Interest rate structured products: can they improve the risk-return profile?

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Abstract

In this paper we investigate the contribution of interest rate structured bonds to portfolios of risk-averse retail investors. We conduct our analysis by simulating the term structure according to a multifactor no-arbitrage interest rate model and comparing the performance of a portfolio consisting of basic products (zero-coupon bonds, coupon bonds and floating rate notes) with a portfolio containing more sophisticated exotic products (like constant maturity swaps, collars, spread and volatility notes). Our analysis, performed under different market environments, as well as volatility and correlation levels, takes into account the combined effects of risk-premiums required by investors and fees that they have to pay. Our results show that capital protected interest rate structured products allow investors to improve risk-return trade-off if no fees are considered. With fees, our simulations show that structured products add value to the basic portfolio in a very limited number of cases. We believe our paper contributes to understanding the role of structured products in investors portfolios also in light of the current regulatory debate on the use of complex financial products by retail investors.

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1 Introduction

Structured products are financial instruments with embedded derivatives designed to provide original payoffs and tailor risk/return profiles for investors. Their payoff is dependent on a formula based on some underlying securities (such as stocks indexes, basket of stocks, interest rates, commodities). They may go by various names (principal protected notes, accelerated return notes, range notes, barrier notes are just a few examples) and different wrappers (notes, certificates, for example). Structured products are built by manufacturers (investment banks) and offered by distributors (usually large commercial banks) as medium term notes with a term that can vary from a few months to several years.

According to ESMA annual report 2020 [21], the total outstanding stock of SRPs held by EU retail investors at the end of 2019 was around EUR 400bn. This is far less than holdings in UCITS (more than EUR 4.5tn) less than half of the holdings in AIFs sold to retail investors (EUR 1tn). Based on this data, the retail market for structured products made up around 2% of the financial net worth of EU households in 2019. According to EUSIPA (European Structured Products Association) the market volume of investment and leverage products issued as securities stood at EUR 281 billion at the end of 2020 for Austria, Belgium, Germany, and Switzerland, was equal to 281 bilion EUR. The total number of exchange traded products was equal to 448.035 for investment products and 1237343 for leverage products. In Italy the amount issued at the end of 2020 was equal to almost 13 bilion EURO down from the pick of almost 17 billion in 2019 but in line with the constant growth in the market since 2016. Structured products are owned by a wide spectrum of clients ranging from institutional to retail individuals. The aim of this paper is to investigate the contribution of structured products to portfolios of retail investors. There are several reasons that could explain why structured products could add values to portfolios. The structuring process generates payoffs able to match any desired wealth distribution. In this regard, structured products increase the investment opportunities available. The literature has proved that stocks and bonds alone cannot provide exposure to all risk factors (such as volatility and price jumps). Derivatives use makes it possible to diversify across risk factors and to receive the associated risk premia. In spite of this consideration, derivatives, alone or combined in a wrapper as structured products, are often not considered by traditional asset allocation models. From a theoretical point of view this could be justified by the consideration that in a complete market, derivatives are redundant since they could be replicated by a dynamic trading strategy in stocks and bonds. But if markets are not complete, then excluding derivatives from asset allocation could lead to suboptimal results On the other hand, structured products are perceived as costly, overly complex and lacking transparency. Investors may have difficulties in understanding all relevant characteristics of complex products. This should not be an issue for institutional users of structured products, but it is so for retail clients. Furthermore, financial institutions and retail clients have strong information asymmetries on pricing the products. This could allow financial institutions to charge higher fees, not fully displayed to investors, which significantly reduce the final performance of the instrument. The increased volumes of structured products issued in the market has drawn the attention of regulators. In July 2013, a report on "Retailisation in the EU" by ESMA highlighted that, "from a consumer protection perspective, retail investors may face difficulties in understanding the drivers of risks and returns of structured products". In the Opinion *Structured Retail Products - Good practices for product governance arrangements*, (March 2014) ESMA writes that *it is good practice for manufacturers to ensure financial products meet the financial needs, investment objectives, knowledge and experience of the target market identified by the manufacturer*. This same idea is behind MIFID2 product governance: financial products have to be designed by the manufacturer with reference to a potential target client.

In this paper we aim to contribute to the current debate on the use of structured products by trying to measure the value added to retail investors by the inclusion of structured products to the efficient frontier and to the risk - return profile. We focus on interest rate linked products. We consider a base portfolio consisting of traditional interest rate products (zero coupons, fixed and floating coupon bonds) and we add structured products (constant maturity swaps, collared floating rate notes, spread notes and volatility notes). In spite of its simplicity, our base portfolio is very significant for retail investors, who are traditionally bondholders and allocate to the equity component only a small portion of their wealth. Then we add to the basic portfolio the structured products mentioned above. They all have capital protection at maturity and they generate coupons throughout their lives. But, compared to traditional bonds, they allow to gain exposure to factors such as the change in the slope of the term structure and the volatility of interest rates. To the best of our knowledge, there are no previous studies on the contribution of interest rate-linked structured products to portfolios of risk averse investors. We investigate how this convenience is robust to different initial market environments like interest rate term structure shapes, as well as volatility and correlation in its changes. Finally we examine how the combined effect of the risk-premium required by investors and fees can change the portfolio allocation with respect to the one consisting only of basic securities. The analysis is conducted by simulating the evolution of the term structure of interest rates using the popular multifactor no-arbitrage Gaussian (G2++) model, see Brigo and Mercurio ([9]). Simulations are used to price the different bonds, using the risk-neutral probability measure, but also to generate real-world scenarios, using the physical or natural measure, and incorporating in the model investors' risk-aversion parameters. Simulations are performed under different scenarios concerning the shape of the term structure of interest rates and investor's risk appetite.

2 Literature review

Most academic papers studying structured products have focused on pricing related issues (Chen and Kesinger ([12]), Wasserfallen and Schenk ([45]), Burth et al. ([10]), Stoimenov and Wilkens ([40], [41]) and Baule et al. ([3]), Wellmeier et al. ([44]), Bernard et al. ([5]) between others). These studies examine the difference between the quoted price and the theoretical fair value of the SP, and they reach the conclusion that they are on average mispriced. Other academic studies have examined the factors that could explain the growth of SPs and what benefits they can offer. Fisher ([25]) finds that rational motives, such as diversification and cost management, as well as irrational motives, such as betting, induce retail investors to buy structured products. Branger and Bruer ([7]) analyze if retail investors with a buy and hold trading strategy can benefit from an investment in structured products. They show that the benefit of investing in typical retail products is equivalent to an annualized risk-free excess return of at most 35 basis points. Taking transaction costs into consideration, benefits are reduced to 14 basis points. Their analysis is however limited to SPs written on a single index. Hens and Rieger ([30]) analyze the benefits in term of utility gains that can be achieved using structured products to deviate from a linear exposure. They show that some of the most used structured products are not optimal for rational retail investors if the utility function is concave. Using a different, non-concave utility function, the gains become significant but still too small to compensate premium costs. They conclude that behavioral factors such as loss-aversion or probability mis-estimation more than utility gains explain the growing demand for structured products. The importance of behavioural factors in explaining structured products demand was also found by Breuer and Perst ([8]). Vanduffel ([43]) shows that structured products allow issuers to gain margins and investors to gain returns. However investors should be very careful to analyse structured products, since potential cash flows can turn into losses in the case of changes in market expectations. Henderson and Pearson ([29]) investigate the dark side of financial innovation, concluding that if investors misunderstand financial markets or suffer from cognitive biases and assign incorrect probability weights to events, then financial institutions can exploit these biases by creating products that pay off in the states that investors overweigh and do not pay off in the states that investors underweigh. In this context, investors mis-price instruments and assign a value that is greater than the fair value. Structured products allow financial institutions to gain from the willingness of investors to overpay. Rieger ([38]) analyses the properties that a product should have to maximize the utility function of an investor. Results show that optimal products should follow the market, that is, they must be co-monotone with the market portfolio (in the case of the CAPM) or with the inverted state price function (in the general case). Chen ([13]) examines the role of derivatives on hedge fund portfolios, showing that funds that use derivatives exhibits lower risk and are less likely to liquidate in poor market condition. Jessen and Jorgensen ([32]) develop an optimal portfolio choice model to describe the role of structured bonds in holdings of small retail investors. The set of investment opportunities available to investors are a risky index, a bank account and a structured product linked to another index. Investors are rational and maximize expected utility at maturity. The results, based on different utility functions, show that structured products have the highest relevance for investors with medium risk aversion. They also show that the portion of structured products is very sensitive to change in the cost of construction. Jessen and Jorgensen conclude that retail investors should include structured products in their portfolios to reach higher diversification. The positive contribution of derivatives use to the performance of pension funds (Cui, Oldenkamp, Vellekoop [14]) and to hedge funds (Chen [13]) has been proved. Ofir and Wiener ([37]) show that behavioral biases explain the investment in structured products among professional investors and claims the importance of a specific regulation to increase investors' protection. Cui et al. ([14]) show that even relatively small investments in derivatives allow pension funds to improve certainty equivalent rates of return and other important performance measures. Derivatives enable pension funds to capture diffusion risk, jump risk and volatility risk and to earn the associated risk premia improving the risk return ratio. Deng et al. ([15]) show that ex-post returns of structured products issued by 13 US brokerage firms since 2007 are highly correlated with the returns of large capitalization equity markets in the aggregate, but individual structured products generally under-perform simple alternative allocations to stocks and bonds. Cèlèrier and Vallèe (2017) investigate why banks design and offer structured products based on two competing theories: risk sharing and catering investors. According to the first theory (Allen and Gale 1994; Duffie and Rahi 1995), innovation in the design of financial products improves risk sharing and contribuites to complete the market offering to investors products that better match their risk return profile. According to the catering theory banks introduce innovation to cater to risk seeking investors. The paper, based on a study of the hurdle rate, the complexity and the risk of structured products, conclude that product complexity has increased from 2002 to 2010, that higher headline rates are associated with higher complexity and higher exposure to the risk of complete losses, that the spread between headline rates and interest rates increase when interest rates are low and finally, that higher markup are associated with higher complexity. Based on these findings they conclude that innovation in product design is coherent only with the catering theory. Maringer, Pohl and Vanini ([33]) analyse structured products with a focus on the Swiss market. They address three main questions: how structured products performed in the period 2008-2014, what the costs for investors at issuance are, how structured products can be used. The study shows that 80% of all issued products generated positive performance. The total expense ratio is estimated in a range between 0.3% and 1.7%. Finally, investors use structured products mainly to take rapid advantage of market opportunities. Entrop et al. ([19]) measure the risk adjusted performance achieved by investors buying structured products. They find that alphas are typically negative, even when transaction costs are ignored. Furthermore the under-performance increases with product complexity, since higher implicit price premiums are charged.

3 The products

Dybvig ([18]) has shown that, in a complete market the most efficient way to achieve a wealth distribution is by purchasing 'simple' structured products, whose payoffs only depend on the value of the underlying asset at maturity and not at intermediate times. Similar results have been obtained also in incomplete markets: see for example Vanduffel et al. ([43]). For this reason, in our analysis we do not consider path-dependent products. However, the products analysed cover a wide spectrum of interest rate products that over the years have been very popular among investors. Interest rate structured products have been little studied in the literature, which is more focused on equity based SPs. Moreover, we assume that: all the SPs considered are default free and have a maturity of five years and annual cash flows. Those are typical expires and payment frequencies for bonds offered to retail clients.

In what follows, we denote with $P(t,T)$ the time t discount factor for maturity T. This quantity is bootstrapped from market quotations of LIBOR and swap rates. LIBOR and swap rates are often also used as reference rates in the determination of the coupon payment. We recall that the (annualized) LIBOR rate $L(t,T)$ quoted at time t for maturity T is related to $P(t,T)$ by the relationship

$$L(t,T) = \frac{1}{\alpha_{t,T}} \left(\frac{1}{P(t,T)} - 1 \right), \tag{1}$$

where $\alpha_{t,T}$ is expressed in years and measures the time fraction (computed according to a given day count convention) between the two dates t and T.

The swap rate $S(t;\tau_n)$ quoted at time t with tenor $\tau_n = t_n - t$ is related to the term structure of discount factors by the relationship

$$S\left(t;t_n\right)=\frac{1-P\left(t,t_n\right)}{\sum\limits_{i=1}^{n}\alpha_{i-1,i}P\left(t,t_i\right)}, \tag{2}$$

where $t_i, i=1,\cdots,n$ are the fixed leg swap payment dates.

In general, SPs cash flows consist in a periodic payment (fixed or variable) $C\left(t_i\right)$ at times t_i, $i=1,...n$, and the payment of the notional N at maturity t_n

$$C\left(t_i\right)=\begin{cases} N\times\alpha_{i-1,i}\times c_i, & i=1,...,n-1, \\ \\ N\times\left(1+\alpha_{n-1,n}\times c_i\right), & i=n, \end{cases}$$

where c_i is the annualized coupon rate, which can be fixed or be floating according to some formula as described below in Section 3.1. Here t_0 refers to the issue date of the bond and corresponds also to the start of the first coupon payment. In particular, if c_i is constant, we have a fixed rate bond. If it randomly changes according to some reference rate, we have a floating rate bond.

A detailed description of the various coupon formulas is provided in the next section.

3.1 Plain Vanilla Bonds

We first consider plain vanilla products, such as zero-coupon bonds, and fixed and floating rate coupon bonds. By construction, the issue price of the bonds considered is set equal to the par value N.

Zero-coupon bond (ZCB) In this case, there are no intermediate cash-flows but a single payment at maturity t_n given by N^{zcb} that is equal to

$$N^{zcb}=\frac{N}{P\left(t,t_n\right)}$$

so that the present value of the payoff is exactly N.

Coupon bond (CB) The fixed rate bond pays at times t_i a fixed coupon $c_i^{cb}=c, i=1,\cdots,n$, and the notional at expiry. The coupon c is chosen according to the formula

$$c^{cb}=\frac{1-P\left(t,t_n\right)}{\sum\limits_{i=1}^{n}\alpha_{i-1,i}P\left(t,t_i\right)},$$

so that the present value of all bond payments is equal to the face value N.

Floating rate note (FRN) This note has at the payment date t_i a variable coupon rate determined according to the level of the LIBOR rate

$$c_i^{frn} = L\left(t_{i-1,}t_i\right).$$

Notice in particular, that we adopt the standard convention of fixing the rate at the beginning of the coupon period, i.e. at time t_{i-1}, whilst the payment is due at t_i (this is the so-called reset in advance pay in arrears convention). Moreover, in this case the coupon tenor is the same as the reference rate tenor, and assuming no default risk, it is well known that the issue price of the bond is exactly equal to the par value. Therefore, in the above coupon formula we do not include a fixed spread component.

3.2 Structured products

The structured bonds considered here are floating rate notes in which the coupon is set according to: a) a swap rate (constant maturity swap), b) a LIBOR or a swap rate, with a collar structure, c) a difference of two swap rates (spread note), d) the absolute value of the difference between a swap rate and a fixed rate (volatility note). Detailed descriptions of these notes follow.

Constant maturity swap (CMS) This structure allows investors to take a position on the long term part of the term structure. The coupon rate is determined according to the formula

$$c_i^{cms} = m \times S\left(t_i; \tau\right),$$

where m is the participation factor. The main difference with respect to the floating rate note is that coupon (annual) and reference rate (a long maturity rate) have different tenors and the fact that the reset and pay in arrears convention applies, i.e. the reset and payment dates coincide. Therefore, this bond at issue is not quoted at par. For this reason, the participation factor m is introduced, so that the CMS fair price at inception is equal to the par value.

