1wk (721)	2yr/1mth (180)	2yr/2mth (90)	Winner
Historical	SR	11.2266%	12.3882% +	9.9810%	10.9923%	8.6750%	9.4493%	12.0782%	11.5431%	10.4293%	0
	CVaR	9.0664%	8.4434%	6.8457%	10.2287%	8.6715%	10.3103%+	9.6026%	7.9987%	9.1033%	0
	Sortino	8.3363%	10.0582%	14.2156%	15.2371%	17.2726%+	12.2620%	3.5526%	4.7785%	9.4497%	0
	Rachev	26.7139%*	14.0297%	17.3294%	27.5344%*	17.6892%	16.6314%	28.1955%*+	18.0668%	13.8439%	3
Gauss	SR	11.3555%	12.3730%	10.3220%	8.4393%	7.7217%	7.9981%	14.6247%+	12.1828%	11.2440%	0
	CVaR	9.8031%+	8.8894%	8.1128%	8.7965%	7.2765%	7.3065%	8.1004%	7.5536%	7.5406%	0
	Sortino	13.4847%	15.5690%	25.6119%*+	16.8144%	17.6990%	11.8878%	5.6646%	5.9189%	7.5433%	1
	Rachev	13.6364%	11.6499%	9.5079%	15.4415%+	10.1850%	14.1848%	12.4470%	9.5544%	10.2552%	0
Gauss1side	SR	13.5729%	12.2498%	12.5984%	13.6385%	12.4475%	10.9878%	14.5945%+	12.5301%	13.1047%	0
	CVaR	7.4526%	11.4281%	12.1853%	10.9059%	10.8726%	10.1133%	14.7947%+	11.8319%	13.0790%	0
	Sortino	16.8983%	12.9041%	14.0015%	11.3853%	9.1609%	9.3402%	18.2292%+	14.9751%	15.9905%	0
	Rachev	12.7261%	12.3283%	3.8039%	11.0708%	6.6991%	7.4229%	17.3068%	19.9915%*+	13.0981%	1
Gauss2side	SR	13.6554%+	13.4564%	11.7746%	12.5541%	12.0631%	11.1349%	13.4970%	12.3117%	12.9736%	0
	CVaR	7.1807%	8.4038%	10.4748%	5.6112%	9.5715%	10.2192%	9.0645%	8.1764%	11.9630%+	0
	Sortino	18.3152%	13.7179%	15.8914%	14.2601%	8.6652%	6.7246%	16.9496%	18.7823%+	21.6733%*	1
	Rachev	6.3832%	10.6535%	12.5767%	14.8927%	13.0983%	11.4715%	20.2021%+	12.7066%	15.5406%	0
GaussSSD	SR	15.1630%	13.7022%	15.6325%	16.4492%	17.0404%	16.3199%	17.2259%+	16.2938%	14.8993%	0
	CVaR	10.1851%	14.9927%	13.8213%	8.5802%	17.4536%+	9.5234%	10.2082%	13.2154%	7.5209%	0
	Sortino	9.1611%	10.8812%	14.4226%	16.9174%	23.8910%+	17.8663%	14.7202%	9.2351%	3.7783%	0
	Rachev	17.5480%+	14.3055%	15.5523%	9.8610%	11.4259%	11.3401%	7.1483%	4.7333%	4.1981%	0
Epa	SR	15.2333%+	13.9903%	13.5328%	8.6617%	8.2596%	12.7341%	12.4431%	11.8960%	12.0904%	0
	CVaR	10.0142%+	8.9100%	7.5754%	8.6295%	6.5279%	6.3922%	8.0411%	7.2869%	7.5512%	0
	Sortino	4.7737%	7.6899%	16.9683%	14.7496%	17.8579%+	10.3010%	3.8552%	2.3897%	7.4644%	0
	Rachev	13.8830%+	13.8246%	12.9482%	10.3210%	9.7895%	11.1208%	11.2897%	8.5476%	9.9400%	0
Epa1side	SR	16.3960%+	15.6885%	14.2701%	13.8862%	12.7890%	10.6983%	14.5335%	12.4756%	13.1202%	0
	CVaR	8.0892%	13.2340%	14.2927%	10.5605%	11.3819%	10.0912%	14.2960%+	11.3733%	12.8414%	0
	Sortino	19.7460%+	17.3507%*	17.9932%	12.3519%	9.9000%	10.6199%	18.5086%	14.9751%	15.9905%	1
	Rachev	4.0653%	11.9533%	9.0257%	3.2676%	4.2288%	9.2820%	8.7334%	16.0729%	18.2559%+	0
Epa2side	SR	14.2144%	13.2495%	11.5997%	13.3030%	14.0958%	14.3405%+	13.5210%	12.3438%	12.9854%	0
	CVaR	7.7391%	10.2049%	12.1405%+	5.1821%	9.4786%	10.8036%	8.8703%	8.3833%	11.9924%	0
	Sortino	18.3333%	16.9274%	18.1825%	16.4584%	10.2833%	10.2453%	17.2591%	18.5235%	21.6733%+	0
	Rachev	11.5602%	12.1012%	11.8256%+	9.4122%	2.8953%	-0.5265%	11.4427%	10.3215%	2.0681%	0
EpaSSD	SR	14.8015%	14.5436%	13.3100%	12.0002%	13.3111%	14.7109%	15.5891%+	13.9033%	13.0492%	0
	CVaR	7.0053%	8.5705%	3.2983%	9.8676%	6.9393%	4.5842%	13.6857%	17.0268%+	11.3348%	0
	Sortino	6.7238%	13.6417%	14.1175%	19.6542%	**30.0183%***+★	23.0347%*	20.4578%	12.4107%	8.6860%	2
	Rachev	11.2413%	14.9660%	11.9909%	11.7539%	7.7254%	11.7235%	16.0946%+	14.2563%	12.0665%	0
Student	SR	6.6690%	7.2769%	5.9196%	4.3086%	3.2987%	3.7524%	9.7599%+	7.9673%	6.9162%	0
	CVaR	9.3245%+	8.4397%	8.1672%	8.4211%	7.3297%	7.3188%	8.1059%	7.5414%	7.5404%	0
	Sortino	11.9124%	11.7775%	21.9390%+	20.1074%	17.4403%	10.5801%	6.7643%	6.2601%	9.3601%	0
	Rachev	11.1704%	8.3437%	5.7221%	11.5844%	8.0976%	3.9896%	18.1074%+	9.5321%	8.1691%	0
Student1side	SR	8.0261%	7.5575%	7.6817%	8.1617%	7.5957%	6.2065%	8.8308%+	7.4202%	8.2539%	0
	CVaR	7.7489%	11.6919%	12.2768%	10.8310%	11.2074%	10.0059%	15.1614%+	11.6589%	13.0668%	0
	Sortino	16.3605%	12.2297%	14.0441%	11.1222%	9.1610%	9.3402%	18.1205%+	14.9751%	15.9905%	0
	Rachev	4.7921%	11.1472%	8.6889%	-1.7422%	5.8581%	6.0202%	14.6354%+	10.7523%	11.7058%	0
Student2side	SR	7.9300%	8.2685%+	6.8089%	6.9631%	7.0704%	6.1628	7.6408%	7.0154%	7.8265%	0
	CVaR	7.1664%	8.4720%	10.3738%	5.2569%	9.6364%	10.0875	9.0707%	8.1962%	11.9755%+	0
	Sortino	16.9224%	14.6723%	16.3128%	14.7070%	9.3726%	7.2141	17.5235%	19.2543%	21.6733%+	0
	Rachev	9.5102%	3.9166%	15.3276%	10.0773%	3.1926%	9.0917	16.6400%+	7.4675%	-0.7346%	0
StudentSSD	SR	10.9937%	9.3932%	10.3508%	10.9690%	11.2671%+	9.6363%	11.0034%	10.7682%	10.3875%	0
	CVaR	7.2523%	12.3287%	9.0210%	10.4993%	20.8491%+	12.7582%	9.1615%	13.7249%	7.1550%	0
	Sortino	7.8073%	12.8066%	13.2857%	14.9467%	22.3679%+	18.8489%	13.3266%	12.8550%	6.8669%	0
	Rachev	11.0165%	9.2622%	5.8789%	12.3126%	18.2351%+	10.6348%	5.0481%	1.7988%	7.3484%	0
SSD	SR	19.0000%+	16.7655%	12.2557%	18.2190%	17.5032%	14.9611%	17.1762%	14.0944%	14.1259%	0
	CVaR	9.1387%	10.9185%	2.4116%	7.6541%	11.1759%+	6.0474%	3.0404%	3.5719%	4.1205%	0
	Sortino	4.6510%	5.7289%	14.3378%	11.9611%+	6.1692%	2.9030%	9.3035%	9.5106%	7.6176%	0
	Rachev	1.9546%	7.3160%	20.4234%+	-0.9326%	10.1543%	0.6379%	-4.0255%	5.1377%	14.3789%	0
Winner		10	2	5	2	10	2	17	3	5	

Note:
* *is the highest ex-post annual return among all models (for each column) among all in-sample/out-of-sample backtesting periods,*
\+ *is the highest ex-post annual return among all models (for each row),*
★ *is the highest ex-post annual return.*

Table 3.3.: The summary of the Sharpe ratio with different penalizations and kernel functions over the backtesting periods.