Floating rate note with a collar (FRNC) This structure protects the buyer and the seller against sudden up or down movements in short term rates. This is made possible by inserting a cap (maximum rate) c, a floor (minimum rate) f and a spread component δ in the coupon formula

$$c_i^{frnc} = \min\left(\max\left(L\left(t_{i-1,}t_i\right), f\right), c\right) + \delta, i = 1, ...n.$$

Here the three components are adjusted to ensure that the issue price is equal to the notional N. Clearly, this can be achieved using different strikes combinations. Therefore, we set the floor f and the cap c equal respectively to 90% and 110% of the average forward LIBOR rate[1]. Then we adjust the spread parameter δ to ensure that the bond is issued at par value.

[1]Given the term structure of discount factors, we compute the simple forward rates with starting dates being the coupon reset dates and as final date the coupon payment date. Given we have different coupon

Here, the standard reset in advance rule applies.

Constant maturity-swap with a collar (CMSC) Similarly to the previous structure, this one protects the seller and the buyer against large changes in the reference rate, here taken to be a long term rate, i.e. a swap rate. As in the previous case, the coupon includes a cap c, a floor f and a spread component

$$c_i^{cmsc}(t_i) = \min\left(\max\left(S\left(t_i;\tau\right), f\right), c\right) + \delta.$$

We set the floor (cap) equal to 90% (110%) of the forward swap rate and we adjust the spread component so that the bond is issued at par value. Here the coupon resets in arrears.

Spread note (SPREAD) This SP pays a coupon related to the difference between a long term swap rate and a short term one. Through this product, the investor bets on the steepening of the swap curve. In general, the rate with shorter tenor is subtracted from the rate with longer tenor. We consider as short term rate the 2-year swap rate and as long term rate the 10-year swap rate. The coupon formula is given by

$$c_i^{spread} = \min\left(\max\left(\left(S\left(t_i;\tau_1\right) - S\left(t_i;\tau_2\right)\right) \times m, f\right), c\right) + \delta,$$

Also in this case, the coupon benefits from a floor rate and a cap rate. In our simulations, the floor f is set at half of the average value of the simulated reference variabile, whilst the cap is set at twice such average value.
Volatility note (VOL). The payoff depends on the absolute value of the difference between a swap rate and a fixed amount c, so that large deviations of the swap rate with respect to a reference value c guarantee large coupons to the bond holder. Small deviations, will pay small amounts. The coupon rate formula is

$$c_i^{vol} = m \times \left|S\left(t_i;\tau\right) - c\right|$$

In our simulations the maturity of the reference swap rate is taken to be $\tau = 10yrs$. The reference value c is set equal to half the expected value of the reference rate in 5 years. The participation factor m is chosen to guarantee that the bond quotes at par at inception.

3.3 Pricing of structured products

According to the no-arbitrage principle, the bond fair value $\pi(t)$ is set equal to the following expected value under the risk-neutral measure

$$\pi(t) = \sum_{i=1}^{n} \alpha_{i-1,i} \times \widetilde{E}_t\left(\frac{c_i}{B(t,t_i)}\right) \times N + P(t,t_n) \times N,$$

dates, we consider the average forward rate.

where $B(t,T)$ is the so called money market account, i.e. the T value of a unit initial investment in a risk-free account at time t

$$B(t,T) = \exp\left(\int_t^T r(s)\,ds\right).$$

In this expression r is the stochastic instantaneous rate whose dynamics is made explicit in section 4.

Given the pricing formula, the various parameters δ and m, in the coupon formula are adjusted to ensure that the fair price at issue is equal to the notional N. For example, in the CMS case the multiplicative factor m is chosen so that

$$m \times \sum_{i=1}^{n} \alpha_{i-1,i} \times \widetilde{E}_t\left(\frac{S(t_i;\tau)}{B(t,t_i)}\right) + P(t,t_n) = 1.$$

The expectation in this expression is estimated via Monte Carlo simulation of the stochastic interest rate model presented in section 4. In some cases, we need to perform some numerical search routine. We proceed as follows. We simulate the state variables of the G2++ model and then we search for the contract parameters such that the contract price is equal to N.

4 The term structure model

For pricing and for conducting our simulations we have adopted the two-additive factors Gaussian G2++ Model, see Brigo and Mercurio [9]. The model is based on the general Heath, Jarrow and Morton [28] framework for the arbitrage-free modelling of the evolution of interest rate curves. In the G2++ specification, the short rate is assumed to be the sum of two mean-reverting correlated Gaussian factors plus a deterministic function, that allows the user to fit exactly the observed term structure of spot rates. The model provides closed form expression for discount bonds, European options on zero-coupon bond, caps and even swaptions via a simple univariate integration. This allows a fast parameter calibration to market quotations. Moreover, due to the factor structure, the model allows for a non perfect correlation between changes of rates of different tenors. This is a well known empirical feature that cannot be captured by one-factor term structure models. Given that different SPs should react differently to the movements of different parts of the interest rate curve, to have a model that captures a non-perfect correlation across different spot rates is of the foremost importance. In addition, the model turns out to be Markovian in the mean-reverting factors. Hence it allows for an efficient and fast Monte Carlo simulation with respect to other model specifications, like the LIBOR market model. Finally, the model allows for negative interest rates, a phenomenon registered in the Eurozone since August 2014.

In the G2++ short rate model, the instantaneous short rate $r(t)$ is given as the sum of two stochastic components, x and y and a deterministic function ϕ

$$r(t) = x(t) + y(t) + \phi(t), \tag{3}$$

with $x(0) = y(0) = 0$, and $r_0 = \phi(0)$. The deterministic function $\phi(t)$ is linked to the observed market forward curve. Indeed, its role is to guarantee that the model zero-coupon bond prices perfectly fit the market ones at the initial time. We discuss in section 6 how this function relates to the observed market discount curve.

The risk-neutral processes $\{x(t), t \geq 0\}$ and $\{y(t), t \geq 0\}$ follow Ornstein-Uhlembeck dynamics

$$\begin{aligned} dx(t) &= -ax(t)\,dt + \sigma d\widetilde{W}_1(t), x(0) = 0, \\ dy(t) &= -by(t)\,dt + \eta d\widetilde{W}_2(t), y(0) = 0. \end{aligned}$$

Here $d\widetilde{W}_1$ and $d\widetilde{W}_2$ are the (risk-neutral) increments of two correlated Brownian motions

$$\widetilde{E}_t\left(d\widetilde{W}_1 d\widetilde{W}_2\right) = \rho dt,$$

where ρ is the correlation coefficient. The parameter restrictions are

$$a, b, \sigma, \eta > 0, \rho \in [-1, 1].$$

Parameters a and b are interpreted as mean-reversion coefficients of the two stochastic factors $x(t)$ and $y(t)$. In order to avoid model identification issues, we must require that the mean reversion coefficients a and b are different. Moreover, if the mean-reversion coefficients a and b are positive, the latent factors, and therefore the short rate as well, have a long-run stationary distribution that is Gaussian. In particular the two stochastic factors revert to 0 under the risk-neutral measure and the short rate reverts to the deterministic function $\phi(t)$. Moreover, the short rate has a stationary distribution given by

$$\lim_{t\to\infty} r(t) \sim \mathcal{N}(M, S_r^2),$$

where

$$M = \lim_{t\leftarrow\infty} \phi(t), \tag{4}$$

and

$$S_r = \sqrt{\frac{\sigma^2}{2a} + \frac{\eta^2}{2b} + 2\rho\frac{\sigma\eta}{a+b}}. \tag{5}$$

Stochastic discount bond prices $P(t,T)$ at a future date t are obtained as product of two quantities

$$P(t,T) = P^{mkt}(0,t,T)\, H(x,y,t,T)$$

where $P^{mkt}(0,t,T)$ is the forward zero-coupon price

$$P^{mkt}(0,t,T) = \frac{P^{mkt}(0,T)}{P^{mkt}(0,t)}$$

and $H(x, y, t, T)$ is an exponential affine function of the two stochastic factors

$$H(x, y, t, T) = exp\left(-\tfrac{1-e^{-a(T-t)}}{a} x\left(t\right) - \tfrac{1-e^{-b(T-t)}}{b} y\left(t\right) + \tfrac{1}{2}\left(V\left(t, T\right) - V\left(0, T\right) + V\left(0, t\right)\right)\right). \tag{6}$$

In the above formulas, $P^{mkt}\left(0, t\right)$ refers to the initial exogeneously specified market term structure of discount factors whose construction is discussed in more detail in Section 6. The expression for the function $V\left(t, T\right)$ is given in formula (4.10) in Brigo and Mercurio [9] and represents the variance of $\int_0^T \left(x(u) + y(u)\right) du$.

Given the jointly Gaussian assumption on x and y, zero-coupon bond prices at any future date have a lognormal distribution. In addition, the risk-neutral dynamics of the zero-coupon bond price is

$$\frac{dP\left(t, T\right)}{P\left(t, T\right)} = r\left(t\right) dt - \sigma D\left(T - t; a\right) d\widetilde{W}_1 - \eta D\left(T - t; b\right) d\widetilde{W}_2,$$

where the function $D\left(\tau; \theta\right)$ is related to the (stochastic) duration of the zero-coupon bond price

$$D\left(\tau; \theta\right) = \frac{1 - e^{-\theta\tau}}{\theta}.$$

Therefore, $\sigma D\left(T - t; a\right)$ and $\eta D\left(T - t; b\right)$ represent the contribution to the bond price volatility of the volatility in the two factors, x and y.

Given the risk-neutral model specification that allows us to price SPs in a way that precludes arbitrage opportunities across all maturities, we now introduce the so called risk-adjusted dynamics that reflect market participants' risk preferences. Indeed, the above dynamics are relevant for pricing the structured products at the initial time. However, to compare the performance of the different products we also need the dynamics under the so called physical (or risk natural or real world) measure. This requires a specification of the risk premia required by the market for taking the risk given by the two Brownian motions. The literature on the specification of this risk-premium is very extensive. A discussion can be found in Singleton [39]. However, for the sake of simplicity and also for better understanding of our results we assume that the risk premia are constant, but then we perform the analysis under different risk aversion scenarios. In practice the specification of the risk-premium consists in introducing two parameters λ_1 and λ_2 and in replacing the risk neutral Brownian motions $\widetilde{W}_i\left(t\right)$ by a new Brownian motions $W_i\left(t\right)$ via the change of drift

$$\widetilde{W}_i\left(t\right) = -\lambda_i t + W_i\left(t\right), i = 1, 2.$$

The dynamics of zero-coupon prices under the true measure becomes[2]

$$\begin{aligned}\frac{dP(t,T)}{P(t,T)} = &\left(r\left(t\right) + \lambda_1 \sigma D\left(T-t;a\right) + \lambda_2 \eta D\left(T-t;b\right)\right) dt \\ &+\sigma D\left(T-t;a\right) dW_1 + \eta D\left(T-t;b\right) dW_2.\end{aligned}$$

where the quantity

$$\lambda_1 \sigma D\left(T-t;a\right) + \lambda_2 \eta D\left(T-t;b\right)$$

represents the the expected return, over $r(t)$, of holding a zcb bond with time to maturity $T-t$ for an instant. By exploiting the risk-neutral and risk-adjusted dynamics, it is now possible to decompose yields into expectations of future interest rates and term premia. In particular, over a period of length dt, the excess return from holding a zero-coupon bond expiring in T with respect to the instantaneous risk-less investment is given by

$$E_t\left(d\ln(P\left(t,T\right))\right) - r\left(t\right) dt = TP\left(t,t+dt,T\right) dt. \tag{7}$$

Here $TP\left(t,t+dt,T\right)$ is the so called (instantaneous) bond term premium and it represents the instantaneous excess return of a zero-coupon bond with time to maturity $T-t$ with respect to the riskless bank account return, see Duffee [17]. Indeed, in general, longer term notes are perceived as riskier and therefore require a premium to compensate for this extra risk. The term premium is then found aggregating temporally and averaging

$$\begin{aligned} TP\left(t,T\right) &= \frac{1}{T-t}\int_t^T TP\left(t,s,T\right) ds \\ &= \frac{1}{T-t}\int_t^T \left(\lambda_1 \sigma D\left(T-s;a\right) + \lambda_2 \eta D\left(T-s;b\right)\right) ds \\ &= \lambda_1 \frac{\sigma}{a}\left(1 + \frac{1}{T-t}\frac{e^{-a(T-t)}-1}{a}\right) + \lambda_2 \frac{\eta}{b}\left(1 + \frac{1}{T-t}\frac{e^{-b(T-t)}-1}{b}\right). \end{aligned}$$

The long-run term premium is then

$$\bar{TP} = \lim_{T-t\to\infty} TP\left(t,T\right) = \lambda_1 \frac{\sigma}{a} + \lambda_2 \frac{\eta}{b}.$$

In the specification (8), the term premium for a given time to maturity $T-t$ is assumed to be time homogeneous, i.e. it depends only on the bond time to maturity and its sign cannot

[2]For the sake of completeness, the dynamics of the two factors, under the new measure are

$$\begin{aligned} dx\left(t\right) &= a\left(\tfrac{-\lambda_1 \sigma}{a} - x\left(t\right)\right) dt + \sigma dW_1\left(t\right), x\left(0\right) = 0, \\ dy\left(t\right) &= b\left(\tfrac{-\lambda_2 \eta}{b} - y\left(t\right)\right) dt + \eta dW_2\left(t\right), y\left(0\right) = 0. \end{aligned}$$

Under the true measure, the two factors will now revert to $-\lambda_1\sigma/a$ and $-\lambda_2\eta/b$. Depending on the sign of λ_i, these long-run values can be negative, null or positive. In addition, the deterministic function $\phi\left(t\right)$ is no longer the unbiased forecast of the future instantaneous rate.

change over time, depending on the fluctuations of the interest rates investors' risk tolerance, see Fama and French [23]. However, we believe that this does not represent a problem for our setup. At first, we are considering bonds having the same maturity and therefore our results will not be affected by the values that the term premium takes at different maturities. The second reason is that our simulations are performed by assigning different values to the parameters λ_i, so that we can generate different shapes and changes of sign in the initial term premium structure.

We stress that portfolio allocation aims at modelling the probability distribution of the market prices at a given future investment horizon under the *true* probability distribution of the market prices, as opposed to the risk-neutral probability measure used for derivatives pricing, see Meucci [35] and Giordano and Siciliano [26]. Based on this distribution, the buy-side community takes decisions on which securities to purchase to improve the prospective payout profile of their position. In practice, the estimation of the true probability distribution, i.e. the estimation of the parameters λ_i (as opposed to the calibration procedure required to obtain the risk-neutral distribution), represents the main quantitative challenge in risk and portfolio management. This is discussed in section 6.

5 Optimal Investing in SPs and Performance Measures

The investor aims to build an optimal portfolio containing plain vanilla bonds (i.e. ZCB, CB and FRN) and structured products as well. The construction of the optimal portfolio is done as follows. Let us define $PV_j^{(k)}$ as the simulated present values of future cash flows of the products j, $j = 1, \cdots, P$ in simulation k, $k = 1, \cdots, K$

$$PV^{(k)} = \sum_{s=1}^{n} \frac{C^{(k)}(s\Delta)}{B^{(k)}(s\Delta)} + \frac{N}{B^{(k)}(n\Delta)}. \tag{8}$$

where $C^{(k)}(s\Delta)$ is the coupon cash flow paid at time $s\Delta$ in simulation k. The actual computation of the coupon is done via Monte Carlo simulation and is discussed in detail in appendix in section A (see in particular formula (16) therein). In the following it is convenient to work in return terms by defining the gross logarithmic return $R_j^{(k)}$

$$R_j^{(k)} = \ln\left(\frac{PV_j^{(k)}}{N}\right),$$

and the net log-return $R_{j,g_j}^{(k)}$

$$R_{j,g_j}^{(k)} = \ln\left(\frac{PV_j^{(k)}}{N(1+g_j)}\right) = R_j^{(k)} - \ln(1+g_j).$$

where the fee g_j is zero if $j = 1, 2, 3$ and non-negative if $j = 4, \cdots, \quad P$. We have decided to set the fee for the basic products to zero because in general they are very liquid instruments, being largely traded in many markets, typically issued by governments and largely available to the vast majority of retail investors. Instead, for the remaining products, the up-front payment required to the investor is $N \times (1 + g)$.

We assume the investor implements a buy and hold strategy and, among all admissible portfolios, choose the optimal portfolio in the subset of mean-variance efficient portfolios. Therefore, using the K simulated scenarios, the investor computes the expected return for each asset j

$$\mu_{j,g_j} = \frac{1}{K} \sum_{k=1}^{K} R_{j,g_j}^{(k)}, \tag{9}$$

and collects them in the mean vector μ_g

$$\mu_g' = \left[\begin{array}{ccc} \mu_{1,g_1} & \mu_{j,g_j} & \mu_{P,g_P} \end{array} \right]. \tag{10}$$

Similarly, we can estimate the covariances $V_{j,i}$ $j, i = 1, \cdots, P$ between R_{j,g_j} and R_{i,g_i}

$$V_{j,i} = \frac{1}{K} \sum_{k=1}^{K} \left(R_{j,g_j}^{(k)} - \mu_{j,g_j} \right) \left(R_{i,g_i}^{(k)} - \mu_{i,g_i} \right), \tag{11}$$

and collect them in the covariance matrix $\mathbf{V}$[3]. Henceforth, the investor solves with respect to the vector of holdings $\mathbf{w}$, $\mathbf{w} \in R_+^n$, the following mean-variance problem with no-short selling constraint

$$\begin{array}{l} \min \frac{1}{2}\mathbf{w}'\mathbf{V}\mathbf{w} \\ sub \\ \mu_g'\mathbf{w} = m \\ \mathbf{1}'\mathbf{w} = 1 \\ \mathbf{w} \geq \mathbf{0}, \end{array} \tag{12}$$

where m is the target expected return required by the investor. The target expected return m in (12) is taken considering twenty equally spaced points in the range $\left[m^{low}, m^{high}\right]$ where, in order to avoid portfolios concentrated in a single product, we set m^{high} to be the gross average of the positive expected returns and m^{low} is the expected return of the global minimum variance (GMV) portfolio obtained solving the problem (12) excluding the budget constraint[4].

Given that the fee amount does not affect the estimation of the covariance matrix, the

[3]Given that the number of simulations is very large, there is no significant difference in using the biased or the unbiased estimate of the covariance matrix. In addition, given the fact that we are using log-returns, notice that the fees do not affect the variances and the covariances. For this reason we omit the dependence of the covariance matrix on g.

[4]In practice, $m_{high} = \frac{\sum_{j=1}^{P} \mu_{j,g_j} 1_{\mu_{j,g_j}>0}}{\sum_{j=1}^{P} 1_{\mu_{j,g_j}>0}}$ and m^{low} is the expected return on the portfolio that solves $\min \frac{1}{2}\mathbf{w}'\mathbf{V}\mathbf{w}$, sub $\mathbf{1}'\mathbf{w} = 1$ and $\mathbf{w} \geq \mathbf{0}$ whilst $m^{high} = \max_{j=1,\cdots,P} \mu_j$, i.e. the largest element in μ. If all the products have a negative expected return, we set m_{high} equal to the average expected returns of the best three products.

GMV composition turns out to be independent of the fee structure. In general, this portfolio will attract an individual with infinite risk-adversion. On the other side, as an investor considers to move away from this portfolio to get some additional return, the charged fee will reduce the expected return of SPs and therefore their attractviness in the efficient portfolio. Therefore, as fees increase, a risk-neutral investor will tend to invest only in basic products, i.e. she will positionate herself on the other extreme of the efficient frontier.

The comparative analysis that we conduct is related to the composition of the optimal portfolios that are solutions of the problem (12) for different values of m as we vary the fee level and the risk-adversion of the investor.

To evaluate the investment strategy in SPs facing fees, we use the two-step mean variance approach in Meucci [34] and proceed as follows.

1. First, we compute the efficient frontier by solving the above optimization problem. We emphasize that to do so we do not need to assume normality of returns or mean-variance preferences. This step only reduces the dimension of the market to the family of efficient portfolios.

2. Second, for a given utility function u, we determine the portfolio belonging to the efficient frontier returning the maximum expected utility. For this portfolio, we determine the amount invested in basic and in structured products and we also compute the expected utility $ExpUt$ associated to it

$$ExpUt(s) = \frac{1}{K}\sum_{k=1}^{K} u\left(\sum_{j=1}^{P} w_j\left(s\right) R_{j,g_j}^{(k)}\right).$$

   where $w_j\left(s\right)$ is the optimal weight of bond j in the efficient portfolio s. We adopt an exponential utility function $u\left(x\right) = -e^{-\lambda x}$, where λ is the (constant) risk-aversion parameter[5].

6 Model calibration and implementation

The simulation of the G2++ model and the performance analysis of investing in SPs requires the following steps.

Term structure of market discount factors The exogeneously specified market discount curve $P^{mkt}\left(0,t\right)$, $t > 0$ is estimated, following standard industry practice, by a bootstrapping procedure of LIBOR money market deposits (with maturities of 1, 3, 6 and 9 months) and interest rate swaps (with maturities of 1, 2, 3, 4, 7, 10, 15 and 20 years) obtained from Bloomberg. Feldhutter and Lando [24] show that swap rates are indeed the most parsimonious proxy for riskless rates. Then, in order to have the spot rate at each simulation step, we have interpolated the derived term structure of continuously compounded

[5]Notice that the utility function is defined in terms of log-returns. This is equivalent to adopt a power utility $u(x) = \frac{x^{1-\gamma}-1}{1-\gamma}$ defined on the terminal wealth, i.e. $W_0 e_p^R$ and by setting $\lambda = -(1-\gamma)$.

spot rates using the parametric Nelson-Siegel functional form. Accordingly, the continuously compounded spot rate is given by

$$R^{NS}(0,\tau) = -\frac{\ln P^{NS}(0,\tau)}{\tau} = \beta_0 + \left(\beta_1 + \frac{\beta_2}{\kappa}\right)\frac{1-e^{-\kappa\tau}}{\kappa\tau} - \frac{\beta_2}{\kappa}e^{-\kappa\tau}.$$

In particular, the parameter β_0 captures the long run level of the spot curve, whilst parameters β_1 and β_2 enable to generate various shapes of the term structure. In detail, β_1 measures the slope of the term structure: a positive (negative) β_1 represents an upward (downward) sloping term structure. β_2 can be positive or negative, and allows to generate a term structure with a hump or a trough, respectively. Finally, the parameter κ determines both the steepness of the slope factor and the location of the maximum. The Nelson-Siegel model parameters have been fitted by minimizing the sum of squared errors between market and model rates

$$\min_{\beta_0,\beta_1,\beta_2,\kappa} \sum_{i=1} \left(R^{NS}(0,\tau_i) - R^{mkt}(0,\tau_i)\right)^2.$$

In particular, we have calibrated the model to five different term structure scenarios (labelled A, B, C, D, and E) representative of different shapes: negatively sloped, positively sloped, average level, near flat and negative rates. In particular, these curves were observed at the following dates: (A) June 6th, 2008; (B) September 28, 2007; (C) average level in the period 1/1/2005 to 30/09/2010; (D) May 20th, 2009; (E) May 2th, 2021. The five different curves are represented in Figure 1. The calibrated parameters are then given in Table 1. The deterministic function $\phi(t)$ appearing in (3) is then related to the calibrated parameters trough the following formula[6]

$$\phi(T) = \beta_0 + \beta_1 e^{-\kappa T} + \beta_2 \kappa T e^{-\kappa T} + \frac{1}{2}\frac{\partial V(0,T)}{\partial T}.$$

The sensitivity of our results to the choice of the deterministic function $\phi(t)$ is then captured by fitting the Nelson-Siegel model to the different term structure shapes given in Table 1 and considering the different model parametrizations given in Table 2.

Parameter calibration of the G2++ model. The analysis by De Jong et al. [16] suggests that the volatility implied by interest rate options, such as caps and swaptions, is a poor predictor of future volatility, because it consistently overestimates realised volatility. For this reason, we calibrate the model using historical volatilities and correlations, estimated using the sample covariance matrix of changes in spot rates with maturities from 1 to 5 years and with reference to the period January 1st 2005 to September 30th, 2010. However, to give

[6] The perfect fit at initial time between model and market zero-coupon bonds is possible if the following restriction is satisfied

$$\int_t^T \phi(s)\,ds = -\ln\frac{P^{mkt}(0,T)}{P^{mkt}(0,t)} + \frac{1}{2}\left(V(0,T) - V(0,t)\right), \tag{13}$$

By taking the partial derivative with respect to T and assuming that the initial discount curve is given by the Nelson-Siegel parametric function, we obtain the expression in the main text.

robustness to our analysis, we consider a market implied calibration as well, choosing the parameters that best fit the implied volatility swaption surface adopting the same procedure as in Brigo and Mercurio [9], page 166. Historically calibrated parameters are reported in scenarios I-III of Table 2. Market implied calibrated parameters are given in the scenarios IV and V in the same Table. We also add two additional parameter settings, i.e. VI and VII that are very similar to the settings V. In the parameter settings VI we set the parameters in order to better reflect the historical volatility of changes in the 5 year Euro spot rate in the period 2010-2020 and generate a lower term premium than in settings V. Indeed, in setting V the long-run term premium is around 15.25% that appears to be too large. So in settings VI, we impose that the long-run term premium takes a more reasonable value of 7.45%. The parameter settings VII instead impose that market participants are risk-neutral because the risk-premium parameters λ_1 and λ_2 are zero.

Risk-Premium Parameters. As previously discussed the size and sign of the risk-premium parameters λ_1 and λ_2 determine the performance of a long term bond with respect to a rolling investment in a short-term zero-coupon bond. Given that all term premia estimates are model-dependent, and also subject to parameter uncertainty we have considered different parametrizations labelled from I to VII that are provided in Table 2. Figure 2 shows how the term premium given in (8) behaves for different time to maturities. In particular, the different parametrizations generate different shapes and signs of the term premium. Among these, we also consider the case where both parameters λ_1 and λ_2 are zero: that is, market participants are assumed to be risk-neutral (scenarios II and VII). The parameter values have been chosen consistently with the values reported in the literature. The evolution of term premia has been of particular interest since the Federal Reserve (FED) and the European Central Bank began large-scale asset purchases. Over this time, short-term interest rates have been close to zero, and the term premium has been compressed and has at times even been negative. The FED term premium estimates are obtained from a five-factor, no-arbitrage term structure model described in detail in Adrian, Crump and Moench [1]. They reports estimates[7] that over the last twenty years average at 1.79%, with a maximum value of 3.45% and a minimum of -0.87%. EUTERPE[8] proposes a model for the estimation of term premia in the Euro Area (EA) that relies on an affine term structure framework with interrelations between yields, volatility and macroeconomic factors. They report a 10 year term premium that varies from 0.9790% in January 2000 to -1.1320% in April 2021. In particular, the maximum reported value is 1.08%, the minimum -1.74% and the average 1.08%. The values of λ_1 and λ_2 considered here allow us to approximately reflect these estimates. In particular, given the calibrated parameters of the G2++ model, we choose the parameters λ_1 and λ_2 so that the 5-year and the long-run term premium, proxied by the 10-year one, are matched. We match the maximum and the average US term premium and the minimum and the average EUR term premium. This last case allows us to deal with a change in the sign of the term premium. In addition, we also consider the case of zero-term premium. The combinations of risk-neutral parameters and term premium parameters are given in Table 2.

Risk-neutral model simulation and product characteristics. Given the initial term structure and the model parametrization, we have simulated the model using the risk-neutral

[7] See `https://www.newyorkfed.org/research/data_indicators/term_premia.html`
[8] See `https://www.unive.it/pag/39846`

specification. This allows us to price the different SPs and to fix the payoff parameters such as the floor, cap and participation ratio so that the bond is priced at par. In order to find a unique solution, we have set the floor and the cap equal respectively to half and twice the expected value of the reference variable, and then solving for the remaining parameter (i.e. the participation factor or the spread) so that the issue price is equal to the face value.

Real world model simulation. Then, we have resimulated the stochastic factors under the true probability measure, i.e. replacing the risk neutral Brownian motions $\tilde{W}_i$ by the the new Brownian motions W_i, to generate the cash flows at each payment date. Then we have computed the returns gross returns $R_j^{(k)}$ and by translation the net returns $R_{j,g_j}^{(k)}$. In this way we have obtained an array of dimensions $K \times N$, where K is the number of Monte Carlo simulations and P is the number of SP'-s considered, $P = 8$. This array contains the simulated present values $PV_j^{(k)}$ of the random cash flows we can achieve in simulation k investing in the jth SP. By varying the fee amount, it is also immediate to generate the simulated present values net of fees.

Fees. From conversations with practitioners and according to the results of previous studies, it appears that subscription fees (implicit and explicit) can have large variations depending on the issuer, on the underlying (interest, index, equity, commodity, etc.), on the presence of exotic components and on the maturity of the contract. According to this discussion, and as said in section 5, we have set the fee for the basic products to zero, whilst, for SPs, we assume that the up-front payment required to the investor is $N \times (1 + g)$. By doing this, we are assuming that there are no further hidden costs due to product mispricing, creditworthiness of the issuer or liquidity costs related to the difficulty of liquidating the holdings prior to maturity(indeed, our investor is using a buy-and-hold strategy).

Instead of specifying *a priori* the fee amount, we have decided to proceed as follows. Notice that the inclusion of fees only reduces the expected return of SPs. Therefore, as we increase the fee level, efficient portfolios, with the exception of the global minimum variance portfolio, will tend to increase the amount allocated to basic products. We call g_{nosp} the fee amount such that the portfolios belonging to the efficient frontier[9] have invested a maximum amount of 3% in SPs, i.e. in practice the SP investment is very residual and the efficient frontier can be built using only basic products. We have considered a 3% threshold, because as illustrated in Table 8 in the next section, this is the minimum percentage invested in SPs included in GMV portfolios, across all possible scenarios: an infinitely risk-adverse individual will invest at least this amount in SPs.

As described in section 5, a risk-adverse individual, once has built the efficient frontier, picks the portfolio that maximes her expected utility. Cleary, her choice should be affected by her risk-adversion coefficient and by the fee level. Therefore, we have (numerically) computed the fee amount g_{basic}^{λ} such that an investor with an exponential utility, picks an efficient portfolio that includes only basic securities. Notice also that in general g_{nosp} is different from g_{basic}^{λ} because in the first case we require that *all* efficient portfolios invest a maximum of 3% in SPs, while in the latter case we require that only the efficient portfolio that is optimal for an individual with exponential utility has a zero investment in SPs.

Additional information can also be obtained by comparing portfolios fully invested only

[9] In those portfolios we do not consider the global minimum variance portfolio

in basic securities (BASIC) or in SPs. We can compute the maximum fee level $g_{sp \succ b}$ such that a portfolio invested only in SPs dominates, i.e. has lower risk for given expected return or larger expected return for given risk, a portfolio invested only in BASIC securities. Similarly, we can compute the minimum fee level $g_{b \succ sp}$ that makes the investment only in BASIC securities more convenient. In both cases, the critical level is found by imposing that the GMV invested only in SPs (BASIC) stays above the efficient frontier made only of BASIC (SP) securities. If there is no dominance the fee level is set 0.

These quantities will provide us with synthetic measures of the maximum cost that makes investment in SPs not convenient. We can then compare these maximum cost with the empirical evidence. In addition, given the above critical levels, we can then investigate what should be the optimal portfolio for an investor that combines BASIC and SP products.