Penalization	Model	In-sample/Out-of-sample: Sharpe Ratio (Number of times we recalibrate the optimal portfolios)									Winner
		6mth/1wk (796)	6mth/1mth (199)	6mth/2mth (99)	1yr/1wk (771)	1yr/1mth (192)	1yr/2mth (96)	2yr/1wk (721)	2yr/1mth (180)	2yr/2mth (90)	
Historical	SR	2.90	3.11	2.78	2.76	2.84	2.43	3.24^{+}	2.78	2.67	0
	CVaR	2.17	1.88	1.88	2.66	2.66	2.93^{+}	2.20	2.18	2.68	0
	Sortino	1.49	0.99	2.88	5.07	5.57^{+}	3.66	-0.62	-0.72	0.82	0
	Rachev	9.64*	5.86	5.46	9.77*	5.70	5.63	10.00^{*+}	5.76*	4.26	4
Gauss	SR	3.18	3.44	3.59	2.39	2.66	2.38	3.90^{+}	3.57	3.24	0
	CVaR	2.84^{+}	2.49	2.49	2.51	2.25	2.26	2.22	2.04	2.04	0
	Sortino	3.05	3.91	8.26^{*+}	5.69	5.65	3.50	1.21	0.74	1.58	1
	Rachev	0.18	2.96	2.74	4.00	3.15	4.87^{+}	2.94	2.49	3.48	0
Gauss1side	SR	4.04	3.52	3.82	4.07	4.01	3.31	4.52	3.80	3.78	0
	CVaR	2.12	3.31	3.56	2.51	3.04	3.10	3.29	4.11	4.85^{+}	0
	Sortino	4.65	4.35	4.48	3.91	3.16	3.28	5.36^{+}	4.48	4.61	0
	Rachev	3.19	3.29	0.80	3.35	1.79	1.58	5.06	5.14^{+}	2.74	0
Gauss2side	SR	4.11^{+}	4.08	3.63	3.58	3.82	3.43	4.02	3.50	3.63	0
	CVaR	2.32	2.59	2.95	1.41	3.00	3.35^{+}	2.84	2.20	3.04	0
	Sortino	4.59	3.13	4.85	3.75	3.77	3.33	5.15	5.29	5.33^{+}	0
	Rachev	2.15	2.93	3.47	3.82	3.34	2.64	4.70^{+}	2.15	2.77	0
GaussSSD	SR	4.02	3.75	4.94	4.09	5.09^{+}	4.94	4.75	4.62	4.28	0
	CVaR	4.47	5.68	5.02	3.03	6.75^{+}	2.25	3.62	3.64	1.59	0
	Sortino	3.77	4.54	5.10	4.88	7.53^{+}	4.97	5.34	2.45	0.13	0
	Rachev	6.33^{+}	4.70	5.89	3.73	4.37	4.07	2.85	2.07	1.23	0
Epa	SR	4.98^{+}	4.40	4.79	2.42	2.82	4.42	3.94	3.73	3.63	0
	CVaR	2.95^{+}	2.72	2.18	2.18	1.91	1.95	2.25	2.01	2.05	0
	Sortino	0.54	0.56	3.91	5.25	6.06^{+}	2.97	-0.53	-1.21	0.69	0
	Rachev	3.64^{+}	3.58	3.17	3.11	3.09	3.19	3.18	2.32	2.56	0
Epa1side	SR	5.36^{+}	5.14	4.63	4.18	4.19	3.18	4.48	3.78	3.78	0
	CVaR	2.40	4.09	4.52	2.41	3.25	3.12	3.11	3.95	4.76^{+}	0
	Sortino	6.06	6.43^{+}	6.37	4.20	3.17	3.30	5.41	4.48	4.61	0
	Rachev	1.54	4.50	1.96	1.30	-0.10	1.31	3.92	4.13	5.09^{+}	0
Epa2side	SR	4.36	3.98	3.54	3.92	4.81	4.98^{+}	4.01	3.51	3.64	0
	CVaR	2.53	3.35	3.74^{+}	1.22	2.84	3.58	2.76	2.27	3.05	0
	Sortino	4.77	4.54	5.63^{+}	4.32	4.49	4.86	5.15	5.07	5.33	0
	Rachev	4.11^{+}	4.02	3.28	3.02	-0.30	-0.86	3.60	1.92	-0.16	0
EpaSSD	SR	4.10	4.08	3.68	3.13	4.10	4.97^{+}	4.73	4.09	3.66	0
	CVaR	2.31	2.68	0.37	2.03	2.10	0.65	3.25	4.24^{+}	2.52	0
	Sortino	2.84	6.66*	6.06	6.40	**10.70**$^{*+\star}$	7.140*	6.00	3.54	1.42	3
	Rachev	3.78	4.28	4.18	3.31	2.19	3.81	4.35^{+}	4.26	3.97	0
Student	SR	1.64	1.75	1.81	1.02	1.29	1.09	2.47^{+}	2.14	1.97	0
	CVaR	2.86^{+}	2.53	2.52	2.44	2.28	2.28	2.22	2.04	2.04	0
	Sortino	3.02	2.59	6.90^{+}	6.90	5.72	3.04	1.19	1.07	2.33	0
	Rachev	2.38	2.71	2.23	2.83	2.21	1.64	5.69^{+}	2.03	1.98	0
Student1side	SR	2.32	2.08	2.20	2.39	2.45	1.79	2.73^{+}	2.22	2.24	0
	CVaR	2.20	3.38	3.59	2.52	3.21	3.06	3.38	4.02	4.83^{+}	0
	Sortino	4.48	4.06	4.50	3.94	3.16	3.28	5.34^{+}	4.48	4.61	0
	Rachev	0.95	2.03	1.32	-0.24	2.12	1.76	3.77^{+}	2.78	2.73	0
Student2side	SR	2.33	2.44^{+}	2.03	1.86	2.20	1.82	2.19	1.83	1.98	0
	CVaR	2.31	2.59	2.89	1.28	2.97	3.29^{+}	2.86	2.23	3.05	0
	Sortino	4.10	3.12	4.78	3.91	3.99	3.49	5.15	5.29	5.33^{*+}	1
	Rachev	2.48	0.83	4.18	2.03	0.74	2.25	5.91^{+}	1.26	-1.12	0
StudentSSD	SR	2.93	2.65	3.20^{+}	2.22	3.01	2.71	2.65	2.64	2.45	0
	CVaR	3.11	5.27	2.95	3.46	7.37^{+}	2.85	3.04	4.13	2.01	0
	Sortino	4.16	5.57	4.78	4.22	7.20^{+}	5.51	4.65	4.00	1.22	0
	Rachev	4.73	4.11	2.63	4.50	6.56^{+}	5.52	1.11	1.05	2.00	0
SSD	SR	5.64^{+}	5.31	4.00	5.25	5.38	4.88	4.70	4.18	4.12	0
	CVaR	2.22	3.78^{+}	0.67	2.06	3.64	1.67	0.24	0.65	0.85	0
	Sortino	1.99	0.84	3.73	4.25	1.15	0.76	4.37^{+}	2.80	1.34	0
	Rachev	2.08	3.23	6.72^{+}	1.45	4.28	1.76	-0.54	0.46	3.72	0
Winner		10	3	6	2	9	6	14	2	6	

Note:
$*$ *is the highest Sharpe ratio among all models (for each column) among all in-sample/out-of-sample backtesting periods,*
$+$ *is the highest Sharpe ratio among all models (for each row),*
$\star$ *is the highest Sharpe ratio.*

Table 3.4.: The summary of the maximum drawdown with different penalizations and kernel functions over the backtesting periods.

Penalization	Model	In-sample/Out-of-sample*+ Annual Return% (Number of times we recalibrate the optimal portfolios) $^{6mth}/_{1wk}$ (796)	$^{6mth}/_{1mth}$ (199)	$^{6mth}/_{2mth}$ (99)	$^{1yr}/_{1wk}$ (771)	$^{1yr}/_{1mth}$ (192)	$^{1yr}/_{2mth}$ (96)	$^{2yr}/_{1wk}$ (721)	$^{2yr}/_{1mth}$ (180)	$^{2yr}/_{2mth}$ (90)	Winner
Historical	SR	0.37	0.35*	0.37	0.37	0.35+	0.38	0.37	0.41	0.37	1
	CVaR	0.48	0.41	0.45	0.43	0.42	0.42	0.46	0.51	0.38+	0
	Sortino	0.83	0.91	0.82	0.61	0.59+	0.71	0.87	0.86	0.82	0
	Rachev	0.39	0.54	0.52	0.39	0.41	0.36*	0.39	0.34+	0.44	1
Gauss	SR	0.43	0.41	0.38+	0.46	0.47	0.49	0.43	0.41	0.40	0
	CVaR	0.54+	0.56	0.56	0.56	0.56	0.56	0.56	0.56	0.56	0
	Sortino	0.84	0.81	0.61+	0.61+	0.66	0.75	0.84	0.85	0.85	0
	Rachev	**0.10***+★	0.64	0.66	0.58	0.50	0.54	0.54	0.65	0.53	1
Gauss1side	SR	0.44	0.43	0.46	0.44	0.45	0.48	0.44	0.44	0.40+	0
	CVaR	0.60	0.49	0.50	0.56	0.57	0.56	0.56	0.45+	0.53	0
	Sortino	0.54	0.47	0.49	0.51	0.42	0.52	0.37+	0.43	0.45	0
	Rachev	0.54	0.62	0.77	0.70	0.80	0.79	0.67	0.52+	0.59	0
Gauss2side	SR	0.46	0.48	0.53	0.46	0.49	0.50	0.46+	0.50	0.49	0
	CVaR	0.56	0.49+	0.55	0.55	0.53	0.55	0.56	0.53	0.55	0
	Sortino	0.58	0.51	0.47	0.37+	0.49	0.67	0.42	0.41	0.37	0
	Rachev	0.75	0.69	0.57+	0.61	0.77	0.80	0.70	0.68	0.78	0
GaussSSD	SR	0.53	0.46	0.46	0.52	0.44	0.43	0.41+	0.42	0.44	0
	CVaR	0.60	0.66	0.68	0.59	0.55+	0.60	0.73	0.68	0.62	0
	Sortino	0.67	0.72	0.73	0.63	0.46+	0.64	0.68	0.71	0.76	0
	Rachev	0.32+	0.40	0.52	0.58	0.65	0.54	0.51	0.54	0.55	0
Epa	SR	0.36	0.39	0.45	0.35*	0.40	0.45	0.34*	0.33*	0.32*+	4
	CVaR	0.51+	0.52	0.59	0.58	0.59	0.59	0.55	0.56	0.56	0
	Sortino	0.85	0.89	0.71	0.64	0.61+	0.76	0.89	0.91	0.84	0
	Rachev	0.49	0.50	0.50	0.54	0.45+	0.54	0.60	0.59	0.58	0
Epa1side	SR	0.44	0.43	0.45	0.44	0.45	0.48	0.44	0.44	0.40+	0
	CVaR	0.60	0.49	0.49	0.55	0.55	0.58	0.56	0.45+	0.53	0
	Sortino	0.54	0.47	0.49	0.50	0.42	0.43	0.37+	0.43	0.45	0
	Rachev	0.78	0.67	0.89	0.78	0.79	0.77	0.64	0.57	0.55+	0
Epa2side	SR	0.46	0.48	0.53	0.46	0.49	0.50	0.46+	0.50	0.49	0
	CVaR	0.57	0.48+	0.55	0.56	0.54	0.55	0.56	0.52	0.55	0
	Sortino	0.60	0.51	0.47	0.37+	0.49	0.65	0.46	0.37	0.37	0
	Rachev	0.59+	0.61	0.85	0.74	0.75	0.75	0.66	0.64	0.68	0
EpaSSD	SR	0.44	0.41	0.37*+	0.47	0.44	0.49	0.44	0.44	0.46	1
	CVaR	0.66	0.58	0.70	0.51+	0.64	0.64	0.77	0.73	0.60	0
	Sortino	0.61	0.72	0.76	0.57	0.46+	0.57	0.64	0.66	0.61	0
	Rachev	0.61	0.61	0.76	0.61	0.62	0.72	0.56+	0.62	0.67	0
Student	SR	0.51	0.50	0.47	0.53	0.52	0.54	0.49	0.51	0.47+	0
	CVaR	0.54+	0.56	0.56	0.57	0.56	0.56	0.56	0.56	0.56	0
	Sortino	0.83	0.80	0.61	0.58+	0.68	0.73	0.84	0.83	0.85	0
	Rachev	0.63	0.62	0.67	0.63	0.65	0.51+	0.59	0.68	0.68	0
Student1side	SR	0.53	0.53	0.55	0.53	0.55	0.58	0.53	0.51	0.49+	0
	CVaR	0.60	0.49	0.49	0.57	0.55	0.57	0.56	0.45+	0.53	0
	Sortino	0.54	0.47	0.49	0.51	0.42	0.52	0.37+	0.43	0.45	0
	Rachev	0.77	0.67	0.69	0.80	0.82	0.75	0.76	0.66	0.63+	0
Student2side	SR	0.56+	0.58	0.62	0.56+	0.59	0.60	0.56+	0.60	0.58	0
	CVaR	0.56	0.48+	0.55	0.56	0.54	0.55	0.56	0.52	0.55	0
	Sortino	0.60	0.51	0.47	0.37+	0.49	0.67	0.45	0.41	0.37+	0
	Rachev	0.73	0.76	0.65+	0.73	0.79	0.77	0.71	0.74	0.87	0
StudentSSD	SR	0.58	0.52+	0.55	0.62	0.60	0.59	0.52+	0.52+	0.54	0
	CVaR	0.59	0.73	0.70	0.59	0.49+	0.63	0.73	0.67	0.61	0
	Sortino	0.69	0.72	0.75	0.62	0.42+	0.64	0.68	0.64	0.64	0
	Rachev	0.53	0.53	0.57	0.36+	0.40	0.69	0.62	0.52	0.54	0
SSD	SR	0.41	0.38	0.45	0.41	0.32*+	0.39	0.41	0.43	0.41	1
	CVaR	0.64	0.50	0.57	0.65	0.49+	0.54	0.66	0.53	0.56	0
	Sortino	0.76	0.86	0.69	0.48+	0.69	0.73	0.68	0.59	0.61	0
	Rachev	0.78	0.82	0.86	0.77	0.74	0.85	0.79	0.78	0.74+	0
Winner		7	4	5	9	11	1	9	6	10	