An illustration of the role of the different critical fee levels is given in figure 3 with reference to an hypothetical scenario. In this figure, the red curve is the efficient frontier built considering ALL products assuming that there are no fees. The cyan frontier is the efficient frontier built using only BASIC securities. If the fee amount is set at g_{nosp}, the ALL efficient becomes the circled red curve: along this curve efficient portfolios will have a maximum weight of 2.92% allocated to SPs. The circled curves are the efficient frontiers built including only SPs varying the fee level. The yellow curve assumes no fees. The purple and green curves are the SP efficient frontiers with a fee level at g_{SPB} and g_{BSP}. g_{SPB} is the maximum fee level such that a portfolio containing only SPs dominates a portfolio made of basic securities. $g_{sp \succ b}$ is the minimum fee level such that a portfolio of basic securities dominates the portfolio of SPs. Figure 3 also suggests that, if no fees are paid, there is a sizeable improvement in the expected return-risk trade-off: the red efficient frontier, that includes SP, dominates the ones built investing in basic securities os in SPs only. If the fee increases up to 3%, a portfolio of basic and SPs still dominates a portfolio made only of basic or SP securities. Indeed, the circled red frontier stays above the cyan and green curves. Investing only in SPs, the circled yellow curve, dominates the investment in basic securities. However, if fees are set at 1% level, the circled yellow curve becomes the purple green curve and the two curves start to cross. If fees increase to 3%, the investment in SPs is now dominated by the investment in basic securities.

7 Numerical Results

We have performed a preliminary analysis examining the mean vector, the standard deviations, the average correlations and the composition of the global minimum variance portfolio across all 35 scenarios, i.e. five initial curves and seven different G2++ parametrization[10]. The above quantities have been estimated by running $K = 500,000$ Monte Carlo simulations with antithetic variates.

Table 3 illustrates the expected return, before fees are paid, for each product in each scenario. We observe that the ZCB and the FRN have a very strong relationship between expected return and term premium but of opposite sign. Indeed, as shown in Appendix B and confirmed in Table 4, where we regress the expected return of the different products

[10] An additional parameters setting, assuming that the term premium is zero has been considered, but not reported here.

on the term premium (TP), the long-run volatility (S_r), the long run (β_2) and the short term interest rates (β_1), the relationship, is perfect and positive for the ZCB. The regression coefficient is approximately equal to the length of investment period (5 years)[11] and the R^2 coefficient is 100%. The FRN contract has also a very strong relationship (R^2 of 96%) with the term premium, but of opposite sign. This makes sense in our setup. The FRN is paying a coupon that is related to the realized LIBOR rate that, on its turn, is determined by the two G2++ factors, whose mean is inversely related to the risk-premium parameters λ_1 and λ_2 and therefore to the term premium. The calculation of the present value of this cash flow depends on the money market account, whose dynamics is also inversely affected by a larger value of the risk premium parameters. In conclusion, a larger term premium reduces both the numerator (the future LIBOR rate) and the denominator (the money market account used as deflator). The net effect turns out to be a lower expected return of the FRN. The remaining interest rate products have still a significant relationship with the term premium, but it is less strong respect to the two previous cases, due to the presence of optionalities in the coupon calculation. In particular, we notice that the expected return of the VOL has a weak dependency on the term premium and a much stronger relationship with the interest rate volatility. Indeed, by construction, the expected cash flow of this product is greater, greater the interest rate volatility. The regression results also shows that there is no significant relationship between expected return and level and slope of the term structure, where these quantities are proxied by the Nelson-Siegel parameters β_0 and β_1.

So the main insight of this Table is that the main driver of the sign of the expected return for the different bonds is the term premium. The ZCB, followed by the FRN, has also a large variability in the expected return across scenarios, given that its sign depends on the sign of the term premium. The performance of the FRNC tracks the performance of the FRN. An exception is represented by the VOL, whose expected return is mainly driven by the interest rate volatility, and by the SPREAD note. We notice that the expected return on this bond has the same sign as the term premium, but it also differs significantly across term structure scenarios. Indeed, in scenario B, i.e. a very steep term structure, the bond has an expected return, in absolute value, larger respect to the other term structure shapes. In particular, this bond, in average, has a large expected return if the term premium is positive and the term structure is very steep. However, a steep curve associated with a negative term premium, e.g. scenario II-B, is very penalising.

This suggests that a combination of plain vanilla coupon bond and floating rate note can be of some appeal to risk-adverse investors whenever there is uncertainty on the sign of the term premium. Products like the VOL note can attract investors with high risk appetite that have strong views on a possible increase of the interest rate volatility. The SPREAD note can appear attractive if there is a view towards a positive term premium and a very steep term structure.

Table 5 reports the standard deviation of each bond in the different scenarios over the five year investment period. In this table, we also report the term premium and the asymptotic volatility S_r of the short rate. In Table 6 we have the results of regressing the standard deviation of the different products on the 5-year Term Premium, the asymptotic volatility S_r, and the short and long term rate implicit in the initial spot curve (i.e. the parameters β_0 and β_1). We observe that S_r explains most of the volatility of the different products

[11] Indeed, the term premium is a time average, whilst the expected return is computed over the full five years period.

(R^2 greater than 90%) and it fully explains the volatility of the ZCB[12] and of the FRN. An exception is the SPREAD note whose volatility is mainly determined by the short and long term rates.

The results of the two regressions therefore suggest that products like CMS, FRNC, CMS, and CMSC have exposures to market and model variables similar to plain vanilla bonds. On the other side, the SPREAD and the VOL could give some benefits to an investor because they are paying off, i.e. they have a higher expected return or a lower volatility, in specific market environments.

Scenarios V and VII differ for the value of the correlation coefficient ρ among the two interest rate factors. In scenario V, we assume a negative correlation, as it is typically the case in the calibration of the G2++ model. In scenario VII, we set it at 0. We see that the most affected is the ZCB both in terms of expected return and variance, as it should be. A negative correlation among the factors lowers the value of the variance term $V(t,T)$ and this affects, as shown in Appendix B, both the expected return and variance of the ZCB return. A change in the correlation of the factors also affects the remaining products, mainly the expected return of the FRN and FRNC and the volatility of the VOL note.

It is also interesting to assess the securities in terms of their contribution to the portfolio diversification, and this is mainly captured by the cross-correlation. In Table 7 we produce, for each scenario, the average correlation of each product with the remainings. We observe that the FRN, followed by the CMS, has in general a significant negative correlation with all the other products. This suggests that these products can have an important role in diversifying the interest rate risk of a portfolio.

Given these preliminary remarks, Table 8 reports the composition of the global minimum variance (GMV henceforth) portfolio[13], across the different scenarios. The last two columns of this Table reports the amount that the GMV portfolio allocates respectively to BASIC and SPs. As the preliminary analysis has just suggested, given the properties of the CB and of the FRN in terms of volatility and correlation, it is not a surprise to see that large proportion of the portfolio is invested in these products, that are also preferred to a ZCB investment. Indeed, across all scenarios, the weight assigned to BASIC products is always greater than 90%. Large part of this investment is allocated to the CB (in the range 70%-85%) and then to the FRN (range 8%-16%). The maximum weight assigned to SPs does not exceed 18% (see last column of Table 8) and in general is below 8%. When this threshold is exceeded, a significant weight is allocated to the SPREAD note (scenarios III-B/C and VI-D).

We have analyzed the determinants of the composition of this portfolio, by regressing the amount invested in basic products on the same independent variables as before. The regression results are illustrated in Table 9 at asset and at aggregate level. In particular, at asset level, the main determinants of the weights of the different products in the GMV are the long-run volatility, and the level of the term structure. At aggregate level, we observe that higher the long-term rate β_0 and lower the short term rate, larger the amount invested in BASIC products.

Then we have analyzed how the above results are affected by the introduction of fees, by computing the critical fee levels g_{nosp}, g^{λ}_{basic}, $g_{sp\succ b}$ and $g_{b\succ sp}$. An illustration of their role is given in Table 10, where we can read: the first two columns refer to the parameter and

[12]This is also proved in Appendix B.

[13]In this Table we do not report the scenario related to negative rates, because we will deal with this case separately later on, due to the particular care we have to use in designing the different products.

curve settings; then we have the fee level g_{nosp}, such that all efficient portfolios will have a maximum amount of 3% invested in SPs (third column), and the actual amount invested in SPs (fourth column); the maximum fee g^{λ}_{basic} such that a risk-adverse investor will have no investment in SPs (fifth column) and then the effective amount that this investor will invest in SPs given the fee g^{λ}_{basic} (sixth column); the maximum fee level $g_{sp\succ b}$ such that there exists a portfolio of SPs that dominates the BASIC portfolio and the weight allocated to SPs in a combined SPs-BASIC portfolio for the given fee level; the minimum fee level $g_{B\succ SP}$ such that there exists a portfolio of BASIC securities that dominates the SP portfolio and the corresponding amount allocated to SPs in a combined portfolio; the last columns give the optimal percentage invested in SPs in a portfolio combining basic and SPs given a fee level of level $g_{B\succ SP}$ (column eight) and a fee level of 1%, 3% and 5% (last column).

A first remark can be made relative to g_{nosp}. In general, the exclusion of SPs from efficient portfolios happens at fees as low as 2-3%. An exception is the parameter scenario II, characterized by a quite exceptional negative term-premium that generates good performances of the FRNC and the FRN, as seen earlier on. Therefore, there is a significant weight assigned to these products. For example in the scenario II-A, SPs receive a weight lower than 5% if the fee is above 6.9%.

Then, we can examine g^{λ}_{basic}, the fee level such that a risk-adverse investor invests only in basic securities. Recall, that, as described in section 5, a risk-adverse individual, once has built the efficient frontier, picks the portfolio that maximizes her expected utility. Cleary, the investment decision should be affected by her risk-adversion coefficient and by the fee level. Therefore, we should compute g^{λ}_{basic} for different values of the risk-adversion coefficient λ. In practice, given the very small differences in the expected return and the volatility of the different products it turns out that investors will pick the same efficient portfolio, whatever the level of their risk-adversion so that it turns out that g^{λ}_{basic} is independent on λ. This implies that when the critical fee level of column five is achieved, all risk-adverse investors will invest only in basic products. In general, g^{λ}_{basic} is not too different from g_{nosp}, and again with the exception of Scenario II, it takes valyes in the range 0-4.5%. Correspondingly, the weight allocated to SPs in the optimal portfolio is less than 1%, a very marginal amount.

Then, we see that in column seven $g_{sp\succ b}$ is always zero. This means that it never happens that a portfolio made of SPs only dominates a portfolio invested in BASIC securities only. However, column 6 says that, given a fee equal to $g_{sp\succ b}$, it can be convenient to hold SPs up to 96% in a combined portfolio of BASIC and SPs. Similarly, in the ninth column we have $g_{b\succ sp}$, i.e. the minimum fee such that a portfolio invested only in BASIC securities dominates the SP portfolio. In agreement with the previous column, this is always the case, albeit it is still optimal to hold BASIC and SPs together. This is illustrated in panel (a) of figure 4. The solid red curve represents the efficient frontier made of BASIC and SPs products. The circled red curve is the same efficient frontier when the fee is set at the maximum level of 6.9%. Given this fee, the maximum weight allocated to SPs along the efficient frontier is 5%. The cyan curve is the efficient frontier made of BASIC securities only. Given that it does not coincide with the circled red curve, it means that efficient portfolios contain SPs. The circled yellow curve is invisible because it is covered by the circled purple curve, that represents the efficient frontier made of SPs with zero fee. Then the circled green represents the efficient frontier made of SPs when the fee is set at 1%. This picture clearly synthetizes that, in our simulations, in general the efficient frontier made of BASIC products stays always above the efficient frontier made of SPs products. However, a combined portfolio, generates an efficient frontier that dominates the BASIC and the SPs frontiers. Panel (b) of figure 4 illustrates instead the case where there is no-added value in SPs: here the efficient frontier contains only BASIC securities.

Finally, the last three columns of the Table give the average amount invested in SPs along the efficient frontier, given different fee levels (1%, 3% and 5%). In scenario III-B, increasing the fee from 1% to 3%, reduces the average investment in SPs from 20% to a low valus as 0.1%. If the fee level is 5% (last column), efficient portfolios, with the exception of cases V and VI and curve setting D, will invest no more than 6.74% in SPs. An 8% fee (not reported in the Table) reduces the average amount invested in SPs to less than 2%.

7.1 Discussion

It is interesting to compare the fee critical levels of our simulations with the empirical evidence. The literature suggests that the greater the complexity of the product, the higher the overpricing, i.e. the (implicit) fees charged to the investor. For example, Henderson and Pearson [29] investigate the overpricing of a popular type of structured products in the U.S. and estimate that it amounts to about 8%, resulting in a negative expected return. They conclude that it is difficult to rationalize purchases of structured products by informed rational investors. Stoimenov and Wilkens [40] find a lack of transparency in the German market of SPs, in the sense that these products appear to be overpriced and thus favor the issuing institutions. Stoimenov and Wilkens [41] consider leverage products in the German retail market and show that these products most guarantee risk-free profits for their issuers. They find that, at issuance, structured products sell at an average of 3.89% above their theoretical values and the overpricing can increase to 5.17% for more complex products. Similarly, significant mispricing in favor of issuers has been found by Benet et al. [4] with reference to reverse-exchangeable securities, which are traded on the AMEX (American Stock Exchange). An analysis of the Italian retail market has been carried out by Billi and Fusai [6]. They have considered around 500 fixed income products issued in the Italian retail market in the year 2009. They estimate, in the primary market, an average premium over theoretical values in the range of 2% to 6%. They also find that mispricing usually has a positive relationship with product complexity. Also consider that this mispricing is on the top of the explicit fees charged to the investor.

On this basis, the critical fee levels discussed in our simulations appear to be below the typical level charged to investors. If we set the fee level at 8% as Stoimenov and Wilkens [40] do, the amount invested in SPs is in average below 2%, again with the exception of scenario II, where the weight averages around 3.5%. In conclusion, our simulation results are quite disappointing regarding the convenience of SP when plausible fees are charged. Indeed, without fees the investor can considerably improve its risk-return trade-off, but in the presence of fees, the diversification benefits of investing in SP completely fades away and the investment in SPs turns out to be negligible or null. The main reason is in large part related to the very small differences in expected returns and standard deviations among the different products. A second reason is due to the fact that basic products have very low volatility and negative cross correlations. Therefore, they allow the investor to achieve large diversification benefits and the additional contribution of other SPs turns out to be completely marginal if fees are charged.

As discussed in our simulations, there are still scenarios where SPs appear to provide significant results if fees are contained. This opens up also another interesting problem related to the role of the financial advisors, that should help their clients to identify which are the most reliable products for given projected scenarios. Instead, Hoechle et al. [31] show that structured products generate substantial profits also for distributors (and not only for issuers) and the high profitability of these products induces financial advisors to promote them strongly to their customers. The revised version of the European Markets in Financial Instruments Directive (MiFID II) tries to cope with this by stating that independent financial advisors must transfer all commissions and fees paid by third parties to their clients. However, these new rules only apply to financial advisors declaring themselves to be independent and leaving the business model of nonindependent advisors largely unchanged.

8 Sensitivity analysis and robustness to model assumptions

In this section we discuss the sensitivity of our results to model parameters and to the model choice. In particular, we analyze the role of mean-reversion, the effect of negative rates and then the choice of a non-Gaussian multifactor interest rate model.

Role of mean-reversion and factor volatility Important model parameters are the mean-reversion coefficients a and b and the volatility of the two factors. According to equations 5 and 8, these parameters determine both the long-term volatility of the short rate and through the function D the bond volatility and then the term premium and its long-run value that, by letting $\tau \to \infty$ in equation 8, is given by $\lambda_1 \sigma / a + \lambda_2 \eta / b$. Therefore ceteris paribus, a larger value of a and b will imply, due to a stronger mean-reversion, a lower long-term volatility, and a lower long-run term-premium.