Note:
* *is the lowest maximum drawdown among all models (for each column) among all in-sample/out-of-sample backtesting periods,*
\+ *is the lowest maximum drawdown among all models (for each row),*
★ *is the lowest maximum drawdown .*

Table 3.5.: Amount of ICX and SSD stochastic dominance relations obtained by maximizing the Sharpe ratio for each in-sample/out-of-sample length period changing penalization typologies and conditional expectation definition.

Penalization		Historical	Gauss	Gauss1side	Gauss2side	GaussSSD	Epa	Epa1side	Epa2side	EpaSSD	Student	Student1side	Student2side	StudentSSD	SSD	Sum
Historical	$\overset{\text{ICX}}{\succ}$	-	0	0	0	0	0	0	0	0	4	0	0	0	0	4
	$\overset{\text{SSD}}{\succ}$	-	1	1	0	0	0	0	0	0	2	6	5	1	0	16
	$\overset{\text{ICX}}{\prec}$	-	3	5	3	5	3	6	9	9	1	2	1	5	6	58
	$\overset{\text{SSD}}{\prec}$	-	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Gauss	$\overset{\text{ICX}}{\succ}$	3	-	0	0	0	2	0	0	0	5	0	2	0	0	12
	$\overset{\text{SSD}}{\succ}$	0	-	1	0	0	0	1	0	0	0	1	0	0	0	3
	$\overset{\text{ICX}}{\prec}$	0	-	2	6	9	1	3	6	9	0	1	4	6	9	56
	$\overset{\text{SSD}}{\prec}$	1	-	0	0	0	1	0	0	0	0	0	0	0	0	2
Gauss1side	$\overset{\text{ICX}}{\succ}$	5	2	-	3	0	0	3	2	0	2	0	4	0	0	21
	$\overset{\text{SSD}}{\succ}$	0	0	-	0	0	0	0	0	0	0	0	0	0	0	0
	$\overset{\text{ICX}}{\prec}$	0	0	-	0	6	1	3	3	4	0	1	0	6	7	31
	$\overset{\text{SSD}}{\prec}$	1	1	-	1	0	0	0	0	0	1	0	0	0	0	4
Gauss2side	$\overset{\text{ICX}}{\succ}$	3	6	0	-	0	0	1	0	0	6	0	2	0	0	18
	$\overset{\text{SSD}}{\succ}$	0	0	1	-	0	0	0	0	0	0	0	0	0	0	1
	$\overset{\text{ICX}}{\prec}$	0	0	3	-	5	1	5	5	6	1	3	0	6	8	43
	$\overset{\text{SSD}}{\prec}$	0	0	0	-	0	1	0	0	0	0	0	0	0	0	1
GaussSSD	$\overset{\text{ICX}}{\succ}$	5	9	6	5	-	6	6	5	3	9	6	3	2	5	70
	$\overset{\text{SSD}}{\succ}$	0	0	0	0	-	0	0	0	0	0	0	0	0	0	0
	$\overset{\text{ICX}}{\prec}$	0	0	0	0	-	0	2	0	0	0	0	0	2	3	7
	$\overset{\text{SSD}}{\prec}$	0	0	0	0	-	1	0	0	0	0	0	0	1	0	2
Epa	$\overset{\text{ICX}}{\succ}$	3	1	1	1	0	-	1	0	0	3	1	3	0	0	14
	$\overset{\text{SSD}}{\succ}$	0	1	0	1	1	-	0	1	0	0	0	0	1	0	5
	$\overset{\text{ICX}}{\prec}$	0	2	0	0	6	-	3	0	4	0	0	0	4	6	25
	$\overset{\text{SSD}}{\prec}$	0	0	0	0	0	-	0	0	0	0	0	0	0	0	0
Epa1side	$\overset{\text{ICX}}{\succ}$	6	3	3	5	2	3	-	2	0	3	3	4	1	1	36
	$\overset{\text{SSD}}{\succ}$	0	0	0	0	0	0	-	0	0	0	0	0	0	0	0
	$\overset{\text{ICX}}{\prec}$	0	0	3	1	6	1	-	2	3	0	3	1	5	7	32
	$\overset{\text{SSD}}{\prec}$	0	1	0	0	0	0	-	0	1	1	0	0	0	0	3
Epa2side	$\overset{\text{ICX}}{\succ}$	9	6	3	5	0	0	2	-	0	6	2	4	0	0	37
	$\overset{\text{SSD}}{\succ}$	0	0	0	0	0	0	0	-	0	0	0	0	0	0	0
	$\overset{\text{ICX}}{\prec}$	0	0	2	0	5	0	2	-	2	1	2	0	3	6	23
	$\overset{\text{SSD}}{\prec}$	0	0	0	0	0	1	0	-	0	0	0	0	0	0	1
EpaSSD	$\overset{\text{ICX}}{\succ}$	9	9	4	6	0	4	3	2	-	6	4	8	0	0	55
	$\overset{\text{SSD}}{\succ}$	0	0	0	0	0	0	1	0	-	0	0	0	0	0	1
	$\overset{\text{ICX}}{\prec}$	0	0	0	0	3	0	0	0	-	0	0	0	0	3	6
	$\overset{\text{SSD}}{\prec}$	0	0	0	0	0	0	0	0	-	0	0	0	2	0	2
Student	$\overset{\text{ICX}}{\succ}$	1	0	0	1	0	0	0	1	0	-	0	1	0	0	4
	$\overset{\text{SSD}}{\succ}$	0	0	1	0	0	0	1	0	0	-	1	0	0	0	3
	$\overset{\text{ICX}}{\prec}$	4	5	2	6	9	3	3	6	6	-	3	6	9	6	68
	$\overset{\text{SSD}}{\prec}$	2	0	0	0	0	0	0	0	0	-	0	0	0	0	2
Student1side	$\overset{\text{ICX}}{\succ}$	2	1	1	3	0	0	3	2	0	3	-	4	0	0	19
	$\overset{\text{SSD}}{\succ}$	0	0	0	0	0	0	0	0	0	0	-	0	0	0	0
	$\overset{\text{ICX}}{\prec}$	0	0	0	0	6	1	3	2	4	0	-	0	6	6	38
	$\overset{\text{SSD}}{\prec}$	6	1	0	0	0	0	0	0	0	1	-	0	0	0	8
Student2side	$\overset{\text{ICX}}{\succ}$	1	4	0	0	0	0	1	0	0	6	0	-	0	0	12
	$\overset{\text{SSD}}{\succ}$	0	0	0	0	0	0	0	0	0	0	0	-	0	0	0
	$\overset{\text{ICX}}{\prec}$	0	2	4	2	3	3	4	4	8	1	4	-	6	6	47
	$\overset{\text{SSD}}{\prec}$	5	0	0	0	0	0	0	0	0	0	0	-	0	0	5
StudentSSD	$\overset{\text{ICX}}{\succ}$	5	6	6	6	2	4	5	3	0	9	6	6	-	1	59
	$\overset{\text{SSD}}{\succ}$	0	0	0	0	1	0	0	0	2	0	0	0	-	0	3
	$\overset{\text{ICX}}{\prec}$	0	0	0	0	2	0	1	0	0	0	0	0	-	2	5
	$\overset{\text{SSD}}{\prec}$	1	0	0	0	0	1	0	0	0	0	0	0	-	0	2
SSD	$\overset{\text{ICX}}{\succ}$	6	9	7	8	3	6	7	6	3	6	6	6	2	-	**75**
	$\overset{\text{SSD}}{\succ}$	0	0	0	0	0	0	0	0	0	0	0	0	0	-	0
	$\overset{\text{ICX}}{\prec}$	0	0	0	0	5	0	1	0	0	0	0	0	1	-	7
	$\overset{\text{SSD}}{\prec}$	0	0	0	0	0	0	0	0	0	0	0	0	0	-	0

Table 3.6.: Amount of ICX and SSD stochastic dominance relations obtained by minimizing the $CVaR_{5\%}$ for each in-sample/out-of-sample length period changing penalization typologies and conditional expectation definition..

Penalization		Historical	Gauss	Gauss1side	Gauss2side	GaussSSD	Epa	Epa1side	Epa2side	EpaSSD	Student	Student1side	Student2side	StudentSSD	SSD	Sum
Historical	$\overset{ICX}{\succ}$	-	1	0	1	0	3	0	0	2	1	0	1	0	1	10
	$\overset{SSD}{\succ}$	-	3	2	4	2	1	2	2	1	0	2	3	2	0	24
	$\overset{ICX}{\prec}$	-	0	4	1	6	0	4	4	3	1	4	2	3	1	33
	$\overset{SSD}{\prec}$	-	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Gauss	$\overset{ICX}{\succ}$	0	-	0	0	0	3	0	0	2	0	0	0	0	0	5
	$\overset{SSD}{\succ}$	0	-	0	0	1	0	0	0	1	0	0	0	2	0	4
	$\overset{ICX}{\prec}$	1	-	5	3	6	0	5	4	3	1	5	3	4	1	41
	$\overset{SSD}{\prec}$	3	-	0	0	0	1	0	0	0	0	0	0	0	0	4
Gauss1side	$\overset{ICX}{\succ}$	4	5	-	4	0	5	3	4	5	3	0	5	0	3	41
	$\overset{SSD}{\succ}$	0	0	-	0	2	0	0	0	0	0	0	0	2	0	4
	$\overset{ICX}{\prec}$	0	0	-	0	4	0	1	0	1	0	5	0	2	0	13
	$\overset{SSD}{\prec}$	2	0	-	0	0	0	0	0	0	0	0	0	0	0	2
Gauss2side	$\overset{ICX}{\succ}$	1	3	0	-	0	3	0	0	3	2	0	0	0	2	14
	$\overset{SSD}{\succ}$	0	0	0	-	0	0	0	0	0	0	0	0	1	0	1
	$\overset{ICX}{\prec}$	1	0	4	-	4	0	1	2	2	0	4	0	4	0	22
	$\overset{SSD}{\prec}$	4	0	0	-	0	0	0	0	0	1	0	1	0	1	7
GaussSSD	$\overset{ICX}{\succ}$	6	6	4	4	-	6	3	6	5	4	4	4	3	4	**59**
	$\overset{SSD}{\succ}$	0	0	0	0	-	0	0	0	0	0	0	0	0	0	0
	$\overset{ICX}{\prec}$	0	0	0	0	-	0	0	0	2	0	0	0	2	0	4
	$\overset{SSD}{\prec}$	2	1	2	0	-	1	3	0	2	0	1	0	0	0	12
Epa	$\overset{ICX}{\succ}$	0	0	0	0	0	-	0	0	2	0	0	0	0	0	2
	$\overset{SSD}{\succ}$	0	1	0	0	1	-	0	0	0	0	0	0	2	0	4
	$\overset{ICX}{\prec}$	3	3	5	3	6	-	5	4	3	1	5	3	4	1	46
	$\overset{SSD}{\prec}$	1	0	0	0	0	-	0	0	0	0	0	0	0	0	1
Epa1side	$\overset{ICX}{\succ}$	4	5	1	1	0	5	-	1	5	2	2	2	1	2	31
	$\overset{SSD}{\succ}$	0	0	0	0	3	0	-	0	0	1	0	0	3	1	8
	$\overset{ICX}{\prec}$	0	0	3	0	3	0	-	0	1	0	3	0	1	0	11
	$\overset{SSD}{\prec}$	2	0	0	0	0	0	-	0	0	1	0	0	0	1	4
Epa2side	$\overset{ICX}{\succ}$	4	4	0	2	0	4	0	-	4	1	0	2	0	1	22
	$\overset{SSD}{\succ}$	0	0	0	0	0	0	0	-	0	0	0	0	1	0	1
	$\overset{ICX}{\prec}$	0	0	4	0	6	0	1	-	2	0	4	0	4	0	21
	$\overset{SSD}{\prec}$	2	0	0	0	0	0	0	-	0	1	0	0	0	1	4
EpaSSD	$\overset{ICX}{\succ}$	3	3	1	2	2	3	1	2	-	3	1	2	2	3	28
	$\overset{SSD}{\succ}$	0	0	0	0	2	0	0	0	-	0	0	0	0	0	2
	$\overset{ICX}{\prec}$	2	2	5	3	5	2	5	4	-	3	5	3	2	3	44
	$\overset{SSD}{\prec}$	1	1	0	0	0	0	0	0	-	0	0	0	0	0	2
Student	$\overset{ICX}{\succ}$	0	2	0	0	0	2	0	0	2	-	0	0	0	0	6
	$\overset{SSD}{\succ}$	0	0	0	0	0	0	0	0	0	-	0	0	2	0	2
	$\overset{ICX}{\prec}$	1	0	5	3	6	0	5	4	3	-	5	3	4	1	40
	$\overset{SSD}{\prec}$	4	3	0	0	0	2	0	0	0	-	0	0	0	0	9
Student1side	$\overset{ICX}{\succ}$	4	5	5	4	0	5	3	4	5	5	-	4	0	3	47
	$\overset{SSD}{\succ}$	0	0	0	0	1	0	0	0	0	0	-	0	2	0	3
	$\overset{ICX}{\prec}$	0	0	0	0	4	0	2	0	1	0	-	0	2	0	9
	$\overset{SSD}{\prec}$	2	0	0	0	0	0	0	0	0	0	-	1	0	0	3
Student2side	$\overset{ICX}{\succ}$	2	3	0	0	0	3	0	0	3	3	0	-	0	2	16
	$\overset{SSD}{\succ}$	0	0	0	1	0	0	0	0	0	0	1	-	1	0	3
	$\overset{ICX}{\prec}$	1	0	5	0	4	0	2	2	2	0	4	-	4	0	24
	$\overset{SSD}{\prec}$	3	0	0	0	0	0	0	0	0	0	0	-	0	1	4
StudentSSD	$\overset{ICX}{\succ}$	3	4	2	4	2	4	1	4	2	4	2	4	-	5	41
	$\overset{SSD}{\succ}$	0	0	0	0	0	0	0	0	0	0	0	0	-	0	0
	$\overset{ICX}{\prec}$	0	0	0	0	3	0	1	0	2	0	0	0	-	0	6
	$\overset{SSD}{\prec}$	2	2	2	1	0	2	3	1	0	2	2	1	-	1	19
SSD	$\overset{ICX}{\succ}$	1	1	0	0	0	1	0	0	3	1	0	0	0	-	7
	$\overset{SSD}{\succ}$	0	0	0	1	0	0	1	1	0	0	0	1	1	-	5
	$\overset{ICX}{\prec}$	1	0	3	2	4	0	2	1	3	0	3	2	5	-	26
	$\overset{SSD}{\prec}$	0	0	0	0	0	0	1	0	0	0	0	0	0	-	1

Table 3.7.: Amount of ICX and SSD stochastic dominance relations obtained by maximizing the Sortino ratio for each in-sample/out-of-sample length period changing penalization typologies and conditional expectation definition.

Penalization		Historical	Gauss	Gauss1side	Gauss2side	GaussSSD	Epa	Epa1side	Epa2side	EpaSSD	Student	Student1side	Student2side	StudentSSD	SSD	Sum
Historical	$\overset{ICX}{\succ}$	-	0	2	2	1	2	2	1	1	0	2	2	2	5	22
	$\overset{SSD}{\succ}$	-	1	1	0	0	0	0	0	1	1	1	0	1	0	6
	$\overset{ICX}{\prec}$	-	6	5	6	5	2	6	6	5	5	4	6	4	0	60
	$\overset{SSD}{\prec}$	-	0	0	0	1	0	0	1	0	0	0	0	0	1	3
Gauss	$\overset{ICX}{\succ}$	6	-	4	4	3	6	3	3	1	2	4	4	3	5	48
	$\overset{SSD}{\succ}$	0	-	0	0	0	0	0	0	1	0	0	0	0	0	1
	$\overset{ICX}{\prec}$	0	-	4	4	3	1	5	4	4	1	4	3	3	0	36
	$\overset{SSD}{\prec}$	1	-	0	0	1	0	0	0	1	0	0	0	0	1	4
Gauss1side	$\overset{ICX}{\succ}$	5	4	-	0	3	5	0	0	2	5	2	0	2	6	34
	$\overset{SSD}{\succ}$	0	0	-	0	0	0	0	0	0	0	0	1	1	0	2
	$\overset{ICX}{\prec}$	2	4	-	3	3	3	3	8	6	3	1	4	2	0	42
	$\overset{SSD}{\prec}$	1	0	-	0	0	0	0	0	0	0	0	0	0	1	2
Gauss2side	$\overset{ICX}{\succ}$	6	4	3	-	3	5	2	0	2	4	4	1	3	7	44
	$\overset{SSD}{\succ}$	0	0	0	-	1	0	0	0	1	0	0	0	1	0	3
	$\overset{ICX}{\prec}$	2	4	0	-	3	2	3	6	4	3	0	4	2	0	33
	$\overset{SSD}{\prec}$	0	0	0	-	0	0	0	0	0	0	0	0	0	0	0
GaussSSD	$\overset{ICX}{\succ}$	5	3	3	3	-	3	3	2	0	3	3	3	4	7	42
	$\overset{SSD}{\succ}$	1	1	0	0	-	1	0	0	1	0	0	0	1	0	5
	$\overset{ICX}{\prec}$	1	3	3	3	-	1	5	5	6	3	3	3	3	0	39
	$\overset{SSD}{\prec}$	0	0	0	1	-	0	0	0	0	1	1	0	0	0	3
Epa	$\overset{ICX}{\succ}$	2	1	3	2	1	-	2	1	0	1	3	2	2	4	24
	$\overset{SSD}{\succ}$	0	0	0	0	0	-	0	0	0	0	0	0	0	0	0
	$\overset{ICX}{\prec}$	2	6	5	5	3	-	6	6	5	5	4	5	4	0	56
	$\overset{SSD}{\prec}$	0	0	0	0	1	-	0	1	1	0	0	0	1	1	5
Epa1side	$\overset{ICX}{\succ}$	6	5	3	3	5	6	-	2	3	5	3	3	4	8	56
	$\overset{SSD}{\succ}$	0	0	0	0	0	0	-	1	1	0	0	1	0	0	3
	$\overset{ICX}{\prec}$	2	3	0	2	3	2	-	4	4	3	0	2	2	0	27
	$\overset{SSD}{\prec}$	0	0	0	0	0	0	-	0	0	0	0	0	0	0	0
Epa2side	$\overset{ICX}{\succ}$	6	4	8	6	5	6	4	-	3	5	8	6	5	9	**75**
	$\overset{SSD}{\succ}$	1	0	0	0	0	1	0	-	1	0	0	0	0	0	3
	$\overset{ICX}{\prec}$	1	3	0	0	2	1	2	-	4	3	0	1	2	0	19
	$\overset{SSD}{\prec}$	0	0	0	0	0	0	1	-	0	0	0	0	0	0	1
EpaSSD	$\overset{ICX}{\succ}$	5	4	6	4	6	5	4	4	-	5	6	4	6	7	66
	$\overset{SSD}{\succ}$	0	1	0	0	0	1	0	0	-	0	0	0	0	0	2
	$\overset{ICX}{\prec}$	1	1	2	2	0	0	3	3	-	3	2	3	0	0	20
	$\overset{SSD}{\prec}$	1	1	0	1	1	0	1	1	-	0	0	1	0	1	8
Student	$\overset{ICX}{\succ}$	5	1	3	3	3	5	3	3	3	-	3	3	3	6	44
	$\overset{SSD}{\succ}$	0	0	0	0	1	0	0	0	0	-	0	0	0	0	1
	$\overset{ICX}{\prec}$	0	2	5	4	3	1	5	5	5	-	5	3	4	0	42
	$\overset{SSD}{\prec}$	1	0	0	0	0	0	0	0	0	-	0	0	0	0	1
Student1side	$\overset{ICX}{\succ}$	4	4	1	0	3	4	0	0	2	5	-	0	2	6	31
	$\overset{SSD}{\succ}$	0	0	0	0	1	0	0	0	0	0	-	1	0	0	2
	$\overset{ICX}{\prec}$	2	4	2	4	3	3	3	8	6	3	-	3	3	0	44
	$\overset{SSD}{\prec}$	1	0	0	0	0	0	0	0	0	0	-	0	0	1	2
Student2side	$\overset{ICX}{\succ}$	6	3	4	4	3	5	2	1	3	3	3	-	2	8	47
	$\overset{SSD}{\succ}$	0	0	0	0	0	0	0	0	1	0	0	-	1	0	2
	$\overset{ICX}{\prec}$	2	4	0	1	3	2	3	6	4	3	0	-	2	0	30
	$\overset{SSD}{\prec}$	0	0	1	0	0	0	1	0	0	0	1	-	0	0	3
StudentSSD	$\overset{ICX}{\succ}$	4	3	2	2	3	4	2	2	0	4	3	2	-	6	37
	$\overset{SSD}{\succ}$	0	0	0	0	0	1	0	0	0	0	0	0	-	0	1
	$\overset{ICX}{\prec}$	2	3	2	3	4	2	4	5	6	3	2	2	-	0	38
	$\overset{SSD}{\prec}$	1	0	1	1	1	0	0	0	0	0	0	1	-	2	7
SSD	$\overset{ICX}{\succ}$	0	0	0	0	0	0	0	0	0	0	0	0	0	-	0
	$\overset{SSD}{\succ}$	1	1	1	0	0	1	0	0	1	0	1	0	2	-	8
	$\overset{ICX}{\prec}$	5	5	6	7	7	4	8	9	7	6	6	8	6	-	84
	$\overset{SSD}{\prec}$	0	0	0	0	0	0	0	0	0	0	0	0	0	-	0