We have considered the scenario I and, we have increased a from to 1.7981 to 3.5836 and b from 0.0517 to 0.1034, and left unchanged all the remaining parameters. This implies that the long-run volatility decreases from 5.892% to 4.167% and the long-run term premium decreases from 3.450% to 1.725%.

Based on the regression results we presented earlier on, given the reduction in the term premium, we expect that the expected return of all products, with the exception of the FRN and FRNC, will decrease. In particular, the most affected will be the ZCB. Similarly, given the reduction in the long-run volatility of the short rate, we expect, according to the regression results, a reduction also in the standard deviation of all products, with the exception of the SPREAD note that has a regression coefficient estimate near zero.

This is indeed reflected in our simulations. In Table 11 we compare the characteristics of the different products in the original Scenario I and in the new setting (scenario VIII), both combined with the term structure A. We observe, as expected, that, given that the mean-reversion coefficients double the mean return of the ZCB is near halved from 12.95% to 6.90%. A similar reduction in the expected return is observed for all the other products, with the exception of all remaining products, FRN and the FRNC. All products see the volatility of their return to be significantly reduced, with the exception of the SPREAD. The practical effect is that the GMV optimal portfolios will see a significant increment of the weigth allocated to the CB and a reduction of weight allocated to SPREAD. Additional information can be found examining the critical fee levels. In particular, in the new setup gives $g_{nosp} = 0.056\%$ versus a value of 2.1% in scenario IA. In addition, if the fee is set at 1%, SPs are completely excluded from the efficient frontier. In conclusion, this analysis shows that a reduction in the interest rate volatility implies a greater investment in BASIC products.

Negative rates The recent experience has seen central banks, mainly in Europe and Japan, imposing negative interest rates. This represents a challenge for sellers of SP. Indeed, negative coupon "payments" are not feasible, therefore issuers have mainly two options. The first option is to add a very large spread to the floating rate, but this will imply that the bond has to be issued well above the par and this would discourage investors that are used to buying around par. Another possibility is to add a floor to the FRN, but like the previous option, the FRN would get expensive as the investor would be required to pay for that protection. According to a report by PIMCO, the American investment management firm that focuses on fixed income and manages more than $1.92 trillion in assets, not surprisingly, most governments and agencies have stopped issuing FRNs, whilst credit FRNs are instead still being issued, given that the credit spreads are typically higher than those on government bonds and high enough to provide a comfortable buffer against negative rates. In our framework the issuer is assumed to be credit-free and the bonds are issued at par, and therefore the only possibility is to assume that investors are not worried about the possibility of receiving negative coupons, and in such a context they still try to maximize their expected utility by investing in products returning a negative interest. Moreover, given that in our setup, we do not consider a currency market that could potentially compensate for the loss caused by the negative return in the domestic currency, and given that there are no reasons linked to the solvency of the issuer, the only plausible reason for investing in SP is that they believe that these rates would decrease further and therefore they would prefer bond structures that protect against such events. This is confirmed in our simulations: also with negative rates, very risk adverse investors still prefer fixed rate bonds to the investment in SPs. Increasing the subscription fees progressively eliminates SPs from the composition of optimal portfolios. As illustrative example, Figure 5 provides an illustration of this case. A fee of 2.7% implies that the efficient frontier contains only BASIC products.

Model assumption In this section, we further analyze the robustness to model assumptions. As alternative to the G2++ model, among the several possibilities offered in the literature, we have considered, for its generality, the flexible stochastic volatility multi-factor model of the term structure introduced by Trolle and Schwartz [42]. As the G2++ model, this model belongs to the Heath, Jarrow and Morton [28] class. The Trolle and Schwartz model has multiple factors driving the forward rate curve with each factor exhibiting stochastic volatility. The model allows for correlation between each factor and the corresponding stochastic volatility. The dynamics of the forward curve can be described in terms of a finite number of state variables which jointly follow an affine diffusion specification. For the purpose of the present paper, we have considered the risk-neutral and real-world model specification with one factor, one stochastic volatility and six state variables. The full model identification under both the risk and real world measures requires the estimation of 11 parameters. In practice, we have considered the parameter estimates reported in column 1 in

Table 1 in [42]. These parameters have been estimated via maximum likelihood using both swaption and cap prices. The stochastic volatility has been simulated using the Andersen method [2], whilst for the remaining state variables we have adopted an Euler scheme with daily time steps (250 steps per year).

For the sake of generality we have considered an investor with preferences described by an utility function defined on the portfolio universe and not only to the ones that are mean-variance efficient. Hence, the problem that the investor is now facing is

$$\begin{array}{l} \max_{\mathbf{w}} \frac{1}{K}\sum_{k=1}^{K} u\left(\mathbf{w}'\mathbf{R}^{(k)}\right) \\ sub \\ \mathbf{w} \geq \mathbf{0} \\ \mathbf{1}'\mathbf{w} = 1 \end{array} \tag{14}$$

where $\mathbf{R}$ is the $K \times n$ array containing the K simulated returns of the n different fixed income products performed using the TS model. We have solved numerically the above problem by assuming an exponential utility function defined in terms of the portfolio return $R = \mathbf{w}'\mathbf{R}^{(k)}$.

The simulations confirm that, whatever the term structure scenario, the investment in basic securities dominates the investment in structured portfolios. For illustration, we provide figure 6 where we present the simulated cumulative distribution function of the best portfolio invested in basic securities or only in structured portfolios. We consider a fee of 0.5%. In figure 6 the simulated cumulative distribution function of basic products, being on the right-most part of the figure, dominates in terms of first order stochastic dominance, see Hadar and Russell [27] the distribution function relative to the portfolio of structured products. In addition, whenever the fees are positive, the optimal portfolio contains only basic securities.

In conclusion, even in the the Trolle and Schwartz model, the simulations confirm the findings we previously obtained with the G2++ model.

9 Conclusion

In this paper we have discussed the relative convenience of investing in a portfolio of fixed income structured products. We have shown that, without fees, structured products can improve the risk-return trade off for a retail investor. This result is in general not robust to the presence of fees: in this case the optimal portfolio consists only of basic products such as zero-coupon bonds, coupon bonds and floating rate notes, and the percentage invested in structured products appears to be marginal or even not significant. Only under very particular configurations of the term premium and shape of the current term structure, investment in SPs can still be convenient, mainly with reference to products like VOL and SPREAD notes.

Investment banks advertise that structured notes guarantee a portfolio diversification that better suits specific investment needs. According to the simulation results presented in this paper, instead fixed income SP are not always designed to be in the best interests of investors and do not always allow to achieve a better risk-reward tradeoff. Additional disadvantages of SP are the pricing of the implicit derivatives components that could lead to potential mispricing. Furthermore investor should consider the credit risk in the event the issuing investment bank forfeits its obligations, and the liquidity of secondary market. Most structured products are nowdays traded in secondary market and market investors wishing to liquidate their holding prior to maturity will need to find a buyer for their investment in the secondary market. The secondary market trading price for a structured product will be subject to a bid-offer spread whose determination depends on several factors and market participants may be disadvantaged, for example, incurring high transaction costs for certain types of trades.

In conclusion, the main point of the present paper is that in an hypothetical transparent market where fees are small and explicit, SP's should be included in the investor portfolio.

However, in opaque markets where the investor is charged hidden fees due to mispricing or credit or liquidity reasons, SPs should not be included.

After the financial crisis, retail clients protection and increased financial markets transparency has been driving the agenda of regulators. In the United States, based on the 2010 Dodd-Frank Wall Street Reform and Consumer Protection Act, much higher attention has been given to payments from product providers to financial advisors.

In Europe, MIFID 2, MIFIR, PRIIPS Regulation and Product governance rules, have contributed to increase transparency on costs and returns as well as on the appropriatness of financial products to real needs of clients. On an advice given on April 2020 by ESMA to the European commission on "inducements and costs and charges disclosure", ESMA states that MiFID II disclosure regime generally works well and helps investors to make informed investment decisions while understanding of inducements by clients should be increased. On March 29 2021 ESMA has clarified that inducement are justifies only in the presence of an additional or higher-level service, to the relevant client, proportional to the level of inducements received. Our paper shows that fees are critical to explain the role of SP in portfolios and that the role of regulators to keep high standard on costs disclosure and to limits inducements is crucial to drive financial markets toward higher efficiency.

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A Monte Carlo Simulation

The G2++ specification is a Markovian model in the two state variables $x(t)$ and $y(t)$, which are jointly Gaussian distributed. This fact implies that Monte Carlo simulation can be performed in a straightforward manner. We detail the steps.

1. Given the model parameters, fix the time step Δ, the bond maturity $T = n\Delta$ and the initial value of the money market account $B(0) = 1$. In the simulations we set $T = 5$ and $\Delta = 1/12$. Set also $x(0) = 0$ and $y(0) = 0$.

2. Simulate the two stochastic factors according to the true probability measure from a bivariate normal distribution

$$\left[\begin{array}{c} x^{(k)}(j\Delta) \\ y^{(k)}(j\Delta) \end{array} \middle| \mathcal{F}_{(j-1)\Delta} \right] \sim \mathcal{N}\left(M^{(k)}(j\Delta), V(\Delta)\right), j = 1, ..., n,$$

   where[14]

$$\begin{aligned} M^{(k)}(j\Delta) &= \left[\begin{array}{c} x^{(k)}((j-1)\Delta)e^{-a\Delta} - \lambda_1\sigma_1\left(1 - e^{-a\Delta}\right) \\ y^{(k)}((j-1)\Delta)e^{-b\Delta} - \lambda_2\eta\left(1 - e^{-b\Delta}\right) \end{array} \right], \\ V(\Delta) &= \left[\begin{array}{cc} \frac{\sigma^2}{2a}\left(1 - e^{-2a\Delta}\right) & \rho\sigma\eta\frac{1-e^{-(a+b)\Delta}}{a+b} \\ \rho\sigma\eta\frac{1-e^{-(a+b)\Delta}}{a+b} & \frac{\eta^2}{2b}\left(1 - e^{-2b\Delta}\right) \end{array} \right]. \end{aligned}$$

3. Simulate the short rate according to

$$r^{(k)}(j\Delta) = x^{(k)}(j\Delta) + y^{(k)}(j\Delta) + \phi(j\Delta),$$

   and the discount curve according to

$$P^{(k)}(j\Delta, T) = \frac{P^{mkt}(0, T)}{P^{mkt}(0, j\Delta)} \exp\left(A^{(k)}(j\Delta, T)\right), \tag{15}$$

   where $A^{(k)}$ is given by the exponent in expression (6).

4. Update the money market account. A possibility is to use the trapezium rule

$$B^{(k)}(j\Delta) = B^{(k)}((j-1)\Delta)e^{\left(r^{(k)}((j-1)\Delta)+r^{(k)}(j\Delta)\right)\frac{\Delta}{2}}.$$

   Unfortunately, this step introduces a discretization error that can be avoided by sampling from a trivariate normal distribution. For sake of clarity and space saving, we do not detail the exact simulation step that we have implemented. The advantage of using the exact simulation scheme rather the above trapezium approximation is that

[14] The formula for the mean vector and covariance matrix are given in Brigo and Mercurio [9].

we can simulate the state variables only on the bond reset dates, with a significant time computational saving.

5. If $j\Delta$ corresponds to a coupon date, given the values of the stochastic factors $x^{(k)}(j\Delta)$ and $y^{(k)}(j\Delta)$ and the discount curve (15), compute the coupon $c^{(k)}(j\Delta)$ and discount it at the initial date using the simulated value of the money market account, i.e. compute

$$\frac{C^{(k)}(j\Delta)}{B^{(k)}(j\Delta)}.$$

For example, if the coupon is tied to a reference rate once we use the simulated discounted curve at the reset date to compute the corresponding value of the reference rate according to formula 1 or 2.

6. For each product, in the simulation k we compute the present value $PV^{(k)}$ of the cash-flows

$$PV^{(k)} = \sum_{j=1}^{n} \frac{C^{(k)}(j\Delta)}{B^{(k)}(j\Delta)} + \frac{N}{B^{(k)}(n\Delta)}. \tag{16}$$

(In the above expression if $j\Delta$ is not a coupon date, we set the coupon to zero).

7. We repeat steps 1-6 for $k = 1, ..., K$, where K is the number of simulations.

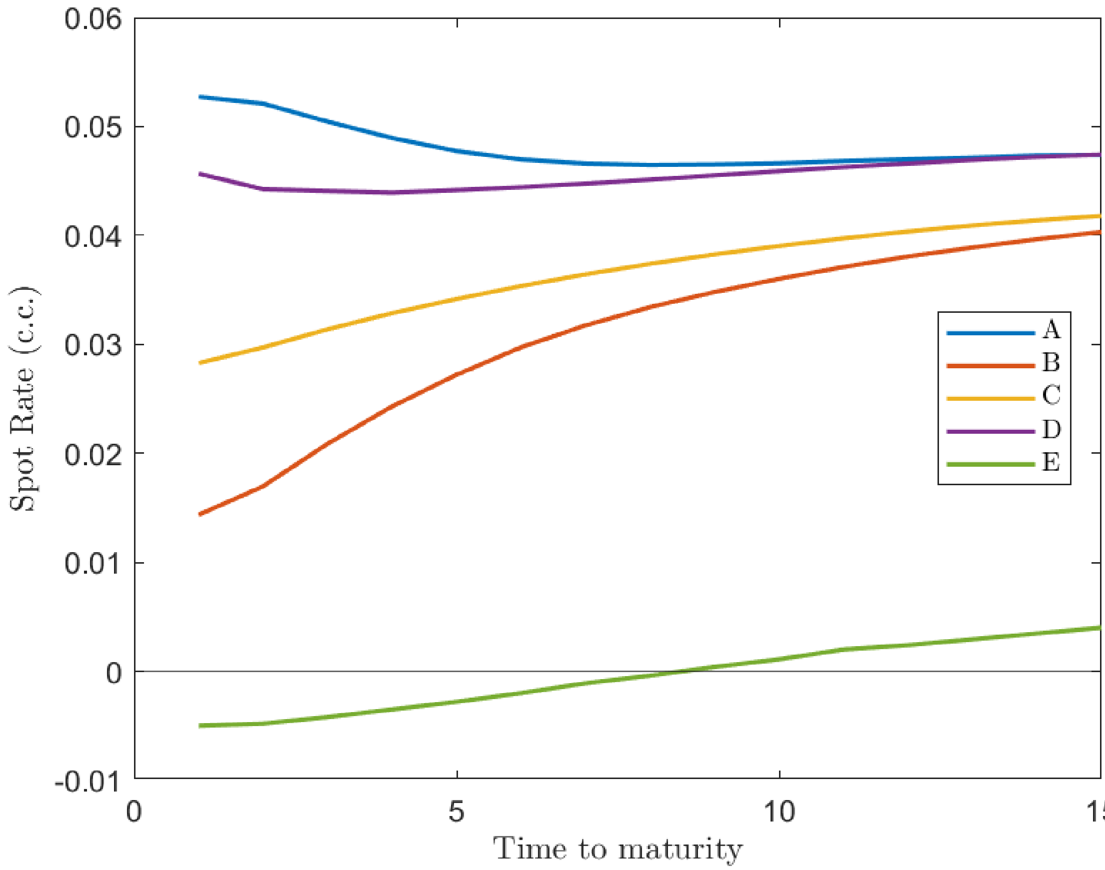


Figure 1: Initial term structures shapes. Four different initial shapes are considered: (A) negatively sloped on June 6th, 2008; (B) positively sloped on September 28th, 2007; (C) average level in the period 1/1/2005 to 30/09/2020; (D) flat on May 20th, 2009. (E) negative rates on May 2nd, 2021.

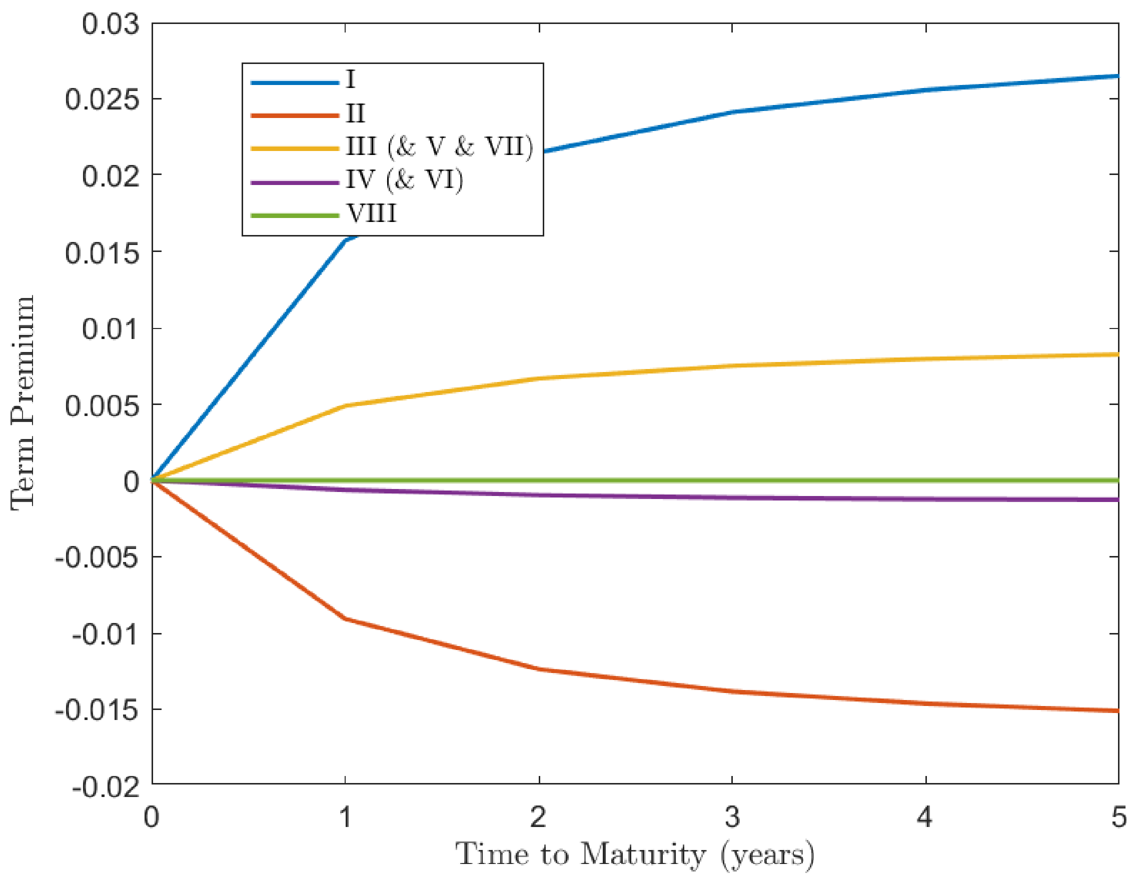


Figure 2: Behavior of the term premium under different G2++ parametrization. Model parameters according to the different scenarios are illustrated in Table 2.

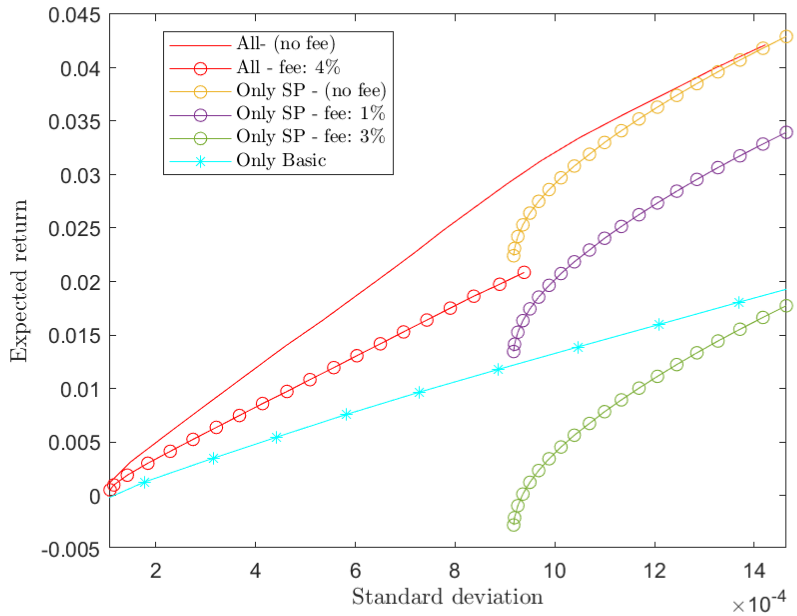
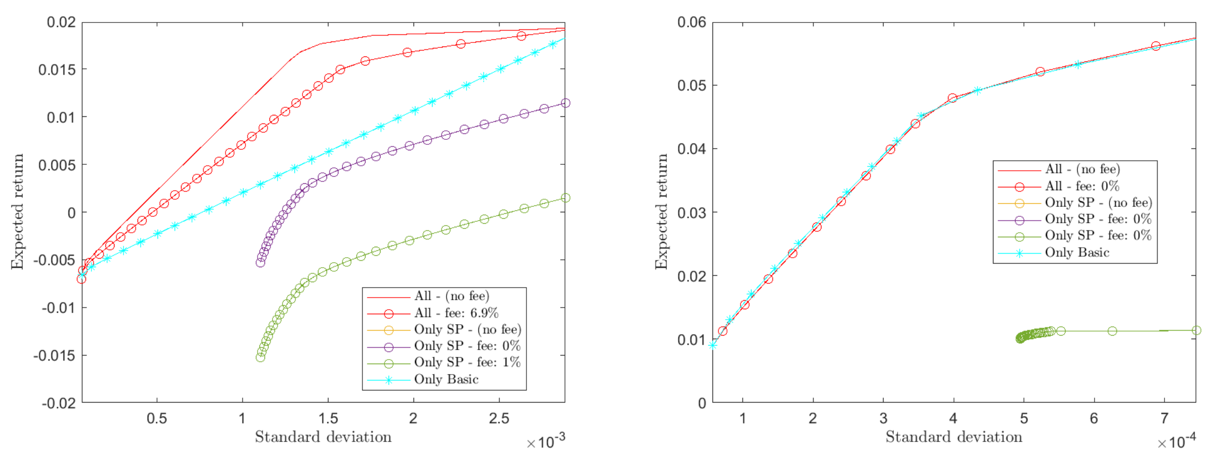


Figure 3: Hypothetical efficient frontiers. ALL: refers to the efficient frontier containing all products; only Basic is the efficient frontier made only of ZCB, CB and FRN; SP are the efficient frontiers containing only SPs varying the fee level (0%, 1% and 3%). The ALL - fee is the efficient frontier built assuming a fee level (4%) such that the maximum amount invested in SPs is 3%.



(a) Scenario II-A (b) Scenario III-A

Figure 4: Efficient frontiers in two different scenarios varying the fee level.

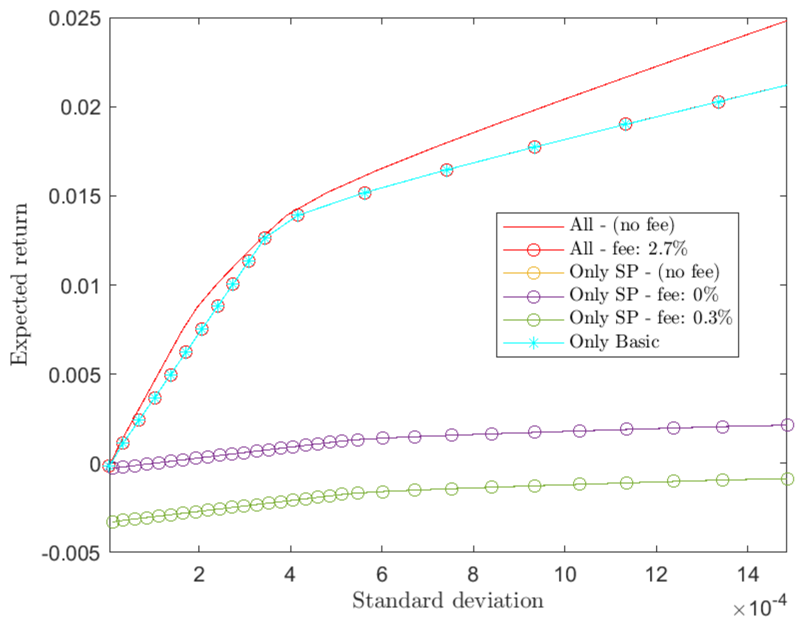


Figure 5: Efficient frontier in scenario III-E (negative rates)

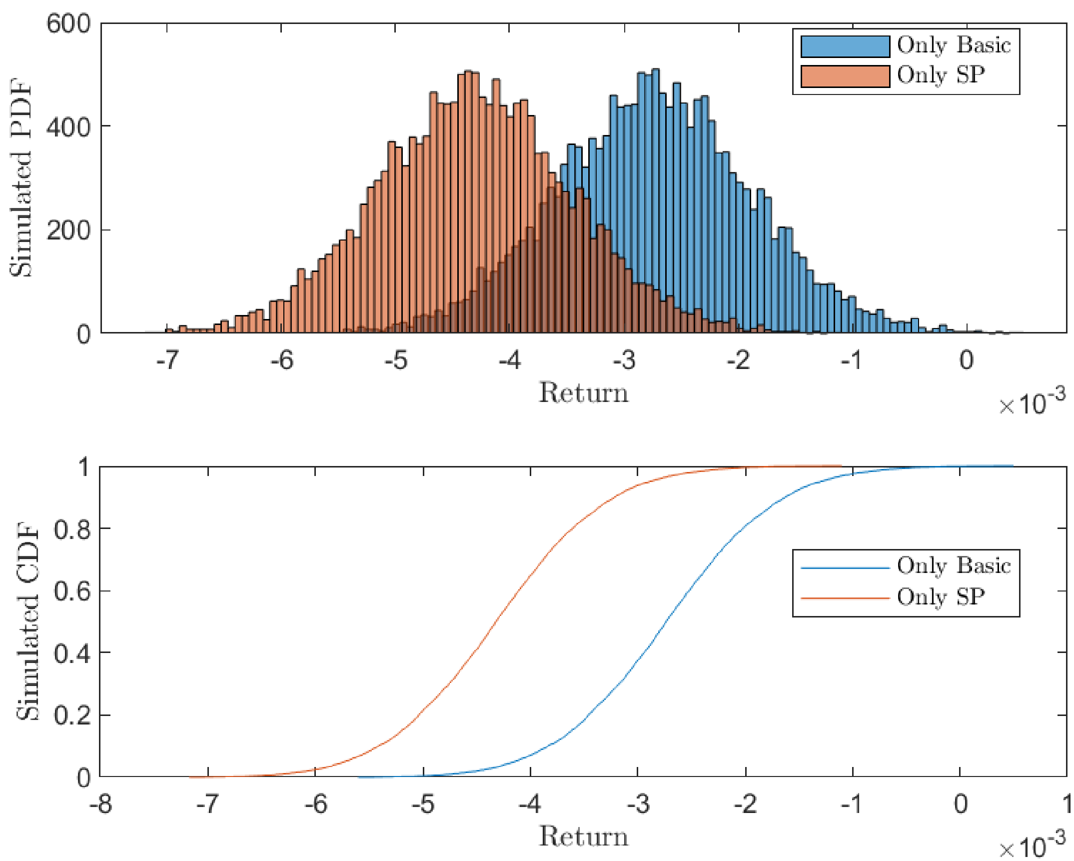


Figure 6: Simulated probability density (top figure) and simulated cumulative distribution (bottom part) of the optimal portfolio for an individual with logarithmic utility function under the term structure scenario A. Simulations are performed using the Trolle and Schwartz [42] model and assuming a 1% fee.

Scenario	A	B	C	D	E
β_0	5.7770%	4.8579%	4.8438%	5.1262%	1.2252%
β_1	-0.1721%	-3.6404%	-2.1799%	-0.3668%	-1.7045%
β_2	-0.6322%	-1.9082%	-0.4342%	-0.6751%	-0.5947%
κ	17.9718%	56.4758%	34.0464%	37.0178%	28.9778%

Table 1: Parameters of the Nelson-Siegel model in five different scenarios of the term structure of spot rates.

B The expected return and variance on a ZCB

In this section, we show how to compute analytically in the G2++ model the expected return on a ZCB expiring in T. For notational convenience we set $t = 0$. The amount that is paid at expiry is $(1 + c)N$, where $c = 1/P(0, T) - 1$, so that the ZCB is issued at par value. The logarithmic return is

$$R_{zcb} = \ln \frac{(1 + c)N}{B(T_n)N}$$

where $B(T) = e^{\int_0^T r(s)ds}$. Therefore

$$R_{zcb} = -\ln(P(0, T)) - \int_0^T r(s)ds = -\ln(P(0, T)) - \int_0^T (x(s) + y(s))ds - \int_0^T \phi(s)ds$$

and, using footnote 6 (with $t = 0$),

$$R_{zcb} = -\int_0^T (x(s) + y(s))ds - \frac{1}{2}V(0, T).$$

We aim to compute the expected return and its variance. If the expectation is taken under the risk-neutral measure, given that the two factors have zero mean, we have

$$\tilde{E}_t (R_{zcb}) = -\frac{1}{2}V(0, T) \tag{17}$$

This term, in the interest rate literature, is called convexity adjustment and is due to the non-linear relationsip between price and return. If the expectation is computed under the real-world measure, the factor dynamics has to be adjusted for the risk-premium, i.e.

$$\begin{aligned} x(t) &= -\lambda_1 \tfrac{\sigma}{a}\left(1 - e^{-at}\right) + \sigma \textstyle\int_0^t e^{-a(t-s)} dW(s), \\ y(t) &= -\lambda_2 \tfrac{\eta}{b}\left(1 - e^{-bt}\right) + \eta \textstyle\int_0^t e^{-b(t-s)} dW(s). \end{aligned}$$

Scenario	I	II	III	IV	V	VI	VII	VIII
λ_1	34.771	-20.229	10.795	-0.086	0.385	-0.086	0.385	34.771
λ_2	0.015	-0.001	0.005	0.026	-0.001	0.026	-0.001	0.015
a	1.7918	1.7918	1.7918	0.7735	0.7735	0.7735	0.7735	3.5836
b	0.0517	0.0517	0.0517	0.082	0.082	0.082	0.082	0.1034
σ_1	0.0015	0.0015	0.0015	0.0223	0.0223	0.0223	0.0223	0.0015
η	0.019	0.019	0.019	0.0104	0.0104	0.0104	0.0104	0.019
ρ	-0.6441	-0.6441	-0.6441	-0.702	-0.702	-0.702	0	-0.6441
S_r	5.892%	5.892%	5.892%	2.450%	2.450%	2.450%	3.132%	4.167%
$TP(0,5)$	2.650%	-1.510%	0.825%	-0.126%	0.825%	-0.126%	0.825%	1.435%
Long Run TP	3.438%	-1.738%	1.090%	0.075%	1.090%	0.075%	1.090%	1.725%

Table 2: Parameters for the G2++ model in eight different scenarios and assuming different values of the risk-premium parameters λ_1 and λ_2. The first three scenarios refer to parameters calibrated using the sample covariance matrix and then assuming different values for the term premium at 5 years and in the long-run. The scenarios IV-VI are characterized by a lower asymptotic volatility of the short rate and by a reduced value of the 5 year term premium, i.e. near 1% and 0%. The scenario VII has been chosen to assess the effect of zero-correlation with respect to scenario V. Paraemters in scenario VIII have been arbitrarily chosen. Concerning the choice of the risk premium: in scenario I, we use the maximum value of the term premium estimated in the US market in the period Jan. 2000 to April 2021, see [1]; in scenario II, we use the minimum value of the term premium estimated in the EURO market in the same period; in scenario III, V and VII, we use the average value of the term premium in the period 2000-2021 in the US market; in scenario IV and VI, the average value in the EUR market; in scenario VIII, we vary the mean-reversion coefficients of Scenario I, keeping constant all the other parameters. The following two scenarios refer to parameters calibrated using market quotations of swaptions and assuming a term premium that can take positive or negative values at different horizons. The row labelled S_r gives the asymptotic volatility of the short rate. The rows labelled $TP(0,5)$, and *Long Run TP* give the 5 and long-run term premiums.