Table 3.8.: Amount of ICX and SSD stochastic dominance relations obtained by maximizing the Rachev ratio for each in-sample/out-of-sample length period changing penalization typologies and conditional expectation definition.

Penalization		Historical	Gauss	Gauss1side	Gauss2side	GaussSSD	Epa	Epa1side	Epa2side	EpaSSD	Student	Student1side	Student2side	StudentSSD	SSD	Sum
Historical	$\overset{ICX}{\succ}$	-	2	3	6	3	6	4	5	3	3	3	4	4	6	**52**
	$\overset{SSD}{\succ}$	-	0	0	0	1	0	0	0	1	0	0	0	1	1	4
	$\overset{ICX}{\prec}$	-	0	0	0	0	0	1	0	0	0	1	1	1	0	4
	$\overset{SSD}{\prec}$	-	0	0	0	1	1	0	0	0	0	0	0	0	0	2
Gauss	$\overset{ICX}{\succ}$	0	-	1	0	1	1	0	0	0	0	0	0	0	4	7
	$\overset{SSD}{\succ}$	0	-	1	0	3	0	1	1	1	0	0	2	1	1	11
	$\overset{ICX}{\prec}$	2	-	2	1	1	0	0	0	2	8	6	2	4	0	28
	$\overset{SSD}{\prec}$	0	-	1	0	0	0	0	0	0	0	0	0	0	0	1
Gauss1side	$\overset{ICX}{\succ}$	0	2	-	3	3	2	5	2	0	0	1	1	0	6	25
	$\overset{SSD}{\succ}$	0	1	-	0	0	0	1	1	1	0	1	0	0	1	6
	$\overset{ICX}{\prec}$	3	1	-	0	0	0	0	0	1	4	4	2	4	0	19
	$\overset{SSD}{\prec}$	0	1	-	0	0	2	0	0	0	0	0	0	0	0	3
Gauss2side	$\overset{ICX}{\succ}$	0	1	0	-	0	3	5	3	0	0	1	1	0	2	16
	$\overset{SSD}{\succ}$	0	0	0	-	0	0	1	0	2	0	1	0	1	1	6
	$\overset{ICX}{\prec}$	6	0	3	-	1	0	2	0	2	3	5	2	3	1	28
	$\overset{SSD}{\prec}$	0	0	0	-	0	2	0	0	0	0	0	0	0	0	2
GaussSSD	$\overset{ICX}{\succ}$	0	1	0	1	-	0	3	2	0	1	2	1	0	5	16
	$\overset{SSD}{\succ}$	1	0	0	0	-	0	1	1	0	0	0	0	1	0	4
	$\overset{ICX}{\prec}$	3	1	3	0	-	0	0	0	3	3	3	2	4	0	22
	$\overset{SSD}{\prec}$	1	3	0	0	-	3	0	0	0	1	0	0	1	0	9
Epa	$\overset{ICX}{\succ}$	0	0	0	0	0	-	3	2	0	0	0	0	0	1	6
	$\overset{SSD}{\succ}$	1	0	2	2	3	-	1	2	4	1	2	1	1	2	22
	$\overset{ICX}{\prec}$	6	1	2	3	0	-	2	0	2	4	5	4	4	1	34
	$\overset{SSD}{\prec}$	0	0	0	0	0	-	0	0	0	0	0	0	0	0	0
Epa1side	$\overset{ICX}{\succ}$	1	0	0	2	0	2	-	0	0	0	0	1	0	4	10
	$\overset{SSD}{\succ}$	0	0	0	0	0	0	-	1	1	0	0	0	0	2	4
	$\overset{ICX}{\prec}$	4	0	5	5	3	3	-	2	6	6	4	3	3	1	45
	$\overset{SSD}{\prec}$	0	1	1	1	1	1	-	2	1	1	0	0	1	0	10
Epa2side	$\overset{ICX}{\succ}$	0	0	0	0	0	0	2	-	0	0	1	0	0	3	6
	$\overset{SSD}{\succ}$	0	0	0	0	0	0	2	-	0	0	0	0	0	0	2
	$\overset{ICX}{\prec}$	5	0	2	3	2	2	0	-	3	3	0	3	3	1	27
	$\overset{SSD}{\prec}$	0	1	1	0	1	2	1	-	1	0	0	0	1	0	8
EpaSSD	$\overset{ICX}{\succ}$	0	2	1	2	3	2	6	3	-	1	1	1	2	7	31
	$\overset{SSD}{\succ}$	0	0	0	0	0	0	1	1	-	0	0	0	0	1	3
	$\overset{ICX}{\prec}$	3	0	0	0	0	0	0	0	-	1	3	2	1	0	10
	$\overset{SSD}{\prec}$	1	1	1	2	0	4	1	0	-	3	1	0	1	0	15
Student	$\overset{ICX}{\succ}$	0	8	4	3	3	4	6	3	1	-	3	1	2	6	44
	$\overset{SSD}{\succ}$	0	0	0	0	1	0	1	0	3	-	0	4	2	0	11
	$\overset{ICX}{\prec}$	3	0	0	0	1	0	0	0	1	-	0	1	3	0	9
	$\overset{SSD}{\prec}$	0	0	0	0	0	1	0	0	0	-	0	0	0	0	1
Student1side	$\overset{ICX}{\succ}$	1	6	4	5	3	5	4	0	3	0	-	4	3	7	45
	$\overset{SSD}{\succ}$	0	0	0	0	0	0	0	0	1	0	-	0	0	0	1
	$\overset{ICX}{\prec}$	3	0	1	1	2	0	0	1	1	3	-	2	3	0	17
	$\overset{SSD}{\prec}$	0	0	1	1	0	2	0	0	0	0	-	0	0	0	4
Student2side	$\overset{ICX}{\succ}$	1	2	2	2	2	4	3	3	2	1	2	-	2	4	30
	$\overset{SSD}{\succ}$	0	0	0	0	0	0	0	0	0	0	0	-	0	0	0
	$\overset{ICX}{\prec}$	4	0	1	1	1	0	1	0	1	1	4	-	4	0	18
	$\overset{SSD}{\prec}$	0	2	0	0	0	1	0	0	0	4	0	-	0	0	7
StudentSSD	$\overset{ICX}{\succ}$	1	4	4	3	4	4	3	3	1	3	3	4	-	5	42
	$\overset{SSD}{\succ}$	0	0	0	0	1	0	1	1	1	0	0	0	-	0	4
	$\overset{ICX}{\prec}$	4	0	0	0	0	0	0	0	2	2	3	2	-	1	14
	$\overset{SSD}{\prec}$	1	1	0	1	1	1	0	0	0	2	0	0	-	0	7
SSD	$\overset{ICX}{\succ}$	0	0	0	1	0	1	1	1	0	0	0	0	1	-	5
	$\overset{SSD}{\succ}$	0	0	0	0	0	0	0	0	0	0	0	0	0	-	0
	$\overset{ICX}{\prec}$	6	4	6	2	5	1	4	3	7	6	7	4	5	-	60
	$\overset{SSD}{\prec}$	1	1	1	1	0	2	2	0	1	0	0	0	0	-	9

4. Impact of Volume on Portfolio Optimization

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4.1. Overview

This study explores the use of volumes of stock returns in portfolio problems. In the analysis, we consider different portfolio strategies applied to the portfolio returns conditional the portfolio of transaction volume using two different estimators of the conditional expectation based either on the Gaussian kernel density function or the Epanechnikov one. In addition, we value some strategies based on penalized returns. To compute the optimal portfolios, we implemented the Sharpe ratio, global minimum $CVaR_{5\%}$, and Rachev ratio optimization, and we found that taking into account volume has an impact on the ex-post wealth. However, this work is not exhaustive but is the starting point for future research.

4.2. Introduction

The relationship between the stock price and trading volume has been studied in several financial works of literature. In early studies, Ying (1966) and Westerfield (1977) found positive relationships between the absolute value of price changes and volume. Further, the evidence of the price-volume relationship can be explained by the rate of information flow into the stock market, as documented by Karpoff (1987). The results provide the behavior of relations between the volume to absolute price ratio and the markets trend. However, the prediction powers have not been investigated. After that, Gervais et al. (2001) revealed that the large trading volumes tend to induce large changes in the stock prices in the next future period.