Param.	Curve	$TP(5)$	S_r	β_0	$\beta_0+\beta_1$	zcb	cb	frn	cms	frnc	cmsc	spread	vol
I	A	2.65%	5.89%	5.78%	5.60%	12.95%	1.53%	-3.44%	0.43%	-2.04%	0.71%	1.95%	0.06%
I	B	2.65%	5.89%	4.86%	1.22%	12.95%	0.93%	-4.21%	0.27%	-1.65%	0.78%	3.26%	0.13%
I	C	2.65%	5.89%	4.84%	2.66%	12.95%	1.14%	-3.95%	0.31%	-1.91%	0.73%	1.13%	0.11%
I	D	2.65%	5.89%	5.13%	4.76%	12.95%	0.93%	-4.28%	0.17%	-1.69%	0.63%	0.80%	0.16%
I	E	2.65%	5.89%	1.23%	-0.48%	12.95%	-0.11%	-5.82%	0.24%	-0.10%	0.17%	0.40%	-0.11%
II	A	-1.51%	5.89%	5.78%	5.60%	-7.85%	-0.86%	1.94%	-0.37%	1.14%	-0.49%	-0.86%	-0.32%
II	B	-1.51%	5.89%	4.86%	1.22%	-7.85%	-0.52%	2.34%	-0.23%	0.90%	-0.54%	-2.69%	-0.24%
II	C	-1.51%	5.89%	4.84%	2.66%	-7.85%	-0.64%	2.21%	-0.27%	1.06%	-0.50%	-1.68%	-0.27%
II	D	-1.51%	5.89%	5.13%	4.76%	-7.85%	-0.52%	2.38%	-0.19%	0.94%	-0.45%	-1.44%	-0.26%
II	E	-1.51%	5.89%	1.23%	-0.48%	-7.85%	0.06%	3.18%	-0.08%	0.05%	-0.13%	-0.83%	0.05%
III	A	0.83%	5.89%	5.78%	5.60%	3.82%	0.45%	-0.82%	0.13%	-0.66%	0.21%	0.70%	-0.08%
III	B	0.83%	5.89%	4.86%	1.22%	3.82%	0.27%	-1.02%	0.08%	-0.53%	0.23%	1.32%	-0.01%
III	C	0.83%	5.89%	4.84%	2.66%	3.82%	0.33%	-0.95%	0.09%	-0.61%	0.21%	0.60%	-0.03%
III	D	0.83%	5.89%	5.13%	4.76%	3.81%	0.27%	-1.04%	0.05%	-0.55%	0.18%	0.42%	-0.04%
III	E	0.83%	5.89%	1.23%	-0.48%	3.82%	-0.03%	-1.45%	0.07%	-0.03%	0.05%	0.21%	-0.03%
IV	A	-0.13%	2.45%	5.78%	5.60%	-0.67%	-0.07%	0.24%	-0.15%	0.20%	-0.14%	-0.13%	-0.23%
IV	B	-0.13%	2.45%	4.86%	1.22%	-0.67%	-0.04%	0.29%	-0.10%	0.21%	-0.16%	-0.33%	-0.15%
IV	C	-0.13%	2.45%	4.84%	2.66%	-0.67%	-0.06%	0.27%	-0.12%	0.22%	-0.15%	-0.18%	-0.19%
IV	D	-0.13%	2.45%	5.13%	4.76%	-0.67%	-0.04%	0.29%	-0.11%	0.22%	-0.14%	-0.16%	-0.17%
IV	E	-0.13%	2.45%	1.23%	-0.48%	-0.67%	0.01%	0.36%	0.06%	0.01%	-0.05%	-0.09%	0.01%
V	A	0.83%	2.45%	5.78%	5.60%	4.09%	0.44%	-1.76%	-0.20%	-1.64%	-0.19%	0.75%	-0.85%
V	B	0.83%	2.45%	4.86%	1.22%	4.09%	0.26%	-2.03%	-0.13%	-1.67%	-0.20%	1.71%	-0.57%
V	C	0.83%	2.45%	4.84%	2.66%	4.09%	0.33%	-1.94%	-0.17%	-1.74%	-0.21%	0.84%	-0.69%
V	D	0.83%	2.45%	5.13%	4.76%	4.09%	0.26%	-2.04%	-0.19%	-1.66%	-0.24%	0.69%	-0.64%
V	E	0.83%	2.45%	1.23%	-0.48%	4.09%	-0.03%	-2.55%	0.27%	-0.06%	-0.14%	0.38%	-0.02%
VI	A	-0.13%	2.45%	5.78%	5.60%	-0.67%	-0.07%	0.24%	-0.15%	0.20%	-0.14%	-0.13%	-0.23%
VI	B	-0.13%	2.45%	4.86%	1.22%	-0.67%	-0.04%	0.29%	-0.10%	0.21%	-0.16%	-0.33%	-0.15%
VI	C	-0.13%	2.45%	4.84%	2.66%	-0.67%	-0.06%	0.27%	-0.12%	0.22%	-0.15%	-0.18%	-0.19%
VI	D	-0.13%	2.45%	5.13%	4.76%	-0.67%	-0.07%	0.25%	-0.14%	0.21%	-0.15%	-0.01%	-0.21%
VI	E	-0.13%	2.45%	1.23%	-0.48%	-0.67%	0.01%	0.36%	0.06%	0.01%	-0.05%	-0.09%	0.01%
VII	A	0.83%	3.13%	5.78%	5.60%	3.95%	0.42%	-1.64%	-0.20%	-1.42%	-0.18%	0.60%	-0.81%
VII	B	0.83%	3.13%	4.86%	1.22%	3.95%	0.26%	-1.89%	-0.13%	-1.27%	-0.19%	1.47%	-0.53%
VII	C	0.83%	3.13%	4.84%	2.66%	3.95%	0.32%	-1.80%	-0.16%	-1.41%	-0.20%	0.70%	-0.66%
VII	D	0.83%	3.13%	5.13%	4.76%	3.95%	0.39%	-1.68%	-0.17%	-1.44%	-0.17%	0.06%	-0.74%
VII	E	0.83%	3.13%	1.23%	-0.48%	3.95%	-0.03%	-2.39%	0.25%	-0.05%	-0.12%	0.34%	-0.02%
					min	-7.85%	-0.86%	-5.82%	-0.37%	-2.04%	-0.54%	-2.69%	-0.85%
					max	12.95%	1.53%	3.18%	0.43%	1.14%	0.78%	3.26%	0.16%

Table 3: Expected return of the different structured products in each term scenario (A, B, C, D, E) and parameters setting (I-VII) over the five year investment period. The expected returns have been estimated using the simulated G2++ interest rate model under the physical measure. In the third column, we have the 5-year term premium. In the fourth column the long-run interest volatility (S_r)and then the long-run rate (β_0) and in the fifth column the short term rate ($\beta_0 + \beta_1$) .

Product	Intercept	$TP(0,5)$	S_r	β_0	$\beta_0+\beta_1$	R^2
zcb	0.001	**4.997**	**-0.072**	0.000	0.000	1
cb	-0.002	**0.331**	0.002	0.049	0.006	0.76
frn	**-0.009**	**-1.689**	0.100	0.084	0.006	0.96
cms	0.000	**0.112**	**0.027**	-0.048	0.001	0.76
frnc	0.001	**-0.641**	0.094	-0.157	0.018	0.76
cmsc	-0.004	**0.218**	0.058	0.002	-0.002	0.78
spread	-0.003	**0.757**	-0.093	0.196	-0.111	0.75
vol	-0.003	0.020	**0.078**	-0.053	-0.014	0.63

Table 4: Coefficient estimates of the regression of the expected return of the different bonds with respect to the 5-year Term Premium (TP(0,5)), asyntotic volatility S_r of the short rate, long-run level of the spot curve (β_0) and short term rate ($\beta_1 + \beta_0$). Bold estimates are significant at 1% level.

Parameters	Curve	$TP(5)$	S_r	zcb	cb	frn	cms	frnc	cmsc	spread	vol
I	A	2.65%	5.89%	0.43%	0.03%	0.39%	0.21%	1.86%	1.18%	0.19%	5.59%
I	B	2.65%	5.89%	0.43%	0.02%	0.44%	0.14%	1.40%	1.29%	0.35%	3.95%
I	C	2.65%	5.89%	0.43%	0.02%	0.42%	0.17%	1.73%	1.26%	0.07%	4.76%
I	D	2.65%	5.89%	0.43%	0.02%	0.44%	0.16%	1.65%	1.33%	0.04%	4.62%
I	E	2.65%	5.89%	0.43%	0.00%	0.52%	0.09%	0.00%	0.66%	0.02%	0.85%
II	A	-1.51%	5.89%	0.43%	0.02%	0.30%	0.17%	0.29%	0.78%	0.31%	4.34%
II	B	-1.51%	5.89%	0.43%	0.01%	0.33%	0.11%	0.78%	0.87%	0.43%	3.11%
II	C	-1.51%	5.89%	0.43%	0.02%	0.32%	0.14%	0.39%	0.85%	0.34%	3.74%
II	D	-1.51%	5.89%	0.43%	0.01%	0.34%	0.13%	0.48%	0.89%	0.19%	3.70%
II	E	-1.51%	5.89%	0.43%	0.00%	0.39%	0.07%	0.00%	0.43%	0.06%	0.74%
III	A	0.83%	5.89%	0.43%	0.03%	0.35%	0.19%	1.21%	0.99%	0.26%	4.97%
III	B	0.83%	5.89%	0.43%	0.02%	0.38%	0.13%	0.72%	1.08%	0.27%	3.54%
III	C	0.83%	5.89%	0.43%	0.02%	0.37%	0.16%	0.94%	1.06%	0.34%	4.26%
III	D	0.83%	5.89%	0.42%	0.02%	0.38%	0.15%	0.90%	1.11%	0.26%	4.14%
III	E	0.83%	5.89%	0.43%	0.00%	0.45%	0.08%	0.00%	0.54%	0.15%	0.80%
IV	A	-0.13%	2.45%	0.08%	0.00%	0.06%	0.05%	0.15%	0.11%	0.24%	0.48%
IV	B	-0.13%	2.45%	0.08%	0.00%	0.07%	0.03%	0.27%	0.12%	0.34%	0.39%
IV	C	-0.13%	2.45%	0.08%	0.00%	0.06%	0.04%	0.12%	0.12%	0.19%	0.48%
IV	D	-0.13%	2.45%	0.08%	0.00%	0.07%	0.04%	0.17%	0.18%	0.25%	0.71%
IV	E	-0.13%	2.45%	0.08%	0.00%	0.08%	0.03%	0.00%	0.30%	0.19%	0.76%
V	A	0.83%	2.45%	0.08%	0.00%	0.06%	0.05%	0.35%	0.13%	0.23%	0.59%
V	B	0.83%	2.45%	0.08%	0.00%	0.07%	0.03%	0.41%	0.14%	0.41%	0.48%
V	C	0.83%	2.45%	0.08%	0.00%	0.07%	0.04%	0.41%	0.15%	0.36%	0.58%
V	D	0.83%	2.45%	0.08%	0.00%	0.07%	0.04%	0.55%	0.22%	0.32%	0.85%
V	E	0.83%	2.45%	0.08%	0.00%	0.08%	0.03%	0.00%	0.39%	0.19%	0.79%
VI	A	-0.13%	2.45%	0.08%	0.00%	0.06%	0.05%	0.15%	0.11%	0.24%	0.49%
VI	B	-0.13%	2.45%	0.08%	0.00%	0.07%	0.03%	0.27%	0.12%	0.34%	0.39%
VI	C	-0.13%	2.45%	0.08%	0.00%	0.06%	0.04%	0.12%	0.12%	0.19%	0.48%
VI	D	-0.13%	2.45%	0.08%	0.00%	0.06%	0.04%	0.13%	0.10%	0.03%	0.40%
VI	E	-0.13%	2.45%	0.08%	0.00%	0.08%	0.03%	0.00%	0.30%	0.19%	0.76%
VII	A	0.83%	3.13%	0.15%	0.01%	0.12%	0.08%	0.82%	0.22%	0.23%	0.94%
VII	B	0.83%	3.13%	0.15%	0.01%	0.14%	0.05%	0.73%	0.25%	0.51%	0.77%
VII	C	0.83%	3.13%	0.15%	0.01%	0.13%	0.06%	0.85%	0.25%	0.40%	0.91%
VII	D	0.83%	3.13%	0.15%	0.01%	0.13%	0.07%	0.82%	0.20%	0.04%	0.80%
VII	E	0.83%	3.13%	0.15%	0.00%	0.16%	0.04%	0.00%	0.40%	0.23%	0.75%
			min	0.08%	0.00%	0.06%	0.03%	0.00%	0.10%	0.02%	0.39%
			max	0.43%	0.03%	0.52%	0.21%	1.86%	1.33%	0.51%	5.59%

Table 5: Standard deviation of the return of the different structured products under different scenarios. The standard deviations have been estimated using the simulated G2++ interest rate model under the physical measure. The column S_r gives the long-run volatility of the short rate.

Product	Intercept	$TP(0,5)$	S_r	β_0	$\beta_0+\beta_1$	R^2
zcb	**-0.002**	0.001	**0.101**	0.000	0.000	1
cb	**0.000**	0.001	**0.004**	*0.002*	0.000	0.92
frn	**-0.001**	**0.023**	**0.090**	**-0.010**	-0.001	1
cms	**-0.001**	**0.006**	**0.030**	0.005	0.006	0.98
frnc	**-0.009**	**0.231**	**0.139**	**0.192**	-0.025	0.9
cmsc	**-0.005**	**0.084**	**0.221**	**0.036**	-0.010	0.94
spread	0.000	-0.020	-0.008	**0.090**	**-0.056**	0.9
vol	**-0.030**	0.173	**0.858**	0.269	0.079	0.91

Table 6: Coefficient estimates of the regression of the standard deviation of the different bonds with respect to the 5-year Term Premium (TP(0,5)), asyntotic volatility S_r of the short rate, long-run level of the spot curve (β_0) and short term rate ($\beta_1+\beta_0$). Bold numbers are significant at 1% level.

Using now the definition of term premium, we have

$$E_0\left(R_{zcb}\right) = (T-0)\times TP(0,T) - \frac{1}{2}V\left(0,T\right),$$

Therefore $T\times TP(0,T)$ is the difference between the expected return under the real world measure and under the risk neutral measure.

Finally, the variance of the return follows quite easily, indeed we have

$$V_0\left(R_{zcb}\right) = V\left(0,T\right).$$

Notice that the expected return and the variance of the ZCB investment do not depend on the shape of the current term structure. This is also reflected in our simulations, see for example Tables 3 and 5: the estimated return and volatility of the ZCB is independent on the term structure scenario.