In the dynamic relationship scheme, the stock returns contribute a positive correlation to volume. The Granger causality tests also show the persistence of its lagged relations; see Chen et al. (2001). Taking a volatility approach to stock return, Lee and Rui (2002) showed that the return volatility reacts to a causal relationship to the trading volume. Moreover, considering the volume as additional information, the forecast volatility model can be explained appropriately by the behavior of the stock returns (Lamoureux and Lastrapes (1990); Gallant et al. (2015)).

In the short-run of stock market behavior, the autocorrelation of stock returns tends to be lower on high-volume days than on low-volume days, as suggested by Campbell et al. (1993). The results underlying this work explained that the buying or selling volume is associated with the stock return. Thus, the basic idea of this work is to implement the effects of volume returns and stock returns in portfolio strategies based on conditional expectation.

Inspired by taking the volume as information to return, we investigate how the stock returns conditional volumes information impacts the portfolio performance. To do so, we apply the conditional expectation using Gaussian and Epanechnikov kernel density function in which the stock returns are conditional to the volumes. The false information may generate if the stock returns are decreasing while the volume returns are positively increasing. Thus we use penalized stock returns to compensate for this effect. We then optimize the portfolio performance by using Sharpe Ratio, global minimum $\text{CVaR}_{5\%}$, and Rachev Ratio applied to the penalized returns.

4.3. Methodology

In this section, we apply the conditional expectation of returns using Gaussian and Epanechnikov kernel functions to the returns and the penalized returns. Then, we use different portfolio optimization models to find optimum choices. In particular, we use the conditional expectation to approximate the returns from the volumes of the portfolio. We set the Nadaraya-Watson as a kernel density estimator:

$$\mathbb{E}\left(y|Vol = x\right) = \frac{\sum_{n=1}^{N} y_n K\left(\frac{x-x_n}{h(N)}\right)}{\sum_{n=1}^{N} K\left(\frac{x-x_n}{h(N)}\right)}$$

where Vol is volume, y is return, $K(.)$ is kernel density function, and $h(.)$ is the bandwidth function defined following the Scott rule (see Scott (2015)) as $h(N) = 3.5N^{(-1/3)}std$ for Gaussian and $h(N) = 3.2N^{(-0.8)}$ for Epanechnikov kernel function.

As for kernel function, we use either the univariate Gaussian:

$$K(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2}$$

or the Epanechnikov one :

$$K(z) = \frac{3}{4}\left(1 - z^2\right) I\left(|z| \leq 1\right)$$

where $I(|z| \leq 1)$ is the indicator function that indicates the value outside $[-1, 1]$ are zero.

As the "bad" news observed, the stock returns tend to be a large decrease. Meanwhile, the volume returns react to the highly increasing trend. This result may influence the choice of optimized returns. To overcome this drawback, we calculate the log return of stock price and the volume as $y_n = \ln\left(\frac{r_n}{r_{n-1}}\right)$ and $Vol_n = \ln\left(\frac{vol_n}{vol_{n-1}}\right)$, where r_n is the return and vol_n is the volume at time $n = 1, \ldots, N$. Then, we penalize the return of asset m as -1 when the return of the stock is decreasing while the return of the volume is increasing. Otherwise, we apply the stock return conditional the volume return as:

$$y_{m,(n)} = \begin{cases} -1 & \text{, for } y_{m,(n)} < 0 \quad \& \quad Vol_{m,n} > 0 \\ \mathbb{E}\left(y_{m,(n)} | Vol_{m,n-1}\right) & \text{otherwise} \end{cases}$$

The basic idea of this penalization is that we want to avoid speculation because we assume that no short sales are allowed. To compare the optimum performance of the portfolio, we use the different optimizations based on the Sharpe ratio, global minimum $\text{CVaR}_{5\%}$, and Rachev ratio. The risk-free rate of the Sharpe ratio defines as the 13-week of daily U.S. treasury bill.

Recall that the Sharpe ratio is given by:

$$\begin{aligned} \underset{\mathbf{w}}{\text{Max}} \quad & \frac{\mathbf{w}^\top \boldsymbol{\mu} - r_f}{\sqrt{\mathbf{w}^\top \sum \mathbf{w}}} \\ \text{s.t.} \quad & \mathbf{w}^\top \mathbf{1} = 1 \\ & \mathbf{w} \geq 0 \end{aligned}$$

the global minimum $\text{CVaR}_{5\%}$ is given by

$$\begin{aligned} \underset{(\mathbf{w}, \gamma, z_n)}{\text{Min}} \quad & \gamma + \frac{1}{(\alpha)N} \sum_{n=1}^{N} z_n \\ \text{s.t.} \quad & z_n \geq -w^\top y_{(n)} - \gamma \\ & \mathbf{w}^\top \mathbf{1} = 1 \\ & \mathbf{w} \geq 0 \\ & z_n \geq 0 \\ & n = 1, 2, 3, \ldots, N \end{aligned}$$

and the Rachev Ratio portfolio optimization problem is given by:

$$
\begin{aligned}
\underset{(\mathbf{w},\alpha,\lambda,z_n,\gamma,t)}{\text{Max}} \quad & \frac{1}{(\alpha)N}\sum_{n=1}^{N} z_n \\
\text{s.t.} \quad & z_n \leq B\lambda_n, \\
& z_n \geq \mathbf{w}^\top y_{(n)} - B\left(1-\lambda_n\right), \\
& z_n \leq \mathbf{w}^\top y_{(n)} + B\left(1-\lambda_n\right), \\
& \gamma + \frac{1}{(\alpha)N}\sum_{n=1}^{N} z_n \leq 1, \\
& z_n \geq -\mathbf{w}^\top y_{(n)} - \gamma, \\
& \mathbf{w}^\top \mathbf{1} = t, \\
& \mathbf{w} \geq 0,\ j = 1,2,\ldots,J \\
& z_n \geq 0 \\
& t \geq 0 \\
& \lambda^\top \mathbf{1} = [\alpha N] \\
& n = 1,2,3,\ldots,N \\
& \lambda_n \in \{0,1\}
\end{aligned}
$$

4.4. Empirical Analysis

In the analysis, we select 30 companies among the components of the S&P500. The companies are the same we select in Chapter 3. The adjusted closing price of daily data and volume retrieve from 01 January 2004 to 31 May 2020. We then convert the data into the log-returns form. To obtain the persistence length of observations, we use backtesting data preparation by setting the in-sample and out-of-sample as 1-year and 1-month. Thus, the dataset contains 250-day for each observation point and rebalances every 20-day before the next analysis.

In the following step, we estimate the data by using the conditional expectation. The stock returns are conditional to the volume returns with Gaussian and Epanechnikov kernel density functions. Furthermore, to compensate for the stock returns by the volume information, we apply the penalization method. We thus have five different returns to analyze, namely, historical return, Gaussian, penalized Gaussian, Epanechnikov, and penalized Epanechnikov. Finally, we optimize the portfolio by Sharpe ratio, global minimum $\text{CVaR}_{5\%}$, and Rachev Ratio methods.

Table 4.1 shows the ex-post annual returns of portfolio performance using different optimization models and approximated returns that:

- The penalized Gaussian shows the highest the ex-post annual returns with the Rachev Ratio optimization model.

- Compared with the historical return, the penalized Gaussian is higher for all optimization models. In particular, by considering the figures 4.1, 4.2, and 4.3, we find that the Sharpe Ratio optimization shows a steady increase in the ex-post annual returns than the other models.

Optimization/Approximation	Historical Return	Gaussian	Penalized Gaussian	Epanechnikov	Penalized Epanechnikov
Sharpe Ratio	18.97 %	11.37 %	**20.00 %**	10.98 %	16.17 %
Global minimum $CVaR_{5\%}$	8.45 %	**15.53 %**	10.30 %	7.85 %	9.37 %
Rachev Ratio	16.53%	16.27 %	**21.66 %**	18.71 %	14.06 %

Table 4.1.: The ex-post annual returns of portfolio performance using different optimization models and approximated returns.

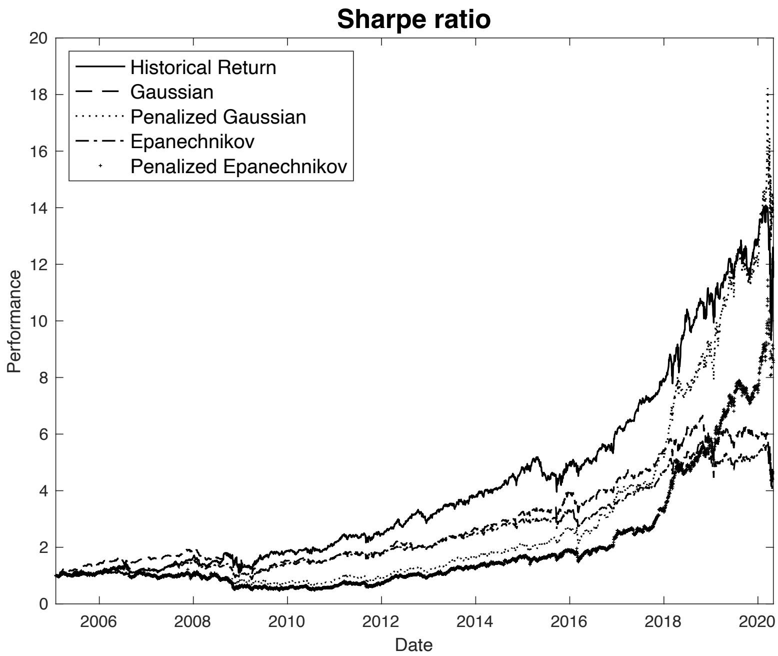

Figure 4.1.: The ex-ante performance of Sharpe ratio optimization among the different of return conditions.

4.5. Conclusion

In this research, we study the impact of volume information on stock returns. In particular, we apply the conditional expectation with Gaussian and Epanechnikov kernel density function and penalization of returns to investigate the impact on the performance of different portfolio selection strategies. We then compute the

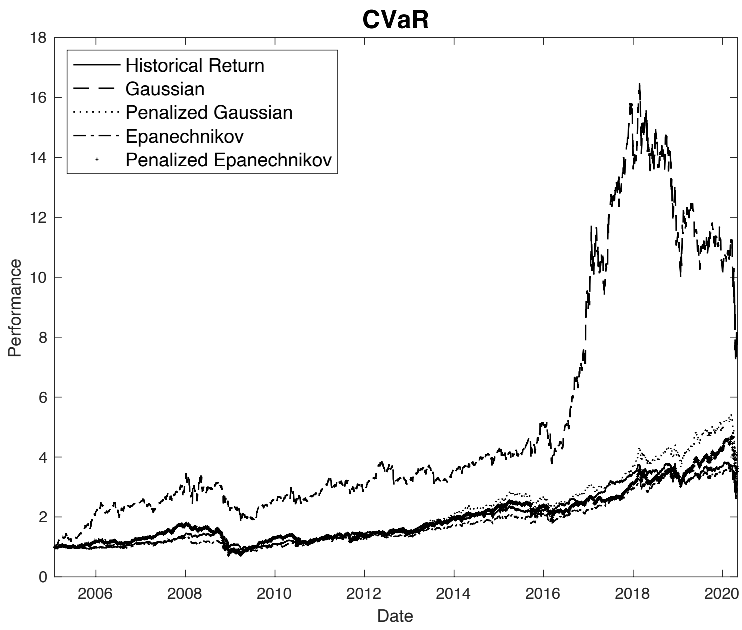

Figure 4.2.: The ex-ante performance of global minimum $\text{CVaR}_{5\%}$ optimization among the different of return conditions.

ex-ante performance by the Sharpe ratio, global minimum $\text{CVaR}_{5\%}$, and Rachev ratio optimizations. From the results, there is evidence that the volume provides some information to the stock return, in particular when we penalize the returns to avoid the speculative strategies. To sum up, the stock return conditional volume information with penalized return can be able to use as a profitable model.

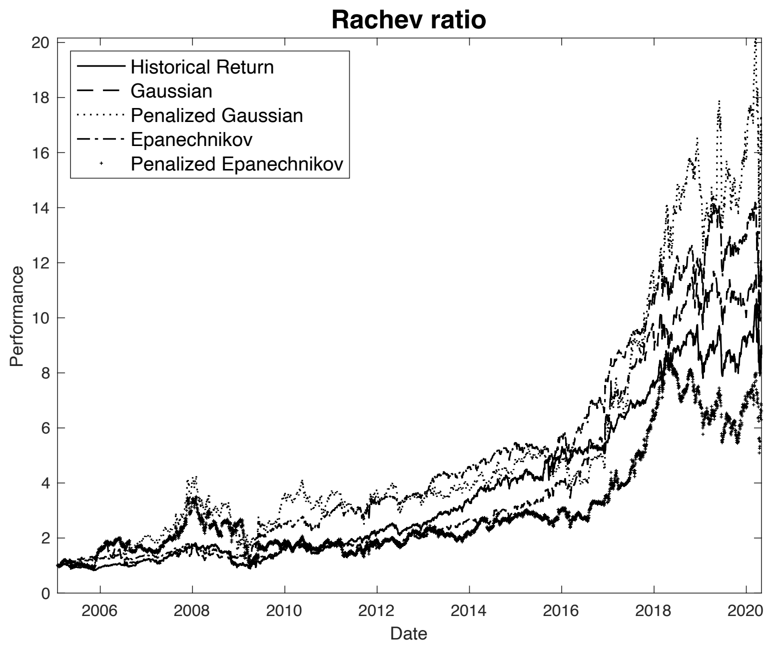

Figure 4.3.: The ex-ante performance of Rachev ratio optimization among the different of return conditions.

5. Summary

In this chapter, we sum up the contributions of this thesis and we discuss the important directions of future work. The principal contribution of this thesis is to address the financial modeling problems, which are the assessment application of credit risk profiles and the enhancement of portfolio performance.

5.1. Conclusion

In Chapter 2, we propose the cosine similarity as a spatial component in the multivariate DCC GARCH(1,1) model. The results can provide more accurate results on the diversification scheme for credit risk application. Moreover, the CoVaR with bivariate Gaussian distribution model can be better in capturing the spillover effects of risk than the ordinary CoVaR model. So, the cosine similarity of banks' structure inferred can be used to explain the spillover effects of credit risk.

Next, in Chapter 3, we construct the conditional expectation with kernel definition and penalized model from the return conditional Google Trends information before applied the portfolio optimization. Furthermore, the mean-variance and mean-$\text{CVaR}_{5\%}$ can be used to enhance the optimum portfolio performance. For the dominance comparison, the best results of each optimization model seem to have characteristics for coving investors' preferences of non-satiable risk-seeking (ICX). Thus, the GT useful information can provide a profitable return on investment.

Following the main idea from the previous chapter, Chapter 4 uses the return that conditional volume instead of Google Trends information. This chapter shows the evidence that the volume provides some information to the stock return, in particular, when we penalize the returns to avoid the speculative strategies.

5.2. Further Extension

Further possible development consists of the financial of credit risk and portfolio optimization. A specific point of each model will describe the suggestion.

5.2.1. Credit Risk

In the credit risk, we focus on the essential features of this research paper. The first feature will be the spatial components on the DCC-GARCH(1,1) model, and the test of CoVaR will be the second feature.

Spatial components:

- Instead of using CDS data, we suggest using the equity data of banks. The advantage can be included the better volatile of data and provided more choice of banks.
- In case we use banks' equity data, we can generate the spatial component from the banks' financial statements.

Estimated parameters and CoVaR:

- To estimate the parameters, we can apply the different multivariate GARCH models, for instance, EGARCH, GJR-GARCH, and TGARCH, to the DCC model to improve the assessment of credit risk profiles.
- In the CoVaR calculation, the bivariate Gaussian density can be used the copulas model to improve the interaction between the correlated VaR.

5.2.2. Portfolio Optimization

For portfolio optimization, the improvement can be extended into the expected returns and the optimization model. The suggestion is as follows:

Expected returns:

- The other useful information that shows a strong relationship with the expected returns can be investigated.
- The multivariate estimation is suggested for the estimation of the conditional expectation model. For example, the multivariate locally weighted least squared regression.

Optimization model:

- The different perspectives of portfolio optimization models may be used—for instance, Treynor Index, Sterling Ratio, and Calmar Ratio.
- The broad spectrum of backtesting samples can be investigated to perceive the best of portfolio performance.

The above suggestion points that have been undertaken for this thesis have highlighted many topics on which further research can be developed.

There is a gap of studies of the CDS data for the credit risk that might have occurred in the spatial components' relationship. Future studies might, for example, look for the equity data and check for similar spatial components. In another case, the cosine similarity might be used the financial statement of each bank to calculate. These include further investigation of the credit risk model of spatial components on how banks' performance influences the contagion of risk interconnectedness. Do the equity returns and the banks' structure inferred under the stress test exercise affect risk mechanisms? The multivariate GARCH(1,1) model might be further investigated as the EGARCH, GJR-GARCH, and TGARCH. These might give better estimation results. Moreover, the bivariate density implementation using the copulas would help provide more accuracy of the CoVaR.

For portfolio optimization, first, the proposed developed techniques would be applied to different useful information. It might give a better performance and also allow a comparison between the different model's performance. Moreover, this thesis has been studied the univariate estimation of conditional expectation. Thus, further investigation of the portfolio optimization might be how the multivariate estimation impacts portfolio performance? Second, there are also several optimization models for further development of the research undertaken in this thesis. The Sharpe ratio, CVaR, Sortino ratio, and Rachev ratio have been used in this thesis; however, the other optimization models might be the different choices such as Treynor Index, Sterling Ratio, and Calmar Ratio. These would provide the inter-model comparison for the portfolio performance. Finally, the backtesting of in-sample/out-of-sample period lengths might be adjusted to obtain the best portfolio performance.

A. Extension Results

In this section, we show the extension of the results of daily equity data. Since weekly CDS data is not enough for the $\text{CoVaR}_{5\%}$ violation test, we consider the daily equity data with the same data period as the weekly CDS data. Then, the descriptive statistics for each bank are presented in Table A.1. Also, in Table A.2, we show the estimation of the Student-t spatial DCC GARCH(1,1) with its confidence intervals from the block bootstrap. Then, the result of the backtesting based on $\text{VaR}_{5\%}$ and the backtesting based on loss functions are shown in Tables A.3, A.4 and A.5.

The descriptive statistics for the white noise test show a lower than 1%. The Ljung-Box test on returns (L-B[r]) shows that the p-values of BCS, SAB, and KBC are less than the 5% significance, rejects the null hypothesis of white noise. The p-value of L-B[r^2] are all less than 1%. Thus, the daily equity returns are containing volatility clustering, as described in Table A.1. Next, the results from Table A.2 show that the estimated parameters are mostly specified within the 5% and 95% confidence ranges. In Table A.3, we found that the GaussDCC, GaussSpDCC, and tSpDCC models show all acceptance cases of the UC test while 6 out of 7 acceptance cases of the CC test at a 99% significant level. At a 95% significant level, we found 1 rejection case for the GaussDCC and GaussSpDCC models and 2 rejection cases for tSpDCC model of the UC test while 1 rejection case only for tSpDCC model of the CC test. The tDCC presents 4 acceptance cases of both UC and CC models at a 99% significant level while 2 acceptance cases of both UC and CC models at a 95% significant level. For the $\text{VaR}_{5\%}$ backtesting based on loss functions, the tDCC model performs the best result for regulator's loss functions and investors' loss functions, as shown in Tables A.4 and A.5.

Bank	Mean	Stdev	Skewness	Kurtosis	Normality	L-B [r]	L-B[r^2]
ISP	-0.0001	0.0254	-0.8170	11.8147	0.0000***	0.0967	0.0000***
ACA	-0.0001	0.0251	-0.3844	11.4020	0.0000***	0.9800	0.0000***
DB	-0.0006	0.0242	0.1201	8.3327	0.0000***	0.5928	0.0000***
BCS	-0.0004	0.0233	-0.6631	13.1494	0.0000***	0.0196**	0.0000***
RAB	-0.0001	0.0234	-0.4745	11.6734	0.0000***	0.0663*	0.0000***
SAB	-0.0008	0.0246	-0.3479	11.1351	0.0000***	0.0013***	0.0000***
KBC	0.0001	0.0252	-0.3025	10.1976	0.0000***	0.0004***	0.0000***

Table A.1.: Descriptive statistics of equity data.

Table A.2.: The Student-t spatial DCC GARCH(1,1) parameters and its confidence intervals from 500 samples of block bootstrap of equity data (2610 data points).