Parameters	Curve	zcb	cb	frn	cms	frnc	cmsc	spread	vol
I	A	12.78%	11.90%	-39.76%	-22.85%	18.74%	19.81%	-38.77%	18.16%
I	B	14.22%	15.47%	-40.89%	-25.11%	17.41%	16.83%	-36.53%	15.12%
I	C	17.00%	17.74%	-43.59%	-26.54%	23.53%	22.88%	-22.80%	19.43%
I	D	18.44%	19.23%	-45.05%	-28.44%	25.57%	26.11%	-13.80%	21.79%
I	E	-36.68%	7.57%	7.93%	-28.09%	7.57%	-21.92%	6.07%	6.87%
II	A	33.21%	31.52%	-60.06%	-30.38%	41.23%	44.88%	40.23%	42.18%
II	B	16.18%	16.51%	-43.35%	-23.52%	-28.53%	23.32%	16.16%	22.44%
II	C	15.18%	15.97%	-42.12%	-21.90%	-29.65%	22.22%	13.13%	20.30%
II	D	12.80%	13.21%	-39.92%	-22.18%	-32.21%	19.32%	6.89%	17.84%
II	E	-27.50%	-0.82%	-1.95%	-18.83%	-0.82%	-12.97%	-14.94%	-3.70%
III	A	18.90%	19.57%	-45.62%	-25.51%	20.52%	22.49%	-29.50%	21.28%
III	B	9.67%	11.05%	-36.61%	-21.45%	11.24%	13.55%	-42.02%	11.89%
III	C	13.05%	13.98%	-39.74%	-22.44%	16.17%	16.73%	-41.79%	15.18%
III	D	12.52%	13.88%	-39.13%	-22.83%	18.07%	16.87%	-38.04%	14.14%
III	E	-45.24%	15.81%	16.40%	-33.69%	15.81%	-25.56%	19.80%	17.17%
IV	A	5.80%	3.49%	-32.50%	-8.70%	20.32%	24.43%	7.06%	23.10%
IV	B	-3.23%	-2.69%	-24.06%	-8.36%	-11.47%	14.36%	-4.57%	12.11%
IV	C	5.72%	4.33%	-33.32%	-10.56%	11.29%	20.18%	-3.56%	18.92%
IV	D	10.27%	9.44%	-37.84%	-16.64%	7.96%	18.73%	-6.60%	18.01%
IV	E	-33.59%	5.32%	4.87%	-20.54%	5.32%	-22.11%	1.12%	0.93%
V	A	9.51%	7.22%	-37.08%	-14.24%	15.72%	22.35%	-4.22%	21.88%
V	B	9.60%	8.68%	-37.62%	-15.60%	8.18%	18.83%	-9.69%	18.51%
V	C	11.14%	9.24%	-39.09%	-16.70%	13.81%	20.97%	-9.13%	20.74%
V	D	14.80%	13.08%	-42.79%	-21.28%	14.09%	21.24%	-9.88%	21.20%
V	E	-33.69%	5.49%	4.94%	-20.00%	5.49%	-23.39%	2.44%	0.14%
VI	A	5.99%	3.65%	-32.76%	-8.92%	20.49%	24.51%	6.97%	23.21%
VI	B	-3.21%	-2.73%	-24.10%	-8.19%	-11.45%	14.35%	-4.56%	12.06%
VI	C	5.93%	4.50%	-33.57%	-10.65%	11.32%	20.28%	-3.51%	19.03%
VI	D	2.43%	0.31%	-29.95%	-7.60%	15.24%	22.35%	-3.13%	21.10%
VI	E	-33.52%	5.26%	4.78%	-20.47%	5.26%	-22.11%	1.05%	0.88%
VII	A	16.09%	15.77%	-38.02%	1.52%	23.98%	28.77%	-9.95%	27.30%
VII	B	12.33%	12.78%	-35.34%	-2.10%	10.77%	24.87%	-24.84%	23.81%
VII	C	13.50%	13.17%	-36.17%	-1.40%	16.63%	26.28%	-23.03%	25.20%
VII	D	12.11%	11.45%	-34.31%	1.70%	19.08%	27.18%	-20.95%	25.76%
VII	E	-39.13%	11.56%	8.31%	-14.61%	11.56%	-27.49%	10.42%	6.56%
	min	-45.24%	-2.73%	-60.06%	-33.69%	-32.21%	-27.49%	-42.02%	-3.70%
	max	33.21%	31.52%	16.40%	1.70%	41.23%	44.88%	40.23%	42.18%

Table 7: The first two columns identify the scenario setting. Then we have the average correlation of each product with the remainings.

Parameters	Curve	zcb	cb	frn	cms	frnc	cmsc	spread	vol	basic	SP
I	A	2.84%	77.94%	9.35%	0.75%	0.25%	0.33%	8.30%	0.23%	90%	10%
I	B	3.02%	82.80%	8.27%	1.09%	0.29%	0.43%	3.73%	0.36%	94%	6%
I	C	4.42%	76.02%	11.20%	1.12%	0.25%	0.34%	6.41%	0.25%	92%	8%
I	D	4.27%	82.56%	10.31%	1.30%	0.25%	0.30%	0.80%	0.22%	97%	3%
II	A	0.14%	86.61%	10.90%	0.28%	0.68%	0.22%	1.03%	0.14%	98%	2%
II	B	1.02%	87.74%	8.50%	0.99%	0.35%	0.33%	0.79%	0.28%	97%	3%
II	C	0.84%	87.24%	8.95%	0.60%	0.81%	0.25%	1.10%	0.22%	97%	3%
II	D	0.23%	90.23%	6.86%	0.42%	0.61%	0.19%	1.25%	0.22%	97%	3%
III	A	3.66%	79.89%	12.91%	0.49%	0.29%	0.28%	2.30%	0.18%	96%	4%
III	B	5.87%	72.10%	8.60%	1.07%	0.59%	0.57%	10.69%	0.50%	87%	13%
III	C	4.51%	72.33%	11.18%	1.01%	0.76%	0.90%	8.75%	0.56%	88%	12%
III	D	3.10%	83.00%	7.84%	0.61%	0.19%	0.26%	4.69%	0.32%	94%	6%
IV	A	2.30%	78.75%	15.06%	0.34%	1.29%	0.96%	0.74%	0.56%	96%	4%
IV	B	0.10%	85.27%	6.63%	2.14%	1.82%	1.49%	1.25%	1.30%	92%	8%
IV	C	2.00%	75.95%	15.42%	0.55%	2.40%	1.28%	1.56%	0.84%	93%	7%
IV	D	0.69%	81.25%	13.36%	0.61%	1.85%	0.72%	1.00%	0.53%	95%	5%
V	A	2.08%	78.35%	13.75%	2.26%	1.03%	0.85%	0.91%	0.76%	94%	6%
V	B	7.62%	69.75%	15.44%	3.61%	0.92%	0.86%	0.96%	0.83%	93%	7%
V	C	5.31%	72.16%	15.59%	3.26%	1.03%	0.87%	0.97%	0.81%	93%	7%
V	D	7.00%	70.90%	16.03%	3.51%	0.65%	0.56%	0.86%	0.48%	94%	6%
VI	A	1.62%	76.32%	16.89%	0.53%	1.72%	1.23%	0.97%	0.72%	95%	5%
VI	B	0.09%	85.33%	6.62%	2.13%	1.81%	1.48%	1.24%	1.29%	92%	8%
VI	C	1.93%	75.66%	15.48%	0.56%	2.49%	1.38%	1.65%	0.85%	93%	7%
VI	D	4.24%	58.14%	19.66%	1.75%	2.54%	2.02%	10.23%	1.42%	82%	18%
VII	A	0.00%	87.67%	8.29%	1.02%	0.57%	0.64%	1.26%	0.55%	96%	4%
VII	B	5.63%	74.67%	11.41%	3.57%	1.07%	1.22%	1.77%	0.65%	92%	8%
VII	C	1.41%	87.52%	7.56%	1.20%	0.42%	0.49%	1.06%	0.34%	96%	4%
VII	D	0.00%	82.20%	6.71%	0.70%	0.51%	0.51%	8.91%	0.47%	89%	11%
	min	0.00%	58.14%	6.62%	0.28%	0.19%	0.19%	0.74%	0.14%	82.04%	2.35%
	max	7.62%	90.23%	19.66%	3.61%	2.54%	2.02%	10.69%	1.42%	97.65%	17.96%

Table 8: Composition of the global minimum variance portfolio

Product	Intercept	$TP(0,5)$	S_r	β_0	$\beta_0 + \beta_1$	R^2
zcb	**0.074**	**0.773**	-0.263	-0.678	-0.152	0.79
cb	0.189	-1.282	**1.581**	**11.903**	-1.919	0.99
frn	**0.064**	0.062	**-0.977**	**1.515**	0.300	0.93
cms	**0.062**	0.070	**-0.309**	**-0.624**	-0.129	0.88
frnc	**0.577**	0.084	-0.303	**-12.136**	**2.007**	0.98
cmsc	**0.021**	-0.044	-0.199	**-0.060**	**-0.055**	0.92
spread	-0.003	0.381	0.590	0.149	-0.004	0.54
vol	**0.016**	-0.045	**-0.120**	-0.070	-0.048	0.91
basic	**0.327**	-0.447	0.340	**12.741**	**-1.771**	0.99
SP	**0.673**	0.447	-0.340	**-12.741**	**1.771**	0.98

Table 9: Coefficient estimates of the regression of the composition of the GMV portfolio with respect to the 5-year Term Premium $TP(0,5)$, asymptotic volatility S_r of the short rate, long-run level of the spot curve (β_0) and short term rate ($\beta_1 + \beta_0$). Bold estimates are significant at 1% level.

Param	Curve	g_{nosp}	%SP	g^{λ}_{basic}	%SP	$g_{sp \succ b}$	%SP	$g_{b \succ sp}$	%SP	%SP ($g = 1\%$)	%SP ($g = 3\%$)	%SP ($g = 5\%$
1	1	2.1%	5.0%	1.1%	0.0%	0.0%	91.1%	0.0%	91.1%	2.3%	0.7%	0.2%
1	2	3.6%	4.7%	3.6%	0.9%	0.0%	77.0%	0.0%	77.0%	12.2%	5.2%	0.0%
1	3	0.0%	0.0%	0.0%	0.0%	0.0%	93.8%	0.0%	93.8%	0.0%	0.0%	0.0%
1	5	0.0%	0.0%	0.0%	0.0%	0.0%	96.0%	0.0%	96.0%	0.0%	0.0%	0.0%
2	1	6.9%	5.0%	32.7%	1.0%	0.0%	0.0%	1.0%	0.0%	6.6%	2.6%	2.6%
2	2	18.0%	5.0%	23.4%	1.0%	0.0%	5.7%	0.2%	5.7%	4.1%	4.0%	3.8%
2	3	17.4%	5.0%	28.5%	1.0%	0.0%	3.8%	1.0%	3.8%	3.3%	3.3%	3.2%
2	5	18.5%	5.0%	28.2%	1.0%	0.0%	3.4%	0.9%	3.4%	3.5%	3.4%	3.3%
3	1	0.2%	1.1%	0.2%	0.7%	0.0%	61.9%	0.0%	61.9%	0.0%	0.0%	0.0%
3	2	2.1%	5.0%	1.6%	0.6%	0.0%	86.1%	0.0%	86.1%	20.0%	0.1%	0.1%
3	3	1.1%	0.2%	1.1%	0.0%	0.0%	75.9%	0.0%	75.9%	0.0%	0.0%	0.0%
3	5	0.5%	0.4%	0.5%	0.3%	0.0%	90.0%	0.0%	90.0%	0.0%	0.0%	0.0%
4	1	0.6%	3.7%	0.7%	0.2%	0.0%	0.0%	0.0%	0.0%	0.1%	0.1%	0.2%
4	2	0.3%	4.4%	0.6%	1.0%	0.0%	6.7%	0.0%	6.7%	0.1%	0.1%	0.2%
4	3	0.5%	4.5%	0.7%	0.7%	0.0%	5.0%	0.0%	5.0%	0.1%	0.1%	0.2%
4	5	0.6%	4.8%	1.3%	1.0%	0.0%	8.7%	0.0%	8.7%	1.2%	0.1%	0.1%
5	1	0.7%	5.0%	0.5%	0.0%	0.0%	50.9%	0.0%	50.9%	0.2%	0.1%	0.1%
5	2	0.8%	2.9%	0.2%	0.8%	0.0%	50.2%	0.0%	50.2%	0.4%	0.1%	0.1%
5	3	0.8%	4.7%	0.3%	0.6%	0.0%	49.6%	0.0%	49.6%	0.2%	0.1%	0.1%
5	5	0.8%	5.0%	0.3%	0.7%	0.0%	50.4%	0.0%	50.4%	0.2%	0.1%	0.0%
6	1	0.6%	4.1%	0.7%	0.2%	0.0%	0.0%	0.0%	0.0%	0.1%	0.1%	0.2%
6	2	0.3%	4.4%	0.6%	1.0%	0.0%	6.7%	0.0%	6.7%	0.1%	0.1%	0.1%
6	3	0.5%	4.3%	0.7%	0.8%	0.0%	5.0%	0.0%	5.0%	0.1%	0.1%	0.2%
6	5	0.6%	4.9%	1.3%	1.0%	0.0%	8.9%	0.0%	8.9%	1.2%	0.1%	0.1%
7	1	0.0%	2.0%	0.3%	0.6%	0.0%	56.1%	0.0%	56.1%	0.0%	0.0%	0.0%
7	2	2.6%	4.8%	4.5%	0.9%	0.0%	61.9%	0.0%	61.9%	4.1%	2.2%	0.3%
7	3	1.4%	4.9%	2.6%	0.9%	0.0%	52.5%	0.0%	52.5%	3.3%	0.3%	0.0%
7	5	1.5%	4.7%	2.6%	0.8%	0.0%	54.2%	0.0%	54.2%	3.4%	0.3%	0.0%

Table 10: The first two columns identify the scenario setting. The third colum gives the maximum fee g_{nosp} such that the amount invested in SP is no greater than 3% across all efficient portfolios. The fourth column gives the actual amount invested in SPs if the fee in column three is applied. The fifth column gives the maximum fee g^{λ}_{basic} such that a risk-adverse investor will minimize the investment in SPs and the adjacent column the actual percentage allocated to SPs given g^{λ}_{basic}. Columns seven (nine) gives the maximum (minimum) fee such that an investment in SPs (BASIC) only dominates an investment in BASIC (SP) products only (the fee is set at 0 if dominance is not possible). The adjacent columns gives the percentage allocated to SPs in a portfolio containing BASIC and SPs if the fees $g_{sp \succ b}$ or $g_{b \succ sp}$ are charged. The last three columns give the average (across all efficient portfolios, excluding the GMV) amount invested in SPs given a 1%, 3% or 5% fee g^{λ}_{basic}.

Scenario I A	zcb	cb	frn	cms	frnc	cmsc	spread	vol
Ex. Return	12.95%	1.53%	-3.44%	0.43%	-2.04%	0.71%	1.95%	0.06%
Std. Deviation	0.43%	0.03%	0.39%	0.21%	1.86%	1.18%	0.19%	5.59%
GMV	2.84%	77.94%	9.35%	0.75%	0.25%	0.33%	8.30%	0.23%
Scenario VIII A								
Ex. Return	6.90%	0.84%	-0.61%	0.43%	-0.42%	0.49%	0.58%	0.14%
Std. Deviation	0.34%	0.02%	0.28%	0.13%	1.15%	0.65%	0.21%	3.05%
GMV	0.00%	92.59%	7.35%	0.00%	0.00%	0.00%	0.00%	0.06%

Table 11: Expected return and standard deviations of the different structured products in the parameter settings V-A and VIII-A.