Parameter/Bank		ISP	ACA	DB	BCS	RAB	SAB	KBC
A_0	5% CI	1.89e-06	-1.93e-04	-2.39e-04	1.86e-05	7.72e-07	-3.54e-04	-1.73e-04
		2.81e-06	**9.57e-15**	**6.59e-06**	**1.06e-05**	**2.20e-06**	**4.48e-15**	**6.43e-16**
	95% CI	7.80e-06	1.39e-04	1.67e-04	2.25e-05	6.46e-06	2.51e-04	1.21e-04
A_1	5% CI	1.15e-01	-1.87e-01	-4.79e-02	1.26e-01	8.23e-02	-1.62e-01	-3.54e-01
		8.52e-02	**2.04e-04**	**5.03e-02**	7.62e-02	6.81e-02	**6.74e-05**	**2.10e-02**
	95% CI	2.10e-01	1.24e-01	1.87e-01	1.70e-01	1.63e-01	1.15e-01	3.01e-01
B_1	5% CI	6.27e-01	-2.78e-01	5.68e-01	4.42e-01	5.85e-01	-4.28e-01	-2.94e-01
		9.17e-01	**4.66e-01**	**9.38e-01**	**9.04e-01**	**9.31e-01**	**4.74e-01**	**4.75e-01**
	95% CI	1.90	1.01	1.92	2.00	2.18	5.77e-01	1.18
A_2	5% CI	-5.83e-01	-5.72e-01	-5.21e-01	-3.66e-01	-1.12e-07	3.06e-08	-7.77e-08
		1.00e-08	**3.45e-08**	**1.06e-08**	**1.05e-08**	**1.04e-08**	**3.28e-08**	**2.67e-08**
	95% CI	3.56e-01	3.49e-01	3.18e-01	2.64e-01	6.42e-08	7.38e-08	1.08e-07
B_2	5% CI	-2.61e-02	2.23e-03	-1.98e-02	-2.60e-02	-3.05e-02	2.52e-02	2.79e-03
		1.01e-08	**1.52e-02**	**1.08e-08**	**1.07e-08**	**1.13e-08**	**1.81e-02**	**1.43e-02**
	95% CI	9.12e-03	4.27e-02	1.21e-02	9.39e-03	5.31e-03	4.41e-02	3.52e-02

γ	5% CI	-2.82e-03
		1.25e-02
	95% CI	2.88e-02
δ	5% CI	9.09e-01
		9.47e-01
	95% CI	1.78
ν	5% CI	5.81
		6.67
	95% CI	1.02e+01

Table A.3.: The p-value of backtesting based $VaR_{5\%}$ tests of equity data.

Bank	Gaussian DCC		Student-t DCC		Gaussian spatial DCC		Student-t spatial DCC	
	UC	CC	UC	CC	UC	CC	UC	CC
ISP	**0.1139**	**0.1389**	0.0015	0.0060	**0.1139**	**0.1989**	**0.1362**	**0.2241**
ACA	**0.5675**	**0.0985**	**0.0782**	0.0233	**0.8572**	**0.6090**	**0.3463**	**0.1095**
DB	**0.7000**	**0.5129**	**0.0947**	0.0466	**0.9173**	**0.2744**	**0.3463**	**0.1978**
BARC	**0.3463**	**0.6416**	0.0170	**0.0577**	**0.3012**	**0.5859**	**0.3012**	**0.5338**
ING	0.0423	0.0046	0.0271	0.0018	0.0423	0.0020	0.0133	0.0007
SAB	**0.2604**	**0.3019**	0.0271	**0.0604**	**0.2604**	**0.3019**	0.0271	0.0429
KBC	**0.0522**	**0.0905**	0.0002	0.0008	**0.9173**	**0.6981**	**0.3463**	**0.4613**

Note: The UC and CC stand for the unconditional coverage and conditional coverage tests. The bold defines as the acceptance at a 95% significance level and the highlighted light-gray defines as the acceptance at a 99% significance level.

Table A.4.: The backtesting based the loss functions of $VaR_{5\%}$ under the regulator's view of equity data.

Regulator's view				
Bank	Lopez			
	Gaussian DCC	Student-t DCC	Gaussian spatial DCC	Student-t spatial DCC
ISP	2228.78	2209.97	**2228.72**	2229.50
ACA	2239.42	**2225.95**	2247.69	2235.49
DB	2241.43	**2227.14**	2244.46	2235.19
BCS	2234.78	**2218.39**	2233.95	2233.98
RAB	2222.84	2220.66	2222.97	**2217.79**
SAB	2233.09	**2220.69**	2233.31	2220.95
KBC	2224.53	**2203.24**	2244.27	2235.79
Bank	Caporin1			
	Gaussian DCC	Student-t DCC	Gaussian spatial DCC	Student-t spatial DCC
ISP	1407.44	**1363.13**	1397.97	1392.85
ACA	1420.49	**1397.45**	1423.48	1421.54
DB	1418.55	**1395.25**	1423.84	1400.72
BCS	1426.45	1394.74	1435.65	**1426.39**
RAB	1398.24	1385.80	1399.20	**1385.11**
SAB	1479.86	1450.17	1479.22	**1452.14**
KBC	1369.20	**1339.61**	1377.38	1375.44
Bank	Caporin2			
	Gaussian DCC	Student-t DCC	Gaussian spatial DCC	Student-t spatial DCC
ISP	41.07	**35.56**	40.29	39.00
ACA	40.29	**37.41**	40.99	40.49
DB	42.13	**39.82**	42.57	40.35
BCS	37.85	**34.88**	39.07	38.54
RAB	35.45	**34.02**	35.80	34.47
SAB	44.62	**41.26**	45.16	42.14
KBC	35.68	**33.31**	34.16	36.86
Bank	Caporin3			
	Gaussian DCC	Student-t DCC	Gaussian spatial DCC	Student-t spatial DCC
ISP	90.94	**83.64**	90.06	88.47
ACA	88.54	**84.18**	89.71	88.58
DB	91.79	**88.65**	92.39	89.27
BCS	81.72	**77.80**	83.41	82.28
RAB	81.00	**79.36**	82.00	80.05
SAB	90.18	**85.97**	91.14	87.60
KBC	84.56	**80.99**	83.13	85.98

Note: The bold defines as the lowest total loss among different models.

Table A.5.: The backtesting based the loss functions of $VaR_{5\%}$ under the investors' view of equity data.

Investors' view				
Bank	Caporin1			
	Gaussian DCC	Student-t DCC	Gaussian spatial DCC	Student-t spatial DCC
ISP	**1469.29**	1441.15	1460.41	1458.24
ACA	1476.34	**1460.93**	1477.31	1482.50
DB	1473.51	**1456.54**	1479.46	1463.55
BCS	1485.04	**1464.91**	1488.98	1487.98
RAB	1457.54	1446.87	1452.62	**1445.22**
SAB	1551.56	1524.49	1548.19	**1524.16**
KBC	1425.78	**1412.82**	1427.64	1435.58
Bank	Caporin2			
	Gaussian DCC	Student-t DCC	Gaussian spatial DCC	Student-t spatial DCC
ISP	43.33	**38.33**	42.61	41.39
ACA	42.28	**39.66**	42.90	42.81
DB	43.93	**41.76**	44.40	42.41
BCS	40.07	**37.44**	41.19	40.66
RAB	37.19	**35.85**	37.39	36.25
SAB	48.24	**44.75**	48.62	46.72
KBC	37.40	**35.53**	35.77	38.70
Bank	Caporin3			
	Gaussian DCC	Student-t DCC	Gaussian spatial DCC	Student-t spatial DCC
ISP	92.99	**86.09**	92.08	90.58
ACA	90.33	**86.24**	91.45	90.46
DB	93.68	**90.71**	94.32	91.33
BCS	**83.50**	79.78	85.07	84.13
RAB	82.74	**81.13**	83.61	81.78
SAB	92.49	**88.42**	93.39	89.98
KBC	86.15	**82.84**	84.64	87.66

Note: The bold defines as the lowest total loss among different models.

B. Conditional Expectation

B.1. Conditional Expectation

In the financial context, the conditional expectation $\mathbb{E}[Y|X]$ is a random variable related to the expectation of Y given the available information of the random variable X. Let us consider two random variable $X : \Omega \to \mathbb{R}$ and $Y : \Omega \to \mathbb{R}$ are integrable in a probability space $(\Omega, \mathcal{F}, \mathbb{P})$ and $\mathcal{F}_X$ is σ-algebra generated by X

$$\mathcal{F}_X = \sigma(X) = X^{-1}(\mathcal{B}) = \left\{X^{-1}(B) : B \in \mathcal{B}\right\}$$

where $\mathcal{B}$ is Borel set of σ-algebra on $\mathbb{R}$. As the approximation of $\mathcal{F}_X$ by σ-algebra generated by the partition of Ω, the $\mathbb{E}[Y|X]$ is equivalent to $\mathbb{E}[Y|\mathcal{F}_X]$. Next, we consider the two-dimensional mapping of (X, Y) from Ω to $\mathbb{R}^2$ that is an independent random observation, $(x_1, y_1), (x_2, y_2), \ldots, (x_n, y_n)$. Then the conditional expectation of Y under $\{X = x\}$ can be defined in the discrete case as

$$\mathbb{E}[Y|X = x] = \sum_{i=1}^{\infty} y_i \mathbb{P}\{Y = y_i | X = x\}, \tag{B.1}$$

where $i = 1, 2, \ldots, n$ and $x \in \mathbb{R}$. To estimate the unknown parameter of the conditional expectation, the Nadaraya–Watson kernel density estimator is used as follows

$$\mathbb{E}(r|GT = x) = \frac{\sum_{j=1}^{J} r_j K\left(\frac{x - x_j}{h(J)}\right)}{\sum_{j=1}^{J} K\left(\frac{x - x_j}{h(J)}\right)}, \tag{B.2}$$

where r is asset return, GT is Google Trends rate, $K(\cdot)$ is the kernel density function, and $h(J)$ is the bandwidth. To avoid the bandwidth choice problem as suggested by Scott (2015), we select the univariate kernel estimator with the bandwidth of $h(J) = 3.5 J^{-1/3} std_{max}$.

B.2. Kernel Density Estimator

According to the asymptotic behavior of kernel density estimator B.2, the sequence of bandwidth h tends to zero as J approaches infinity. Thus, the univariate kernel function can be considered. In this study, we use the Gaussian, Student-t and Epanechnikov as defined follows:

$$K_{Gaussian}(u) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}u^2}, \tag{B.3}$$

$$K_{Epanechnikov}(u) = \frac{3}{4}\left(1 - u^2\right) I\left(|u| \leq 1\right), \tag{B.4}$$

$$K_{Student-t}(u) = \frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\sqrt{\nu\pi}\,\Gamma\left(\frac{\nu}{2}\right)} \left(1 + \frac{u^2}{\nu}\right)^{-\frac{\nu+1}{2}}, \tag{B.5}$$

where $I\left(|u| \leq 1\right)$ in Epanechnikov kernel function defines as an indicator function that any values outside the domain $[-1, 1]$ are zero. For the Student-t kernel function, Γ is the Gamma function and ν is the number of degrees of freedom.

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